#LAMIN JAMMEH
#EE597
#HW3

#Pr4.2 Saddle Point

restart;

$sys := D(y1)(t) = 2 \cdot y1(t) - 3 \cdot y2(t), \quad D(y2)(t) = \frac{3}{4} \cdot y1(t) - 3 \cdot y2(t);$

$$sys := D(y1)(t) = 2\, y1(t) - 3\, y2(t), D(y2)(t) = \frac{3\, y1(t)}{4} - 3\, y2(t) \tag{1}$$

$sys1 := sys[1];$

$$sys1 := D(y1)(t) = 2\, y1(t) - 3\, y2(t) \tag{2}$$

$sys2 := sys[2];$

$$sys2 := D(y2)(t) = \frac{3\, y1(t)}{4} - 3\, y2(t) \tag{3}$$

$sol := dsolve([sys]);$

$$sol := \left\{ y1(t) = c_1\, e^{\frac{3t}{2}} + c_2\, e^{-\frac{5t}{2}}, y2(t) = \frac{c_1\, e^{\frac{3t}{2}}}{6} + \frac{3\, c_2\, e^{-\frac{5t}{2}}}{2} \right\} \tag{4}$$

$sol[1];$

$$y1(t) = c_1\, e^{\frac{3t}{2}} + c_2\, e^{-\frac{5t}{2}} \tag{5}$$

$sol[2];$

$$y2(t) = \frac{c_1\, e^{\frac{3t}{2}}}{6} + \frac{3\, c_2\, e^{-\frac{5t}{2}}}{2} \tag{6}$$

#particular solution in real time

$yp := dsolve(\{sys, y1(0) = 10, y2(0) = 0\}, [y1(t), y2(t)]);$

$$yp := \left\{ y1(t) = \frac{45\, e^{\frac{3t}{2}}}{4} - \frac{5\, e^{-\frac{5t}{2}}}{4}, y2(t) = \frac{15\, e^{\frac{3t}{2}}}{8} - \frac{15\, e^{-\frac{5t}{2}}}{8} \right\} \tag{7}$$

