

Computer Vision

Chapter 3. Image Processing

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Chapter 3 – Content (cont)

- Remind: Digital image representation
- Point processing
- · Convolution and linear filtering
- More neighborhood operators
 - Median/max/min filters
 - Arithmetical/ Logical operations
 - Binary image and morphological operations
- Image transforms



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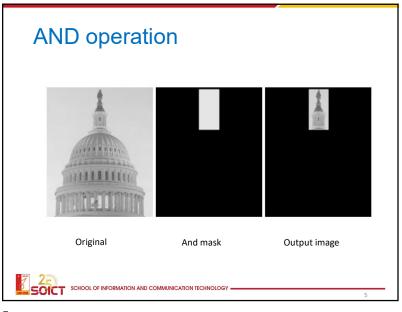
Arithmetical Operations Logical Operations

- AND operation
- OR operation
- Image subtraction
- · Image addition

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}



OR operation

Original OR mask Output image

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Image Addition

 If f and g are two images, the pixelwise addition R is defined as:

-R(x,y) = Min(f(x,y)+g(x,y); 255)

- · Image addition is used to
 - lower the noise in a serie of images
 - increase the luminance by adding the image to itself





Source: Eric Favier. L'analyse et le traitement des images. ENISE

Average Images

• g(x,y) is the addition of f(x,y) and noise $\eta(x,y)$

$$g(x, y) = f(x, y) + \eta(x, y)$$

 If we have several images {g(x,y)}, we can compute the average one

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$

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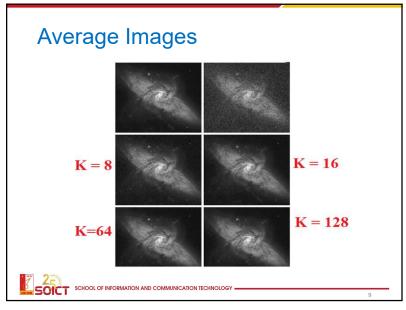
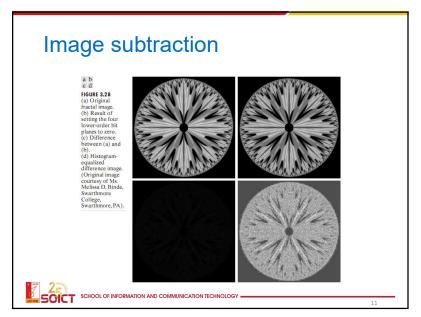


Image subtraction
The pixelwise substraction of two images f and g is:
 S(x,y) = Max(f(x,y)-g(x,y); 0)
Image substraction is used to
 – detect defaults, detect difference between images
 – detect motion in images

Source: Eric Favier. L'analyse et le traitement des images. ENISE.

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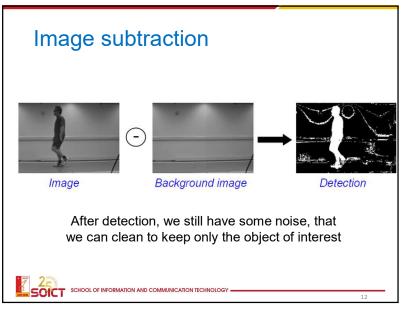


Image multiplication

- The multiplication S of an image f by a ratio (factor) is defined as:
 - -S(x,y) = Max(f(x,y)*ratio; 255)
- · Image multiplication can be used to increase the

contrast or the luminosity









Source: Eric Favier.
SOICT school of Information and Communication technology. Source : Eric Favier. L'analyse et le traitement des images. ENISE.

