```
% Name: Lamin Jammeh
% Class: EE480 Online
% Semster: Fall 2023
% HW_9

% Basic Problemsclear;
%% ********** Q4.24b **********
clc;
syms t s;
r = t.*heaviside(t);
r_d = (t-1).*heaviside(t-1);
x = r-r_d-heaviside(t-1);
X_s = laplace(x)

%% ********** Q4.24c **********
clear;
clc;
% Define the time range
t = 0:0.001:10;  % Adjust the range as needed
T = 1;
% Define the ramp function r(t)
r = t.*heaviside(t); % unit ramp interms of unit step r(t)=t*u(t)
r_d = (t-1).*heaviside(t-1); %r(t-1)
x = r-r_d-heaviside(t-1); %x1(t)=r(t)-r(t-1)-u(t-1)
N = 40; % Number of Harmonics
X_dc = 0.5; % dc term or a_0
x_i = zeros(size(t)); %signal starts at t=0
for k = 1:N
    %Xk = 1/T * integral(@(t) t .*exp(-j*2*pi*k*(t/T)),0,T);
    Xk = 1/T *(j/(2*pi.*k));
    x_i = x_i + Xk * exp(j * 2 * pi * k * t / T);
end
X_f = X_dc + x_i; %fourier Term
figure;
subplot(2,1,1);
plot(t, X_f, 'b', 'LineWidth', 2);
xlabel('t');
ylabel('x(t)');
title('Plot of Fourier Series');
grid on;

%for the Magnitude frequency response determine Magnitude of FS and the
%frequency axis

%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum = abs(X_f);

%Step 2 determine the freq axis
f0 = 1/T; %fundamental Freq=1/fundamental period
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num_coefficients = length(magnitude_spectrum);
f_axis = (0:num_coefficients-1) / num_coefficients * f0;

% Plot the magnitude spectrum
subplot(2,1,2);
stem(f_axis, magnitude_spectrum, 'LineStyle', 'none');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Magnitude Line Spectrum of Fourier Series (40 Harmonics, T=1s)');
grid on;

%% ********** Q4.26 a **********
clc;
syms t s;
x = heaviside(t)-heaviside(t-1);
X_s = laplace(x)

%% ********** Q4.26b **********
clear;
clc;
% Define the time range
t = 0:0.001:10;  % Adjust the range as needed
T = 2;

x = heaviside(t)-heaviside(t-1);
N = 40; % Number of Harmonics
X_dc = 1; % dc term or a_0
x_i = zeros(size(t)); %signal starts at t=0
for k = 1:N
    Xk = (1-exp(-j*pi.*k)) / (j*pi.*k);
    x_i = x_i + Xk * exp(j * 2 * pi * k * t / T);
end
X_f = X_dc + x_i; %fourier Term

figure;
subplot(2,1,1);
plot(t, X_f, 'b', 'LineWidth', 2);
xlabel('t');
ylabel('x(t)');
title('Plot of Fourier Series');
grid on;

%for the Magnitude frequency response determine Magnitude of FS and the
%frequency axis

%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum = abs(X_f);

%Step 2 determine the freq axis
```

```
f0 = 1/T; %fundamental Freq=1/fundamental period
num_coefficients = length(magnitude_spectrum);
f_axis = (0:num_coefficients-1) / num_coefficients * f0;

% Plot the magnitude spectrum
subplot(2,1,2);
stem(f_axis, magnitude_spectrum, 'LineStyle', 'none');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Magnitude Line Spectrum of Fourier Series (40 Harmonics, T=2s)');
grid on;

%% ********** Q4.28 part 1 **********
clear;
clc;
%find the laplace of x(t) and y(t)
syms t s;
x1 = heaviside(t)-heaviside(t-1);
X_s = laplace(x1)

r = t.*heaviside(t);
r_1 = (t-1).*heaviside(t-1);
r_2 = (t-2).*heaviside(t-2);
y1 = r - 2*r_1 + r_2;
Y_s = laplace(y1)

%% ********** Q4.28 **********
clear;
clc;
% Define the time range
t = 0:0.001:5;  % Adjust the range as needed
T = 2;

%Part 1 x1(t) = u(t)-u(t-1)
x = heaviside(t)-heaviside(t-1);
N = 40; % Number of Harmonics
X_dc = 1; % dc term or a_0
x_i = zeros(size(t)); %signal starts at t=0
for k = 1:N
    Xk = 0.5*(1-exp(-j*pi.*k)) / (j*pi.*k);
    x_i = x_i + Xk * exp(j * 2 * pi * k * t / T);
end
X_f = X_dc + x_i; %fourier Term

%for the Magnitude frequency response determine Magnitude of FS and the
%frequency axis

%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum_x = abs(X_f);
```

```
%Step 2 determine the freq axis
f0 = 1/T; %fundamental Freq=1/fundamental period
num_coefficients = length(magnitude_spectrum_x);
f_axis1 = (0:num_coefficients-1) / num_coefficients * f0;

%Part 2 y1(t) = r(t)-2r(t-1)+r(t-2)
r = t.*heaviside(t);
r_1 = (t-1).*heaviside(t-1);
r_2 = (t-2).*heaviside(t-2);
y = r - r_1 + r_2;
Y_dc = 1; % dc term or a_0
y_i = zeros(size(t)); %signal starts at t=0
for k = 1:N
    Yk = 0.5*(exp(-j*2*pi.*k)-exp(-j*pi.*k)+1) / (j*pi*k)^2;
    y_i = y_i + Yk * exp(j * 2 * pi * k * t / T);
end
Y_f = Y_dc + y_i; %fourier Term

%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum_y = abs(Y_f);

%Step 2 determine the freq axis
f0 = 1/T; %fundamental Freq=1/fundamental period
num_coefficients = length(magnitude_spectrum_y);
f_axis2 = (0:num_coefficients-1) / num_coefficients * f0;

%Plot the Fourier Series of x(t) and y(t)
figure;
subplot(2,1,1);
plot(t,X_f,'r','LineWidth',2);
xlabel('time');
ylabel('amplitude');
title('Fourier Series plot of x(t)')
subplot(2,1,2);
plot(t,Y_f,'LineWidth',2);
xlabel('time');
ylabel('amplitude');
title('Fourier Series plot of y(t)')

%Plot the Magnitude Line Spectra of x(t) and y(t)
figure;
stem(f_axis1, magnitude_spectrum_x, 'LineStyle', 'none');
hold on;
stem(f_axis2, magnitude_spectrum_y, 'LineStyle', 'none');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Magnitude Line Spectrum of Fourier Series X(t) vs Y(t) (40 Harmonics, T=2s)');
grid on;
```

```
legend('X(t)','Y(t)')
hold off

%% ********** Q4.29 part 1 **********
clear;
clc;
%find the laplace of x(t) and y(t)
syms t s;
x1 = heaviside(t)-heaviside(t-1);
X_s = laplace(x1)

y1 = heaviside(t)-heaviside(t-0.5);
Y_s = laplace(y1)

%% ********** Q4.29b **********
clear;
clc;
t = 0:0.001:2;
w = j*2*pi;
W = (80*pi)/(2*pi);
T0 = 2;
T1 = 1;
%calculate the x1(t) terms
X_dc = 1; % dc term or a_0
x_i = zeros(size(t)); %signal starts at t=0
for k = 1:W
    Xk = 0.5*(1-exp(-w.*k/T0)) / (w.*k/T0);
    x_i = x_i + Xk * exp(j * 2 * pi * k * t / T0);
end
X_f = X_dc + x_i; %fourier Term

%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum_x = abs(X_f);

%Step 2 determine the freq axis
f0 = 1/T0; %fundamental Freq=1/fundamental period
num_coefficients = length(magnitude_spectrum_x);
f_axis1 = (0:num_coefficients-1) / num_coefficients * f0;


%calculate the y1(t) terms
Y_dc = 1; % dc term or a_0
y_i = zeros(size(t)); %signal starts at t=0
for k = 1:W
    Yk = (1/T1)*((1-exp(-w.*k/2))/w.*k);
    y_i = y_i + Yk * exp(j * 2 * pi * k * t / T1);
end
Y_f = Y_dc + y_i; %fourier Term
```

