From: Signals and Systems Using Matlab, Luis Chaparro, Academic Press, 2015

3.5

$$Y(s) = \mathcal{L}[(\cos(t)u(t))e^{-2t}] = \mathcal{L}[\cos(t)u(t)]_{s\to s+2} = \frac{s+2}{(s+2)^2+1}$$

(b) The zero is $s = -2$ and the poles $s_{1,2} = -2 \pm j$. Because the poles are in the left-hand s-plane $x_1(t) \to 0$ as $t \to \infty$. Indeed, we have that $X_1(s)$ is the Laplace transform of $x_1(t) = \cos(t)e^{-2t}u(t)$ which tends to zero as $t \to \infty$.

(c) i. $z(t) = -e^{-t}u(t) + \delta(t)$, so

$$Z(s) = \frac{-1}{s+1} + 1 = \frac{s}{s+1}$$

ii. If $f(t) = e^{-t}u(t)$, then $z(t) = df(t)/dt$ so

$$Z(s) = s\mathcal{L}[f(t)] - f(0-) = \frac{s}{s+1} - 0 = \frac{s}{s+1}.$$

3.8 (a) The signal $x(t)$ and its first derivative are shown in Fig. (3.1). The second derivative is

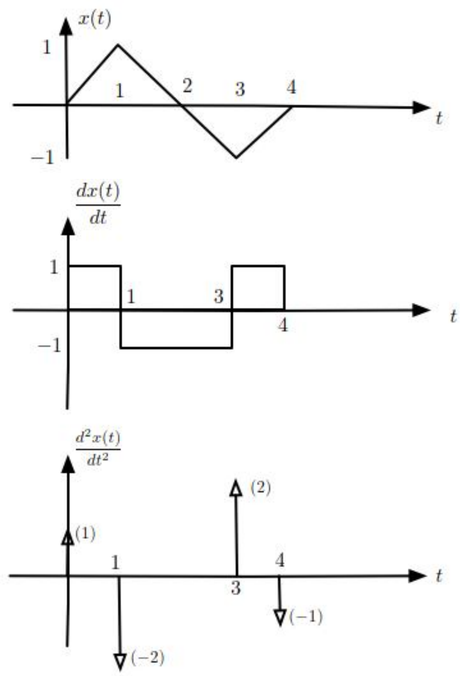

Figure 3.1: Problem 8

$$\frac{d^2x(t)}{dt^2} = \delta(t) - 2\delta(t-1) + 2\delta(t-3) - \delta(t-4)$$

(b) The Laplace transform of the second derivative is

$$s^2X(s) = 1 - 2e^{-s} + 2e^{-3s} - e^{-4s}$$

from which we obtain

$$X(s) = \frac{e^{-2s}}{s^2}[e^{2s} - 2e^{s} + 2e^{-s} - e^{-2s}] = \frac{e^{-2s}}{s^2}[2\sinh(2s) - \sinh(s)]$$

3.9 (a) Writing

$$Y_1(s) = \frac{e^{-2s}}{s^2+1} + \frac{1}{s+2} + \frac{2}{(s+2)^3}$$

then from the table of Laplace transforms:

$$y_1(t) = \sin(t-2)u(t-2) + e^{-2t}u(t) + t^2 e^{-2t}u(t)$$

(b) Expressing $Y_2(s)$ in terms $X_2(s)$ and $I(s)$ (due to the initial conditions) as:

$$\begin{aligned} Y_2(s) &= \frac{X(s)}{s^2+3s+2} + \frac{I(s)}{s^2+3s+2} \\ &= \frac{1}{s(s^2+3s+2)} + \frac{-s-1}{s^2+3s+2} \end{aligned}$$

we have that $I(s) = -s-1$, and $(s^2+3s+2)Y_2(s) = X(s)$ would give the ordinary differential equation

$$\frac{d^2y_2(t)}{dt^2} + 3\frac{dy_2(t)}{dt} + 2y_2(t) = x_2(t)$$

Its Laplace transform gives

$$(s^2+3s+2)Y_2(s) - sy_2(0) - dy_2(0)/dt - 3y_2(0) = X(s)$$

so that $I(s) = sy_2(0) + (dy_2(0)/dt + 3y_2(0))$ which compared with $I(s) = -s-1$ gives $y_2(0) = -1$ and $dy_2(0)/dt = -1 - 3y_2(0) = 2$.

(c) We have that

$$Y(s) = \frac{A}{s} + \frac{Bs+C}{(s+1)^2+4}$$

$A = Y(s)s|_{s=0} = 1/5$, then

$$Y(s) - \frac{A}{s} = \frac{-s/5 - 2/5}{(s+1)^2+4} = \frac{Bs+C}{(s+1)^2+4}$$

gives $B = -1/5$ and $C = -2/5$, so

$$Y(s) = \frac{1}{5s} - \frac{1}{5}\frac{s+1}{(s+1)^2+4} - \frac{1}{10}\frac{2}{(s+1)^2+4}$$

The steady state response is $y_{ss} = 1/5$, while the transient is

$$y_t(t) = [-(1/5)e^{-t}\cos(2t) - (1/10)e^{-t}\sin(2t)]u(t).$$

3.13 (a) Zeros: $s = \pm j2$, poles: $s_1 = 0$, $s_{2,3} = -1 \pm j$. The system is not BIBO because of the pole at zero.

(b) For $x(t) = \cos(2t)u(t)$

$$Y(s) = \frac{s^2+4}{s((s+1)^2+1)}\,\frac{s}{s^2+4} = \frac{1}{(s+1)^2+1}$$

$y(t) = e^{-t}\sin(t)u(t)$

$\cancel{y(t) = e^{-t}\cos(t)u(t)}$, and $\lim_{t\to\infty} y(t) = 0$.

3.15 (a) $S(s) = H(s)X(s)$, so $H(s) = S(s)/X(s) = sS(s)$

$$H(s) = s\left(\frac{0.5}{s} - \frac{1}{s+1} + \frac{0.5}{s+2}\right) = \frac{1}{(s+1)(s+2)}$$

3.17 (a) $H(s) = Y(s)/X(s) = 1/(s^2+4)$ so that $(s^2+4)Y(s) = X(s)$ giving

$$\frac{d^2y(t)}{dt^2} + 4y(t) = x(t)$$

(b) The Laplace transform including initial conditions is

$$\begin{aligned}
&s^2Y(s) - y(0)s - \frac{dy(0)}{dt} + 4Y(s) = X(s) \\
&(s^2+4)Y(s) = 1 + X(s) \\
&Y(s) = \frac{1+X(s)}{s^2+4} = 0 \Rightarrow X(s) = -1 \\
&\text{or } \; x(t) = -\delta(t)
\end{aligned}$$