Final Project: Minimizing Cost for a Weekly Diet Plan

MSE 426 Introduction to Engineering Design and Optimization

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Due Date: April 9th, 2022

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Introduction

In optimization, our goal is to find the biggest or smallest value of a function that is constrained in some way. The constraint will be a condition that is commonly expressed as an equation and must be absolute and positive regardless of our answer. When a function is maximized or minimized in relation to a set, it frequently indicates a range of options possible in a certain situation. An objective function allows you to compare different options to see which is the "best" one to make. The values of parameters can be chosen in the engineering design of an object, system, or structure, subject to some conditions (constraints) describing their ranges and interrelationships. Furthermore, number of variables affecting the product's acceptability are influenced by elements such as cost, weight, speed, and so on. Along with the choices of design parameters that meet certain performance specifications one must apply quantitative methods and identify the best solution within given constraints to find the optimization of a system.

Objective

The objective of this project report is to observe study for the cost of a diet plan for an active person over 7 days. The person would be restricted to eating from a blend of four types of food only. However, the person has an alternative to eat from outside if his likeness factor of eating similar blend decreases throughout the trial period.

Problem Description

In addition to providing for healthy growth and development, nutrition is an important aspect of human performance. Macronutrients, micronutrients, and fluids must be consumed in the right amounts to supply energy for development and activity. An individual must learn what, when, and how to eat and drink before, during, and after action to maximize performance. Macronutrients, such as carbohydrates, protein, and fats, provide the fuel for physical activity and sports participation.

A healthy diet should consist of 45 to 65 percent carbohydrates, 10 to 30 percent protein, and 25 to 35 percent fat. Food consumption should be timed to maximise performance.

Meal planning is one of the most difficult things to do. Meal timing is critical and must be tailored to the individual. It's critical for athletes as well as for a common person, to figure out which foods they enjoy and that also help them perform better and keep the minimum requirements of daily nutrients.

For this problem, the subject is an 18-year-old male weighing 140lbs (63.5kgs). The person exercise regularly for about three hours each day and, our goal is to minimize the cost of the 7-day meal plan for the person by keeping a blend of four products while providing all the necessary nutrients for his body.

Objective Function Formulation

For this study brown rice, chicken, fish, and vegetables are the selected as the common staple.

Table 1 shows the food composition and price based on per serving:

Table 1: Food Composition and Prices per serving

Food Type	Fat (g)	Carbohydrate (g)	Protein (g)	Cost ($)	Cost ($/gram)
Rice (320g-350g)	0.59	79.77	6.65	1.36	0.0038
Chicken (225g-250g)	15.44	0	59.1	2.58	0.01032
Fish (120g-150g)	7.12	0	25.94	11.99	0.0799
Vegetables (100g-120g)	0.62	16.15	4	0.32	0.0027

The figure 1 provides information on the recommended serving intake for a healthy diet.

