

Sediment transport in cold region catchments: the MESH-SED model

Luis A. Morales-Marín

Global Institute for Water Security (GIWS)

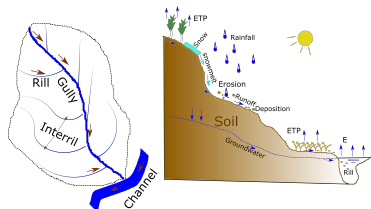
- 1 Introduction
- 2 Controls and boundary conditions
 - System representation of solute transport
- 3 Model structure
- 4 Input data
 - Static input data
 - Dynamic input data
- 5 Output data
- 6 GitHub & References

Introduction

- Solutes are transported across the catchment via surface runoff and groundwater flow.
- Nitrogen (N) and phosphorus are abundant nutrient especially in agricultural areas.
- Nutrients can cause eutrophication and water quality deterioration. Pollutants can also deteriorate the water quality.
- Sediments can be considered as proxies of solutes.

Sediment transport processes

- Overland and in-stream erosion and deposition.
- Soil leaching of sediments.
- Atmospheric deposition
- In cold region:
 - Soil freeze-thaw cycles enhance rock weathering and soil leaching
 - Frozen soil-enhance runoff and erosion in surface layer

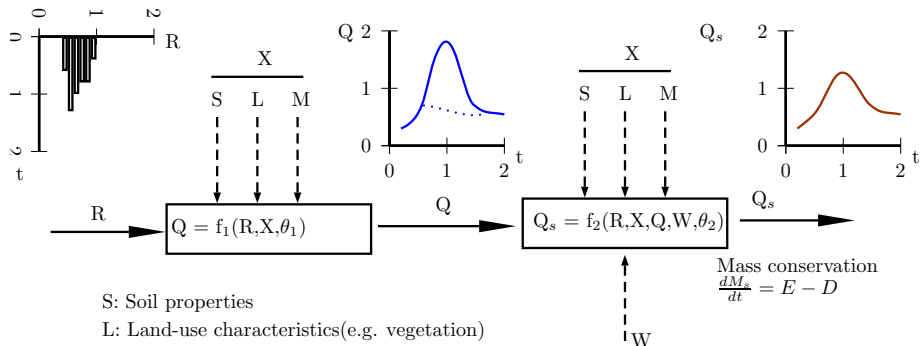


Controls in overland and groundwater transport.

- Slope, land type, soil type (diameter, resistance, specific weight, etc)
- Rainfall, air temperature, soil water content, soil conductivity

Controls in in-stream transport:

- Channel slope and section geometry
- Bank and bottom material properties (diameter, resistance, specific weight), discharge, shear stress (function of slope and discharge) and



S: Soil properties

L: Land-use characteristics(e.g. vegetation)

M: Meteorological variables

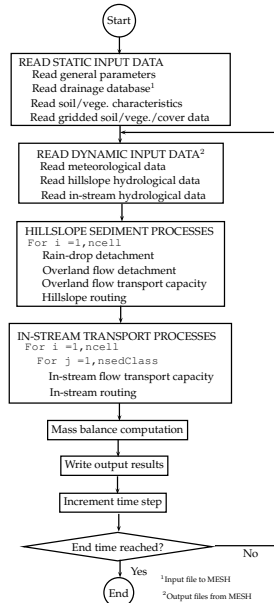
W: Cold region processes (e.g. freeze-thaw cycle)

E: Erosion

D: Deposition

Some **MESH-SED** features:

- loosely coupled with MESH [1].
- physically-based watershed sediment transport model.
- developed based on SHETRAN [2] and SHESED [3].
- semi-distributed model that work on a orthogonal grid (MESH grid).
- suitable for large-scale catchment to run on a continuous basis.
- include different sediment classes.



