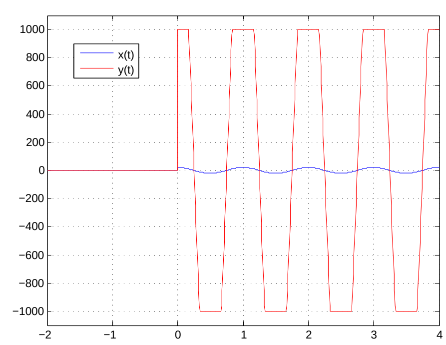
ee480 hw 4 solns

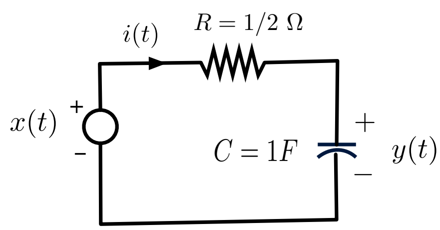
2.1 (a) The $y(t)$-$x(t)$ relation is a line through the origin between -10 to 10 and a constant before and after that. The system is non-linear, for instance if $x(t) = 7$ the output is $y(t) = 700$ but if we double the input, the output is not $2y(t) = 1400$ but 1000.
(b) If the inputs is always between -10 and 10 the system behaves like a linear system. In this case the output is chopped whenever $x(t)$ is above 10 or below -10. Se Fig. 2.1.
(c) Whenever the input goes below -10 or above 10 the output is -1000 and 1000, otherwise the output is $2000\cos(2\pi t)u(t)$.
(d) If the input is delayed by 2 the clipping will still occur, simply at a later time. So the system is time invariant.



input and output of amplifier.

2.5 (a) See Figure below. The circuit is a series connection of a voltage source $x(t)$ with a resistor $R = 1/2\ \Omega$, and capacitor $C = 1$F. Indeed, the mesh current is $i(t) = dy(t)/dt$ so

$$x(t) = Ri(t) + y(t) = Rdy(t)/dt + y(t)$$



(b) The output is

$$y(t) = e^{-2t}\, 0.5\, e^{2\tau}|_0^t = 0.5(1 - e^{-2t})u(t)$$

and

$$\begin{aligned}
\frac{dy(t)}{dt} &= e^{-2t}u(t) + 0.5(1 - e^{-2t})\delta(t) \\
&= e^{-2t}u(t) \\
\frac{dy(t)}{dt} + 2y(t) &= e^{-2t}u(t) + u(t) - e^{-2t}u(t) \\
&= u(t)
\end{aligned}$$

2.7 (a) The charge is

$$q(t) = C(t)v(t)$$

so that

$$i(t) = \frac{dq(t)}{dt} = C(t)\frac{dv(t)}{dt} + v(t)\frac{dC(t)}{dt}$$

(b) If $C(t) = 1 + \cos(2\pi t)$ and $v(t) = \cos(2\pi t)$, the current is

$$\begin{aligned}
i_1(t) &= C(t)\frac{dv(t)}{dt} + v(t)\frac{dC(t)}{dt} \\
&= (1 + \cos(2\pi t))(-2\pi\sin(2\pi t)) - \cos(2\pi t)(2\pi\sin(2\pi t)) \\
&= -2\pi\sin(2\pi t)[1 + 2\cos(2\pi t)]
\end{aligned}$$

(c) When the input is

$$v(t - 0.25) = \cos(2\pi(t - 1/4)) = \sin(2\pi t)$$

the output current is

$$\begin{aligned}
i_2(t) &= C(t)\frac{dv(t-0.25)}{dt} + v(t - 0.25)\frac{dC(t)}{dt} \\
&= (1 + \cos(2\pi t))(2\pi\cos(2\pi t)) - 2\pi\sin^2(2\pi t) \\
&= 2\pi\cos(2\pi t) + 2\pi[\cos^2(2\pi t) - \sin^2(2\pi t)]
\end{aligned}$$

which is not

$$i_1(t - 0.25) = 2\pi\cos(2\pi t)[1 + \sin(2\pi t)]$$

so the system is time varying.

