0.1 Basic Problems

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by Luis Chaparro, Academic Press

0.1 (a) i. $\mathcal{R}e(z) + \mathcal{I}m(v) = 8 - 2 = 6$

ii. $|z + v| = |17 + j1| = \sqrt{17^2 + 1}$

iii. $|zv| = |72 - j16 + j27 + 6| = |78 + j11| = \sqrt{78^2 + 11^2}$

iv. $\angle z + \angle v = \tan^{-1}(3/8) - \tan^{-1}(2/9)$

v. $|v/z| = |v|/|z| = \sqrt{85}/\sqrt{73}$

vi. $\angle(v/z) = -\tan^{-1}(2/9) - \tan^{-1}(3/8)$

(b) i. $z + v = 17 + j = \sqrt{17^2 + 1}e^{j\tan^{-1}(1/17)}$

ii. $zv = 78 + j11 = \sqrt{78^2 + 11^2}e^{j\tan^{-1}(11/78)}$

iii. $z^* = 8 - j3 = \sqrt{64 + 9}(e^{-j\tan^{-1}(3/8)})^* = \sqrt{73}e^{j\tan^{-1}(3/8)}$

iv. $zz^* = |z|^2 = 73$

v. $z - v = -1 + j5 = \sqrt{1 + 25}e^{-j\tan^{-1}(5)}$

0.2 (a) $z = 6e^{j\pi/4} = 6\cos(\pi/4) + j6\sin(\pi/4)$

i. $\mathcal{R}e(z) = 6\cos(\pi/4) = 3\sqrt{2}$

ii. $\mathcal{I}m(z) = 6\sin(\pi/4) = 3\sqrt{2}$

(b) i. Yes, $\mathcal{R}e(z) = 0.5(z + z^*) = 0.5(2\mathcal{R}e(z)) = \mathcal{R}e(z) = 8$

ii. Yes, $\mathcal{I}m(v) = -0.5j(v - v^*) = -0.5j(2j\mathcal{I}m(v)) = \mathcal{I}m(v) = -2$

iii. Yes, $\mathcal{R}e(z + v^*) = \mathcal{R}e(\mathcal{R}e(z) + \mathcal{R}e(v^*) + \mathcal{I}m(z) - \mathcal{I}m(v)) = \mathcal{R}e(z + v) = 17$

iv. Yes, $\mathcal{I}m(z + v^*) = \mathcal{I}m(17 + j5) = \mathcal{I}m(z - v) = \mathcal{I}m(-9 + j5) = 5$

0.9 (a) If $w = e^z$ then

$$\log(w) = z = 1 + j1$$

given that the log and e functions are the inverse of each other.
The real and imaginary of w are

$$w = e^z = e^1 e^{j1} = \underbrace{e\cos(1)}_{\text{real part}} + j \underbrace{e\sin(1)}_{\text{imaginary part}}$$

(b) The imaginary parts are cancelled and the real parts added twice in

$$w + w^* = 2\mathcal{R}e[w] = 2e\cos(1)$$

(c) Replacing z

$$w = e^z = e^1 e^{j1}$$

so that $|w| = e$ and $\angle w = 1$.
Using the result in (a)

$$|\log(w)|^2 = |z|^2 = 2$$

(d) According to Euler's equation

$$\cos(1) = 0.5(e^j + e^{-j}) = 0.5\left(\frac{w}{e} + \frac{w^*}{e}\right)$$

which can be verified using $w + w^*$ obtained above.

$\boxed{10(e)}$

i. Phasor $4e^{j\pi/3}$

ii. $-4\sin(2t+\pi/3) = 4\cos(2t+\pi/3+\pi/2)$ with phasor $4e^{j5\pi/6}$

iii. We have

$$
\begin{aligned}
4\cos(2t+\pi/3) - 4\sin(2t+\pi/3) &= \mathcal{R}e[(4e^{j\pi/3} + 4e^{j(\pi/2+\pi/3)})e^{j2t}] \\
&= \mathcal{R}e[4e^{j\pi/3}\underbrace{(1+e^{j\pi/2})}_{\sqrt{2}e^{j\pi/4}}e^{j2t}] \\
&= \mathcal{R}e[4\sqrt{2}e^{j7\pi/12}e^{j2t}]
\end{aligned}
$$