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Example code

- I1 = imread('images/ball1.png');
- >> I1 = rgb2gray(I1);
- >> I1 = 255 I1;
- >> I2 = imread('images/ball2.png');
- >> I2 = rgb2gray(I2);
- >> 12 = 255 12;
- >> I3 = I1 + I2;
- >> I4 = image_substract(I1, I2);
- >> 15 = 14;
- >> I5(find(I4==-255))=255;
- >> 15 = uint8(15);
- I6 = I3 I5;
- 17 = 11 16;
- 18 = 12 16;



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Operations on images













Source: www.nte.montaigne.u-bordeaux.fr/SuppCours/5314/Dai/TraitImage01-02.ppt

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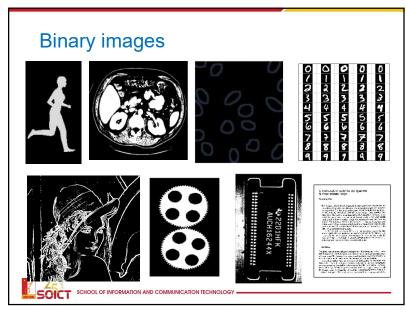
Binary images

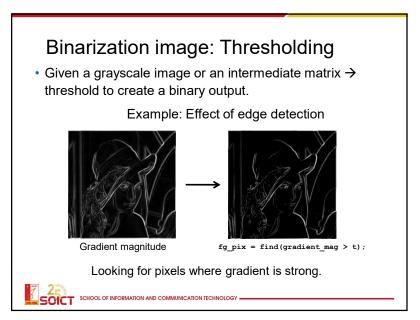
- Two types of pixels: foreground pixel (object, note 1) and background pixel (note 0)
- Be used
 - To mark region(s) of interest
 - As results of thresholding method

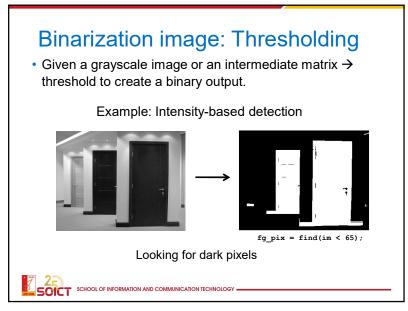
1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1

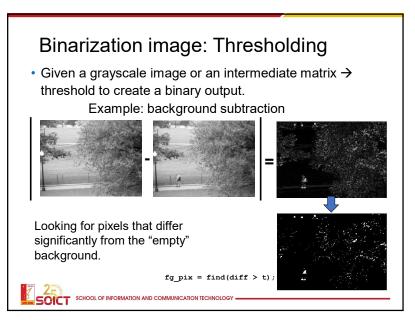








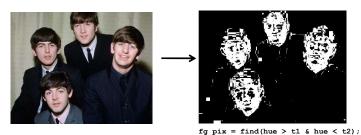




Binarization image: Thresholding

 Given a grayscale image or an intermediate matrix → threshold to create a binary output.

Example: color-based detection



Looking for pixels within a certain hue range.



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Slide credit: Kristen Grauman

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Morphological operators

- Change the shape of the foreground regions via intersection/union operations between a scanning structuring element and binary image.
- Useful to clean up result from thresholding
- Main components
 - -Structuring element
 - Operators:
 - Basic operators: Dilation, Erosion
 - · Others: Opening, Closing, ...



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Issues

- What to do with "noisy" binary outputs?
 - Holes
 - Extra small fragments



- How to demarcate multiple regions of interest?
 - Count objects
 - Compute further features per object





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Slide credit: Kristen Grauman

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Structuring elements

 Masks of varying shapes and sizes used to perform morphology, for example:

