Name of Students	
TA Hai Tung (VIE)	**ZHENG Ziyang (CHN)**
Studend IDs	
134278	**134277**

GNSS Introduction

Homework 1—Definitions and basics

Exercise HW1-1

1. Magellan® Meridian Marine

- **Type of receiver**: handed
- **Accuracy provided**:

Horizontal Accuracy (meters)	< 7
Horizontal Accuracy (%RMS)	95% 2D
Horizontal Accuracy -RMS w/ WAAS (meters)	< 3
Horizontal Accuracy(%RMS/WAAS)	95% 2D
Vertical Accuracy (meters RMS)	10
Velocity (knots RMS)	0.1

- **Additional features**: Seven customizable graphic navigation displays show a map, compass, speedometer, and text readouts of heading, bearing, speed, direction, ETA, and more. Choose from 12 built-in coordinate systems and 76 datums including Latitude/Longitude, Universal Transverse Mercator, and Military Grid Reference System.
- **Price**: $319.95

2. DGPS Max 3019

- **Type of receiver**: professional
- **Accuracy provided:**

  Sub metre (3ft) 95% confidence.

  Typical +/- 30cm (1ft) pass to pass
- **Additional features**:
  - The DGPS Max receiver is ideal for guidance and autosteer in spraying and spreading or yield mapping applications. This multi band receiver offers the flexibility to utilize free WAAS/EGNOS (US and EU only) OR Marine Beacon OR OmniSTAR VBS differential correction services.
  - Easy Setup
  - Fast 5Hz Update Rate
- **Price**: $2000

3. Magellan SporTraktm Pro GPS receiver

- **Type of receiver**: handed, professional
- **Accuracy provided:**

Horizontal Accuracy (meters)	<7
Horizontal Accuracy (%RMS)	95% 2D
Horizontal Accuracy -RMS w/ WAAS (meters)	<3
Horizontal Accuracy(%RMS/WAAS)	95% 2D w/WAAS
Vertical Accuracy (meters RMS)	10
Velocity (knots RMS)	0.1

- **Additional features:**

Position Update Rate (per second)	1
Time to First Fix: Cold (seconds)	300
Time to First Fix: Warm (seconds)	60
Time to First Fix: Hot (seconds)	15
Maximum Velocity (mph)	951
Maximum Velocity (km/h)	1530
Unit Width (Inches)	2.2
Unit Height (inches)	5.6
Unit Depth (inches)	1.2
Weight (oz)	6.1
Weight (gm	227
Display Size Width (inches)	2.3
Display Size Height (inches)	1.4
Display Size Width (mm)	44.0
Display Size Height (mm)	55.9
Display Resolution	120 X 160
Display Resolution Color	Greyscale

- **Price**: $269.95

4. XE1610-PVT GPS receiver

- **Type of receiver**: OEM
- **Accuracy provided**: < 5m CEP without SA (horizontal)
- Additional features:
  - Sensitivity: down to -143 dBm tracking, urban canyon capabilities
  - Warm Start is under 32 seconds
  - Hot Start is under 12 seconds
  - Ultra low power: 19mA @ 3 Volts, full tracking

- Embedded ARM7TDMI
- Small form factor and low cost solution
- Ready-to-plug, fully autonomous PVT solution. Easily integrated into existing systems
- On-board RAM for GPS navigation data, on-board Flash memory back-up
- PPS output
- Bidirectional NMEA interface
- Real Time Clock with separate back-up power supply

• **Price**: unknown

5. Hiper XT

• **Type of Receiver**: Professional

• **Accuracy provided**:

- Static, Rapid Static H: 3mm + 0.5ppm
  V: 5mm + 0.5ppm
- RTK H: 10mm + 1ppm
  V: 15mm + 1ppm

• **Additional features**:

Tracking Specifications

Tracking Channels,standard: 40 L1 GPS (20 GPS L1+L2 on Cinderella2 days)

Signals Tracked: L1/L2 C/A and P Code & Carrier and GLONASS

GPS+ Antenna Specifications: GPS / GLONASS Antenna Integrated

I/O: Communication Ports 4x serial (RS232), USB

Status Indicator 4x3-color LED's, two-function keys (MINTER)

Control & Display Unit: External Field Controller

Memory & Recording:

Internal Memory Up to 128 MB

Data Update Rate Up to 20 times per second (20Hz)

Data Type Code & Carrier from L1 and L2, GPS & GLONASS

Environmental Specifications

Enclosure Aluminum extrusion, waterproof

Operating Temperature -30°C to 55°C

Dimensions W:159 x H:172 x D:88 mm / 6.25 x 6.75 x 3.5 in

Weight 1.65 kg / 3.64 lbs

• **Price:** $44100

Exercise HW1-2

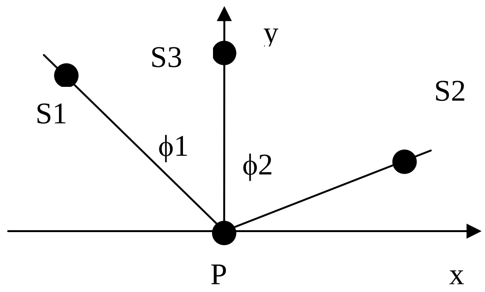

Figure 1

1. ***Evaluate the geometric matrix H of the system for a user in P***

The coordinate of position P in xPy coordination system is (0,0). At this position, we have the matrix H:

Because user's clock is synchronized with transmitters' clocks, H is a 2x2 matrix

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{a}_{x1} & \boldsymbol{a}_{y1} \\ \boldsymbol{a}_{x2} & \boldsymbol{a}_{y2} \end{bmatrix}$$

With: $$\boldsymbol{a}_{x1} = \frac{\boldsymbol{x}_1 - \boldsymbol{x}_P}{\boldsymbol{r}_{1,P}} = \sin(\phi_1) \qquad \boldsymbol{a}_{y1} = \frac{\boldsymbol{y}_1 - \boldsymbol{y}_P}{\boldsymbol{r}_{1,P}} = \cos(\phi_1)$$

$$\boldsymbol{a}_{x2} = \frac{\boldsymbol{x}_2 - \boldsymbol{x}_P}{\boldsymbol{r}_{2,P}} = \sin(\phi_2) \qquad \boldsymbol{a}_{y2} = \frac{\boldsymbol{y}_2 - \boldsymbol{y}_P}{\boldsymbol{r}_{2,P}} = \cos(\phi_2)$$

$$\boxed{\boldsymbol{H} = \begin{bmatrix} \sin(\phi_1) & \cos(\phi_1) \\ \sin(\phi_2) & \cos\phi_2) \end{bmatrix}}$$

