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#EE597
#FINAL TERM

#QUESTION 1

restart;
$f(x) := x;$

$$f := x \mapsto x \tag{1}$$

#STEP 1 determine the [a0] term

$a[0] := \frac{1}{2 \cdot L} \cdot int(f(x), x = 0 ..L);$

$$a_0 := \frac{L}{4} \tag{2}$$

#STEP 2 determine the [an] cosine term

$a[n] := \frac{1}{L} \cdot int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right);$

$$a_n := \frac{L\,(n\,\pi \sin(n\,\pi) + \cos(n\,\pi) - 1)}{n^2\,\pi^2} \tag{3}$$

#STEP 3 determine the [bn] sine term

$b[n] := \frac{1}{L} \cdot int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = 0 ..L\right);$

$$b_n := \frac{L\,(-n\,\pi \cos(n\,\pi) + \sin(n\,\pi))}{n^2\,\pi^2} \tag{4}$$

#The Fourier series of a function f(x) of period p = L is obtained by typing

$f := a[0] + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1 ..\mathrm{infinity}\right);$

$$f := \frac{L}{4} + \sum_{n=1}^{\infty} \left(\frac{L\,(n\,\pi \sin(n\,\pi) + \cos(n\,\pi) - 1) \cos\left(\frac{n\,\pi\,x}{L}\right)}{n^2\,\pi^2} + \frac{L\,(-n\,\pi \cos(n\,\pi) + \sin(n\,\pi)) \sin\left(\frac{n\,\pi\,x}{L}\right)}{n^2\,\pi^2} \right) \tag{5}$$

#QUESTION 2

restart;

#Part A

$$f(x) := \frac{(2 \cdot z^3 + z^2 + 4)}{z^4 - 4 \cdot z^2};$$

$$f := x \mapsto \frac{2 \cdot z^3 + z^2 + 4}{z^4 - 4 \cdot z^2} \tag{6}$$

#define center c and radius r of the circle

$c := 5;$ $\quad$ $r := 2;$

$$c := 5$$

$$r := 2 \tag{7}$$

#define z(t)=c+r·e^{I·t}

$z := c + r \cdot \exp(I \cdot t);$

$$z := 5 + 2\,\mathrm{e}^{\mathrm{I}\,t} \tag{8}$$

#deine zdt

$zdt := diff(z, t);$

$$zdt := 2\,\mathrm{I}\,\mathrm{e}^{\mathrm{I}\,t} \tag{9}$$

#define f(z(t))

$F := f(z);$

$$F := \frac{2\,(5 + 2\,\mathrm{e}^{\mathrm{I}\,t})^3 + (5 + 2\,\mathrm{e}^{\mathrm{I}\,t})^2 + 4}{(5 + 2\,\mathrm{e}^{\mathrm{I}\,t})^4 - 4\,(5 + 2\,\mathrm{e}^{\mathrm{I}\,t})^2} \tag{10}$$

#integration of f=int[(F·zdt), t=0..2·PI]

$f\_int := int(evalc(F \cdot zdt), t = 0\,..2 \cdot \mathrm{PI});$

$$\begin{aligned} f\_int := -\frac{1}{28\,(9\tan(\Pi)^2 + 49)\,(20\cos(2\,\Pi) + 29)}\Big(&-1176 \\ &+ 7560\,\mathrm{I}\tan(\Pi)^2\cos(2\,\Pi)\arctan\left(\frac{\tan(\Pi)}{5}\right) \\ &+ 2520\,\mathrm{I}\tan(\Pi)^2\cos(2\,\Pi)\arctan\left(\frac{5\tan(\Pi)}{9}\right) \\ &- 10080\,\mathrm{I}\tan(\Pi)^2\cos(2\,\Pi)\arctan(\tan(\Pi)) + 10962\ln(5)\tan(\Pi)^2 \\ &+ 216\tan(\Pi)^2\cos(2\,\Pi) - 5481\tan(\Pi)^2\ln(12\cos(2\,\Pi) + 13) \\ &- 1827\tan(\Pi)^2\ln(28\cos(2\,\Pi) + 53) + 7308\tan(\Pi)^2\ln(3) + 41160\ln(5)\cos(2\,\Pi) \\ &- 20580\cos(2\,\Pi)\ln(12\cos(2\,\Pi) + 13) - 6860\cos(2\,\Pi)\ln(28\cos(2\,\Pi) + 53) \\ &+ 27440\cos(2\,\Pi)\ln(3) + 3248\,\mathrm{I}\tan(\Pi) + 59682\,\mathrm{I}\arctan\left(\frac{\tan(\Pi)}{5}\right) \\ &+ 19894\,\mathrm{I}\arctan\left(\frac{5\tan(\Pi)}{9}\right) - 79576\,\mathrm{I}\arctan(\tan(\Pi)) + 59682\ln(5) \end{aligned} \tag{11}$$