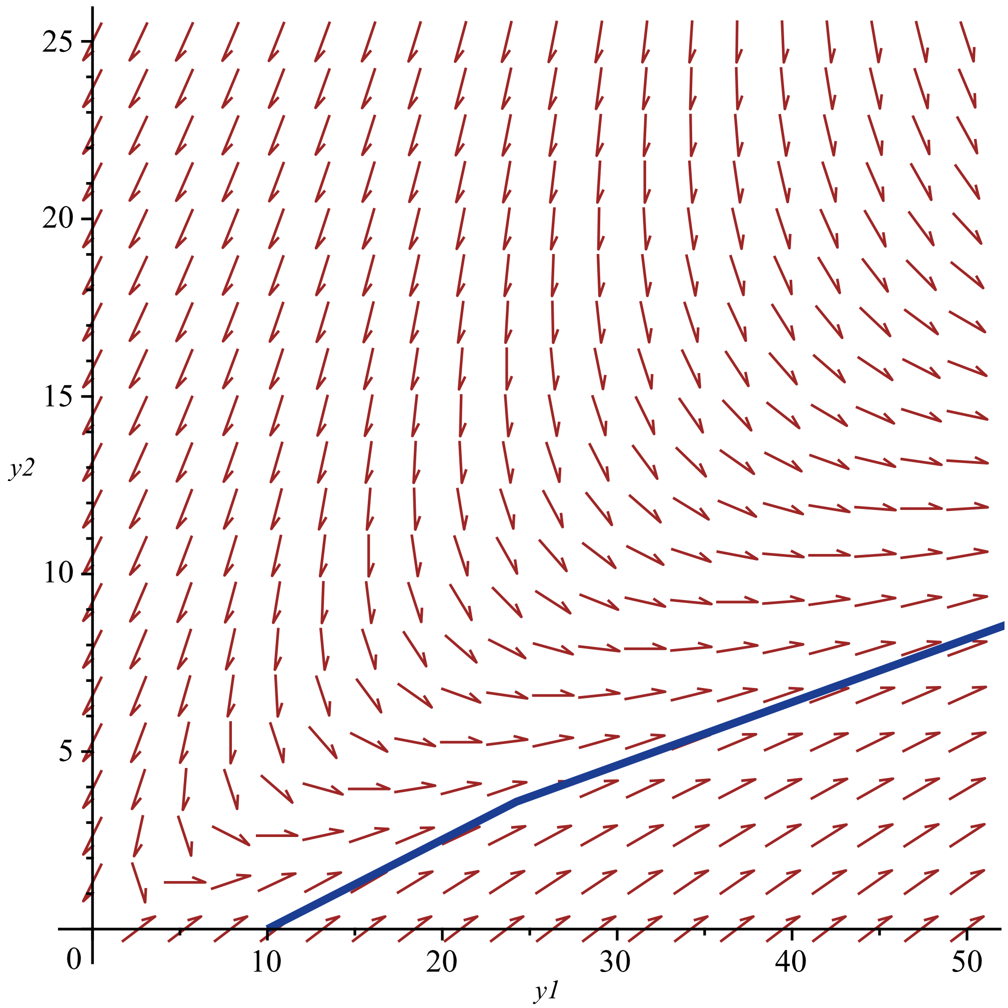
$yp[1];$

$$y1(t) = -\frac{5\, e^{-\frac{5t}{2}}}{4} + \frac{45\, e^{\frac{3t}{2}}}{4} \tag{8}$$

$yp[2];$

$$y2(t) = -\frac{15\,\mathrm{e}^{-\frac{5t}{2}}}{8} + \frac{15\,\mathrm{e}^{\frac{3t}{2}}}{8} \qquad \textbf{(9)}$$

with(*DEtools*) :
DEplot([*sys1*, *sys2*], [*y1*(*t*), *y2*(*t*)], *t* = 0 ..25, *y1* = 0 ..50, *y2* = 0 ..25, *number* = 2, [[0, 10, 0], [0, 0, 0]]);



#Pr4.4 Spiral Point

restart;
#deifne the 2 systems
sys := D(*y1*)(*t*) = 2·*y1*(*t*) + 9·*y2*(*t*),   D(*y2*)(*t*) = 3·*y2*(*t*);

$$sys := \mathrm{D}(y1)(t) = 2\,y1(t) + 9\,y2(t), \mathrm{D}(y2)(t) = 3\,y2(t) \qquad \textbf{(10)}$$

$sys1 := sys[1];$

$$sys1 := \mathrm{D}(y1)(t) = 2\, y1(t) + 9\, y2(t) \tag{11}$$

$sys2 := sys[2];$

$$sys2 := \mathrm{D}(y2)(t) = 3\, y2(t) \tag{12}$$

#obtain a general solution
$sol := dsolve([sys]);$

$$sol := \left\{ y1(t) = 9\, c_2\, \mathrm{e}^{3\,t} + c_1\, \mathrm{e}^{2\,t},\, y2(t) = c_2\, \mathrm{e}^{3\,t} \right\} \tag{13}$$

$sol[1];$

$$y1(t) = 9\, c_2\, \mathrm{e}^{3\,t} + c_1\, \mathrm{e}^{2\,t} \tag{14}$$

$sol[2];$

$$y2(t) = c_2\, \mathrm{e}^{3\,t} \tag{15}$$

#particular solution in real time

$yp := dsolve(\{sys, y1(0) = 1, y2(0) = 1\}, [y1(t), y2(t)]);$

$$yp := \left\{ y1(t) = 9\, \mathrm{e}^{3\,t} - 8\, \mathrm{e}^{2\,t},\, y2(t) = \mathrm{e}^{3\,t} \right\} \tag{16}$$

