4.2a

Thursday, October 26, 2023 4:35 AM

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\, d\tau$$

$$x(t) = e^{j\Omega_0 t}$$

$$y(t) = \frac{1}{T}\int_{t-T}^{t} e^{j\Omega_0 t}\, dt$$

$$y(t) = \frac{1}{T}\cdot\frac{1}{j\Omega_0} e^{j\Omega_0 t}\Big|_{t-T}^{t}$$

$$y(t) = \frac{1}{j\Omega_0 T}\left[e^{j\Omega_0 t} - e^{j\Omega_0 (t-T)}\right]$$

$$y(t) = \frac{1}{j\Omega_0 T}\left[e^{j\Omega_0 t} - \left(e^{j\Omega_0 t}\cdot e^{-j\Omega_0 T}\right)\right]$$

$$y(t) = \frac{e^{j\Omega_0 t}}{j\Omega_0 T}\left[1 - e^{-j\Omega_0 T}\right]$$

$$y(t) = e^{j\Omega_0 t}\left[\frac{1}{j\Omega_0 T} - \frac{e^{-j\Omega_0 T}}{j\Omega_0 T}\right]$$

Using Eigen function $y(t) = x(t) \cdot H(j\Omega_0)$

$$y(t) = \underbrace{e^{j\Omega_0 t}}_{x(t)} \cdot \underbrace{\left[\frac{1 - e^{-j\Omega_0 T}}{j\Omega_0 T} \right]}_{H(j\Omega_0)}$$

$$\boxed{\therefore \; H(j\Omega_0) = \frac{1 - e^{-j\Omega_0 T}}{j\Omega_0 T}}$$

Ans

4.3a

Thursday, October 26, 2023 5:20 AM

i) $x_1(t) = 1 + \cos(\overset{\Omega_0}{2\pi t}) - \cos(\overset{\Omega_0}{6\pi t})$

$\Omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{\Omega_0}$

$x_1(t) \Rightarrow$ has 2 T_0 at $2\pi t$ & $6\pi t$

$T_0 @ 2\pi t = \frac{2\pi}{2\pi} = 1$

$T_0 @ 6\pi t = \frac{2\pi}{6\pi} = \frac{1}{3}$

} Common factor of 1 & $\frac{1}{3}$ is 1

$\boxed{T_0 \text{ of } x_1(t) = 1}$

$x_2(t) = 1 + \cos(2\pi t) - \cos(6t)$

T_0 @ $2\pi t$ & $6t$

$T_0 @ 2\pi t = 1$ from above

$T_0 @ 6t = \frac{2\pi}{6} = \frac{\pi}{3} \Rightarrow \pi$ is irrational $\therefore x_2(t)$ is aperiodic

ii) $a_0 = \frac{1}{T_0}\int_0^{T_0} x_1(t)\,dt = \frac{1}{T_0}\int_0^{T_0}\left(1 + \cos(2\pi t) + \cos(6\pi t)\right)dt$

$T_0 = 1$

$a_0 = \int_0^1 (1)\,dt + \int_0^1 \cos(2\pi t)\,dt + \int_0^1 \cos(6\pi t)\,dt$

$a_0 = t\Big|_0^1 + \frac{1}{2\pi}\sin(2\pi t)\Big|_0^1 + \frac{1}{6\pi}\sin(6\pi t)\Big|_0^1$

$a_0 = 1 + \left[\frac{1}{2\pi}\left(\sin(2\pi) - \sin(0)\right)\right] + \left[\frac{1}{6\pi}\left(\sin(6\pi) - \sin(0)\right)\right]$

$1 + \left[\frac{1}{2\pi}(0 - 0)\right] + \left[\frac{1}{6\pi}(0 - 0)\right]$

$\boxed{a_0 = 1}$

$a_n = \frac{2}{T_0}\int_0^{T_0} x_1(t)\cos(nt)\,dt$

$\left[1 + \cos(2\pi t) + \cos(6\pi t)\right]\cos(nt)\,dt$

$a_n = \frac{2}{1}\int_0^1 \left[1 + \cos(2\pi t) + \cos(6\pi t)\right]dt$

$a_n = 2\int_0^1 \left[\cos(nt) + \cos(nt)\cos(2\pi t) + \cos(nt)\cos(6\pi t)\right]$

$= 2\left[\int_0^1 \cos(nt)\,dt + \underbrace{\int_0^1 \cos(nt)\cos(2\pi t)\,dt}_{0} + \underbrace{\int_0^1 \cos(nt)\cos(6\pi t)\,dt}_{0}\right]$