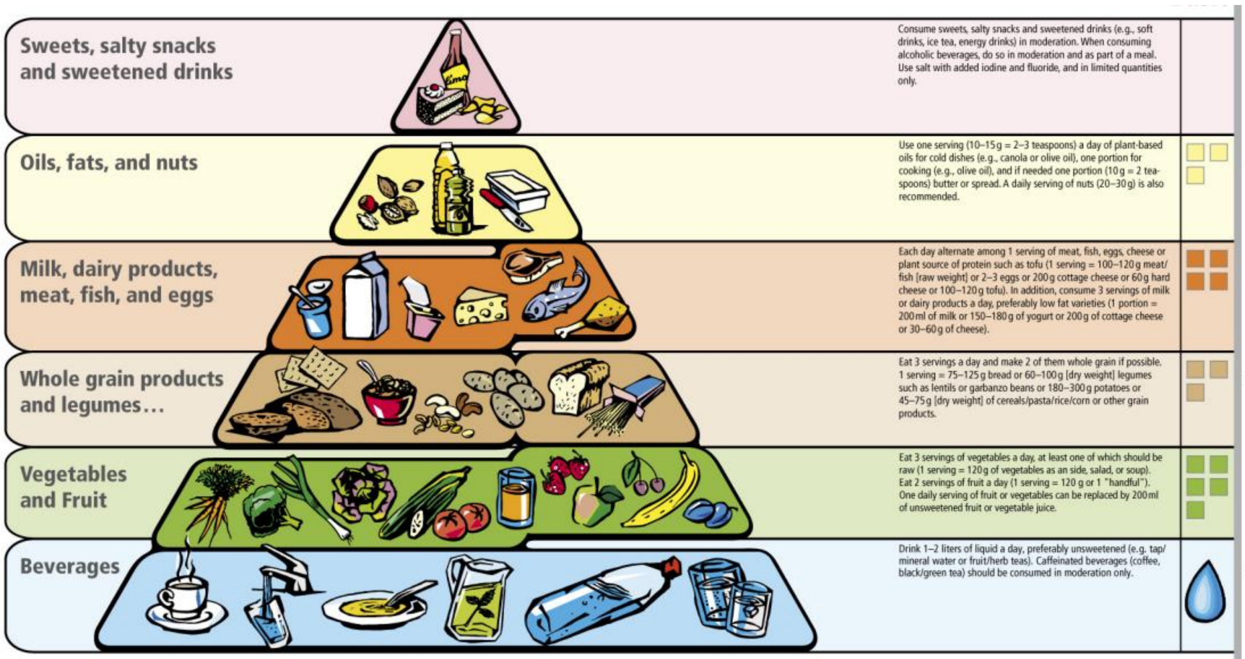


Figure 1: Diet Chart

According to the study of 2015-2020 dietary guidelines a healthy diet should include fish twice per week and at least one serving of meat per day. It is also recommended to have three servings of mixed vegetables each day, along with three servings of whole grain products each day.

Based on the guidelines the frequency for a week's meal plan for the person is shown in table 2:

Table 2: Individual Food Frequency over 7 days

Food Type	**Frequency (for 7 days)**	**Freq/N (Frequency over 7 days)**
Rice (320g-350g)	21	3
Chicken (225g-250g)	7	1
Fish (120g-150g)	2	0.2857
Vegetables (100g-120g)	21	3

Choice of Variables

For this problem the choice of variables would be the proportion of each type of food in grams required per day.

However, having same type of blend everyday exhausts the subject which prompts him to stray from the diet plan and have some meal from outside. This likeness factor is denoted by the following equation 1:

$$likeness\ factor: \sum_{i=1}^{n}(x_i)^{\beta}$$

$$where\ \beta = \frac{Frequency}{N * 100}, range\ of\ 0.0028 - 0.03$$

Equation 1: Likeness Factor

The function highlights the cost of decreasing likeness and increasing probability that the subject is likely to eat from outside. In theory, β can be any number greater than 2 but for simplicity and understanding this problem, β is chosen such that the food with most proportions and servings over the course of 7 days would have larger penalty (0.03) and vice versa. C_2 is the average cost that subject would most likely spend if he ate from outside. For this problem we assume C_2 to be \$15. The overall objective function is shown in equation 2:

$$Objective\ Function: \sum_{i=1}^{n}(\frac{Freq_i}{N} C_i * x_i) + C_2 * \sum_{i=1}^{n}(x_i)^{\beta}$$

Equation 2: Objective Function

Expanding the Objective function by substituting the corresponding values from table 1 and table 2 we get:

$$Min_{cost}: (0.0114 * x_1 + 0.01032 * x_2 + 0.0228 * x_3 + 0.0081 * x_4) + 15 * (x_1^{0.03} + x_2^{0.01} + x_3^{0.0028} + x_4^{0.03})$$