In general: G = $(H^TH)^{-1}$ if (H^TH) is non-singular

$$\boldsymbol{G} = \left(\begin{bmatrix} \sin\phi_1 & \sin\phi_2 \\ \cos\phi_1 & \cos\phi_2 \end{bmatrix} \begin{bmatrix} \sin(\phi_1) & \cos(\phi_1) \\ \sin(\phi_2) & \cos(\phi_2) \end{bmatrix} \right)^{-1}$$

$$\boxed{\boldsymbol{G} = \frac{1}{\sin^2(\phi_1 - \phi_2)} \begin{bmatrix} \cos^2(\phi_1) + \cos^2(\phi_2) & -\left(\sin(\phi_1)\cos(\phi_1) + \sin(\phi_2)\cos(\phi_2)\right) \\ -\left(\sin(\phi_1)\cos(\phi_1) + \sin(\phi_2)\cos(\phi_2)\right) & \sin^2(\phi_1) + \sin^2(\phi_2) \end{bmatrix}}$$

2. ***Evaluate the horizontal dilution of precision HDOP in the following cases:***
   - **ϕ1=-π/4 , ϕ2=π/4**

With these values, the matrix G is:

$$\boldsymbol{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So $\boxed{\text{HDOP} = \sqrt{1+1} = \sqrt{2}}$

- **$\phi1=-\pi/2$, $\phi2=\pi/2$**

With these values of angles, we have:

$$\boldsymbol{H} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$\boldsymbol{H^T H} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ is singular so H^TH cannot have the inverse matrix. It means that HDOP in this case is worst.

3. Justify the results discussing for which case of ϕ1e ϕ2 the minimum value of HDOP is reached

In general, if H^TH is non-singular, we have HDOP = $\sqrt{g_{11} + g_{22}}$

So we have:

$$\text{HDOP} = \sqrt{\frac{\cos^2(\phi_1)+\cos^2(\phi_2)}{\sin^2(\phi_1-\phi_2)} + \frac{\sin^2(\phi_1)+\sin^2(\phi_2)}{\sin^2(\phi_1-\phi_2)}} = \sqrt{\frac{2}{\sin^2(\phi_1-\phi_2)}}$$

HDOP goes to minimum value iff $\sin^2(\phi_1-\phi_2)$ goes to maximum value, it means

$$\sin^2(\phi_1-\phi_2) = 1 \Leftrightarrow \sin(\phi_1-\phi_2) = \pm 1 \Leftrightarrow \boxed{(\phi_1-\phi_2) = \pm\frac{\pi}{2}}$$

4. Supposing the user is not synchronized with the transmitters. Write the H matrix and evaluate the H matrix and the HDOP in this case

$$\boldsymbol{H} = \begin{bmatrix} a_{x1} & a_{y1} & 1 \\ a_{x2} & a_{y2} & 1 \\ a_{x3} & a_{y3} & 1 \end{bmatrix}$$

$$a_{x1} = \frac{x_1 - x_P}{r_{1,P}} = \sin(\phi_1) \qquad a_{y1} = \frac{y_1 - y_P}{r_{1,P}} = \cos(\phi_1)$$

$$a_{x2} = \frac{x_2 - x_P}{r_{2,P}} = \sin(\phi_2) \qquad a_{y2} = \frac{y_2 - y_P}{r_{2,P}} = \cos(\phi_2)$$

$$a_{x3} = \frac{x_3 - x_P}{r_{3,P}} = 0 \qquad a_{y1} = \frac{y_1 - y_P}{r_{1,P}} = 1$$

$$\boxed{\boldsymbol{H} = \begin{bmatrix} \sin\phi_1 & \cos\phi_1 & 1 \\ \sin\phi_2 & \cos\phi_2 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$

$$\boldsymbol{G} = (\boldsymbol{H}^T\boldsymbol{H})^{-1} \quad = \quad \left(\begin{bmatrix} \sin\phi_1 & \sin\phi_2 & 0 \\ \cos\phi_1 & \cos\phi_2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \sin\phi_1 & \cos\phi_1 & 1 \\ \sin\phi_2 & \cos\phi_2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \right)^{-1}$$

$$= \left(\begin{bmatrix} \sin^2\phi_1 + \sin^2\phi_2 & \sin\phi_1\cos\phi_1 + \sin\phi_2\cos\phi_2 & \sin\phi_1 + \sin\phi_2 \\ \sin\phi_1\cos\phi_1 + \sin\phi_2\cos\phi_2 & \cos^2\phi_1 + \cos^2\phi_2 + 1 & \cos\phi_1 + \cos\phi_2 + 1 \\ \sin\phi_1\sin\phi_2 & \cos\phi_1\cos\phi_2 & 3 \end{bmatrix} \right)^{-1}$$

- **In case $\phi 1 = -\pi/4$, $\phi 2 = \pi/4$ we have matrix G:**

$$\boldsymbol{G} \quad = \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 17.4853 & -14.0711 \\ 0 & -14.0711 & 11.6569 \end{bmatrix}$$

So $\boxed{\text{HDOP = 5.4902}}$

- In case $\phi 1 = -\pi/2$, $\phi 2 = \pi/2$ we have matrix G:

$$\boldsymbol{G} \quad = \quad \begin{bmatrix} 0.7071 & 0 & 0 \\ 0 & 5.1213 & -4.1213 \\ 0 & -1.7071 & 1.7071 \end{bmatrix}$$

So $\boxed{\text{HDOP = 2.7451}}$

Exercise HW1-3

1. Plot DOPs

IN CASE OF CHINA

Latitude of a place in China = 32.10

Longitude of a place in China = 118.12

❖ **Nominal GPS constellation provided by `gpssoft`**

- ***Mask angle =10^0***

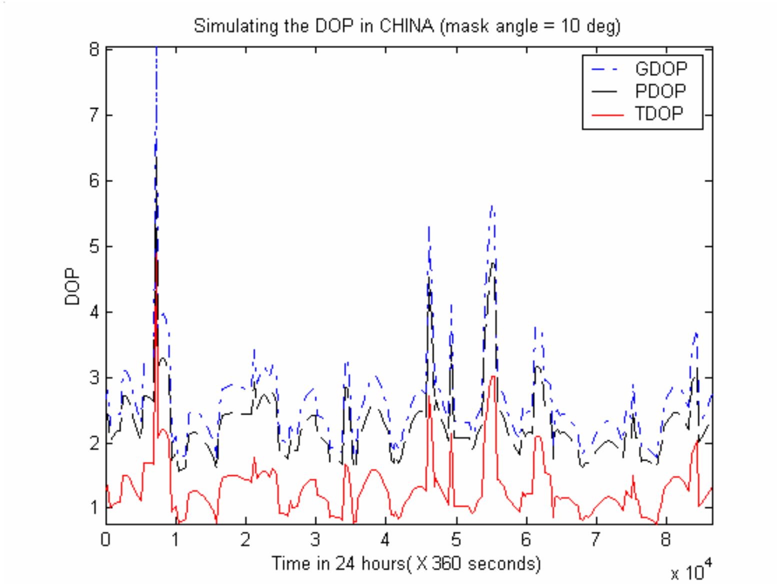

Figure 1. DOPs of a place in China along 24h

(in case: mask angle=10^0 and gpssoft constellation)

