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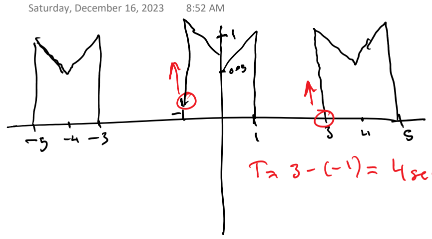
Saturday, December 16, 2023 8:52 AM



$T = 3-(-1) = 4\,sec$

$u(t) = 1 \quad t \geq 0$

$r = t \cdot u(t)$

a) $T = \text{period} = \boxed{4\ sec}$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}\ rad/sec}$

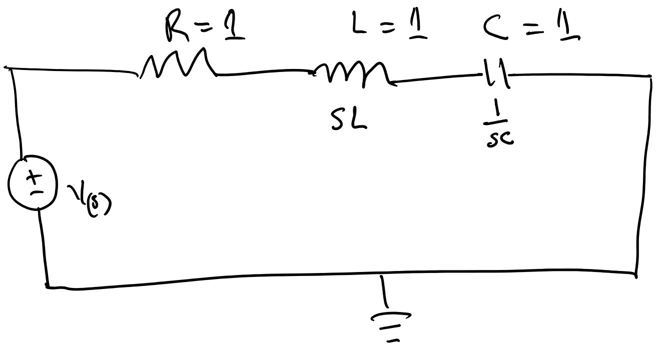
b) $$\boxed{f(t) = \begin{cases} (0.5t \cdot u(t)) - u(t-1) & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 2 \end{cases}}$$

c) $$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_0 = \frac{1}{4}\int_0^4 \left[(0.5t \cdot u(t) - u(t-1)\right]dt$$

$$\boxed{a_0 = \frac{1}{4}}$$

Check Matlab for a_n, b_n & fourier series



$$V_{(s)}(t) = V_R(t) + V_L(t) + V_C(t)$$

$$V_C(t) = \frac{1}{C}\int i(t)$$

$$i(t) = \frac{V_{(s)}(t)}{Z}$$

$$z = R + j\left(sL - \frac{1}{s}\right)$$

$$z = 1 + j\left[s - \frac{1}{s}\right]$$

$$Z = 1 + j\left(\frac{s^2 - 1}{s}\right)$$

$$Z = \frac{s + js^2 - j}{s}$$

$$V_C(t) = \frac{1}{sC}\left[\frac{V_s(t) \cdot s}{s + js^2 - j}\right]$$

$$V_C(t) = \frac{V_s(t)}{s + js^2 - j} \quad \text{let } V_{(s)}(t) = 1$$

$$\boxed{V_C(t) = \frac{1}{s + s^2 - 1}}$$

$$V_L(t) = \left(\frac{s}{s^2 + s - 1} \right) sL \quad \text{wher } L = 1$$

$$\boxed{V_L(t) = \frac{s^2}{s^2 + s - 1}}$$

$$V_R(t) = R \left(\frac{s}{s^2 + s - 1} \right)$$

$$\boxed{V_R(t) = \frac{s}{s^2 + s - 1}}$$

Check Matlab code for freq response

Saturday, December 16, 2023 7:32 AM

Z- transform

$x[n] = u[n-1] \Rightarrow$ input

$h[n] = -0.25^n \cdot u[n] \Rightarrow$ Impulse response

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$y[n] = \sum_{k=\infty}^{\infty} u[k-1] \cdot \left[0.25^{n-(k-1)} \cdot u[n-k-1]\right]$$

for $k < 1 \quad u[k-1] = 0$

$k \geq 1 \quad u[k-1] = 1$

$$y[n] = \sum_{k=1}^{\infty} 1 \cdot \left[0.25^{n-(k-1)} u[n-k]\right]$$

geometric Sum $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$

$$y[n] = 0.25^n \cdot \frac{1-0.25^n}{1-0.25} = 0.25^n \cdot \frac{1-0.25^n}{0.75}$$

$$\boxed{y[n] = \frac{0.25^n \cdot (1-0.25^n)}{0.75}}$$

$$2\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^3} + 6\frac{dy}{dt} + 8y = 10\,u(s)$$

$$y\left(2s^3+4s^2+6s+8\right) = 10.u(s)$$

$$\underbrace{\dddot{y} + 4\overset{\ddot{y}}{\underset{\varphi_3}{\downarrow}} + 6\overset{\dot{y}}{\underset{\varphi_2}{\downarrow}} + 8\overset{y}{\underset{\varphi_1}{\downarrow}}}_{\text{state variables}} = k\,u(s) \xrightarrow{\text{note}}$$

$s.y \to \dot{y} \to \dot{Q}_1$

$s.y \to \ddot{y} \to \dot{Q}_2$

$s^3.y \to \dddot{y} \to \dot{Q}_3$

$\Rightarrow$ step 2 get ~~the~~ state variables $\varphi_1(t)$ & $\varphi_2(t)$ & $\varphi_3(t)$

$$\left.\begin{aligned} \varphi_1(t) &= y(t) \\ \varphi_2(t) &= \dot{y}(t) = \dot{\varphi}_1(t) \\ \varphi_3(t) &= \ddot{y}(t) = \dot{\varphi}_2(t) \end{aligned}\right\} \text{1}^{\text{st}} \text{ order equation for the variable}$$

$\dot{\varphi}_1(t)$, $\dot{\varphi}_2(t)$, and $\dot{\varphi}_3(t)$ are given by

$$\left.\begin{aligned} \dot{\varphi}_1 &= \varphi_2 \\ \dot{\varphi}_2 &= \varphi_3 \end{aligned}\right\} \text{step 3}$$

$$\dot{\varphi}_3 = \dddot{y} = -2\ddot{y} - 3\dot{y} - 4y + 10\,u(t)$$

now put $\dot{\varphi}_3$, $\dot{\varphi}_2$ & $\dot{\varphi}_1$ in terms of state variable

$$\left.\begin{aligned} \dot{\varphi}_3 &= -2\varphi_3 - 3\varphi_2 - 4\varphi_1 + 10.u(t) \\ \dot{\varphi}_2 &= \varphi_3 + 0.\varphi_2 + 0.\varphi_1 + 0.u(t) \\ \dot{\varphi}_1 &= 0.\varphi_3 + \varphi_2 + 0.\varphi_1 + 0.u(t) \end{aligned}\right\} \text{step 4}$$

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & -3 & -4 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \cdot \begin{bmatrix} u(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$