$yp[1];$

$$y1(t) = 9\, \mathrm{e}^{3\,t} - 8\, \mathrm{e}^{2\,t} \tag{17}$$

$yp[2];$

$$y2(t) = \mathrm{e}^{3\,t} \tag{18}$$

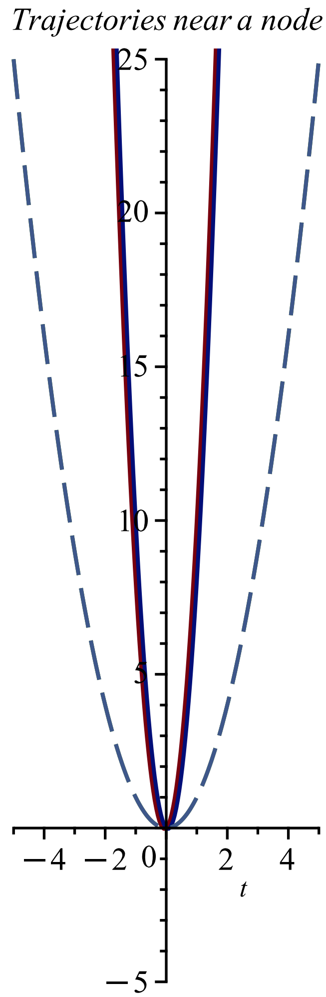
$with(plots):$
$c1s := [1, 0]: \quad c2s := [9, 1]:$
$Traj := [c1s[i] \cdot t + c2s[i] \cdot t^2, \; -c1s[i] \cdot t + c2s[i] \cdot t^2]:$
$i := 1: \; Traj; \; p1 := plot(Traj, t = -5..5, \; y = -5..5):$

$$[9\, t^2 + t, 9\, t^2 - t] \tag{19}$$

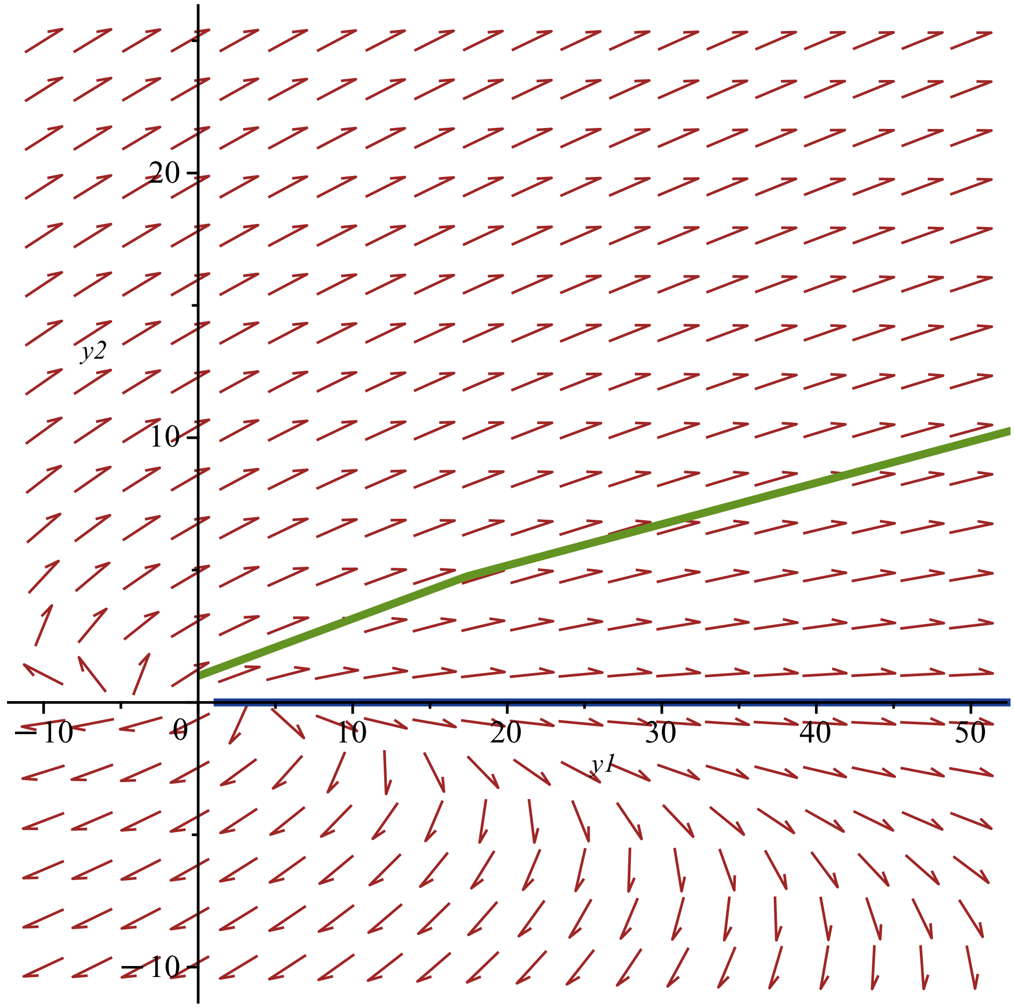
$i := 2: \; Traj; p2 := plot(Traj, t = -5..5, linestyle = dash):$

