GPS and Galileo receivers

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Correlation functions: Các hàm tương quan

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ACF and CCF

- ❑ A software receiver is mainly based on correlation functions:
  - Auto-Correlation Function (ACF): Hàm tự tương quan
  - Cross-Correlation Function (CCF): Hàm tương quan chéo
- ❑ Given *x[n]* and *y[n]*, where $0 \le n \le L-1$, we define

ACF

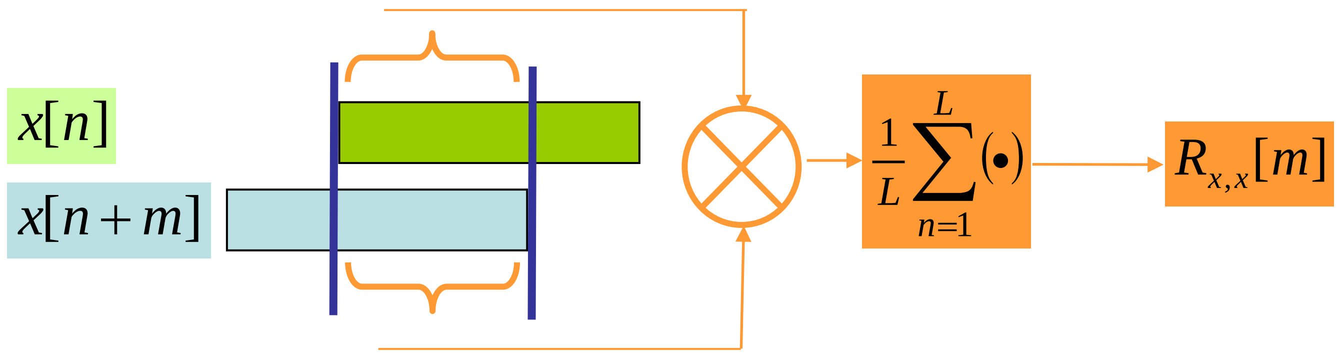
$$R_{x,x}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]x[n+m]$$

CCF

$$R_{x,y}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]y[n+m]$$

Linear Auto-Correlation

$$R_{x,x}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]x[n+m]$$



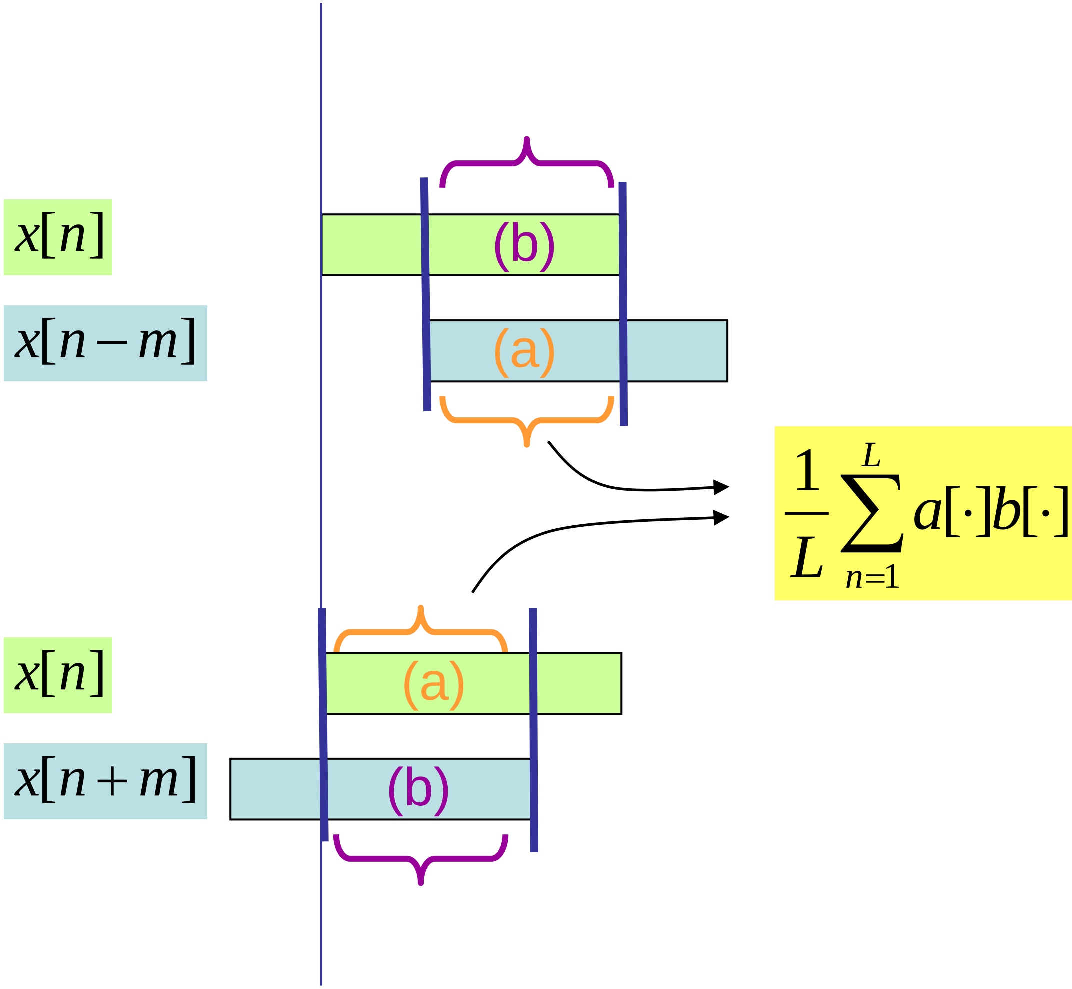
$$\boxed{R_{x,x}[m] \neq 0 \quad \text{for } |m| \leq 2L-1}$$

