

1)

x	y	z	p
0	0	0	0.17
0	0	1	0.18
0	1	0	0.09
0	1	1	0.04
1	0	0	0.25
1	0	1	0.1
1	1	0	0.09
1	1	1	0.08

- a) $p(x)=0.08+0.10+0.04+0.18=0.40$
b) $p(\sim x|y)=(0.09+0.09)/(0.08+0.09+0.04+0.09)=0.6$
c) $p(\sim y|x, \sim z)=P(\sim y, \sim x, \sim z)/p(x, \sim z)=0.18/(0.04+0.18)=0.8181$
d) $p(x|z,y)=0.08/(0.08+0.09)=0.47$ and $p(x|y)=(0.08+0.04)/(0.08+0.09+0.04+0.09)=0.4$
because $p(x|z,y)=p(x|y)$, $p(z|x,y)=p(z|y)$, $p(x,z|y)=p(x|y)*p(z|y)$ so we conclude that they are not conditionally independent

$$p(x,z|y)=p(x|y)p(z|y)$$

$$p(x,z|y)=p(x,z,y)/p(y)=0.08/0.3=0.27$$

$$p(x|y)p(z|y)=(1-0.6)*p(z,y)/0.3=4/3(0.8+0.9)=0.23 \neq 0.27 \rightarrow \text{not conditionally}$$

independent

- e) $p(x|y,z)=0.47$, $p(x|z)=(0.08+0.1)/(0.08+0.09+0.1+0.25)=0.35$ so not conditionally independent

$$p(x,y|z)=p(x|z).p(y|z)$$

$$p(x,y|z)=0.08/0.52=0.154$$

$$p(x|z)p(y|z)=18/52 * 17/52=0.113 \neq 0.154 \rightarrow \text{not conditionally independent}$$

2)

$$p(\text{lightning})=p(l)$$

$$p(\text{storm})=p(s)$$

$$p(\text{thunder})=p(t)$$

$$p(\text{camp fire})=p(c)$$

$$p(\text{forest fire})=p(f)$$

$$\text{a) } P(t)=p(t=T, l=T, s=T)+p(t=T, l=T, s=F)+p(t=T, l=F, s=F)+p(t=T, l=F, s=T)=0.1316+0.015+0.0376+0.19=0.3742$$

b) $p(t,f)=p(t|\sim l)p(f|\sim l,\sim c)p(\sim l)p(\sim c)+p(t|\sim l)p(f|\sim l,c)p(\sim l)p(c)+p(t|l)p(f|l,\sim c)p(l)p(\sim c)+p(t|l)p(f|l,c)p(l)p(c)=0.25*0.01*0.82*0.4+0.25*0.15*0.82*0.6+0.94*0.5*0.18*0.4+0.94*0.6*0.18*0.6=\sim 0.64$

c) $p(s|t)=p(s,t)/p(t)=(p(s,l,t)+p(s,\sim l,t))/p(t)=(p(s)p(l|s)p(t|l)+p(s)p(\sim l|s)p(t|\sim l))/p(t)$
 $p(s,t)=\text{sigma over } l=T,F \text{ of } (p(s)*p(l=|s)*p(t|l)=0.2*0.7*0.94+0.2*0.3*0.25=0.1466$
 $\rightarrow p(s|t)=0.1466/0.3742=\sim 0.39$

d)

$p(c|\sim t)=p(c,\sim t)/p(\sim t)=p(c,\sim t,l,f)+p(c,\sim t,l,\sim f)+p(c,\sim t,\sim l,f)+p(c,\sim t,\sim l,\sim f)=p(c)/p(t) * (p(\sim t,l)+p(\sim t,\sim l))$
 $= (p(c)/p(\sim t)) * p(\sim t)=p(c)=0.6 \rightarrow \text{independent}$

e)

$P(l,f)=p(l,f,c)+p(l,f,\sim c)=p(f|l,c)p(l,c)+(f|l,\sim c)p(l,\sim c)$
 $p(l,c)=p(l,c,\sim t,\sim f)+p(l,c,\sim t,f)+p(l,c,t,\sim f)+p(l,c,t,f)$
 $p(l,p)=0.6*0.18*0.6+0.5*0.18*0.4=0.1008$