$where, \quad x_i = Proportion\ of\ food$

$Freq = Number\ of\ eating\ occasions\ of\ food\ "n"\ over\ "N"\ survey\ days$

$C = Cost\ of\ food$

$N = Number\ of\ days\ (7)$

$C_2 = Cost\ of\ food\ from\ outside$

Equation 3: Expanded Objective Function with Parameter Definition

Constraints

Based on the problem description table 1, following constraints of fat, carbohydrates and protein is formed as shown in equation 4:

$$subject\ to: -$$

$$fat : 0.59 * x_1 + 15.44 * x_2 + 7.12 * x_3 + 0.62 * x_4 \geq 70$$

$$carbohydrates : 79.77 * x_1 + 16.15 * x_4 \geq 560$$

$$protein : 6.65 * x_1 + 59.1 * x_2 + 25.94 * x_3 + 4 * x_4 \geq 114.3$$

Equation 4: Constraints for Macronutrients

It is also recommended that a healthy person eats between 3 to 5 pounds of food per day as constrained in equation 5:

$$total\ weight\ of\ food\ per\ day\ in\ grams : 1360 \leq x_1 + x_2 + x_3 + x_4 \leq 2268$$

Equation 5: Constraint for Maximum Allowance of Food per day

The bounds for the variables for x_i are calculated with the help of table 2 and the recommended portion per serving, the calculated bounds are shown in equation 6:

$$Per\ day\ gram\ serving\ by\ product: 960 \leq x1 \leq 1080, 225 \leq x2 \leq 250, 35 \leq x3 \leq 43, 300 \leq x4 \leq 360$$

Equation 6: Bounds for the Variable

Application of Tools

For this problem Oasis and MATLAB Optimization Toolbox (fmincon) is used to minimize the objective function.

Oasis

Oasis is an optimization software that can compute the full range of design possibilities and uses intelligent sampling which pinpoints the focus areas that contain the most promising options.

Stopping Criteria

For the stopping criteria of running our optimization function in Oasis, the maximum number of iterations were selected to be 500. The objective function normally converged within 500 iterations when tested with 1000 maximum iterations.

MATLAB Optimization Toolbox (fmincon)

The MATLAB optimization toolbox is a useful MATLAB add-on that provides a library of solvers that can be utilized directly from the MATLAB environment. It enables design optimization tasks to be completed. It has functions for locating parameters that minimize or maximize objectives while remaining constrained. For this project's constrained problem, "*fmincon*" function was used. Optimal solution was found using the objective function and constraints provided.

Stopping Criteria

Default conditions were used for the stopping criteria of fmincon as shown in table 3.

Table 3: Stopping Criteria for fmincon

Properties	**Limit**
Optimality Tolerance	1e-6
Maximum Iterations	1000
Maximum Function Evaluations	3000

Results

Oasis

Optimization function was executed multiple times and the overall function converged to a value of 82.4626. The results obtained by Oasis is shown in figure 2 and 3:

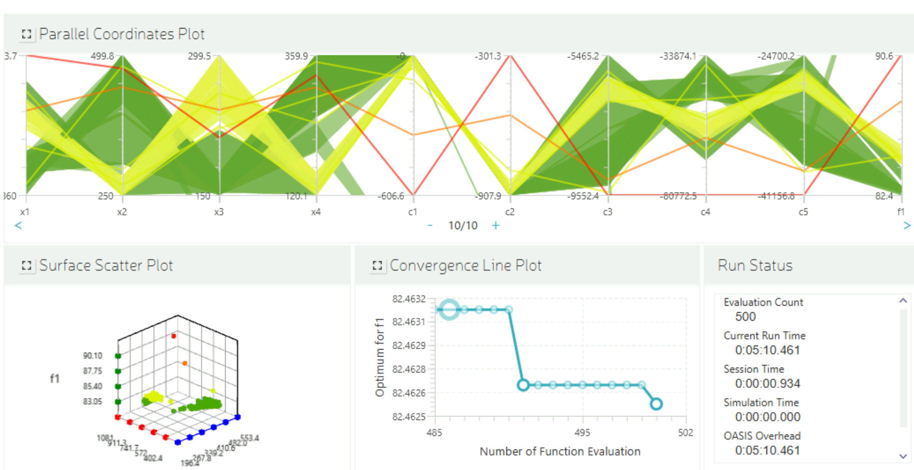
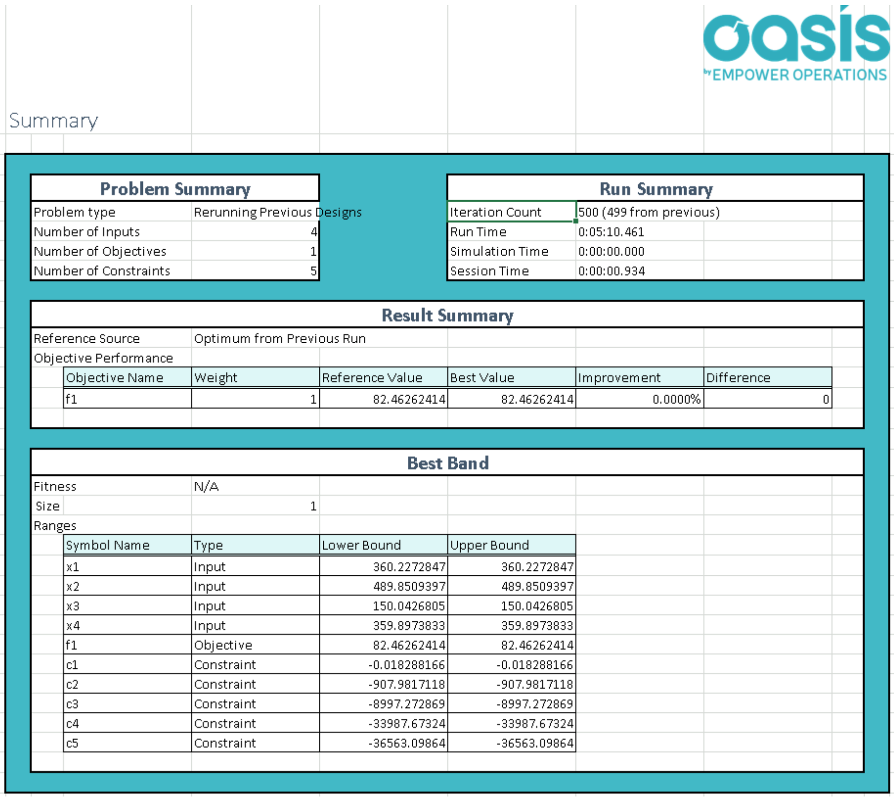


Figure 2: Convergence Plot Oasis



Summary

Problem Summary

Problem type	Rerunning Previous Designs
Number of Inputs	4
Number of Objectives	1
Number of Constraints	5

Run Summary

Iteration Count	500 (499 from previous)
Run Time	0:05:10.461
Simulation Time	0:00:00.000
Session Time	0:00:00.934

Result Summary

Reference Source: Optimum from Previous Run

Objective Performance

Objective Name	Weight	Reference Value	Best Value	Improvement	Difference
f1	1	82.46262414	82.46262414	0.0000%	0

Best Band

Fitness: N/A

Size: 1

Ranges

Symbol Name	Type	Lower Bound	Upper Bound
x1	Input	360.2272847	360.2272847
x2	Input	489.8509397	489.8509397
x3	Input	150.0426805	150.0426805
x4	Input	359.8973833	359.8973833
f1	Objective	82.46262414	82.46262414
c1	Constraint	-0.018288166	-0.018288166
c2	Constraint	-907.9817118	-907.9817118
c3	Constraint	-8997.272869	-8997.272869
c4	Constraint	-33987.67324	-33987.67324
c5	Constraint	-36563.09864	-36563.09864

Figure 3: Optimal Solution for 500 Iterations

The most optimum proportion obtained from the function are shown in table 4:

Table 4: Optimized Values using Oasis

Proportion (x_i)	**Value**
x_1	960.0028712
x_2	225.0000519
x_3	35.00001
x_4	300.0018299
Function value	83.7097683

We can conclude that the given values for x_i's satisfy all the constraints while keeping the cost minimal. It can be observed that the minimum quantity of food should give us the minimum cost hence we did observe that the values of x_i's sums to the minimum acceptable amount of food per day, hence it is a logical answer for our objective function.