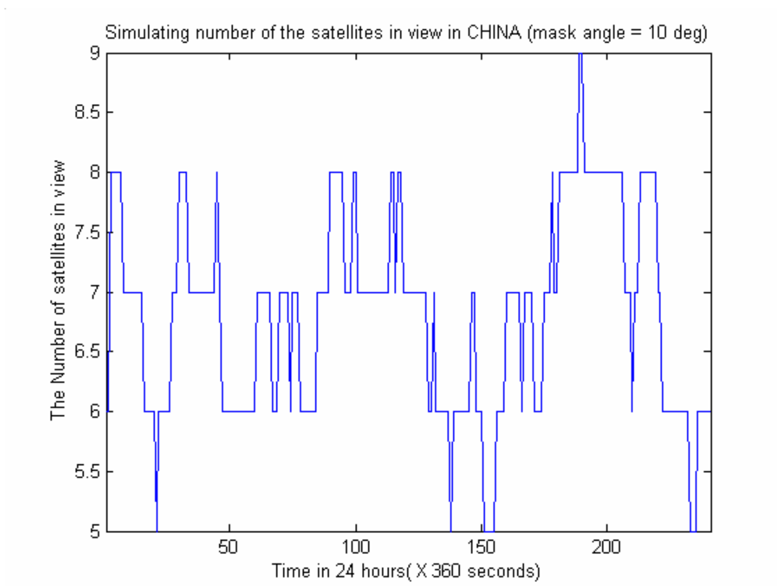

Figure 2. Number of satellites in view along 24 hours in a place in China

(in case: mask angle=10^0 and gpssoft constellation)

	PDOP	TDOP
Mean values	2.2910	1.3038
Maximum values	6.4037	4.8973

- ***Mask angle = 60^0***

In the simulation using gpsoft matlab toolbox, the number of visual satellites is less than 4. Therefore, we cannot estimate the DOPs.

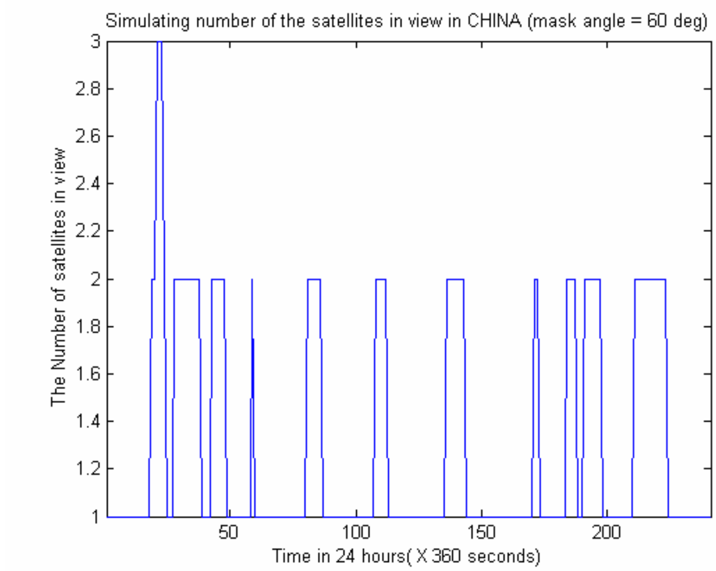

Figure 3. Number of satellites in view along 24 hours in a place in China
(in case: mask angle=60^0 and gpssoft constellation)

❖ **Present real GPS constellation from yuma file (20/10/2005)**

- ***Mask angle =10^0***

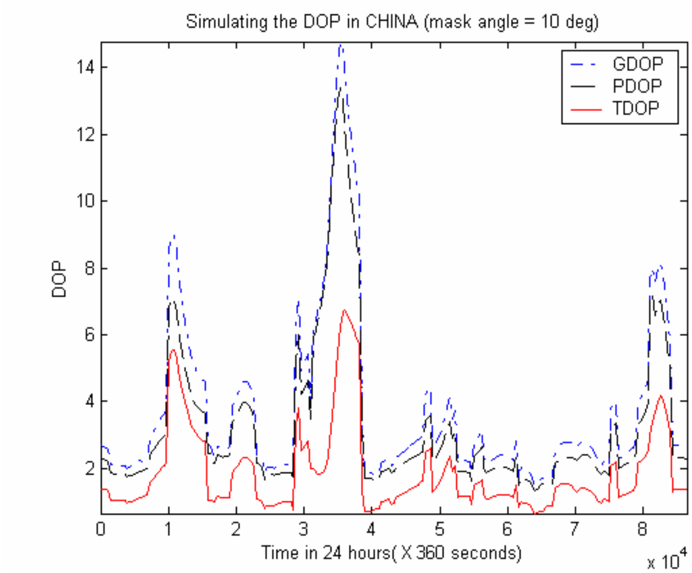

Figure 4. DOPs of a place in China along 24h
(in case: mask angle=10^0 and real constellation)

