Pr.10.2 This simple problem illustrates that a line integral in general depends not only on the endpoints, but also on the shape of the path. The commands are as in Pr.10.1.

```
> FC3 := subs(x = r3[1], y = r3[2], z = r3[3], F3);
```

$$FC3 := F3$$

```
> F3 := [2·z, 7·x, −3·y];
```

$$F3 := [2\,z,\ 7\,x,\ -3\,y]$$

```
> r3 := [cos(t), sin(t), 2·t];
```

$$r3 := [\cos(t),\ \sin(t),\ 2\,t]$$

```
> FC3 := subs(x = r3[1], y = r3[2], z = r3[3], F3);
```

$$FC3 := [4\,t,\ 7\cos(t),\ -3\sin(t)]$$

```
> rprime3 := VectorCalculus[diff](r3, t);
```

$$rprime3 := [-\sin(t),\ \cos(t),\ 2]$$

```
> with(LinearAlgebra) :
> integrand3 := DotProduct(FC3, rprime3);
```

$$integrand3 := -4\sin(\bar{t})\,t + 7\cos(\bar{t})\cos(t) - 6\sin(t)$$

```
>
> int(integrand3, t = 0..2·π)
```

$$15\,\pi$$

Pr.10.4 The vanishing of the curl of $\mathbf{F} = [ze^x,\ 2y,\ e^x]$ shows path independence. Find a potential by integration. Maple will not give "constants" of integration (functions in the present case), but this is not essential; simply integrate the three functions in the form (the components of $\mathbf{F}$) with respect to x, y, z, respectively, and find out whether you can find a common expression f for all three results such that $\mathbf{F} = \operatorname{grad} f$. Then calculate $f(a, b, c) - f(0, 0, 0)$.

```
> with(LinearAlgebra):
  VectorCalculus[SetCoordinates]('cartesian'[x, y, z]):
> F := VectorCalculus[VectorField](<z*exp(x)  2*y  exp(x)>);
```

$$F := (ze^x)\,\bar{e}_x + 2\,y\bar{e}_y + (e^x)\,\bar{e}_z$$

```
> VectorCalculus[Curl](F);                          # Resp. 0ē_z
> int(F[1], x);                                     # Resp. ze^x
> int(F[2], y);                                     # Resp. y^2
> int(F[3], z);                                     # Resp. ze^x
```

Noting that the last expression is the same as the first, $\mathbf{F} = \operatorname{grad} \mathbf{f}$ can be obtained from

```
> f := int(F[1], x) + int(F[2], y);                 # Resp. f := ze^x + y^2
> answer := subs(x = a, y = b, z = c, f) - subs(x = 0, y = 0, z = 0, f);
```

$$answer := ce^a + b^2$$

Pr.10.10 Type the given representation of S and keep in mind that $\mathbf{r} = [x, y, z]$.

```
> r := <a*cos(v)*cos(u)  b*cos(v)*sin(u)  c*sin(v)>;
```

$$r := [\ a\cos(v)\cos(u) \quad b\cos(v)\sin(u) \quad c\sin(v)\ ]$$

Observe that you can combine the first two components by using $\cos^2 u + \sin^2 u = 1$, namely,

```
> E1 := r[1]^2/a^2 + r[2]^2/b^2;
```

$$E1 := \cos(v)^2\cos(u)^2 + \cos(v)^2\sin(u)^2$$

```
> E2 := simplify(E1);                          # Resp.  E2 := cos(v)^2
```

From this you see that by adding the square of the third component divided by c^2 you obtain

```
> E := E1 + r[3]^2/c^2;
```

$$E := \cos(v)^2\cos(u)^2 + \cos(v)^2\sin(u)^2 + \sin(v)^2$$

```
> simplify(E);                                 # Resp.  1
```

Because `r[1]`, `r[2]`, `r[3]` equal x, y, z, respectively, your result is $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. This procedure is typical; the transition from one type of representation to another usually requires some trials.

A normal vector $\mathbf{N}$ is obtained from the given representation by differentiating and taking the cross product,

```
> ru := VectorCalculus[diff](r, u);
```

$$ru := -a\cos(v)\sin(u)\,e_x + (b\cos(v)\cos(u))\,e_y$$

```
> rv := VectorCalculus[diff](r, v);
```

$$rv := -a\sin(v)\cos(u)\,e_x - b\sin(v)\sin(u)\,e_y + (c\cos(v))\,e_z$$

```
> with(LinearAlgebra):
```

```
> N := CrossProduct(ru, rv);
```

$$N := [\ b\cos(v)^2\cos(u)c \quad a\cos(v)^2\sin(u)c \quad a\cos(v)\sin(u)^2 b\sin(v) + b\cos(v)\cos(u)^2 a\sin(v)\ ]$$

```
> N := simplify(%);
```

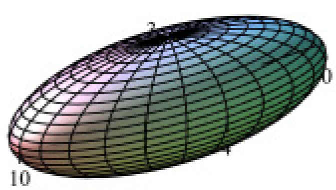
$$N := [\ b\cos(v)^2\cos(u)\,c \quad a\cos(v)^2\sin(u)\,c \quad a\cos(v)\,b\sin(v)\ ]$$

For plotting, type the following. You can turn the ellipsoid by clicking anywhere on the figure and then dragging.

```
> R := subs(a = 10, b = 4, c = 3, r);
```

$$R := [\ 10\cos(v)\cos(u) \quad 4\cos(v)\sin(u) \quad 3\sin(v)\ ]$$

```
> plot3d(R, u = 0..2*Pi, v = -Pi/2..Pi/2, axes = NORMAL,
    orientation = [45, 70], scaling = constrained);
```



Example 10.10. Ellipsoid

Pr.10.16 The plane $x + y + z = 1$ gives the upper portion of the surface, and $x + y = 1$ its intersection with the xy-plane $z = 0$. Hence $z = 1 - x - y$ and $y = 1 - x$, respectively. These are the upper limits of integration over z and y, respectively. Finally, integrate over x from 0 to 1.

```
> with(LinearAlgebra):  VectorCalculus[SetCoordinates]('cartesian'[x,y,z]):
> F := <3*x  z^3*y^5  y^3*x^4>;          # Resp.
```

$F := \begin{bmatrix} 3x & z^3y^5 & y^3x^4 \end{bmatrix}$

```
> Div := VectorCalculus[Divergence](VectorCalculus[VectorField](F));
```

$$Div := 3 + 5\,z^3y^4$$

```
> int(int(int(Div, z = 0..1 - x - y), y = 0..1 - x), x = 0..1);
```

$$\frac{2521}{5040}$$