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% Name: Lamin Jammeh
% Class: EE480 Online
% Semster: Fall 2023
% HW_10

% Basic Problems
%% *********** 5.16a **********
clear;
clc;
% STEP 1 defien the syms function and signal x(t) with p(t)
syms t s w;
t_range = -2:0.001:10;% duration of the signal captures
x = @(t) ((1+cos(pi.*t)).*heaviside(t+1)-heaviside(t-1)); %signal x(t)
p = @(t) (heaviside(t+1)-heaviside(t-1)); %pulse signal

%STEP2 determine the x(t) values with the define  time range
x_values = subs(x(t),t,t_range);

%STEP3 Plot the x(t) with the define time range
plot(t_range,x_values,'r','LineWidth',2);
xlabel('t');
ylabel('x(t)');
title("Plot of x(t) in the time domain")
grid on;

% *********** 5.16b *********
% determine he fourier Transform with fourier function in matlab
P = fourier(p(t), t, w);
%% ********** 5.29 **********
clear;
clc;
% STEP 1 define the duration of the signal and the N(number of Fourier
% coeeficients)
t =0:0.01:10;
N = 50;

% STEP 2 Define the function or signal x(t)
x = @(t) (heaviside(t+0.5)-heaviside(t-0.5));

%STEP 2 create a matrix for the Periods and Cn values to be used
T_values = [2,4,8,16];
Cn_values = cell(1,numel(T_values)); % stores Cn values at each T0

% Step 3 create a Loop to deffine w=2pi/T at each T values
for T = 1:numel(T_values) % accesses the T_values or index through T_values
    T0 = T_values(T);
    w = 2*pi/T0;

    % STEP 4 create an empty array for the Cn [1 x Number of Harmonics]
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    Cn = zeros(1,N); %opens an array from C1...CN

% STEP 5 calculate the Cn from -N:N using the complex integral method
for n = -N:N
    n_index = n + N + 1; % index through -N:N in increments of +1
    Cn(n_index) = (1 / T) * integral(@(t) x(t) .* exp(-1j * n * w * t), 0, T0); %↙
determine Cn from -N:1:N
end

% STEP 6 Determine the Cn values for each T0 value
Cn_values{T} = Cn; %determine Cn values for each T0
end

% STEP 7 Plot Cn_mag @ each T0
figure;
for T = 1:numel(T_values) % accesses the T_values or index through T_values
    T0 = T_values(T);
    subplot(numel(T_values), 1, T);
    stem(-N:N, abs(Cn_values{T}), 'r');
    title(['T = ' num2str(T0)]);
    xlabel('Harmonics(n)');
    ylabel('|Fourier Coefficients (c_n)|');
end

%% ********** 5.31 matlab **********
clear;
clc;
% STEP 1 define syms function interms of t and w
syms t w;

t_range = -10:0.01:10; % duration of the signal x(t)
w_range = -10:0.01:10; %freq range of X(w)

x = @(t) (2*exp(-2*abs(t)));% signal x(t)

% Step 2 determine the FT using the fourier function of matlab
X = fourier(x(t),t,w);

% STEP 3 determine the magnitude of the FT using abs(FT)
X_mag = abs(X);

%STEP 4 determine the Xmag and x(t) values for the given freq and time range using↙
sub(func,old,new)
x_tValue = subs(x(t), t, t_range);
X_magValue = subs(X_mag, w, w_range);

%TEP4 plot the signals to visualize
subplot(2,1,1)
% Plot of signal X(t)
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plot(t_range,x_tValue,'r','LineWidth',2)
xlabel('Time (t)');
ylabel('x(t)')
title('Plot to visualize signal x(t)')
grid on;