$$[t^2, t^2] \tag{20}$$

$display(p1, p2, title = \text{'Trajectories near a node'}, scaling = constrained);$



with(*DEtools*) :
DEplot([*sys1*, *sys2*], [*y1*(*t*), *y2*(*t*)], *t* = 0 ..25, *y1* = −10 ..50, *y2* = −10 ..25, *number* = 2, [[0, 1, 0], [0, 0, 1]]);



#Pr4.10 Electrical Network

restart;

node1 := D(*i1*)(*t*) = −3·*i1*(*t*) + 3·*i2*(*t*) + 24;

$$node1 := \mathrm{D}(i1)(t) = -3\ i1(t) + 3\ i2(t) + 24 \quad \textbf{(21)}$$

node2 := 8·*i2*(*t*) + 3(*i2*(*t*) − *i1*(*t*)) + 4·*int*(*i2*(*t*), *t*) = 0;

$$node2 := 11\ i2(t) - 3\ i1(t) + 4\left(\int i2(t)\,\mathrm{d}t\right) = 0 \quad \textbf{(22)}$$

nodef2 := *diff*(*node2*, *t*);

$$nodef2 := 11\ \frac{\mathrm{d}}{\mathrm{d}t}\ i2(t) - 3\ \frac{\mathrm{d}}{\mathrm{d}t}\ i1(t) + 4\ i2(t) = 0 \quad \textbf{(23)}$$

with(*LinearAlgebra*) :
A := *Matrix*([[−3, 3], [9, 5]]);

$$A := \begin{bmatrix} -3 & 3 \\ 9 & 5 \end{bmatrix} \quad \textbf{(24)}$$

Eigenvalues(*A*);

#this the lamda which is L1 and L2

$$\begin{bmatrix} 1+\sqrt{43} \\ 1-\sqrt{43} \end{bmatrix} \qquad (25)$$

$eig := Eigenvectors(A);$

$$eig := \begin{bmatrix} 1+\sqrt{43} \\ 1-\sqrt{43} \end{bmatrix}, \begin{bmatrix} \frac{3}{4+\sqrt{43}} & \frac{3}{4-\sqrt{43}} \\ 1 & 1 \end{bmatrix} \qquad (26)$$

$eig[1];$

$$\begin{bmatrix} 1+\sqrt{43} \\ 1-\sqrt{43} \end{bmatrix} \qquad (27)$$

$L1 := eig[1][1];$

$$L1 := 1+\sqrt{43} \qquad (28)$$

$x1 := eig[2][1];$

$$x1 := \begin{bmatrix} \frac{3}{4+\sqrt{43}} & \frac{3}{4-\sqrt{43}} \end{bmatrix} \qquad (29)$$

$L2 := eig[1][2];$

$$L2 := 1-\sqrt{43} \qquad (30)$$

$x2 := eig[2][2];$

$$x2 := \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad (31)$$

#general solution for the system $y=c1x1e^{L1t}+c2x2e^{L2t}$

$y := c1\cdot x1\cdot \exp(L1\cdot t) + c2\cdot x2\cdot \exp(L2\cdot t);$

$$y := \begin{bmatrix} \frac{3\,c1\,\mathrm{e}^{(1+\sqrt{43})\,t}}{4+\sqrt{43}} + c2\,\mathrm{e}^{(1-\sqrt{43})\,t} & \frac{3\,c1\,\mathrm{e}^{(1+\sqrt{43})\,t}}{4-\sqrt{43}} + c2\,\mathrm{e}^{(1-\sqrt{43})\,t} \end{bmatrix} \qquad (32)$$

#Pr4.14 Electrical Network

$restart;$

$\mathrm{mu} := \frac{1}{2};$

$$\mu := \frac{1}{2} \qquad (33)$$

$sys := \mathrm{D}(y1)(t) = y2(t),\ \mathrm{D}(y2)(t) = \mathrm{mu}\cdot\left(1 - y1(t)^2\right)\cdot y2(t) - y1(t);$