If *x[n]* is real → $\boxed{R_{x,x}[-m] = R_{x,x}[m]}$

Even ACF (x[n] real)

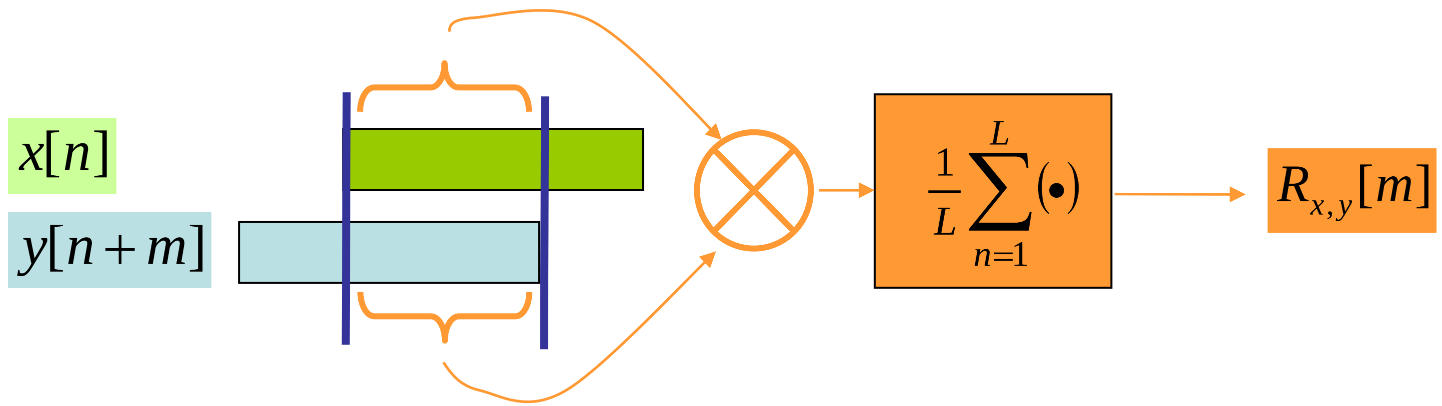
Proof

$$\boxed{R_{x,x}[-m] = R_{x,x}[m]}$$



Linear CCF

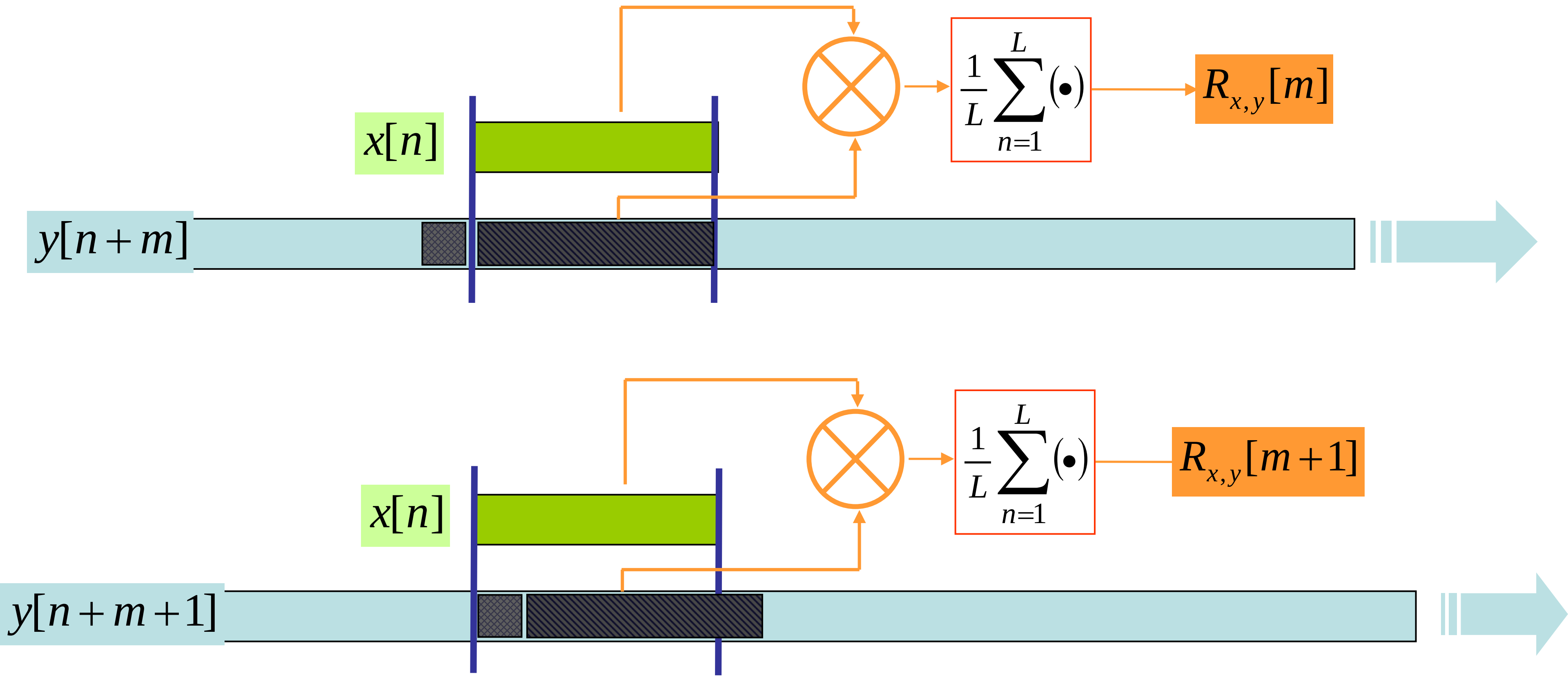
$$R_{x,y}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]y[n+m]$$



