TUTORIJAL 2

Zad 1

Loptica mase m sa nabojem $+Q$

Ravnu pokretnu ploča M sa nabojem $-Q$

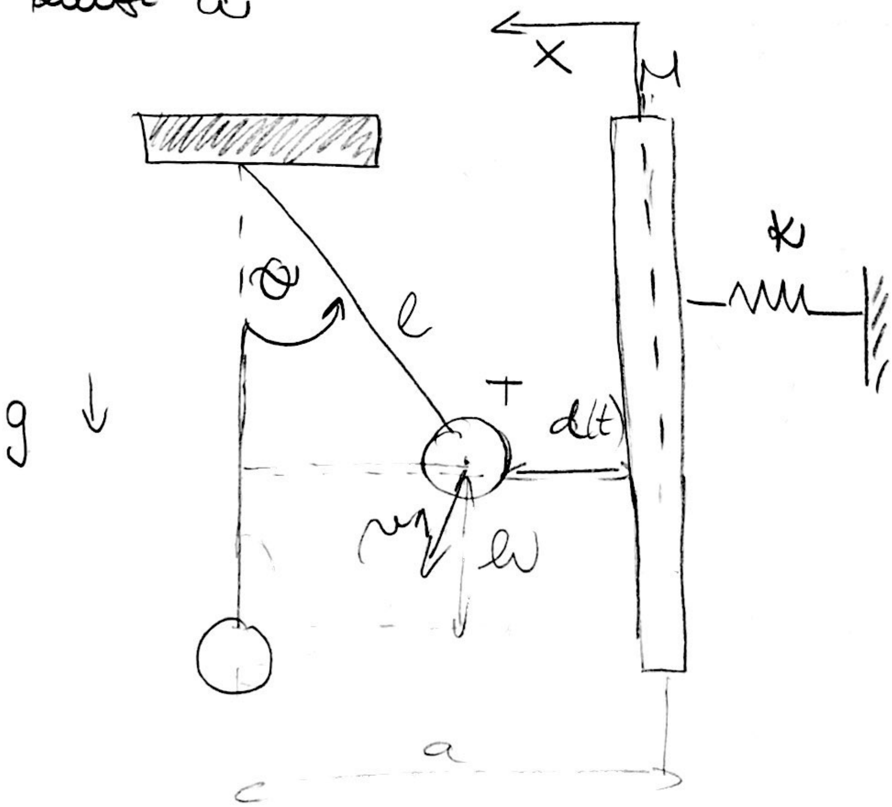
Električna energija sadržana u tom sistemu iznosi:

$$E_e = \frac{Q^2}{16\pi \varepsilon d}$$

ε – dielektrična konstanta

d – udaljenost loptice od ploče

Kad je opruga nenapregnuta i klatno u vertikalnom položaju, razmak loptice i ploče iznosi a



$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_e$$

mehanički dio — električni dio

$$\mathcal{L} = E_k^* - E_p + E_m^* - E_e$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = 0$$

$q_j \Rightarrow$ generalizirane koordinate.

θ, x

E_k^* – kinetička koenergija

$v_1, v_2 \Rightarrow$

ukupna brzina u nekom radnom stanju.

$$E_k^* = \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2$$

$$v_1 = l\dot{\theta}$$

$$v_2 = \dot{x}$$

$E_m^* = 0 \Rightarrow$ nema induktiviteta

Ovdje smo pokupili energiju opruge i mase m (mase M jer god da se postavi referentna točka uvijek nula)

$$E_p = \frac{1}{2} k x^2 + mgh$$

$$h = l(1-\cos\theta)$$

$$d = a - x - l\sin\theta$$

vremenske funkcije $\theta(t)$, $x(t)$

$$E_e = \frac{Q^2}{16\pi\varepsilon(a-x-l\sin\theta)}$$

$$x:\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\dot{x}$$

$$\mathcal{L} = \frac{1}{2} m l^2 \dot{\theta}^2 + \frac{1}{2} M \dot{x}^2 - \frac{1}{2} k x^2 - mgl(1-\cos\theta) - \frac{Q^2}{16\pi\varepsilon(a-x-l\sin\theta)}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = M\ddot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x} = -kx - \frac{Q^2}{16\pi\varepsilon(a-x-l\sin\theta)^2} \quad ?$$

$$\boxed{x:\quad M\ddot{x} + kx + \frac{Q^2}{16\pi\varepsilon(a-x-l\sin\theta)^2} = 0}$$

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θ: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$

$$\frac{\partial \mathcal{L}}{\partial \dot\theta} = ml^2\dot\theta$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) = ml^2\ddot\theta$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -mgl\sin\theta - \frac{Q^2 l\cos\theta}{16\pi\varepsilon(a-x-l\sin\theta)^2}$$

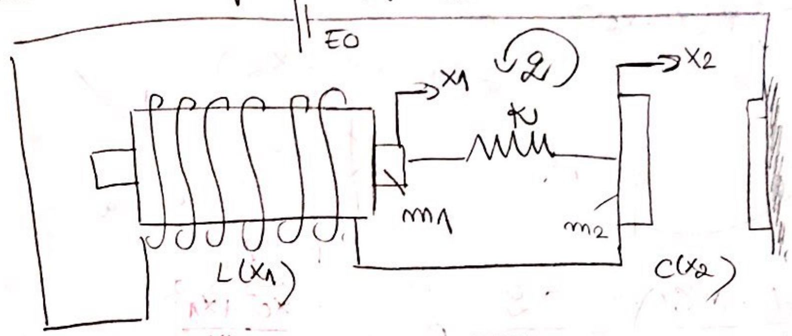
$$\theta: \; ml^2\ddot\theta + mgl\sin\theta + \frac{l\cos\theta Q^2}{16\pi\varepsilon(a-x-l\cos\theta)^2} = 0$$

Zad 2 Pokretna ploča kondenzatora mase m_2 i nelinearna spruga linearno spregnuta konstantom krutosti k s pokretnom jezgrom elektromagneta mase m_1. Ova dva konvertora su međusobno povezana električki u seriju s idealnim izvorom konstantnog napona E_0. Promjene induktivnosti i kapaciteta u funkciji mehaničkih koordinata

$$L(x_1) = \frac{L_0}{1+\left(\frac{x_1}{x_0}\right)^2}$$

$$C(x_2) = \frac{C_0 d_0}{d_0 - x_2}$$

L_0, C_0, x_0 i d_0 konstante
Položaji $x_1 = 0$ i $x_2 = 0$ kad je opruga nenapregnuta
Zanemari sve otpore u sustavu



$x_1, x_2, Q \rightarrow$ generalizirane koordinate.

$$\mathcal{L} = \mathcal{L}_m + \mathcal{L}_e$$

$$\mathcal{L} = E_k^* - E_p + E_m^* - E_c$$

$j = \overline{1,2}$

$Q_1 = x_1$
$Q_2 = x_2$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Q}_j}\right) - \frac{\partial \mathcal{L}}{\partial Q_j} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Q}}\right) - \frac{\partial \mathcal{L}}{\partial Q} = E_0$$

$$E_k^* = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$E_m^* = \frac{1}{2} L(x_1)\dot{Q}^2$$

$$E_p = \frac{1}{2} k (x_1 - x_2)^2$$

$x_1 = 0$
$x_2 = 0$

$$E_c = \frac{Q^2}{2C(x_2)}$$

$$\mathcal{L} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} \frac{L_0}{1+\left(\frac{x_1}{x_0}\right)^2} \dot{q}^2 - \frac{1}{2} k (x_1 - x_2)^2 - \frac{q^2}{2 C_0 d_0} (d_0 - x_2)$$

$E_p = 0$

... pogledala

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{d}{dt}\left(\frac{L_0 \dot{q}}{1+\left(\frac{x_1}{x_0}\right)^2}\right)$$

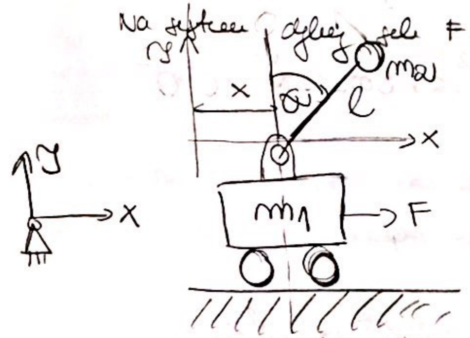
$$= \frac{(L_0 \dot{q})'\left(1+\left(\frac{x_1}{x_0}\right)^2\right) - L_0 \dot{q}\left(1+\left(\frac{x_1}{x_0}\right)^2\right)'}{\left[1+\left(\frac{x_1}{x_0}\right)^2\right]^2}$$

$$= \frac{L_0 \ddot{q}\left(1+\left(\frac{x_1}{x_0}\right)^2\right) - L_0 \dot{q} \frac{2 x_1}{x_0^2} \dot{x}_1}{\left[1+\left(\frac{x_1}{x_0}\right)^2\right]^2}$$

$$= L_0 \ddot{q} \ldots \frac{(d_0 - x_2)}{C_0 d_0} q = E_0$$

$\ldots^2 = 0$

Zad 3 Masa m_1 ... pričvršćen štap dužine l na masi m_2. Masa m_1 se kreće po x-osi ... i štap se okreće slobodno oko tačke O. Na ... djeluje sila F.



