1.3_b

Friday, September 8, 2023 6:44 PM

Using Euler's Identity

$x(t) = e^{jt} = \cos(t) + j\sin(t)$

Even Component $\Rightarrow x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$x_e(t) = \frac{1}{2}\left[\cos(t) + j\sin(t) + \cos(-t) + j\sin(-t)\right]$

note $\cos(\theta) = \cos(-\theta)$

$\therefore \cos(-t) = \cos(t)$

$\sin(-t) = -\sin(t)$

$x_e(t) = \frac{1}{2}\left[\cos(t) + \cancel{j\sin(t)} + \cos(t) - \cancel{j\sin(t)}\right]$

$x_e(t) = \frac{1}{\cancel{2}}\left[\cancel{2}\cos(t)\right]$

$\boxed{x_e(t) = \cos(t)}$

Odd Component $\Rightarrow x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$

$x_o(t) = \frac{1}{2}\left[\cos(t) + j\sin(t) - \left(\cos(-t) + j\sin(-t)\right)\right]$

$= \frac{1}{2}\left[\cos(t) + j\sin(t) - \left(\cos(t) - j\sin(t)\right)\right]$

$= \frac{1}{2}\left[\cancel{\cos(t)} - \cancel{\cos(t)} + j\sin(t) + j\sin(t)\right]$

$= \frac{1}{\cancel{2}}\left[\cancel{2}j\sin(t)\right]$

$\boxed{x_o(t) = j\sin(t)}$

1.6_a

Saturday, September 9, 2023 1:13 AM

Find Ω_0 rad/sec, f_0 Hz, & T_0 sec from $-\infty < t < \infty$

i) $\cos(2\pi t)$

$$\boxed{\Omega_0 = 2\pi \text{ rad/sec}}$$

$$f_0 = \frac{\Omega_0}{2\pi}$$

$$f_0 = \frac{2\pi}{2\pi} = \boxed{1 \text{ Hz}}$$

$$T_0 = \frac{1}{f_0} = \frac{1}{1} = \boxed{1 \text{ sec}}$$

ii) $\sin\left(t - \frac{\pi}{4}\right)$

$$\boxed{\Omega_0 = 1 \text{ rad/sec}}$$

$$f_0 = \frac{\Omega_0}{2\pi} = \boxed{\frac{1}{2\pi} \text{ Hz}}$$

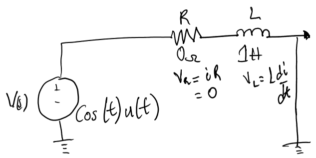
$$T_0 = \frac{1}{f_0} = \frac{1}{\frac{1}{2\pi}} = \boxed{2\pi \text{ sec}}$$

iii) $\tan(\pi t)$

$$\boxed{\Omega_0 = \pi \text{ rad/sec}}$$

$$f_0 = \frac{\Omega_0}{2\pi} = \frac{\pi}{2\pi} = \boxed{\frac{1}{2} \text{ Hz}}$$

$$T_0 = \frac{1}{f_0} = \frac{1}{\frac{1}{2}} = \boxed{2 \text{ sec}}$$



$P_L(t) = V_L(t) \cdot i_L(t)$

note Resistor is 0Ω $\therefore V_L(t) = V_s(t)$

$V_L(t) = L\frac{di_L(t)}{dt}$

$\int V_L(t) = L\int \frac{di_L(t)}{dt}$

$\frac{1}{L}\int V_L(t) = i_L(t)$

$i_L(t) = \frac{1}{1H}\int \cos(t)u(t)\,dt$

note $u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$

$i_L(t) = \sin(t) + C$

$P_L(t) = \cos(t)u(t)\left[\sin(t) + C\right]$

$t \geq 0 \quad u(t) = 1$

$\boxed{P_L(t) = \cos(t)\left[\sin(t) + C\right]}$

$P_{L\,ave} = \frac{1}{T}\int_0^T p(t)\,dt$

$V_s(t) = \cos(t)u(t)$

$2\pi f = 1$ rad/sec

$f = \frac{1}{2\pi}$

$T = \frac{1}{f}$

$T = 2\pi$

$P_{L\,ave} = \frac{1}{2\pi}\int_0^{2\pi}\cos(t)\left[\sin(t) + C\right]dt$

$= \frac{1}{2\pi}\left[\boxed{\int_0^{2\pi}\cos(t)\sin(t)\,dt} + \boxed{C\int_0^{2\pi}\cos(t)\,dt}\right]$

