HA NOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

Computer Vision

Chapter 6: Motion detection and tracking

Chapter 6 - Content

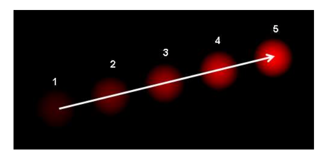
- Introduction
- Approaches of motion detection
- Moving object tracking

Introduction

- Motion detection: Action of sensing physical movement in a give area
- Motion can be detected by measuring change in speed or vector of an object
- Goals of motion detection and tracking
  - Moving objects detection: Optical Flow.
  - Motion detection: Change detection (Background subtraction)
  - Tracking: Computing trajectories of moving objects
- Applications of motion detection
  - Indoor/outdoor security
  - Real time crime detection
  - Traffic monitoring
  - Many intelligent video analysis systems are based on motion detection

Approaches to Motion Detection

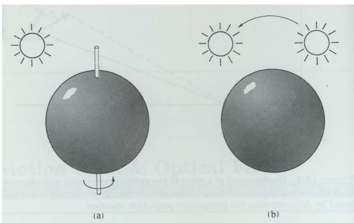
- Optical Flow
  - Compute motion within region or the frame as a whole



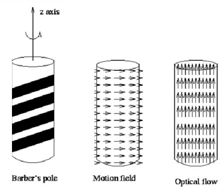
- Change detection
  - Detect objects within a scene (Background subtraction)
  - Track object across a number of frames

Optical flow vs. motion field

- Optical flow does not always correspond to motion field



Barber pole illusion



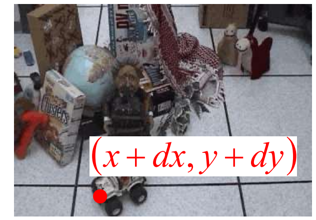
- Optical flow is an approximation of the motion field
- The error is small at points with high spatial gradient under some simplifying assumptions

Estimating optical flow

- Assume the image intensity *I* is constant

Time = *t*

Time = *t*+*dt*

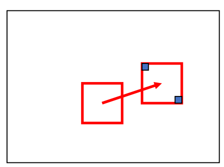


$I_0(x,y,t) \approx I_1(x+\delta x, y+\delta y, t+\delta t)$

Formulation: Optical flow constraint

$$I(x,y,t) \approx I(x+\delta x, y+\delta y, t+\delta t)$$
$$I(x,y,t) \approx I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

$$\frac{\partial I}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial I}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial I}{\partial t} = 0, \text{ and let } \delta t \to 0$$
$$I_x u + I_y v + I_t = 0$$
$$I_x u + I_y v = -I_t,$$
$$\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t, \nabla I^T \mathbf{u} = \mathbf{b}; A\mathbf{u} = \mathbf{b}$$
$$E(u,v) = (I_x u + I_y v + I_t)^2$$



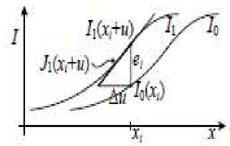
Lucas-Kanade algorithm

$$E(u,v) = \sum_{x,y\in\Omega} (I_x(x,y)u + I_y(x,y)v + I_t)^2$$

$$\|A\mathbf{u} - \mathbf{b}\|^2$$
$$\mathbf{u} = (A^T A)^{-1} A^T \mathbf{b}$$
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$
$$\left(\sum \nabla I \cdot \nabla I^T\right) \vec{u} = -\sum \nabla I \cdot I_t$$

Matrix form

$$E(\mathbf{u}+\Delta\mathbf{u}) = \sum_i [I_1(\mathbf{x}_i+\mathbf{u}+\Delta\mathbf{u}) - I_o(\mathbf{x}_i)]^2$$
$$\approx \sum_i [I_1(\mathbf{x}_i+\mathbf{u}) + \mathbf{J}_1(\mathbf{x}_i+\mathbf{u})\Delta\mathbf{u} - I_o(\mathbf{x}_i)]^2$$
$$= \sum_i [\mathbf{J}_1(\mathbf{x}_i+\mathbf{u})\Delta\mathbf{u} + e_i]^2$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \nabla I_1(\mathbf{x}_i+\mathbf{u}) = \left(\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y}\right)(\mathbf{x}_i+\mathbf{u})$$
$$e_i = I_1(\mathbf{x}_i+\mathbf{u}) - I_o(\mathbf{x}_i)$$



$$y = f(\mathbf{x}+\Delta\mathbf{x}) \approx f(\mathbf{x}) + J(\mathbf{x})\Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^{\mathrm{T}} H(\mathbf{x})\Delta\mathbf{x}$$