$$\boxed{R_{x,y}[m] \neq 0 \quad \text{for } |m| \leq 2L-1}$$

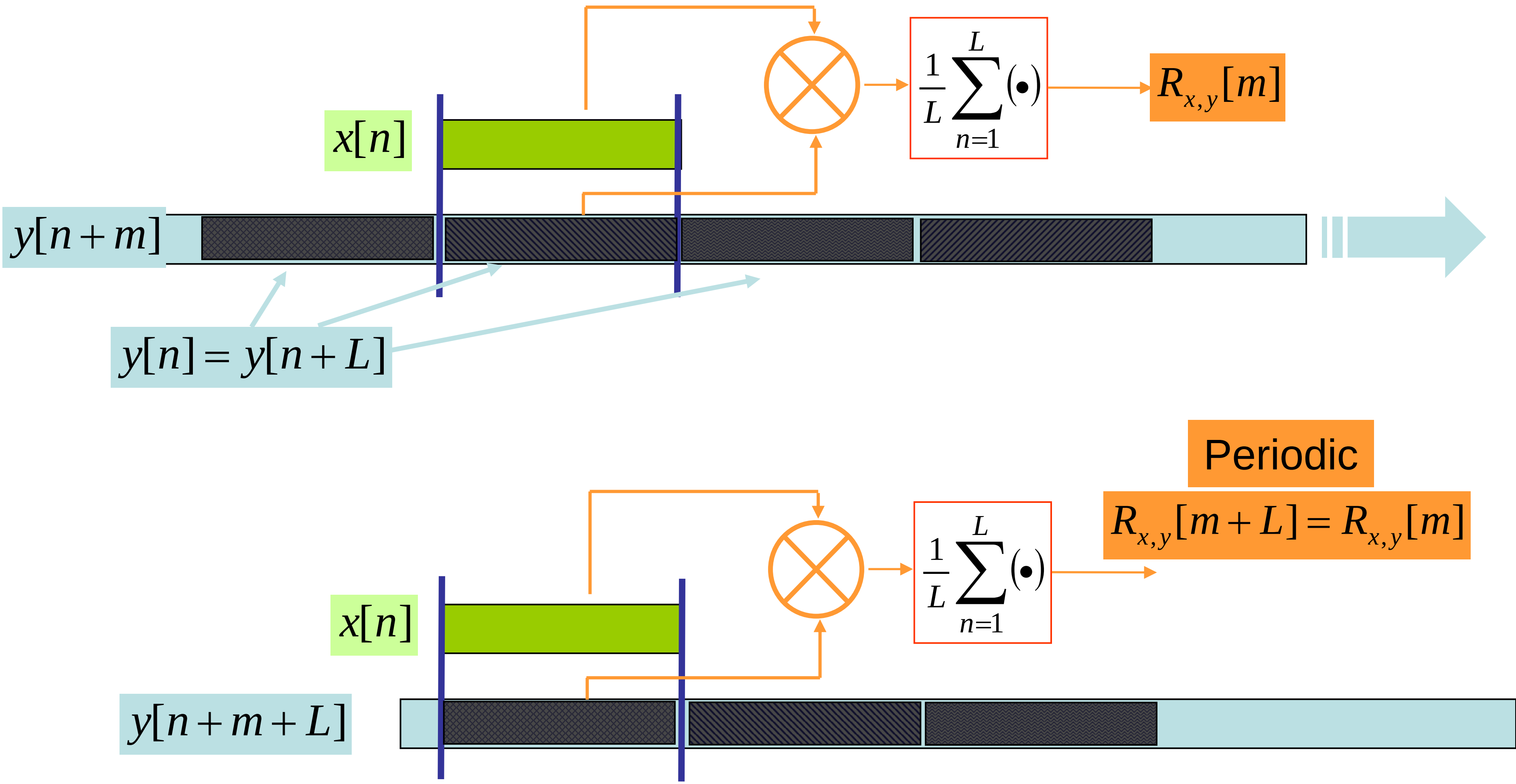
$$\boxed{R_{x,y}[-m] \neq R_{x,y}[m]}$$

Linear CCF (short & long)



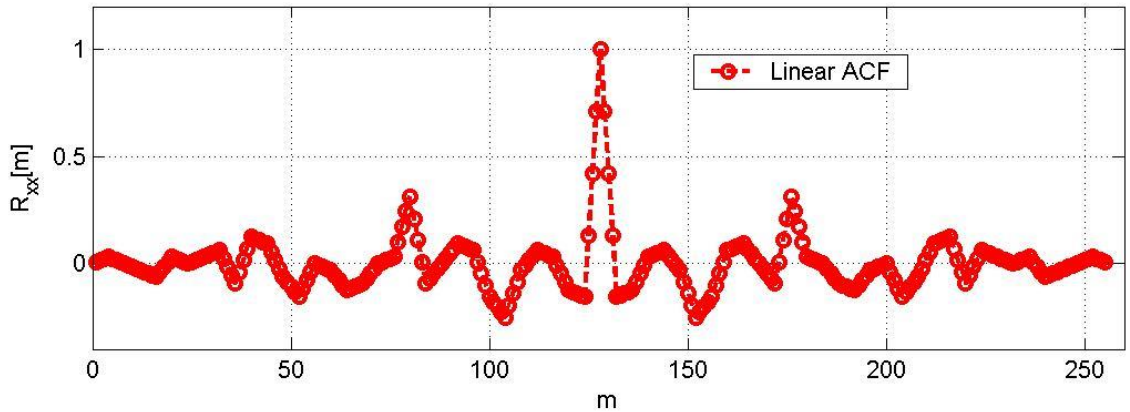
The CCF is evaluated sample by sample

Linear CCF (short & periodic)