$\Rightarrow m_1$ ide u pravcu x

$\theta, x \Rightarrow$ generalisane koordinate

$$\theta: \frac{d}{dx}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$x: \frac{d}{dx}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = F$$

$$\mathcal{L} = E_k^* - E_p$$

$$E_k^* = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 v_2^2$$

$$v_2^2 = \left(\frac{d}{dt}(x_2(t))\right)^2 + \left(\frac{d}{dt}(y_2(t))\right)^2$$

$x_2(t) = x + l\sin\theta$

$y_2(t) = l\cos\theta$

$v_2^2 = (\dot{x} + l\cos\theta \cdot \dot{\theta})^2 + (-l\sin\theta\,\dot{\theta})^2$

$v_2^2 = \dot{x}^2 + 2\dot{x}l\cos\theta\,\dot{\theta} + l^2\cos^2\theta\,\dot{\theta}^2 + l^2\sin^2\theta\,\dot{\theta}^2$

$E_k^* = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + 2\dot{x}l\cos\theta\,\dot{\theta} + l^2\dot{\theta}^2$

$E_p = m_2 g l\cos\theta$

$\mathcal{L} = E_k^* - E_p$

$\mathcal{L} = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + 2\dot{x}\dot{\theta}l\cos\theta + l^2\dot{\theta}^2)$

$-m_2 g l\cos\theta$

x: $(m_1+m_2)\ddot{x} + m_2 l(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = F$

θ: $m_2(l^2\ddot{\theta} + l\ddot{x}\cos\theta) = m_2 g l\sin\theta$

$m_2 l^2\ddot{\theta} + m_2 l(\ddot{x}\cos\theta - \dot{x}\sin\theta\,\dot{\theta})$

$+ m_2 l\dot{x}\dot{\theta}\sin\theta + m_2 g l\sin\theta = 0$

$m_2 l^2\ddot{\theta} + m_2 l\ddot{x}\cos\theta - \cancel{m_2 l\dot{x}\sin\theta\dot{\theta}}$

$+ \cancel{m_2 l\dot{x}\dot{\theta}\sin\theta} + m_2 g l\sin\theta = 0$

$m_2 l^2\ddot{\theta} + m_2 l\ddot{x}\cos\theta + m_2 g l\sin\theta = 0.$

2014/15

) Napisati čuvene metode za konzervativne mehaničke sistem, a zatim za takav sistem izvesti princip održanja energije.

(isto pitanje na roku 2013/14)

Čuvena varijaciona analiza konzervativnih mehaničkih sistema

- Za mehaničke sisteme bez disipacij, bez vanjskih sila, čuvene varijacione metode poprima jednostavna formu.

$$\delta V = \int_{t_0}^{t_1} \delta L\,dt = \delta\int_{t_0}^{t_1} L\,dt$$

$$L = E_f^* - E_e$$

- Odgovarajuće Lagrangeove jednačine dobijaju formu:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \qquad j = 1,2,\ldots,l$$

Održavanje energije

- Posmatrajmo vremenske derivacije Lagranžijana:

$$\frac{d\mathcal{L}}{dt} = \sum_{i=1}^{m}\left(\frac{\partial \mathcal{L}}{\partial q_i}\dot{q}_i + \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\ddot{q}_i\right)$$

- S druge strane iz Lagrangeove jednačine slijedi:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = \frac{\partial \mathcal{L}}{\partial q_i}$$

- Osim tog vrijedi:

$$\sum_{i=1}^{m} \frac{d}{dt}\left(\dot{q}_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = \sum_{i=1}^{m} \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\ddot{q}_i + \sum_{i=1}^{m} \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right)\dot{q}_i$$

$$= \sum_{i=1}^{m} \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\ddot{q}_i + \sum_{i=1}^{m} \frac{\partial \mathcal{L}}{\partial q_i}\dot{q}_i$$

$$\Rightarrow \sum_{i=1}^{m} \frac{d}{dt}\left(\dot{q}_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{d\mathcal{L}}{dt} = 0$$

- Zadnja jednačina se može integraliti a može se iskoristiti činjenica da je generalisani moment p_i dat sa:

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = \frac{\partial E_f^*}{\partial \dot{q}_i}$$

- Može se dobiti:

$$\sum_{i=1}^{m} p_i \dot{q}_i - \mathcal{L} = const$$

- Imajući u vidu konstitutivne relacije, suma u postupku jednako predstavlja zbir svih kinetičkih energija i koenergija tako da imamo:

$$E_f^* + E_f - \mathcal{L} = E_f^* + E_f - (E_f^* - E_e) = const$$

$$\Rightarrow E_f + E_e = const$$

Rok 2012/13

Pitanje 2 Napisati izraze za otvorene i zatvorene varijacione indikatore za fluidne sisteme. Kratko objasniti koji varijable figurišu u datim indikatorima. Za oba pristupa (otvoreni i zatvoreni) napisati odgovarajuće sisteme Lagrangeove jednačine.

isto pitanje kao na roku 2013/14

Varijaciona analiza fluidnih sistema – otvoreni pristup

- Otvorena varijaciona analiza fluidnih sistema zahtjeva potpun nezavisan skup generalisanih koordinata momenta pritiska koji određuju konfiguraciju sistema.

- Generalisane koordinate → momenti pritiska $(\Gamma_1, \Gamma_2 \ldots \Gamma_m)$

$$\delta V = \int_{t_0}^{t_1}\left[\delta L - \sum_{j=1}^{e}\left(\frac{\partial J}{\partial \dot{\Gamma}_j} - Q_i\right)\delta \Gamma_j\right]dt$$

Varijaciona analiza fluidnih sistema

– čvorni pristup

∘ Lagranžijan analognog sistema:

$$\mathcal{L} = E_f^* - E_e$$

E_f^* – Totalna koenergija skladište toka

E_e – Totalna energija skladište napora

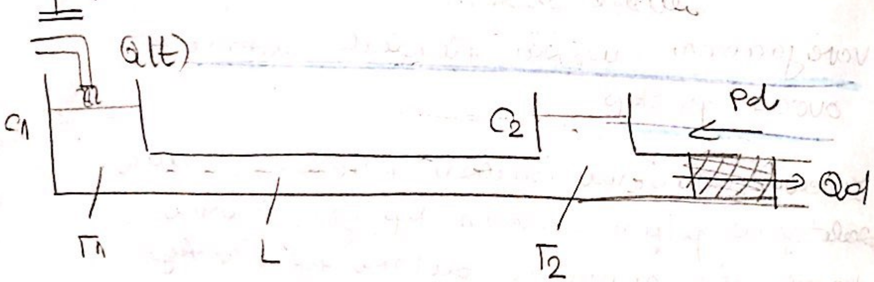
$\mathcal{J}$ – Totalni kosadržaj disipatora u kolu

Q_j – izvore generalisanog toka koji odgovara j-toj koordinati generalisanog nula pritiska.

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Gamma}_j}\right) - \frac{\partial L}{\partial \Gamma_j} + \frac{\partial \mathcal{J}}{\partial \dot{\Gamma}_j} = Q_j}$$

$j = 1, 2 \ldots \ell$

(*) Pr:



∘ Fluidni sistem se sastoji iz dva otvorena rezervoara, i [illegible] inertans i disipacija na otvoru ka atmosferskom pritisku.

Konstitutivna relacija disipatora:

$$P_D = \frac{1}{D} Q_d^2$$

∘ Sistem ima dva koordinatna pritiska (Γ_1, Γ_2) koje formiraju potpun nezavisan skup. Ovi momenti su zapravo integrali pritiska u bazama rezervoara.

∘ potencijalna koenergija sistema

$$E_f^* = \frac{1}{2} C_1 \dot{\Gamma}_1^2 + \frac{1}{2} C_2 \dot{\Gamma}_2^2$$

∘ kinetička energija sistema:

$$E_e = \frac{1}{2L} (\Gamma_1 - \Gamma_2)^2$$

• Lagranžijan sistema

$$\mathcal{L} = \frac{1}{2} C_1 \dot{\Gamma}_1^2 + \frac{1}{2} C_2 \dot{\Gamma}_2^2 - \frac{1}{2L} (\Gamma_1 - \Gamma_2)^2$$

∘ kosadržaj sistema

$$\mathcal{J} = \int_0^{P_d} Q_d \, dP_d = \ldots = \frac{2}{3} \sqrt{D} \sqrt{P_d^3} = \frac{2}{3} D^{1/2} (\dot{\Gamma}_2)^{3/2}$$

∘ Lagranževa jednačina kretanja:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Gamma}_1}\right) - \frac{\partial \mathcal{L}}{\partial \Gamma_1} + \frac{\partial \mathcal{J}}{\partial \dot{\Gamma}_1} = Q(t)$$

$$\Rightarrow C_1 \ddot{\Gamma}_1 + \frac{1}{L} \Gamma_1 - \frac{1}{L} \Gamma_2 = Q(t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Gamma}_2}\right) - \frac{\partial \mathcal{L}}{\partial \Gamma_2} + \frac{\partial J}{\partial \dot{\Gamma}_2} = 0$$

$$\Rightarrow C_2 \ddot{\Gamma}_2 + \frac{1}{L}\Gamma_2 - \frac{1}{L}\Gamma_1 + \sqrt{D\Gamma_2} = 0$$

Varijaciona analiza fluidnih sistema
- kontinuum pristup

• Kontinuirana varijaciona analiza fluidnih sistema zahtjeva potpuno nezavisan skup generalisanih koordinata volumena koje određuju konfiguraciju sistema.

