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#EE597
#HW12

#Pr 19.1 (Quadratic equation)

restart;

$f := x^2 - 77 \cdot x + 0.1 = 0;$

$$f := x^2 - 77\,x + 0.1 = 0 \tag{1}$$

$Sol1 := evalf[4](\,solve(f, x)\,);$

$$Sol1 := 77.00, 0.001299 \tag{2}$$

$Sol2 := evalf[4](RootOf(f, x));$

$$Sol2 := 0.001299 \tag{3}$$

#Pr 19.4 (Fixed-point iteration)

restart;

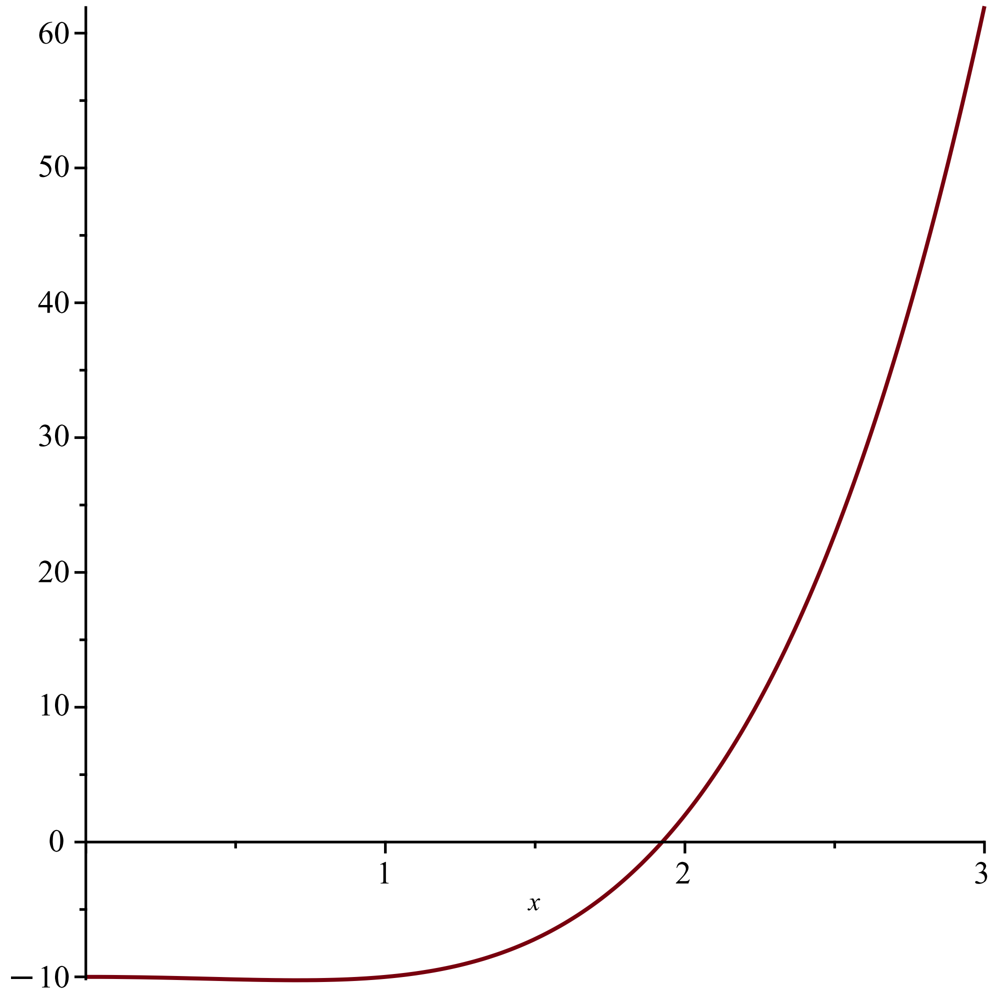
#defien the function f as $x^4 - x^2 - 10$

