2.1(b)

Friday, September 15, 2023 10:10 PM

Time Invariant

this condition is true when $\boxed{y(t-\tau) = 100\,x(t-\tau)}$ where $\tau = 2$

$y(t) = 100\,x(t)$

$x(t) = 20\cos(2\pi t)\,u(t)$

$x(t-\tau) = 20\cos[2\pi(t-\tau)]\,u(t-\tau)$

$100\,x(t-\tau) = 100[20\cos[2\pi(t-\tau)]u(t-\tau)]$

$100\,x(t-\tau) = 2000\cos[2\pi(t-\tau)\,u(t-\tau)]$

$y(t-\tau) = 100\cdot[20\cos(2\pi(t-\tau))\,u(t-\tau)]$

$y(t-\tau) = 2000\cos[2\pi(t-\tau)]\,u(t-\tau)]$

$\therefore$ The system is time Invariant

2.5

Saturday, September 16, 2023 5:03 PM

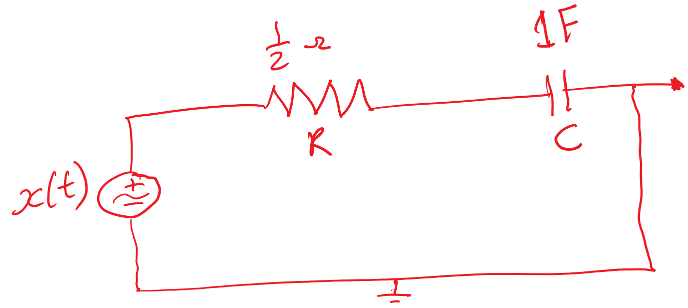
ODE of RC circuit $\Rightarrow \dfrac{dv(t)}{dt} + \dfrac{1}{RC} v(t) = 0$

for the said circuit ODE $\Rightarrow \dfrac{dy(t)}{dt} + 2\,y(t) = 2x(t)$

$\dfrac{1}{RC} = 2$

$R = \dfrac{1}{2C}$ when $C = 1F$

$\boxed{R = \frac{1}{2}\,\Omega}$



2.5b

initial condition $x(t) = u(t)$

output $y(t) = e^{-2t} \int_0^t e^{2\tau}\, d\tau = \dfrac{1}{e^{2t}} \boxed{\int_0^t e^{2\tau}\, d\tau}$

$\int_0^t e^{2\tau}\, d\tau = \left[\frac{1}{2} e^{2\tau} + C\right]_0^t$ let $u = 2\tau$ $\quad \frac{du}{d\tau} = 2 \quad \therefore d\tau = \frac{1}{2} du$

$= \frac{1}{2} e^{2t} + C - \frac{1}{2} e^{0} - C$

$= \frac{1}{2}\left[e^{2t} - 1\right]$

$y(t) = \dfrac{1}{e^{2t}} \cdot \dfrac{1}{2}\left[e^{2t} - 1\right]$

$y(t) = \dfrac{1}{2}\left[\dfrac{e^{2t}}{e^{2t}} - \dfrac{1}{e^{2t}}\right]$

$y(t) = \dfrac{1}{2}\left[1 - \dfrac{1}{e^{2t}}\right]$

Since the initial condition entails a unit step function

$\boxed{y(t) = \frac{1}{2}\left[1 - e^{-2t}\right] u(t)}$

2.7

Sunday, September 17, 2023 9:38 PM

a]

$$q(t) = c(t)\,v(t)$$

$$i(t) = \frac{dq(t)}{dt} \Rightarrow \int i(t)\,dt = \int \frac{dq(t)}{\cancel{dt}}\,\cancel{dt}$$

$$\int i(t)\,dt = q(t)$$

Since $q(t) = c(t)\,v(t)$ & $q(t) = \int i(t)\,dt$

$$\therefore\ c(t)\,v(t) = \int i(t)\,dt$$

$$\boxed{v(t) = \frac{1}{c(t)} \int i(t)\,dt}$$

OK

$$\boxed{i(t) = \frac{d}{dt}\,c(t)\,v(t) \quad \text{product Rule} \quad i(t) = c(t)\frac{dv(t)}{dt} + v(t)\frac{dc(t)}{dt}}$$

b]

$$c(t) = 1 + \cos(2\pi t) \qquad v(t) = \cos(2\pi t)$$

$$i(t) = \left[1 + \cos(2\pi t)\right]\frac{d}{dt}\left[\cos(2\pi t)\right] + \cos(2\pi t)\frac{d}{dt}\left[1 + \cos(2\pi t)\right]$$

$$i(t) = -2\pi \sin(2\pi t)\left[1 + \cos(2\pi t)\right] + \cos(2\pi t)\left[0 - 2\pi \sin(2\pi t)\right]$$

$$i(t) = -2\pi \sin(2\pi t) - 2\pi \sin(2\pi t)\cos(2\pi t) - 2\pi \cos(2\pi t)\sin(2\pi t)$$

$$i(t) = -2\pi \sin(2\pi t) - 4\pi \sin(2\pi t)\cos(2\pi t)$$

$$\boxed{i(t) = -2\pi \sin(2\pi t)\left[1 + 2\cos(2\pi t)\right]}$$

c) when $c(t)$ stays the same and $v(t)$ is delayed by 0.25 sec

$$i_2(t) = c(t)\frac{d}{dt}v(t - \tfrac{1}{4}) + v(t - \tfrac{1}{4})\frac{d}{dt}c(t)$$

$$i_2(t) = \left[1 + \cos(2\pi t)\right]\frac{d}{dt}\left[\cos\left(2\pi(t - \tfrac{1}{4})\right)\right] + \cos\left(2\pi(t - \tfrac{1}{4})\right)\frac{d}{dt}\left[1 + \cos(2\pi t)\right]$$

$$i_2(t) = \left[1 + \cos(2\pi t)\right]\frac{d}{dt}\left[\cos(2\pi t - \tfrac{\pi}{2})\right] + \cos(2\pi t - \tfrac{\pi}{2})\frac{d}{dt}\left[1 + \cos(2\pi t)\right]$$

$$i_2(t) = -2\pi \sin\left(2\pi t - \frac{\pi}{2}\right)\left[1 + \cos(2\pi t)\right] + \cos\left(2\pi t - \frac{\pi}{2}\right)\left[0 - 2\pi \sin(2\pi t)\right]$$

note $\cos(2\pi \cdot n) = 1$ where $-\infty < n < \infty$

$$i_2(t) = 2\pi \sin\left(2\pi t - \frac{\pi}{2}\right)\left[1 + 1\right] + \cos\left(2\pi t - \frac{\pi}{2}\right)\left[-2\pi \sin(2\pi t)\right]$$