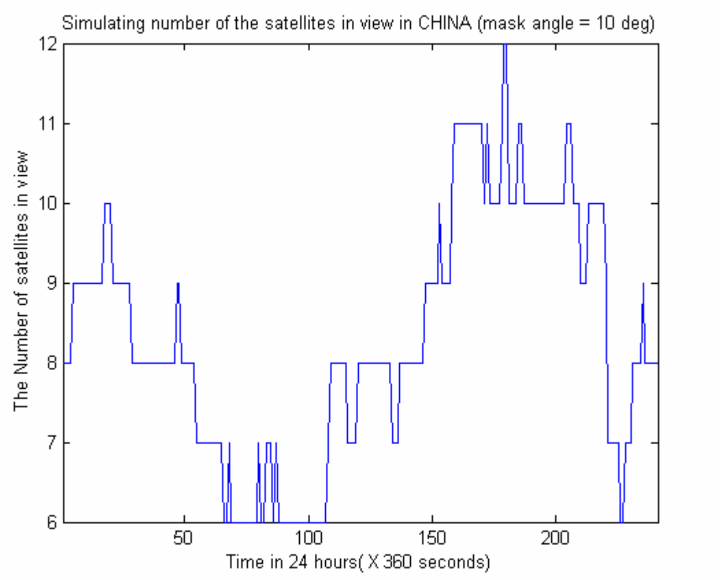

Figure 5. Number of satellites in view along 24 hours in a place in China
(in case: mask angle=10^0 and real constellation)

	PDOP	TDOP
Mean values	3.2902	1.8849
Maximum values	13.3766	6.7438

- ***Mask angle =60^0***

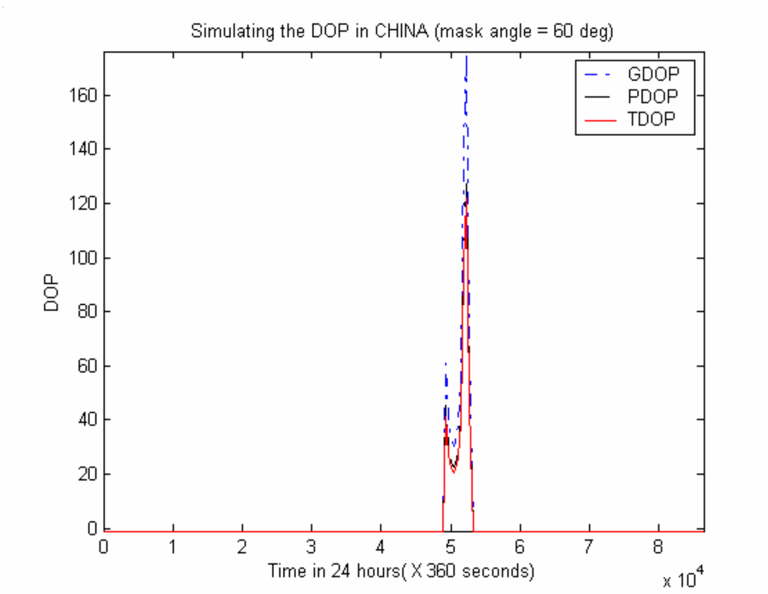

Figure 6. DOPs of a place in China along 24h
(in case: mask angle=60^0 and real constellation)

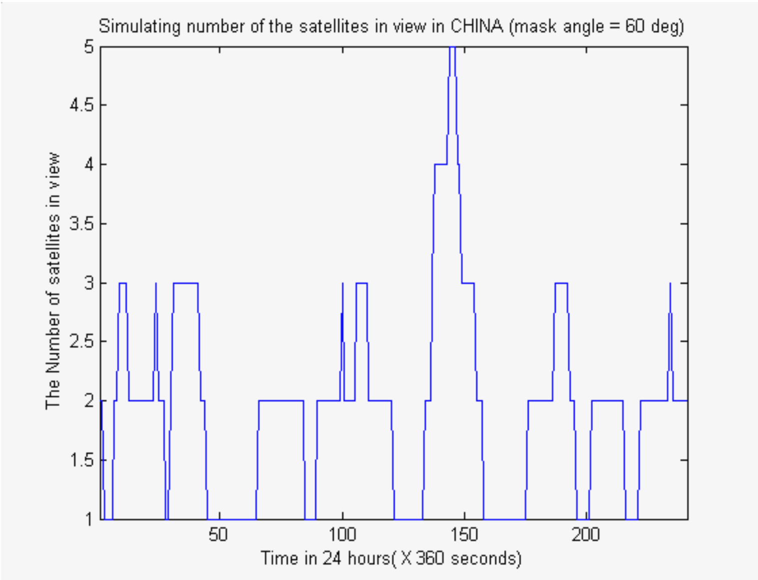

Figure 7. Number of satellites in view along 24 hours in a place in China
(in case: mask angle=60^0 and real constellation)

IN CASE OF VIETNAM

Latitude of a place in Vietnam = 21.03333

Longitude of a place in Vietnam = 105.85

❖ **Nominal GPS constellation provided by `gpssoft`**

- ***Mask angle =10^0***

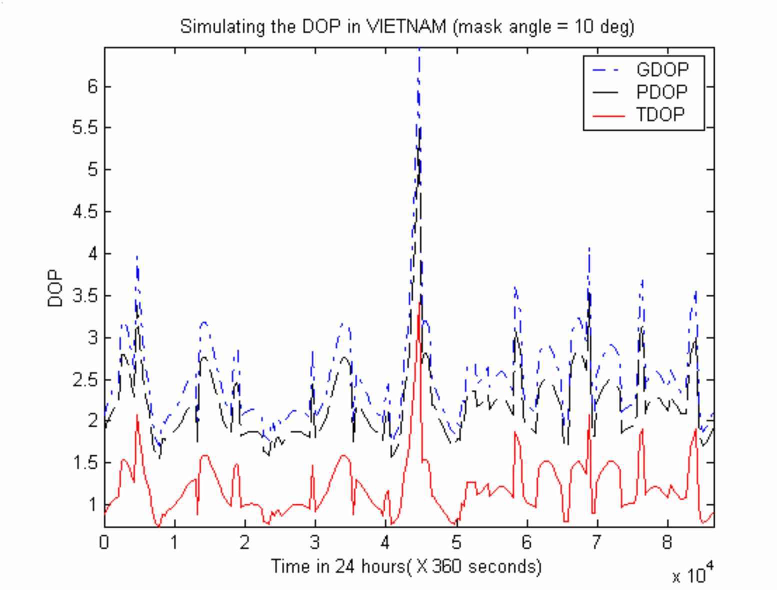

Figure 8. DOPs of a place in Vietnam along 24h
(in case: mask angle=10^0 and gpssoft constellation)