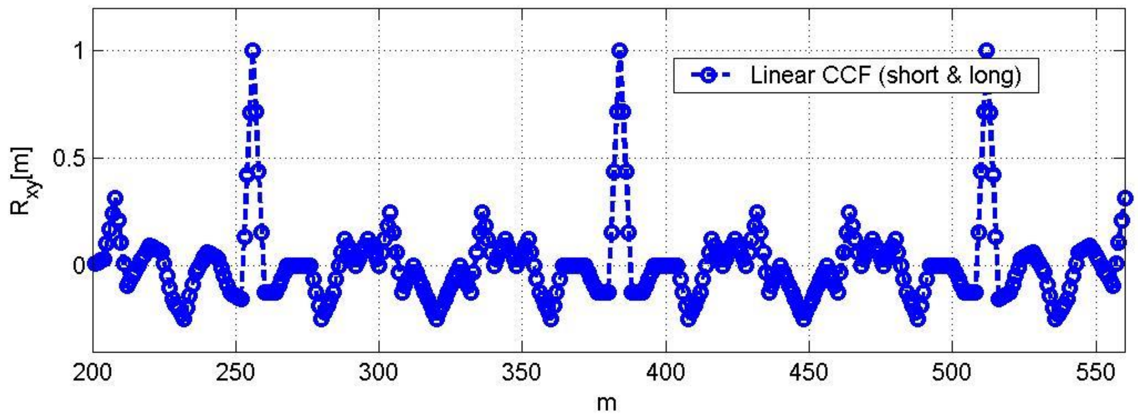


Example of linear correlations

- ❑ ACF of a short PN sequence $x[n]$



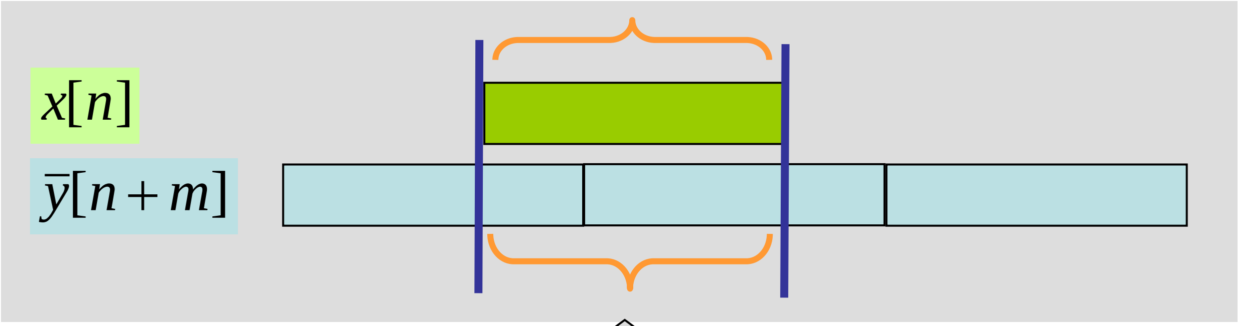
- ❑ CCF of $x[n]$ with a periodic version of $x[n]$

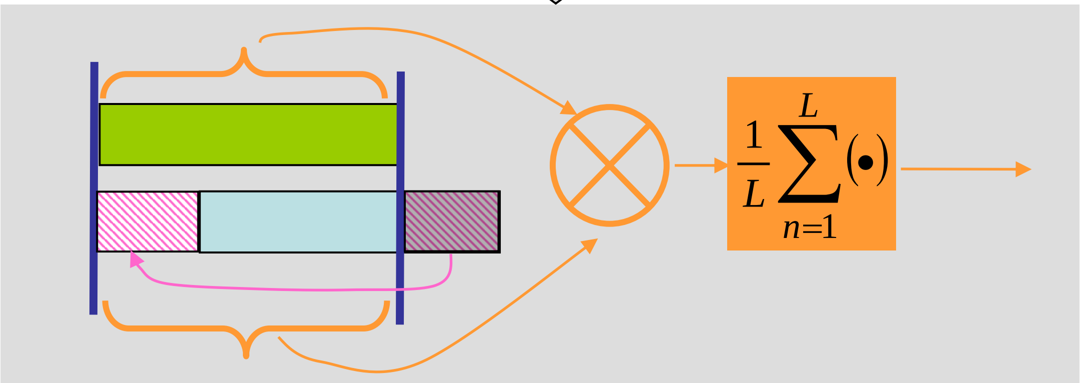


Circular correlation

$$\overline{R}_{x,y}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]\bar{y}[n+m]$$

