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#EE597
#MID TERM

#QUESTION 1

restart;

#Define ODE1

$ODE1 := diff(i1(t), t) + 3\cdot(i1(t) - i2(t)) = 24$

$$ODE1 := \frac{d}{dt}\, i1(t) + 3\, i1(t) - 3\, i2(t) = 24 \qquad (1)$$

#Define ODE2

$ODE2 := 8\cdot i2(t) + 3\cdot(i2(t) - i1(t)) + 4\cdot int(i2(t), t) = 0;$

$$ODE2 := 11\, i2(t) - 3\, i1(t) + 4\left(\int i2(t)\, dt\right) = 0 \qquad (2)$$

#Diff ODE2 to get rid of the int

$ODE\_2 := diff(ODE2, t);$

$$ODE\_2 := 11\, \frac{d}{dt}\, i2(t) - 3\, \frac{d}{dt}\, i1(t) + 4\, i2(t) = 0 \qquad (3)$$

#Model the RLC circuit as one system

$sys\_ode := ODE1,\ ODE\_2;$

$$sys\_ode := \frac{d}{dt}\, i1(t) + 3\, i1(t) - 3\, i2(t) = 24, 11\, \frac{d}{dt}\, i2(t) - 3\, \frac{d}{dt}\, i1(t) + 4\, i2(t) = 0 \qquad (4)$$

#define initial conditions

$ics := i1(0) = 0,\ i2(0) = 0;$

$$ics := i1(0) = 0, i2(0) = 0 \qquad (5)$$

#solve for the i1 and i2(t) respective using the dsolve fucntion

$sol := dsolve([sys\_ode, ics]);$

$$sol := \left\{ i1(t) = \frac{11\, e^{-\frac{6\,t}{11}}}{2} - \frac{27\, e^{-2\,t}}{2} + 8, i2(t) = \frac{9\, e^{-\frac{6\,t}{11}}}{2} - \frac{9\, e^{-2\,t}}{2} \right\} \qquad (6)$$

#QUESTION 2

#Part A

restart;

#using the inttrans package for laplace and integration

$with(inttrans):$

$ODE := L\cdot diff(i(t),\ t) + R\cdot i(t) = (1 - \mathrm{Heaviside}(t - 3\cdot \mathrm{Pi}))\cdot \sin(t);$

$$ODE := L\left(\frac{d}{dt}\, i(t)\right) + R\, i(t) = \left(1 - \mathrm{Heaviside}(t - 3\,\pi)\right) \sin(t) \qquad (7)$$

#solve ODE with initial condition

$yp := dsolve(\{ODE, i(0) = 0\}, i(t), method = laplace);$

$$yp := i(t) = \frac{\left(\sin(t)\, R - L\left(\mathrm{e}^{-\frac{R\,(t-3\,\pi)}{L}} + \cos(t)\right)\right) \mathrm{Heaviside}(-t+3\,\pi) + L\left(\mathrm{e}^{-\frac{R\,(t-3\,\pi)}{L}} + \mathrm{e}^{-\frac{R\,t}{L}}\right)}{L^2+R^2} \tag{8}$$

#convert to piecewise

yp1 := *convert*(%, *piecewise*, *t*);

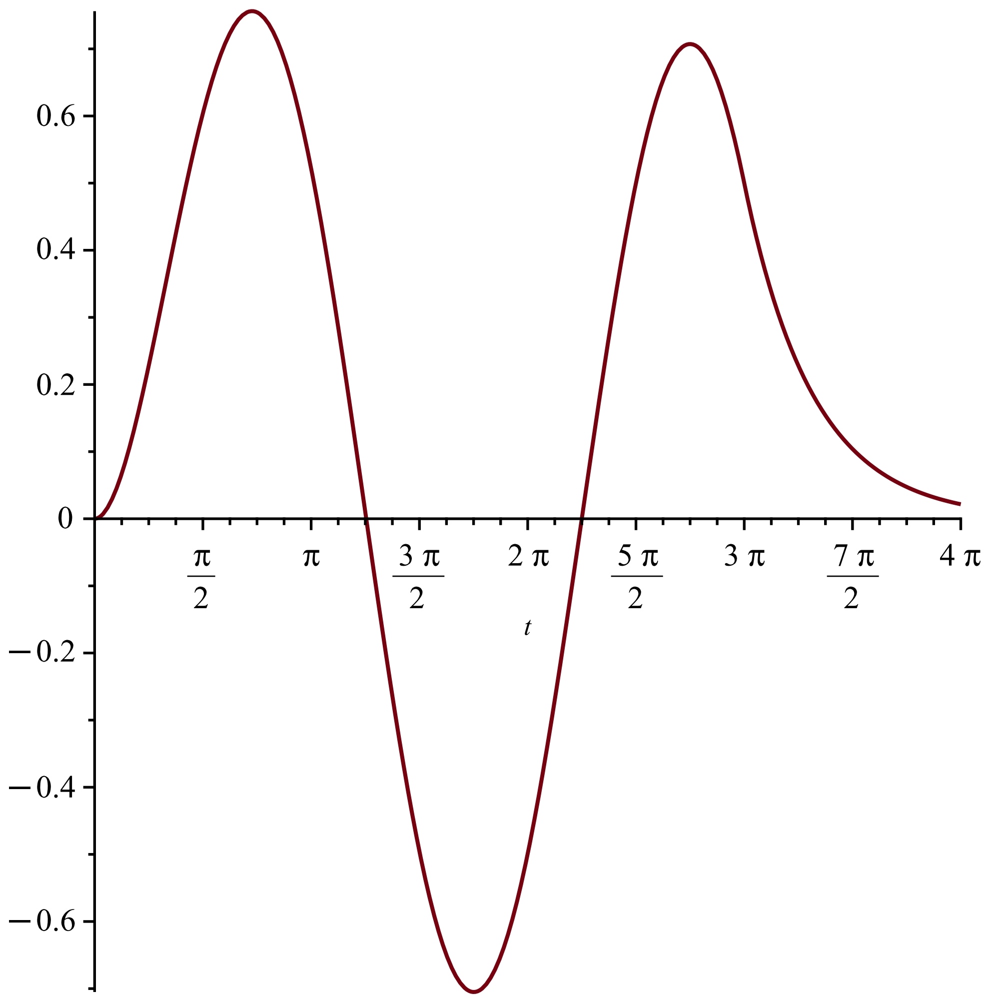
$$yp1 := i(t) = \begin{cases} \dfrac{L\left(\mathrm{e}^{-\frac{R\,(t-3\,\pi)}{L}} + \mathrm{e}^{-\frac{R\,t}{L}}\right) + \sin(t)\, R - L\left(\mathrm{e}^{-\frac{R\,(t-3\,\pi)}{L}} + \cos(t)\right)}{L^2+R^2} & t < 3\,\pi \\ \dfrac{L\left(1 + \mathrm{e}^{-\frac{3\,R\,\pi}{L}}\right) + \mathit{undefined}}{L^2+R^2} & t = 3\,\pi \\ \dfrac{L\left(\mathrm{e}^{-\frac{R\,(t-3\,\pi)}{L}} + \mathrm{e}^{-\frac{R\,t}{L}}\right)}{L^2+R^2} & 3\,\pi < t \end{cases} \tag{9}$$