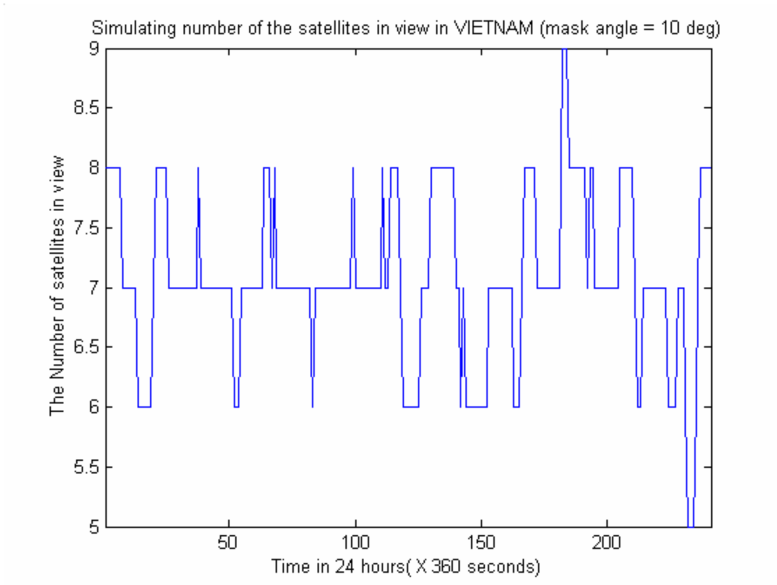

Figure 9. Number of satellites of a place in Vietnam along 24h
(in case: mask angle=10^0 and gpssoft constellation)

	PDOP	TDOP
Mean values	3.2902	1.8849
Maximum values	13.3766	6.7438

- ***Mask angle =60^0***

In the simulation using gpsoft matlab toolbox, the number of visual satellites is less than 4. Therefore, we cannot estimate the DOPs.

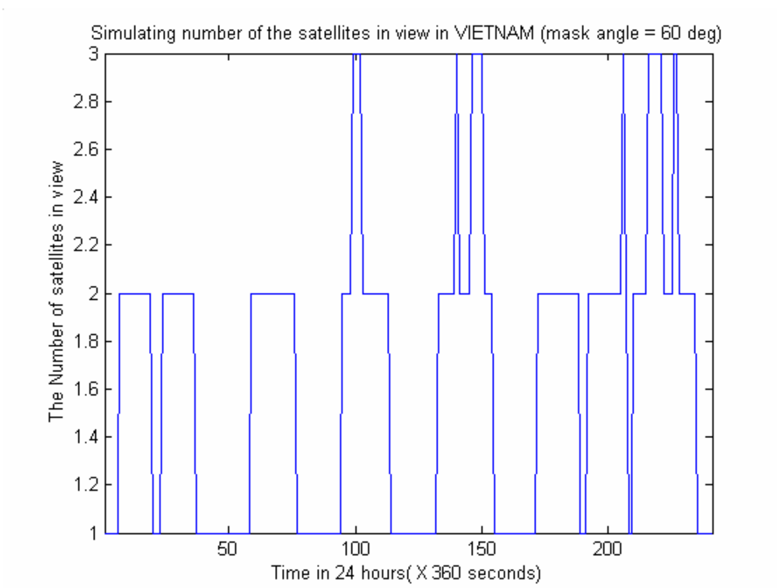

Figure 10. Number of satellites of a place in Vietnam along 24h
(in case: mask angle=60^0 and gpssoft constellation)

❖ Present real GPS constellation from yuma file (20/10/2005)

- ***Mask angle =10^0***

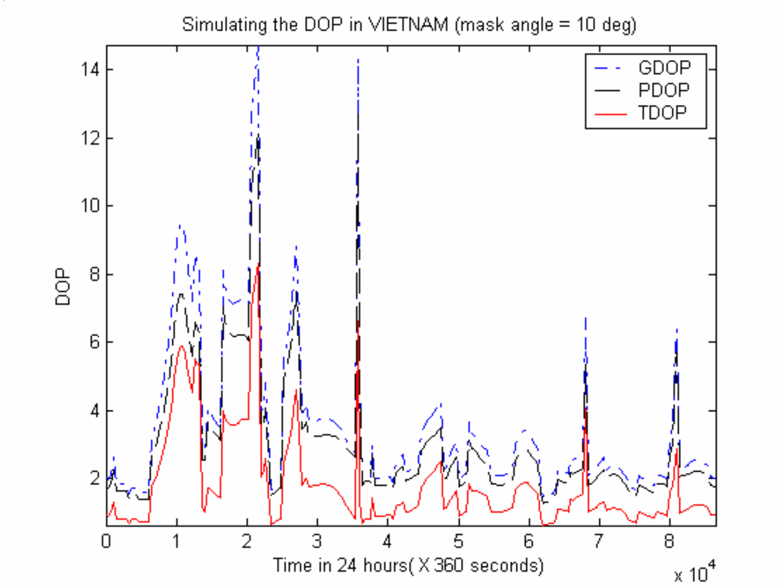

Figure 11. DOPs of a place in Vietnam along 24h
(in case: mask angle=10^0 and real constellation)

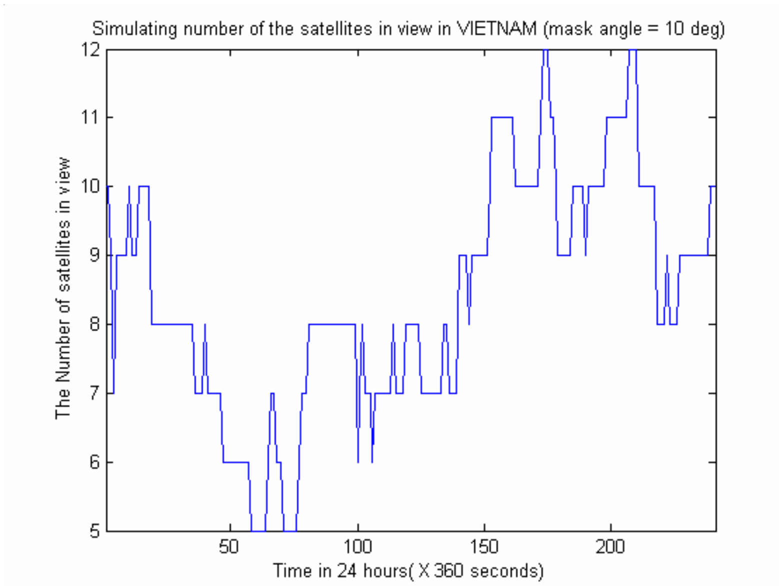

Figure 12. Number of satellites of a place in Vietnam along 24h
(in case: mask angle=10^0 and real constellation)

	PDOP	TDOP
Mean values	3.0619	1.8238
Maximum values	12.7045	8.3145

- ***Mask angle =60^0***

In this case, the number of satellites in view is always less than 4, so we cannot optain any DOPs.

