#LAMIN JAMMEH
#EE597
#HW6

_#Pr9.2 (Addition, scalar multiplication)

restart;
with(*LinearAlgebra*) :
#define vectors a and c
$a := \langle -2 \mid -3 \mid 5\rangle;\ c := \langle 7 \mid -2 \mid 8\rangle;$

$$a := \begin{bmatrix} -2 & -3 & 5 \end{bmatrix}$$

$$c := \begin{bmatrix} 7 & -2 & 8 \end{bmatrix} \tag{1}$$

$4 \cdot a + 8 \cdot c;$

$$\begin{bmatrix} 48 & -28 & 84 \end{bmatrix} \tag{2}$$

$4 \cdot (a + 2 \cdot c);$

$$\begin{bmatrix} 48 & -28 & 84 \end{bmatrix} \tag{3}$$

#Pr 9.4 (Equilibrium)

restart;
with(*LinearAlgebra*) :
#deifne all 3 points P,q, and u as a 3 point vector
$p := Vector([x, y, z]);\quad q := Vector([-5, 7, -1]);\ u := Vector([4, -4, -1]);$

$$p := \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$q := \begin{bmatrix} -5 \\ 7 \\ -1 \end{bmatrix}$$

$$u := \begin{bmatrix} 4 \\ -4 \\ -1 \end{bmatrix} \tag{4}$$

 # at equilibrium the Vector Sum is the zero vector
$P := p + q + u = \langle 0, 0, 0\rangle;$

$$P := \begin{bmatrix} x-1 \\ y+3 \\ z-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(5)}$$