so that the phasor is $4\sqrt{2}e^{j7\pi/12}$

0.13 As we will see later, the sampling period of $x(t)$ with a frequency of $\Omega_{max} = 2\pi f_{max} = 2\pi$ should satisfy the Nyquist sampling condition

$$f_s = \frac{1}{T_s} \geq 2f_{max} = 2 \text{ samples/sec}$$

so $T_s \leq 1/2$ (sec/sample). Thus when $T_s = 0.1$ the continuous-time and the discrete-time signals look very much like each other, indicating the signals have the same information — such a statement will be justified in the chapter on sampling where we will show that the continuous-time signal can be recovered from the sampled signal. It is clear that when $T_s = 1$ the information is lost. Although it is not clear from the figure that when we let $T_s = 0.5$ the discrete-time signal keeps the information, this sampling period satisfies the Nyquist sampling condition and as such the original signal can be recovered from the sampled signal. The following MATLAB script is used.

```
% Pr. 0._13
clear all; clf
T=3; Tss= 0.0001; t=[0:Tss:T];
 xa=4*cos(2*pi*t);  % continuous-time signal
 xamin=min(xa);xamax=max(xa);
 figure(1)
 subplot(221)
 plot(t,xa); grid
 title('Continuous-time Signal');  ylabel('x(t)');  xlabel('t sec')
 axis([0  T 1.5*xamin 1.5*xamax])
 N=length(t);

 for k=1:3,
      if k==1,Ts= 0.1;  subplot(222)
        t1=[0:Ts:T];  n=1:Ts/Tss: N; xd=zeros(1,N);  xd(n)=4*cos(2*pi*t1);
        plot(t,xa); hold on; stem(t,xd);grid;hold off
        axis([0  T 1.5*xamin 1.5*xamax]); ylabel('x(0.1 n)'); xlabel('t')
        elseif  k==2, Ts=0.5;  subplot(223)
        t2=[0:Ts:T]; n=1:Ts/Tss: N;  xd=zeros(1,N);  xd(n)=4*cos(2*pi*t2);
        plot(t,xa); hold on; stem(t,xd); grid; hold off
        axis([0  T 1.5*xamin 1.5*xamax]); ylabel('x(0.5 n)'); xlabel('t')
        else,Ts=1; subplot(224)
        t3=[0:Ts:T];  n=1:Ts/Tss: N;  xd=zeros(1,N);  xd(n)=4*cos(2*pi*t3);
        plot(t,xa); hold on;  stem(t,xd); grid; hold off
         axis([0  T 1.5*xamin 1.5*xamax]); ylabel('x(n)'); xlabel('t')
      end
  end
```

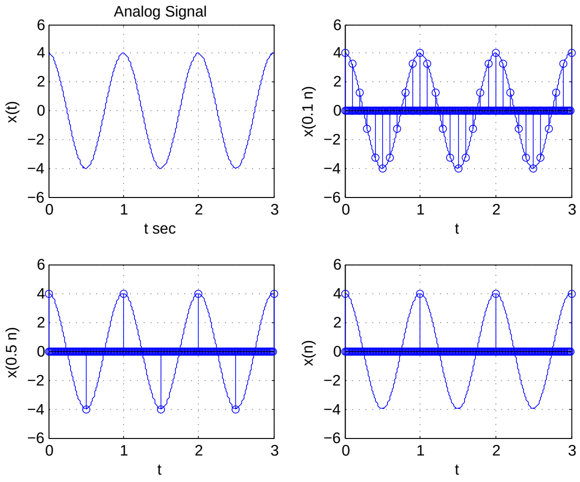

Figure 2: Problem 13: Analog continuous-time signal (top left); continuous-time and discrete-time signals superposed for $T_s = 0.1$ sec (top right) and $T_s = 0.5$ sec and $T_s = 1$ sec (bottom left to right).