note $\cos(t)\cdot\sin(t) = \frac{1}{2}\left[\sin(t-t) + \sin(t+t)\right]$

$= \frac{1}{2}\sin(2t)$

$\boxed{\text{Part 1}}$ $\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\sin(2t) = \frac{1}{4\pi}\left[-\cos(2t)\right]_0^{2\pi} = \frac{1}{4\pi}\left[-\cos(4\pi) - \left[-\cos(0)\right]\right]$

$= \frac{1}{4\pi}(-1 + 1)$

$\boxed{= \frac{1}{4\pi}(0) = 0}$

$\boxed{\text{Part 2}}$ $\frac{1}{2\pi}C\int_0^{2\pi}\cos(t)\,dt = \frac{C}{2\pi}\left[\sin(t)\right]_0^{2\pi}$

$= \frac{C}{2\pi}\left[\sin(2\pi) - \sin(0)\right]$

$= \frac{C}{2\pi}\left[0 - 0\right]$

$\boxed{= 0}$

$\boxed{\therefore P_{L\,ave} = 0}$

1.18_a

Saturday, September 9, 2023 3:10 PM

$x(t) = 2e^{j2\pi t}$ $y(t) = e^{j\pi t}$

$z(t) = x(t) + y(t)$

$z(t) = 2e^{j2\pi t} + e^{j\pi t}$

$z(t) = 3e^{j\pi t}$

$z(t) = 3[\cos(\pi t) + j\sin(\pi t)]$

$\Omega_0 = \frac{2\cancel{\pi}}{T_0} = \cancel{\pi}$

$\boxed{T_0 = 2 \text{ seconds}}$

1.24

Saturday, September 9, 2023 4:15 PM

$x(t) = \cos(\pi t)$ $T_0 = 2$ sec

a) Expand by $x\left(\frac{t}{2}\right)$ $-2 < t < 2$

t	$x(t)$
-2	$\cos(-2\pi) = 1$
-1	$\cos(-\pi) = -1$
0	$\cos(0) = 1$
1	$\cos(\pi) = -1$
2	$\cos(2\pi) = 1$

t	$x\left(\frac{t}{2}\right)$
-2	$\cos\left(\pi \frac{-2}{2}\right) = \cos(-\pi) = -1$
-1	$\cos\left(-\frac{\pi}{2}\right) = 0$
0	$\cos(0) = 1$
1	$\cos\left(\frac{\pi}{2}\right) = 0$
2	$\cos(\pi) = -1$

$x(t) = \cos(\pi t)$

$\Omega_0 = \frac{2\pi}{T_0} = \pi$

$T_0 = 2$ sec

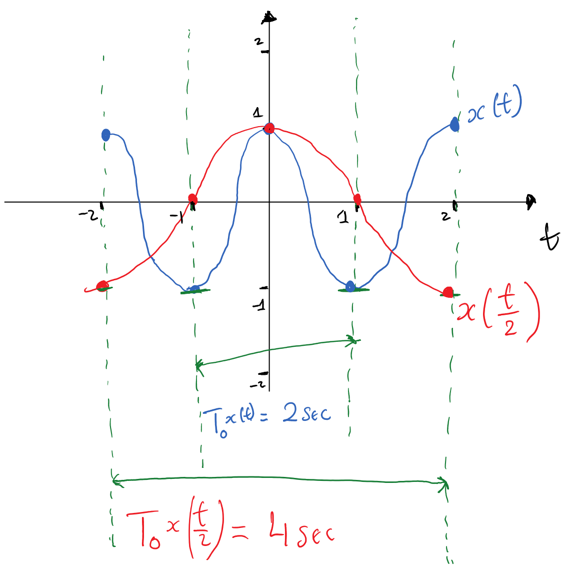
$x\left(\frac{t}{2}\right) = \cos\left(\frac{\pi}{2} t\right)$