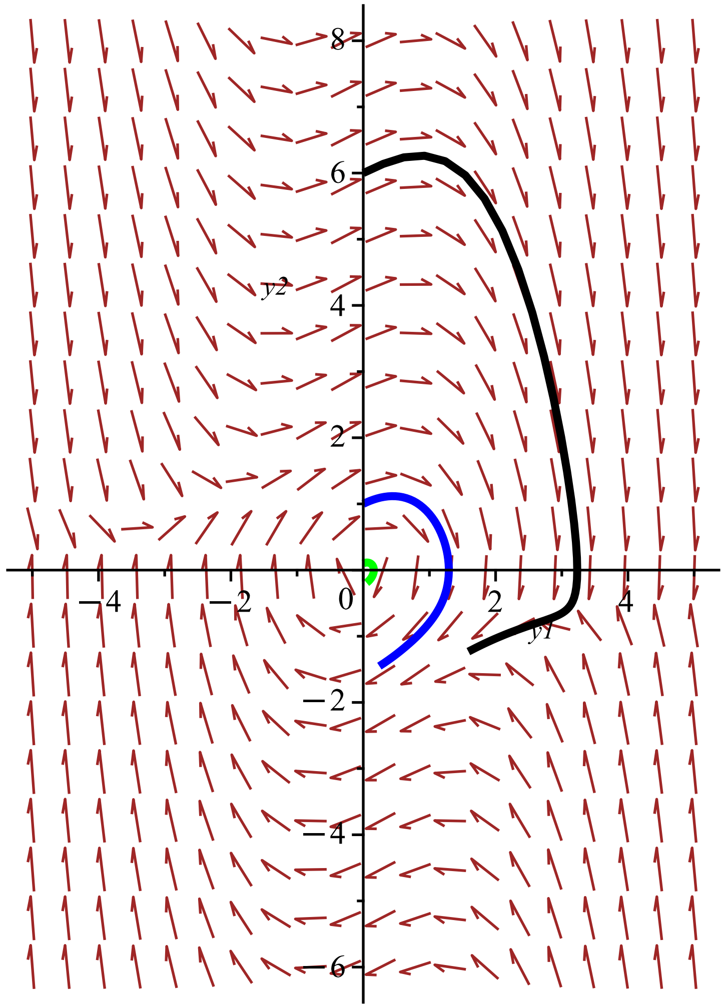
$$sys := \mathrm{D}(y1)(t) = y2(t),\ \mathrm{D}(y2)(t) = \frac{(1 - y1(t)^2)\, y2(t)}{2} - y1(t) \tag{34}$$

inits := [0, 0, 6], [0, 0, 1], [0, 0, 0.1];

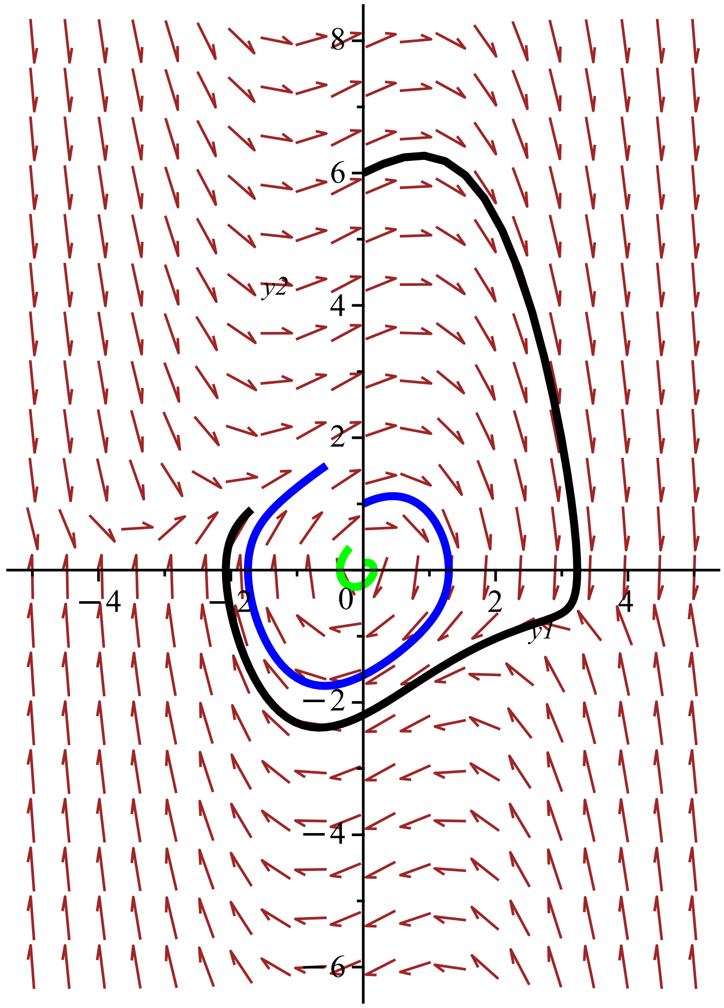
$$inits := [0, 0, 6], [0, 0, 1], [0, 0, 0.1] \tag{35}$$