$= 2\int_0^1 \cos(nt)\,dt$

$= 2\left[\frac{1}{n}\sin(nt)\Big|_0^1\right]$

$= \frac{2}{n}\left[\sin(n) - \sin(0)\right]$

b

$$a_n = \frac{2}{n}\sin(n) \quad = \begin{cases} 0 & @\ n=0 \\ \frac{2}{n}\sin(n) & @\ n \neq 0 \end{cases}$$ Ans

$$b_n = \frac{2}{T_0}\int_0^{T_0} x_1(t)\sin(nt)\,dt$$

$$b_n = 2\int_0^1 \left[\sin(nt) + \sin(nt)\cos(2\pi t) + \sin(nt)\cos(6\pi t)\right]$$

$$b_n = 2\left[\int_0^1 \sin(nt)\,dt + \underbrace{\int_0^1 \sin(nt)\cos(2\pi t)\,dt}_{0} + \underbrace{\int_0^1 \sin(nt)\cos(6\pi t)\,dt}_{0}\right]$$

$$b_n = 2\int_0^1 \sin(nt)\,dt$$

$$b_n = -\frac{2}{n}\cos(nt)\Big|_0^1$$

$$b_n = -\frac{2}{n}\left[\cos(n) - \cos(0)\right]$$

$$b_n = \frac{-2}{n}\cos(n) \quad = \begin{cases} 0 & @ \\ \frac{-2}{n}\cos(n) & @\ n \neq 0 \end{cases}$$ Ans

Fourier Series Representation of $x_1(t) = 1 + \sum_{n=1}^{\infty}\left[\frac{2}{n}\sin(n)\cos(\Omega_0 n t) - \frac{2}{n}\cos(n)\cos(\Omega_0 n t)\right]\Big|_{\Omega_0 = 2\pi f_0}$

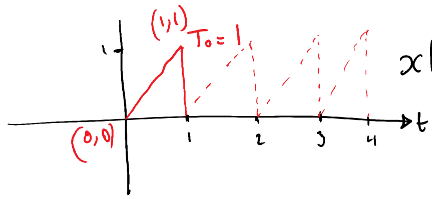
iii) It is hard to obtain fourier series with aperiodic signals because aperiodic signal does not repeat themselves at specific interval

eg $x_2(t) = 1 + \cos(2\pi t) + \cos(6t)$ repeat itself but at non-specific intervals

4.6

Saturday, October 28, 2023 1:46 PM

a) $x_1(t) = t\left[u(t) - u(t-1)\right]$



$x(t) = m(t) + b$

$b = 0$

$m = \frac{1-0}{1-0} = 1$

$$\boxed{x(t) = t}$$

b) $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$

$a_0 = \frac{1}{1}\int_0^1 t\, dt$

$a_0 = \frac{t^2}{2}\Big|_0^1 = 1 - 0$

$$\boxed{a_0 = \frac{1}{2}}$$

$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos(mt)\, dt$

$a_n = 2\int_0^1 (t)\cos(mt)\, dt$

note $\int \cos(mt)\, dt = \frac{1}{m}\sin(mt)$

where $m = n\Omega_0 = n2\pi$

$\therefore \int \cos(n2\pi t)\, dt = 0$

$a_n = 2\left[0\right]$

$$\boxed{a_n = 0}$$

$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\sin(mt)\, dt$

$b_n = 2\int_0^1 (t)\sin(mt)\, dt$

$b_n = 2\left[\frac{1}{m} t\cos(mt) + \frac{1}{m^2}\sin(mt)\right]_0^1$

$m = n\Omega_0 = n2\pi$

$b_n = 2\left\{\left[\frac{1}{n2\pi}\underbrace{\cos(n2\pi)}_{-1^n} + \frac{1}{(n(2\pi))^2}\underbrace{\sin(n2\pi)}_{0}\right] - \left[\frac{1}{n2\pi}\underbrace{\cos(0)}_{1} + \frac{1}{(n(2\pi))^2}\underbrace{\sin(0)}_{0}\right]\right\}$

$b_n = 2\left\{\left[\frac{-1^n}{n2\pi} + 0\right] - \left[\frac{1}{n2\pi} + 0\right]\right\}$

$$\boxed{b_n = 2\left(\frac{-1^n - 1}{n2\pi}\right) = \begin{cases} 0 & @\ n = +\text{ even integer} \\ 2\left(\frac{-1^n - 1}{n2\pi}\right) & @\ +n\text{ integers} \end{cases}}$$

Ans

c)

$$X[k] = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k \frac{t}{T}}\, dt$$

