3.20 (a) Letting $x(t) = \delta(t)$ so that $y(t) = h(t)$ we get

$$h(t) = \alpha\delta(t-T) + \alpha^3\delta(t-3T)$$

(b) $H(s) = \alpha e^{-sT} + \alpha^3 e^{-s3T}$
$H(s)$ has no poles, its zeros satisfy $\alpha e^{-sT} + \alpha^3 e^{-s3T} = 0$ or (dividing by $\alpha e^{-sT} > 0$)

$$\begin{aligned}
1 &= e^{-j\pi}\alpha^2 e^{-2Ts} = \alpha^2 e^{-2T(\sigma+j(\Omega+\pi/(2T))}\ \text{ letting }\ s=\sigma+j\Omega \\
1e^{-j2\pi k} &= (\alpha^2 e^{-2T\sigma})e^{-j2T(\Omega+\pi/(2T))},\ \ k = 0, \pm 1, \pm 2, \cdots
\end{aligned}$$

so that

$$\begin{aligned}
&1 = \alpha^2 e^{-2T\sigma} \Rightarrow\ \ \sigma = \frac{\log\alpha^2}{2T},\ \ \text{real part of zeros} \\
&-j2T(\Omega+\pi/(2T)) = -jk2\pi \Rightarrow\ \ \Omega = \frac{(2k-1)\pi}{2T},\ \ \text{imaginary part of zeros} \\
&\text{zeros}\ \ \ s = \sigma + j\Omega = \frac{\log\alpha^2}{2T} + j\left(\frac{(2k-1)\pi}{2T}\right)
\end{aligned}$$

System is BIBO stable since $\sigma = \log(\alpha)/T < 0$ because $0 < \alpha < 1$.

3.22 (a)

$$X(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1 - e^{-s})^2$$

with the whole s-plane as ROC. Pole $s = 0$ is cancelled by zero $s = 0$. The transfer function is

$$H(s) = \frac{Y(s)}{X(s} = \frac{s+2}{(s+1)^2} = \frac{1}{s+1} + \frac{1}{(s+1)^2} \qquad \text{ROC } \sigma > 0$$

Using the frequency shift property, the inverse of $1/(s+1)^2$ is $e^{-t}r(t)$, so

$$h(t) = e^{-t}(1+t)u(t)$$

3.23 (a) Using the Laplace transform

$$H(s) = \frac{1}{s+2}$$
$$X(s) = \frac{1-e^{-3s}}{s}$$
$$Y(s) = H(s)X(s) = \frac{1-e^{-3s}}{s(s+2)}$$

Letting

$$F(s) = \frac{1}{s(s+2)} = \frac{0.5}{s} - \frac{0.5}{s+2} \Rightarrow f(t) = 0.5(1-e^{-2t})u(t)$$

then

$$y(t) = f(t) - f(t-3) = 0.5(1-e^{-2t})u(t) - 0.5(1-e^{-2(t-3)})u(t-3)$$

Graphically, we plot $h(\tau)$ and $x(t-\tau)$ as functions of τ and shift $x(t-\tau)$ from the left to the right and integrate the overlapping areas to get

$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t e^{-2\tau}\,d\tau = 0.5(1-e^{-2t}) & 0 \leq t < 3 \\ \int_{t-3}^t e^{-2\tau}\,d\tau = 0.5(e^{-2(t-3)} - e^{-2t}) & t \geq 3 \end{cases}$$

which can be written as

$$\begin{aligned} y(t) &= 0.5(1-e^{-2t})[u(t)-u(t-3)] + 0.5[e^{-2(t-3)} - e^{-2t}]u(t-3) \\ &= 0.5(1-e^{-2t})u(t) - 0.5(1-e^{-2(t-3)})u(t-3) \end{aligned}$$

coinciding with the result obtained using the Laplace transform.

3.32 (a) Although the denominator can be obtained by multiplying by hand the three different polynomials, the function *conv* can be used to obtain the coefficients of the product of the three polynomials. The steady–state is obtained by looking at the residue corresponding to the poles $s = 0$, in this case $1/50 = 0.02$

```
% Pr. 3_32 (a)
clear all; clf
den=conv([1 0],[1 1]); den=conv(den,[1 10 50])
syms s t w
disp('>>>>> Inverse Laplace <<<<<')
x=ilaplace((s^2+2*s+1)/(den(1)*s^4+den(2)*s^3+den(3)*s^2+den(4)*s+den(5)))
figure(1)
ezplot(x,[0,5])
axis([0 5 0 .1]); grid

>>>>> Inverse Laplace <<<<<
x = -1/50*exp(-5*t)*cos(5*t)+9/50*exp(-5*t)*sin(5*t)+1/50
```

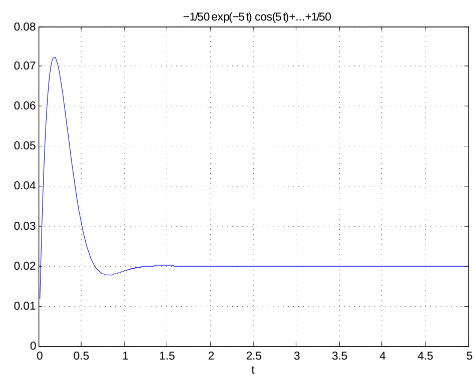

Figure 3.7: Problem 3.32(a)