$$i_2(t) = 4\pi \sin\left(2\pi t - \frac{\pi}{2}\right) - 2\pi \sin(2\pi t)\cos\left(2\pi t - \frac{\pi}{2}\right)$$

$$\boxed{i_2(t) = 2\pi\left[2\sin\left(2\pi t - \frac{\pi}{2}\right) - \sin(2\pi t)\cos\left(2\pi t - \frac{\pi}{2}\right)\right]}$$

```
%create a custom function for system in question 2.1 HW4 EE480 Fall 2023
%PSU World Campus
function y = mySystem1(x)
y = 100*x;
end
```

```
% Name: Lamin Jammeh
% Class: EE480 Online
% Semster: Fall 2023
% HW_4

% Basic Problems
%% ********** question 2.1 **********
% ********** question 2.1(ai)**********
 clear;
 clc;
x = -20:20; %define the range for the X-axis
y = 100 * x; %y(t) @ -10<x(t)<10
y(x>10) = 1000; % y(t) when x(t)>10
y(x<-10) = -1000; % y(t) when x(t)<-10
plot(x, y, 'LineWidth',2); %plot x(t) vs y(t) at all stages
xlabel('x(t)'); %label x-axis
ylabel('y(t)'); %labels y-axis
title('x(t) vs y(t) for Q2.1a'); %assign a plot title
grid on;

%% ********** question 2.1(aii)**********
%test for linearity using the scaling method since there is only one input
%x(t)
%perform the scale test for Output scenario 1

%Scale Test alpha*y(t) = alpha*S[x(t)]
 clear;
 clc;
x = -10:10; % define x(t) from -10:10
alpha = 10; %scaling factor is 10


y = mySystem1(x); %mySystem1 is a function defined as y=100x(t)
y_alpha = alpha * mySystem1(x);

 %condition for linearity
 System_is_Linaer = isequal(y_alpha,alpha * y);
  if System_is_Linaer
      disp('The System is Linear @ -10<x(t)<10');
  else
      disp('System is not Linear @ -10<x(t)<10');
  end

 %% ********** question 2.1(b) **********
 clear;
 clc;
 %notee am changing the y(t) from 100x(t) to be 2x(t) for better resultion on input x
(t)
 t = -2:0.001:4; %range of value for t
```

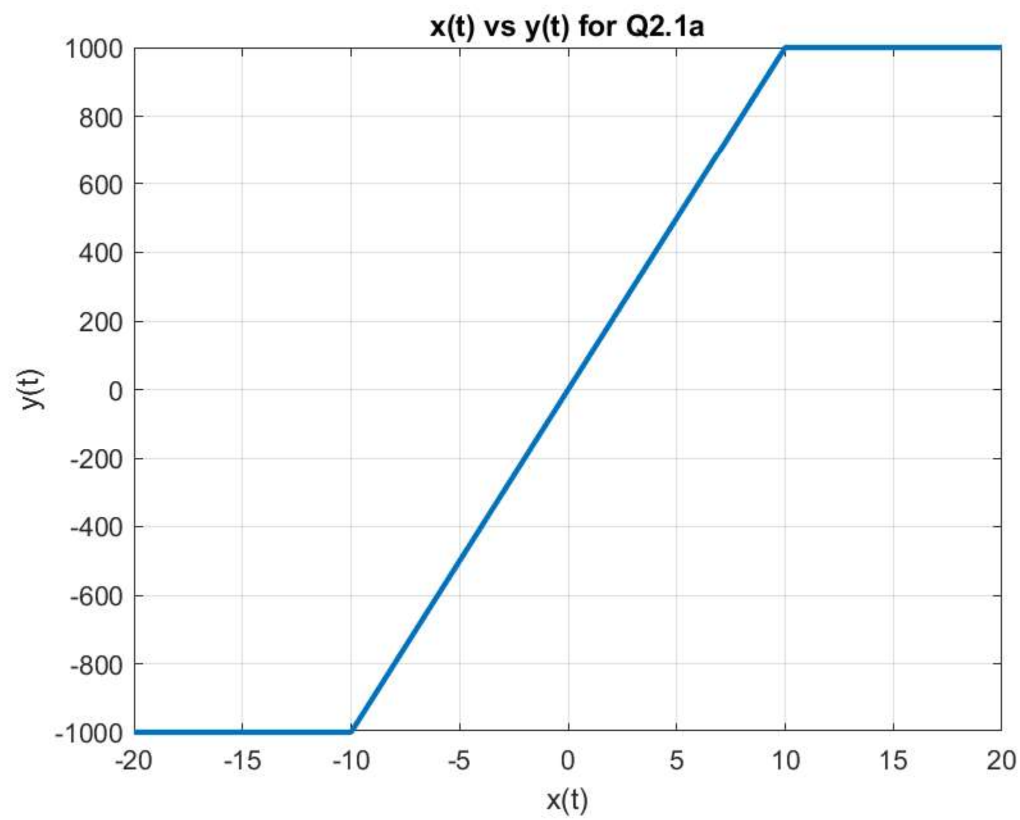
```
 u = heaviside(t); % use unit step function already build in matlab as heaviside
 x = 20 * cos(2*pi*t) .* u; %define Input of the system with the sinusoid wave
 y = 2.*(x);%define the output of the system with the system
 figure;
 plot(t,x,'r');
 hold on;
 plot(t,y,'b','LineWidth',2);
 xlabel('t');
 ylabel('Amplitude');
 legend('input=x(t)','output=y(t)')
 title('x(t) vs y(t)');
 grid on;
hold off;

 % *********** question 2.1(c) **********

 t = -2:0.001:4; %range of value for t
 u1 = heaviside(t-2); % use unit step function already build in matlab as heaviside
 x1 = 20 * cos(2*pi*t-2) .* u1; %define Input of the system with the sinusoid wave
 y1 = 2.*(x1);%define the output of the system with the system
 figure;
 plot(t,x1,'r');
 hold on;
 plot(t,y1,'b','LineWidth',2);
 xlabel('t');
 ylabel('Amplitude');
 legend('input=x1(t)','output=y1(t)')
 title('x1(t) vs y1(t)');
 grid on;
 hold off;

 % ********** compare the y(t) to y1(t) or the delayed response **********
 figure;
 plot(t,y,'g','LineWidth',2);
 hold on;
 plot(t,y1,'r','LineWidth',3);
 xlabel('t');
 ylabel('Amplitude');
  legend('input=x1(t)','output=y1(t)')
 title('y(t) vs y1(t)');
 grid on;
 hold off;

 %% ********** question 2.5(b) **********
 clear;
 clc
t = -10:0.01:10; % t range should show peak of y(t) of not increase the range
y = 1/2 .* (1-exp(-2.*t)) .* heaviside(t); %use matrix multiplication '.*' to include
all values of t
```