subplot(2,1,2)
% Plot the magnitude spectrum
stem(w_range, X_magValue);
title('Magnitude Spectrum of x(t)');
xlabel('Frequency (w)');
ylabel('Magnitude');
grid on;
```

5.4

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$$X(\Omega) = \frac{2}{1+\Omega^2}$$

$$x(t) \xleftrightarrow{F} X(\Omega)$$

$$\boxed{\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \\ X(j\omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \end{aligned}}$$

a)

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

$$\left.\frac{2}{1+\Omega^2}\right|_{\Omega=0} = \int x(t)\, e^{-j\Omega t}\, dt$$

$$\frac{2}{1+(0)^2} = \int x(t)\, e^{-j0t}\, dt$$

$$\boxed{2 = \int_{\infty}^{\infty} x(t)\, dt}$$

b) $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{0}\, d\Omega$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2}{1+\Omega^2}\, d\Omega$$

$$x(0) = \frac{2}{2\pi}\left[\tan^{-1}(\Omega)\right]_{-\infty}^{\infty}$$

$$x(0) = \frac{1}{\pi}\left[\tan^{-1}(\Omega)\right]_{-\infty}^{\infty}$$

$$x(0) = \frac{1}{\pi}\left[\pi - 0\right]$$

$$\boxed{x(0) = 1}$$

c)

$X_{(\Omega)} = \mathcal{L}\, x(t)$

$x(t) = \mathcal{L}^{-1} X(\Omega)$

$\Omega = \frac{s}{j} \qquad X\left(\frac{s}{j}\right) = \dfrac{2}{1+\left(\frac{s}{j}\right)} = \dfrac{2}{1-s^2}$

$x(t) = \mathcal{L}^{-1} \dfrac{2}{1-s^2}$

$x(t) = \mathcal{L}^{-1} \dfrac{2(1)}{1^2 - s^2}$