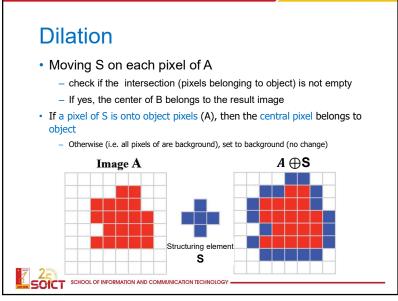


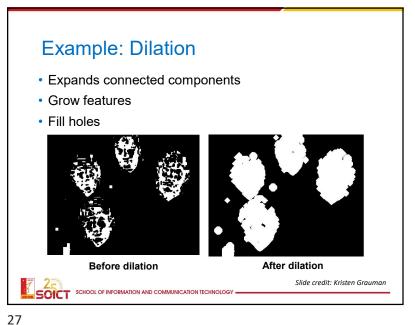


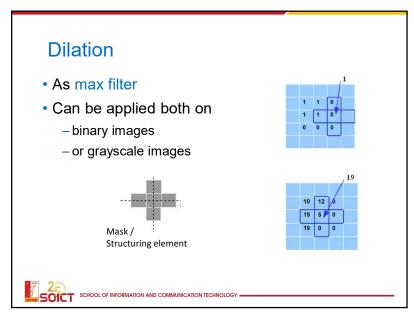
 Scan mask (structuring element) over the object (foreground) borders (inside and outside) and transform the binary image

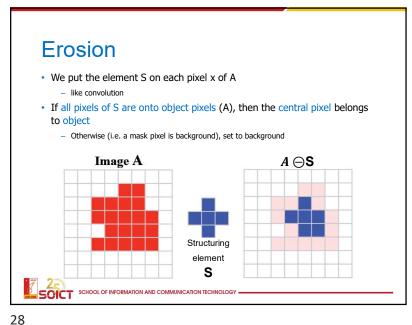


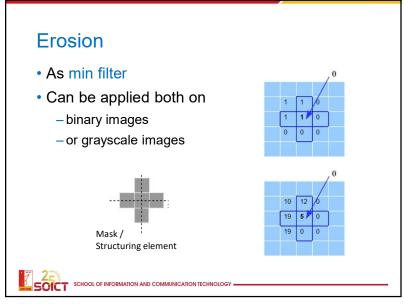
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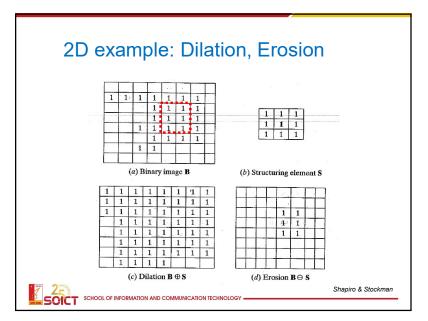