$f := x^4 - x^2 - 10;$

$$f := x^4 - x^2 - 10 \tag{4}$$

#sketch f as a plot f vs x

$plot(\,f, x = 0\,..3\,);$



$x(0) := 1;$ *#roughly*

$$x(0) := 1 \tag{5}$$

$g := x \rightarrow \mathrm{sqrt}(x^4 - 10);$

$$g := x \mapsto \sqrt{x^4 - 10} \tag{6}$$

$evalf(subs(x = 1, diff(g(x), x)));\quad evalf(subs(x = 1.346, diff(g(x), x)));$

$$-0.6666666666\,\mathrm{I}$$

$$-1.881721527\,\mathrm{I} \tag{7}$$

#create a loop to define x(n)

for n **from** 1 **to** 60 **do**

$x(n) := evalf[8](g(x(n-1))):$

end:

$seq(x(n), n = 0..60);$

$$1, 3.\,\mathrm{I}, 8.4261498, 70.929543, 5030.9991, 2.5310952 \times 10^{7}, 6.4064429 \times 10^{14}, 4.1042511 \times 10^{29}, 1.6844877 \times 10^{59}, 2.8374988 \times 10^{118}, 8.0513994 \times 10^{236}, 6.4825032 \times 10^{473}, 4.2022847 \times 10^{947}, 1.7659197 \times 10^{1895}, 3.1184724 \times 10^{3790}, 9.7248701 \times 10^{7580}, 9.4573099 \times 10^{15161}, 8.9440711 \times 10^{30323}, 7.9996408 \times 10^{60647}, 6.3994253 \times 10^{121295}, 4.0952645 \times 10^{242591}, 1.6771191 \tag{8}$$

$\times 10^{485183}, 2.8127285 \times 10^{970366}, 7.9114416 \times 10^{1940732}, 6.2590908 \times 10^{3881465}, 3.9176217$
$\times 10^{7762931}, 1.5347760 \times 10^{15525863}, 2.3555374 \times 10^{31051726}, 5.5485565 \times 10^{62103452}, 3.0786479$
$\times 10^{124206905}, 9.4780729 \times 10^{248413810}, 8.9833866 \times 10^{496827621}, 8.0701235 \times 10^{993655243},$
$6.5126893 \times 10^{1987310487}, 4.2415122 \times 10^{3974620975}, 1.7990426 \times 10^{7949241951}, 3.2365543$
$\times 10^{15898483902}, 1.0475284 \times 10^{31796967805}, 1.0973158 \times 10^{63593935610}, 1.2041019 \times 10^{127187871220},$
$1.4498614 \times 10^{254375742440}, 2.1020981 \times 10^{508751484880}, 4.4188165 \times 10^{1017502969760}, 1.9525939$
$\times 10^{2035005939521}, 3.8126230 \times 10^{4070011879042}, 1.4536094 \times 10^{8140023758085}, 2.1129803$
$\times 10^{16280047516170}, 4.4646858 \times 10^{32560095032340}, 1.9933419 \times 10^{65120190064681}, 3.9734119$
$\times 10^{130240380129362}, 1.5788002 \times 10^{260480760258725}, 2.4926101 \times 10^{520961520517450}, 6.2131051$
$\times 10^{1041923041034900}, 3.8602675 \times 10^{2083846082069801}, 1.4901665 \times 10^{4167692164139603}, 2.2205962$
$\times 10^{8335384328279206}, 4.9310475 \times 10^{16670768656558412}, 2.4315229 \times 10^{33341537313116825}, 5.9123036$
$\times 10^{66683074626233650}, 3.4955334 \times 10^{133366149252467301}, 1.2218754 \times 10^{266732298504934603}$

$evalf(solve(f, x));$

$$1.643642941\,I, -1.643642941\,I, 1.923944416, -1.923944416 \tag{9}$$

#Pr 19.5 (Newton's iteration method)