MESH-SED input/output structure

abc Output from MESH

abc Input to MESH

Notes: In the output files:

ts: indicate time series

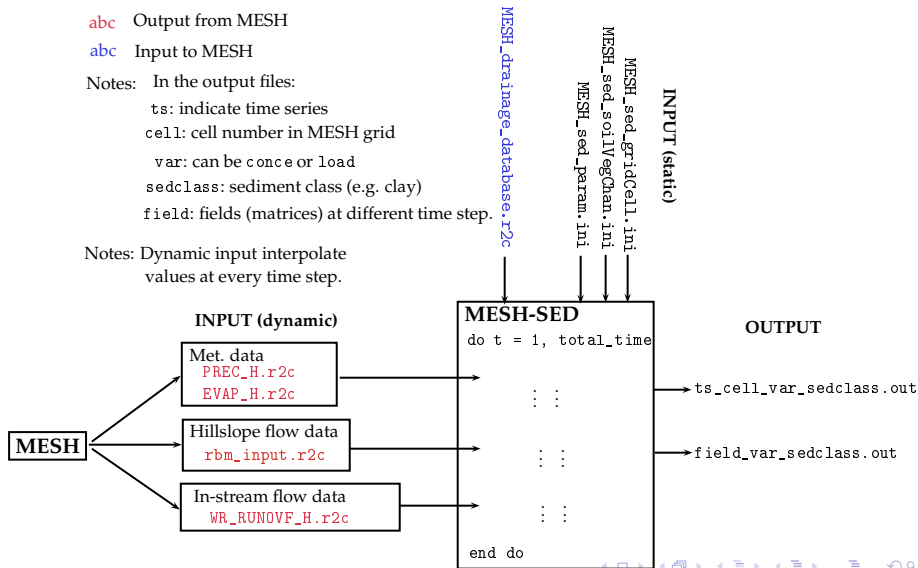
cell: cell number in MESH grid

var: can be conc or load

sedclass: sediment class (e.g. clay)

field: fields (matrices) at different time step.

Notes: Dynamic input interpolate values at every time step.



MESH drainage database data

The information regarding catchment topology, cells connectivity, and model 'skeleton' is contained in `MESH_drainage_database.r2c`. This file is an input file for MESH and it is usually produced by Green Kenue. In **MESH-SED**, `MESH_drainage_database.r2c` is read at the beginning of the model.

Sediment transport model data

The following input files are read at the beginning of the program.

- `MESH_sed_param.ini`: Contains general information require to run de model.
- `MESH_sed_soilVegChan.ini`: Contains the soil and vegetation type characteristics.
- `MESH_sed_gridCell.ini`: Soil, vegetation and cover information for every cell of the grid.

MESH_sed_param.ini

```

Line 1  Comment
Line 2  MESH study case directory path
Line 3  Comment
Line 4  MESH study case directory path that contain output fields
Line 5  Comment
Line 6  Hillslope.time.weighting_factor in-stream.time.weighting_factor in-stream.space.weighting_factor
Line 7  Comment
Line 8  Hillslope transport capacity method: 1[Yalin, 1963][4] 2[Engelund-Hansen, 1967][5]
Line 9  Comment
Line 10  In-stream transport capacity method: 1[Engelund-Hansen, 1967][5], 2[Ackers and White, 1973][6], 3[Day, 1980][]
Line 11  Comment
Line 12  Simulation time step (sec). It must be a multiple or factor of 60 sec
Line 13  Comment
Line 14  Comment
Line 15  Start time: year julian_day hour minutes seconds
Line 16  End time: year julian_day hour minutes seconds
Line 17  Comment
Line 18  Upper limit of volumetric suspended sediment concentration
Line 19  Comment
Line 20  Output directory
Line 21  Comment
Line 22  Number of output grid points (ngr)
Line 23  Comment
Line 24  Comment
Line 25  cell_number1  variable1  particle_class1

```

```

:
:
:
cell_numberngr  variablengr  particle_classngr

```

where *cell_number* is equal to *rank* in MESH_drainage_database.r2c, *variable* can be *conce* (concentration) or *load* and *particle_class* is listed in Table 1.