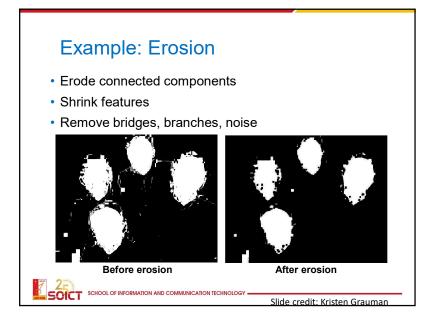


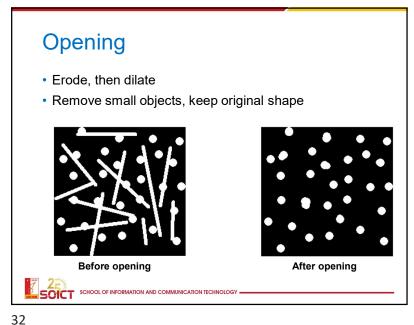


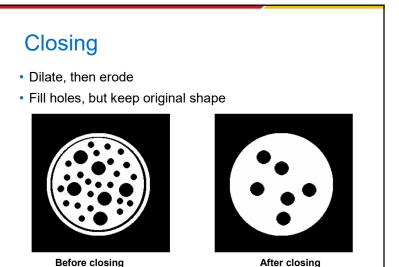












Opening vs Closing

A ° B = (A ⊙ B) ⊕ B

Opening

A • B = (A ⊕ B) ⊙ B

Closing

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Content

• Remind: digital image representation

demo: http://bigwww.epfl.ch/demo/jmorpho/start.php

- Point processing
- · Convolution and linear filtering

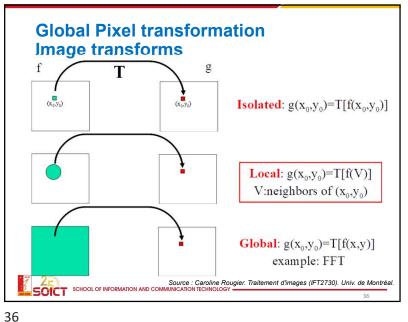
demo: http://digwww.epti.cn/demo/jm

- More neighborhood operators
- Image transforms
 - Frequency domain
 - Frequencies in images: Spectral image
 - · Fourier transform
 - Frequential Processing (frequential filters)
 - KL transform, PCA



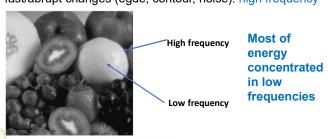
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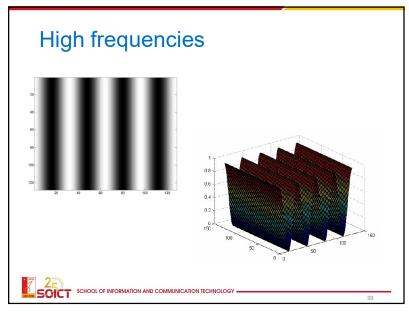


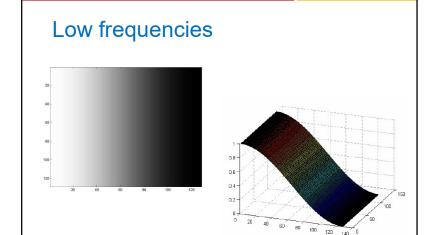
Frequency domain Frequencies in images

- · What are the (low/high) frequencies in an image?
 - Frequency = intensity change
 - Slow changes (homogeneous /blur regions): low frequency
 - fast/abrupt changes (egde, contour, noise): high frequency



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Fourier Transforms (FT) 2D FT of Continuous signal

Continuous FFT:

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-2\pi i(xu+yv)} dx dy$$

Inverse FFT:

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{2\pi i(xu+yv)} du dv$$

• FFT: Fast Fourier Transform - Fast FT algorithm



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Discrete Fourier Transforms (DFT)

- Discrete two-dimensional Fourier Transform (DFT) of an image array is defined in series form as follows;
- Direct Fourier transform:

$$X(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X(m,n) \exp\left(-j2\pi \left(\frac{um}{M} + \frac{vn}{N}\right)\right)$$

Invers Fourier transform:

$$X(m,n) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{M-1} X(u,v) \exp\left(j2\pi \left(\frac{um}{M} + \frac{vn}{N}\right)\right)$$

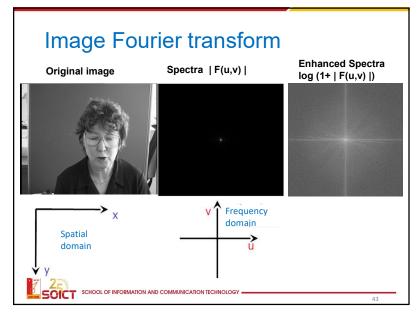
X(m,n): Original image MxN; X(u,v): Spectral image

- The indices (u,v) are called the spatial frequencies
- |X(u,v)|: magnitude spectral function, $\Phi(u,v)$: phase functions Both the magnitude and the phase functions are necessary for the complete reconstruction of an image from its Fourier transform
- FFT: an efficient computational algorithm for the discrete Fourier Transform



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2D FFT - discrete

Direct transform

$$F(u,v) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \exp \left[-2\pi i \left(\frac{mu}{M} + \frac{nv}{N} \right) \right] ,$$

$$u = 0, 1, \dots, M - 1$$
, $v = 0, 1, \dots, N - 1$,

Inverse transform

$$f(m,n) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) \exp \left[2\pi i \left(\frac{mu}{M} + \frac{nv}{N} \right) \right] ,$$

$$m = 0, 1, \dots, M - 1$$
, $n = 0, 1, \dots, N - 1$.