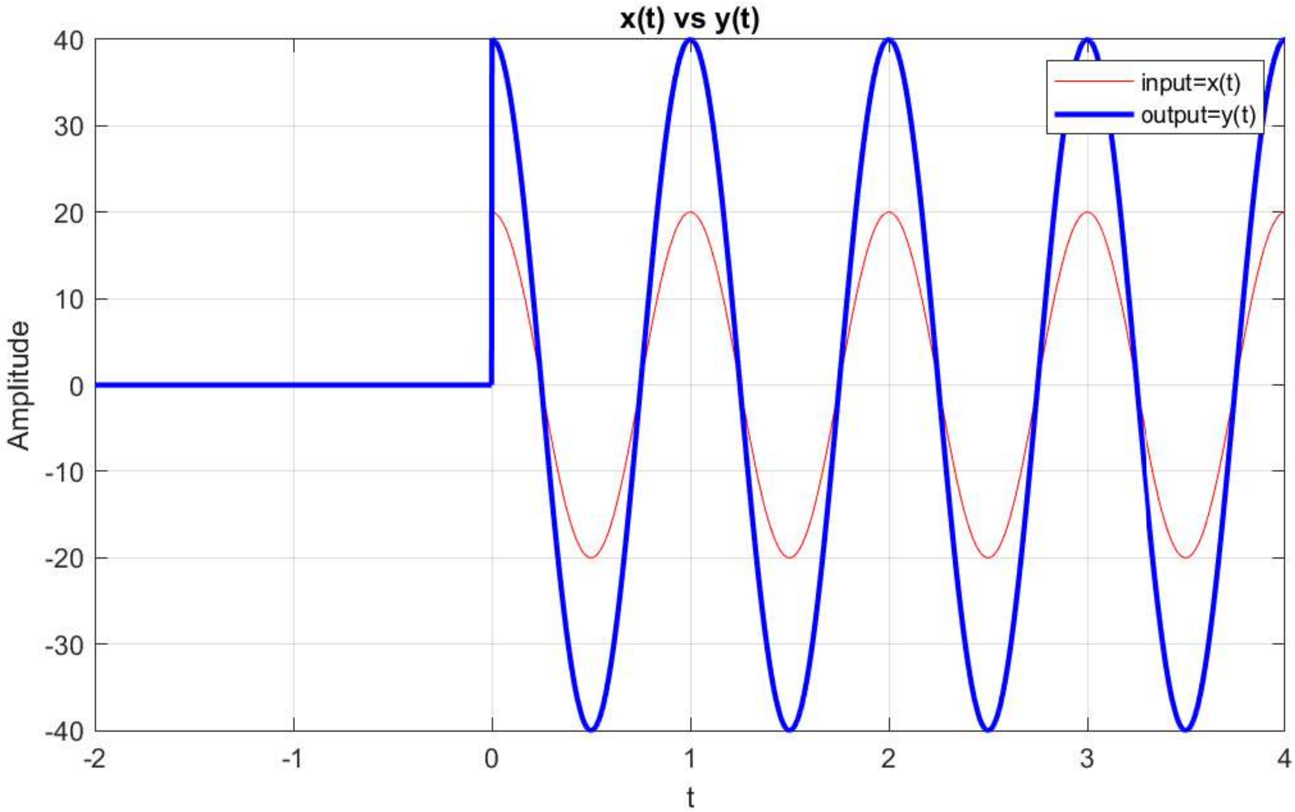
```
plot(t,y,'LineWidth',2);
xlabel('t');
ylabel('y(t)');
title('y(t) plot for question 2.5(b)')
grid on;

%% ********** question 2.22(a) **********
clear;
clc;
% Vo(t) = R(t)Vi(t)
t = 0:0.01:0.2; % time range
u = heaviside (t);
R = (1 + 0.5*cos(20*pi*t)).*u;
Vi = 1; %input voltage
Vo = -R * Vi;
figure;
plot(t,Vo, 'LineWidth',2);
xlabel('t');
ylabel('Vo');
title('Output Vo = R * Vi')
grid on;

% ********** question 2.22(b) **********
%if the switch close at t0=50msec
% Vo(t+(0.05) = R(t+0.05)Vi(t+0.05)
t0 = 0.05;        % Starting time in seconds (50 ms)
t1 = t0:0.01:0.2; % Time vector from t0 to 0.2 seconds with a step of 1 ms

u1 = heaviside (t0); % Unit step function starting at t0


% Calculate the values of R(t) for the given time vector:
R1 = (1 + 0.5 * cos(20 * pi* (t1 - t0))) .* u1;

% Calculate Vo(t) using the equation Vo(t) = R(t) * Vi(t):
Vo1 = -R1 .* Vi;

% Plot Vo(t):
figure;
plot(t1, Vo1);
xlabel('Time (s)');
ylabel('Vo(t) (V)');
title('Plot of Vo(t) with t0 = 50 ms');
grid on;



% ********** Reason for Time invariant **********
```

```
% the unit step response u(t) makes the system Time Invariant because it
% only allows the system to be active at t>=0 irrespective of the anytime
% shifting

% Vi(t) =1V which is time independent so the input voltage of the system won't change ↙
with time

%R(t) has a cos(20pit) which is equal to cos(10*2pi*t) with make the R(t)
%time invariant sine cos(2pi)=cos(0)=1 and any number multiply by zero is
%zero regarles of time shifting

%% ********** question 2.25 **********
clear;
clc;
% Note the Impulse response concoluted with the Input of the system gives
% the output
%Step 1 convolute h(t) with x(t)
%Step 2 Apply the diode effect

% Define Impulse Response h(t)=e^(-2t).u(t)
figure;
t = 0:0.001:20;
u = heaviside(t); % u(t)
h = exp(-2*t).*u;
subplot(2,2,1)
plot(t,h,'LineWidth',2);
xlabel('Time (s)');
ylabel('h(t)');
title("Impulse Response h(t)=e^(-2t).u(t)");
grid on;

% Define input x1(t)=cos(2pi*t)[u(t)-u(t-20)]
u1 = heaviside(t-20); % defines u(t-20)
x1_t = cos(2*pi*t).*(u-u1); %Defines x1(t)
subplot(2,2,2)
plot(t,x1_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('x1(t)');
title("Input x1(t)=cos(2pi*t)[u(t)-u(t-20)]");
grid on;

% Step 1 convolute x(t) with h(t)
y_t = conv(x1_t,h,"same"); % Output without diode effect
subplot(2,2,3)
plot(t,y_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('y(t)');
title('Output without Diode effect');
grid on;
```

