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#EE597
#HW10

#Pr 15.2 (Complex sequence)

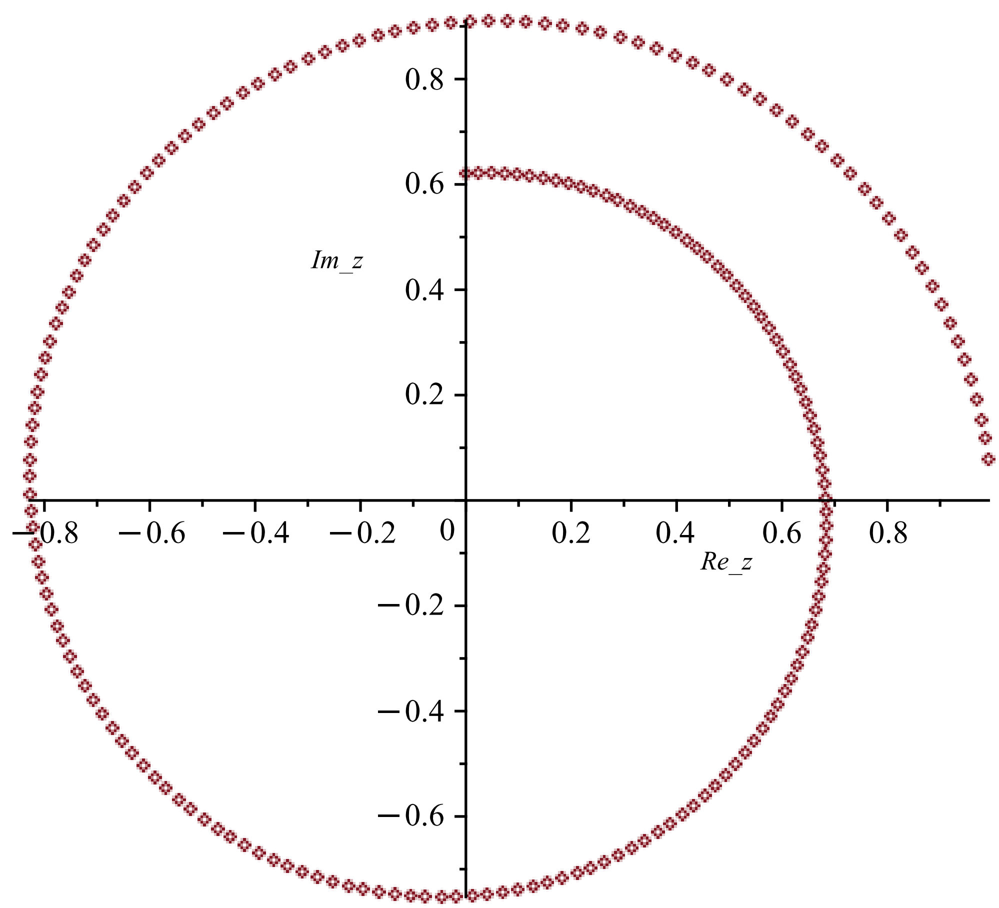
restart;

$zn := \left(\frac{10 \cdot I}{11}\right)^{\frac{n}{20}};$

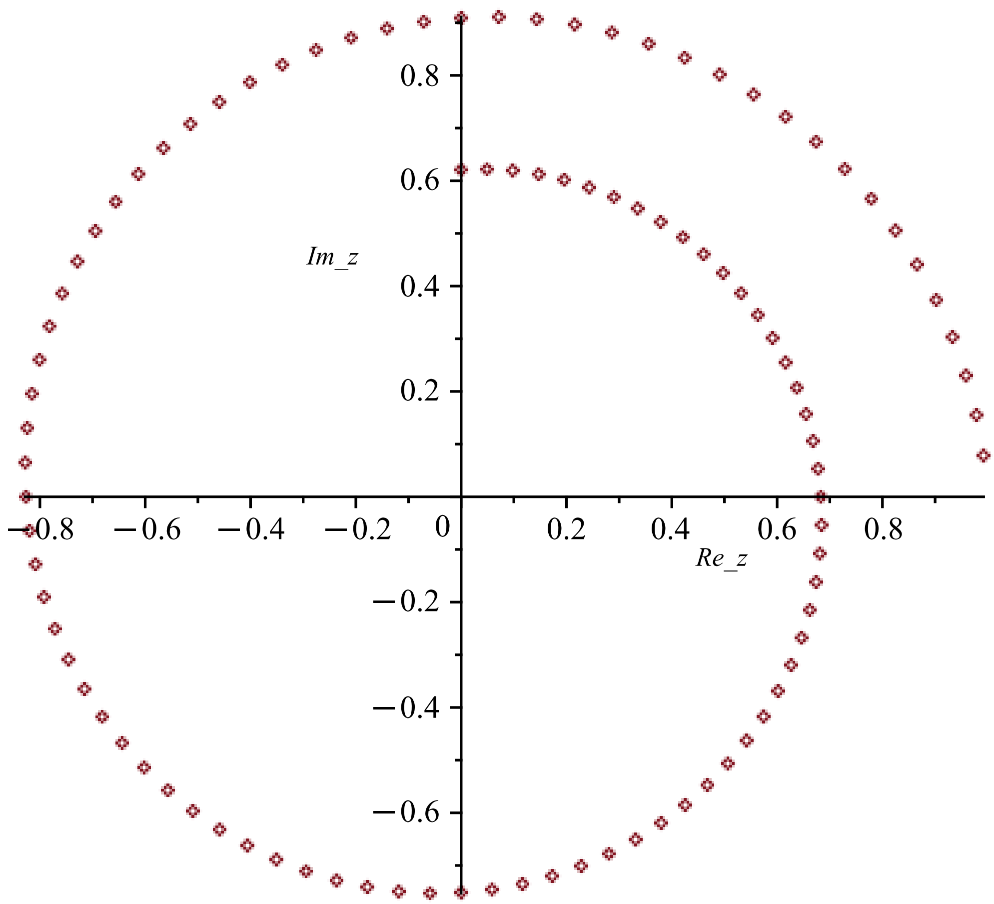
$$zn := \left(\frac{10\,\mathrm{I}}{11}\right)^{\frac{n}{20}} \qquad \textbf{(1)}$$