• generalisane koordinate volumena $(V_1, V_2 \dots V_g)$

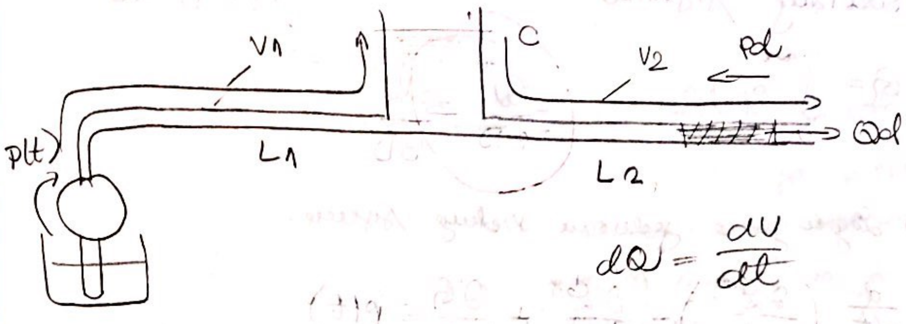
$$\delta J = \int_{t_0}^{t_1} \left[\delta L^* - \sum_{j=1}^{g} \left(\frac{\partial G}{\partial \dot{V}_j} - P_j\right)\delta V_j\right] dt$$

⇒ ko-Lagrangian analognog kola:

$$\mathcal{L}^* = E_c^* - E_f$$

E_c^* – Totalna koenergija skladištena naporom
E_f – Totalna energija skladištena tokom
G – Totalni sadržaj disipacije u sistemu
P_j – broj generalisanog pritiska koji odgovara j-toj koordinati generalisanog volumena.

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{V}_j}\right) - \frac{\partial \mathcal{L}^*}{\partial V_j} + \frac{\partial G}{\partial \dot{V}_j} = P_j}$$



$$dQ = \frac{dV}{dt}$$

⇒ Fluidni sistem se sastoji od izvora pritiska dvije inertanse, otvorenog rezervoara i disipacije na otvoru kao n. pr.

konstitutivna relacija disipatora

$$P_d = \frac{1}{D} Q_d^2$$

⇒ dvije koordinate volumena (V_1, V_2) formiraju potpuno nezavisan skup. Volumni integrali protoka fluida kroz odg. inertanse.

• kinetička koenergija

$$E_c^* = \frac{1}{2} L_1 \dot{V}_1^2 + \frac{1}{2} L_2 \dot{V}_2^2$$

• Potencijalna energija sistema

$$E_f = \frac{1}{2C}(V_1 - V_2)^2$$

∘ko-Lagranžian sistema:

$$\mathcal{L}^* = \frac{1}{2} L_1 \dot{v}_1^2 + \frac{1}{2} L_2 \dot{v}_2^2 - \frac{1}{2C}(v_1 - v_2)^2$$

$$P_D = \frac{1}{D} Q_D^2$$

∘ Sadržaj sistema

$$G = \int_0^{Qd} P_d \, d Q_d = \dots = \frac{\dot{Q}_d^3}{3D} = \frac{\dot{v}_2^3}{3D} \Rightarrow$$

∘ Lagrangeove jednadžbe kretanja sustava:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{v}_1}\right) - \frac{\partial \mathcal{L}^*}{\partial v_1} + \frac{\partial G}{\partial \dot{v}_1} = p(t)$$

$$\Rightarrow L_1 \ddot{v}_1 + \frac{1}{C} v_1 - \frac{1}{C} v_2 = \cancel{\varepsilon(t)}\ P(t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{v}_2}\right) - \frac{\partial \mathcal{L}^*}{\partial v_2} + \frac{\partial G}{\partial \dot{v}_2} = 0$$

$$\Rightarrow L_2 \ddot{v}_2 + \frac{1}{C} v_2 - \frac{1}{C} v_1 + \frac{\dot{v}_2^2}{D} = 0$$

u okviru ovog odgovora riješimo i pitanje 3/ sa roka 2014/15. zadatak 3

$$Q_D = \sqrt{P_d \cdot D}$$

$$P_d = \mathring{T}_2$$

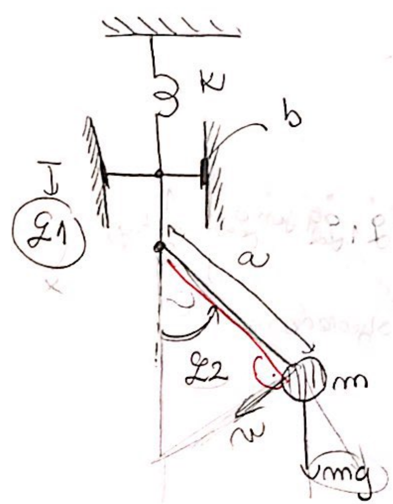
$$\int_0^{P_d} \sqrt{D}\sqrt{P_d}\, dP_d = \sqrt{D}\,\frac{2}{3}\, P_d^{3/2} = \sqrt{D}\,\frac{2}{3}\,\mathring{T}_2$$

∘

ROK 2013/14

Pitanje / Zadatak 3

koristeći Lagrangeov pristup, izvesti mat. model za mehanički sistem.



∘ generalizirane koordinate (ξ_1, ξ_2) tvore potpun, nezavisan skup koordinata. Sistem nije ograničen vanjskim kretanjem brzine, ali je podvrgnut djelovanju povratne gravitacijske sile na masu m.

∘ Generalizirane sile F_1, F_2 sile se dobiju pomicanjem kao sila koja djeluje na generalizirane koordinate.

$$F_1 = mg$$

∘ Gravitacijska sila ima komponentu $-mg\sin\xi_2$ koja je tangencijalna na smjer rotacije stoga je generalizirana sila vezana za moment:

$$J_2 = -mga\sin\xi_2$$

$$J_2 = -mga\sin\xi_2$$

- Kinetička energija sistema:

$$E_k^* = \frac{1}{2} m v^2 = \frac{1}{2} m \left[\dot{q}_1^2 + (a\dot{q}_2)^2 - 2\dot{q}_1 \dot{q}_2 a \sin q_2 \right]$$

? FALI

- Potencijalna energija sistema:

$$E_p = \frac{1}{2} k q_1^2$$

- Lagranžijan:

$$\mathcal{L} = E_k^* - E_p = \frac{1}{2} m \left[\dot{q}_1^2 + (a\dot{q}_2)^2 - 2\dot{q}_1 \dot{q}_2 a \sin q_2 \right] - \frac{1}{2} k q_1^2$$

- Ko sadržaj sistema je definisan sljedećom relacijom:

$$J = \frac{1}{2} b \dot{q}_1^2$$

- Za koordinatu q_1, Lagrangeova jednačina:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_1}\right) - \frac{\partial \mathcal{L}}{\partial q_1} + \frac{\partial J}{\partial \dot{q}_1} = mg$$

$$\boxed{m\ddot{q}_1 - ma\ddot{q}_2 \sin q_2 - ma\dot{q}_2^2 \cos q_2 + kq_1 + b\dot{q}_1 = mg}$$

- Za koordinatu q_2:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_2}\right) - \frac{\partial \mathcal{L}}{\partial q_2} + \frac{\partial J}{\partial \dot{q}_2} = -mga \sin q_2$$

$$m(a^2\ddot{q}_2 - \dot{q}_1\dot{q}_2 a \cos q_2 - \ddot{q}_1 a \sin q_2) + m\dot{q}_1\dot{q}_2 a \cos q_2 = -mga \sin q_2$$ ✓

$$\boxed{a\ddot{q}_2 - \ddot{q}_1 a \sin q_2 + g \sin q_2 = 0}$$

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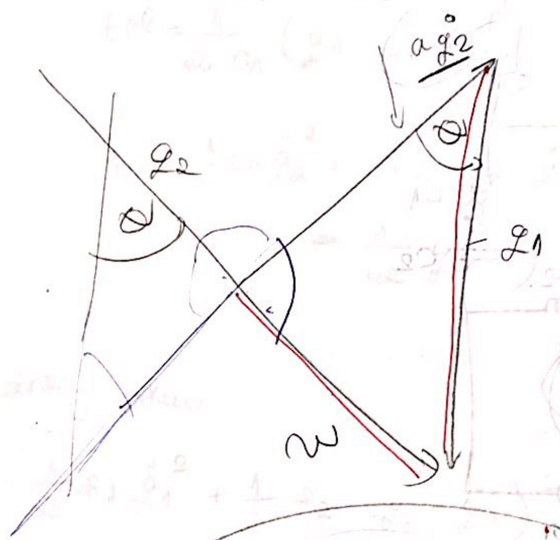


$$\theta = \frac{\pi}{2} - q_2$$

~~$\cos \theta = \sin q_2$~~ sin

$\boxed{\sin(90 - \theta) = \cos \theta}$

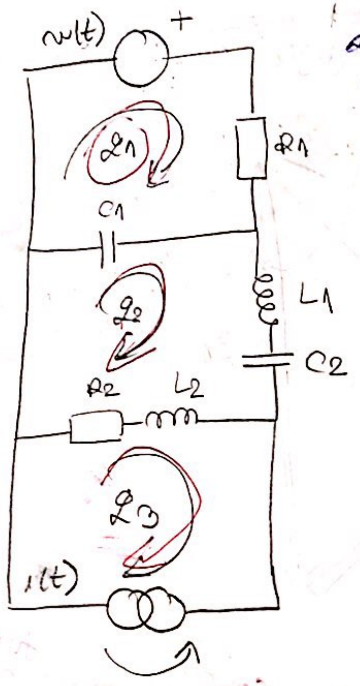
$$v^2 = \dot{q}_1^2 + \dot{q}_2^2 a^2 - 2\dot{q}_1 \dot{q}_2 a \sin q_2$$

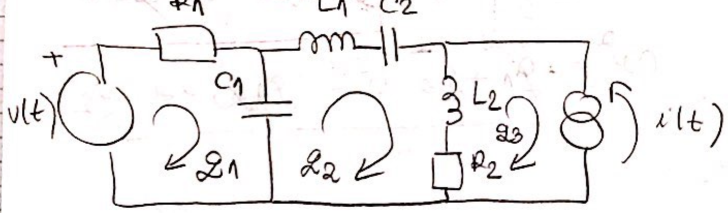
$$E_k^* = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\dot{q}_1^2 + (a\dot{q}_2)^2 - 2\dot{q}_1\dot{q}_2 a \sin q_2 \right) - \frac{1}{2} k q_1^2$$

Pitanje / zadatak 4

Rok 2013/14
Koristeći Lagrangeov postupak izvesti i nacrtati električno kolo.