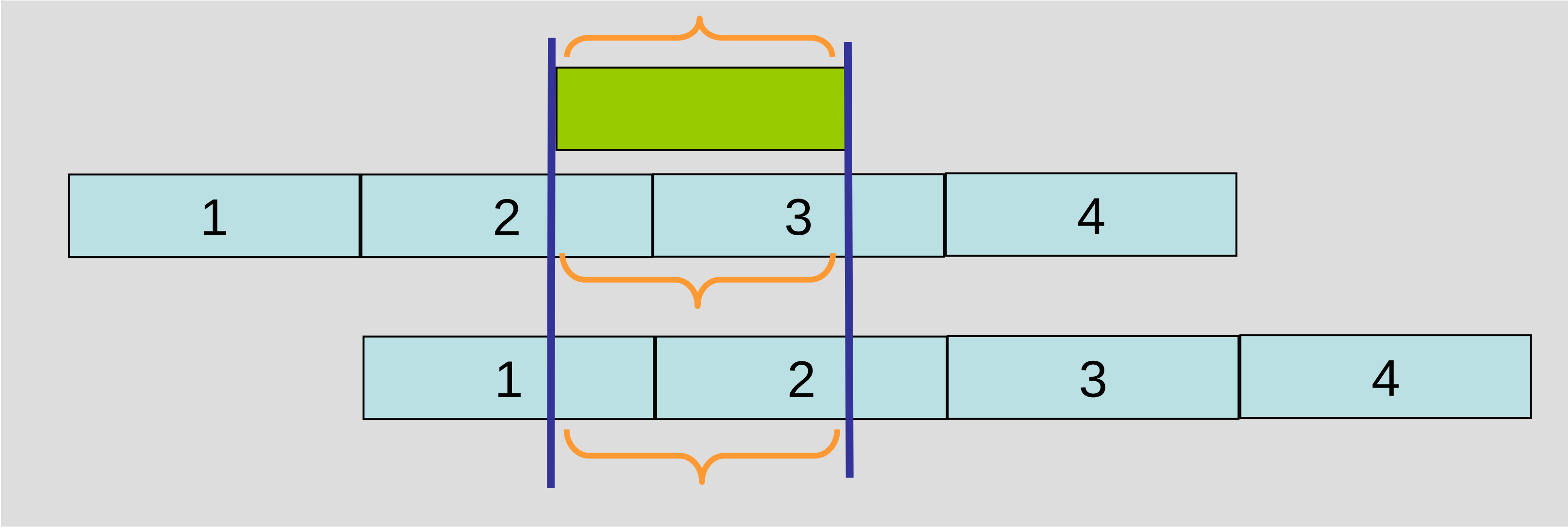
where $\bar{\boldsymbol{y}}[\boldsymbol{n}]$ is a periodic version of ***y[n]*** with period ***L***





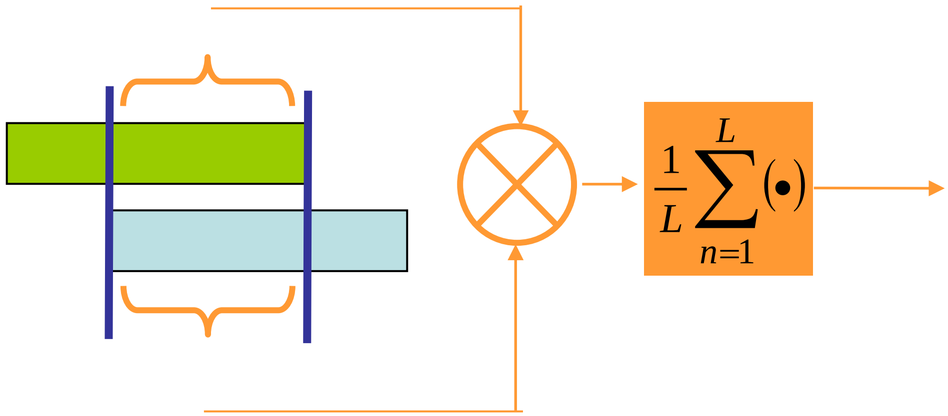
Periodicity of the Circular correlation

$$\overline{R}_{x,y}[m] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]\overline{y}[n+m]$$