with(*DEtools*) :

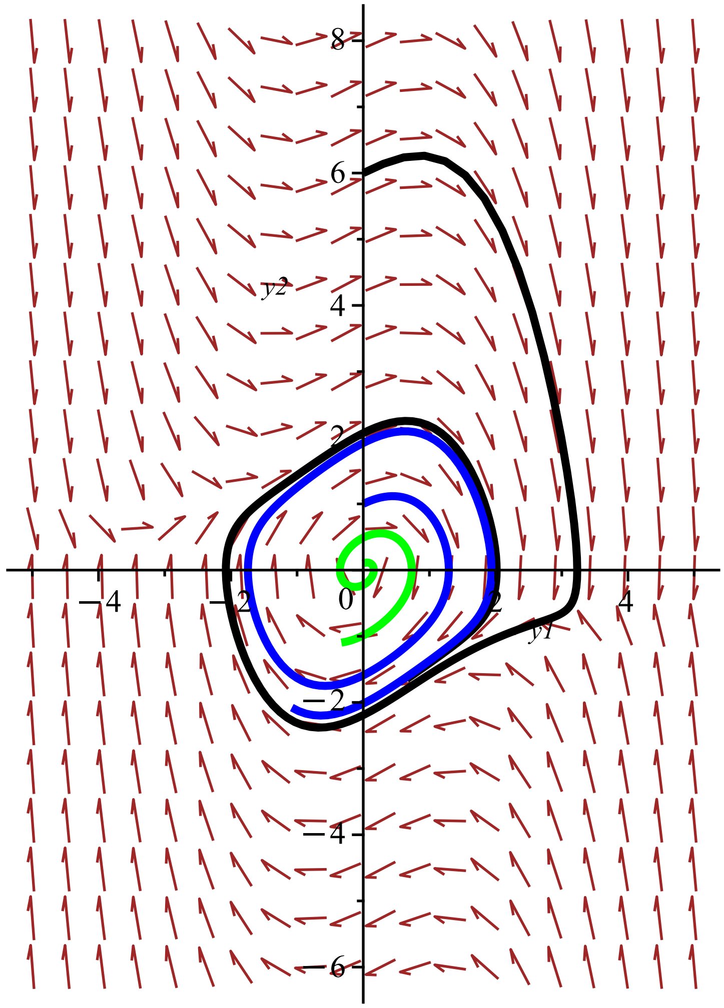
DEplot([*sys*[1], *sys*[2]], [*y1*(*t*), *y2*(*t*)], *t* = 0 ..3, *y1* = −5 ..5, *y2* = −6 ..8, [*inits*], *linecolor* = [*black*, *blue*, *green*], *scaling* = *constrained*, *stepsize* = 0.05);



DEplot([*sys*[1], *sys*[2]], [*y1*(*t*), *y2*(*t*)], *t* = 0 ..6, *y1* = −5 ..5, *y2* = −6 ..8, [*inits*], *linecolor* = [*black*, *blue*, *green*], *scaling* = *constrained*, *stepsize* = 0.05);



$DEplot([sys[1], sys[2]], [y1(t), y2(t)], t = 0..10, y1 = -5..5, y2 = -6..8, [inits], linecolor = [black, blue, green], scaling = constrained, stepsize = 0.05);$



at $u=\frac{1}{2}$ the spirals are forming faster than at $u = 2$

#Pr5.2 Power Series

restart;

