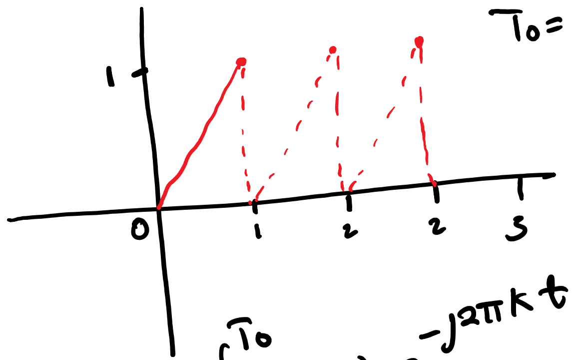
4.24

Saturday, November 4, 2023 12:54 PM

$$x_1(t) = r(t) - r(t-1) - u(t-1)$$

a)



$T_0 = 1$

$$X[k] = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k t}\, dt$$

$$= 1\int_0^1 t e^{-j2\pi k t}\, dt$$

$$= \frac{-j2\pi k\, e^{-j2\pi k}}{(j2\pi k)^2}$$

$$= \frac{-j2\pi k}{(j2\pi k)^2} = \frac{-1}{j2\pi k}$$

$$\boxed{X_k = \frac{j}{2\pi k}}$$

b) using laplace transform

$$x_1(t) = r(t) - r(t-1) - u(t-1)$$

$$X_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$ ⇒ from Matlab Code

$$X_1(s) = \frac{1 - e^{-s} - se^{-s}}{s^2}$$

$$X_{[k]}\Big|_{s=j2\pi k} = \frac{1 - e^{-j2\pi k} - j2\pi k\, e^{-j2\pi k}}{(j2\pi k)^2}$$

$$e^{j2\pi k} = 1$$

$$= \frac{1 - 1 - j2\pi k(1)}{(j2\pi k)^2}$$

$$= \frac{-j2\pi k}{(j2\pi k)^2} = \frac{-1}{j2\pi k}$$

$$\boxed{X[k]\Big|_{s=j2\pi k} = \frac{j}{2\pi k}}$$

4.26

Saturday, November 4, 2023 2:54 PM

$x_1(t) = u(t) - u(t-1)$

$X_1(s) = \frac{1}{s} - \frac{e^{-s}}{s}$ from matlab

$X_1(s) = \frac{1 - e^{-s}}{s}$

$X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{T_0}\left(X_1(s)\right)$

$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0}$

$X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{2}\left(\frac{1 - e^{-j\frac{2\pi k}{2}}}{j\frac{2\pi k}{2}}\right)$

$$\boxed{X[k]\Big|_{s=j\frac{2\pi k}{T_0}} = \frac{1}{2}\left(\frac{1 - e^{-j\pi k}}{j\pi k}\right)}$$

demitapp Cold & Cough 4 yrs old

$X_{dc} = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$

$X_{dc} = \frac{1}{2}\int_0^{2} 1\, dt$

$X_{dc} = \frac{1}{2}\left[\, t\Big|_0^2 \right]$

$$\boxed{X_{dc} = \frac{1}{2}(2) = 1}$$

4.28

Saturday, November 4, 2023 5:28 PM

$$y_1(t) = r(t) - 2r(t-1) + r(t-2)$$

$$Y_1(s) = \frac{e^{-2s}}{s^2} - \frac{2e^{-s}}{s^2} + \frac{1}{s^2}$$

$$Y_1(s) \quad \frac{e^{-2s} - 2e^{-s} + 1}{s^2}$$

$$Y[k] = \frac{1}{T_0}\left(Y_1(s)\right)$$

$$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{2} = j\pi$$

$$Y[k] = \frac{1}{2}\left[\frac{e^{-2j\pi k} - e^{-j\pi k} + 1}{(j\pi k)^2}\right]$$

$$Y\_dc = \frac{1}{T_0}\int_0^{T_0} y(t)\,dt$$

$$Y\_dc = \frac{1}{2}\left(\frac{t^2}{2}\right)\Big|_0^2$$

$$Y\_dc = \frac{1}{2}\left(\frac{2^2}{2}\right)$$

$$\boxed{Y\_dc = 1}$$

$$\boxed{\begin{aligned} X[k]\Big|_{s=j\frac{2\pi k}{T_0}} &= \frac{1}{2}\left(\frac{1-e^{-j\pi s}}{j\pi k}\right) \\ X_n\, dc &= 1 \end{aligned}}$$

from Q 4.26

4.29

Sunday, November 5, 2023 12:28 PM

$$x_1(t) = u(t) - u(t-1) \quad 0 \le t \le 2$$

$$X_1(s) = \frac{1}{s} - \frac{e^{-s}}{s}$$ from matlab $\mathcal{L}\{x_1(t)\}$

$$X[k] = \frac{1}{T_0}\left[X_1(s)\right] = \frac{1}{T_0}\left[\frac{1-e^{-s}}{s}\right]$$

$$T_0 = 2 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{2} = j\pi$$

$$\boxed{X[k]\Big|_{s=j\omega k} = \frac{1}{2}\left(\frac{1-e^{-j\pi k}}{j\pi k}\right)}$$

fourier Coefficent

$y_1(t) = u(t) - u(t-0.5) \quad 0 \leq t \leq 1$

$Y_1(s) = \frac{1}{s} - \frac{e^{-\frac{s}{2}}}{s}$ from matlab $\mathcal{L}\{y_1(t)\}$

$Y_{[k]} = \frac{1}{T_0}\left(Y_1(s)\right) = \frac{1}{T_0}\left[\frac{1 - e^{-\frac{s}{2}}}{s}\right]$

$T_0 = 1 \qquad s = j\omega = j\frac{2\pi}{T_0} = j\frac{2\pi}{1} = j2\pi$

$Y_{[k]}\Big|_{s=j\omega k} = \left(\frac{\cancel{1}}{\cancel{1}}\left(\frac{1 - e^{-j2\pi k}}{j2\pi k}\right)\right)$

$$\boxed{Y_{[k]}\Big|_{s=j\omega k} = \frac{1 - e^{-j2\pi k}}{j2\pi k}}$$

fourier Coefficent