$T_0 = 1$ $\quad x(t) = t$ from above graph

$$X[k] = 1\int_0^1 t e^{-j2\pi k \frac{t}{1}}\, dt$$

$$X[k] = \frac{-t}{j2\pi k} e^{-j2\pi k t} + \frac{e^{-j2\pi k t}}{(j2\pi)^2}\Bigg|_0^1$$

$$X[k] = \left[\frac{-e^{-j2\pi k}}{j2\pi k} + \frac{e^{-j2\pi k}}{(j2\pi)^2}\right]_{t=1} - \left[0 + \frac{1}{(j2\pi)^2}\right]_{t=0}$$

note $e^{2\pi k} = 1$

$$X[k] = \left[\frac{-1}{j2\pi k} + \frac{1}{(j2\pi)^2} - \frac{1}{(j2\pi)^2}\right]$$

$$\boxed{X[k] = \frac{-1}{j2\pi k} = \frac{j}{2\pi k}}$$ Ans

d)

$$y(t) = \frac{d}{dt}x(t) \qquad x(t) = t$$

$$y(t) = \frac{d}{dt}t = 1$$

$$Y[k] = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-j2\pi k t/T}\, dt$$

$$Y[k] = 1\int_0^1 e^{-j2\pi k t}\, dt$$

$$Y[k] = -j2\pi k t\, e^{-j2\pi k t}\Big|_0^1$$

$$Y[k] = \left[-j2\pi k\, e^{-j2\pi k}\right]_{t=0} - [0]_{t=0}$$

$e^{2\pi k} = 1$

$$\boxed{Y[k] = -j2\pi k}$$

4.7a

Saturday, October 28, 2023 7:12 PM

i) $x(t) = \sum_{k=-\infty}^{\infty} \frac{3}{4+(k\pi)^2} e^{jk\pi t}$

$\omega = 2\pi f_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{\omega}$

$T_0 = \frac{2\pi}{k\pi} = \frac{2}{k}$ @ T_0 $k=1$

$\boxed{T_0 = 2 \text{ sec}}$

ii) Average or dc value of $x(t)$ is the first coefficent of the fourier series a_0

$x(t)$ @ $a_0 = \sum_{k=0}^{\infty} \frac{3}{4+(0\cdot\pi)^2} e^{j0\pi t}$

$\boxed{a_0 = \frac{3}{4}}$

iii) $x(t) = \sum_{k=-\infty}^{\infty} \underbrace{\frac{3}{4+(k\pi)^2}}_{\text{scale factor} = S_k} e^{-jk\pi t}$

$\boxed{\text{if } S_k = S_{-k} = \frac{3}{4+(k\pi)^2} \therefore x(t) \text{ is even}}$

iv) $x(t) = \sum_{k=-\infty}^{\infty} \frac{3}{4+(k\pi)^2} e^{-jk\pi t}$ note $\Omega_0 = k\pi$

when $x(t) = A\cos(3\pi t)$ $\Omega_0 = k\cancel{\pi} = 3\cancel{\pi}$

$k = 3$

Since $x(t)$ is even

$$\boxed{A = 2\left(\frac{3}{4+(3\pi)^2}\right) = \frac{6}{4+9\pi^2}}$$

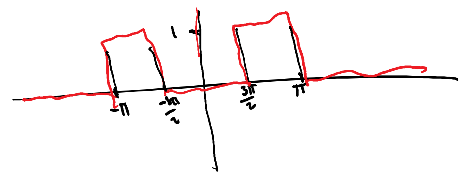
4.14a

Saturday, October 28, 2023 8:01 PM

$\Omega_0 = \frac{\pi}{4}$

$$|H(j\Omega)| = \begin{cases} 1 & \pi \le \Omega \le \frac{3\pi}{2} \\ 1 & -\frac{3\pi}{2} \le \Omega \le -\pi \\ 0 & \text{otherwise} \end{cases}$$

$$\angle H(j\Omega) = \begin{cases} -\Omega & \pi \le \Omega \le \frac{3\pi}{2} \\ \Omega & -\frac{3\pi}{2} \le \Omega \le -\pi \\ 0 & \text{otherwise} \end{cases}$$



$$x(t) = \sum_{n=-\infty}^{\infty} X[n]\, e^{j\Omega_0 t}$$

$$= 2e^{-j\frac{5\pi}{4}t} - j e^{-j\frac{\pi}{4}t} + 2e^{j\frac{\pi}{4}t} + j e^{j\frac{\pi}{4}t}$$

$$= j\left(e^{j\frac{\pi}{4}t} - e^{j\frac{\pi}{4}t}\right) + 2\left(e^{-j\frac{5\pi}{4}t} + e^{j\frac{\pi}{4}t}\right)$$

$$= -2\left[e^{j\frac{\pi}{4}t} - e^{-j\frac{\pi}{4}t}\right] + 4\left[\frac{e^{j\frac{5\pi}{4}t} + e^{-j\frac{5\pi}{4}t}}{2}\right]$$