$restart;$

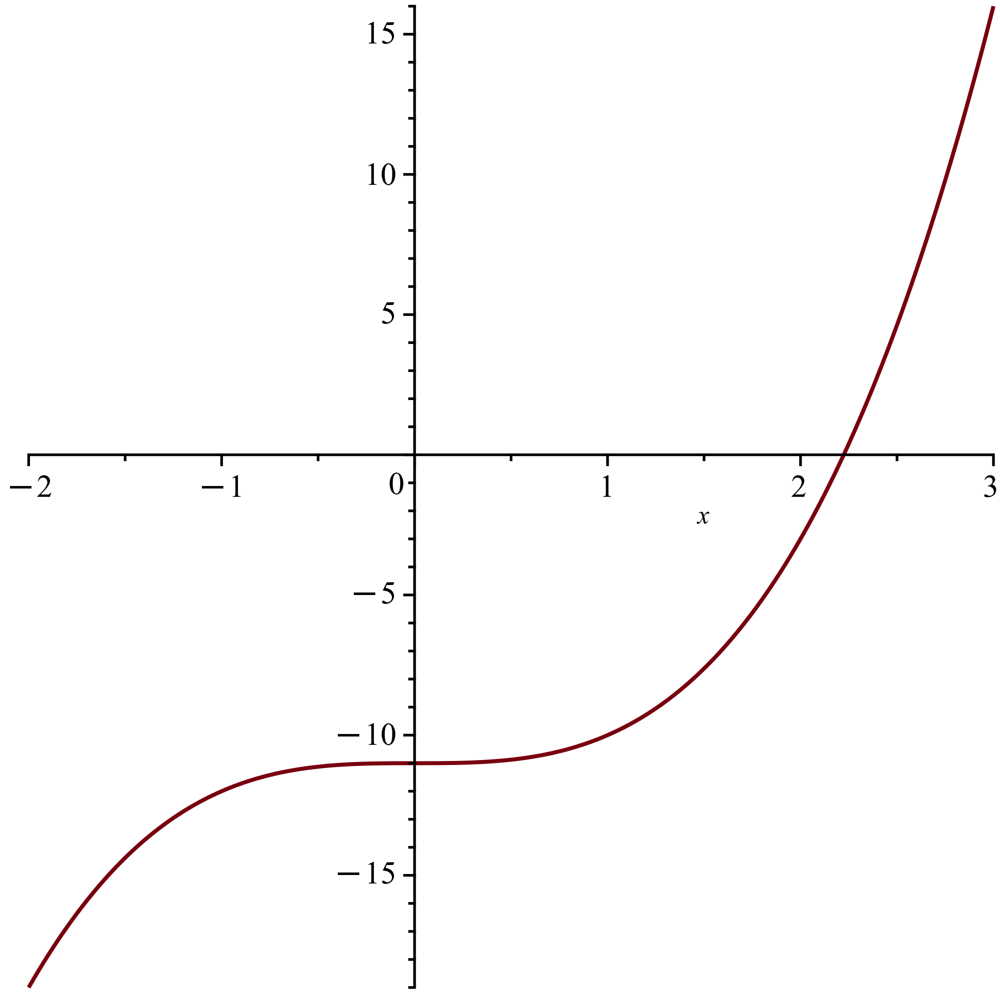
$f := x^3 - 11;$

$$f := x^3 - 11 \tag{10}$$

$df := diff(f, x);$

$$df := 3\,x^2 \tag{11}$$

$plot(f, x = -2\,..3);$



#From the graph p0 is roughly 2.25

$p0 := 2.25$

$$p0 := 2.25 \quad (12)$$

#using the Newton function from the code editor

$NEWTON(f, df, p0, 10^{-7}, 20);$

```
P(2) = 2.224280
P(3) = 2.223980
P(4) = 2.223980
```

$$2.223980091 \quad (13)$$

#Pr 19.7 (Polynomial interpolation)

restart;

$u := [1.00000, 0.97844, 0.96874, 0.95973];$

$$u := [1.00000, 0.97844, 0.96874, 0.95973] \quad (14)$$

$v := [1.00, 1.04, 1.06, 1.08];$

$$v := [1.00, 1.04, 1.06, 1.08] \quad (15)$$

$p := interp(u, v, x);$

$$p := -45.47491505\, x^3 + 140.6308262\, x^2 - 146.5803734\, x + 52.42446228 \quad (16)$$