$\Omega_0 = \frac{2\cancel{\pi}}{T_0} = \frac{\cancel{\pi}}{2}$

$T_0 = 4$ sec

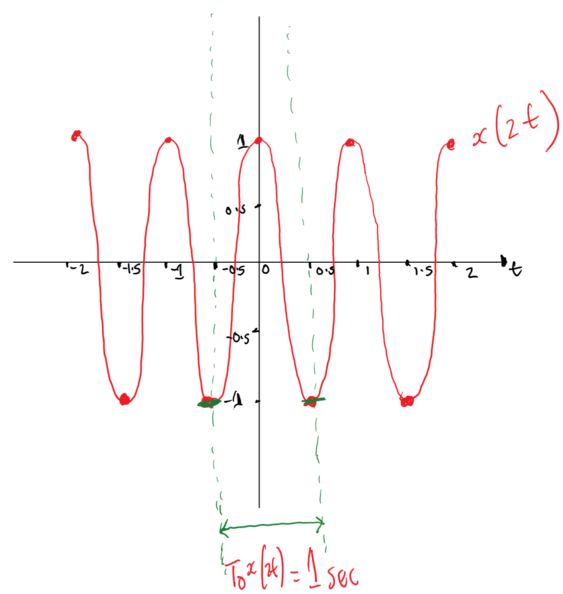
the signal is expanded by 2

$x\left(\frac{t}{2}\right)$ is period because it repeats itself every $T_0 = 4$ sec where $x\left(\frac{t}{2}\right) = -1$



b)

t	$x(2t)$
-2	$\cos(2\pi(-2)) = \cos(-4\pi) = 1$
-1.5	$\cos(-3\pi) = -1$
-1	$\cos(-2\pi) = 1$
-0.5	$\cos(-\pi) = -1$
0	$\cos(0) = 1$
0.5	$\cos(\pi) = -1$
1	$\cos(2\pi) = 1$
1.5	$\cos(3\pi) = -1$
2	$\cos(4\pi) = 1$