$$+ 10962 \, I \tan(\Pi)^2 \arctan\left(\frac{\tan(\Pi)}{5}\right) + 3654 \, I \tan(\Pi)^2 \arctan\left(\frac{5 \tan(\Pi)}{9}\right)$$

$$- 14616 \, I \tan(\Pi)^2 \arctan(\tan(\Pi)) + 2240 \, I \tan(\Pi) \cos(2 \Pi)$$

$$+ 41160 \, I \cos(2 \Pi) \arctan\left(\frac{\tan(\Pi)}{5}\right) + 13720 \, I \cos(2 \Pi) \arctan\left(\frac{5 \tan(\Pi)}{9}\right)$$

$$- 54880 \, I \cos(2 \Pi) \arctan(\tan(\Pi)) + 7560 \ln(5) \tan(\Pi)^2 \cos(2 \Pi)$$

$$- 3780 \tan(\Pi)^2 \cos(2 \Pi) \ln(12 \cos(2 \Pi) + 13)$$

$$- 1260 \tan(\Pi)^2 \cos(2 \Pi) \ln(28 \cos(2 \Pi) + 53) + 5040 \tan(\Pi)^2 \cos(2 \Pi) \ln(3)$$

$$- 29841 \ln(12 \cos(2 \Pi) + 13) - 9947 \ln(28 \cos(2 \Pi) + 53) + 1176 \cos(2 \Pi)$$

$$\left. - 216 \tan(\Pi)^2 + 39788 \ln(3) \right)$$

restart;

#Part B

$f := \dfrac{(2 \cdot z^3 + z^2 + 4)}{z^4 - 4 \cdot z^2}$;

$$f := \frac{2 z^3 + z^2 + 4}{z^4 - 4 z^2} \tag{12}$$

 # convert f into partial fraction

convert(*f*, *fullparfrac*);

$$\frac{1}{2 (z + 2)} + \frac{3}{2 (z - 2)} - \frac{1}{z^2} \tag{13}$$

 # Find the residue 'res' at z = 2 since that is the point in the contour

res := *residue*(*f*, *z* = 2);

$$res := \frac{3}{2} \tag{14}$$

 # multiply the res by 2PiI to get the integral of the contour F

$F := res \cdot 2 \cdot \text{Pi} \cdot I$;

$$F := 3 \, I \, \pi \tag{15}$$

#QUESTION 4

restart;

$Digits := 5;$

$$Digits := 5 \tag{16}$$

$with(Statistics):$
$sample := [0.5, -0.7, 0.3, 1.1, 0.9, -1.2, 0.5, 1.3, 1.0];$

$$sample := [0.5, -0.7, 0.3, 1.1, 0.9, -1.2, 0.5, 1.3, 1.0] \tag{17}$$

$n := Count(sample);$

$$n := 9 \tag{18}$$

$mu0 := Mean(sample);$

$$\mu 0 := 0.41111 \tag{19}$$

$var := Variance(sample);$

$$var := 0.71362 \tag{20}$$

$s := StandardDeviation(sample);$

$$s := 0.84476 \tag{21}$$

$a := 0.05; \quad r := \frac{a}{2}; \qquad accept := 1 - r;$

$$a := 0.05$$

$$r := 0.025000$$

$$accept := 0.97500 \tag{22}$$

 # calculat the t test for the sample

$t0 := \dfrac{mu0}{\dfrac{s}{\mathrm{sqrt}(n)}};$

$$t0 := 1.4600 \tag{23}$$

#Two-sided test

$c1 := Quantile('StudentT'(n-1), r);$

$$c1 := -2.3060 \tag{24}$$

$c2 := Quantile('StudentT'(n-1), accept);$

$$c2 := 2.3060 \tag{25}$$

-

 # The t0 falls in the critical range c1=[-2.306 to +2.306] therefore reject the Null hypothesis that the mean of the samples equal