- konfiguracija [illegible] i ograničenja izvora struje $i(t)$ tako da vrijedi sljedeće:

$$q_3 = -\int_0^t i(\tau)\,d\tau$$

$$\delta q_3 = 0$$

- Induktivna koenergija, kapacitivna energija, Lagrangian funkcija:

$$E_m^* = \frac{1}{2} L_1 \dot{q}_2^2 + \frac{1}{2} L_2 (\dot{q}_2 - \dot{q}_3)^2$$

$$E_el = \frac{1}{2C_1}(q_1 - q_2)^2 + \frac{1}{2C_2} q_2^2$$

$$L = \frac{1}{2}\left(L_1 \dot{q}_2^2 + L_2(\dot{q}_2 - \dot{q}_3)^2 - \frac{1}{C_1}(q_1 - q_2)^2 - \frac{1}{C_2} q_2^2\right)$$

- Sadržaj [illegible]

$$G = \frac{1}{2} R_1 \dot{q}_1^2 + \frac{1}{2} R_2 (\dot{q}_2 - \dot{q}_3)^2$$

- Za koordinatu q_1 Lagrangeova jednadžba je:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{q}_1}\right) - \frac{\partial \mathcal{L}^*}{\partial q_1} + \frac{\partial G}{\partial \dot{q}_1} = v(t)$$

$$\frac{1}{C_1}(q_1 - q_2) + R_1 \dot{q}_1 = v(t)$$

- Za koordinatu q_2 Lagrangeova jednadžba

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}^*}{\partial \dot{q}_2}\right) - \frac{\partial \mathcal{L}^*}{\partial q_2} + \frac{\partial G}{\partial \dot{q}_2} = 0$$

$$L_1\ddot{q}_2 + L_2(\ddot{q}_2-\ddot{q}_3) + \frac{1}{C_1}(q_1-q_2)(-1) + \frac{1}{C_2}q_2$$

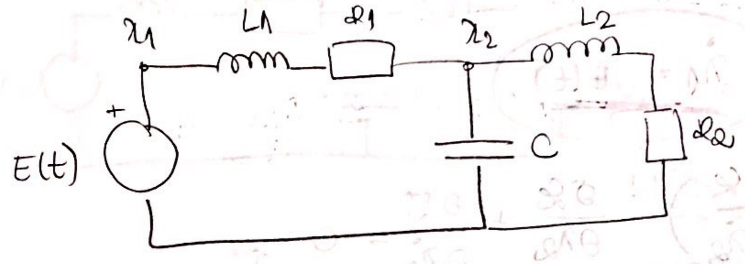
$$+ R_2(\dot{q}_2-\dot{q}_3) = 0$$

$$L_1\ddot{q}_2 + L_2(\ddot{q}_2-\ddot{q}_3) - \frac{1}{C_1}(q_1-q_2) + \frac{1}{C_2}q_2$$

$$+ R_2(\dot{q}_2-\dot{q}_3) = 0$$

RCK 2012/2013

Zad 2 Koristeći Lagrangeov pristup izvesti jednadžbe kretanja za električno kolo sa slike:



Uvodimo pristup

x_1, x_2 – generalizirane koordinate.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_j}\right) - \frac{\partial \mathcal{L}}{\partial x_j} + \frac{\partial \mathcal{J}}{\partial \dot{x}_j} = I_j$$

$$\mathcal{L} = E_f^* - E_e$$

E_f^* → elektrost.; E_e → el. magn.

$$E_f^* = \frac{1}{2}C\dot{x}_2^2$$

$$E_e = \frac{1}{2L_1}(x_1-x_2)^2 + \frac{1}{2L_2}x_2^2$$

$$\mathcal{J} = \frac{1}{2R_1}(\dot{x}_1-\dot{x}_2)^2 + \frac{1}{2R_2}\dot{x}_2^2$$

Sređivanjem:

$$\mathcal{L} = \frac{1}{2}C\dot{x}_2^2 - \left(\frac{1}{2L_1}(x_1-x_2)^2 + \frac{1}{2L_2}x_2^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\dot{x}_1}\right) - \frac{\partial \mathcal{L}}{\partial x_1} + \frac{\partial J}{\partial \dot{x}_1} = E(t)$$

$$x_1 = \int_0^t E(t)\,dt \qquad \delta x_1 = 0$$

$$\dot{x}_1 = E(t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) - \frac{\partial \mathcal{L}}{\partial x_2} + \frac{\partial J}{\partial \dot{x}_2} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) = C\ddot{x}_2$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = -\frac{1}{L_1}(x_1 - x_2)(-1) - \frac{1}{L_2}x_2$$

$$= \frac{1}{L_1}(x_1 - x_2) - \frac{1}{L_2}x_2$$

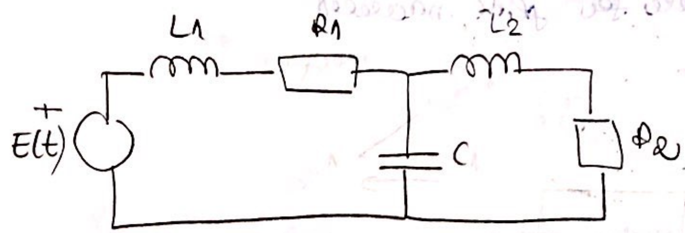
$$\frac{\partial J}{\partial \dot{x}_2} = \frac{1}{R_1}(\dot{x}_1 - \dot{x}_2)(-1) + \frac{1}{R_2}\dot{x}_2$$

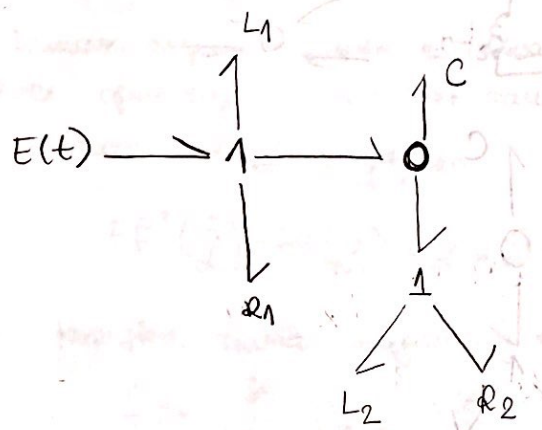
x_2:

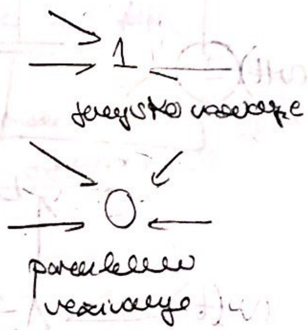
$$C\ddot{x}_2 - \frac{1}{L_1}(x_1 - x_2) + \frac{1}{L_2}x_2 - \frac{1}{R_1}(\dot{x}_1 - \dot{x}_2) + \frac{1}{R_2}\dot{x}_2 = 0$$

ROK 2012/13

Zad3 Za električni krug nacrtati odg. graf.





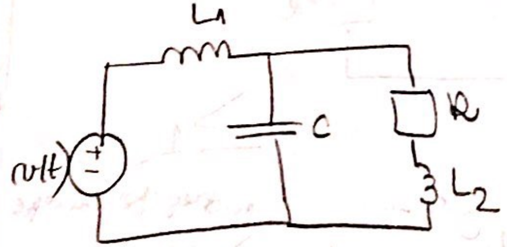


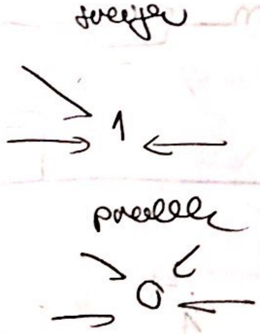
Rok 2014/15

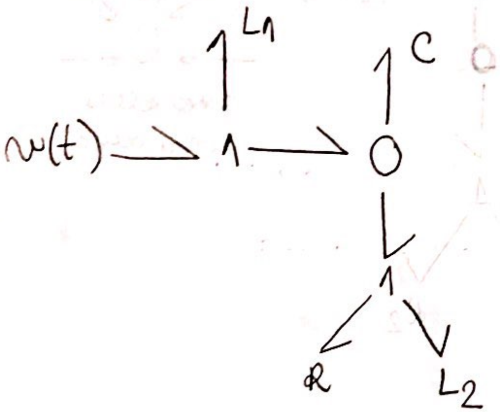
(*) dodatno uvodi

Pitanje / Zad 9

Za električni krug sa slike nacrtaj bond graf.







Rok 2012/13

Pitanje 1) Izvedi da je u općem varijacioni indikator za sustav s jednim stupnjem slobode oblika

$$\delta V = \int_{t_0}^{t_1} \left[\delta E_f^* - \delta E_p - \left(\frac{\partial J}{\partial \dot{q}} - f_0 \right) \delta q \right] dt$$

Izvedi odgovarajuću Lagrangeovu jednadžbu za taj sistem.