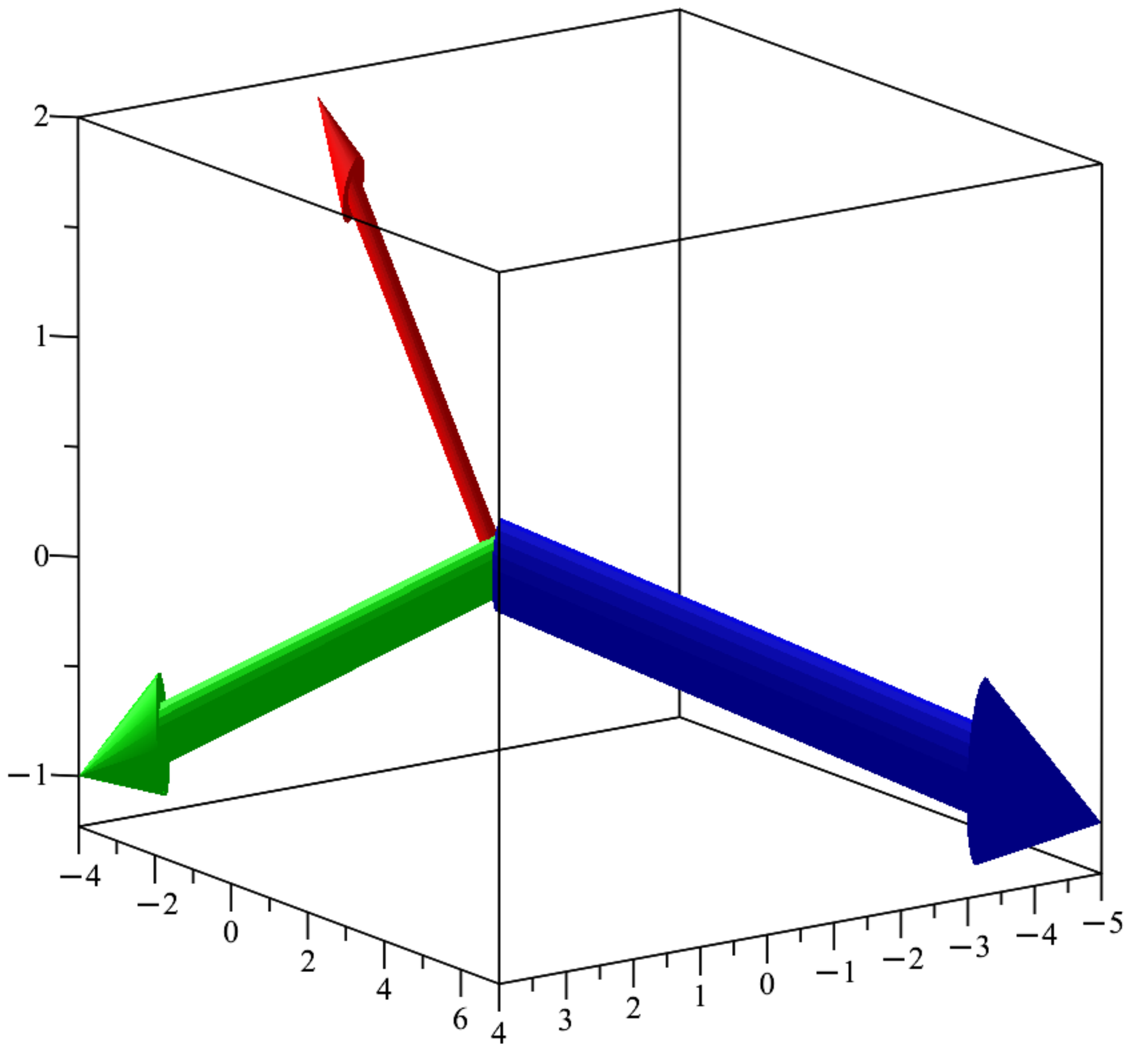
$solve(P);$

$$\{x=1, y=-3, z=2\} \quad \textbf{(6)}$$

$P := Vector([1, -3, 2]);$

$$P := \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \quad \textbf{(7)}$$

$with(VectorCalculus):$
$PlotVector([P, q, u], color = [red, blue, green]);$
#plots the tail to tail Vectors of P, q, and u



#Pr 9.6 (Angle)

restart;
with(*LinearAlgebra*) :
 # define the points b and c
$b := \langle 6 \mid -2 \mid -4 \rangle;\ c := \langle 2 \mid -1 \mid -1 \rangle;$

$$b := \begin{bmatrix} 6 & -2 & -4 \end{bmatrix}$$

$$c := \begin{bmatrix} 2 & -1 & -1 \end{bmatrix} \tag{8}$$

VectorAngle(*b*, *c*); *#angle in radians*

$$\arccos\left(\frac{3\sqrt{14}\sqrt{6}}{28}\right) \tag{9}$$

evalf[4](*convert*(%, *degrees*)); #*convert and evaluate to 4 sig figures*

$$11.04\ degrees \tag{10}$$

#Pr 9.8 (Vector product)

restart;
with(*LinearAlgebra*) :
$a := \langle 5 \mid 3 \mid -4 \rangle;\ c := \langle 7 \mid -4 \mid 3 \rangle;$

$$a := \begin{bmatrix} 5 & 3 & -4 \end{bmatrix}$$

$$c := \begin{bmatrix} 7 & -4 & 3 \end{bmatrix} \tag{11}$$

$w := CrossProduct(a, c);$

$$w := \begin{bmatrix} -7 & -43 & -41 \end{bmatrix} \tag{12}$$

$W := evalf[4](Norm(w, 2));$

$$W := 59.82 \tag{13}$$

$v := CrossProduct(c, a);$

$$v := \begin{bmatrix} 7 & 43 & 41 \end{bmatrix} \tag{14}$$

$V := evalf[4](Norm(v, 2));$

$$V := 59.82 \tag{15}$$

$x := DotProduct(a, c);$

$$x := 11 \tag{16}$$

#Pr 9.10 (Tetrahedron)

restart;
with(*LinearAlgebra*) :
·#*volume of a tetrahedron*$=\frac{1}{6}\cdot[vector(AB)\ X\ vector(AC)]\bullet vectorAD$
Define the vertices
$a := \langle 5 \mid 7 \mid 8 \rangle;\ b := \langle 1 \mid 5 \mid 11 \rangle;\ c := \langle 6 \mid 7 \mid 8 \rangle;\ d := \langle 4 \mid 6 \mid 7 \rangle;$

$$a := \begin{bmatrix} 5 & 7 & 8 \end{bmatrix}$$

$$b := \begin{bmatrix} 1 & 5 & 11 \end{bmatrix}$$

$$c := \begin{bmatrix} 6 & 7 & 8 \end{bmatrix}$$

$$d := \begin{bmatrix} 4 & 6 & 7 \end{bmatrix} \tag{17}$$

#*Using vertice 'a'as refrence*
$AB := a - b;\ AC := a - c;\ AD := a - d;$

$$AB := \begin{bmatrix} 4 & 2 & -3 \end{bmatrix}$$

$$AC := \begin{bmatrix} -1 & 0 & 0 \end{bmatrix}$$

$$AD := \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \tag{18}$$

$Volume := \frac{1}{6} \cdot (CrossProduct(AB, AC)).AD;$ #using "." as a DotProduct operator