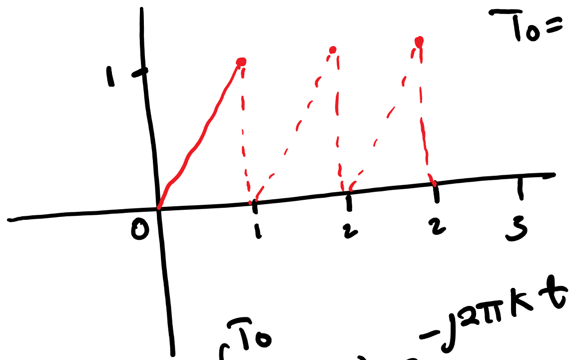
```
%step 1 determine the Magnitude of Fourier Series (FS) of the signal
magnitude_spectrum_y = abs(Y_f);

%Step 2 determine the freq axis
f0 = 1/T1; %fundamental Freq=1/fundamental period
num_coefficients = length(magnitude_spectrum_y);
f_axis2 = (0:num_coefficients-1) / num_coefficients * f0;

%Plot the Fourier Series of x(t) and y(t)
figure;
subplot(2,1,1);
plot(t,X_f,'LineWidth',2);
xlabel('time');
ylabel('amplitude');
title('Fourier Series plot of x(t)')
subplot(2,1,2);
plot(t,Y_f,'LineWidth',2);
xlabel('time');
ylabel('amplitude');
title('Fourier Series plot of y(t)')

%Plot the Magnitude Line Spectra of x(t) and y(t)
figure;
subplot(2,1,1)
stem(f_axis1, magnitude_spectrum_x, 'LineStyle', 'none');
subplot(2,1,2)
stem(f_axis2, magnitude_spectrum_y, 'LineStyle', 'none');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Magnitude Line Spectrum of Fourier Series Y(t) (0:80pi, T=1s)');
```

4.24

Saturday, November 4, 2023 12:54 PM

$x_1(t) = r(t) - r(t-1) - u(t-1)$

a)



