Pr.1.2

```
> with(DEtools):
```

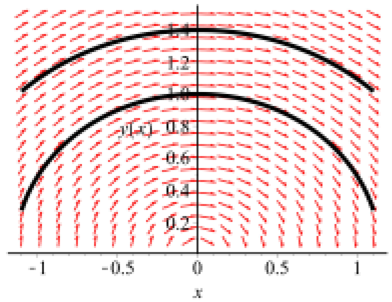
```
> ode := diff(y(x), x) = -13*x/(17*y(x));
```

$$ode := \frac{d}{dx}y(x) = -\frac{13}{17}\frac{x}{y(x)}$$

```
> inits := [0, 1], [0, 1.4];                # Resp.
```

$inits := \{[0, 1], [0, 1.4]\}$

```
> DEplot(ode, y(x), x = -1.1..1.1, y = 0.08..1.5, inits,
   stepsize = 0.01, scaling = constrained, linecolor = black);
```



Problem 1.2. Direction field of $y' = -13x/17y$

```
> dsolve(diff(y(x), x) = -4*x/(9*y(x)));
```

$$y(x) = -\frac{1}{3}\sqrt{-4x^2 + 9\_C1},\ y(x) = \frac{1}{3}\sqrt{-4x^2 + 9\_C1}$$

The general solution is $\frac{x^2}{17} + \frac{y^2}{13} = \text{const}$. The command `scaling = constrained` (optional) gives equal scales on both axes, so that the solution curves appear as ellipses. The choice of x- and y-intervals in plots is usually a matter of trial and error, until one obtains a satisfactory figure. In this case, the lower y-limit prevents plotting errors that arise from trying to plot the ends of the ellipses.

Pr.1.4

```
> ode := diff(y(t), t) + 0.5*y(t) = 1;
```

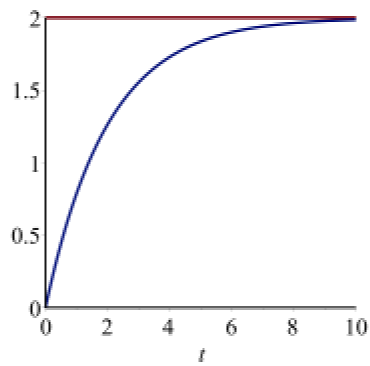
$$ode := \frac{\mathrm{d}}{\mathrm{d}x} y(t) + 0.5\, y(t) = 1$$

Solve the initial value problem by `dsolve`,

```
> sol := dsolve(ode, y(0) = 0);           # Resp.
```

$sol := y(t) = 2 - 2\,\mathrm{e}^{-\frac{1}{2}t}$

```
> plot(rhs(sol), 2, t = 0..10, ytickmarks = [0, 0.5, 1.0, 1.5, 2]);
```



Problem 1.4. Exponential approach to the limit $y = 2$

Pr.1.12 Type the given ODE in the form

```
> ode := csc(x)*diff(y(x), x) - y(x)*cot(x)*csc(x)
  - 100*cos(30*x) = 0;
```

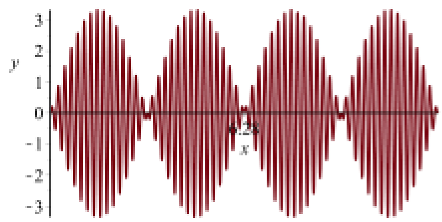
$$ode := \csc(x)\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right) - y(x)\cot(x)\csc(x) - 100\cos(30\,x) = 0$$

Solve the initial value problem by `dsolve`,

```
> sol := dsolve(ode, y(3*Pi/2) = 0);
```

$$sol := y(x) = \frac{10}{3}\sin(x)\sin(30\,x)$$

```
> plot(rhs(sol), x = 0..4*Pi, scaling = constrained, labels = [x, y],
  xtickmarks = [ evalf[3](2*Pi)]);
```



Problem 1.12. Beats given by $y(x) = 10/3\sin x\sin 30x$

The x-axis shows an approximate tickmark for $x = 2\pi$. Try to type 2π; it does not seem to work. Drop `scaling = constrained`, to see how the figure changes without this optional part of the command.

Pr.1.18 The current is governed by the ODE

```
> ode := 0.5*diff(i(t), t) + 7*i(t) = 5;
```

$$ode := 0.5 \left(\frac{d}{dt} i(t)\right) + 7\, i(t) = 5$$

The required solutions are obtained by

```
> it := dsolve({ode, i(0) = i0});
```

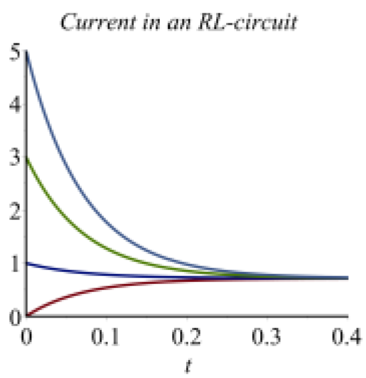
$$i1 := i(t) = \frac{5}{7} + e^{-14\,t} \left(i0 - \frac{5}{7}\right)$$

```
> S := seq(it, i0 in [5, 3, 1, 0]);
```

$$S := i(t) = \frac{5}{7} + \frac{30}{7} e^{-14\,t}, i(t) = \frac{5}{7} + \frac{16}{7} e^{-14\,t}, i(t) = \frac{5}{7} + \frac{2}{7} e^{-14\,t}, i(t) = \frac{5}{7} - \frac{5}{7} e^{-14\,t}$$

Plotting is now done by the command

```
> plot(seq(rhs(S[j]), j = 1..4), t = 0..0.4,
   title = `Current in an RL-circuit`);
```



Problem 1.18. Current $i(t)$ in an RL-circuit for three different initial values $i(0)$