Table: Sediment/soil particles classes and diameters [7]

	particle_class		ϕ range (mm)
Mud	1	clay	$0.00006 \leq \phi < 0.0039$
	2	veryFineSilt	$0.0039 \leq \phi < 0.0078$
Silt	3	fineSilt	$0.0078 \leq \phi < 0.0156$
	4	mediumSilt	$0.0156 \leq \phi < 0.0313$
	5	coarseSilt	$0.0313 \leq \phi < 0.0625$
Sand	6	veryFineSand	$0.0625 \leq \phi < 0.125$
	7	fineSand	$0.125 \leq \phi < 0.25$
	8	mediumSand	$0.25 \leq \phi < 0.5$
	9	coarseSand	$0.5 \leq \phi < 1.0$
	10	veryCoarseSand	$1.0 \leq \phi < 2.0$
Gravel	11	granule	$2.0 \leq \phi < 4.0$
	12	pebble	$4.0 \leq \phi < 64.0$
	13	cobble	$64.0 \leq \phi < 256.0$
	14	boulder	$256.0 \leq \phi < 4096.0$

Example

```

! MESH study case directory path
'/media/giws_research/Luis/MESH/Graham_River_WFDEI_Luis/07FA005_Graham_Riv/'
! MESH study case directory name that contain output fields
'Out_Fields_R2C'
! Hillslope time weigh. factor          in-stream time weigh. factor          in-stream space weigh. factor
0.65          0.50          0.50
! Flag for overland sediment transport capacity
2
! Choose transport capacity method: 1[Yalin, 1963] 2[Engelund-Hansen, 1967]
! Flag for instream sediment transport capacity
3
! Choose transport capacity method: 1[Engelund-Hansen, 1967], 2[Ackers and White, 1973], 3[Day, 1980]
! Simulation time step sec
60
! Simulation Run Times
---#---#---#---#---#
2002 365  23   0   0          ! Start year, jday, hour, minutes, seconds
2003   1   1   0   0          ! Stop year, jday, hour, minutes, seconds
! Upper limit of volumetric suspended sediment concentration
0.9
! Output Directory
'/media/giws_research/Luis/MESH/MESH_SED/test/output_ts/'
! Number of output grid points
5
! Grid numbersranks          variable_name          sed_class_name
12345678901234567890123456789012345678901234567890
1          load          clay
2          conce          veryCoarseSand
4          conce          cobble
10         conce          boulder
15         load          clay
! Others parameters?????
! Printing selection

```

MESH_sed_soilVegChan.ini

Line 1 Comment

Line 2 Number of soil types (ns)

Line 3 For each soil type i

$\text{frac}_1^1 \dots \text{frac}_j^1 \dots \text{frac}_{14}^1$
 $\rho_s^1 \lambda^1 k_R^1 k_F^1 k_b^1$

\vdots
 \vdots
 \vdots

$\text{frac}_1^i \dots \text{frac}_j^i \dots \text{frac}_{14}^i$
 $\rho_s^i \lambda^i k_R^i k_F^i k_b^i$

\vdots
 \vdots
 \vdots

$\text{frac}_1^{ns} \dots \text{frac}_j^{ns} \dots \text{frac}_{14}^{ns}$
 $\rho_s^{ns} \lambda^{ns} k_R^{ns} k_F^{ns} k_b^{ns}$

Line 4 Comment

Line 5 Number of vegetation classes (nv)

Line 6 For each vegetation type j

$\text{vege}_1 \dots \text{vege}_j \dots \text{vege}_{nv}$
 $X_1 \dots X_j \dots X_{nv}$
 $d_1 \dots d_j \dots d_{nv}$
 $\text{DRIP}_1 \dots \text{DRIP}_j \dots \text{DRIP}_{nv}$

Line 7 Comment

Line 8 $\beta \delta_{max} \alpha$

frac = Fraction occupied for each particle class.

ρ_s = Soil density (kg m^{-3}).

λ = Soil surface porosity ($0. \leq \lambda \leq 1.$).

k_R = Soil detachability (J^{-1}).

k_F = Overland flow detachment ($\text{mg m}^{-2} \text{s}^{-1}$).

k_b = Channel bank flow detachment ($\text{mg m}^{-2} \text{s}^{-1}$).

vege = Vegetation class.