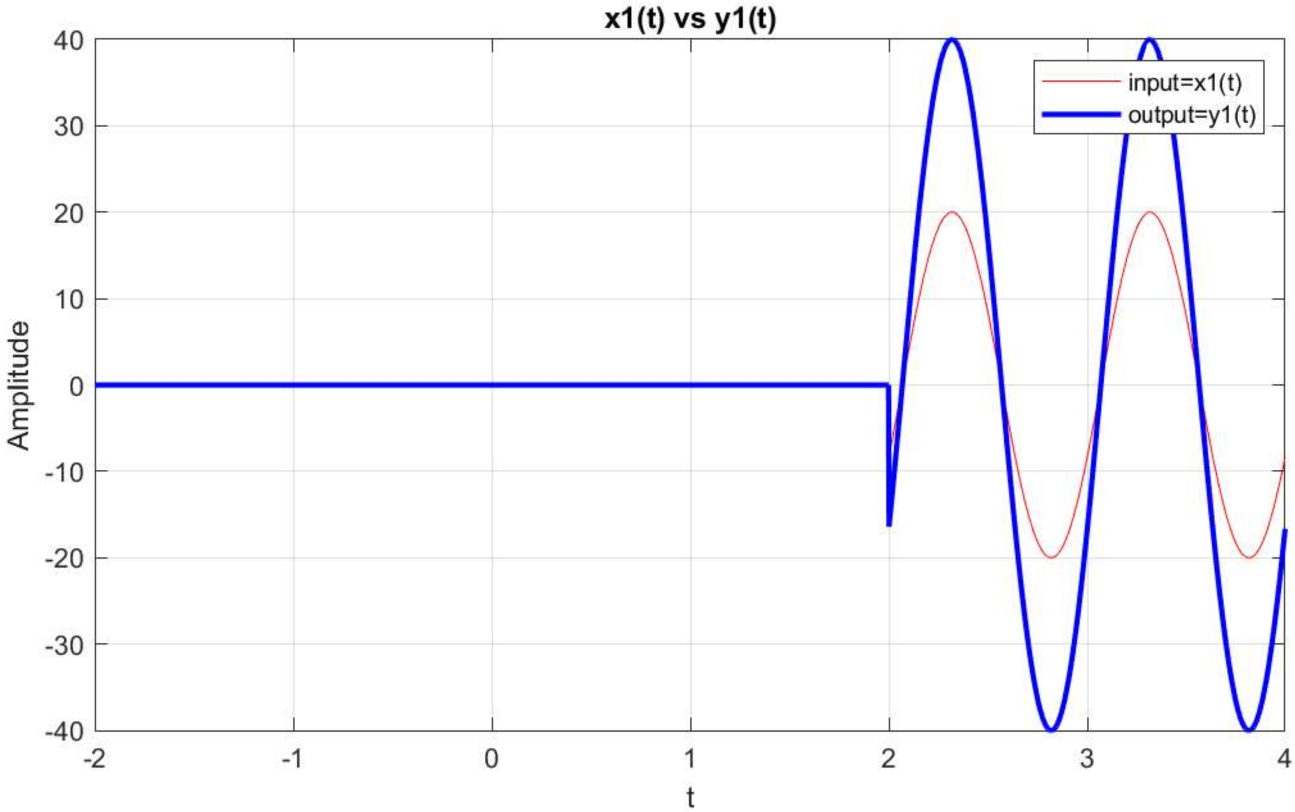
```
% Apply Diode effect this will clip all negative values of y(t)
d = max(y_t,0); % Diode effect
y1_t=(y_t).*d;   % Apply diode effect to the output
subplot(2,2,4)
plot(t,y1_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('y1(t)');
title('Output with Diode effect');
grid on;


% ********** using input x2(t) **********

% Define input x2(t)=sin(pi*t)e^(-20t)[u(t)-u(t-20)]
figure;
x2_t = sin(pi*t).*exp(-20*t).*(u-u1); %Defines x1(t)
subplot(2,2,1)
plot(t,x2_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('x2(t)');
title("Input x2(t)=sin(pi*t)e^(-20t)[u(t)-u(t-20)]");
grid on;

% Step 1 convolute x(t) with h(t)
y_t2 = conv(x2_t,h,"same"); % Output without diode effect
subplot(2,2,2)
plot(t,y_t2,'LineWidth',2);
xlabel('Time (s)');
ylabel('y(t)');
title('Output without Diode effect');
grid on;

% Apply Diode effect this will clip all negative values of y(t)
d = max(y_t2,0); % Diode effect
y2_t=(y_t2).*d;   % Apply diode effect to the output
subplot(2,2,3)
plot(t,y2_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('y2(t)');
title('Output with Diode effect');
grid on;

% ********** using input x3(t) **********
%input x3(t)=r(t)-2r(t-2)+r(t-4)

% Step 1 define input x3(t) into 3 sections r(t),-2r(t-2), and r(t-4)
figure;
x3(t >= 0) = 1; % defines the r(t) as unit step
```

```
x3(t >= 2) = -2 * exp(-2 * (t(t >= 2)-2)); % defines -2r(t-2) @t>=2
x3(t >= 4) = exp(-2*(t(t >= 4)-4)); % defines r(t-4) @t>=4

subplot(3,1,1)
plot(t,x3,'LineWidth',2);
xlabel('Time (s)');
ylabel('x3(t)');
title("Input x3(t)=r(t)-2r(t-2)+r(t-4)");
grid on;

% Step2 defines output y3(t) for input x3(t)
y3 = conv(x3, h, 'full'); % convolute x3(t)*h(t) for all parts of x3(t)

y3 = y3(1:length(t)); % define length of y3 to be the same as range of t

subplot(3,1,2)
plot(t,y3,'LineWidth',2);
xlabel('Time (s)');
ylabel('y(t)');
title('Output without Diode effect');
grid on;

% Step3 Apply Diode effect this will clip all negative values of y(t)
d = max(y3,0); % Diode effect
y3_t=(y3).*d;   % Apply diode effect to the output
subplot(3,1,3)
plot(t,y3_t,'LineWidth',2);
xlabel('Time (s)');
ylabel('y3(t)');
title('Output with Diode effect');
grid on;
```