$x(2t) = \cos(2\pi t)$

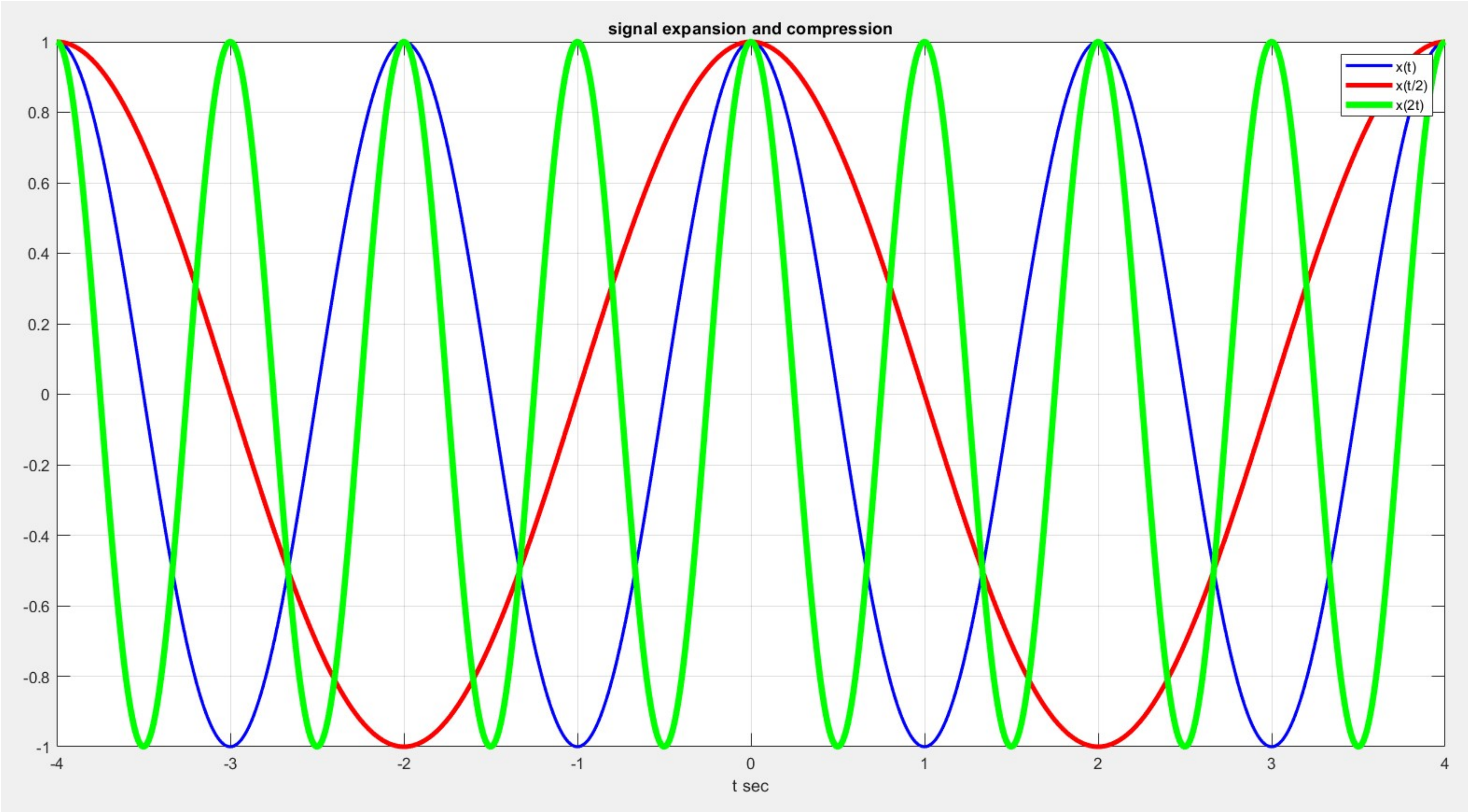
$\Omega_0 = \frac{2\pi}{T_0} = 2\pi$

$T_0 = 1$ sec

