1.3_b

Friday, September 8, 2023 6:44 PM

Using Euler's Identity

$x(t) = e^{jt} = \cos(t) + j\sin(t)$

Even Component $\Rightarrow x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$x_e(t) = \frac{1}{2}[\cos(t) + j\sin(t) + \cos(-t) + j\sin(-t)]$

note $\cos(\theta) = \cos(-\theta)$

$\therefore \cos(-t) = \cos(t)$

$\sin(-t) = -\sin(t)$

$x_e(t) = \frac{1}{2}[\cos(t) + \cancel{j\sin(t)} + \cos(t) - \cancel{j\sin(t)}]$

$x_e(t) = \frac{1}{\cancel{2}}[\cancel{2}\cos(t)]$

$\boxed{x_e(t) = \cos(t)}$

odd Component $\Rightarrow x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$x_o(t) = \frac{1}{2}[\cos(t) + j\sin(t) - (\cos(-t) + j\sin(-t))]$

$= \frac{1}{2}[\cos(t) + j\sin(t) - (\cos(t) - j\sin(t))]$

$= \frac{1}{2}[\cancel{\cos(t)} - \cancel{\cos(t)} + j\sin(t) + j\sin(t)]$

$= \frac{1}{\cancel{2}}[\cancel{2}j\sin(t)]$

$\boxed{x_o(t) = j\sin(t)}$

1.6_a

Saturday, September 9, 2023 1:13 AM

find Ω_0 rad/sec, f_0 Hz, & T_0 sec from $-\infty < t < \infty$

i) $\cos(2\pi t)$

$\boxed{\Omega_0 = 2\pi \text{ rad/sec}}$

$f_0 = \frac{\Omega_0}{2\pi}$

$f_0 = \frac{2\pi}{2\pi} = \boxed{1 \text{ Hz}}$

$T_0 = \frac{1}{f_0} = \frac{1}{1} = \boxed{1 \text{ sec}}$

ii) $\sin(t - \frac{\pi}{4})$

$\boxed{\Omega_0 = 1 \text{ rad/sec}}$

$f_0 = \frac{\Omega_0}{2\pi} = \boxed{\frac{1}{2\pi} \text{ Hz}}$

$T_0 = \frac{1}{f_0} = \frac{1}{\frac{1}{2\pi}} = \boxed{2\pi \text{ sec}}$

iii) $\tan(\pi t)$

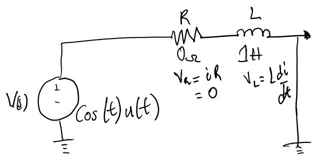
$\boxed{\Omega_0 = \pi \text{ rad/sec}}$

$f_0 = \frac{\Omega_0}{2\pi} = \frac{\pi}{2\pi} = \boxed{\frac{1}{2} \text{ Hz}}$

$T_0 = \frac{1}{f_0} = \frac{1}{\frac{1}{2}} = \boxed{2 \text{ sec}}$

1.9_a

Saturday, September 9, 2023 1:26 AM



$P_L(t) = V_L(t) \cdot i_L(t)$

note Resistor is 0Ω $\therefore V_L(t) = V_s(t)$

$V_L(t) = L \frac{di_L(t)}{dt}$

$\int V_L(t) = L \int \frac{di_L(t)}{dt}$

$\frac{1}{L}\int V_L(t) = i_L(t)$

$i_L(t) = \frac{1}{1H} \int \cos(t)\, u(t)\, dt$

note $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$i_L(t) = \sin(t) + C$

$P_L(t) = \cos(t)\, u(t) \left[\sin(t) + C\right]$

$t \geq 0 \quad u(t) = 1$

$\boxed{P_L(t) = \cos(t)\left[\sin(t) + C\right]}$

$P_L\text{ ave} = \frac{1}{T}\int_0^T P(t)\, dt$

$V_s(t) = \cos(t)\, u(t)$

$2\pi f = 1$ rad/sec

$f = \frac{1}{2\pi}$

$T = \frac{1}{f}$

$T = 2\pi$

$P_L\text{ ave} = \frac{1}{2\pi}\int_0^{2\pi} \cos(t)\left[\sin(t) + C\right] dt$

$= \frac{1}{2\pi}\left[\boxed{\int_0^{2\pi} \cos(t)\sin(t)\, dt} + \boxed{C\int_0^{2\pi} \cos(t)\, dt}\right]$

note $\cos(t) \cdot \sin(t) = \frac{1}{2}\left[\sin(t - t) + \sin(t + t)\right]$

$= \frac{1}{2}\sin(2t)$

Part 1 $\frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\sin(2t) = \frac{1}{4\pi}\left[-\cos(2t)\right]_0^{2\pi} = \frac{1}{4\pi}\left[-\cos(4\pi) - \left[-\cos(0)\right]\right]$

$= \frac{1}{4\pi}(-1 + 1)$

$\boxed{= \frac{1}{4\pi}(0) = 0}$

Part 2 $\frac{1}{2\pi} C\int_0^{2\pi} \cos(t)\, dt = \frac{C}{2\pi}\left[\sin(t)\right]_0^{2\pi}$