Matrix form

$$A\Delta\mathbf{u} = \mathbf{b}$$
$$A = \sum_i \mathbf{J}_1^T(\mathbf{x}_i+\mathbf{u})\mathbf{J}_1(\mathbf{x}_i+\mathbf{u})$$
$$\mathbf{b} = -\sum_i e_i \mathbf{J}_1(x_i+u)$$
$$A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}, \quad \mathbf{b} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \nabla I_1(\mathbf{x}_i+\mathbf{u}) = \left(\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y}\right)(\mathbf{x}_i+\mathbf{u})$$
$$e_i = I_1(\mathbf{x}_i+\mathbf{u}) - I_o(\mathbf{x}_i)$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \mathbf{J}_0(\mathbf{x}_i)$$

Computing gradients in X-Y-T

$$I_x = \frac{1}{4\delta x}[(I_{i+1,j,k} + I_{i+1,j,k+1} + I_{i+1,j+1,k} + I_{i+1,j+1,k+1}) - (I_{i,j,k} + I_{i,j,k+1} + I_{i,j+1,k} + I_{i,j+1,k+1})]$$

likewise for I_y and I_t

The aperture problem

Let $A = \sum \nabla I \cdot \nabla I^T$, and $\mathbf{b} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$

- Algorithm: At each pixel compute u by solving $A\mathbf{u} = \mathbf{b}$
- A is singular if all gradient vectors point in the same direction
  - e.g., along an edge
  - of course, trivially singular if the summation is over a single pixel or there is no texture
  - i.e., only *normal flow* is available (aperture problem)
- Corners and textured areas are OK

Error functions

- Robust error function

$$E(\mathbf{u}) = \sum_i \rho(I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x})), \quad \rho(x) = \frac{x^2}{1 + x^2/a^2}$$

- Spatially varying weights

$$E(\mathbf{u}) = \sum_i w_0(\mathbf{x}_i) w_1(\mathbf{x}_i + \mathbf{u}) [I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x}_i)]^2$$

- Bias and gain: images taken with different exposure

$$I(\mathbf{x} + \mathbf{u}) = (1 + \alpha) I(\mathbf{x}) + \beta, \alpha \text{ is the gain and } \beta \text{ is the bias}$$

$$E(\mathbf{u}) = \sum_i [I(\mathbf{x}_i + \mathbf{u}) - (1 + \alpha) I(\mathbf{x}_i) - \beta]^2$$

- Correlation (and normalized cross correlation)

Horn-Schunck algorithm

- Global method with smoothness constraint to solve aperture problem
- Minimize a global energy functional with calculus of variations

$$E = \int \left(\left(I_x u + I_y v + I_t \right)^2 + \alpha^2 (|\nabla u|^2 + |\nabla v|^2) \right) dxdy$$

$$\frac{\partial L}{\partial u} - \frac{\partial}{\partial x} \frac{\partial L}{\partial u_x} - \frac{\partial}{\partial y} \frac{\partial L}{\partial u_y} = 0$$

$$\frac{\partial L}{\partial v} - \frac{\partial}{\partial x} \frac{\partial L}{\partial v_x} - \frac{\partial}{\partial y} \frac{\partial L}{\partial v_y} = 0$$

where L is the integrand of the energy function

Horn-Schunck algorithm

$$I_x (I_x u + I_y v + I_t) - \alpha^2 \Delta u = 0$$

$$I_y (I_x u + I_y v + I_t) - \alpha^2 \Delta v = 0$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator

$$\Delta u(x, y) = \bar{u}(x, y) - u(x, y)$$

$$(I_x^2 + \alpha^2) u + I_x I_y v = \alpha^2 \bar{u} - I_x I_t$$

$$I_x I_y u + (I_y^2 + \alpha^2) v = \alpha^2 \bar{v} - I_y I_t$$

Horn-Schunck algorithm

- Iterative scheme

$$u^{k+1} = \bar{u}^k - \frac{I_x (I_x \bar{u}^k + I_y \bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

$$v^{k+1} = \bar{v}^k - \frac{I_y (I_x \bar{u}^k + I_y \bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

- Yields high density flow
- Fill in missing information in the homogenous regions
- More sensitive to noise than local methods

Motion detection using Background subtraction

- Uses a reference background image for comparison purposes.
- Current image (containing target object) is compared to reference image pixel by pixel.
- Places where there are differences are detected and classified as moving objects.