- Dinamički varijacijski indikator koji izražava se preko $\delta V = \int_{t_0}^{t_1} \delta I \, dt$
- Za opisivanje dinamičkog ponašanja sustava, možemo definirati Lagrangian:

$$E_f^*(\dot{q}) - E_p(q) = \mathcal{L} = \mathcal{L}(q, \dot{q})$$

- Koristeći pravila varijacionog računa

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \dot{q}$$

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}} \delta q \right) - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}} \right) \delta q$$

- Koristeći prethodno uvođenje izraza za varijaciju Lagrangiana možemo predstaviti varijacioni indikator:

$$\delta V = \int_{t_0}^{t_1} \left[\delta E_f^* - \delta E_p - \left(\frac{\partial J}{\partial \dot{q}} - f_0 \right) \delta q \right] dt =$$

$$= \int_{t_0}^{t_1} \left[\delta L - \left(\frac{\partial J}{\partial \dot{q}} - f_0 \right) \delta q \right] dt$$

$$\delta V = \int_{t_0}^{t_1} \left[\frac{\partial \mathcal{L}}{\partial q}\delta q + \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\delta q\right) - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right)\delta q - \left(\frac{\partial \mathcal{J}}{\partial \dot q} - f_e\right) \right.$$

$$\delta V = \int_{t_0}^{t_1} \left[\frac{\partial \mathcal{L}}{\partial q}\delta q + \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\delta q\right) - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right)\delta q - \left(\frac{\partial \mathcal{J}}{\partial \dot q} - f_e\right) \right.$$

$$\delta V = \frac{\partial \mathcal{L}}{\partial \dot q}\delta q \Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} \left[\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right) - \frac{\partial \mathcal{L}}{\partial q} + \left(\frac{\partial \mathcal{J}}{\partial \dot q} - f_e\right)\right)\delta q \, dt$$

- prvi član se ne javlja pošto je jednak nuli jer vrijedi: $\delta q(t_0) = \delta q(t_1) = 0$

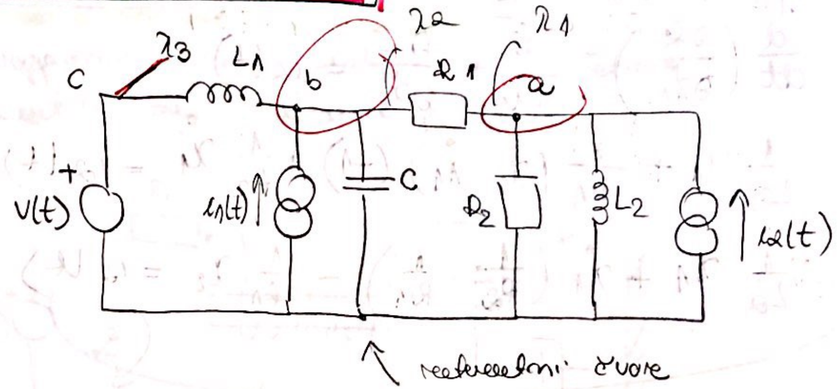
- varijacija nestaje jedino ako:

$$\delta V = -\int_{t_0}^{t_1} \left[\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right) - \frac{\partial \mathcal{L}}{\partial q} + \left(\frac{\partial \mathcal{J}}{\partial \dot q} - f_e\right)\right)\delta q \, dt$$

- da ~~početni~~ prethodni integral mora biti zadovoljena sljedeća jednadžba (Lagrangeova jednadžba za jedan stupanj slobode)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right) - \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial \mathcal{J}}{\partial \dot q} = f_e$$

PRIMJER IZ PREDAVANJA



$$\Rightarrow \lambda_3 = \int_0^t v(\tau)\,d\tau$$

$$\delta\lambda_3 = 0$$

$$\dot\lambda_3 = v(t)$$

- kapacitivna koenergija

$$E_{el}^* = \frac{1}{2} C \dot\lambda_2^2$$

- induktivna energija

$$E_m = \frac{1}{2L_1}(\lambda_3 - \lambda_2)^2 + \frac{1}{2L_2}\lambda_1^2$$

$$\mathcal{L} = E_{el}^* - E_m = \frac{1}{2} C \dot\lambda_2^2 - \left[\frac{1}{2L_1}(\lambda_3 - \lambda_2)^2 + \frac{1}{2L_2}\lambda_1^2\right]$$

- ko sadržaj dat sa:

$$\mathcal{J} = \frac{1}{2R_1}(\dot\lambda_2 - \dot\lambda_1)^2 + \frac{1}{2R_2}\dot\lambda_1^2$$

- po koord. x_1:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) - \frac{\partial \mathcal{L}}{\partial x_1} + \frac{\partial \mathcal{J}}{\partial \dot{x}_1} = i_2(t)$$

$$\frac{1}{L_2} x_1 + \frac{1}{R_1}(\dot{x}_2 - \dot{x}_1)(-1) + \frac{1}{R_2}\dot{x}_1 = i_2(t)$$

$$\frac{1}{L_2} x_1 + \dot{x}_1\left(\frac{1}{R_2} + \frac{1}{R_1}\right) - \frac{1}{R_1}\dot{x}_2 = i_2(t)$$

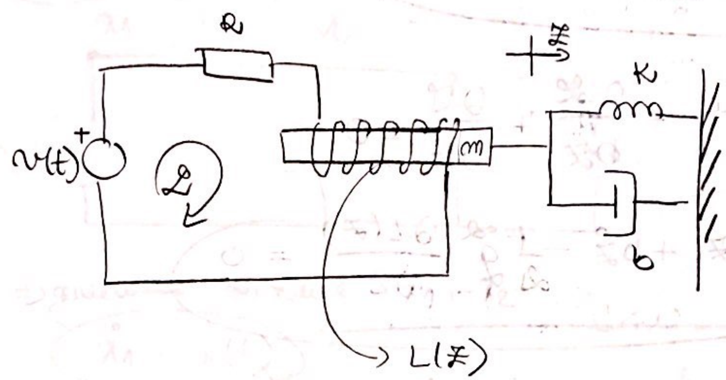
- po x_2: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) - \frac{\partial \mathcal{L}}{\partial x_2} + \frac{\partial \mathcal{J}}{\partial \dot{x}_2} = i_1(t)$

$$C\ddot{x}_2 + \frac{1}{L_1}(x_3 - x_2)(-1) + \frac{1}{R_1}(\dot{x}_2 - \dot{x}_1) = i_1(t)$$

$$C\ddot{x}_2 - \frac{1}{L_1}(x_3 - x_2) + \frac{1}{R_1}(\dot{x}_2 - \dot{x}_1) = i_1(t)$$

PRIMJER IZ PREDAVANJA

Varijaciona analiza sistema sa dvopolnim elementima koji skladište energiju



generalisane koordinate z i q

=> Ko-Lagranžijan električnog podsistema je:

$$\mathcal{L}^*_{elec} = \frac{1}{2} L(z)\dot{q}^2 \quad E_m^*$$

induktivna koenergija (nema kapacitivne energije)

=> Lagranžijan mehaničkog podsistema:

$$\mathcal{L}_{meh} = \frac{1}{2} m\dot{z}^2 - \frac{1}{2} k z^2$$

$E_k^* - E_P$

=> Kompletni Lagranžijan je odnosno:

$$\mathcal{L} = \mathcal{L}_{meh} + \mathcal{L}^*_{elec} = \frac{1}{2} m\dot{z}^2 - \frac{1}{2} k z^2 + \frac{1}{2} L(z)\dot{q}^2$$

=> Kompletni kosadržaj sistema.

$$\mathcal{J} = \frac{1}{2} R\dot{q}^2 + \frac{1}{2} b\dot{z}^2$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial \mathfrak{J}}{\partial \dot{q}} = v(t)$$

$$L(z)\ddot{q} + R\dot{q} = v(t)$$

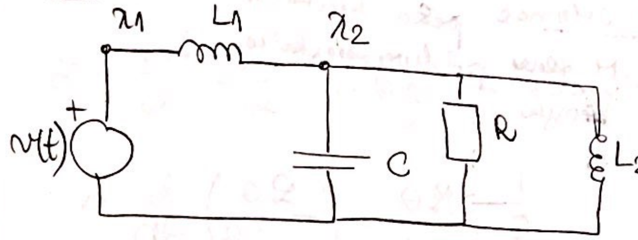
$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{z}}\right) - \frac{\partial \mathcal{L}}{\partial z} + \frac{\partial \mathfrak{J}}{\partial \dot{z}} = 0$$

$$m\ddot{z} + kz + b\dot{z} - \frac{1}{2}\dot{q}^2 \frac{\partial L(z)}{\partial z} = 0$$

