GNSS Introduction

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Part III

Errors

Pseudorange errors

- control system: ephemerides, clocks, codes, measurement errors
- ionosphere: the propagation delay depends on the frequency on on the density of electrons along the path
- troposphere: the propagation delay depends on the pressure, temperature, humidity of the air
- multipath
- receiver noise
- uncompensated relativistic effects
- selective availability (SA): the signal has been intentionally disturbed to limit the accuracy of the GPS to civil user ("ended" May 1st 2000)

User Equivalent Range Error

- The pseudorange errors can be modeled as random variables
  - gaussian with zero mean and variance σ^2_j
  - identically distributed
  - independent
  - with variance σ^2_{UERE}

- Under these hypoteses:

$$\sigma_{UERE} = \sqrt{\sum_j \sigma_j^2} \quad [m]$$

User equivalent range error (UERE)

segment	error source	PPS	SPS (SA)	SPS (no SA)
		1σ [m]	1σ [m]	1σ [m]
space	satellite clock stability	3.0	3.0	3.0
	satellite perturbations	1.0	1.0	1.0
	selective availability	-	32.3	-
	other	0.5	0.5	0.5
control	ephemeris prediction	4.2	4.2	4.2
	other	0.9	0.9	0.9
user	ionospheric delay	2.3	5.0	5.0
	tropospheric delay	2.0	1.5	1.5
	receiver noise	1.5	1.5	1.5
	multipath	1.2	2.5	2.5
	other	0.5	0.5	0.5
UERE	total (rss)	6.6	33.3	8.0

The received pseudorange

- Considering the error contribution on the pseudorange from the *k-th* satellite

$$\rho^k = r_k + c\cdot(\delta t_u - \delta t^k) + I_\rho^k + T_\rho^k + \varepsilon_\rho^k$$

- and on the phase

$$\Phi^k = \lambda^{-1}\left[r_k + I_\Phi^k + T_\Phi^k\right] + \frac{c}{\lambda}\cdot(\delta t_u - \delta t^k) + N + \varepsilon_\Phi^k$$

- iono and troposphere induced errors (in meters)

$$I_\rho, I_\Phi, T_\rho, T_\Phi$$

Ionosphere

- Ionosphere (50-1000 km) is a region of ionized gases (free electrons and ions)
- Ionization is caused by the sun's radiation
- The speed of propagation of a radio signal depends upon the number of free electrons in its path
- Total electron content (TEC) is defined as the *number of electrons in a tube of 1m^2 cross section from receiver to satellite*

Ionospheric model

- Ionosphere induces a pseudorange delay

$$I_{\rho} = \frac{40.3 \cdot TEC}{f^2}$$

- the delay in phase measurement have same magnitude but opposite sign
- can be estimated and compensated for, using double frequency receivers.

Ionosphere free measurements

- Assuming to be able to measure the pseudorange at two frequencies

$$\rho_1 = \rho^* + \frac{40.3 \cdot TEC}{f_1^2}$$
$$\rho_2 = \rho^* + \frac{40.3 \cdot TEC}{f_2^2}$$

- being ρ^* the ionosphere-free pseudorange
- The set of equations can be solved for ρ^* and TEC obtaining

$$\rho^* = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2$$

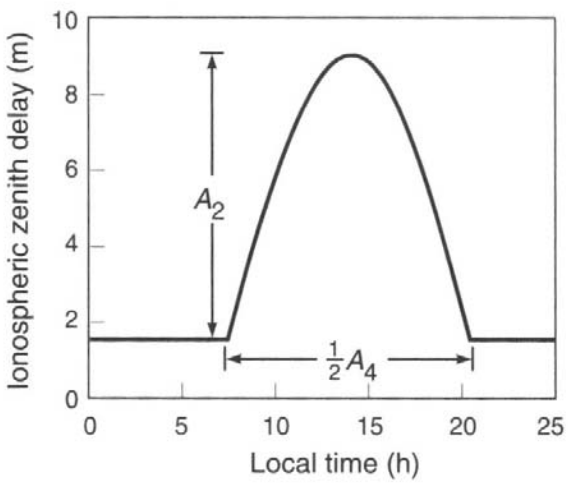
Ionosphere model

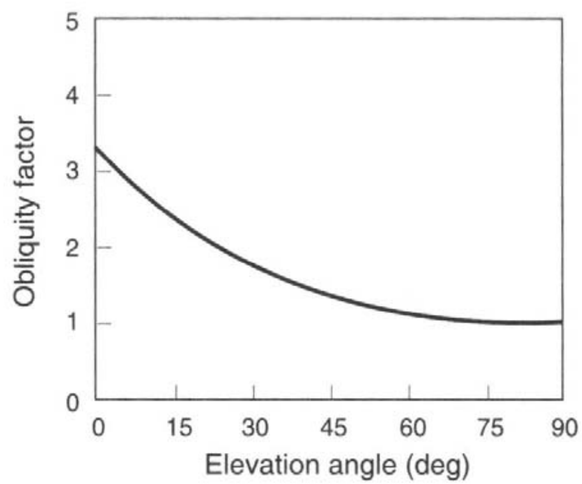
- Single frequency receivers use models whose parameters are broadcast by the satellites
- The Klobuchar model is an empirical model using a reduced number of parameters to be described

The Klobuchar model

$$\frac{\hat{I}_{z,L1}}{c} = \begin{cases} A_1 + A_2 \cos\left(\dfrac{2\pi(t - A_3)}{A_4}\right), & |t - A_3| < A_4 / 4 \\ A_1 & otherwise \end{cases}$$

- Values A_1 *and* A_3 are fixed and A_2 *and* A_4 are specified in the navigation message broadcast by each satellite





Troposphere

- Tropsphere is a non dispersive medium
- Refractivity: $N=(n-1)\times10^{-6}$
- It can be split in dry and wet refractivity

$$N=N_d+N_w$$

$$T= 10^{-6}\int N(l)dl=T_d+T_w$$

(associated to the zenith)

- The delay depends on an obliquity factor

$$T=T_d\, m_d(E)+T_w\, m_w(E)$$

- where $m_d(E)$ *and* $m_w(E)$ are called mapping functions
- Saastamoinen model
- Hopfield model

Remarks on the errors

- The theory of the navigation solution and the error budgets are valid under the hypothesis of errors
  - statistically independent for the different SV
  - error modeled as random gaussian variables with zero mean
- Such hypotheses are seldom verified in practical cases, and rely on the use of error models within the receivers able to estimate at least the average contribution for some error sources

- Receivers apply corrections for the expected
  - bias of the satellite clock
  - tropospheric delay
  - ionospheric delay

End of Part III