$T_0 = 1$

$$X_{[k]} = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k t}\, dt$$

$$= 1\int_0^1 t e^{-j2\pi k t}\, dt$$

$$= \frac{-j2\pi k\, e^{-j2\pi k}}{(j2\pi k)^2}$$

$$= \frac{-j2\pi k}{(j2\pi k)^2} = \frac{-1}{j2\pi k}$$

$$\boxed{X_k = \frac{j}{2\pi k}}$$

b) Using laplace transform

$$x_1(t) = r(t) - r(t-1) - u(t-1)$$

$$X_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$ ⇒ from Matlab Code

$$X_1(s) = \frac{1 - e^{-s} - s e^{-s}}{s^2}$$

$$X[k]\Big|_{s=j2\pi k} = \frac{1 - e^{-j2\pi k} - j2\pi k\, e^{-j2\pi k}}{(j2\pi k)^2}$$

$$e^{j2\pi k} = 1$$

$$= \frac{1 - 1 - j2\pi k(1)}{(j2\pi k)^2}$$

$$= \frac{-j2\pi k}{(j2\pi k)^2} = \frac{-1}{j2\pi k}$$

$$\boxed{X[k]\Big|_{s=j2\pi k} = \frac{j}{2\pi k}}$$

4.26

Saturday, November 4, 2023 2:54 PM

$x_1(t) = u(t) - u(t-1)$

$X_1(s) = \frac{1}{s} - \frac{e^{-s}}{s}$ from matlab

$X_1(s) = \frac{1 - e^{-s}}{s}$

$X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{T_0}\left(X_1(s)\right)$

$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0}$

$X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{2}\left(\frac{1 - e^{-j\frac{2\pi k}{2}}}{j\frac{2\pi k}{2}}\right)$

$$\boxed{X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{2}\left(\frac{1 - e^{-j\pi k}}{j\pi k}\right)}$$

demitapp Cold & Cough 4 yrs old

$X_{dc} = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$

$X_{dc} = \frac{1}{2}\int_0^{2} 1\,dt$

$X_{dc} = \frac{1}{2}\left[t\Big|_0^2\right]$

$$\boxed{X_{dc} = \frac{1}{2}(2) = 1}$$

4.28

Saturday, November 4, 2023 5:28 PM

$$y_1(t) = r(t) - 2r(t-1) + r(t-2)$$

$$Y_1(s) = \frac{e^{-2s}}{s^2} - \frac{2e^{-s}}{s^2} + \frac{1}{s^2}$$

$$Y_1(s) \quad \frac{e^{-2s} - 2e^{-s} + 1}{s^2}$$

$$Y[k] = \frac{1}{T_0}\left(Y_1(s)\right)$$

$$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{2} = j\pi$$

$$Y[k] = \frac{1}{2}\left[\frac{e^{-2j\pi k} - e^{-j\pi k} + 1}{(j\pi k)^2}\right]$$

$$Y\_dc = \frac{1}{T_0}\int_0^{T_0} y(t)\, dt$$

$$Y\_dc = \frac{1}{2}\left(\frac{t^2}{2}\right)\Big|_0^2$$

$$Y\_dc = \frac{1}{2}\left(\frac{2^2}{2}\right)$$

$$\boxed{Y\_dc = 1}$$

$$\boxed{\begin{aligned} &X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{2}\left(\frac{1-e^{-j\pi s}}{j\pi k}\right) \\ &X_{\sim DC} = 1 \end{aligned}}$$

from Q 4.26

4.29

Sunday, November 5, 2023 12:28 PM

$x_1(t) = u(t) - u(t-1) \quad 0 \le t \le 2$

$X_1(s) = \frac{1}{s} - \frac{e^{-s}}{s}$ from matlab $\mathcal{L}\{x_1(t)\}$

$X[k] = \frac{1}{T_0}\left[X_1(s)\right] = \frac{1}{T_0}\left[\frac{1-e^{-s}}{s}\right]$

$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{2} = j\pi$

$$\boxed{X[k]\Big|_{s=j\omega k} = \frac{1}{2}\left(\frac{1-e^{-j\pi k}}{j\pi k}\right)}$$

fourier Coefficent

$y_1(t) = u(t) - u(t-0.5) \quad 0 \le t \le 1$

$Y_1(s) = \frac{1}{s} - \frac{e^{-\frac{s}{2}}}{s}$ from matlab $\mathcal{L}\{y_1(t)\}$

$Y[k] = \frac{1}{T_0}\left(Y_1(s)\right) = \frac{1}{T_0}\left[\frac{1 - e^{-\frac{s}{2}}}{s}\right]$