MATLAB Optimization Toolbox (fmincon)

Optimization function was executed multiple times using different starting points and the overall function converged to a value of 70.03. The results obtained by using fmincon function is shown in table 5 on default stopping criteria:

Table 5: Results with different starting points for fmincon (Default Stopping Criteria)

Starting value (x_i)	**Function Value**	**Num of Iterations**	**Optimum Value for x_i**	**Optimum Value for x_i**
(1080,230,41,315)	70.0296	27	(960,225,35,300)	(960,225,35,300)
(960,225,35,300)	70.0296	18	(960,225,35,300)	(960,225,35,300)
(1080,250,43,360)	70.0296	29	(960,225,35,300)	(960,225,35,300)
(1074,236,39,310)	70.0296	28	(960,225,35,300)	(960,225,35,300)
(1010,230,40,316)	70.0296	27	(960,225,35,300)	(960,225,35,300)

However, changing the optimal tolerance stopping criteria from (1e-06) to (1e-08), we observe that the function still converges at 70.0296 as shown in table 6:

Table 6: Results with different starting points for fmincon (Optimal Tolerance of 1e-08)

Starting value (x_i)	**Function Value**	**Num of Iterations**	**Optimum Value for x_i**
(1080,230,41,315)	70.0296	28	(960,225,35,300)
(960,225,35,300)	70.0296	19	(960,225,35,300)
(1080,250,43,360)	70.0296	30	(960,225,35,300)
(1074,236,39,310)	70.0296	29	(960,225,35,300)
(1010,230,40,316)	70.0296	28	(960,225,35,300)

We noticed that a decrease in optimal tolerance increased the number of iterations but the overall convergence for the function remained unchanged.

Analysis

From the results obtained we can analyse that for the cost of the diet study to be minimal we need the minimum amount of proportions for each staple but within the constraints of providing all the nutrients for the body. Theoretically, minimal points for x_i should be at the lower bound which was confirmed by the optimization using both Oasis and MATLAB Optimization Toolbox (fmincon).

Comparison

Table 7 shows the comparison between using Oasis and MATLAB Optimization Toolbox (fmincon) function:

Table 7: Comparison between Oasis and fmincon

Oasis			**MATLAB Optimization Toolbox (fmincon)**		
Function Value	Max Iterations	Time	Function Value	Max Iterations	Time
82.4626	490	5 min 10 sec	70.0296	30	5 sec

It can be said that visually Oasis is more engaging than MATLAB. But based on the problem defined in this report MATLAB Optimization Toolbox (fmincon) performed better than Oasis. Fmincon converged to a more minimal point with less iterations and in faster time.

Conclusion

To summarize, we applied the optimization methods that we learned in class into practice using MATLAB and Oasis. Through this project, we learned that MATLAB's Optimization Toolbox performed well for the problem described in this report, but Oasis was visually more pleasing. Fmincon function is a rigid optimization tool to evaluate constrained or unconstrained equations. Each application tool is explicitly efficient for different types of problems.

Based on the project results, we can conclude that using fmincon we reached a minimum value for the objective function. We also noticed that by using various starting points and different stopping criteria the convergence for the function didn't change.

References

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[5] "Desmos | Graphing Calculator", *Desmos*, 2022. [Online]. Available: https://www.desmos.com/calculator. [Accessed: 10- Apr- 2022].

Appendix

myrun

```
myfun = @(x) (0.0114*x(1)+0.01032*x(2)+0.0228*x(3)+0.0081*x(4))+15*(x(1)^0.03 +
x(2)^0.01 + x(3)*0.0028 + x(4)^0.03);
x0 = [1080;230;41;315];
A = [-1 -1 -1 -1;
     1 1 1 1;
     -0.59 -15.44 -7.12 -0.62;
     -79.77 0 0 -16.15;
     -6.65 -59.1 -25.94 -4];
b = [1360;
     2268;
     70;
     560;
     114.3];
Aeq = [];
beq = [];
lb = [960,225,35,300];
ub = [1080,250,43,360];
```

```
options = optimoptions(@fmincon,'Display','iter','OptimalityTolerance',1e-08)
[x,fval,exitflag, output] = fmincon(myfun,x0,A,b,Aeq,beq,lb,ub,[],options);
x
fval
exitflag
```