Define the position of x at which p should be obtain
$X := [1.02, 1.04];$

$$X := [1.02, 1.04] \tag{17}$$

$P1 := evalf[7](subs(x = X[1], p));$

$$P1 := 0.96641 \tag{18}$$

$P2 := evalf[7](subs(x = X[2], p));$

$$P2 := 0.93406 \tag{19}$$

#Pr.20.2 (Gauss elimination)

restart;
with(LinearAlgebra) :
$A := Matrix([[4, 4, 2], [3, -1, 2], [3, 7, 1]]);$

$$A := \begin{bmatrix} 4 & 4 & 2 \\ 3 & -1 & 2 \\ 3 & 7 & 1 \end{bmatrix} \tag{20}$$

$b := \langle 0, 0, 0 \rangle;$

$$b := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

$A1 := \langle A \mid b \rangle;$

$$A1 := \begin{bmatrix} 4 & 4 & 2 & 0 \\ 3 & -1 & 2 & 0 \\ 3 & 7 & 1 & 0 \end{bmatrix} \tag{22}$$

$x := LinearSolve(A, b);$

$$x := \begin{bmatrix} -5\,\_t_2 \\ \_t_2 \\ 8\,\_t_2 \end{bmatrix} \tag{23}$$

with(Student[NumericalAnalysis]) :
#join A and b to form one Matrix A1
$B := GaussianElimination(A1);$

$$B := \begin{bmatrix} 4 & 4 & 2 & 0 \\ 0 & -4 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

#Pr.20.3 (Doolittle factorization)

restart;
with(*LinearAlgebra*) :
$A := Matrix([[2, 5, 2], [6, 16, 7], [10, 32, 21]]);$

$$A := \begin{bmatrix} 2 & 5 & 2 \\ 6 & 16 & 7 \\ 10 & 32 & 21 \end{bmatrix} \tag{25}$$

$b := \langle 5.1, 7.3, 15.7\rangle;$

$$b := \begin{bmatrix} 5.1 \\ 7.3 \\ 15.7 \end{bmatrix} \tag{26}$$

#define p,L,and U

$(p, L, U) := LUDecomposition(A);$

$$p, L, U := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 7 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 5 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix} \tag{27}$$

#define using Ly=b
$y := LinearSolve(L, b);$

$$y := \begin{bmatrix} 5.10000000000000 \\ -8.00000000000000 \\ 46.2000000000000 \end{bmatrix} \tag{28}$$

#define using Ux=y
$x := LinearSolve(U, y);$

$$x := \begin{bmatrix} 39.8750000000000 \\ -19.5500000000000 \\ 11.5500000000000 \end{bmatrix} \tag{29}$$

#Pr.20.6 (Gauss-Jordan elimination)

restart;
with(*LinearAlgebra*) :
$A := Matrix([[16, 9, 8], [9, 16, 12], [8, 12, 13]]);$

$$A := \begin{bmatrix} 16 & 9 & 8 \\ 9 & 16 & 12 \\ 8 & 12 & 13 \end{bmatrix} \tag{30}$$

$b := \langle 12.3, 13.6, 20.7\rangle;$

$$b := \begin{bmatrix} 12.3 \\ 13.6 \\ 20.7 \end{bmatrix} \tag{31}$$

$A1 := \langle A \mid b \rangle;$

$$A1 := \begin{bmatrix} 16 & 9 & 8 & 12.3 \\ 9 & 16 & 12 & 13.6 \\ 8 & 12 & 13 & 20.7 \end{bmatrix} \tag{32}$$

$B := ReducedRowEchelonForm(A1);$

$$B := \begin{bmatrix} 1. & 0. & 0. & 0.129777777777778 \\ 0. & 1. & 0. & -1.16133333333333 \\ 0. & 0. & 1. & 2.58444444444444 \end{bmatrix} \tag{33}$$

$Ans := \langle B[1,4], B[2,4], B[3,4] \rangle$ *#the answer is the last column of the matrix B*

$$Ans := \begin{bmatrix} 0.129777777777778 \\ -1.16133333333333 \\ 2.58444444444444 \end{bmatrix} \tag{34}$$