X = Fall height (m).

d = Leaf drip diameter (mm).

DRIP = % drainage as leaf drip.

Example

! Soil type characterization

5

0.	.05	.10	.30	.30	.25	0.	0.	0.	0.	0.
2650.	0.2	25.	2.	2.						
0.	.05	.20	.25	.30	.20	0.	0.	0.	0.	0.
2650.	0.2	25.	2.	2.						
0.	0.	.10	.35	.25	.30	0.	0.	0.	0.	0.
2650.	0.2	25.	2.	2.						
0.05	0.05	0.05	0.20	0.30	0.15	0.10	0.10	0.10	0.	0.
2650	0.2	25.	2.	2.						
0.05	0.05	0.05	0.20	0.30	0.15	0.10	0.10	0.10	0.	0.
2650.	0.2	25.	2.	2.						

! Canopy vegetation parameters

8

maize	bsprout	subeet	potato	tforest	soybean	spruce	sycamore	! # of vegetation type
0.5	1.1	0.5	1.1	6.	0.87	18.	20.	! Vegetation type
4.5	6.3	4.6	5.9	6.	5.	7.	3.	! Fall height m
35	23	18	2	3	4	10	5	! Drip diameter mm
								! % drainage as leaf drip

! In-stream sediment transport parameters

5.0 0.01 0.4

! 3

! 0.0	0.50	0.40	0.10	0.0	0.0	0.0	0.0	0.0	0.0
! 2650.	0.3								
! 0.30	0.40	0.30	0.0	0.0	0.0	0.0	0.0	0.0	0.0
! 2650.	0.3								
! 0.05	0.10	0.40	0.30	0.15	0.0	0.0	0.0	0.0	0.0
! 2650.	0.3								

MESH_sed_gridCell.ini

```

Line 1 Comment
Line 2 Initial depth of loose soil
       $\delta_{11}$     $\delta_{12}$    ...    $\delta_{1n}$ 
       $\delta_{21}$     $\delta_{22}$    ...    $\delta_{2n}$ 
      .       .       .       .
      .       .       .       .
       $\delta_{m1}$     $\delta_{m2}$    ...    $\delta_{mn}$ 
Line 3 Comment
Line 4 Ground cover
       $G_{11}$     $G_{12}$    ...    $G_{1n}$ 
       $G_{21}$     $G_{22}$    ...    $G_{2n}$ 
      .       .       .       .
      .       .       .       .
       $G_{m1}$     $G_{m2}$    ...    $G_{mn}$ 
Line 5 Comment
Line 6 Hillslope soil type
       $hst_{11}$    $hst_{12}$   ...    $hst_{1n}$ 
       $hst_{21}$    $hst_{22}$   ...    $hst_{2n}$ 
      .       .       .       .
      .       .       .       .
       $hst_{m1}$    $hst_{m2}$   ...    $hst_{mn}$ 
Line 7 Comment
Line 8 Steady-state sediment flux boundary
conditions
       $bc_{11}$     $bc_{12}$    ...    $bc_{1n}$ 
       $st_{21}$     $bc_{22}$    ...    $bc_{2n}$ 
      .       .       .       .
      .       .       .       .
       $bc_{m1}$     $bc_{m2}$    ...    $bc_{mn}$ 
Line 9 Comment

```

m = Number of rows (latitudes coordinates) of the MESH grid.

n = Number of columns (longitudes coordinates) of the MESH grid.

δ = Initial depth of hillslope loose soil (m) at each cell.

G = Proportion [$0 \leq G \leq 1$] of impervious area in the cell.

hst = Index ($hst = 1...ns$) of the predominant hillslope soil type at each cell.

Steady-state sediment flux boundary conditions at each cell .

Note: **MESH-SED** can be modified to introduce dynamically time-dependant G and bc fields.