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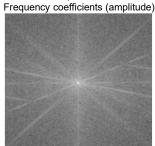
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Example: FFT of an image

Natural image



Fourier decomposition



|F(ω)|

f(x,y)What does it mean to be at pixel x,y?

What does it mean to be more or less bright in the Fourier decomposition image?



Slide by Steve Seitz

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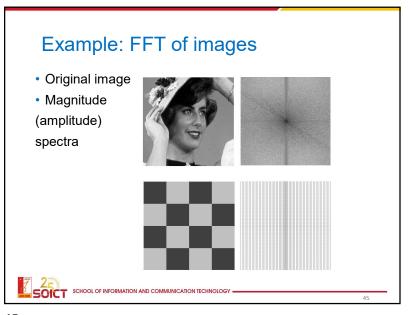
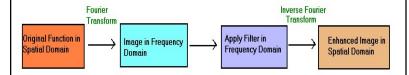


Image filtering in the frequency domain

 We will be dealing only with functions (images) of finite duration so we will be interested only in Fourier Transform



Linear filters in frequency domain:
 Low Pass Filter, High Pass Filter, Band Pass Filter

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Image spectral analysis

- An image is a visual signal
 - We can analysis the frequencies of the signal
- How?
 - we will create a new « image » which will contains all frequencies of the image
 - Like a 2D frequency graphic
 - The basic tool for it is the Fourier Transform
- We talk about the frequency domain, opposing to the spatial domain (image)
- Image filtering in the frequency domain



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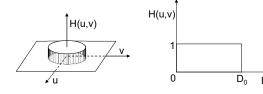
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H(u,v) - Ideal low pass filter

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \sqrt{u^2 + v^2}$$

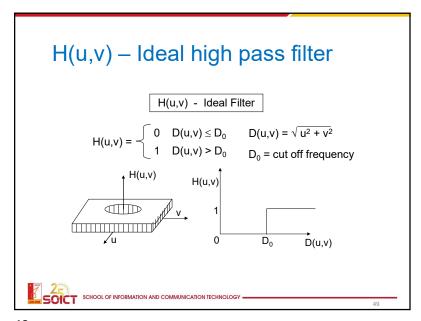
$$D_0 = \text{cut off frequency}$$



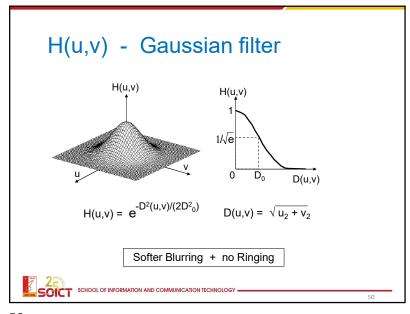


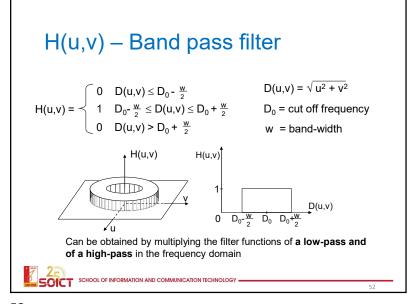
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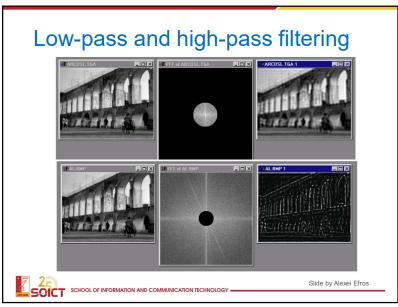
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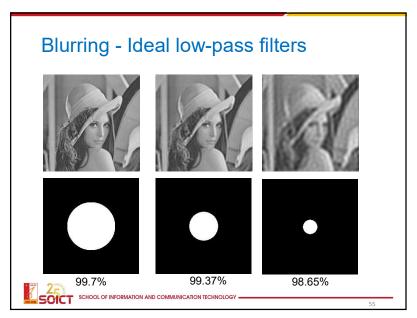