Motivation: simple difference of two images shows moving objects

a. Original scene

b. Same scene later

Subtraction of scene a from scene b

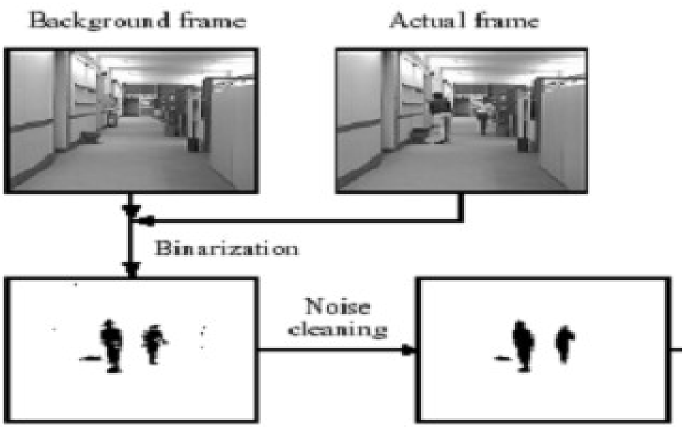
Subtracted image with threshold of 100

Approaches to Background Modeling

- Background Subtraction
- Statistical Methods
  (e.g., Gaussian Mixture Model, Stauffer and Grimson 2000)

Background Subtraction:

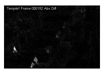
1. Construct a background image B as average of few images
2. For each actual frame I, classify individual pixels as foreground if |B-I| > T (threshold)
3. Clean noisy pixels

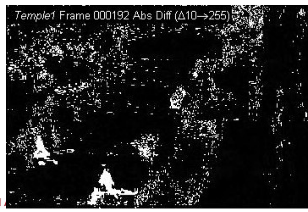


Background Subtraction

Background Image

Current Image





Static scene Object Detection

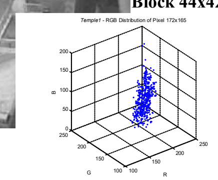
- Model the background and subtract to obtain object mask
- Filter to remove noise
- Group adjacent pixels to obtain objects
- Track objects between frames to develop trajectories

Statistical Methods

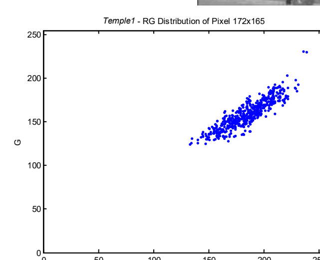
- Pixel statistics: average and standard deviation of color and gray level values
- Gaussian Mixture Model
  – Model the color values of a particular pixel as a mixture of Gaussians
  – Multiple adaptive Gaussians are necessary to cope with acquisition noise, lighting changes, etc.
  – Pixel values that do not fit the background distributions (Mahalanobis distance) are considered foreground

Gaussian Mixture Model

Block 44x42 Pixel 172x165



R-G-B Distribution



R-G Distribution

Detection of Moving Objects Based on local variation

For each block location (*x,y*) in the video plane

- Consider texture vectors in a symmetric window *[t-W, t+W]* at time *t*
- Compute the covariance matrix
- ***Motion measure*** is defined as the largest eigenvalue of the covariance matrix

Dynamic Distribution Learning and Outlier Detection

(1) $$\frac{f(t)-mean(t-1)}{std(t-1)} > C_1$$ Detect Outlier

(2) $$\frac{f(t)-mean(t-1)}{std(t-1)} < C_2$$ Switch to a nominal state

(3) $$mean(t) = u \cdot mean(t-1) + (1-u) \cdot f(t)$$

(4) $$std(t) = \sqrt{\sigma^2(t)}$$

Update the estimates of mean and standard deviation only when the outliers are not detected

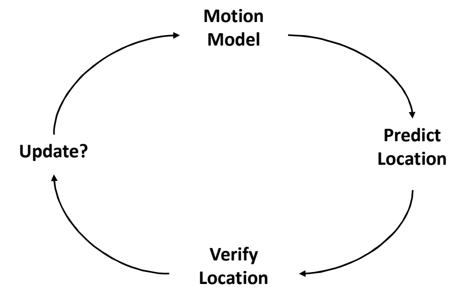
(5) $$\sigma^2(t) = u \cdot \sigma^2(t-1) + (1-u) \cdot (f(t) - mean(t-1))^2$$

Moving object tracking

- Model of object motion
- Kalman filter
- Mean Shift

Model of object motion

- Mathematical model of objects' motions:
  - position, velocity (speed, direction), acceleration
- Can predict objects' positions