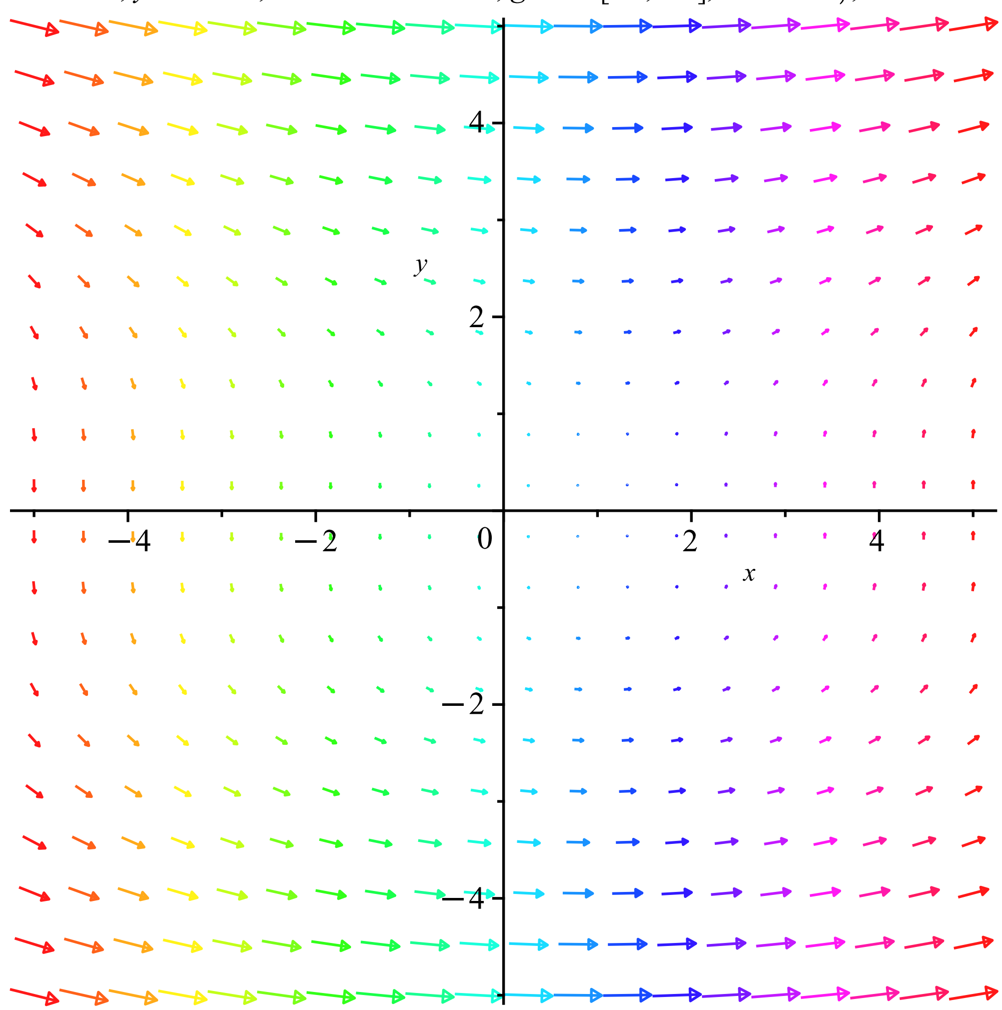
$$Volume := \frac{5}{6} \tag{19}$$

#Pr 9.14 (Vector field)

restart;
with(*plots*) :
$v := [y^2, x - 1];$

$$v := [y^2, x - 1] \tag{20}$$

$fieldplot(v, x = -5..5, y = -5..5, arrows = medium, grid = [20, 20], color = x);$



#Pr 9.18 (Tangential acceleration)

restart;

 # Tangential component of accelaration $aT = \frac{a(t) \cdot v(t)}{\| v(t) \|} \cdot \frac{v(t)}{\|v(t)\|}$ **also velocity** $(v) = r(t)'$ **and** $a = v'$

with(*LinearAlgebra*) :

#define the curve r(t) as r

$r := Vector([\sin(t), \cos(2 \cdot t), -\sin(2 \cdot t)]);$

$$r := \begin{bmatrix} \sin(t) \\ \cos(2\,t) \\ -\sin(2\,t) \end{bmatrix} \quad \textbf{(21)}$$

 # Determine the Velocity (v)=r' and acce (a)= r''

$v := VectorCalculus[diff](r, t);\ a := VectorCalculus[diff](r, t, t);$

$$v := (\cos(t))e_x + (-2\sin(2\,t))e_y + (-2\cos(2\,t))e_z$$

$$a := (-\sin(t))e_x + (-4\cos(2\,t))e_y + (4\sin(2\,t))e_z \quad \textbf{(22)}$$