0.16 (a) According to Kirchoff's current law

$$i_s(t) = i_R(t) + i_L(t) = \frac{v_L(t)}{R} + i_L(t)$$

but $v_L(t) = L di_L(t)/dt$ so that the ordinary differential equation relating the input $i_s(t)$ to the output current in the inductor $i_L(t)$ is

$$\frac{di_L(t)}{dt} + i_L(t) = i_s(t)$$

after replacing $L = 1$ and $R = 1$. Notice that this d.e. is the dual of the one given in the Chapter, so that the difference equation is

$$i_L(nT_s) = \frac{T_s}{2+T_s}[i_s(nT_s) + i_s((n-1)T_s)] + \frac{2-T_s}{2+T_s} i_L((n-1)T_s) \qquad n \geq 1$$

$$i_L(0) = 0$$

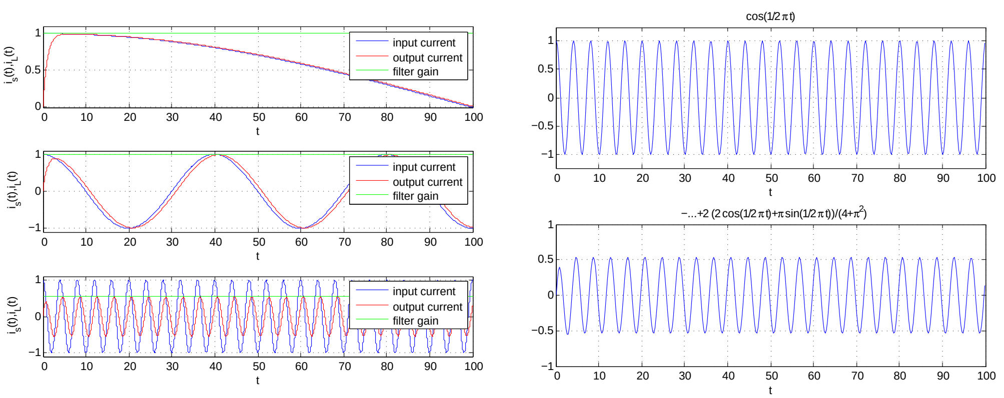

Figure 5: Problem 16: Left (top to bottom): solution of difference equation for $\Omega_0 = 0.005,\ 0.05,\ 0.5$ (rad/sec). Right: input (top), solution of ordinary differential equation (bottom).

(b)(c) The scripts to solve the difference and ordinary differential equations are the following.

```
% Pr.  0_16
clear all
% solution of difference equation
Ts=0.01;
t=[0:Ts:100];
figure(4)
for k=0:2;
    if k==0, subplot(311)
     elseif k==1, subplot(312)
     else, subplot(313)
    end
 W0= 0.005*10^k*pi; % frequency of source
 is=cos(W0*t); % source
```

```
 a=[1 (-2+Ts)/(2+Ts)]; % coefficients of i_L(n), i_L(n-1)
 b=[Ts/(2+Ts) Ts/(2+Ts)]; % coefficients of i_s(n), i_s(n-1)
 il=filter(b,a,is);  % current in inductor computed by
                     % MATLAB function 'filter'
 H=1/sqrt(1+W0^2)*ones(1,length(t)); % filter gain at W0
 plot(t,is,t,il,'r',t,H,'g'); xlabel('t'); ylabel('i_s(t),i_L(t)')
 axis([0 100 1.1*min(is) 1.1*max(is)])
 legend('input current','output current','filter gain'); grid
 pause(0.1)
end
%%%%
% solution of ordinary differential equation for cosine input of frequency  0.5pi
clear all
syms t x y
x=cos(0.5*pi*t);
y=dsolve('Dy+y=cos(0.5*pi*t)','y(0)=0','t')
figure(5)
 subplot(211)
 ezplot(x,[0 100]);grid
 subplot(212)
 ezplot(y,[0 100]);grid
 axis([0 100 -1 1])
```

0.19 The indefinite integral equals $0.5t^2$. Computing it in $[0, 1]$ gives the same value as the sum of the integrals computed between $[0, 0.5]$ and $[0.5, 1]$.