Simple motion model

- Newton's laws

$$s(t) = s_0 + ut + \frac{1}{2} a t^2$$

- s = position
- u = velocity
- a = acceleration
  - all vector quantities
  - measured in image co-ordinates

Prediction

- Can predict position at time t knowing
  - Position
  - Velocity
  - Acceleration
- At t=0

Uncertainty

- If some error in a - Δa or u - Δu
- Then error in predicted position - Δs

$$\Delta s(t) = s_0 + \Delta ut + \frac{1}{2} \Delta a\, t^2$$

Verification

- Is the object at the predicted location?
  - Matching
    - How to decide if object is found
  - Search area
    - Where to look for object

Object Matching

- Compare
  - A small bitmap derived from the object vs.
  - Small regions of the image
- Matching?
  - Measure differences

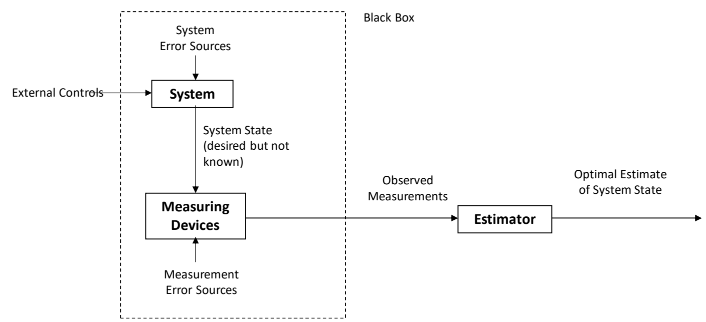
Search Area

- Uncertainty in knowledge of model parameters
  - Limited accuracy of measurement
  - Values might change between measurements
- Define an area in which object could be
  - Centred on predicted location, s ± Δs

Update the Model

- Is the object at the predicted location?
- Yes
  - No change to model
- No
  - Model needs updating
  - Kalman filter is a solution
    - Mathematically rigorous methods of using uncertain measurements to update a model

Kalman Filter



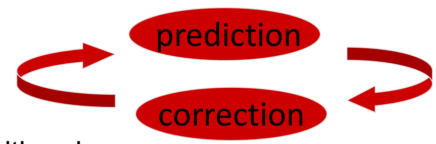
- Problem formulation
  - System state cannot be measured directly
  - Need to estimate “optimally” from measurementsh

Kalman Filter

- Recursive data processing algorithm
- Generates optimal estimate of desired quantities given the set of measurements
- Optimal?
  - For linear system and white Gaussian errors, Kalman filter is "best" estimate based on all previous measurements
  - For non-linear system optimality is 'qualified'
- Recursive?
  - Doesn't need to store all previous measurements and reprocess all data each time step

Kalman filter

- Matrix description of system state, model and measurement
- Progressive method



- Proper dealing with noise

Kalman filter

- Advantages of using KF in particle tracking
- Progressive method
  - No large matrices has to be inverted
- Proper dealing with system noise
- Track finding and track fitting
- Detection of outliers
- Merging track from different segments

Kalman filter assumptions

- Linear system
  - System parameters are linear function of parameters at some previous time
  - Measurements are linear function of parameters
- White Gaussian noise
  - White: uncorrelated in time
  - Gaussian: noise amplitude

$\Rightarrow$ **KF is the optimal filter**

Kalman filter

- Relates
  - Measurements **y**[k]
    - e.g. positions
  - System state **x**[k]
    - Position, velocity of object, etc
  - Observation matrix **H**[k]
    - Relates system state to measurements
  - Evolution matrix **A**[k]
    - Relates state of system between epochs
  - Measurement noise **n**[k]
  - Process noise **v**[k]

Example

- Tracking two corners of a minimum bounding box
- Matching using colour
- Image differencing to locate target

Condensation Tracking

- So far considered single motions
- What if movements change?
  - Bouncing ball
  - Human movements
- Use multiple models
  - plus a model selection mechanism

Selection and Tracking

- Occur simultaneously
- Maintain
  - A distribution of likely object positions plus weights
- Predict
  - Select N samples, predict locations
- Verify
  - Match predicted locations against image
  - Update distributions