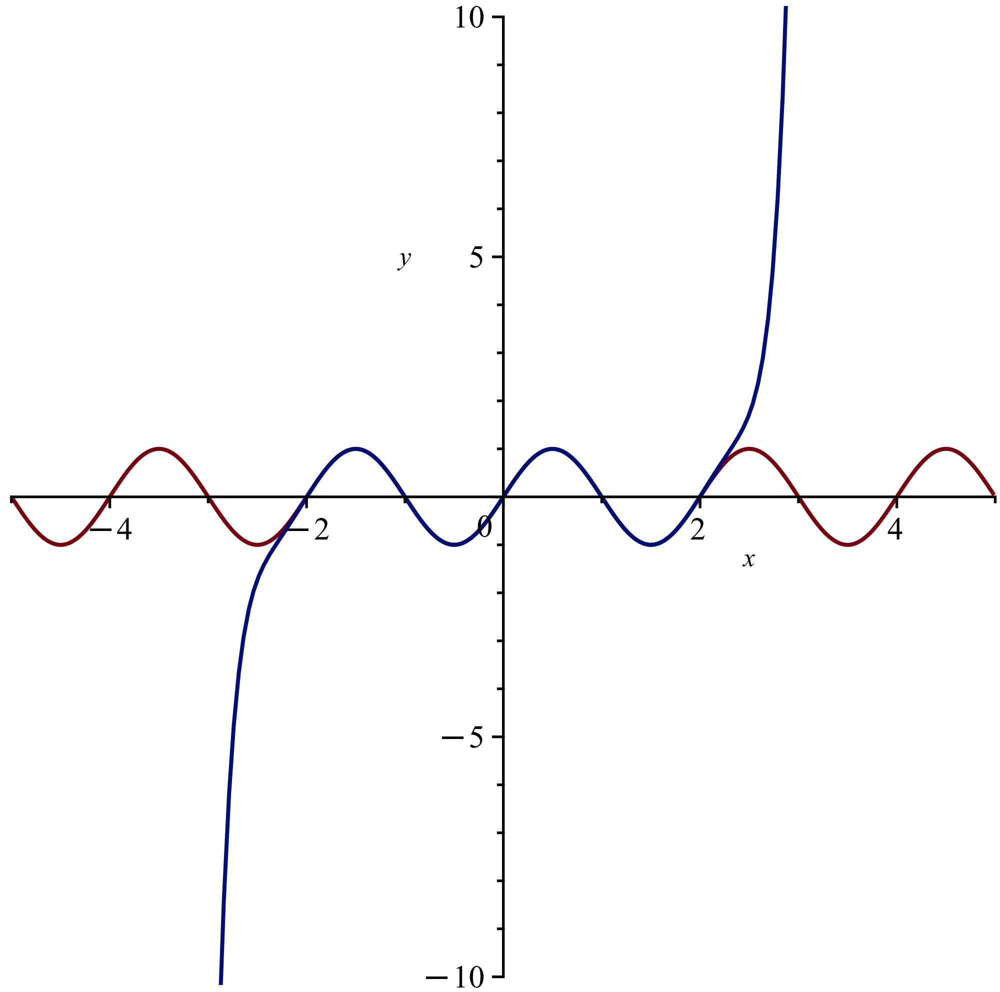
$f := \sin(\mathrm{Pi}\cdot x)$;

$$f := \sin(\pi x) \tag{36}$$

Maclaurin_series $:= \mathit{series}(f, x=0, 19)$;

$$\mathit{Maclaurin\_series} := \pi x - \frac{1}{6}\pi^3 x^3 + \frac{1}{120}\pi^5 x^5 - \frac{1}{5040}\pi^7 x^7 + \frac{1}{362880}\pi^9 x^9 - \frac{1}{39916800}\pi^{11} x^{11} + \frac{1}{6227020800}\pi^{13} x^{13} - \frac{1}{1307674368000}\pi^{15} x^{15} + \frac{1}{355687428096000}\pi^{17} x^{17} + O(x^{19}) \tag{37}$$