ZADACI SA LABA

LAB4

Zad 1



$\Rightarrow$ generalisane koordinate x_1, x_2

$$\dot{x}_1 = v(t)$$

$$\mathcal{L} = E_{el}^* - E_m$$

$$E_{el}^* = \frac{1}{2} C \dot{x}_2^2$$

$$E_m = \frac{1}{2L_1}(x_1 - x_2)^2 + \frac{1}{2L_2} x_2^2$$

$$\mathfrak{J} = \frac{1}{2R}\dot{x}_2^2$$

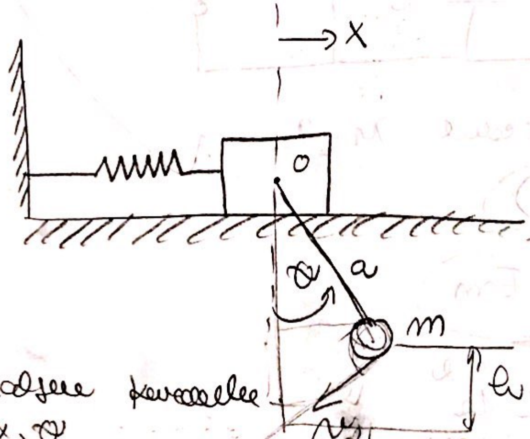
$$\mathcal{L} = \frac{1}{2} C \dot{x}_2^2 - \left[\frac{1}{2L_1}(x_1 - x_2)^2 + \frac{1}{2L_2} x_2^2\right]$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) - \frac{\partial \mathcal{L}}{\partial x_2} + \frac{\partial \mathfrak{J}}{\partial \dot{x}_2} = 0$$

$$C\ddot{x}_2 + \frac{1}{L_1}(x_1 - x_2)(-1) + \frac{1}{L_2}x_2 + \frac{1}{R}\dot{x}_2 = 0$$

$$\boxed{C\ddot{x}_2 + \frac{1}{R}\dot{x}_2 + x_2\left(\frac{1}{L_2} + \frac{1}{L_1}\right) - \frac{1}{L_1}x_1 = 0}$$

Zad2) Prosto klatno duljine na čijem kraju se nalazi masa m vezano je za tijelo mase M u točki O, oko koje može da se okreće. Masa M je vezana za nepomični oslonac preko opruge krutosti k. Tijelo mase M klizi po horizontalnoj ravni bez trenja



$\Rightarrow$ generalisane koordinate x, θ

$$\mathcal{L} = E_k^* - E_p$$

$$E_k = \frac{M v_1^2}{2} + \frac{m v_2^2}{2}$$

$\cos(90-\theta) = \sin\theta$

$\cos(180-\theta) = -\cos(\theta)$

$$v_1 = \dot{x}$$

$$v_2^2 = \dot{x}^2 + (a\dot{\theta})^2 - 2\dot{x}a\dot{\theta}\cos(180-\theta)$$

$$v_2^2 = \dot{x}^2 + (a\dot{\theta})^2 + 2\dot{x}a\dot{\theta}\cos(\theta)$$

$$E_p = \frac{kx^2}{2} + mga(1-\cos\theta)$$

$$\mathcal{L} = E_k^* - E_p$$

$$\mathcal{L} = \frac{M\dot{x}^2}{2} + \frac{m}{2}\left(\dot{x}^2 + (a\dot{\theta})^2 + 2\dot{x}a\dot{\theta}\cos(\theta)\right) - \frac{kx^2}{2} - mga(1-\cos\theta)$$

$$x:\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = M\ddot{x} + m\ddot{x} + ma\ddot{\theta}\cos\theta - ma\dot{\theta}\sin\theta\,\dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial x} = -kx$$

$$\underline{M\ddot{x} + m\ddot{x} + ma(\ddot{\theta}\cos\theta - \dot{\theta}\sin\theta\,\dot{\theta}) + kx = 0}$$

$$\theta:\ \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = ma^2\ddot{\theta} + ma\left(\ddot{x}\cos(\theta) - \dot{x}\sin(\theta)\dot{\theta}\right)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -ma\dot{x}\dot{\theta}\sin(\theta) - mga\sin(\theta)$$

$$\theta:\ ma^2\ddot{\theta} + ma\ddot{x}\cos\theta - \cancel{ma\dot{x}\sin(\theta)\dot{\theta}} + \cancel{ma\dot{x}\dot{\theta}}\sin(\theta) + mga\sin(\theta) = 0$$

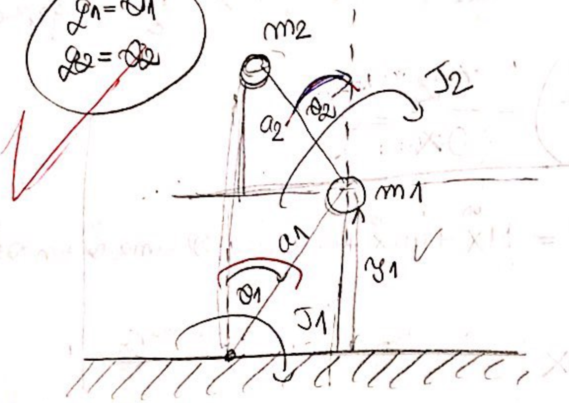
$$\boxed{ma^2\ddot{\theta} + ma\ddot{x}\cos\theta + mga\sin(\theta) = 0}$$

LAB 5

Zad 1) Robotski manipulator sa dva zgloba predstaviti u vektorskom prostoru

generalisane koordinate

$q_1 = \theta_1$
$q_2 = \theta_2$



$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = J_j \qquad (j = 1,2)$$

$$\mathcal{L} = E_k^* - E_p$$

$$E_k^* = \frac{m_1 v_1^2}{2} + \frac{m v_2^2}{2}$$

$$E_p = m_1 g a_1 \cos\theta_1 + m_2 g a_2 \cos\theta_2 \quad ?$$

$x(t)$, $y(t)$ } vremenske funkcije.

$$v^2 = \dot{x}^2 + \dot{y}^2$$

$x_1(t) = a_1 \sin\theta_1 \qquad y_1(t) = a_1 \cos\theta_1$

$$v_1^2 = \left[\frac{d}{dt}(a_1 \sin\theta_1)^2 + \frac{d}{dt}(a_1 \cos\theta_1)\right]^2$$

$$= a_1^2 \dot{\theta}_1^2 (\sin^2\theta_1 + \cos^2\theta_1) = a_1^2 \dot{\theta}_1^2$$

$x_2(t) = a_1 \sin\theta_1 + a_2 \sin\theta_2$

$y_2(t) = a_1 \cos\theta_1 + a_2 \cos\theta_2$

$$v_2^2 = [\dot{x}_2(t)]^2 + [\dot{y}_2(t)]^2$$

$$= [a_1 \cos\theta_1 \dot{\theta}_1 + a_2 \cos\theta_2 \dot{\theta}_2]^2 + [-a_1 \sin\theta_1 \dot{\theta}_1 - a_2 \sin\theta_2 \dot{\theta}_2]^2$$

$$= (a_1 \cos\theta_1 \dot{\theta}_1)^2 + 2 a_1 \cos\theta_1 \dot{\theta}_1 a_2 \cos\theta_2 \dot{\theta}_2 + (a_2 \cos\theta_2 \dot{\theta}_2)^2$$

$$+ (a_1 \sin\theta_1 \dot{\theta}_1)^2 + 2 a_1 \sin\theta_1 \dot{\theta}_1 a_2 \sin\theta_2 \dot{\theta}_2 + (a_2 \sin\theta_2 \dot{\theta}_2)^2$$

$\sin 2\alpha = 2 \sin\alpha \cos\alpha$

$$= a_1^2 \dot{\theta}_1^2 + a_2^2 \dot{\theta}_2^2 + 2 a_1 a_2 \dot{\theta}_1 \dot{\theta}_2 (\underbrace{\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2}_{\cos(\theta_1 - \theta_2)})$$

$$\mathcal{L} = \frac{m_1}{2} a_1^2 \dot{\theta}_1^2 + \frac{m_2}{2}\left[a_1^2 \dot{\theta}_1^2 + a_2^2 \dot{\theta}_2^2 + 2 a_1 a_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)\right]$$

$$- (m_1 + m_2) g a_1 \cos\theta_1 - m_2 g a_2 \cos\theta_2$$

?

$$\frac{\partial \mathcal{L}}{\partial \dot\theta_1} = m_1 a_1^2 \dot\theta_1 + m_2 a_1^2 \dot\theta_1 + 2 a_1 a_2 \dot\theta_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \dot\theta_2} = m_2 a_2^2 \dot\theta_2 + 2 a_1 a_2 \dot\theta_1 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = m_2 a_1 a_2 \dot\theta_1 \dot\theta_2 \left(-\sin(\theta_1 - \theta_2)\right) + (m_1 + m_2) g a_1 (\sin(\theta_1))$$

$$\frac{\partial \mathcal{L}}{\partial \theta_2} = m_2 a_1 a_2 \dot\theta_1 \dot\theta_2 \left(-\sin(\theta_1 - \theta_2)\right)(-1) + m_2 g a_2 \sin\theta_2 \ (1)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta_1}\right) = m_1 a_1^2 \ddot\theta_1 + m_2 a_1^2 \ddot\theta_1 + 2 a_1 a_2 \left(\ddot\theta_2 \cos(\theta_1 - \theta_2) - \dot\theta_2 \sin(\theta_1 - \theta_2)(\dot\theta_1 - \dot\theta_2)\right)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta_2}\right) = m_2 a_2^2 \ddot\theta_2 + 2 a_1 a_2 \left(\ddot\theta_1 \cos(\theta_1 - \theta_2) - \dot\theta_1 \sin(\theta_1 - \theta_2) \cdot (\dot\theta_1 - \dot\theta_2)\right)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta_1}\right) - \frac{\partial \mathcal{L}}{\partial \theta_1} = 0$$

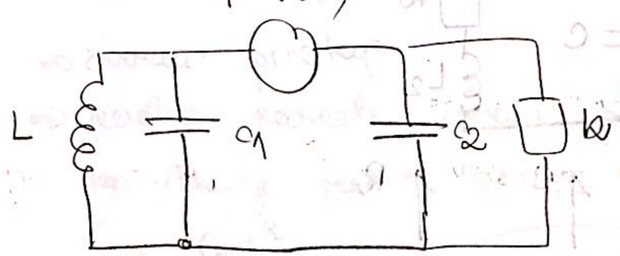
$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2} = 0$$

