5.4

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$$X(\Omega) = \frac{2}{1+\Omega^2}$$

$$x(t) \xleftrightarrow{F} X(\Omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

a)

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

$$\left.\frac{2}{1+\Omega^2}\right|_{\Omega=0} = \int x(t)\, e^{-j\Omega t}\, dt$$

$$\frac{2}{1+(0)^2} = \int x(t)\, e^{-j0t}\, dt$$

$$\boxed{2 = \int_{\infty}^{\infty} x(t)\, dt}$$

b) $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{0}\, d\Omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2}{1+\Omega^2}\, d\Omega$$

$$x(0) = \frac{2}{2\pi}\left[\tan^{-1}(\Omega)\right]_{-\infty}^{\infty}$$

$$x(0) = \frac{1}{\pi}\left[\tan^{-1}(\Omega)\right]_{-\infty}^{\infty}$$

$$x(0) = \frac{1}{\pi}\left[\pi - 0\right]$$

$$\boxed{x(0) = 1}$$

c)

$$X(\Omega) = \mathcal{L}\, x(t)$$

$$x(t) = \mathcal{L}^{-1} X(\Omega)$$

$$\Omega = \frac{s}{j} \qquad X\left(\frac{s}{j}\right) = \frac{2}{1+\left(\frac{s}{j}\right)} = \frac{2}{1-s^2}$$

$$x(t) = \mathcal{L}^{-1} \frac{2}{1-s^2}$$

$$x(t) = \mathcal{L}^{-1} \frac{2(1)}{1^2 - s^2}$$

$$x(t) = e^{-a|t|} \qquad a = 1$$

$$\boxed{x(t) = e^{-|t|}}$$

5.6

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$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

a) $x(t) = \cos(t) \quad 0 \le t \le 1$

$$X(\Omega) = \int_0^1 x(t)\, e^{-j\Omega t}\, dt$$

$$X(\Omega) = \int_0^1 \cos(t)\, e^{-j\Omega t}\, dt$$

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

$$X(\Omega) = \frac{1}{2} \int_0^1 \left(e^{jt} + e^{-jt}\right) e^{-j\Omega t}\, dt$$

$$X(\Omega) = \frac{1}{2} \int_0^1 e^{jt - j\Omega t} + e^{-(jt + j\Omega t)}\, dt$$

$$X(\Omega) = \frac{1}{2}\left[\int_0^1 e^{jt(-\Omega+1)}\, dt + \int_0^1 e^{-jt(\Omega+1)}\, dt\right]$$

$$X(\Omega) = \frac{1}{2}\left[\frac{e^{j(1-\Omega)} - 1}{j(1-\Omega)} - \frac{e^{-j(1+\Omega)} - 1}{j(1+\Omega)}\right]$$

$$X(\Omega) = e^{-j\frac{(\Omega-1)}{2}} \cdot \frac{\sin(\Omega-1)}{(\Omega-1)} + e^{-j\frac{(\Omega+1)}{2}}\, \frac{\sin(n+1)}{n+1}$$

b) $y(t) = x(2t)$ use scaling property of fourier Transform

$$x(at) = \frac{1}{|a|} X\left(\frac{\Omega}{a}\right) \quad a = 2$$

let $P_{(\Omega)} = 2e^{-j\frac{\Omega}{2}} \frac{\sin(\Omega)}{2}$

$$X_{(\Omega)} = \frac{1}{2}\left[P(\Omega+1) + P(\Omega-1)\right]$$

$$Y_{(\Omega)} = \frac{1}{|2|} X(\Omega)$$

$$Y_{(\Omega)} = \frac{1}{2} \cdot \frac{1}{|2|}\left[P\left(\frac{\Omega}{2}+1\right) + P\left(\frac{\Omega}{2}-1\right)\right]$$

$$\boxed{Y_{\Omega} = \frac{1}{4}\left[P\left(\frac{\Omega}{2}+1\right) + P\left(\frac{\Omega}{2}-1\right)\right]}$$

$$z(t) = x\left(\frac{t}{2}\right)$$

$$x(at) \xleftrightarrow{F} \frac{1}{|a|} X\left(\frac{\Omega}{a}\right) \quad a = \frac{1}{2}$$

$$x\left(\frac{t}{2}\right) \xleftrightarrow{F} \frac{1}{\left|\frac{1}{2}\right|} X\left(\frac{\Omega}{\frac{1}{2}}\right)$$

$$x\left(\frac{t}{2}\right) \xleftrightarrow{F} 2X(2\Omega)$$

$$Z(\Omega) = 2X(2\Omega)$$

$$Z(\Omega) = 2 \cdot \frac{1}{2}\left[P(2\Omega+1) + P(2\Omega-1)\right]$$

$$\boxed{Z(\Omega) = P\left[P(2\Omega+1) + P(2\Omega-1)\right]}$$

c)

$Y(\Omega)$ is an $\boxed{\text{expanded}}$ signal of $X(\Omega)$ in the frequency domain

$Z(\Omega)$ is a $\boxed{\text{compressed}}$ signal of $X(\Omega)$ in the frequency domain

5.16

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$$p(t) = u(t+1) - u(t-1)$$

$$F\{p(t)\} = F\{u(t+1)\} - F\{u(t-1)\}$$

$$= \left[\frac{-1}{j\omega}\left(1-e^{-j\omega}\right)\right] - \left[\frac{-1}{j\omega}\left(1-e^{j\omega}\right)\right]$$

$$= \frac{e^{-j\omega}-1}{j\omega} + \frac{1-e^{j\omega}}{j\omega}$$

$$\boxed{F\{p(t)\} = \frac{e^{-j\omega}-e^{j\omega}}{j\omega}}$$