Q1

Saturday, March 23, 2024 3:18 PM

Bus #	Type	Rating	Details
1	Slack	400 MW	13.8 kV
2	load	13.8 kV	$S_2 = 150 + j30$ MVA
3	load	13.8 kV	$S_3 = 200 + j50$ MVA, p.f = 22 MVA

Power base of system $S_{base} = 100$ MVA

Voltage base of System $V_{base} = 13.8$ kV

$$Z_{base} = \frac{(V_{base})^2}{S_{base}} = \frac{(13.8\text{kV})^2}{100\text{ MVA}} = 1.90\,\Omega$$

Bus 1 – 3 Voltages 1.0 p.u

$$P_1 = \frac{400\text{ MW}}{100\text{ MVA}} = 4\text{ p.u}$$

$$P_2 = \frac{150\text{MW}}{100\text{ MVA}} = 1.5\text{ p.u} \qquad Q_2 = \frac{30\text{ MVAR}}{100\text{ MVA}} = 0.3\text{ p.u}$$

$$P_3 = \frac{200\text{ MW}}{100\text{ MVA}} = 0.2\text{ p.u}$$

$$Q_{3\text{-final}} = Q_3 - Q_{3\,(\text{p.f correction})}$$

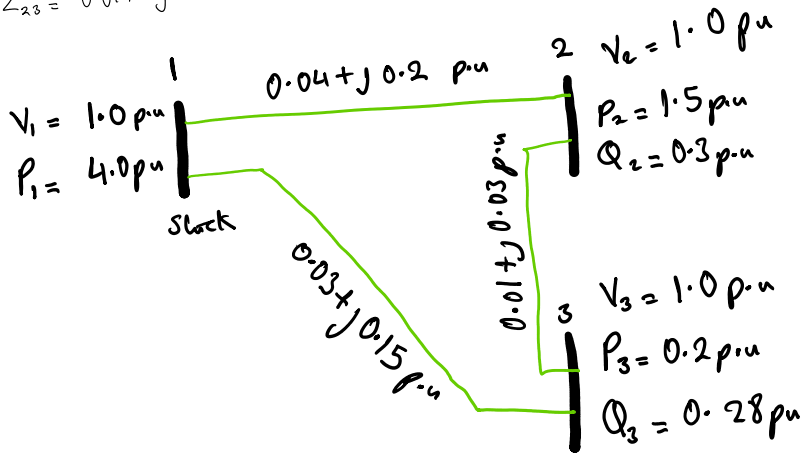
$$Q_{3\text{-final}} = 50 - 22\ [\text{MVAR}] = 28\text{ MVAR}$$

$$Q_3 = \frac{28\text{MVAR}}{100\text{ MVA}} = 0.28\text{ p.u}$$

$$Z_{12} = 0.076 + j0.38\,\Omega = 0.04 + j0.2\text{ p.u}$$

$$Z_{13} = 0.057 + j0.285\,\Omega = 0.03 + j0.15\text{ p.u}$$

$$Z_{23} = 0.019 + j0.057\,\Omega = 0.01 + j0.03\text{ p.u}$$



1b 3x3 Admittance Matrix

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{11} = \frac{1}{Z_{12}} + \frac{1}{Z_{13}} \qquad Y_{22} = \frac{1}{Z_{12}} + \frac{1}{Z_{23}} \qquad Y_{33} = \frac{1}{Z_{23}} + \frac{1}{Z_{13}}$$

$$Y_{12} = Y_{21} = \frac{1}{Z_{12}}$$

$$Y_{13} = Y_{31} = \frac{1}{Z_{13}}$$

$$Y_{23} = Y_{32} = \frac{1}{Z_{23}}$$

$$Y = \begin{bmatrix} 2.243 - j11.218 & 0.962 - j4.808 & 1.282 - j6.410 \\ 0.962 - j4.808 & 10.962 - j34.808 & 10.0 - j30.0 \\ 1.282 - j6.410 & 10.0 - j30.0 & 11.282 - j36.410 \end{bmatrix}$$

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Y =

   2.2436 -11.2179i   0.9615 - 4.8077i   1.2821 - 6.4103i
   0.9615 - 4.8077i  10.9615 -34.8077i  10.0000 -30.0000i
   1.2821 - 6.4103i  10.0000 -30.0000i  11.2821 -36.4103i
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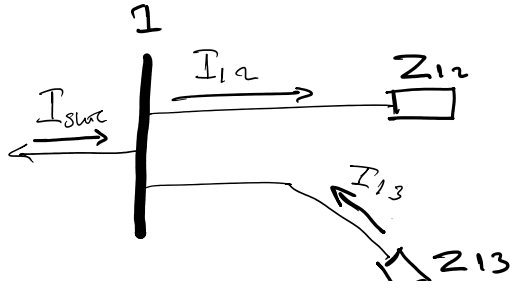
1c

$$V_k(i+1) = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*(i)} - \sum_{n=1}^{k-1} Y_{kn} V_n(i+1) - \sum_{n=k+1}^{N} Y_{kn} V_n(i) \right]$$

where $k = 1:3$ $n = 1:3$

Voltage at Bus 1



Z_{13}

Sum of Current at a node is Zero

$$I_{slack} + I_{12} + I_{13} = 0$$

$$I_{slack} = \frac{S_{Loads}}{V_1} = \frac{P - jQ}{V_1}$$

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = Y_{12}(V_1 - V_2)$$

$$I_{13} = \frac{V_1 - V_3}{Z_{13}} = Y_{13}(V_1 - V_3)$$

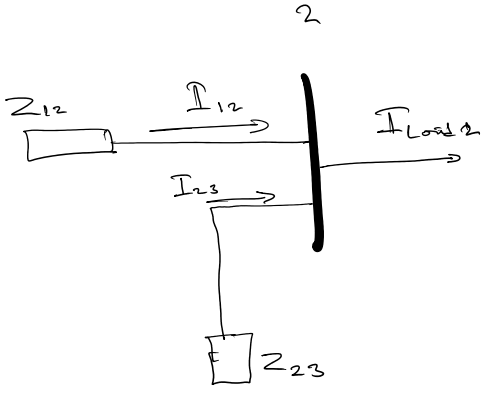
$$\frac{P - jQ}{V_1} = Y_{12}V_1 + Y_{13}V_1 - (Y_{12}V_2 + Y_{13}V_3)$$

$$\frac{P - jQ}{V_1} = V_1(Y_{12} + Y_{13})$$

$$\frac{P - jQ}{V_1} = V_1(Y_{11}) - (Y_{12}V_2 + Y_{13}V_3)$$

$$V_1 = \frac{1}{Y_{11}}\left[\frac{P_1 - jQ_1}{V_1^*} - (Y_{12}V_2 + Y_{13}V_3)\right]$$

At Bus 2



$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - \left(Y_{21} V_1 + Y_{23} V_3 \right) \right]$$

At Bus 3

$$V_3 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*} - \left(Y_{31} V_1 + Y_{32} V_2 \right) \right]$$