$$\boxed{x(t) = -2\sin\left(\frac{\pi}{4}t\right) + 4\cos\left(\frac{5\pi}{4}t\right)}$$

for bandpass filter has 2 component

@ $\Omega = \frac{\pi}{4}$ $\quad 0\angle 0^\circ \quad 1\angle -\frac{\pi}{4}$

@ $\Omega = \frac{5\pi}{4}$ $\quad j\angle -\frac{5\pi}{4}$

$$y(t) = \underbrace{0\angle 0}_{\phi}\left(-2\sin\left(\frac{\pi}{4}t\right)\right) + \underbrace{j\angle -\frac{5\pi}{4}}_{\phi}\cos\left(\frac{5\pi}{4}t\right)$$

$$y(t) = 4\cos\left(\frac{5\pi}{4}t + \left(-\frac{5\pi}{4}\right)\right)$$

$$\boxed{y(t) = 4\cos\left[\frac{5\pi}{4}(t-1)\right]}$$

15a

Saturday, October 28, 2023 9:05 PM

$$x(t) = |\cos(\pi t)| \quad -\infty < t < \infty$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(n 2\pi \frac{t}{T_0} + \phi_n\right)$$

$$C_0 = \frac{2 V_m}{\pi} = \frac{2}{\pi}$$

$$y(t) = A_0 C_0 = 1$$

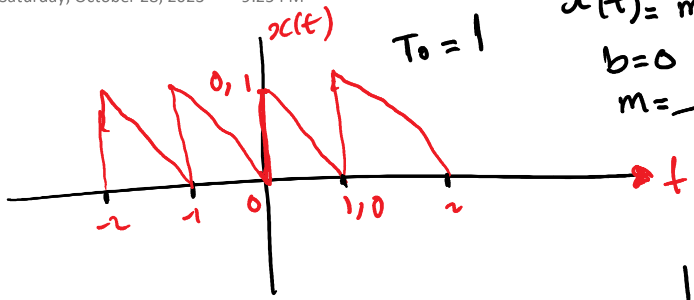
$$A_0 = \frac{1}{C_0} = \frac{1}{\frac{2}{\pi}}$$

$$A_0 = \frac{\pi}{2}$$

$$\boxed{A_0 = \text{dc gain} = \frac{\pi}{2}}$$

4.17a

Saturday, October 28, 2023 9:25 PM



$T_0 = 1$

$x(t) = mt + b \quad \therefore x(t) = -t$

$b = 0$

$m = \frac{1-0}{0-1} = -1$

Using Laplace Transform $X[k] = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j2\pi k \frac{t}{T_0}} dt$

$$X[k] = \frac{1}{1} \int_0^1 -t \, e^{-j2\pi k t} dt$$

$$X[k] = \frac{t}{j2\pi k} e^{-j2\pi k t} - \frac{e^{-j2\pi k t}}{(j2\pi)^2} \Bigg|_0^1$$

$$e^{2\pi} = 1$$

$$X[k] = \frac{t}{j2\pi k} - \frac{1}{(j2\pi)^2} \Bigg|_0^1$$

$$X[k] = \left[\frac{1}{j2\pi k} - \frac{1}{(j2\pi)^2} \right] - \left[0 - \frac{1}{(j2\pi)^2} \right]$$

$$X[k] = \left[\frac{1}{j2\pi k} - \frac{1}{(j2\pi)^2} + \frac{1}{(j2\pi)^2} \right]$$

$$\boxed{X[k] = \frac{1}{j2\pi k} = \frac{-j}{2\pi k}}$$

4.18a

Saturday, October 28, 2023 9:46 PM

LTI System $H(s) = \dfrac{Y(s)}{X(s)} \Rightarrow Y(s) = X(s) \cdot H(s)$

$y(t) = x(t) * h(t)$

$H(s) = \dfrac{s+1}{s^2+3s+2}$

$x(t) = 1 + \cos(t + \frac{\pi}{4})$

$\omega = 1$

$\frac{\pi}{4} = 45$

$H(0) = \dfrac{0+1}{0+0+2} = \dfrac{1}{2}$

$$H(j\omega) = \frac{j\omega + 1}{(j\omega)^2 + 3(j\omega) + 2}$$

$$= \frac{j+1}{(j)^2 + 3j + 2} = \frac{j+1}{3j+1} = 0.447 \angle 26.6^\circ$$

$y(t) = h(t) * x(t)$

$y(t) = \frac{1}{2} + 0.447 \cos(t + 45^\circ - 26.6^\circ)$

$\boxed{y(t) = \frac{1}{2} + 0.447 \cos(t - 18.4^\circ)}$ Ans