MESH_sed_gridCell.ini

Line 10 Predominant vegetation type
 vt_{11} vt_{12} \dots vt_{1n}
 vt_{21} bc_{22} \dots vt_{2n}
 \vdots \vdots \ddots \vdots
 vt_{m1} vt_{m2} \dots vt_{mn}

Line 11 Comment

Line 12 Canopy cover
 C_{11} C_{12} \dots C_{1n}
 C_{21} C_{22} \dots C_{2n}
 \vdots \vdots \ddots \vdots
 C_{m1} C_{m2} \dots C_{mn}

Line 13 Comment

Line 14 Channel soil type
 cst_{11} cst_{12} \dots cst_{1n}
 cst_{21} cst_{22} \dots cst_{2n}
 \vdots \vdots \ddots \vdots
 cst_{m1} cst_{m2} \dots cst_{mn}

Line 15 Comment

vt = Predominant vegetation class at each cell (see *vege*)

C = Proportion [$0 \leq G \leq 1$] of the cell area covered by vegetation (canopy) at each cell.

cst = Index ($hst = 1...ns$) of the predominant channel bed soil type at each cell.

Note: **MESH-SED** can be modified to introduce dynamically time-dependent vt and C fields

MESH_sed_gridCell.ini

Line 16 Channel bank soil type

$cbst_{11}$	$cbst_{12}$	\dots	$cbst_{1n}$
$cbst_{21}$	$cbst_{22}$	\dots	$cbst_{2n}$
\vdots	\vdots	\ddots	\vdots
$cbst_{m1}$	$cbst_{m2}$	\dots	$cbst_{mn}$

Line 17 Comment

Line 18 Initial thickness of channel bed material

δc_{11}	δc_{12}	\dots	δc_{1n}
δc_{21}	δc_{22}	\dots	δc_{2n}
\vdots	\vdots	\ddots	\vdots
δc_{m1}	δc_{m2}	\dots	δc_{mn}

- $cbst$ = Index ($hst = 1...ns$) of the predominant channel-bank soil type at each cell.
- δc = Initial thickness of channel bed material (m). Usually $0.5m \leq \delta c \leq 5m$ for small to medium size streams, $\delta c \geq 5m$ for large rivers.

Example

! Initial depth of loose soil m

1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03

1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03

1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03

1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03 1.0E-03

! Ground cover: %[0. -> 1.] of cell area covered by snow.g. glaciers, rock or impervious

0.1 0. 0.2 0.6 0.3 0.5 0.9 0.8 0.1 0.2 0.8 0.9 0.1

0.1 0. 0.2 0.6 0.3 0.5 0.9 0.8 0.1 0.2 0.8 0.9 0.1

0.1 0. 0.2 0.6 0.3 0.5 0.9 0.8 0.1 0.2 0.8 0.9 0.1

0.1 0. 0.2 0.6 0.3 0.5 0.9 0.8 0.1 0.2 0.8 0.9 0.1

! Soil type

1 2 3 4 5 1 2 3 4 5 1 2 3

1 2 3 4 5 1 2 3 4 5 1 2 3

1 2 3 4 5 1 2 3 4 5 1 2 3

1 2 3 4 5 1 2 3 4 5 1 2 3

! Sediment flux boundary conditions

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

! Predominant vegetation type in each cell

maize bsprout subeet potato tforest soybean spruce sycamore maize spruce tforest sub

maize bsprout subeet potato tforest soybean spruce sycamore maize spruce tforest sub

maize bsprout subeet potato tforest soybean spruce sycamore maize spruce tforest sub

maize bsprout subeet potato tforest soybean spruce sycamore maize spruce tforest sub

! Canopy cover: %[0. -> 1.] of cell area covered by canopy see the canopy types above