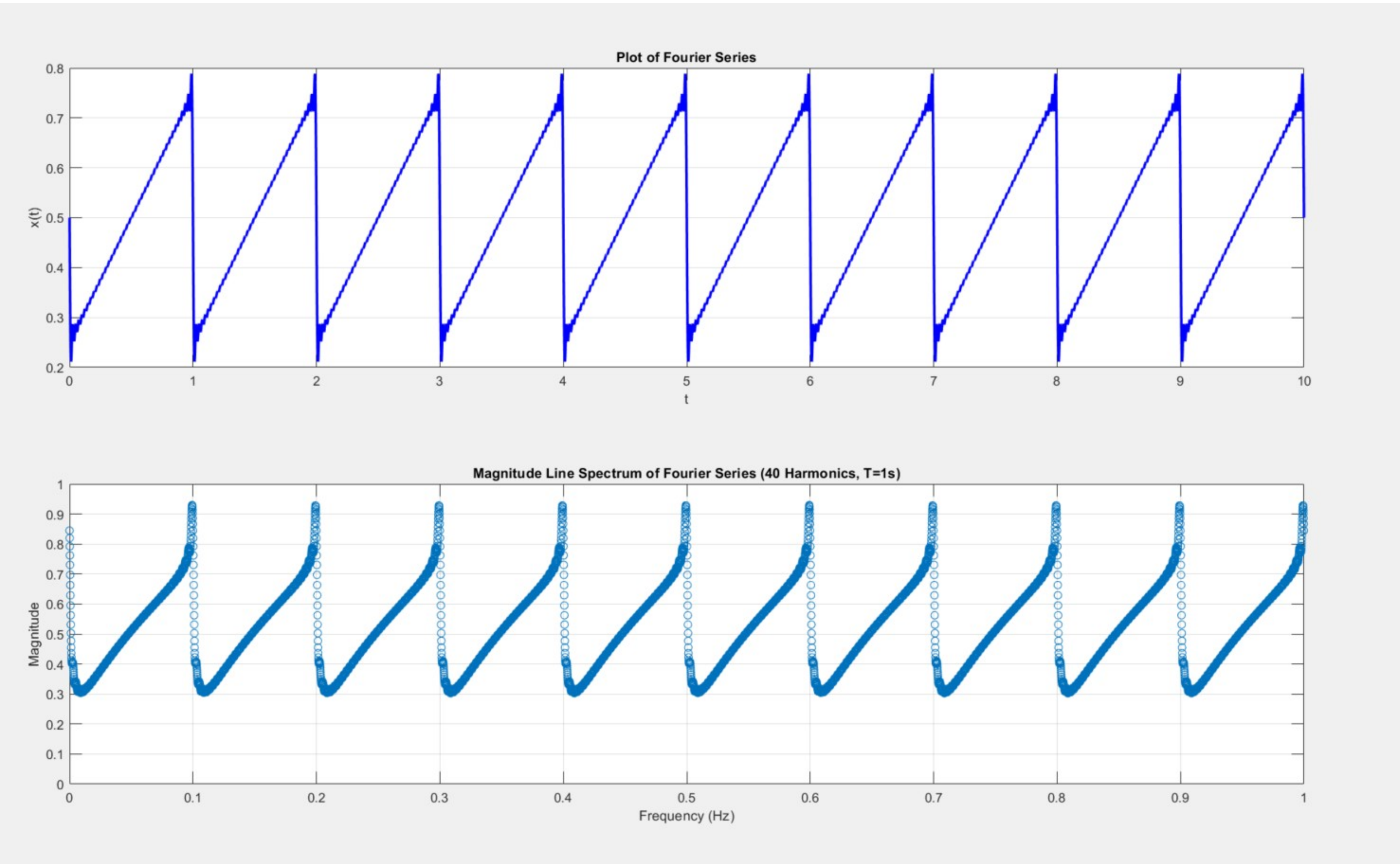
$T_0 = 1 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{1} = j2\pi$

$Y[k]\Big|_{s=j\omega k} = \left(\cancel{\frac{1}{1}}\left(\frac{1 - e^{-j2\pi k}}{j2\pi k}\right)\right)$