$= \frac{C}{2\pi}\left[\sin(2\pi) - \sin(0)\right]$

$= \frac{C}{2\pi}\left[0 - 0\right]$

$\boxed{= 0}$

$\boxed{\therefore P_L\text{ ave} = 0}$

1.18_a

Saturday, September 9, 2023 3:10 PM

$x(t) = 2e^{j2\pi t}$ $y(t) = e^{j\pi t}$

$z(t) = x(t) + y(t)$

$z(t) = 2e^{j2\pi t} + e^{j\pi t}$

$z(t) = 3e^{j\pi t}$

$z(t) = 3[\cos(\pi t) + j\sin(\pi t)]$

$\Omega_0 = \frac{2\cancel{\pi}}{T_0} = \cancel{\pi}$

$\boxed{T_0 = 2 \text{ seconds}}$

1.24

Saturday, September 9, 2023 4:15 PM

$x(t) = \cos(\pi t) \quad T_o = 2\text{ sec}$

a) Expand by $x\left(\frac{t}{2}\right) \quad -2 < t < 2$

t	$x(t)$
-2	$\cos(-2\pi) = 1$
-1	$\cos(-\pi) = -1$
0	$\cos(0) = 1$
1	$\cos(\pi) = -1$
2	$\cos(2\pi) = 1$

$x(t) = \cos(\pi t)$

$\Omega_o = \frac{2\pi}{T_o} = \pi$

$T_o = 2\text{ sec}$

t	$x\left(\frac{t}{2}\right)$
-2	$\cos\left(\pi \frac{-2}{2}\right) = \cos(-\pi) = -1$
-1	$\cos\left(-\frac{\pi}{2}\right) = 0$
0	$\cos(0) = 1$
1	$\cos\left(\frac{\pi}{2}\right) = 0$
2	$\cos(\pi) = -1$

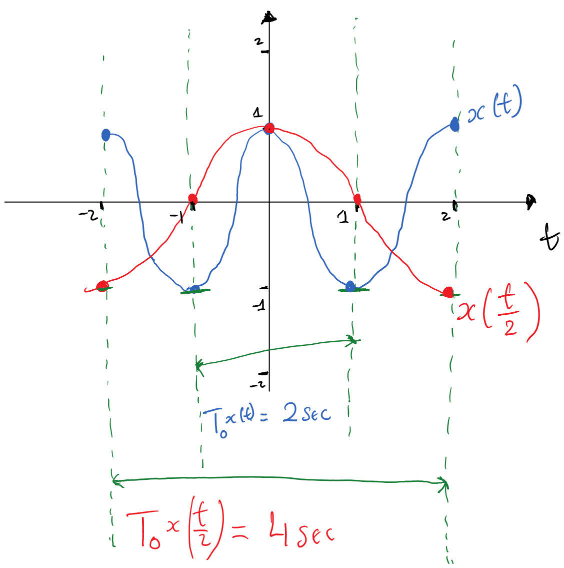
$x\left(\frac{t}{2}\right) = \cos\left(\frac{\pi}{2} t\right)$

$\Omega_o = \frac{2\cancel{\pi}}{T_o} = \frac{\cancel{\pi}}{2}$

$T_o = 4\text{ sec}$

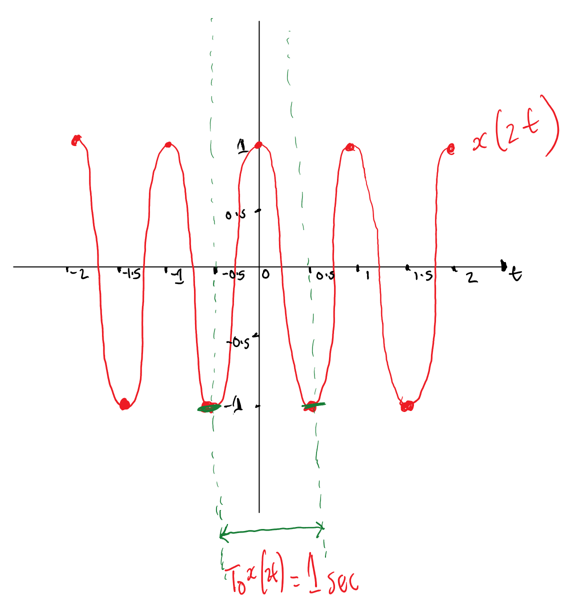
the signal is expanded by 2

$x\left(\frac{t}{2}\right)$ is period because it repeats itself every $T_o = 4\text{sec}$ where $x\left(\frac{t}{2}\right) = -1$



b)

t	$x(2t)$
-2	$\cos(2\pi(-2)) = \cos(-4\pi) = 1$
-1.5	$\cos(-3\pi) = -1$
-1	$\cos(-2\pi) = 1$
-0.5	$\cos(-\pi) = -1$
0	$\cos(0) = 1$
0.5	$\cos(\pi) = -1$
1	$\cos(2\pi) = 1$
1.5	$\cos(3\pi) = -1$
2	$\cos(4\pi) = 1$



$x(2t) = \cos(2\pi t)$

$\Omega_0 = \frac{2\pi}{T_0} = 2\pi$

$T_0 = 1$ sec

$x(2t)$ is periodic because it repeats itself at every $T_0 = 1$ sec where $x(2t) = 1$