As seen before, the sum

$$S = \sum_{n=0}^{100} n = \frac{100(101)}{2} = 5050$$

while

$$S_1 = S + 50 = 5100$$
$$S_2 = S$$

the first sum has an extra term when $n = 50$ while the other does not. To verify this use the following script:

```
% Pr. 0_19
clear all
N=100;
syms n,N
S=symsum(n,0,N)
S1=symsum(n,0,N/2)+symsum(n,N/2,N)
S2=symsum(n,0,N/2)+symsum(n,N/2+1,N)
```

giving

```
S = 5050
S1 = 5100
S2 = 5050
```

0.21 (a) The point (1,1) in the two-dimensional plane corresponds to $z = 1 + j$. The magnitude and phase are

$$|z| = \sqrt{1+1} = \sqrt{2}$$
$$\angle z = \tan^{-1}(1) = \pi/4$$

(b) For the other complex numbers:

$$|w| = \sqrt{2}, \quad \angle w = \pi - \pi/4 = 3\pi/4$$
$$|v| = \sqrt{2}, \quad \angle v = \pi + \pi/4 = 5\pi/4$$
$$|u| = \sqrt{2}, \quad \angle u = -\pi/4$$

The sum of these complex numbers

$$z + w + v + u = 0$$

(c) The ratios

$$\frac{z}{w} = \frac{1+j}{-1+j} = \frac{\sqrt{2}e^{j\pi/4}}{\sqrt{2}e^{j3\pi/4}} = 1e^{-j\pi/2} = -j$$
$$\frac{w}{v} = \frac{-1+j}{-1-j} = \frac{\sqrt{2}e^{j3\pi/4}}{\sqrt{2}e^{j5\pi/4}} = 1e^{-j\pi/2} = -j$$
$$\frac{u}{z} = \frac{1-j}{1+j} = \frac{\sqrt{2}e^{-j\pi/4}}{\sqrt{2}e^{j\pi/4}} = 1e^{-j\pi/2} = -j$$

Also, multiplying numerator and denominator by the by the conjugate of the denominator we get the above results. For instance,

$$\frac{z}{w} = \frac{1+j}{-1+j} = \frac{(1+j)(-1-j)}{2} = \frac{-1-j-j-j^2}{2} = \frac{-2j}{2} = -j$$

and similarly for the others. Using these ratios we have

$$\frac{u}{w} = \frac{u}{z} \times \frac{z}{w} = (-j)(-j) = -1.$$

(d) $y = 10^{-6} = j10^{-6} = 10^{-6}z$ so that

$$|y| = 10^{-6}|z| = 10^{-6}$$
$$\angle y = \pi/4$$

Although the magnitude of y is negligible, its phase is equal to that of z.

The results are verified by the following script:

```
% Pr. 0_21
z=1+j; w=-1+j; v=-1-j;u=1-j;
figure(1)
compass(1,1)
hold on
compass(-1,1,'r')
hold on
compass(-1,-1,'k')
```

```
hold on
compass(1,-1,'g')
hold off
% part (a)
abs(z)
angle(z)
% part (b)
abs(w)
angle(w)
abs(v)
angle(v)
abs(u)
angle(u)
r=z+w+v+u
%part (c)
r1=z/w
r2=w/v
r3=u/z
r4=u/z
r5=u/w
figure(2)
compass(real(r1),imag(r1))
hold on
compass(real(r2),imag(r2),'r')
hold on
compass(real(r3),imag(r3),'k')
hold on
compass(real(r4),imag(r4),'g')
hold on
compass(real(r5),imag(r5),'b')
hold off
% part (c)
z
y=z*1e-16
abs(y)
angle(y)/pi
```

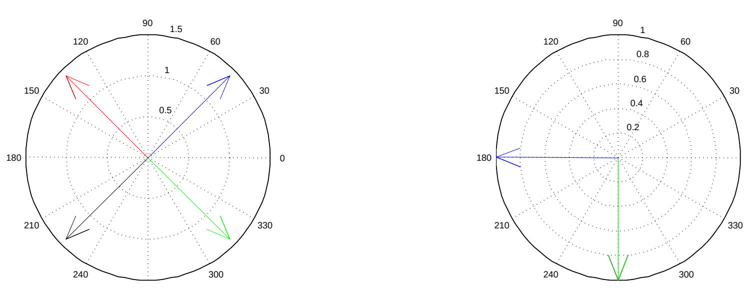

Figure 8: Problem 21: Results of complex calculations in parts (a) z, w, v, u and (b) $z/w, w/v, u/z, z/w$