$$\boxed{Y[k]\Big|_{s=j\omega k} = \frac{1 - e^{-j2\pi k}}{j2\pi k}}$$

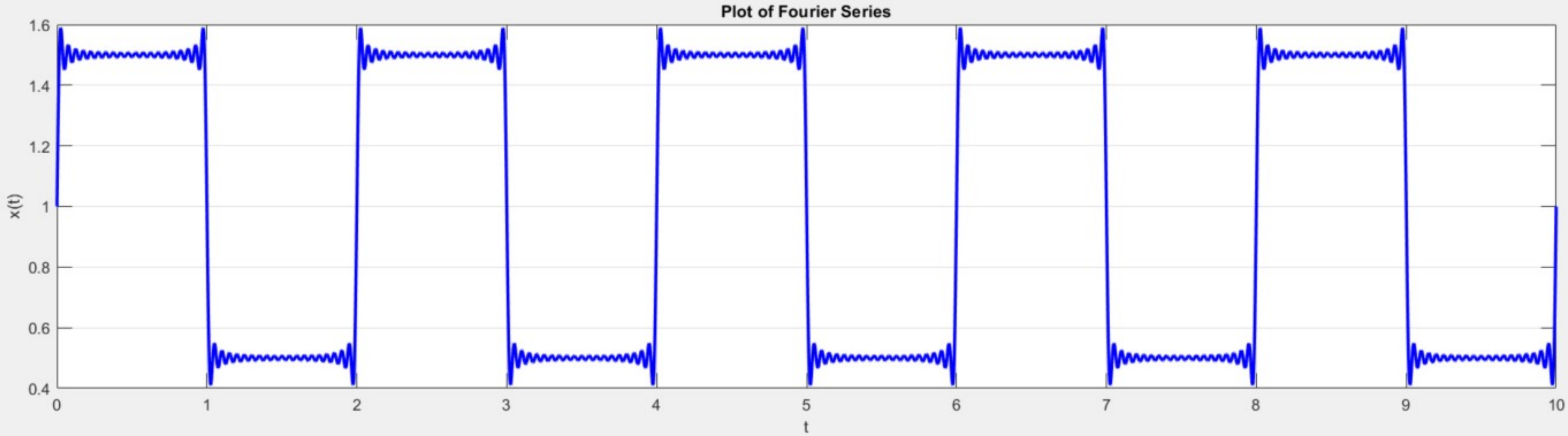
fourier Coefficent

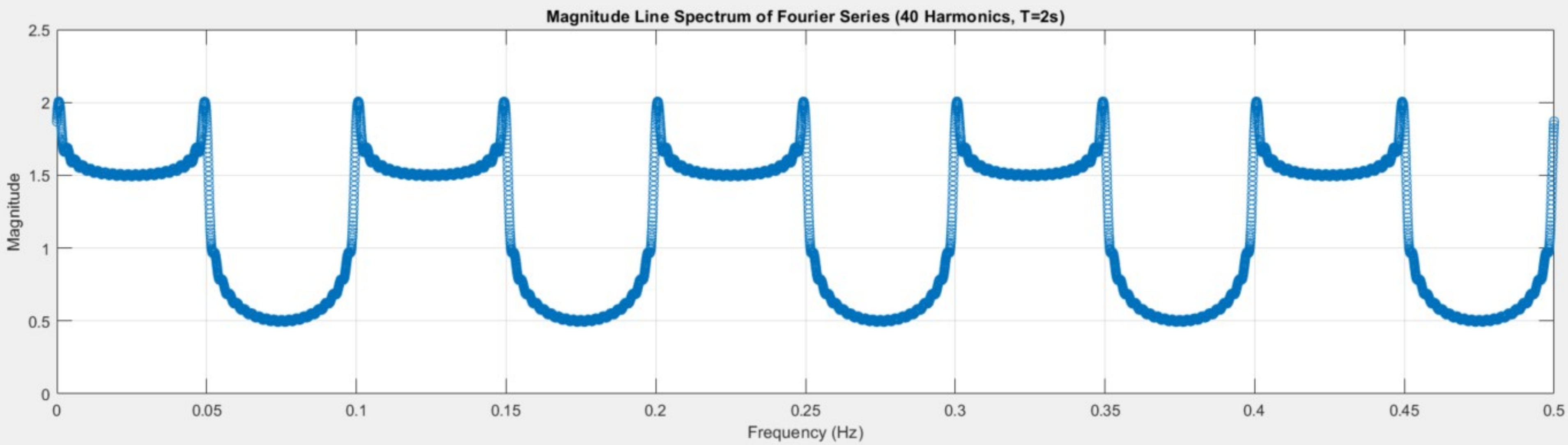
```
>> syms t s;
r = t.*heaviside(t);
r_d = (t-1).*heaviside(t-1);
x = r-r_d-heaviside(t-1);
X_s = laplace(x)

X_s =

1/s^2 - exp(-s)/s^2 - exp(-s)/s

>>
```



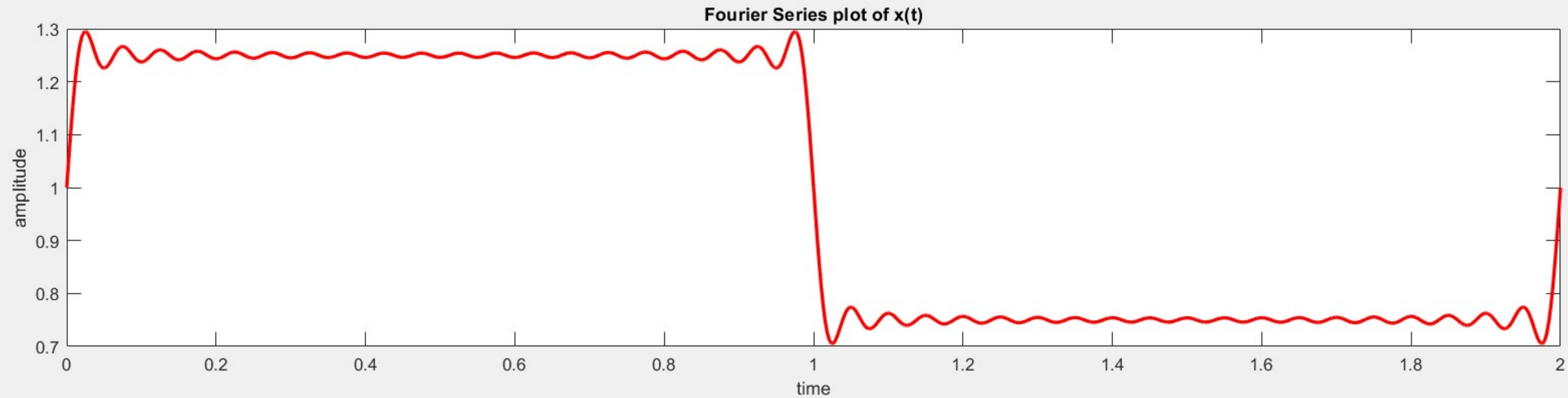
Plot of Fourier Series
x(t)
t
Magnitude Line Spectrum of Fourier Series (40 Harmonics, T=1s)
Magnitude
Frequency (Hz)

```
>> syms t s;
x = heaviside(t)-heaviside(t-1);
X_s = laplace(x)

X_s =

1/s - exp(-s)/s

>>
```