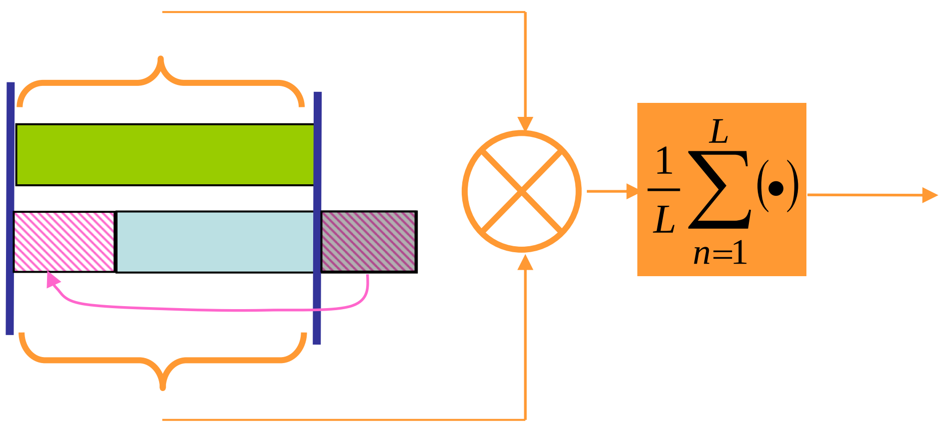
ACF and CCF are periodic of period *L*

Linear & Circular correlation (tính thông qua FFT – thuật toán nhanh phù hợp mềm hóa bộ thu



$$\overline{R}_{x,y}[0] = R_{x,y}[0]$$

$$\overline{R}_{x,y}[m] \cong R_{x,y}[m]$$
$$\text{for } |m| << L$$



Circular correlation and DFT

- Given the DFT and IDFT definition of a generic *x[n]*

$$X[k] = \mathrm{DFT}\{x[n]\} = \sum_{n=0}^{L-1} x[n]\,\mathrm{e}^{-j2\pi nk/L} \qquad 0 \le k < L$$

$$x[n] = \mathrm{IDFT}\{X[k]\} = \frac{1}{L}\sum_{k=0}^{L-1} X[k]\,\mathrm{e}^{j2\pi nk/L} \qquad 0 \le n < L$$

- It is possible to show that

$$\boxed{\mathrm{IDFT}\{X[k]Y^*[k]\} = \overline{R}_{x,y}[m]}$$

Circular correlation and DFT (proof)

- Observe that

$$\sum_{n=0}^{L-1} \bar{x}[n]\,\mathrm{e}^{-j2\pi nk/L} = \bar{X}[k]$$

$-\infty \le k < +\infty$

$$\frac{1}{L}\sum_{k=0}^{L-1} \bar{X}[k]\,\mathrm{e}^{j2\pi nk/L} = \bar{x}[n]$$

$-\infty \le n < +\infty$

- Proof

$$X[k]Y^*[k] = \sum_{n=0}^{L-1} x[n]\,\mathrm{e}^{-j2\pi nk/L} \sum_{n=0}^{L-1} y[n]\,\mathrm{e}^{+j2\pi nk/L}$$

$$= \sum_{l=0}^{L-1} \sum_{n=0}^{L-1} x[n]\, y[l]\mathrm{e}^{-j2\pi(n-l)k/L}$$

$$\sum_{k=0}^{L-1} \bar{X}[k]\bar{Y}^*[k]\mathrm{e}^{+j2\pi mk/L} = \frac{1}{L^2}\sum_{l=0}^{L-1} \bar{y}[l] \sum_{n=0}^{L-1} \bar{x}[n] \sum_{k=0}^{L-1} \mathrm{e}^{-j2\pi(n-l-m)k/L}$$

$$= \frac{1}{L^2}\sum_{l=0}^{L-1} \bar{y}[l] \sum_{n=0}^{L-1} \bar{x}[l+m]\,L$$

$$= \sum_{l=0}^{L-1} \bar{y}[l]\bar{x}[l+m]$$

Linear & Circular ACF (Example)

ACF of a short PN sequence