(b) Separating the given expression into two,

$$\begin{aligned} X(s) &= \frac{1}{s(s+2)} - \frac{se^{-s}}{s(s+2)} \\ &= \frac{0.5}{s} - \frac{0.5}{s+2} - \frac{e^{-s}}{s+2} \end{aligned}$$

giving

$$x(t) = 0.5u(t) - 0.5e^{-2t}u(t) - e^{-2(t-1)}u(t-1)$$

Using the MATLAB script we compute the inverse.

```
% Pr. 3_32 (b)
 clear all; clf
 syms s t w
 x=ilaplace((1-s*exp(-s))/(s^2+2*s))
 figure(2)
```

```
ezplot(x,[0,5])
axis([0 5 -0.6 0.6])
grid

x =  exp(-t)*sinh(t)-heaviside(t-1)*exp(-2*t+2)
```

Notice that the expression in the solution obtained from *ilaplace*

$$e^{-t}\sinh(t)u(t) = e^{-t}\frac{e^{t}-e^{-t}}{2}u(t) = 0.5u(t) - 0.5e^{-2t}u(t)$$

is identical to the first inverse given above.

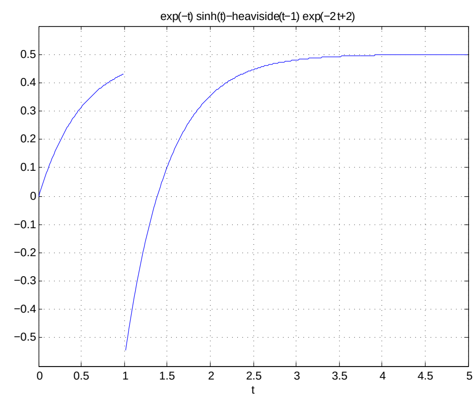

Figure 3.8: Prob. 3.32(b)

(c) Letting the initial conditions be zero, the inverse Laplace transform of

3.35

$$Y(s) = \frac{X(s)}{s^2+2s+3}$$

with $X(s)=1$ gives the impulse response

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2+2} \quad \Rightarrow \quad h(t) = \frac{1}{\sqrt{2}} e^{-t} \sin(\sqrt{2}t)u(t)$$

The poles, computed by the following script, are in the open left-hand s-plane so the system is BIBO stable.

```
%% Pr. 3_35
num=[0 0 1];
den=[1 2 3];
syms s t h
figure(1)
subplot(121)
[r,p]=pfeLaplace(num,den)
disp('>>>>> Inverse Laplace <<<<<')
h=ilaplace(num(3)/(den(1)*s^2+den(2)*s+den(3)))
```

```
subplot(122)
ezplot(h,[0,8])
axis([0 8  -0.2 0.4])
grid

 >>>>>  Poles <<<<<
 p =  -1.0000 + 1.4142i
        -1.0000 - 1.4142i

 r =     0 - 0.3536i
          0 + 0.3536i
```

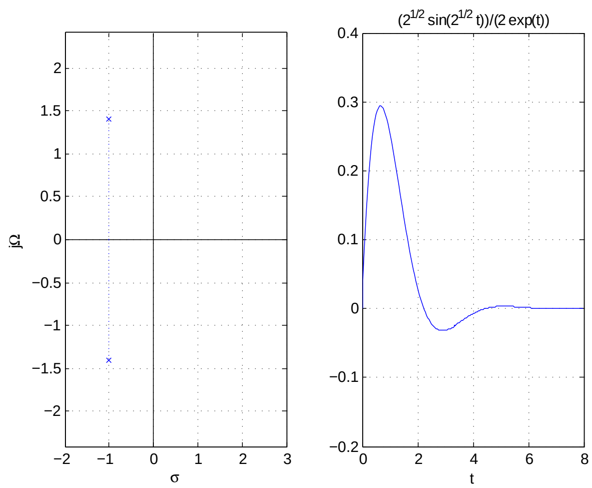

Figure 3.11: Prob. 35

3.37 (a) The transfer function is

$$H(s) = \frac{1}{s^2+5s+6} = \frac{1}{(s+2)(s+3)}$$

so that

$$Y(s) = \frac{1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

where

$$A = Y(s)s|_{s=0} = \frac{1}{6}$$

$$B = Y(s)(s+2)|_{s=-2} = -\frac{1}{2}$$

$$C = Y(s)(s+3)|_{s=-3} = \frac{1}{3}$$

so that the total response is

$$y(t) = \left[\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}\right]u(t)$$

which starts at $y(0) = 0$ and in steady state it is $1/6$, and in between it increases smoothly from 0 to $1/6$.