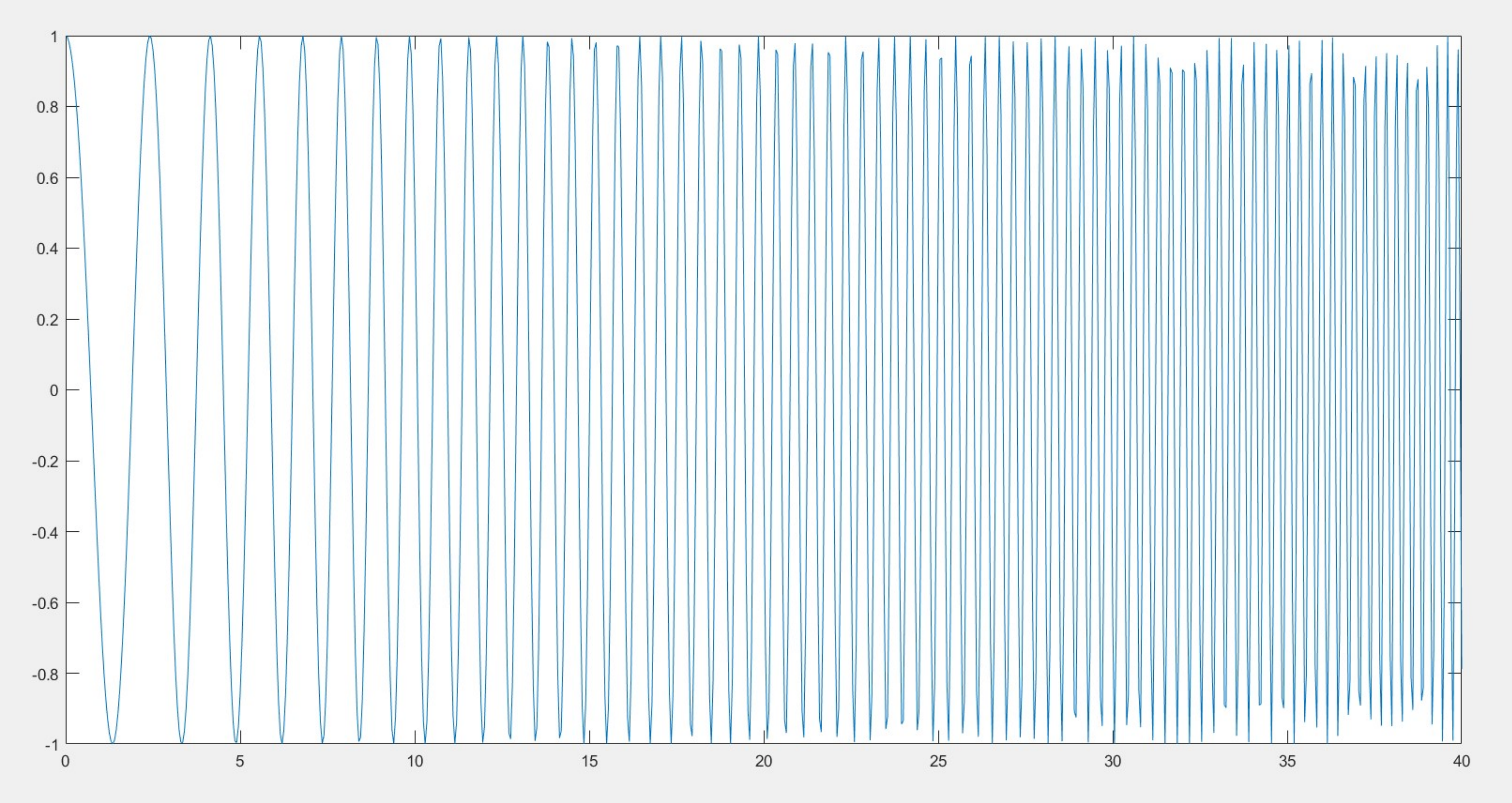
$x(2t)$ is periodic because it repeats itself at every $T_0 = 1$ sec where $x(2t) = 1$