I_t := *subs*(*R* = 1, *L* = 1, *yp1*);

$$I\_t := i(t) = \begin{cases} \dfrac{\mathrm{e}^{-t}}{2} + \dfrac{\sin(t)}{2} - \dfrac{\cos(t)}{2} & t < 3\,\pi \\ \mathit{undefined} + \dfrac{\mathrm{e}^{-3\,\pi}}{2} & t = 3\,\pi \\ \dfrac{\mathrm{e}^{-t+3\,\pi}}{2} + \dfrac{\mathrm{e}^{-t}}{2} & 3\,\pi < t \end{cases} \tag{10}$$

with(*plots*) :

plot(*rhs*(*I_t*), *t* = 0 ..4·Pi);



#QUESTION 3

restart;

with(*LinearAlgebra*) :

$f := 10\cdot\ln(x^4 + y^4);$

$$f := 10\ln(x^4 + y^4) \tag{11}$$

$v := VectorCalculus[Gradient](f, [x, y]);$

$$v := \left(\frac{40\,x^3}{x^4 + y^4}\right)\bar{e}_x + \left(\frac{40\,y^3}{x^4 + y^4}\right)\bar{e}_y \tag{12}$$

#b is the unit vector of the vector a

$a := \langle 1 \mid -1\rangle;\ b := \frac{a}{Norm(a, 2)};$

$$a := \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$b := \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \tag{13}$$

#directional derivative vector(v).b

$deriv := DotProduct(b, Vector(v), conjugate = false);$

$$deriv := \frac{20\sqrt{2}\,x^3}{x^4+y^4} - \frac{20\sqrt{2}\,y^3}{x^4+y^4} \tag{14}$$

#substitude the points (0,4)into deriv` `

subs(*x* = 0, *y* = 4, *deriv*);

$$-5\sqrt{2} \tag{15}$$

#evaluate to 4 sig figures

evalf[4](%);

$$-7.070 \tag{16}$$

#QUESTION 4

restart;

$r := \langle u \mid u^2 \mid v \rangle$;

$$r := \begin{bmatrix} u & u^2 & v \end{bmatrix} \tag{17}$$

r_u := *VectorCalculus*[*diff*](*r*, *u*);   *r_v* := *VectorCalculus*[*diff*](*r*, *v*);

$$r\_u := (1)e_x + (2\,u)e_y + (0)e_z$$

$$r\_v := (0)e_x + (0)e_y + (1)e_z \tag{18}$$

with(*LinearAlgebra*) :

N := *CrossProduct*(*r_u*, *r_v*);

$$N := \begin{bmatrix} 2\,u & -1 & 0 \end{bmatrix} \tag{19}$$

$F := \langle \exp(z) \mid \exp(z)\cdot\sin(y) \mid \exp(z)\cdot\cos(y) \rangle$;

$$F := \begin{bmatrix} e^z & e^z \sin(y) & e^z \cos(y) \end{bmatrix} \tag{20}$$

 # on S the vector function takes the form F(r(u,v)) call it FS

FS := *subs*(*x* = *r*[1], *y* = *r*[2], *z* = *r*[3], *F*);

$$FS := \begin{bmatrix} e^v & e^v \sin(u^2) & e^v \cos(u^2) \end{bmatrix} \tag{21}$$

integrand := *DotProduct*(*FS*, *N*, *conjugate* = *false*);

$$integrand := 2\,e^v\,u - e^v \sin(u^2) \tag{22}$$

#Now integrate over u and v

answer := *int*(*int*(*integrand*, *u* = 0 ..2), *v* = 0 ..3);

$$answer := -4 + \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\right)}{2} - \frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}\,e^3}{2} + 4\,e^3 \tag{23}$$