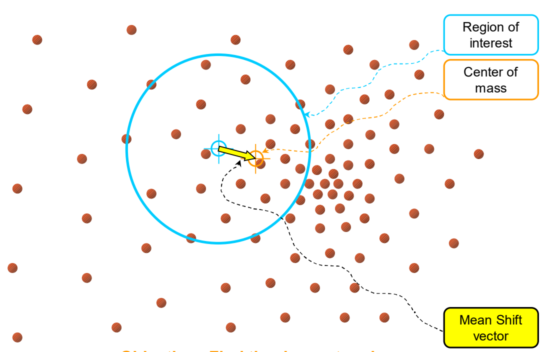
Mean-shift algorithm

- The mean-shift algorithm is an efficient approach to tracking objects whose appearance is defined by histograms. (not limited to only color)
- Motivation – to track non-rigid objects, (like a walking person), it is hard to specify an explicit 2D parametric motion model.
- Appearances of non-rigid objects can sometimes be modeled with color distributions

Mean Shift

- Mean-Shift in tracking task:
  - ➢ track the motion of a cluster of interesting features.
- 1. choose the feature distribution to represent an object (e.g., color + texture),
- 2. start the mean-shift window over the feature distribution generated by the object
- 3. finally compute the chosen feature distribution over the next video frame
  - Starting from the current window location, the mean-shift algorithm will find the new peak or mode of the feature distribution, which (presumably) is centered over the object that produced the color and texture in the first place.
  - In this way, the mean-shift window tracks the movement of the object frame by frame.

Intuitive Description



<u>Objective</u> : Find the densest region
Distribution of identical billiard balls

Stolen from: *www.wisdom.weizmann.ac.il/~deniss/vision_spring04/files/**mean_shift/mean_shift**.ppt*

Mean Shift vector

- Given:

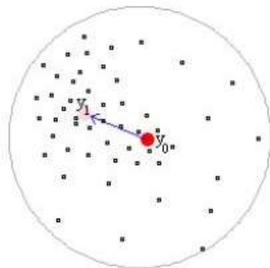
Data points and approximate location of the mean of this data:

- Task:

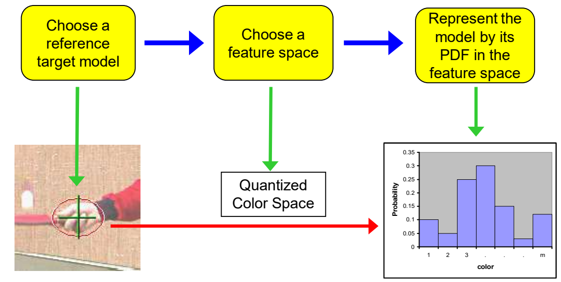
Estimate the exact location of the mean of the data by determining the shift vector from the initial mean.

Mean Shift vector

$$M_h(\mathbf{y}) = \left[\frac{1}{n_x}\sum_{i=1}^{n_x}\mathbf{x}_i\right] - \mathbf{y}_0$$

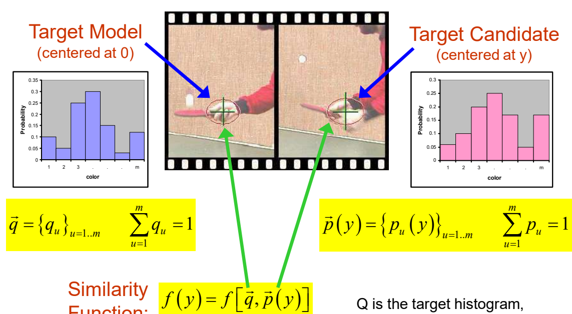


Mean-Shift Object Tracking
Target Representation



Stolen from: www.cs.wustl.edu/~pless/559/lectures/lecture22_tracking.ppt

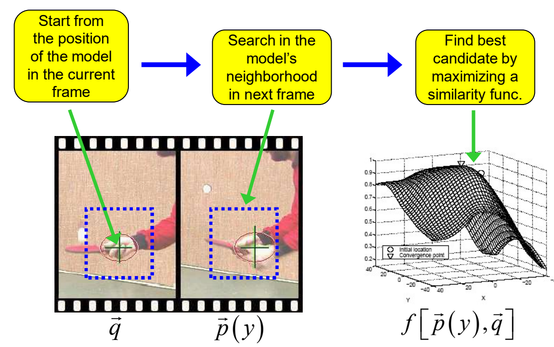
Mean-Shift Object Tracking
PDF Representation



Q is the target histogram,
P is the object histogram
(depends on location y)

Stolen from: www.cs.wustl.edu/~pless/559/lectures/lecture22_tracking.ppt

Mean-Shift Object Tracking
Target Localization Algorithm



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