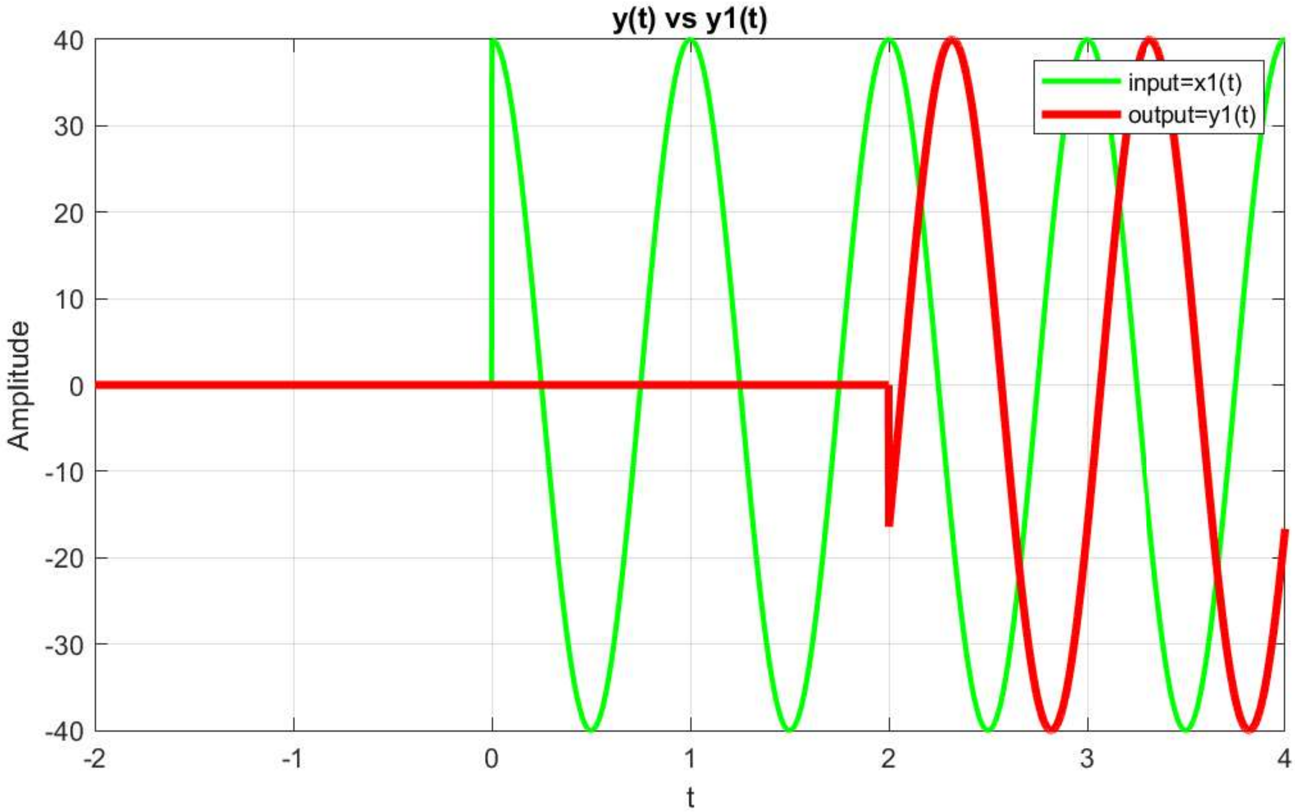

x(t) vs y(t) for Q2.1a
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
y(t)
-20
-15
-10
-5
0
5
10
15
20
x(t)



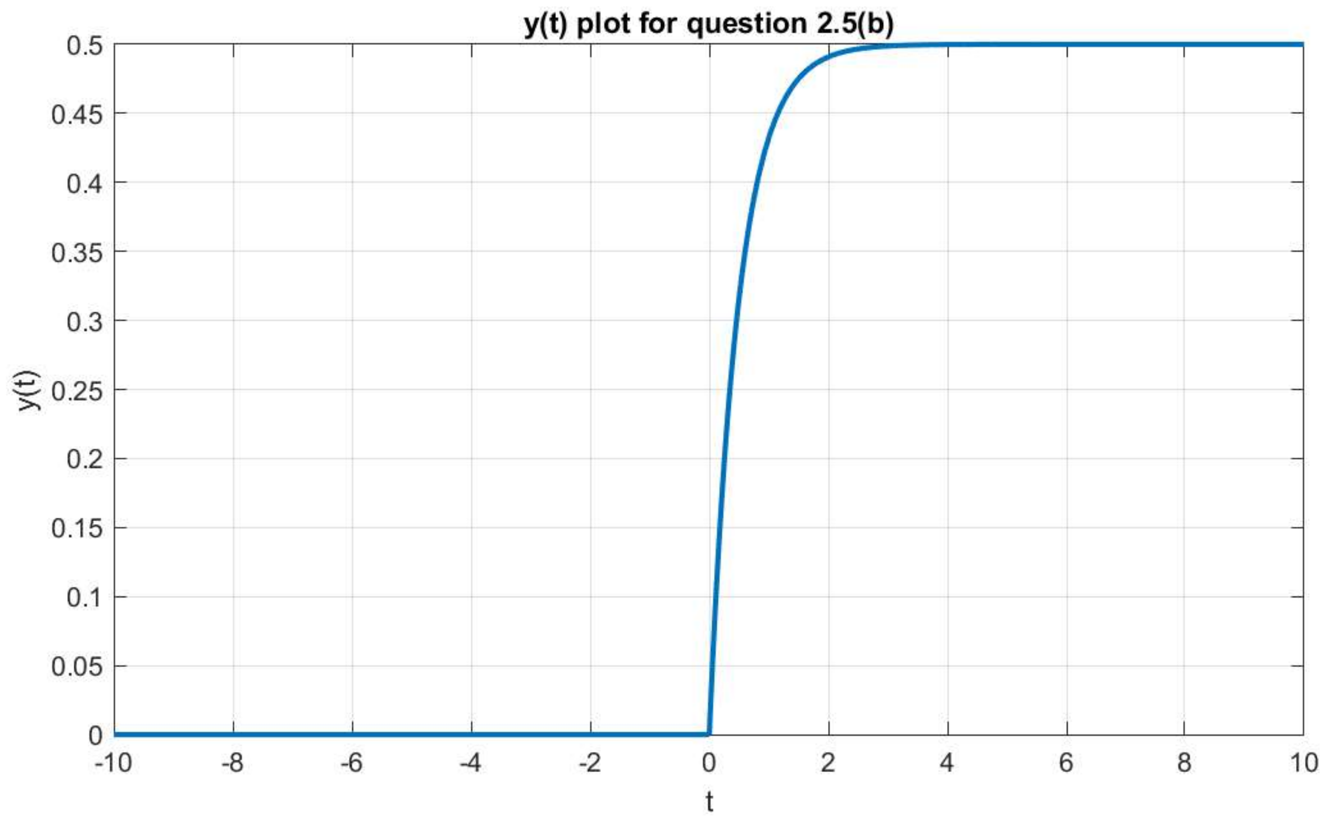
x(t) vs y(t)
Amplitude
t
input=x(t)
output=y(t)
40
30
20
10
0
-10
-20
-30
-40
-2
-1
0
1
2
3
4