θ_1: $m_1 a_1^2 \ddot\theta_1 + m_2 a_1^2 \ddot\theta_1 + 2 a_1 a_2 \left(\ddot\theta_2 \cos(\theta_1 - \theta_2) - \dot\theta_2 \sin(\theta_1 - \theta_2)(\dot\theta_1 - \dot\theta_2)\right) + m_2 a_1 a_2 \dot\theta_1 \dot\theta_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g a_1 \sin\theta_1 = 0$

θ_2:

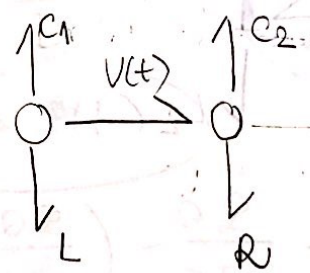
$m_2 a_2^2 \ddot\theta_2 + 2 a_1 a_2 \left(\ddot\theta_1 \cos(\theta_1 - \theta_2) - \dot\theta_1 \sin(\theta_1 - \theta_2)(\dot\theta_1 - \dot\theta_2)\right) - m_2 a_1 a_2 \dot\theta_1 \dot\theta_2 \cdot \sin(\theta_1 - \theta_2) - m_2 g a_2 \sin\theta_2 = 0$

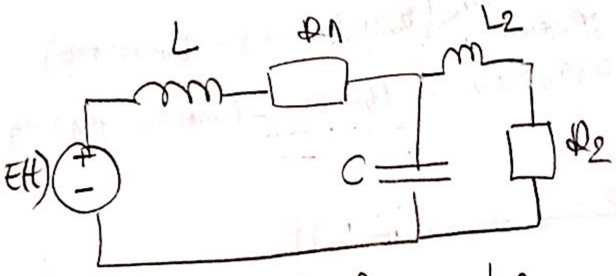
BOND GRAF od Lele

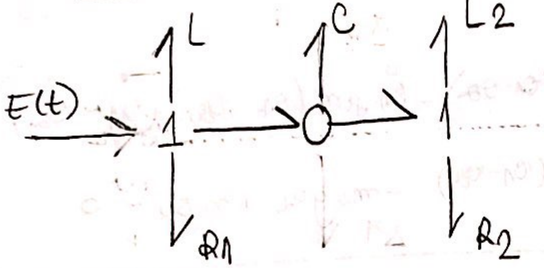


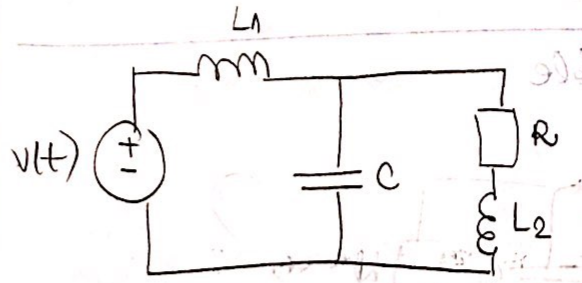
?

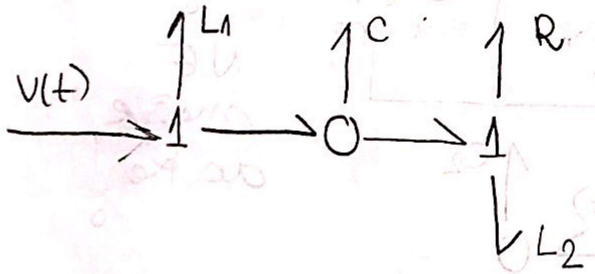
NE može dalje!





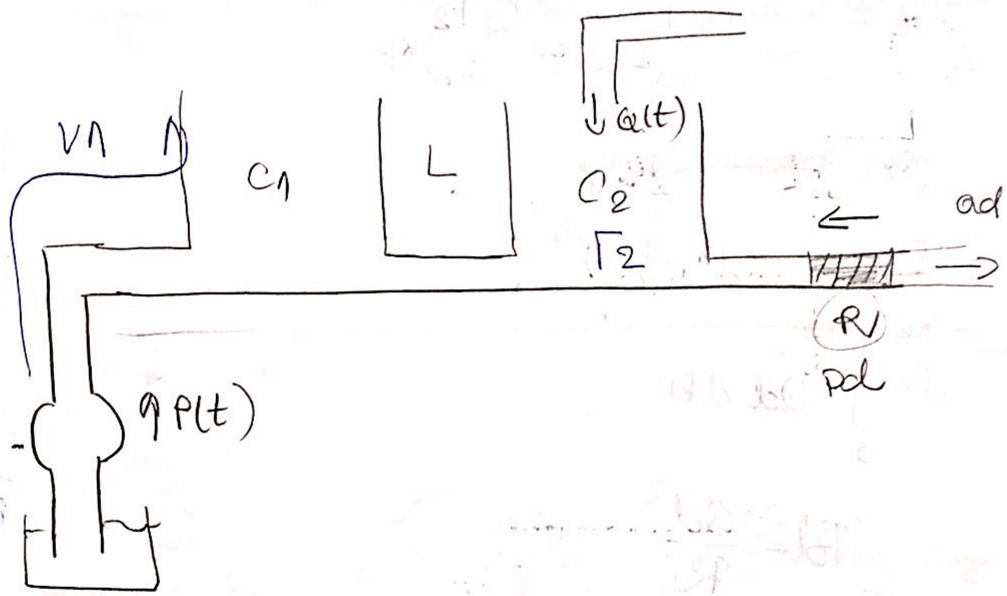






LAB 5

Zad2 Kombinovana i čvorna metoda



⇒ čvorni pristup

⇒ generalisane koordinate momenti - preko Γ_1 i Γ_2 koje akumuliraju pritiska p1 i p2

$$P_d = \frac{Q d^2}{D}$$

$$\dot{V}_1 = P(t)$$

$$\dot{\Gamma}_1 = P(t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Gamma}_2}\right) - \frac{\partial \mathcal{L}}{\partial \Gamma_2} + \frac{\partial J}{\partial \dot{\Gamma}_2} = Q(t)$$

$\mathcal{L} = E_f^* - E_c$ ~~potencjalna koenergia~~

$$E_f^* = \frac{1}{2} C_1 \dot{\Gamma}_1^2 + \frac{1}{2} C_2 \dot{\Gamma}_2^2$$

V_1^2

~~Kinetyczna energia~~:

$$E_C = \frac{1}{2L} (\Gamma_1 - \Gamma_2)^2$$

– ko sadzaj:

$$J = \int_0^{R} Q_d \, dR$$

$$P_d = \frac{Q_d^2}{R}$$

$$J = \int_0^{P_d} \sqrt{R}\sqrt{P_d}\, dP_d = \sqrt{R}\,\frac{2}{3} P_d^{3/2} = \sqrt{R}\,\frac{2}{3} \dot{\Gamma}_2^{3/2}$$

$$J = \frac{2}{3} \sqrt{R}\, \sqrt{\dot{\Gamma}_2}^{\,3}$$

$$\mathcal{L} = \frac{1}{2} C_1 \dot{\Gamma}_1^2 + \frac{1}{2} C_2 \dot{\Gamma}_2^2 - \frac{1}{2L} (\Gamma_1 - \Gamma_2)^2$$

$$C_2 \ddot{\Gamma}_2 - \frac{1}{L} (\Gamma_1 - \Gamma_2) + \sqrt{D}\, \frac{\cancel{2}}{\cancel{3}} \frac{\cancel{3}}{\cancel{2}} \dot{\Gamma}_2^{1/2} = Q(t)$$

$$C_2 \ddot{\Gamma}_2 - \frac{1}{L} (\Gamma_1 - \Gamma_2) + \sqrt{D}\, \sqrt{\dot{\Gamma}_2} = Q(t)$$

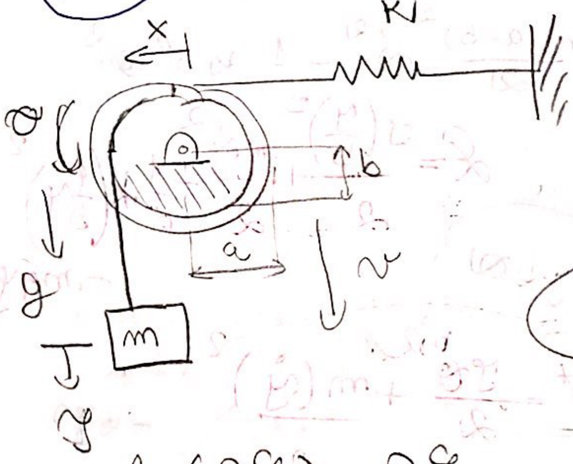
$$J = \frac{1}{2R} \dot{\Gamma}_2^2$$

odwrotny efekt

P_d

Rok 2012/13

Zad 1



a – promień koła większego

b – mniejszego ?

$$\omega = \dot{\theta}$$

θ generalizowana koordinata

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q} = F_i$$

$$L = E_f^* - E_e$$

$$= E_f^*(\dot{\theta}) - E_e(\theta)$$

$x = a \cdot \theta$

$y = b\,\theta$

$\dot{y} = b\,\dot{\theta}$

$\mathcal{L} = E_k^* - E_p$

$E_k^* \rightarrow$ kinetička koenergija

$E_k^* = \frac{1}{2} m b \dot{\theta}^2 + \frac{1}{2} J_0 \dot{\theta}^2$ $\frac{m\dot{y}^2}{2}$

translacija rotacija tijela

$J_0 = \frac{m(a-b)^2}{2}$ moment inercije

sustav

$E_p = \frac{1}{2} k x^2 = \frac{1}{2} k a^2 \theta^2$

$\mathcal{L} = \frac{1}{2} m b \dot{\theta}^2 + \frac{1}{2} \frac{m(a-b)^2}{2} \dot{\theta}^2 - \frac{1}{2} k a^2 \theta^2$

$\cancel{J = mgb}$

$\mathcal{L} = \frac{J\left(\frac{\dot{y}}{b}\right)^2}{2} + \frac{m\dot{y}^2}{2} - \frac{1}{2} k \left(\frac{a}{b} y\right) + mg$

$mgy = mgb\theta$

$\theta = \text{const}$

$x = a \cdot \theta$

$y = b \cdot \theta$

$E_k^* = \frac{J\dot{\theta}^2}{2} + \frac{m(\dot{y})^2}{2}$

$E_p = \frac{1}{2} k x^2 + mg(-y)$?

