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#EE597
#HW7

#Pr 10.2 (Path dependence, same endpoints)

restart;
 # Step1 Definne the Force vector F and the curve path r
$F := \langle 2\cdot z \mid 7\cdot x \mid -3\cdot y\rangle;\qquad r := \langle \cos(t) \mid \sin(t) \mid 2\cdot t\rangle;$

$$F := \begin{bmatrix} 2\,z & 7\,x & -3\,y \end{bmatrix}$$

$$r := \begin{bmatrix} \cos(t) & \sin(t) & 2\,t \end{bmatrix} \tag{1}$$

 # Step 2 define the the F on curve r as FC
$FC := eval(subs(x=r[1],\ y=r[2],\ z=r[3],\ \langle F[1] \mid F[2] \mid F[3]\rangle));$

$$FC := \begin{bmatrix} 4\,t & 7\cos(t) & -3\sin(t) \end{bmatrix} \tag{2}$$

 # Step3 define displacement along the curve r as r1
$r1 := VectorCalculus[diff](r,\ t);$

$$r1 := (-\sin(t))e_x + (\cos(t))e_y + (2)e_z \tag{3}$$

 # Step4 define the work equation (force · displacement) as the DotProduct of FC and r1
$with(LinearAlgebra):$
$work := DotProduct(FC,\ r1,\ conjugate=false);$

$$work := -4\,t\sin(t) + 7\cos(t)^2 - 6\sin(t) \tag{4}$$

 # Step 5 find the work done at the given cordinate from (1,0,0) to (1,0,4·Pi)
$Work\_Done := int(work,\ t=0..4\cdot\mathrm{Pi});$

$$Work\_Done := 30\,\pi \tag{5}$$

#Straight Line Method

$r\_new := \langle t \mid t \mid t\rangle;\quad r1\_new := VectorCalculus[diff](r\_new,\ t);$

$$r\_new := \begin{bmatrix} t & t & t \end{bmatrix}$$

$$r1\_new := (1)e_x + (1)e_y + (1)e_z \tag{6}$$

$FC\_new := eval(subs(x=r\_new[1],\ y=r\_new[2],\ z=r\_new[3],\ \langle F[1] \mid F[2] \mid F[3]\rangle));$

$$FC\_new := \begin{bmatrix} 2\,t & 7\,t & -3\,t \end{bmatrix} \tag{7}$$

$new\_work := int(DotProduct(FC\_new, r1\_new), t=0..4\cdot\mathrm{Pi});$

$$new\_work := 48\,\pi^2 \tag{8}$$

#Pr 10.4 (Independence of path)

restart;
$with(LinearAlgebra):$

$VectorCalculus[SetCoordinates]('cartesian'[x, y, z]);$

$$cartesian_{x, y, z} \tag{9}$$

Determine independent of path if F'[x,y,z] = f = $z \cdot e^x\, dx + 2 \cdot y\, dy + e^z\, dz$

$f := int(z \cdot \exp(x), x) + int(2 \cdot y, y) + int(\exp(z), z);$

$$f := z\,e^x + y^2 + e^z \tag{10}$$

$F := \langle diff(f, x) \mid diff(f, y) \mid diff(f, z)\rangle;$

$$F := \left[\begin{array}{ccc} z\,e^x & 2\,y & e^x + e^z \end{array}\right] \tag{11}$$

use either the culr of VectorField(F) or the gradient of f to determine independent of path

$VectorCalculus[Curl](VectorCalculus[VectorField](F));$

$$(0)\bar{e}_x + (0)\bar{e}_y + (0)\bar{e}_z \tag{12}$$

$gradf := VectorCalculus[Gradient](f);$

$$gradf := \left(z\,e^x\right)\bar{e}_x + (2\,y)\bar{e}_y + \left(e^x + e^z\right)\bar{e}_z \tag{13}$$

apply limits of integration to 'f' from [0,0,0] to [a,b,c]