$S := seq([\mathrm{Re}(zn),\ \mathrm{Im}(zn)],\ n = 1..100):$

with(*plots*) :
complexplot([*zn*], *n* = 1 ..100, *style* = *point*, *labels* = [*Re_z*, *Im_z*], *scaling* = *constrained*);



plot([*S*], *style* = *point*, *labels* = [*Re_z*, *Im_z*], *scaling* = *constrained*);



#Pr 15.3 (Convergence test)

restart;

$z := \frac{(33+22\cdot I)^n}{n!}$;

$$z := \frac{(33+22\,\mathrm{I})^n}{n!} \qquad \textbf{(2)}$$

$z\_n := subs(n=n+1,\ zn)$;

$$z\_n := zn \qquad \textbf{(3)}$$

#using the ratio test $\frac{z\_n}{z}$

$ratio := \left(\frac{z\_n}{z}\right)$;

$$ratio := \frac{zn\,n!}{(33+22\,\mathrm{I})^n} \qquad \textbf{(4)}$$

simplify(*ratio*);

$$zn\,(33+22\,\mathrm{I})^{-n}\,n! \qquad \textbf{(5)}$$

$(\mathrm{abs}(z))^{\frac{1}{n}}$;

$$\left|\frac{(33+22\,\mathrm{I})^n}{n!}\right|^{\frac{1}{n}} \tag{6}$$

limit(%, n = infinity);

$$0 \tag{7}$$

#Pr 15.6 (Radius of convergence)

restart;
 # define the series
$a := n(n-1)\cdot 3^n \cdot z^{2n}$;

$$a := n(n-1)\,3^n\,z^{2n} \tag{8}$$

 # define a(n+1) as a_n
$a\_n := subs(n = n+1, a)$;

$$a\_n := (n+1)(n)\,3^{n+1}\,z^{2n+2} \tag{9}$$

$\frac{a}{a\_n}$;

$$\frac{n(n-1)\,3^n\,z^{2n}}{(n(n)+1)\,3^{n+1}\,z^{2n+2}} \tag{10}$$

simplify(%);

$$\frac{n(n-1)}{3\,z^2\,(n(n)+1)} \tag{11}$$

 # Radius of Convergence =limit($\frac{a}{an}$, n = infinity$\Big)$
limit(%, n = infinity);

$$\lim_{n\to\infty}\frac{n(n-1)}{3\,z^2\,(n(n)+1)} \tag{12}$$

#Pr 15.7 (Radius of convergence)

restart;
 # define the series
$a := \frac{(4\cdot n)!}{5^n\cdot(n!)^4}\cdot z^n$;

$$a := \frac{(4\,n)!\,z^n}{5^n\,n!^4} \tag{13}$$

 # define a(n+1) as a_n
$a\_n := subs(n = n+1, a)$;

$$a\_n := \frac{(4\,n+4)!\,z^{n+1}}{5^{n+1}\,(n+1)!^4} \tag{14}$$

Radius of Convergence =limit$\left(\frac{a}{an}, n = \text{infinity}\right)$

convergence_test $:= limit\left(\frac{a}{a\_n},\ n = \text{infinity}\right);$

$$convergence\_test := \frac{5}{256\,z} \tag{15}$$

#Pr 15.10 (Taylor series)

restart;

#define the function f=cos$^2(z)$

$f := \cos^2(z);$

$$f := \cos(z)^2 \tag{16}$$

 # find the zero term as a0

$a0 := eval\left(subs\left(z = \frac{\text{Pi}}{2}, f\right)\right);$

$$a0 := 0 \tag{17}$$

#find the taylor series

$Taylor\_Series := seq\left(\left(eval\left(subs\left(z = \frac{\text{Pi}}{2},\ \frac{diff(f,\ z\$n)}{n!}\right)\right)\right), n = 1\,..4\right);$

$$Taylor\_Series := 0, 1, 0, -\frac{1}{3} \tag{18}$$