$x(t) = e^{-a|t|} \qquad a = 1$

$$\boxed{x(t) = e^{-|t|}}$$

5.6

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$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

a) $x(t) = \cos(t) \quad 0 \le t \le 1$

$$X(\Omega) = \int_0^1 x(t)\, e^{-j\Omega t}\, dt$$

$$X(\Omega) = \int_0^1 \cos(t)\, e^{-j\Omega t}\, dt$$

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

$$X(\Omega) = \frac{1}{2}\int_0^1 \left(e^{jt} + e^{-jt}\right) e^{-j\Omega t}\, dt$$

$$X(\Omega) = \frac{1}{2}\int_0^1 e^{jt - j\Omega t} + e^{-(jt + j\Omega t)}\, dt$$

$$X(\Omega) = \frac{1}{2}\left[\int_0^1 e^{jt(-\Omega+1)}\, dt + \int_0^1 e^{-jt(\Omega+1)}\, dt\right]$$

$$\boxed{X(\Omega) = \frac{1}{2}\left[\frac{e^{j(1-\Omega)} - 1}{j(1-\Omega)} - \frac{e^{-j(1+\Omega)} - 1}{j(1+\Omega)}\right]}$$

$$X(\Omega) = e^{-j\frac{(\Omega-1)}{2}} \cdot \frac{\sin(\Omega-1)}{(\Omega-1)} + e^{-j\frac{(\Omega+1)}{2}} \frac{\sin(n+1)}{n+1}$$

b) $y(t) = x(2t)$ use scaling property of fourier Transform

$$x(at) = \frac{1}{|a|} X\left(\frac{\Omega}{a}\right) \quad a = 2$$

let $P_{(\Omega)} = 2e^{-j\frac{\Omega}{2}} \frac{\sin(\Omega)}{2}$

$$X_{(\Omega)} = \frac{1}{2}\left[P(\Omega+1) + P(\Omega-1)\right]$$

$$Y_{(\Omega)} = \frac{1}{|2|} X(\Omega)$$

$$Y_{(\Omega)} = \frac{1}{2} \cdot \frac{1}{|2|}\left[P\left(\frac{\Omega}{2}+1\right) + P\left(\frac{\Omega}{2}-1\right)\right]$$