3.39 (a) If $X(s) = 1/s$ then

$$Y_1(s) = \frac{X(s)(s+1)}{s^2+2s+4}$$

so that the differential equation connecting the input $x(t)$ and the output $y_1(t)$ is

$$\frac{d^2y_1(t)}{dt^2} + 2\frac{dy_1(t)}{dt} + 4y_1(t) = x(t) + \frac{dx(t)}{dt}$$

Similarly,

$$Y_2(s) = \frac{X(s)s}{s^2+4s+4}$$

so that the differential equation connecting the input $x(t)$ and the output $y_2(t)$ is

$$\frac{d^2y_2(t)}{dt^2} + 4\frac{dy_2(t)}{dt} + 4y_2(t) = \frac{dx(t)}{dt}$$

Finally,

$$Y_3(s) = \frac{X(s)(s-1)}{s(s^2+2s+10)}$$

so that the differential equation connecting the input $x(t)$ and the output $y_3(t)$ is

$$\frac{d^3y_3(t)}{dt^3} + 2\frac{d^2y_3(t)}{dt^2} + 10\frac{dy_3(t)}{dt} = -x(t) + \frac{dx(t)}{dt}$$

(b) For $Y_1(s)$ its poles are $s = 0$ and $s = -1 \pm j\sqrt{3}$ and a zero at $s = -1$, its general solution is of the form

$$y_1(t) = [A + Be^{-t}\cos(\sqrt{3}t + \theta)]u(t)$$

For $Y_2(s)$ its poles are $s = -2$ (double) and no zero, its general solution is of the form

$$y_2(t) = [Be^{-2t} + Cte^{-2t}]u(t)$$

For $Y_3(s)$ its poles are $s = 0$ (double) and $s = -1 \pm j3$ and a zero at $s = 1$, its general solution is of the form

$$y_3(t) = [A + Bt + Ce^{-t}\cos(3t + \theta)]u(t)$$

The values of the coefficients of the partial fraction expansion (also called residues), the inverse and the plot of the response is computed using MATLAB.

```
%% Prob 3_39
num=[0 0  1 1]; % numerator of Y1
% num=[0 0 1];    % numerator of Y2
% num=[0 0 0 1 -1];  % numerator of Y3
den=[1 2 4 0];  % denominator of Y1
% den=[1 4 4];     % denominator of Y2
% den=[1  2 10 0 0]; % denominator of Y3
syms s t y
figure(1)
subplot(121)
[r,p]=pfeLaplace(num,den)
disp('>>>>> Inverse Laplace <<<<<')
y1=ilaplace((num(3)*s+num(4))/(den(1)*s^3+den(2)*s^2+den(3)*s));y=y1
```

```
% y2=ilaplace(num(3)/(den(1)*s^2+den(2)*s+den(3)));y=y2
% y3=ilaplace((num(4)*s+num(5))/(den(1)*s^4+den(2)*s^3+den(3)*s^2));y=y3
subplot(122)
ezplot(y,[0,10])
axis([0 10 -1 1])
grid
```

For the three cases we have the residuals (r) the corresponding poles (p) and the complete responses.

```
%% case 1
r = -0.1250 - 0.2165i
     -0.1250 + 0.2165i
      0.2500
p = -1.0000 + 1.7321i
      -1.0000 - 1.7321i
         0
y1 = 1/4-1/4*exp(-t)*cos(3^(1/2)*t)+1/4*3^(1/2)*exp(-t)*sin(3^(1/2)*t)
%% case 2
r =   0
       1
p = -2
      -2
y2 = t*exp(-2*t)
%% case 3
r = -0.0600 + 0.0033i
     -0.0600 - 0.0033i
      0.1200
     -0.1000
p= -1.0000 + 3.0000i
     -1.0000 - 3.0000i
         0
         0
y3= 3/25-3/25*exp(-t)*cos(3*t)-1/150*exp(-t)*sin(3*t)-1/10*t
```

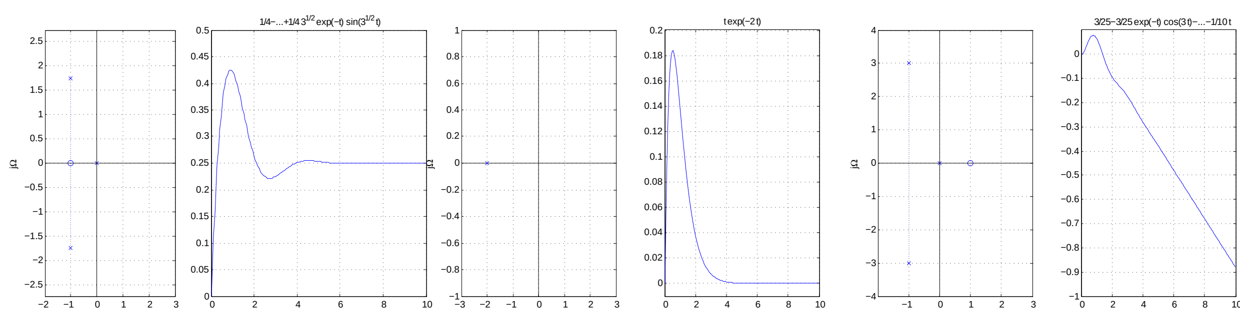

Figure 3.15: Problem 39: Poles/zeros and response for cases (1) (left) to (3) (right).