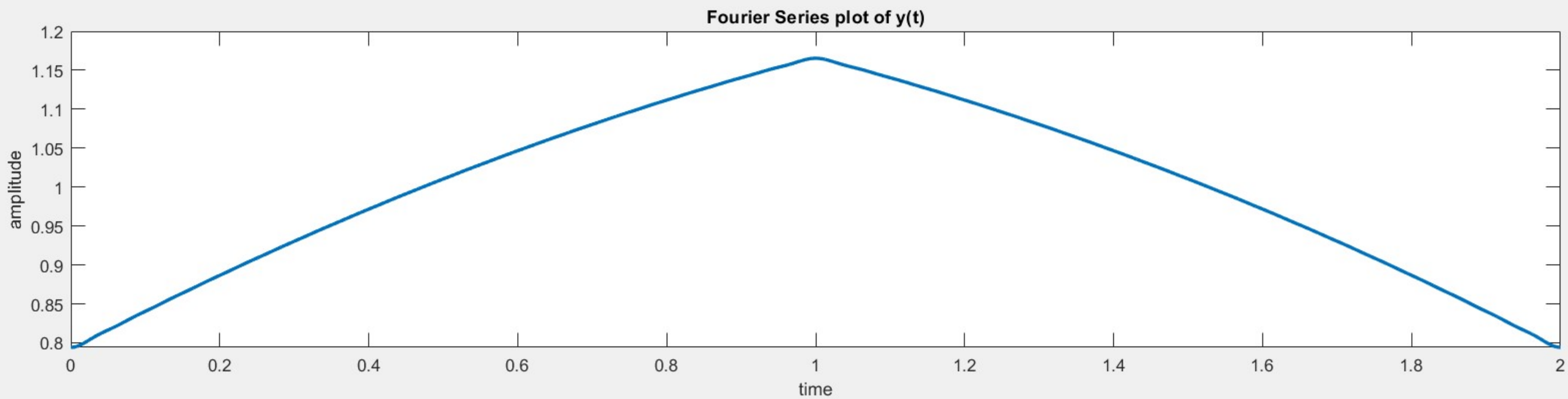

Plot of Fourier Series
x(t)
t
1.6
1.4
1.2
1
0.8
0.6
0.4
0 1 2 3 4 5 6 7 8 9 10


Magnitude Line Spectrum of Fourier Series (40 Harmonics, T=2s)
Magnitude
Frequency (Hz)
2.5
2
1.5
1
0.5
0
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5

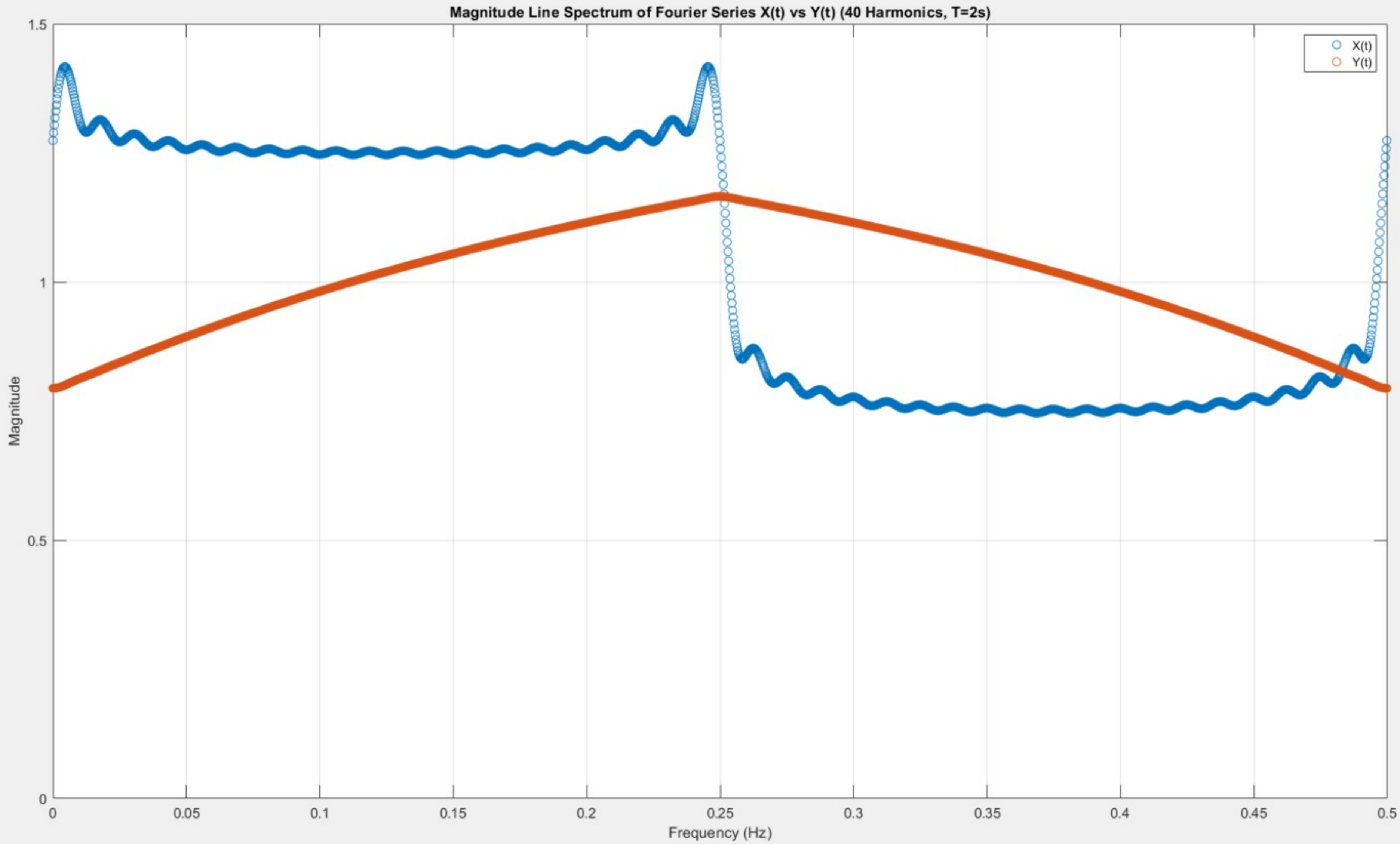
```
>> syms t s;
x1 = heaviside(t)-heaviside(t-1);
X_s = laplace(x1)

r = t.*heaviside(t);
r_1 = (t-1).*heaviside(t-1);
r_2 = (t-2).*heaviside(t-2);
y1 = r - 2*r_1 + r_2;
Y_s = laplace(y1)

X_s =

1/s - exp(-s)/s


Y_s =

exp(-2*s)/s^2 - (2*exp(-s))/s^2 + 1/s^2

>>
```