$$\boxed{Y_{\Omega} = \frac{1}{4}\left[P\left(\frac{\Omega}{2}+1\right) + P\left(\frac{\Omega}{2}-1\right)\right]}$$

$$z(t) = x\left(\frac{t}{2}\right)$$

$$x(at) \overset{F}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\Omega}{a}\right) \quad a = \frac{1}{2}$$

$$x\left(\frac{t}{2}\right) \overset{F}{\longleftrightarrow} \frac{1}{\left|\frac{1}{2}\right|} X\left(\frac{\Omega}{\frac{1}{2}}\right)$$

$$x\left(\frac{t}{2}\right) \overset{F}{\longleftrightarrow} 2X(2\Omega)$$

$$Z(\Omega) = 2X(2\Omega)$$

$$Z(\Omega) = 2 \cdot \frac{1}{2}\left[P(2\Omega+1) + P(2\Omega-1)\right]$$

$$\boxed{Z(\Omega) = P\left[P(2\Omega+1) + P(2\Omega-1)\right]}$$

c)

$Y(\Omega)$ is an $\boxed{\text{expanded}}$ signal of $X(\Omega)$ in the frequency domain

$Z(\Omega)$ is a $\boxed{\text{compressed}}$ signal of $X(\Omega)$ in the frequency domain

5.16

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$$p(t) = u(t+1) - u(t-1)$$

$$F\{p(t)\} = F\{u(t+1)\} - F\{u(t-1)\}$$

$$= \left[\frac{-1}{j\omega}\left(1-e^{-j\omega}\right)\right] - \left[\frac{-1}{j\omega}\left(1-e^{j\omega}\right)\right]$$

$$= \frac{e^{-j\omega}-1}{j\omega} + \frac{1-e^{j\omega}}{j\omega}$$

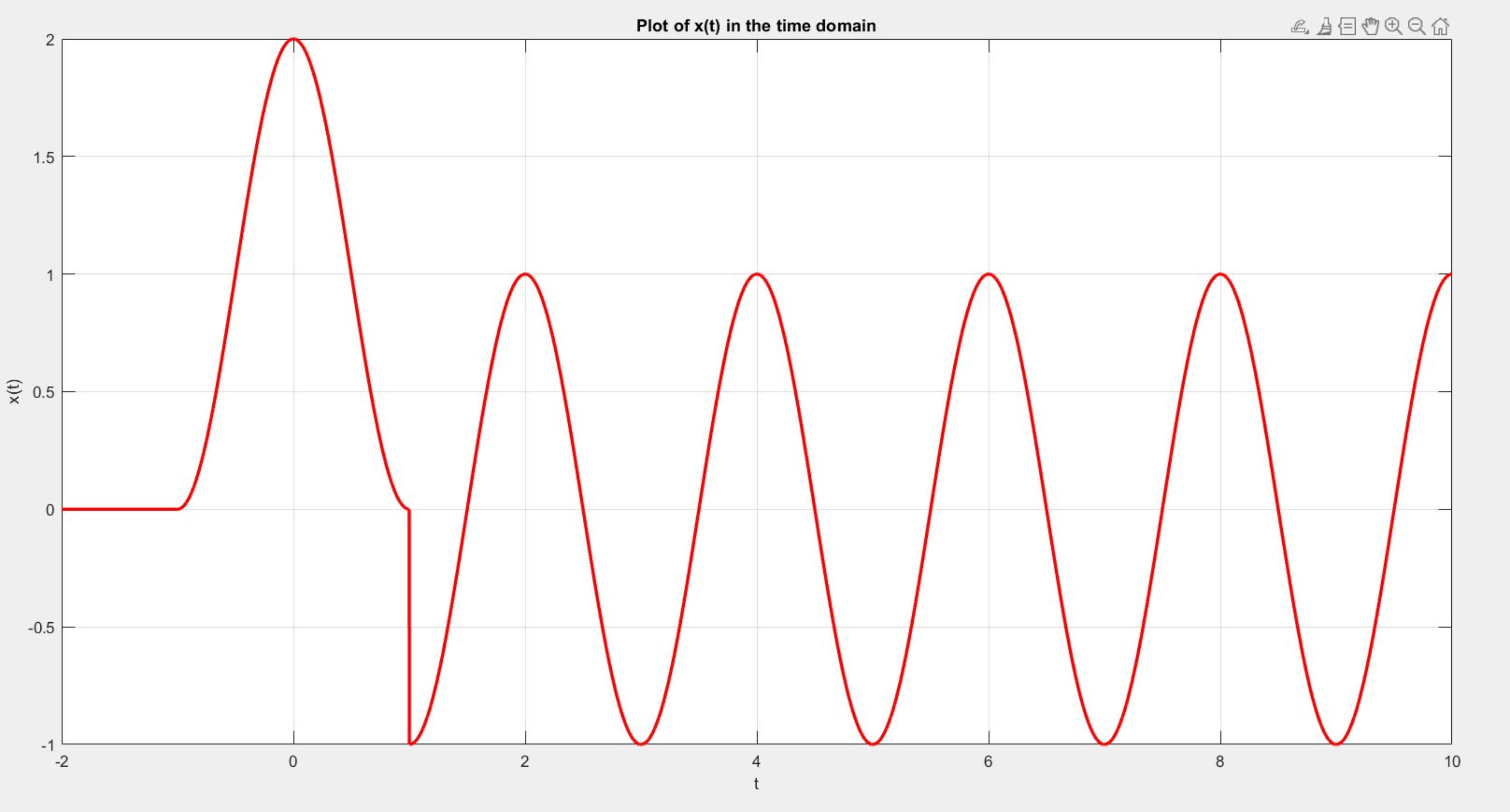
$$\boxed{F\{p(t)\} = \frac{e^{-j\omega} - e^{j\omega}}{j\omega}}$$

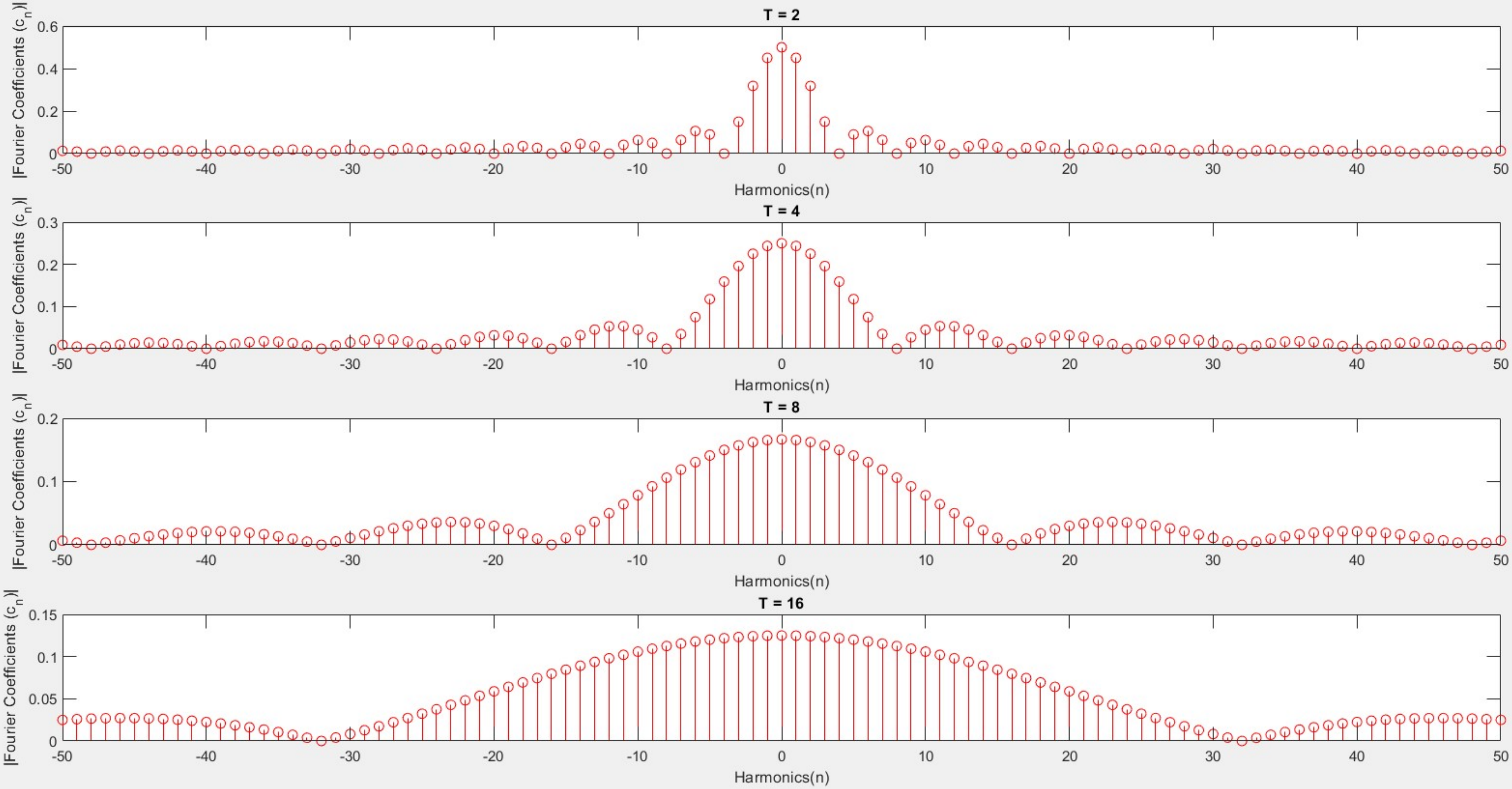
```
>> P = fourier(p(t), t, w)