$\mathit{plot}([f, \mathit{Maclaurin\_series}], x=-5..5, y=-10..10)$;



#Pr5.8 Power Series

restart;
with(*orthopoly*) :

(38)

$P6 := P(6, x);$

$$P6 := -\frac{5}{16} + \frac{231}{16}x^6 - \frac{315}{16}x^4 + \frac{105}{16}x^2 \quad \textbf{(39)}$$

#Pr5.8 Frobenius method

restart;
$ODE := (x-1)^2 \cdot diff(y(x), x, x) + (x-1) \cdot diff(y(x), x) - 9 \cdot y(x) = 0;$

$$ODE := (x-1)^2 \left(\frac{d^2}{dx^2} y(x)\right) + (x-1)\left(\frac{d}{dx} y(x)\right) - 9\, y(x) = 0 \quad \textbf{(40)}$$

$dsolve(ODE);$

$$y(x) = \frac{c_1}{(x-1)^3} + c_2 (x-1)^3 \tag{41}$$

Series5 := *add*$\left(a[m]\cdot x^{(m+r-1)}, m=0..4\right)$;

$$Series5 := a_0 x^{-1+r} + a_1 x^r + a_2 x^{1+r} + a_3 x^{2+r} + a_4 x^{3+r} \tag{42}$$

NewSeriesODE := $(x-1)^2 \cdot diff(Series5, x, x) + (x-1)\cdot diff(Series5, x) - 9\cdot Series5$;

$$NewSeriesODE := (x-1)^2 \left(\frac{a_0 x^{-1+r} (-1+r)^2}{x^2} - \frac{a_0 x^{-1+r} (-1+r)}{x^2} + \frac{a_1 x^r r^2}{x^2} - \frac{a_1 x^r r}{x^2} + \frac{a_2 x^{1+r} (1+r)^2}{x^2} - \frac{a_2 x^{1+r} (1+r)}{x^2} + \frac{a_3 x^{2+r} (2+r)^2}{x^2} - \frac{a_3 x^{2+r} (2+r)}{x^2} + \frac{a_4 x^{3+r} (3+r)^2}{x^2} - \frac{a_4 x^{3+r} (3+r)}{x^2}\right) + (x-1)\left(\frac{a_0 x^{-1+r} (-1+r)}{x} + \frac{a_1 x^r r}{x} + \frac{a_2 x^{1+r} (1+r)}{x} + \frac{a_3 x^{2+r} (2+r)}{x} + \frac{a_4 x^{3+r} (3+r)}{x}\right) - 9 a_0 x^{-1+r} - 9 a_1 x^r - 9 a_2 x^{1+r} - 9 a_3 x^{2+r} - 9 a_4 x^{3+r} \tag{43}$$

#Pr5.16 Bessel's equation

restart;

ODE := $x^2 \cdot diff(y(x), x, x) + x\cdot diff(y(x), x) + \left(9\cdot x^6 - \frac{1}{9}\right)\cdot y(x) = 0$;

$$ODE := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + \left(\frac{d}{dx} y(x)\right) x + \left(9 x^6 - \frac{1}{9}\right) y(x) = 0 \tag{44}$$

sol := *dsolve*(*ODE*);

$$sol := y(x) = c_1 \operatorname{BesselJ}\left(\frac{1}{9}, x^3\right) + c_2 \operatorname{BesselY}\left(\frac{1}{9}, x^3\right) \tag{45}$$

subs(nu = 0, *ODE*) / *x*;

$$\frac{x^2 \left(\frac{d^2}{dx^2} y(x)\right) + \left(\frac{d}{dx} y(x)\right) x + \left(9 x^6 - \frac{1}{9}\right) y(x)}{x} = 0 \tag{46}$$

GenForm := *simplify*(%);

$$GenForm := \frac{x^2 \left(\frac{d^2}{dx^2} y(x)\right) + \left(\frac{d}{dx} y(x)\right) x + \left(9 x^6 - \frac{1}{9}\right) y(x)}{x} = 0 \tag{47}$$

subs(c_1 = 1, c_2 = 0, nu = *n*, *sol*);   *#general solution for the bessel equation @ Jn*

$$y(x) = \operatorname{BesselJ}\left(\frac{1}{9}, x^3\right) \tag{48}$$