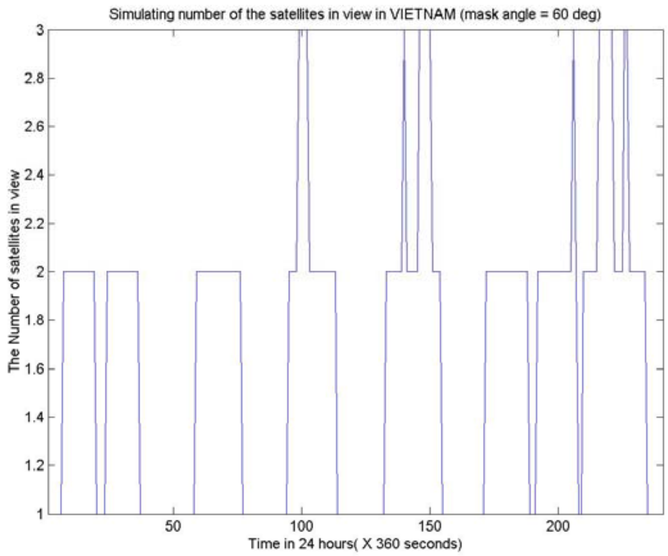

Figure 13. Number of satellites of a place in Vietnam along 24h
(in case: mask angle=60^0 and real constellation)

2. Provide an explanation for the changes in the PDOP variation

PDOP is the abbreviation of Position Dilution of Precision and as its definition, this parameter depends on the matrix $G=(H^TH)^{-1}$, with H is geomatric matrix. Because the geometry of satellites relating to a position changes along time, so matrix H also changes causing the PDOP variation.

Exercise HW1-4

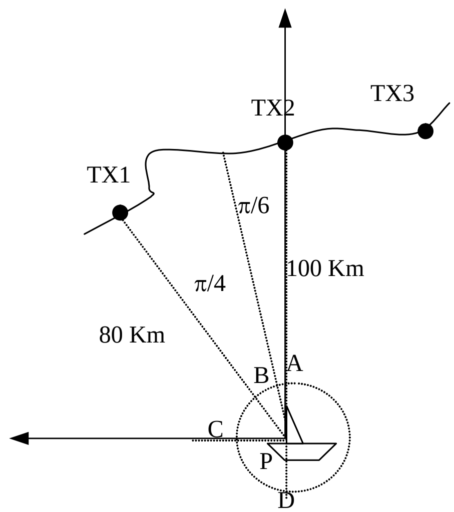

Figure 14.

In this excercise, we divide into two cases:

- The clock's boat is synchronized with transmitters' clocks
- The clock's boat is non-synchronized with transmitters' clocks

In case "non-sync" in time

We choose the coordinate system so that P is root with the coordinate (0,0) as figure.

In general, TX1(x1,y1) TX2(x2,y2) and auxiliary transmiter, denoted by TXa (xa,ya)

We have H matrix:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{a}_{x1} & \boldsymbol{a}_{y1} & 1 \\ \boldsymbol{a}_{x2} & \boldsymbol{a}_{y2} & 1 \\ \boldsymbol{a}_{xa} & \boldsymbol{a}_{ya} & 1 \end{bmatrix}$$

$$\boldsymbol{a}_{x1} = \frac{\boldsymbol{x}_1 - \boldsymbol{x}_P}{\boldsymbol{r}_{1,P}} = \sin(\boldsymbol{\phi}_1) \qquad \boldsymbol{a}_{y1} = \frac{\boldsymbol{y}_1 - \boldsymbol{y}_P}{\boldsymbol{r}_{1,P}} = \cos(\boldsymbol{\phi}_1)$$

$$\boldsymbol{a}_{x2} = \frac{\boldsymbol{x}_2 - \boldsymbol{x}_P}{\boldsymbol{r}_{2,P}} = \sin(\boldsymbol{\phi}_2) \qquad \boldsymbol{a}_{y2} = \frac{\boldsymbol{y}_2 - \boldsymbol{y}_P}{\boldsymbol{r}_{2,P}} = \cos(\boldsymbol{\phi}_2) \qquad \mathbf{x}$$

$$\boldsymbol{a}_{x2} = \frac{\boldsymbol{x}_a - \boldsymbol{x}_P}{\boldsymbol{r}_{a,P}} = \sin(\boldsymbol{\phi}_a) \qquad \boldsymbol{a}_{y2} = \frac{\boldsymbol{y}_a - \boldsymbol{y}_P}{\boldsymbol{r}_{a,P}} = \cos(\boldsymbol{\phi}_a)$$

With $\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \boldsymbol{\phi}_a$ are respectively angle between the lines connecting transmitters to P and Oy,

$$\boldsymbol{H} = \begin{bmatrix} \sin\boldsymbol{\phi}_1 & \cos\boldsymbol{\phi}_1 & 1 \\ \sin\boldsymbol{\phi}_2 & \cos\boldsymbol{\phi}_2 & 1 \\ \sin\boldsymbol{\phi}_a & \cos\boldsymbol{\phi}_a & 1 \end{bmatrix}$$

$$G = (H^T H)^{-1}$$

- **If A is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_2 = 0, \phi_a = 0$

$$\text{So } \boldsymbol{H} = \begin{bmatrix} 0.7071 & 0.7071 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

In this case ($H^T H$) is singular so we cannot get the inversion matrix of it, so GDOP has the infinite value and point A cannot be chosen to put auxiliary transmitter.

- **If B is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \frac{\pi}{6}$

$$\text{So } \boldsymbol{H} = \begin{bmatrix} 0.7071 & 0.7071 & 1 \\ 0 & 1 & 1 \\ 0.5 & 0.8660 & 1 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} 42.2633 & 114.8698 & -117.9434 \\ 114.8695 & 296.6541 & -300.6600 \\ -117.9434 & -300.6600 & 305.6659 \end{bmatrix}$$

So $\boxed{GDOP_B = 25.5065}$

- **If C is the position that is used to put the auxiliary transmiter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \frac{\pi}{2}$

$$\boldsymbol{G} = \begin{bmatrix} 9.2432 & -8.2432 & 9.9504 \\ -8.2432 & 9.2432 & -9.9504 \\ 9.9504 & -9.9504 & 11.6575 \end{bmatrix}$$

we have $\boxed{GDOP=5.4903}$

- **If D is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \pi$

$$\boldsymbol{G} = \begin{bmatrix} 3.5001 & -0.5 & -0.7071 \\ -0.5 & 0.5 & 0 \\ -0.7071 & 0 & 0.5 \end{bmatrix}$$

we have $\boxed{GDOP = 2.1213}$

In this case (non-synchronization between user's clock and transmitters' clocks), because the GDOP of D is minimum value in all GDOP values so at D the affection of geometry in error is minimum, so D is the best place to put auxiliary transmitter.

