

Pr.9.2 `[> a := [-2, -3, 5]; c := [7, -2, 8];`
`[> 4*a + 8*c; 4*(a + 2*c);` # Resp. $[48, -28, 84]$ $[48, -28, 84]$

Pr.9.4 `[> p := [-5, 7, -1] - [4, -4, -1];` # Resp. $p := [-9, 11, 0]$

Pr.9.6 Type ?angle.

```
[> b := <6, -2, -4>; c := <2, -1, -1>;
[> with(LinearAlgebra):
[> DotProduct(b, c)/sqrt(DotProduct(b, b)*DotProduct(c, c));

$$\frac{3}{14} \sqrt{21}$$

The length (Euclidean norm) of b can also be obtained by the command Norm(b, 2). Similarly for c. Thus,
[> DotProduct(b, c)/(Norm(b, 2)*Norm(c, 2)); # Resp.  $\frac{3}{28} \sqrt{14} \sqrt{6}$ 
[> evalf[5](arccos(%)); # Resp. 0.19018
[> VectorAngle(b, c); # Resp.  $\arccos\left(\frac{3}{28} \sqrt{14} \sqrt{6}\right)$ 
[> evalf[5](%); # Resp. 0.19018
```

Pr.9.8 Use the notations $\mathbf{v} = \mathbf{a} \times \mathbf{c}$ and $\mathbf{w} = \mathbf{c} \times \mathbf{a}$.

```
[> a := <5 3 -4>; c := <7 -4 3>;
[> with(LinearAlgebra):
[> v := CrossProduct(a, c); w := CrossProduct(c, a);

$$v := \begin{bmatrix} -7 & -43 & 41 \end{bmatrix}$$


$$w := \begin{bmatrix} 7 & 43 & 41 \end{bmatrix}$$

```

For the absolute value try the command `abs`, which will not give what you want. (Type ?abs.)

```
[> evalf(abs(v)); # Resp.  $\begin{bmatrix} 7. & 43. & 41. \end{bmatrix}$ 
```

The length or Euclidean norm of a vector **v** is obtained by the command `Norm(v, 2)`. Thus,

```
[> Norm(v, 2); Norm(w, 2); DotProduct(a, c); # Resp.  $\sqrt{3579}$   $\sqrt{3579}$  11
```

Pr.9.10 Choose one of the points, say, $P = (1, 3, 6)$, as the common initial point of the three edge vectors \mathbf{a} , \mathbf{b} , \mathbf{c} that determine the tetrahedron.

```
[ > P := <5  7  8>; A := <1  5 11>; B := <6  7  8>;
  C := <4  6  7>;

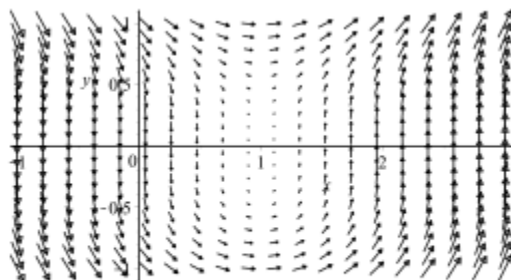
[ > a := A - P;                                # Resp. a := [ -4  -2  3 ]
[ > b := B - P;                                # Resp. b := [  1  0  0 ]
[ > c := C - P;                                # Resp. c := [ -1  -1  -1 ]
[ > with(LinearAlgebra):

The volume is 1/6 of the absolute value of the scalar triple product of  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ ,

[ > DotProduct(CrossProduct(a, b), c);          # Resp. -5
[ > 1/6*abs(%);                                # Resp. 5/6
```

Pr.9.14 Try other x and y intervals. For longer intervals the arrows may differ in length to an extent that you can no longer distinctly see the directions of the shorter arrows.

```
[ > with(plots):
[ > fieldplot( [y^2, x - 1], x = -1..3, y = -1..1, scaling = constrained,
               arrows = SLIM, fieldstrength = average(0.8));
```



Problem 9.14. Vector field $\mathbf{v} = [y^2, 1]$

Pr.9.18 Type `?spacecurve` for information on plotting of curves. Type the position vector \mathbf{r} , the velocity vector \mathbf{v} , and the acceleration vector \mathbf{a} .

```
[ > with(LinearAlgebra):
> r := <sin(t)  cos(2*t)  -sin(2*t)>;
      r := [ sin(t)  cos(2t)  -sin(2t) ]
> v := VectorCalculus[diff](r, t);
      v := (cos(t)) e_x - 2 sin(2t) e_y - 2 cos(2t) e_z
> a := VectorCalculus[diff](v, t);
      a := -sin(t) e_x - 4 cos(2t) e_y + 4 sin(2t) e_z
```

From this you obtain the tangential acceleration

```
[ > atan := DotProduct(a, v, conjugate = false)/DotProduct(v, v,
      conjugate = false)*v;
atan := [ -frac(sin(t) cos(t)^2, cos(t)^2 + 4 sin(2t)^2 + 4 cos(2t)^2),
          2 sin(t) cos(t) sin(2t),
          2 cos(t) sin(t) cos(2t) ]
          cos(t)^2 + 4 sin(2t)^2 + 4 cos(2t)^2 ]
```

This formula for the tangential acceleration (representing a vector in the direction of \mathbf{v}) is explained in Sec. 9.5 of AEM.

```
[ > atan := simplify(%);
atan := [ -frac(sin(t) cos(t)^2, cos(t)^2 + 4),
          4 sin(t)^2 cos(t)^2,
          2 sin(t) cos(t)(2 cos(t)^2 - 1) ]
          cos(t)^2 + 4 ]
```

The commands for the plotting are as shown. The curve is closed because the components of \mathbf{r} are periodic. To visualize the curve and understand the plot, you may wish to plot the projections into the three coordinate planes as well as portions of the curve in space corresponding to subintervals of the interval from 0 to 2π . Click on the figure and change the viewpoint to get a better idea of the curve.

```
[ > with(plots):
> spacecurve(r, t = 0..2*Pi, axes = NORMAL);
```



Problem 9.18. Curve represented by $\mathbf{r}(t)$