#Determine the unit velocity of the curve

$u := Normalize(v, 2);$

$$u := \begin{bmatrix} \frac{\cos(t)}{\sqrt{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2}} \\ -\frac{2\sin(2\,t)}{\sqrt{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2}} \\ -\frac{2\cos(2\,t)}{\sqrt{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2}} \end{bmatrix} \quad \textbf{(23)}$$

 # determine the tengential acceleration $aT = \frac{a}{\| v \|} \cdot u$

$aT := \frac{a}{Norm(v, 2)} \cdot u;$

$$aT := -\frac{\sin(\bar{t})\cos(t)}{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2} + \frac{8\cos(2\,\bar{t})\sin(2\,t)}{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2} - \frac{8\sin(2\,\bar{t})\cos(2\,t)}{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2} \quad \textbf{(24)}$$

#define aT as a 3 point vector[i, j, k] for easy ploting

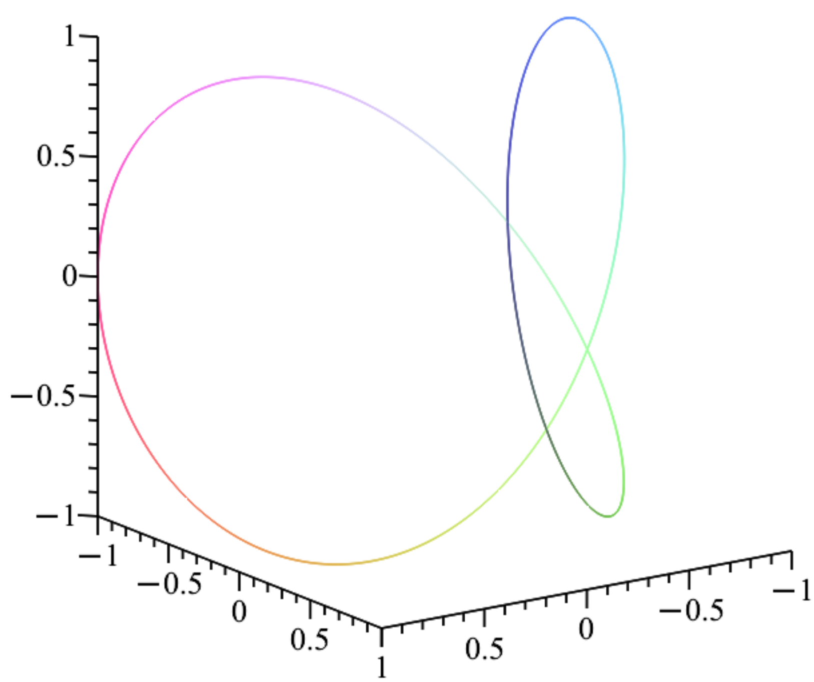
$aT := Vector\Bigg(\Bigg[-\frac{\sin(\bar{t})\cos(t)}{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2}, \frac{8\cos(2\,\bar{t})\sin(2\,t)}{|\cos(t)|^2 + 4\,|\sin(2\,t)|^2 + 4\,|\cos(2\,t)|^2},$

$$- \frac{8 \sin(2 \bar{t}) \cos(2 t)}{|\cos(t)|^2 + 4 |\sin(2 t)|^2 + 4 |\cos(2 t)|^2} \Bigg]\Bigg)$$

$$aT := \begin{bmatrix} -\frac{\sin(\bar{t}) \cos(t)}{|\cos(t)|^2 + 4 |\sin(2 t)|^2 + 4 |\cos(2 t)|^2} \\ \frac{8 \cos(2 \bar{t}) \sin(2 t)}{|\cos(t)|^2 + 4 |\sin(2 t)|^2 + 4 |\cos(2 t)|^2} \\ -\frac{8 \sin(2 \bar{t}) \cos(2 t)}{|\cos(t)|^2 + 4 |\sin(2 t)|^2 + 4 |\cos(2 t)|^2} \end{bmatrix} \tag{25}$$

 # remove the VectorCalculus library
with(*plots*) :
?*spacecurve*
spacecurve([*r*[1], *r*[2], *r*[3], *t* = 0 ..2·Pi], *axes* = *FRAME*, *numpoints* = 1000, *title*
= "plot of curve r(t)");

plot of curve r(t)



spacecurve([*aT*[1], *aT*[2], *aT*[3], *t* = 0 ..10·Pi], *axes* = *FRAME*, *numpoints* = 100, *title*
= "Plot of tangnetial acceleration");

Plot of tangnetial acceleration