HW 2 solutions

1.1 Notice that $0.5[x(t)+x(-t)]$, the even component of $x(t)$, is discontinuous at $t=0$, it is 1 at $t=0$ but 0.5 at $t \pm \epsilon$ for $\epsilon \to 0$. Likewise the odd component of $x(t)$, or $0.5[x(t)-x(-t)]$, must be zero at $t=0$ so that when added to the even component one gets $x(t)$.
$z(t)$ equals $x(t)$. See Fig. 1.

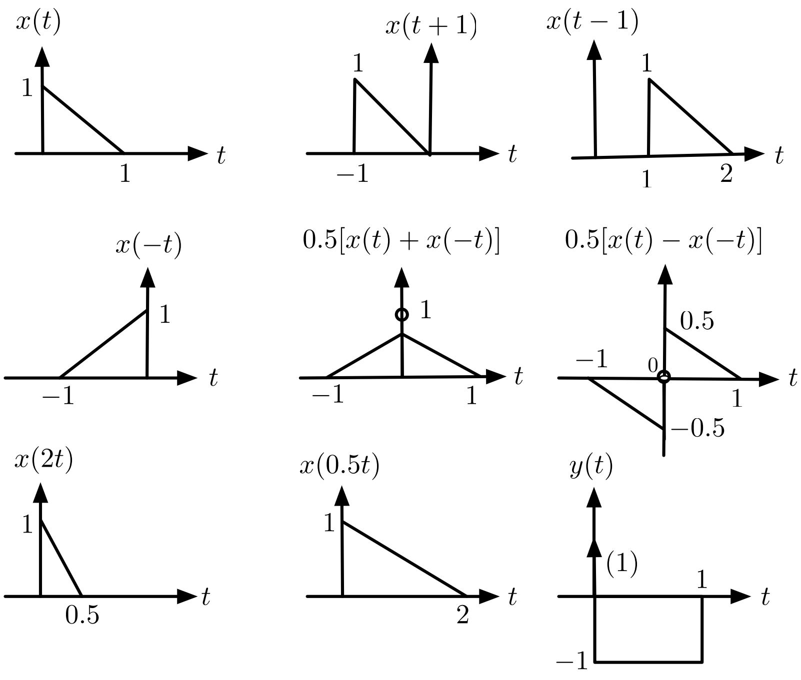

Figure 1.1: Problem 1

1.2 (a) If $x(t) = t$ for $0 \leq t \leq 1$, then $x(t+1)$ is $x(t)$ advanced by 1, i.e., shifted to the left by 1 so that $x(0) = 0$ occurs at $t = -1$ and $x(1) = 1$ occurs at $t = 0$.

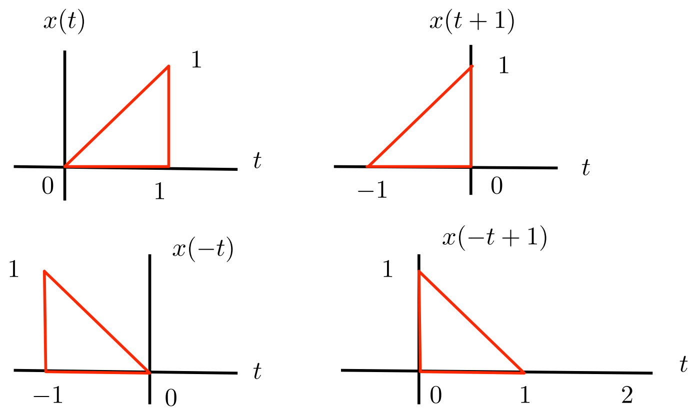

Figure 1.2: Problem 2: Original signal $x(t)$, shifted versions $x(t+1)$, $x(-t)$ and $x(-t+1)$.

The signal $x(-t)$ is the reversal of $x(t)$ and $x(-t+1)$ would be $x(-t)$ advanced to the right by 1. Indeed,

t	$x(-t+1)$
1	$x(0)$
0	$x(1)$
-1	$x(2)$

The sum $y(t) = x(t+1) + x(-t+1)$ is such that at $t = 0$ it is $y(0) = 2$; $y(t) = x(t+1)$ for $t < 0$; and $y(t) = x(-t+1)$ for $t > 0$. Thus,

$$y(t) = x(t+1) = t+1 \qquad 0 \leq t+1 < 1 \quad \text{or} \quad -1 \leq t < 0$$
$$y(0) = 2$$
$$y(t) = x(-t+1) = -t+1 \qquad 0 \leq -t+1 < 1 \quad \text{or} \quad 0 < t \leq 1$$

or

$$y(t) = \begin{cases} t+1 & -1 \leq t < 0 \\ 2 & t = 0 \\ -t+1 & 0 < t \leq 1 \end{cases}$$

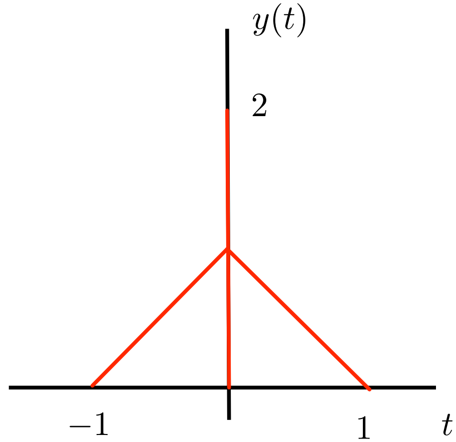

Figure 1.3: Problem 2: Triangular signal $y(t)$ with discontinuity at the origin.

identical as the discontinuity of $y(t)$ does not add any area.