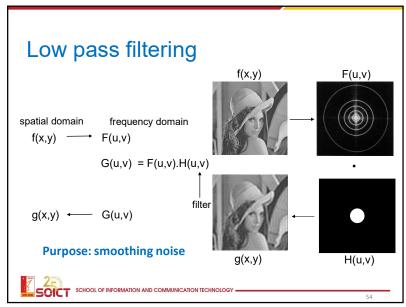
High-pass gaussian filter $H(u,v) = 1 - e^{-D^2(u,v)/(2D^2_0)}$ $D(u,v) = \sqrt{u_2 + v_2}$ School of information and communication technology











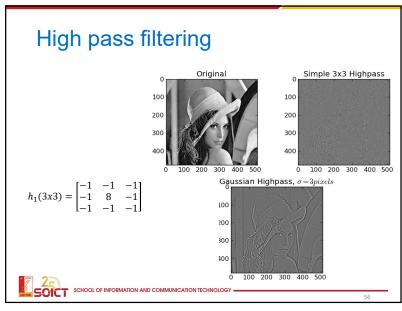


Image filtering: Hybrid Images

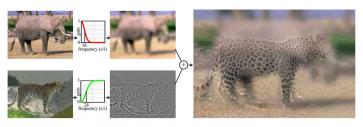


Figure 2: hybrid images are generated by superimposing two images at two different spatial scales: the low-spatial scale is obtained by filtering one image with a low-pass filter, and the high spatial scale is obtained by filtering a second image with a high-pass filter. The final hybrid image is composed by adding these two filtered images.

A. Oliva, A. Torralba, P.G. Schyns, SIGGRAPH 2006



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Principle Component Analysis (cont.)

- PCA enables transform a number of possibly correlated variables into a smaller number of uncorrelated variables called principal components
- The first principal component accounts for as much of the variability in the data as possible
- Each succeeding component (orthogonal to the previous ones) accounts for as much of the remaining variability as possible

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Principle Component Analysis (PCA) (Karhunen-Loeve transform)

- PCA transforms the original input space into a lower dimensional space
 - By constructing dimensions that are linear combinations of the given features
- The objective: consider independent dimensions along which data have largest variance (i.e., greatest variability - uncorrelation)

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Principal Component Analysis (cont.)

- PCA is the most commonly used dimension reduction technique.
- Data samples

$$x_1,\dots,x_N$$

Compute the mean

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Computer the covariance matrix:

$$\Sigma_{x} = \frac{1}{N} \sum_{i=1}^{N} (x_{i} - \overline{x})(x_{i} - \overline{x})^{T}$$



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Principal Component Analysis (cont.)

- Compute the eigenvalues λ and eigenvectors e of the matrix Σ_{r}
- Solve $\Sigma_x x = \lambda x$
- · Order them by magnitude:

$$\lambda_1 \geq \lambda_2 \geq .\lambda_N.$$

- PCA reduces the dimension by keeping direction e such that $\lambda < T$.

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Principal Component Analysis (cont.)

 For many datasets, most of the eigenvalues are negligible and can be discarded.

The eigenvalue λ measures the variation In the direction of corresponding eigenvector

Example:

$$\lambda_1 \neq 0, \lambda_2 = 0.$$



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Principal Component Analysis (cont.)

