#Pr 6.2 (Transform by integration)

restart :
with(*inttrans*) : #apply the integration transform library
$f := \cos^2 \text{omega}[0] \cdot t;$ #define the function

$$f := \cos^2 \omega_0\, t \tag{1}$$

$F := laplace(f, t, s);$ #obtain the laplace

$$F := \frac{\cos^2 \omega_0}{s^2} \tag{2}$$

#using integral method to verify the answer $Fs = \int e^{-st} \cdot f(t) dt\ [t = 0 .. \infty]$
$Fs := int(\exp(-s \cdot t) \cdot f,\ t = 0 ..\text{infinity});$

$$Fs := \lim_{t \to \infty} \left(-\frac{\cos^2 \omega_0 \left(e^{-s\,t}\, s\, t + e^{-s\,t} - 1 \right)}{s^2} \right) \tag{3}$$

#Pr 6.4 (Inverse transform)

restart;
with(*inttrans*) :
$F := \frac{s-1}{s^2-9};$ #dedine F(s)

$$F := \frac{s-1}{s^2-9} \tag{4}$$

$f := invlaplace(F,\ s,\ t);$ #find f(t) using the inverse laplace of F(s)

$$f := \frac{e^{3\,t}}{3} + \frac{2\, e^{-3\,t}}{3} \tag{5}$$

#Pr 6.6 (Initial value problem, subsidiary equation)

restart;
with(*inttrans*) :
$ODE := \mathrm{D}(y)(t) + 5 \cdot y(t) = 3.5 \cdot \exp(-5\, y \cdot t);$ **#define the ODE**

$$ODE := \mathrm{D}(y)(t) + 5\, y(t) = 3.5\, e^{-5\, y\, t} \tag{6}$$

$subsid := laplace(ODE,\ t, s);$

$$subsid := s\, \mathcal{L}(y(t), t, s) - 1.\, y(0) + 5.\, \mathcal{L}(y(t), t, s) = \frac{3.500000000}{s + 5.\, y} \tag{7}$$

$subsid2 := subs(y(0) = 1, subsid);$
#apply the Initial condition y(0)=0 and y'(0)=0

$$subsid2 := s\, \mathcal{L}(y(t), t, s) - 1. + 5.\, \mathcal{L}(y(t), t, s) = \frac{3.500000000}{s + 5.\, y} \tag{8}$$

$Y := solve(subsid2,\ laplace(y(t),\ t,\ s));$ **#solve the laplace**

$$Y := \frac{0.5000000000\ (7. + 2.\, s + 10.\, y)}{(s + 5.\, y)\ (s + 5.)} \tag{9}$$

$yp\ := invlaplace(Y, s, t);$
 # *take the inverse laplace to get the particular solution or y(t)*

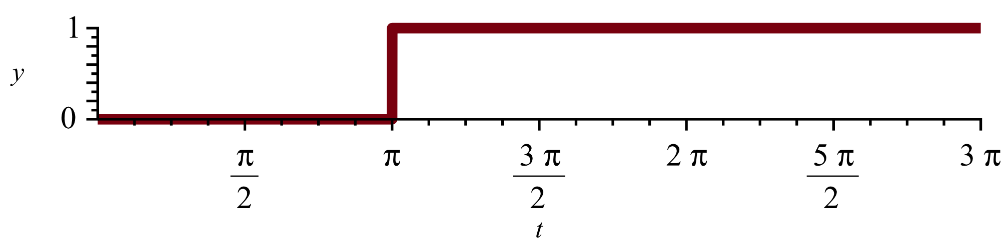
$$yp := \frac{0.1000000000\ \left(e^{-5.\, t}\ (-3. + 10.\, y) - 7.\, e^{-5.\, y\, t}\right)}{y - 1.} \tag{10}$$

#Pr 6.8 (t-shifting)

#plot f(t)=u$(t - \pi)$ **and** *find it's Transform*
restart;
with(*inttrans*) :
$f\ := \mathrm{Heaviside}(t - \mathrm{Pi});$ *#define the function (f)*

$$f := \mathrm{Heaviside}(t - \pi) \tag{11}$$

$plot(f, t = 0\,..3 \cdot \mathrm{Pi}, labels = [t, y], scaling = constrained);$ *#plot the function*



$F\ := laplace(f,\ t,\ s);$ *#find the Transform*

$$F := \frac{e^{-s\,\pi}}{s} \tag{12}$$

#Pr6.12 (RC -circuit, Dirac's delta)

restart;
with(*inttrans*) :
#define the circuit elements

$R := 2; C = \frac{1}{2};$

$$R := 2$$

$$C = \frac{1}{2} \tag{13}$$

#obtain the equivalent impedance Z=R+Xc

$Xc := -\frac{1}{s \cdot C};$

$$Xc := -\frac{1}{s\,C} \tag{14}$$

$Z := R + Xc;$

$$Z := 2 - \frac{1}{s\,C} \tag{15}$$

#define v(t)
$Vt := K \cdot \mathrm{Dirac}(t-1);$

$$Vt := K\,\mathrm{Dirac}(t-1) \tag{16}$$

 # convert vt to the s domain using laplace
$Vs := laplace(Vt, t, s);$

$$Vs := K\,\mathrm{e}^{-s} \tag{17}$$

#use Kirchoff's law to find the current in S-domain $I\_s=\frac{Vs}{Z}$

$Is := \frac{Vs}{Z};$

$$Is := \frac{K\,\mathrm{e}^{-s}}{2 - \frac{1}{s\,C}} \tag{18}$$

#find i(t) using inverse laplace
$i := invlaplace(Is,\ s,\ t);$

$$i := \frac{\mathrm{Heaviside}(t-1)\,K\,\mathrm{e}^{\frac{t-1}{2\,C}}}{4\,C} \tag{19}$$

$i\_t := subs\left(K = 110, C = \frac{1}{2}, i\right);$

$$i\_t := 55\,\mathrm{Heaviside}(t-1)\,\mathrm{e}^{t-1} \tag{20}$$

#plot i(t) at time range for which K is constants

$plot(i\_t, t = 0..3, labels = [t, "i(t)"]);$