In case "sync" in time

$$\boldsymbol{H} = \begin{bmatrix} \sin\phi_1 & \cos\phi_1 \\ \sin\phi_2 & \cos\phi_2 \\ \sin\phi_a & \cos\phi_a \end{bmatrix}$$

- **If A is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_2 = 0, \phi_a = 0$ **so:**

$\boldsymbol{G} = \begin{bmatrix} 2.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$ so PDOP = 1.7321 (in this case, because of synchronization in time so we only have PDOP)

- **If B is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \frac{\pi}{6}$

$\boldsymbol{G} = \begin{bmatrix} 2.754 & -1.142 \\ -1.142 & 0.9180 \end{bmatrix}$ so PDOP = 1.9162

- **If C is the position that is used to put the auxiliary transmiter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \frac{\pi}{2}$

$\boldsymbol{G} = \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.75 \end{bmatrix}$ so PDOP = 1.2247

- **If D is the position that is used to put the auxiliary transmitter, we have:**

$\phi_1 = \frac{\pi}{4}, \phi_1 = 0, \phi_a = \pi$

$\boldsymbol{G} = \begin{bmatrix} 2.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$ so PDOP = 1.7321

<u>So in this case (synchronization among all clocks), C with the minimum PDOP is the best place to put auxiliary transmitter.</u>

Exercise HW1-5

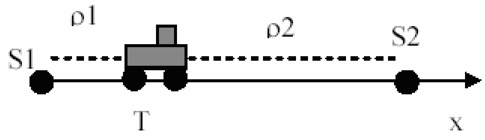

Figure 15

ρ1 [Km]	ρ2 [Km]
4.9813	6.9567
5.0726	6.8334
4.9412	7.0125
5.2183	7.0288
4.9864	6.8854
5.0114	7.1191
5.1067	7.1189
5.0059	6.9962
4.9904	7.0327

Table 1

This is one dimension positioning system, and call S1(0)is root of axis x, we have :

$$\boxed{\mathbf{H} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}}$$

$$\mathbf{G} = (\mathbf{H}^T\mathbf{H})^{-1} = \begin{bmatrix} 0.5000 & 0 \\ 0 & 0.5000 \end{bmatrix}$$

$$GDOP = \sqrt{g_{11} + g_{22}} = 1$$

$$TDOP = \sqrt{g_{22}} = \sqrt{0.5} = 0.707$$

$$XDOP = \sqrt{g_{11}} = \sqrt{0.5} = 0.707$$

We have S1(0), S2(10), so if call T(x_T) is the position of train, T can be calculated through these equations :

$$\begin{cases} \rho_1 = \boldsymbol{x_T} - \boldsymbol{x_{S1}} + \boldsymbol{c} \times \partial \boldsymbol{t} \\ \rho_2 = \boldsymbol{x_{S2}} - \boldsymbol{x_T} + \boldsymbol{c} \times \partial \boldsymbol{t} \end{cases} = \begin{cases} \rho_1 = \boldsymbol{x_T} + \boldsymbol{c} \times \partial \boldsymbol{t} \\ \rho_2 = 10 - \boldsymbol{x_T} + \boldsymbol{c} \times \partial \boldsymbol{t} \end{cases}$$ so with each row of table we have one estimated value of T and time bias:

ρ1 (Km)	ρ2 (Km)	x (Km)	∂t (µs)
4.9813	**6.9567**	**4.0123**	**3.23**
5.0726	**6.8334**	**4.1196**	**3.1767**
4.9412	**7.0125**	**3.9644**	**3.2562**
5.2183	**7.0288**	**4.0947**	**3.7452**
4.9864	**6.8854**	**4.0505**	**3.1197**
5.0144	**7.1191**	**3.9476**	**3.5558**
5.1067	**7.1189**	**3.9939**	**3.7093**
5.0059	**6.9962**	**4.0048**	**3.3368**
4.9904	**7.0327**	**3.9789**	**3.3718**
		E(x)=4.0185	**E(∂t)=3.3891**
		σ_x**=0.0568** Công thức tính phương sai:= 1/N * ∑(x - E(x))²	

So the estimated position of train is 4.0185±0.0568(km) far from S1.
And the average bias of the train's clock is 3.3891(µs)

Exercise HW1-6

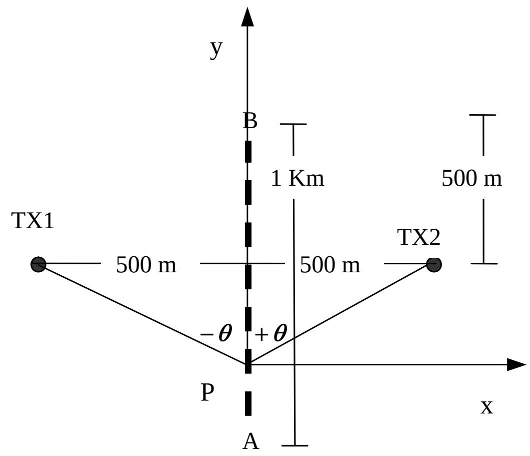

Figure 15.

Suppose the coordinate system like the Fig.15, and call the point with maximum positioning error is point P.

We have the geometric matrix at P:

$$\boldsymbol{H} = \begin{bmatrix} -\sin(\phi) & \cos(\phi) \\ \sin(\phi) & \cos\phi) \end{bmatrix}$$

And then we have $G=(H^TH)^{-1}$

$$\boldsymbol{H} = \begin{bmatrix} \frac{1}{2\sin^2(\phi)} & 0 \\ 0 & \frac{1}{2\cos^2(\phi)} \end{bmatrix}$$

So the estimation of error is:

$$\sigma_x^2 = \sigma_{UERE}^2 \times \frac{1}{2\sin^2(\phi)}$$

Following Fig.15, we have $\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$. In this range, σ_x^2 goes to maximum when $\phi = \frac{\pi}{4}$. In addition, because of pseudorange errors is modeled as a random variable with uniform distribution in the range (-10,10). So we have:

$$\sigma_{UERE}^2 = \frac{1}{12}(a-b)^2 = \frac{400}{12}$$

$$\sigma_x^2 = \sigma_{UERE}^2 \times 1 = \frac{400}{12}$$

Because $\phi = \frac{\pi}{4}$ so there are two positions having the maximum error in positioning, these are A and B.