#QUESTION 3

$restart;$
$Digits : 5;$

$$5 \tag{26}$$

$with(DEtools):$

#Part 1 Solve the ODEs to use for evaluating the error of the RKS

$eq1 := diff(y1(x), x) = 2 \cdot y1(x) - 4 \cdot y2(x);$

$$eq1 := \frac{d}{dx}\, y1(x) = 2\, y1(x) - 4\, y2(x) \tag{27}$$

$eq2 := diff(y2(x), x) = y1(x) - 3 \cdot y2(x);$

$$eq2 := \frac{d}{dx}\, y2(x) = y1(x) - 3\, y2(x) \tag{28}$$

$ics := y1(0) = 3, y2(0) = 0;$

$$ics := y1(0) = 3, y2(0) = 0 \tag{29}$$

$sol := dsolve([eq1, eq2, ics]);$

$$sol := \{y1(x) = -e^{-2x} + 4\, e^{x}, y2(x) = -e^{-2x} + e^{x}\} \tag{30}$$

$sol1 := sol[1];$

$$sol1 := y1(x) = -e^{-2x} + 4\, e^{x} \tag{31}$$

$sol2 := sol[2];$

$$sol2 := y2(x) = -e^{-2x} + e^{x} \tag{32}$$

#Part 2 use code editor to define the RKS procdure and solve for eq1

$f := (x, y) \rightarrow [y[1], 2 \cdot y[1] - 4 \cdot y[2]];$

$$f := (x, y) \mapsto [y_1, 2 \cdot y_1 - 4 \cdot y_2] \tag{33}$$

#define the constants for both ODEs

$y[0] := [3, 0];$

$$y_0 := [3, 0] \tag{34}$$

$x[0] := 0 : h := 0.1 : N := 5 :$

#Run the RKS procedure for eq1

$RKS(f, x, y, h, N) :$

#Display the results for eq1

```
printf("\n Displays Results for Eq1 using the RKS procedure\n");
printf("  x        y1         y2        err y1        err y2\n");
for n from 0 to N do
    printf("%6.3f  %10.6f  %10.6f  %12.8f  %12.8f\n",  x[n], y[n][1],  y[n][2],  subs(x=x[n],
   rhs(sol[1])) − y[n][1],  subs(x=x[n], rhs(sol[2])) − y[n][2]);
end;
```

```
Displays Results for Eq1 using the RKS procedure
 x          y1         y2          err y1         err y2
0.000     3.000000    0.000000    0.00000000    0.00000000
0.100     3.315512    0.521725    0.28644042   -0.23528484
0.200     3.664208    0.926360    0.55108327   -0.37527698
0.300     4.049575    1.258268    0.80104810   -0.45722062
0.400     4.475473    1.547798    1.04249711   -0.50530192
0.500     4.946162    1.815966    1.28084373   -0.53512372
```

#Part 3 use code editor to define the RKS procdure and solve for eq2

$f := (x, y) \to [y[2],\ y[1] - 3 \cdot y[2]];$

$$f := (x, y) \mapsto [y_2, y_1 - 3 \cdot y_2] \quad (35)$$

#Run the RKS procedure for eq2

$RKS(f, x, y, h, N):$

#Display the results for eq2

```
printf("\n Displays Results for Eq2 using the RKS procedure\n");
printf("  x         y1         y2        err y1        err y2\n");
for n from 0 to N do
    printf("%6.3f %10.6f %10.6f %12.8f %12.8f\n", x[n], y[n][1], y[n][2], subs(x=x[n],
   rhs(sol[1])) - y[n][1], subs(x=x[n], rhs(sol[2])) - y[n][2]);
end;
```

```
 Displays Results for Eq2 using the RKS procedure
  x           y1           y2           err y1         err y2
 0.000    3.000000     0.000000     0.00000000     0.00000000
 0.100    3.013625     0.259588     0.58832792     0.02685266
 0.200    3.049774     0.454147     1.16551722     0.09693546
 0.300    3.102922     0.602213     1.74770179     0.19883390
 0.400    3.169123     0.717115     2.34884657     0.32538121
 0.500    3.245568     0.808439     2.98143796     0.47240278
```