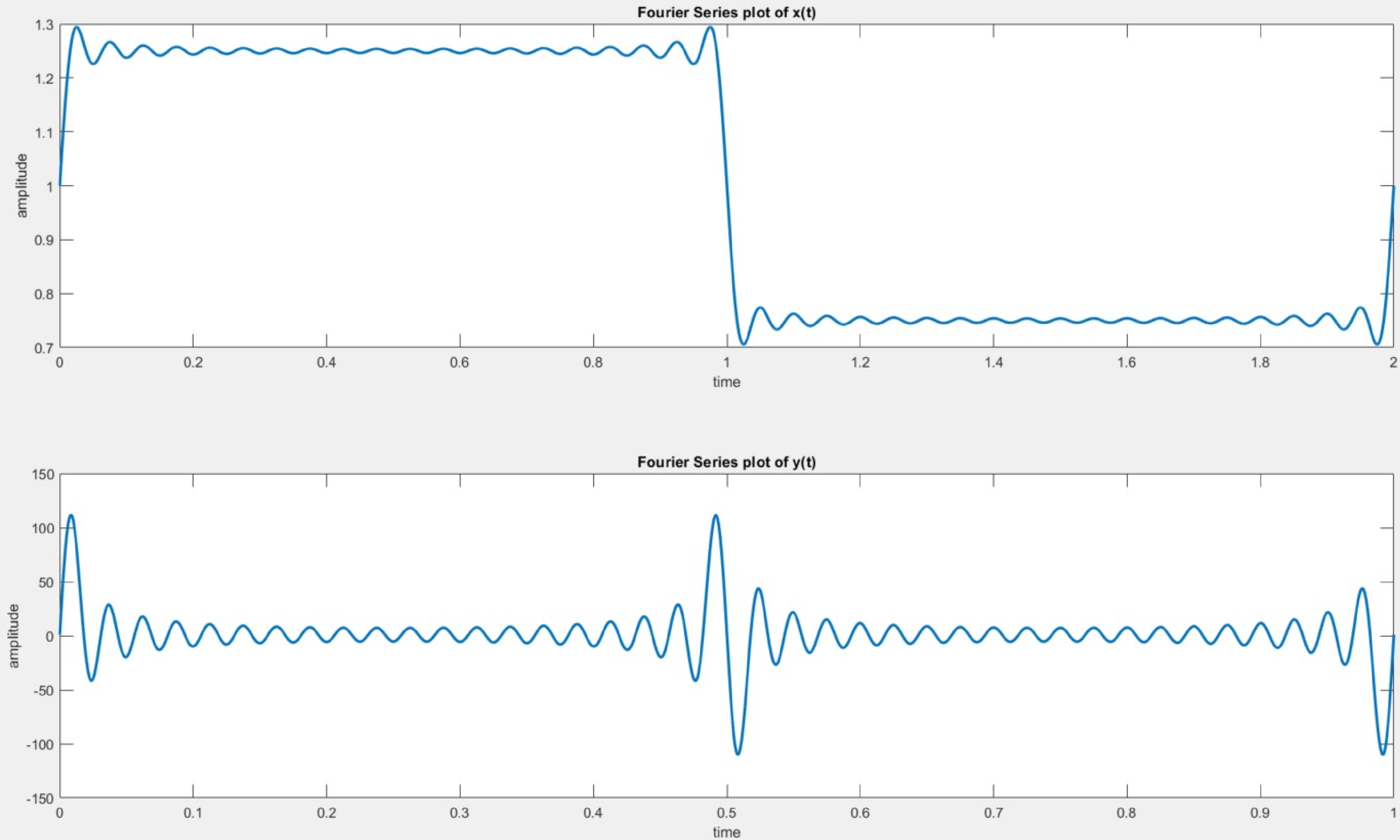
Fourier Series plot of x(t)
amplitude
time
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