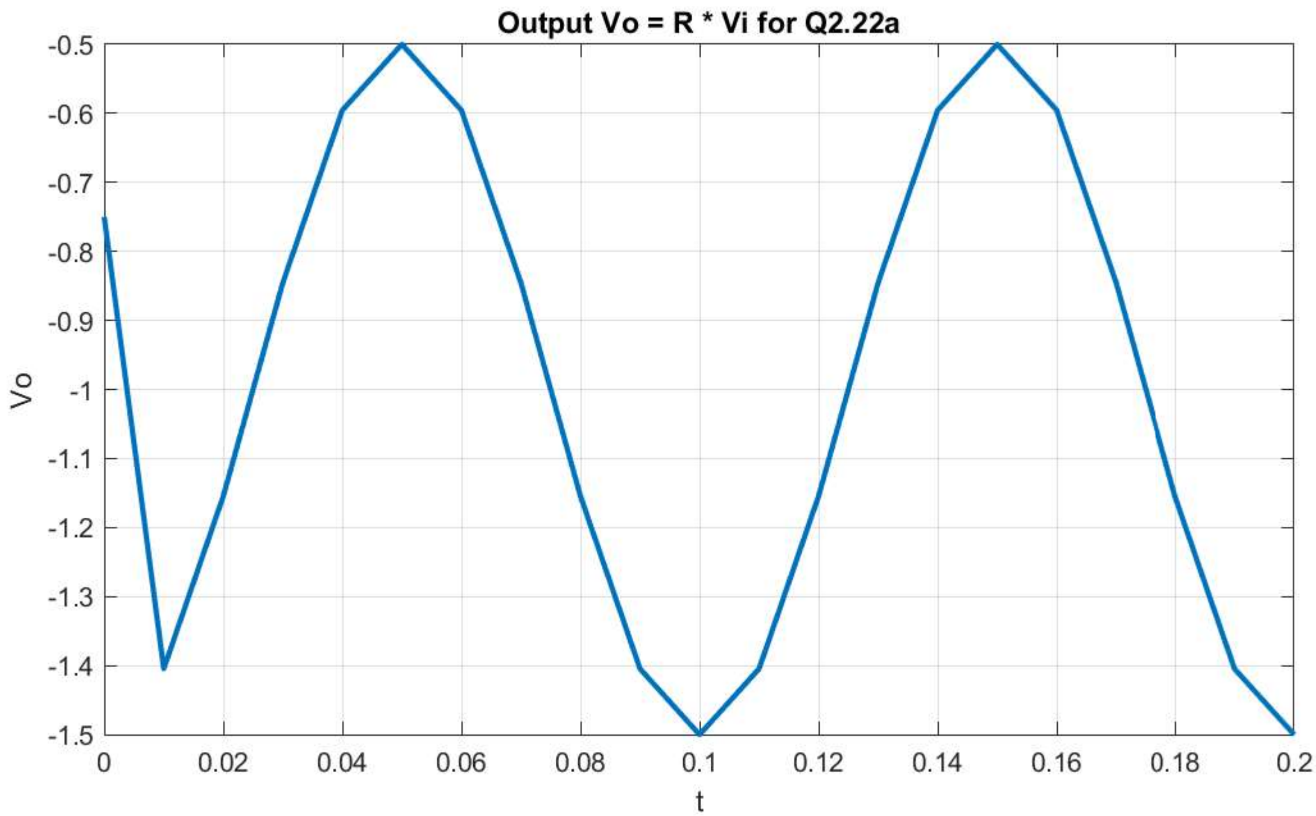


x1(t) vs y1(t)
input=x1(t)
output=y1(t)
Amplitude
t
40
30
20
10
0
-10
-20
-30
-40
-2
-1
0
1
2
3
4