- How to get uncorrelated components which Capture most of the variance
- · Project the data onto the selected eigenvectors:

$$y_i = e_i^T (x_i - \overline{x})$$

• If we consider first M eigenvectors we get new lower dimensional representation

$$[y_1, \dots, y_M]$$

· Proportion covered by first M eigenvalues

$$\frac{\sum_{i=1}^{M} \lambda_i}{\sum_{i=1}^{N} \lambda_i}$$

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Determining the number of components

- Plot the eigenvalues
 - each eigenvalue is related to the amount of variation explained by the corresponding axis (eigenvector)
 - If the points on the graph tend to level out (show an "elbow" shape), these eigenvalues are usually close enough to zero that they can be ignored

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Phép biến đổi KL – PCA (Phân tích thành các thành phần chính - Principle Component Analysis)

- Không gian vector dữ liệu n chiều: $\chi = \{X\}, X = [x_i] \forall x_i \in R, i \in [1, n].$
- PCA là phép biến đổi đơn vị theo công thức: $\mathbf{Y} = \Phi \mathbf{X}$ sao cho thỏa mãn:

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

Vector Y bao gồm các thành phần không tương quan, nghĩa là mọi thành phần $y_i, y_k, \forall i, k \in [1,n]$ thỏa mãn hiệp phương sai bằng 0:

$$C_{ii} = E[y_i y_k] - E[y_i] \cdot E[y_k] = 0$$

 Φ là ma trận của phép biến đổi, còn gọi ma trận chiếu, được định nghĩa như sau: $\Phi = [e_i^T]$ với $i \in [1, n]$.

 $\mathbf{e}_{\mathbf{i}}$: là các vecto riêng của \mathbf{ma} trận hiệp phương sai của các vector X

e_i: là các vecto co sở trực giao của không gian đặc trưng

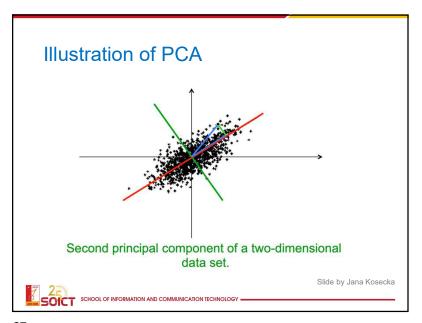
KL-PCA giảm thứ nguyên: chọn *m*<*n* tương ứng với *m* giá trị riêng lớn nhất của ma trận hiệp phương sai. Kết quả "*m* - dimensional subspace"

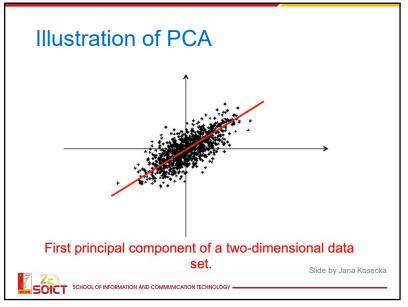
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PCA application: Using PCA for face recognition

- When viewed as vectors of pixel values, face images are extremely high-dimensional
 - -100x100 image = 10,000 dimensions
- However, relatively few 10,000-dimensional vectors correspond to valid face images
- We want to effectively model the subspace of face images



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Eigenfaces: Key idea

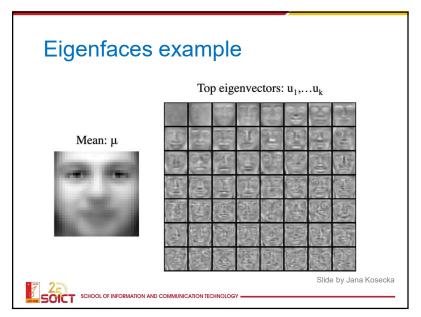
- · Assume that most face images lie on a low-dimensional subspace determined by the first k (k < d) directions of maximum variance
- Use PCA to determine the vectors or "eigenfaces" $\mathbf{u}_1,...,\mathbf{u}_k$ that span that subspace
- · Represent all face images in the dataset as linear combinations of eigenfaces