0.9 1.0 0.8 0.4 0.7 0.5 0.1 0.2 0.9 0.8 0.0

0.9 1.0 0.8 0.4 0.7 0.5 0.1 0.2 0.9 0.8 0.0

0.9 1.0 0.8 0.4 0.7 0.5 0.1 0.2 0.9 0.8 0.0

0.9 1.0 0.8 0.4 0.7 0.5 0.1 0.2 0.9 0.8 0.0

! Channel-bed soil type

1 2 3 1 2 3 1 2 3 1 2 3 1

1 2 3 1 2 3 1 2 3 1 2 3 1

1 2 3 1 2 3 1 2 3 1 2 3 1

1 2 3 1 2 3 1 2 3 1 2 3 1

Meteorological data

Hourly precipitation ($PREC$) data and evapotranspiration ($EVAP$) data are interpolated at every time step from the following MESH output files:

$PREC_H.r2c$

$EVAP_H.r2c$

$[-2.7em]$

HEADER

$time_1$			
$PREC_{11}$	$PREC_{12}$	\dots	$PREC_{1n}$
$PREC_{21}$	$PREC_{22}$	\dots	$PREC_{2n}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$PREC_{m1}$	$PREC_{m2}$	\dots	$PREC_{mn}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$time_n$			
$PREC_{11}$	$PREC_{12}$	\dots	$PREC_{1n}$
$PREC_{21}$	$PREC_{22}$	\dots	$PREC_{2n}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$PREC_{m1}$	$PREC_{m2}$	\dots	$PREC_{mn}$

HEADER

$time_1$			
$EVAP_{11}$	$EVAP_{12}$	\dots	$EVAP_{1n}$
$EVAP_{21}$	$EVAP_{22}$	\dots	$EVAP_{2n}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$EVAP_{m1}$	$EVAP_{m2}$	\dots	$EVAP_{mn}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$time_n$			
$EVAP_{11}$	$EVAP_{12}$	\dots	$EVAP_{1n}$
$EVAP_{21}$	$EVAP_{22}$	\dots	$EVAP_{2n}$
\vdots	\vdots	\ddots	\vdots
\vdots	\vdots	\ddots	\vdots
$EVAP_{m1}$	$EVAP_{m2}$	\dots	$EVAP_{mn}$

Hillslope flow data

Hourly overland flow (*RUNOFF*) data are interpolated at every time step from *WR_RUNOVF.r2c*

```

HEADER
time_1
RUNOFF11  RUNOFF12  ...  RUNOFF1n
RUNOFF21  RUNOFF22  ...  RUNOFF2n
...
...
RUNOFFm1  RUNOFFm2  ...  RUNOFFmn
...
...
time_n
RUNOFF11  RUNOFF12  ...  RUNOFF1n
RUNOFF21  RUNOFF22  ...  RUNOFF2n
...
...
RUNOFFm1  RUNOFFm2  ...  RUNOFFmn

```

Other overland flow variables are calculated as:

- water depth (m) $h = 0.34 Q^{0.341}$ according to [8]
- velocity (m/s) $V = Q/A$, assuming a rectangular channel cross section, $A = h * width$.

In-stream flow data

From `rbm_input.r2c` MESH output file the following hourly variables are interpolated at every time step:

- *DISC* = In-stream discharge (m^3/s)
- *DEPT* = Channel water depth (m)
- *WIDT* = Channel width (m)
- *VELO* = Channel flow velocity (m/s)

HEADER

time_1

*DISC*₁₁

*DISC*₁₂

...

*DISC*_{1n}

*DISC*₂₁

*DISC*₂₂

...

*DISC*_{2n}

⋮

⋮

⋮

⋮

*DISC*_{m1}

*DISC*_{m2}

...

*DISC*_{mn}

*DEPT*₁₁

*DEPT*₁₂

...

*DEPT*_{1n}

*DEPT*₂₁

*DEPT*₂₂

...

*DEPT*_{2n}

⋮

⋮

⋮

⋮

*DEPT*_{m1}

*DEPT*_{m2}

...

*DEPT*_{mn}

*WIDT*₁₁

*WIDT*₁₂

...

*WIDT*_{1n}

*WIDT*₂₁

*WIDT*₂₂

...

*WIDT*_{2n}

⋮

⋮

⋮

⋮

*WIDT*_{m1}

*WIDT*_{m2}

...