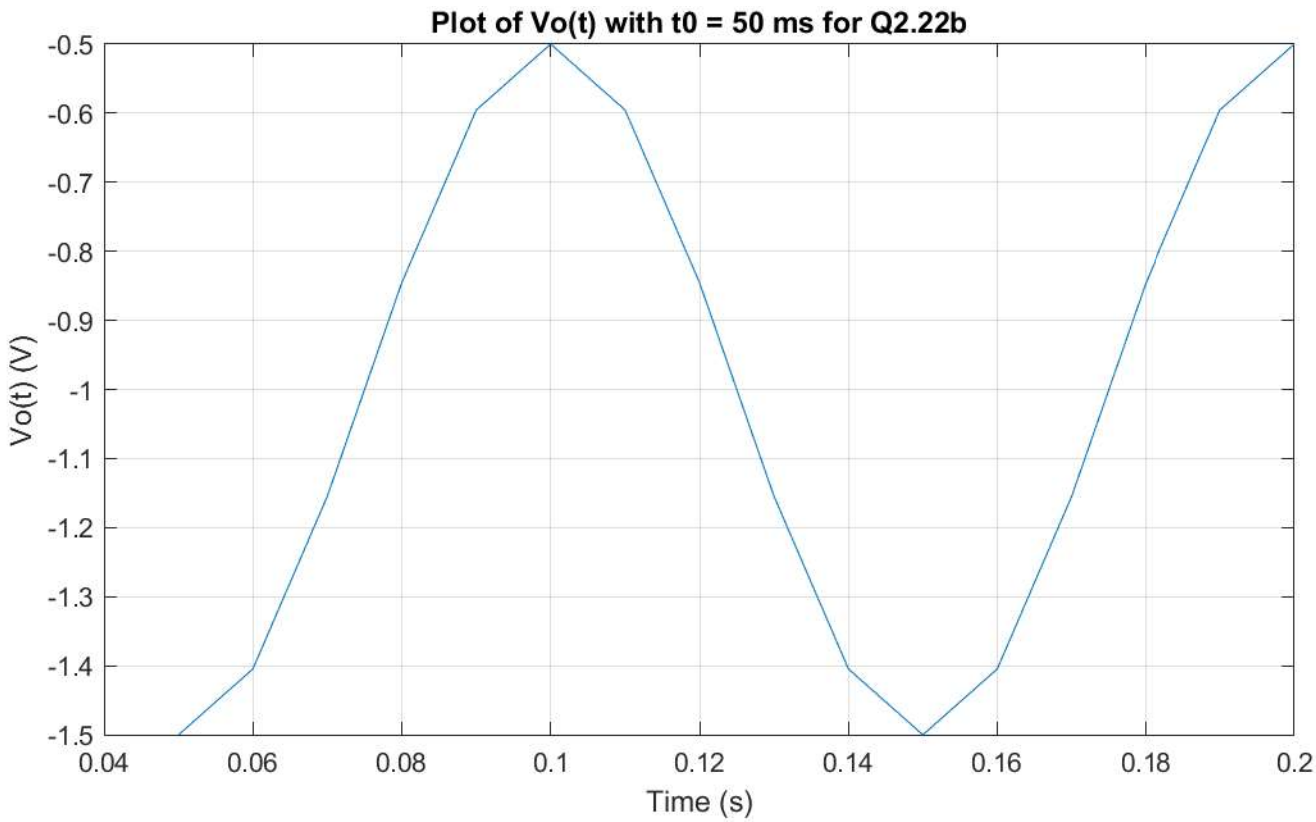


y(t) vs y1(t)
input=x1(t)
output=y1(t)
Amplitude
t
40
30
20
10
0
-10
-20
-30
-40
-2
-1
0
1
2
3
4

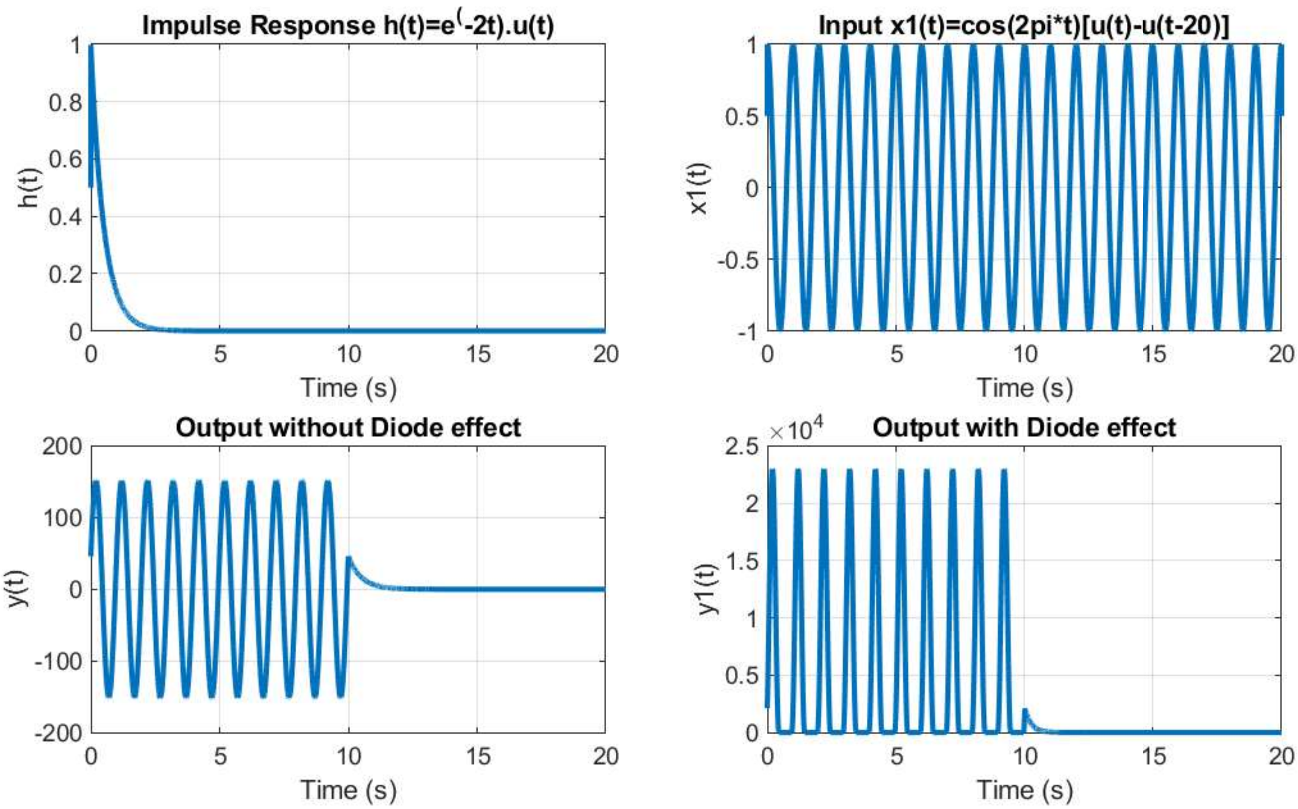

y(t) plot for question 2.5(b)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
y(t)
-10
-8
-6
-4
-2
0
2
4
6
8
10
t