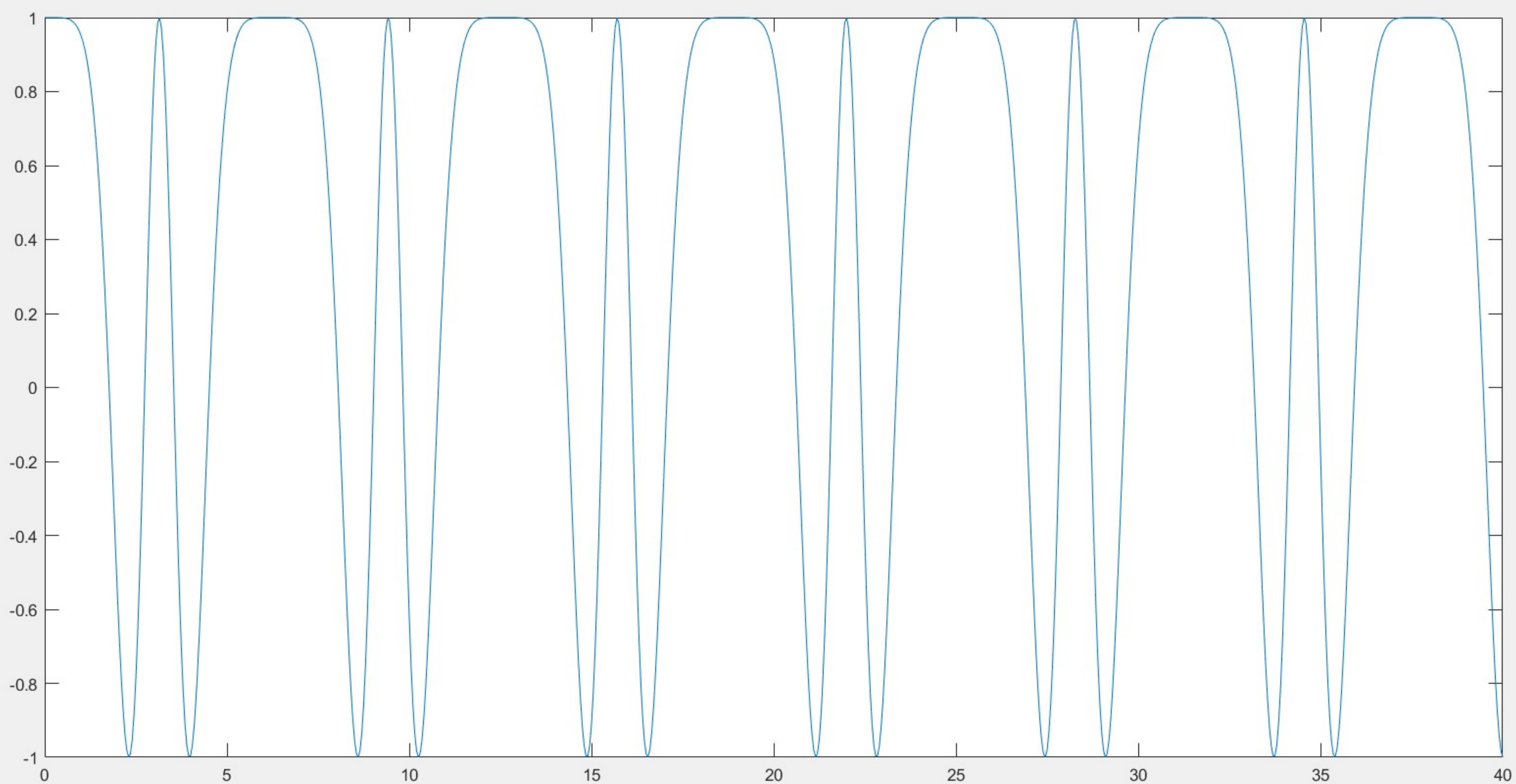
```
% Name: Lamin Jammeh
% Class: EE480 Online
% Semster: Fall 2023
% HW_3

% Basic Problems
%% ********** question 1.24 **********
clear
clc
% grab enough sample period of the signal and expand if necessary
t = -4:0.01:4; %sample from -2 to 2 in steps of 0.01
x_t = cos(pi * t); % define the x(t)
plot(t,x_t, 'b',LineWidth=2); % x(t) is the blue line
hold on;

%expanded signal
x_t2 = cos(pi * t/2); % define x(t/2)
plot(t,x_t2, 'r',LineWidth=3); % x(t/2) is the red line

%compressed signal
x_t3 = cos(pi * 2*t); %define x(2t)
plot (t,x_t3, 'g',LineWidth=4); % x(2t) is the green line

%label the graph
title('signal expansion and compression');
xlabel('t_ sec');
legend('x(t)','x(t/2)','x(2t)');
grid on;
hold off;


%% ********** question 1.29_a **********
clear
clc
f_s = 20000;
t = 0:0.05:40; %sampling period
A = 1;
omega = 2;
s_t = (t.^2)/4;
y_t = A * cos(omega * t + s_t);
plot(t,y_t)
sound(y_t,f_s)
%% ********** question 1.29_b **********
clear
clc
f_s = 20000;
t = 0:0.05:40; %sampling period
A = 1;
omega = 2;
```

```
s_t = -2 * sin(t);
y_t = A * cos(omega * t + s_t);
plot(t,y_t)
sound(y_t,f_s)
```



signal expansion and compression
x(t)
x(t/2)
x(2t)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-3
-2
-1
0
1
2
3
4
t sec


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40