*WIDT*_{mn}

*VELO*₁₁

*VELO*₁₂

...

*VELO*_{1n}

*VELO*₂₁

*VELO*₂₂

...

*VELO*_{2n}

⋮

⋮

⋮

⋮

*VELO*_{m1}

*VELO*_{m2}

...

*VELO*_{mn}

⋮

⋮

⋮

⋮

Output data

MESH-SED estimates sediment concentration and loads at every cell of the computational grid and for every sediment class at every time step.

MESH-SED also estimates changes of the land surface due to accumulation or erosion of sediments. The format of the output information is specified in `MESH_sed_param.ini`. The typical output files are:

`ts_cell_var_sedclass.out`
(e.g. `ts_10_load_veryFineSand.out`)

<i>time</i> ₁	<i>var</i> ₁
<i>time</i> ₂	<i>var</i> ₂
⋮	⋮
<i>time</i> _{<i>n</i>}	<i>var</i> _{<i>n</i>}

- ts: indicate time series.
- cell: cell number in MESH grid (rank).
- var: it can be conce or load.
- sedclass: sediment class (see Table 1)
- field: field (matrices) at different time steps.

`field_var_sedclass.out`
(e.g. `field_conce_coarseSilt.out`)

<i>time</i> ₁	<i>var</i> ₁₁	<i>var</i> ₁₂	⋯	<i>var</i> _{1<i>n</i>}
	<i>var</i> ₂₁	<i>var</i> ₂₂	⋯	<i>var</i> _{2<i>n</i>}
⋮	⋮	⋮	⋮	⋮
	<i>var</i> _{<i>m</i>1}	<i>var</i> _{<i>m</i>2}	⋯	<i>var</i> _{<i>m</i><i>n</i>}
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
<i>time</i> _{<i>n</i>}	<i>var</i> ₁₁	<i>var</i> ₁₂	⋯	<i>var</i> _{1<i>n</i>}
	<i>var</i> ₂₁	<i>var</i> ₂₂	⋯	<i>var</i> _{2<i>n</i>}
⋮	⋮	⋮	⋮	⋮
	<i>var</i> _{<i>m</i>1}	<i>var</i> _{<i>m</i>2}	⋯	<i>var</i> _{<i>m</i><i>n</i>}

References I



A. Pietroniro, V. Fortin, N. Kouwen, C. Neal, R. Turcotte, B. Davison, D. Versegny, E. Soulis, R. Caldwell, N. Evora, *et al.*, “Using the mesh modelling system for hydrological ensemble forecasting of the laurentian great lakes at the regional scale,” *Hydrology and Earth System Sciences Discussions*, vol. 3, no. 4, pp. 2473–2521, 2006.








J. Ewen, G. Parkin, and P. E. O’Connell, “Shetran: distributed river basin flow and transport modeling system,” *Journal of hydrologic engineering*, vol. 5, no. 3, pp. 250–258, 2000.



J. Wicks and J. Bathurst, “Shesed: a physically based, distributed erosion and sediment yield component for the she hydrological modelling system,” *Journal of Hydrology*, vol. 175, no. 1-4, pp. 213–238, 1996.

References II

-  M. S. Yalin, "An expression for bed-load transportation," *Journal of the Hydraulics Division*, vol. 89, no. 3, pp. 221–250, 1963.
-  F. Engelund and E. Hansen, "A monograph on sediment transport in alluvial streams," *Technical University of Denmark Østervoldgade 10, Copenhagen K.*, 1967.
-  P. Ackers and W. R. White, "Sediment transport: new approach and analysis," *Journal of the Hydraulics Division*, vol. 99, no. hy11, 1973.
-  C. K. Wentworth, "A scale of grade and class terms for clastic sediments," *The Journal of Geology*, vol. 30, no. 5, pp. 377–392, 1922.
-  P. M. Allen, J. C. Arnold, and B. W. Byars, "Downstream channel geometry for use in planning-level models," *JAWRA Journal of the American Water Resources Association*, vol. 30, no. 4, pp. 663–671, 1994.