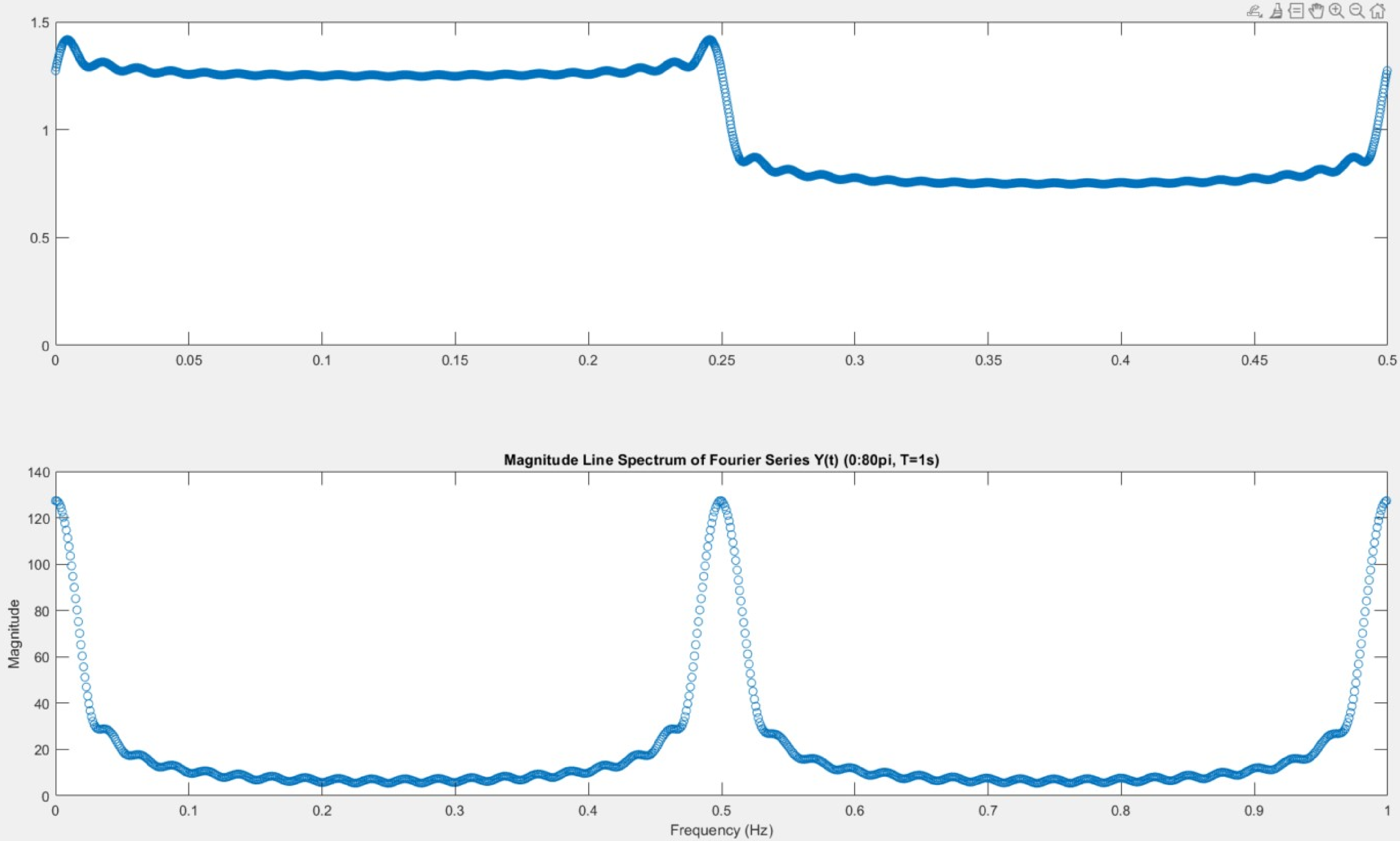
Fourier Series plot of y(t)
amplitude
time
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Magnitude Line Spectrum of Fourier Series X(t) vs Y(t) (40 Harmonics, T=2s)
X(t)
Y(t)
Magnitude
Frequency (Hz)
0
0.5
1
1.5
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

```
>> syms t s;
x1 = heaviside(t)-heaviside(t-1);
X_s = laplace(x1)

y1 = heaviside(t)-heaviside(t-0.5);
Y_s = laplace(y1)

X_s =

1/s - exp(-s)/s


Y_s =

1/s - exp(-s/2)/s

>>
```



Fourier Series plot of x(t)
amplitude
time
Fourier Series plot of y(t)
amplitude
time



Magnitude Line Spectrum of Fourier Series Y(t) (0:80pi, T=1s)
Magnitude
Frequency (Hz)