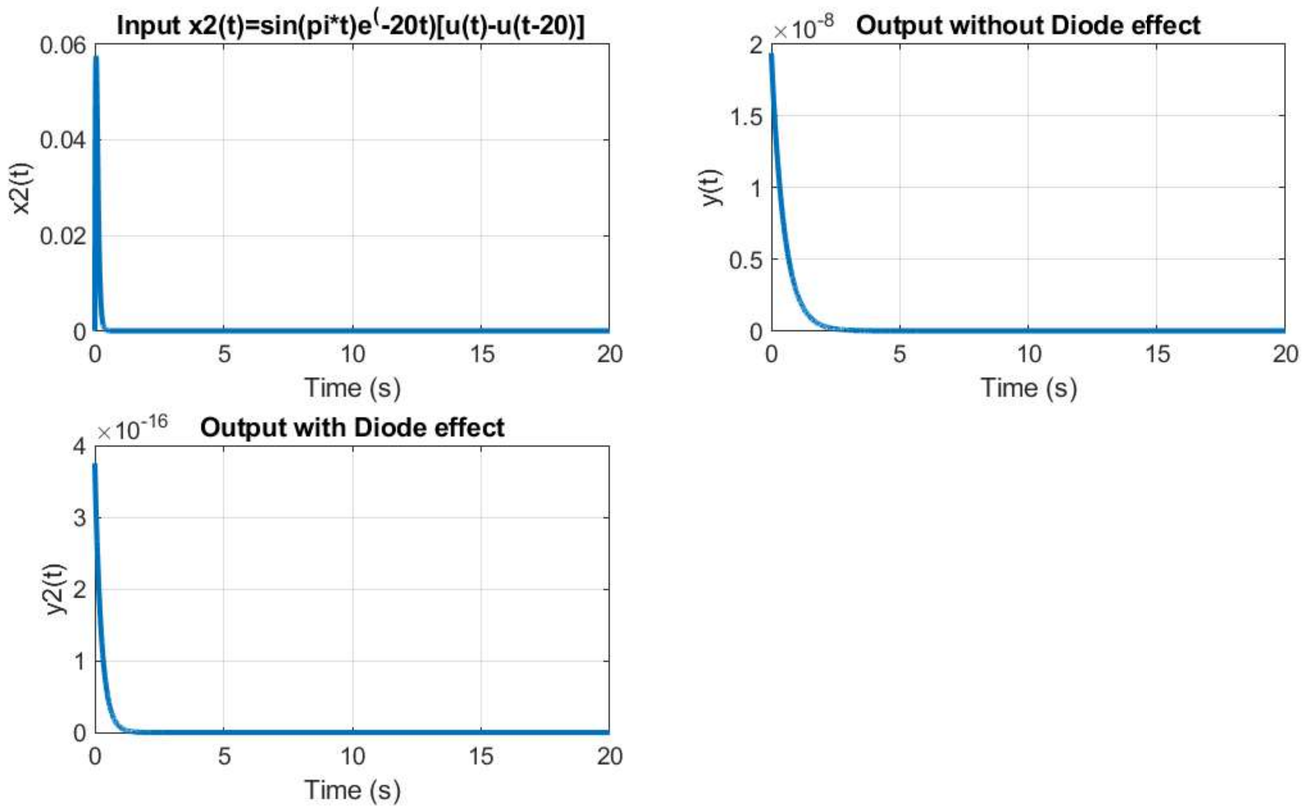


Output Vo = R * Vi for Q2.22a
Vo
t
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-1.4
-1.5
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

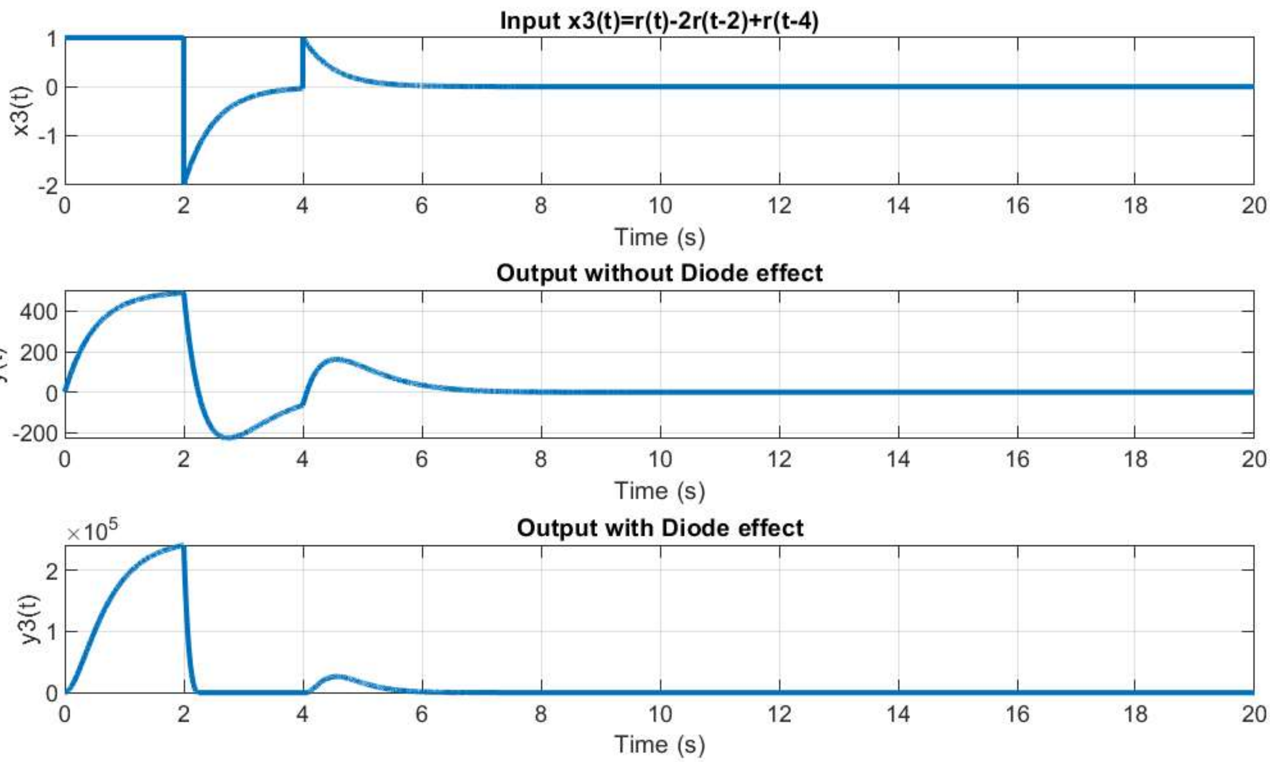


Plot of Vo(t) with t0 = 50 ms for Q2.22b
Vo(t) (V)
Time (s)
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-1.4
-1.5
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2


Impulse Response h(t)=e^(-2t).u(t)
h(t)
Time (s)
Input x1(t)=cos(2pi*t)[u(t)-u(t-20)]
x1(t)
Time (s)
Output without Diode effect
y(t)
Time (s)
Output with Diode effect
×10^4
y1(t)
Time (s)



Input x2(t)=sin(pi*t)e^(-20t)[u(t)-u(t-20)]
0.06
0.04
0.02
0
x2(t)
0
5
10
15
20
Time (s)
×10^-8
Output without Diode effect
2
1.5
1
0.5
0
y(t)
0
5
10
15
20
Time (s)
×10^-16
Output with Diode effect
4
3
2
1
0
y2(t)
0
5
10
15
20
Time (s)


Input x3(t)=r(t)-2r(t-2)+r(t-4)
x3(t)
Time (s)
Output without Diode effect
y(t)
Time (s)
Output with Diode effect
×10^5
y3(t)
Time (s)