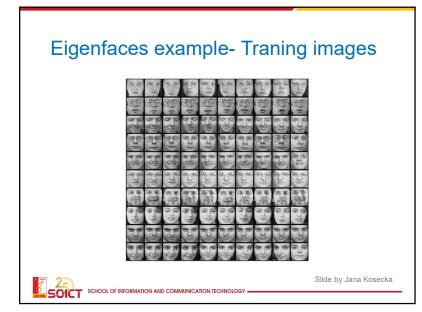
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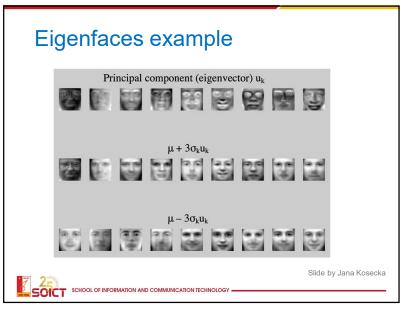
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Trích chọn đặc trưng dùng PCA

Các bước thực hiện PCA

B1: Tính vector trung bình $\mu = E\{X_k\}$ và tính các vector :

$$X_k = X_k - \mu$$

B2: Tính ma trận hiệp phương sai của các vector X trong bộ dữ liệu quan sát (ảnh ban đầu): $C_{X} = E\left\{XX^{T}\right\} = \frac{1}{M}(XX^{T})$

B3: Giải phương trình đặc tính tìm được các trị riêng của ma trận hiệp phương sai C_X , và kết hợp điều kiện trực giao được n vector riêng ứng n trị riêng. Các vector riêng trực giao ở trường hợp nhận dạng khuôn mặt được gọi là **Eigenfaces.**

B4: Trích chọn thành phần chính: Chọn ra m vector riêng ứng với m trị riêng lớn nhất (m < n) để thiết lập ma trận phép biến đổi hay ma trận chiếu Φ

B5: Thực hiện phép biến đổi PCA theo quan hệ: $Y = \Phi X$

Kết quả đạt được là bộ dữ liệu mới với các vector Y là các vector đặc trưng của đối tượng.



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Exercise 1

• Given an 8-bit image – 8 x 8

 52
 55
 61
 66
 70
 61
 64
 73

 63
 59
 55
 90
 109
 85
 69
 72

 62
 59
 68
 113
 144
 104
 66
 73

 63
 58
 71
 122
 154
 106
 70
 69

 67
 61
 68
 104
 126
 88
 68
 70

 79
 65
 60
 70
 77
 68
 58
 75

 85
 71
 64
 59
 55
 61
 65
 83

 87
 79
 69
 68
 65
 76
 78
 94

1) Compute and show the histogram:

2) Compute the Power-law Transformation using $s=c.r^{\gamma}$

with
$$c = 0.1$$
; $r = 1$, $\gamma = 1$

- 3) Comment about the contrast of the image and make a modification of contrast
- 4) Equalize the histogram for above image with 8-bins
- 5) Compute the filtered image by the filters such as: Mean filter, Laplacian filter, Median filter, Min/ Max filter



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Recognition with eigenfaces

- · Process labeled training images:
- Find mean μ and covariance matrix Σ
- Find k principal components (eigenvectors of Σ) $\mathbf{u}_1, \dots \mathbf{u}_k$
- Project each training image x_i onto subspace spanned by principal components:
 (w_{i1},...,w_{ik}) = (u₁^T(x_i-μ),...,u_k^T(x_i-μ))
- Given novel image x:
- Project onto subspace: $(\mathbf{w}_1, \dots, \mathbf{w}_k) = (\mathbf{u}_1^T(\mathbf{x} - \boldsymbol{\mu}), \dots, \mathbf{u}_k^T(\mathbf{x} - \boldsymbol{\mu}))$
- Optional: check reconstruction error $\mathbf{x} \hat{\mathbf{x}}$ to determine whether image is really a face
- · Classify as closest training face in k-dimensional subspace



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Exercise 2

Given two images as bellow





- 1. Transform images to negative ones
- Process to have an image which has only the "ball"



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