2.22 (a) The output voltage when the switch closes at $t = 0$ is

$$v_o(t) = -R(t)u(t) = -(1 + 0.5\cos(20\pi t))u(t)$$

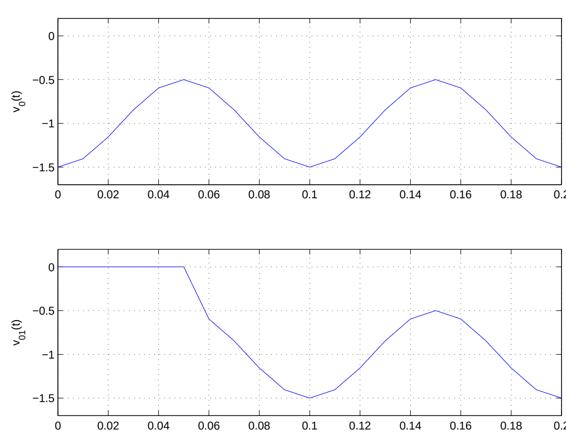
The initial value of the voltage is $v(0) = -1.5$.
(b) If the switch closes at $t = 50$ msec, the output voltage is

$$\begin{aligned} v_{o1}(t) &= -R(t)u(t - 50 \times 10^{-3}) \\ &= \begin{cases} -(1 + 0.5\cos(20\pi t)) & t \geq 50 \times 10^{-3} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

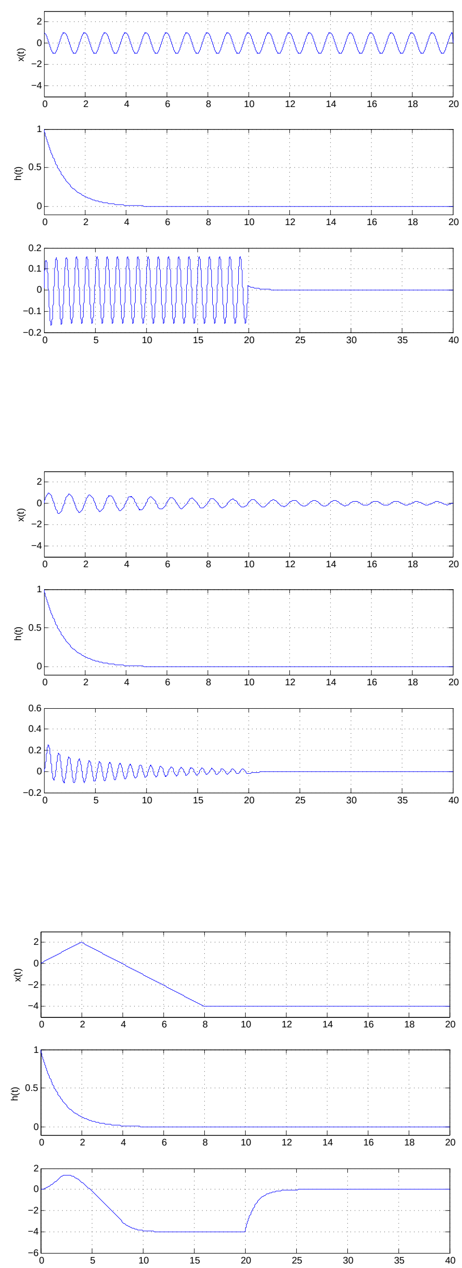
with an initial value of $v_{o1}(50 \times 10^{-3}) = -0.5$.
(c) The initial values are different, and $v_{01}(t) \neq v_o(t - 50 \times 10^{-3})$ so the system is time varying.

The following script is used for the plotting in (a) and (b)

```
%% Pr 2_22
t=0:0.01:0.2;M=length(t);
v0=-(1+0.5*cos(20*pi*t));
t1=0:0.01:0.05;
N=length(t1);
v01=[zeros(1,N) v0(N+1:M)];
figure(1)
subplot(211)
plot(t,v0); grid; axis([0 max(t) -1.7 0.2]); ylabel('v_0(t)')
subplot(212)
plot(t,v01);  grid;axis([0 max(t) -1.7 0.2]); xlabel('t'); ylabel('v_{01}(t)')
```



2.25
```
% Pr 2_25
clear all; clf
Ts=0.01; delay=1; Tend=20;
t=0:Ts:Tend;
%x=cos(2*pi*t).*(ustep(t,0)-ustep(t,-20));
%x=sin(2*pi*t).*exp(-0.1*t).*(ustep(t,0)-ustep(t,-20));
x=ramp(t,1,0)+ramp(t,-2,-2)+ramp(t,1,-8);
h=exp(-t);
y=Ts*conv(x,h);
% plots
t1=0:Ts:length(y)*Ts-Ts;
figure(1)
subplot(311)
plot(t,x); axis([0 20 -5 3]);grid;ylabel('x(t)');
subplot(312)
plot(t,h); axis([0 20 -0.1 1]);grid;ylabel('h(t)');
subplot(313)
plot(t1,y);
grid
```



input, impulse response and output of convolution integral.