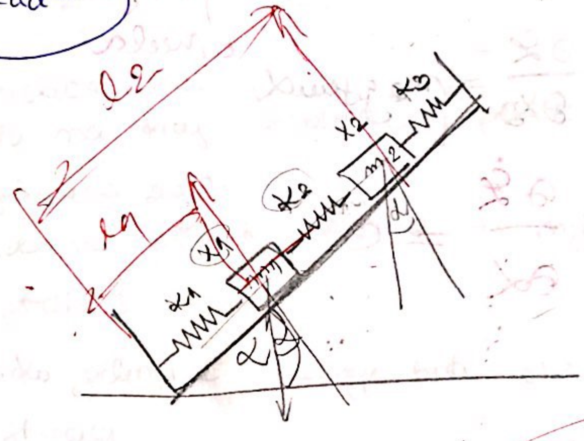
$\dot{y} = b\dot{\theta}$

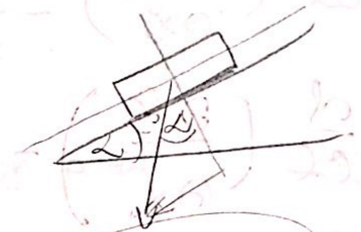
$\theta = \frac{x}{a} = \frac{y}{b}$

$\dot{\theta} = \frac{\dot{y}}{b}$

rok 2014/15

Zad 2





$F_{1x} = m_1 g \sin\alpha$

$F_{2x} = m_2 g \sin\alpha$

x_1, x_2 - generalizirane koordinate

$v_1^2 = (\dot{x}_1)^2 + (l_1\dot{\alpha})^2$

$v_2^2 = (\dot{x}_2)^2 + (l_2\dot{\alpha})^2$

$\mathcal{L} = E_k^* - E_p$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial J}{\partial \dot{q}} = I_j$

$E_p = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 x_2^2$
$+ m_1 g x_1 (1 - \sin\alpha) + m_2 g x_2 (1 - \sin\alpha)$

$E_k = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$

Ako nema disipacije $J = 0$

$\mathcal{L} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \left(\frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_1 - x_2)^2 + \frac{1}{2} k_3 x_2^2\right)$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) - \frac{\partial \mathcal{L}}{\partial x_1} = m g \sin\alpha$$

uzeo ovdje nula

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right) - \frac{\partial \mathcal{L}}{\partial x_2} = m_2 g \sin\alpha$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}}\right) - \frac{\partial \mathcal{L}}{\partial \alpha} = 0$$

ište radeš samo do po $\dot{x}_1$ i $\dot{x}_2$ i sređeni pile lavel

BOND grafovi

PRAVILA:

strelica za L:
ide na kraj strelice tj pored L

strelica za C:
ide na početak strelice tj nije pored C

strelice
ako jedinice trebaju biti sve crtice na jednoj strelici

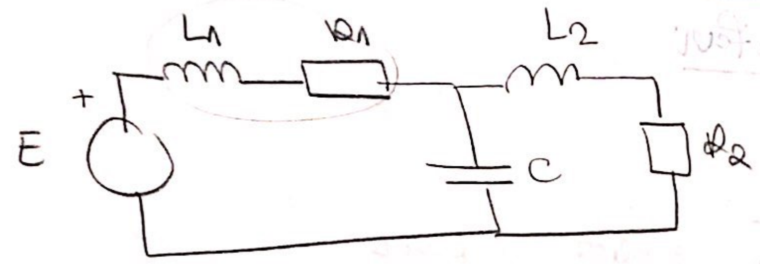
strelice ako nule – trebaju biti sve prema crtom jedne crtice strelice

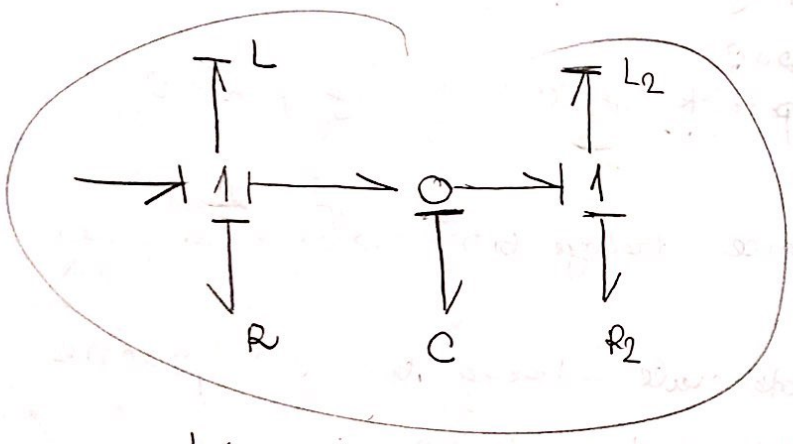
strelice ⊕ –
naštimaj da vrijede prethodno za 0 i 1 jer ovise tj kod je

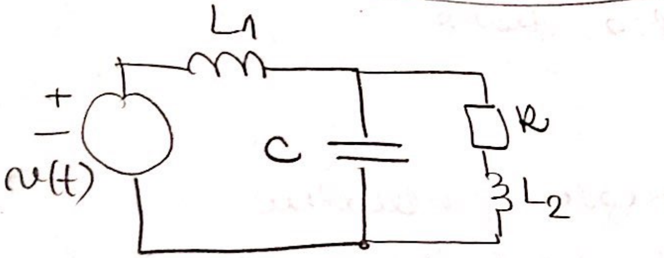
(*) ako ne može biti zadovoljeno sve od ovog – onda je greška da budu zadovoljena pravila za 0 i 1, a da je prekršeno nešto od pravila za L i C.

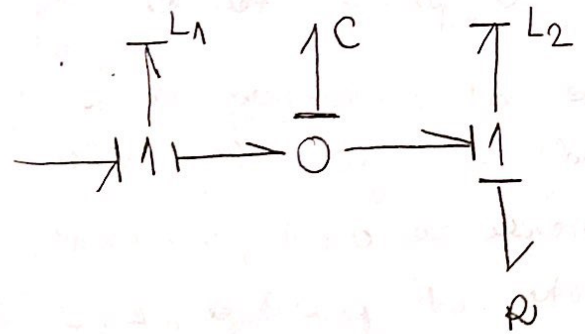
⇓

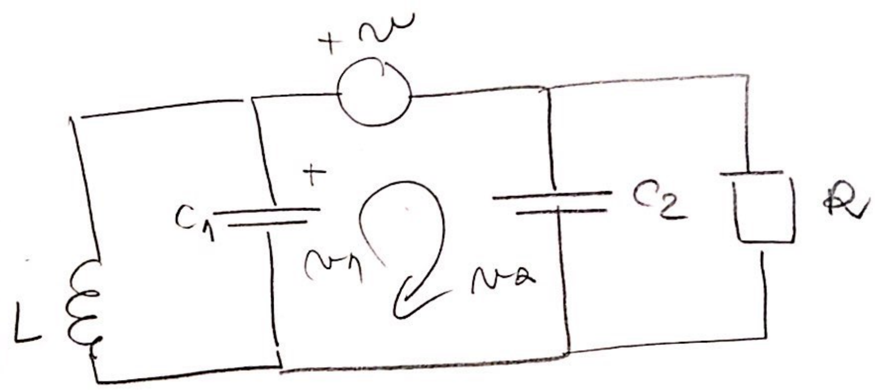
diferencijalna kauzalnost.

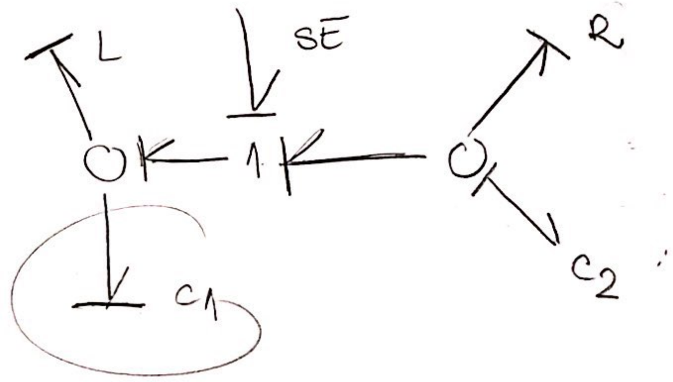












Nouvelle
difeer
Keudelf

$v + v_2 - v_1 + 0$

$\Rightarrow v + \frac{q_2}{C_2} - \frac{q_1}{C_1} = 0$