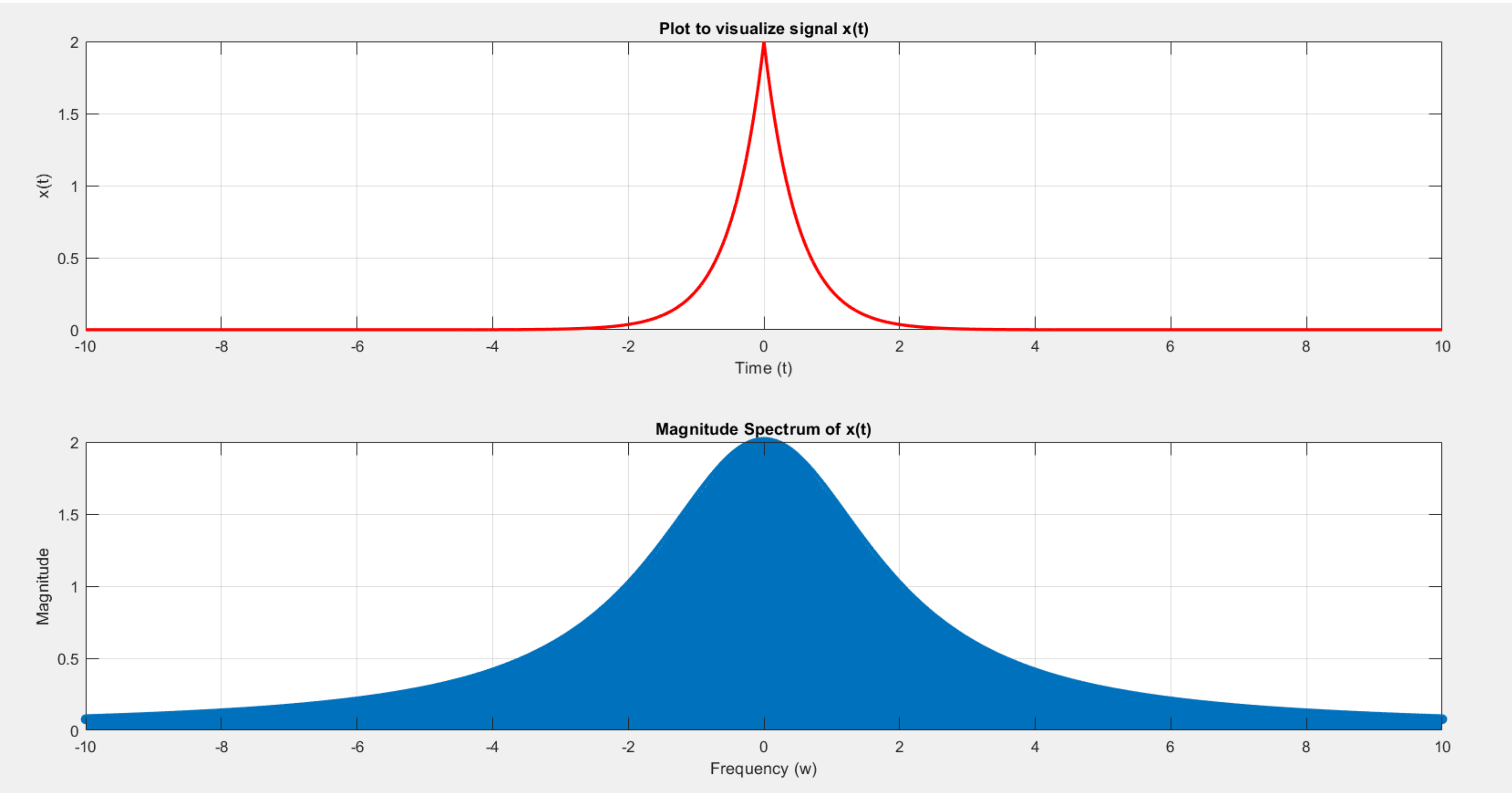
P =

- (- sin(w) + cos(w)*1i)/w + (sin(w) + cos(w)*1i)/w

>>
```


Plot of x(t) in the time domain
x(t)
t
2
1.5
1
0.5
0
-0.5
-1
-2
0
2
4
6
8
10


T = 2
T = 4
T = 8
T = 16
|Fourier Coefficients (c_n)|
Harmonics(n)



Plot to visualize signal x(t)
x(t)
Time (t)
Magnitude Spectrum of x(t)
Magnitude
Frequency (w)