$f\_value := subs(x = a, y = b, z = c, f) - subs(x = 0, y = 0, z = 0, f);$

$$f\_value := c\,e^a + b^2 + e^c - e^0 \tag{14}$$

#Pr 10.10 (Experiment on surface normal)

$restart;$

define the S : r(u, v) = [a cos v cos u, b cos v sin u, c sin v] where a=10, b=4, and c=3

$R := \langle a \cdot \cos(v) \cdot \cos(u) \mid b \cdot \cos(v) \cdot \sin(u) \mid c \cdot \sin(v)\rangle;$

$$R := \left[\begin{array}{ccc} a\cos(v)\cos(u) & b\cos(v)\sin(u) & c\sin(v) \end{array}\right] \tag{15}$$

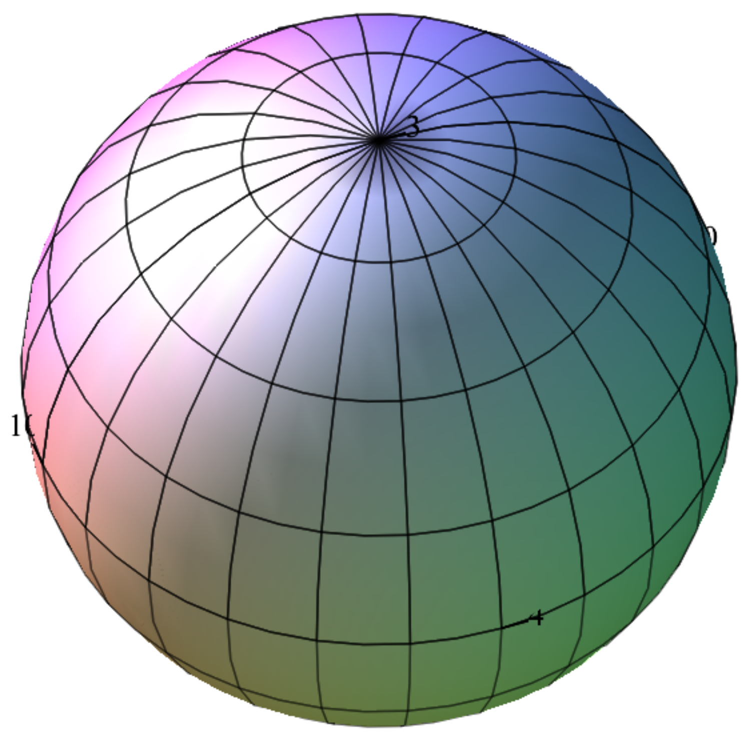
$r := subs(a = 10, b = 4, c = 3, R);$

$$r := \left[\begin{array}{ccc} 10\cos(v)\cos(u) & 4\cos(v)\sin(u) & 3\sin(v) \end{array}\right] \tag{16}$$

#r :=subs(a = 25, b = 15, c = 30, R);

$plot3d(\langle r[1] \mid r[2] \mid r[3]\rangle, u = 0..2 \cdot \text{Pi}, v = 0..3 \cdot \text{Pi}, axes = Normal, labels = [x, y, z],$
$orientation = [70, 40], title = \text{" Surface of ellipsoid"});$

Surface of ellipsoid



#Pr 10.16 (Surface integral. Divergence theorem)

restart;

$F := \langle 3 \cdot x \mid x^3 \cdot y^5 \mid y^3 \cdot z^4 \rangle;$

$$F := \begin{bmatrix} 3x & x^3 y^5 & y^3 z^4 \end{bmatrix} \quad \textbf{(17)}$$

$with(LinearAlgebra):$

$VectorCalculus[SetCoordinates]('cartesian'[x, y, z]):$

$divF := VectorCalculus[Divergence](VectorCalculus[VectorField](F));$

$$divF := 5\,x^3\,y^4 + 4\,y^3\,z^3 + 3 \tag{18}$$

 # determine the Integral of divF using these vertices [0,0,0], [0,1,0], [0,0,1]

$Integral := int(int(int(divF,\ x=0..1),\ y=0..1),\ z=0..1);$

$$Integral := \frac{7}{2} \tag{19}$$