Exercise HW 1-7

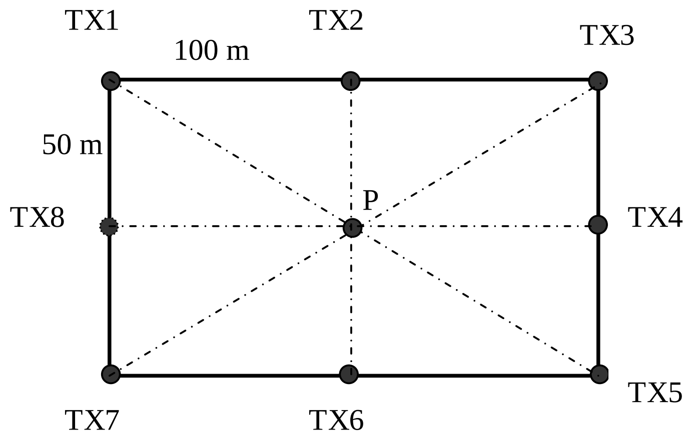

Figure 16.

Assuming the position of point P is (0, 0):

$$a_{x1} = \frac{-100}{\sqrt{100^2+50^2}} = -\frac{2}{\sqrt{5}} \quad a_{y1} = \frac{1}{\sqrt{5}} \qquad a_{x2} = 0 \quad a_{y2} = 1 \qquad a_{x3} = \frac{2}{\sqrt{5}} \qquad a_{y2} = \frac{1}{\sqrt{5}}$$

So, the geomatric matrix H is:

$$\mathbf{H} = \begin{bmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ 0 & 1 \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix},$$

$$\mathbf{G} = (\mathbf{H}^T\mathbf{H})^{-1} = \begin{bmatrix} 0.6250 & 0 \\ 0 & 0.7143 \end{bmatrix}, GDOP = 1.1573$$

$$\Delta\rho = \rho - \hat{\rho} = \begin{bmatrix} 112.5 - \sqrt{100^2+50^2} \\ 52-50 \\ 105 - \sqrt{100^2+50^2} \end{bmatrix} = \begin{bmatrix} 0.70 \\ 2.00 \\ -6.80 \end{bmatrix}$$

$$\Delta x = (H^T H)^{-1} H^T \Delta\rho = \begin{bmatrix} 0.6250 & 0 \\ 0 & 0.7143 \end{bmatrix} \begin{bmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ 0 & 1 \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}^T \begin{bmatrix} 0.70 \\ 2.00 \\ -6.80 \end{bmatrix} = \begin{bmatrix} -4.1926 \\ -0.5200 \end{bmatrix}$$

So the position of user is $\boxed{(-4.1926,-0.5200)}$.

The vertical and horizontal uncertainties are $\sigma_x^2 = 0.625 \times \sigma_{UERE}^2$ and $\sigma_y^2 = 0.7143 \times \sigma_{UERE}^2$

Exercise HW 1-8

From two following equations:

$$\rho_{f_1} = \rho^* + \frac{40.3 \times TEC}{f_1^2} \tag{1}$$

$$\rho_{f_2} = \rho^* + \frac{40.3 \times TEC}{f_2^2} \tag{2}$$

We have: $$\boxed{\rho^* = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2} \tag{3}$$

In case the pseudorange measurements ρ_{f1}, ρ_{f2} are affected by errors (not given by the propagation in the ionosphere) the resulting ρ^* is affected by an increased error contribution, because:

Since these errors are <u>Gaussian, uncorrelated, zero mean and unitary variance</u> so if we call variances of noise of ρ^*, ρ_{f1}, ρ_{f2} are $\sigma_*^2, \sigma_1^2 = \sigma_2^2 = \sigma^2$ respectively, we have:

$$\sigma_*^2 = E(\rho^{*2}) = E\left(\left(\frac{f_1^2}{f_1^2 - f_2^2}\right)^2 \rho_1^2 + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)^2 \rho_2^2 - 2\frac{f_1^2}{f_1^2 - f_2^2}\frac{f_2^2}{f_1^2 - f_2^2}\rho_1\rho_2\right)$$

$$= \left(\frac{f_1^2}{f_1^2 - f_2^2}\right)^2 E(\rho_1^2) + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)^2 E(\rho_2^2) - 2\frac{f_1^2}{f_1^2 - f_2^2}\frac{f_2^2}{f_1^2 - f_2^2}E(\rho_1\rho_2)$$

$$\sigma_*^2 = \left(\left(\frac{f_1^2}{f_1^2 - f_2^2}\right)^2 + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)^2\right)\sigma^2 \tag{4}$$

With f_1 = L1 = 1575.420MHz f_2= L2 = 1227.60MHz

$$\sigma_*^2 = (1.546^2 + 2.546^2)\sigma^2$$

$$\boxed{\sigma_* \approx 3\sigma} \tag{5}$$

From (4,5) we conclude that the noise in ionosphere-free pseudorange are increased (larger than in ρ_{f1}, ρ_{f2}).

- ***<u>Choose the best couple of frequencies in Galileo system to correcting inospheric error.</u>***

With this problem, we divide into two cases:

- **With applications that need more accuracy of measurement:** in this case, we must choose a couple of frequencies that make ρ^* less noisy. From (4), we have:

$$\sigma_*^2 = \left(\left(\frac{f_1^2}{f_1^2 - f_2^2}\right)^2 + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)^2\right)\sigma^2$$

$$\sigma_* \approx M\sigma$$

f_1	f_2	M
1176.450	1207.140	27.4730
1176.450	1278.750	8.5249
1176.450	1575.420	2.5883
1207.140	1278.750	12.3039
1207.140	1575.420	2.8086
1278.750	1575.420	3.5101

From this table, we can conclude that, the couple of frequencies:

$\boxed{\text{E5a = 1176.450 MHz, L1 = 1575.420MHz}}$ is the best.

- **With applications that need realtime processing:** Because I_{L1},I_{L2} relate to delay caused by ionosphere, so the less these values are, the less these delay are. Hence, we can obtain the position in almost real time. Following this conclusion, in Galileo we should choose two maximum frequencies E6 and L1 in order to correct the ionospheric error because with following equation:

$$I_i = \frac{40.3 \times TEC}{f_i^2}$$

So the bigger value of frequency, the less delay we have:

$$\boxed{\text{E6=1278,750 MHz} \quad \text{L1=1575,420 MHz}}$$