#Pr.20.14 (Least squares, straight line)

restart;
with(*CurveFitting*) :
data := [[300, 400, 500, 600, 700, 750], [470, 580, 1030, 1420, 1880, 2000]] :
$p1 := LeastSquares(data[1], data[2], x);$

$$p1 := -\frac{55420}{73} + \frac{6702\,x}{1825} \tag{35}$$

$p2 := LeastSquares(data[1], data[2], x, curve = a \cdot x^2 + b \cdot x + c);$

$$p2 := -\frac{193310}{3489} + \frac{43349}{58150}\,x + \frac{193}{69780}\,x^2 \tag{36}$$

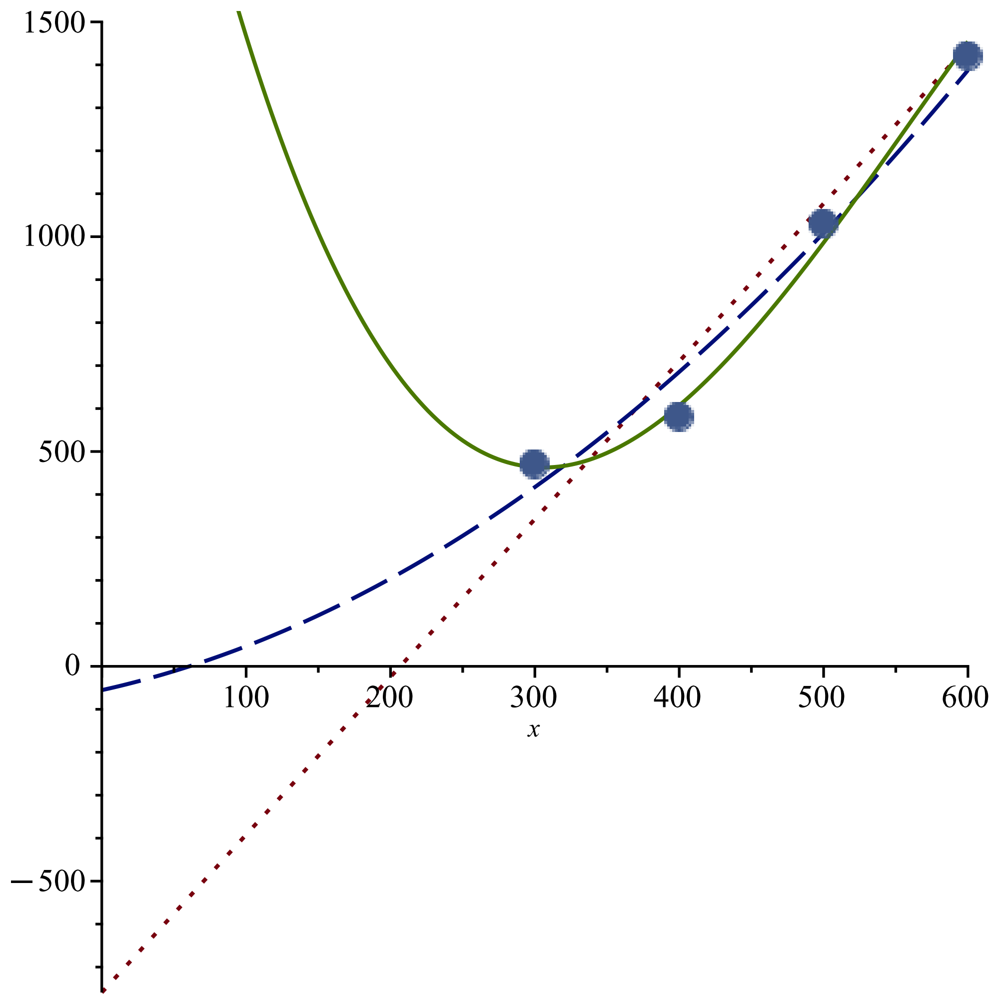
$p3 := LeastSquares(data[1], data[2], x, curve = a \cdot x^3 + b \cdot x^2 + c \cdot x + d);$

$$p3 := \frac{345820870}{118999} - \frac{65148289}{3569970}\,x + \frac{487877}{11899900}\,x^2 - \frac{108709}{4462462500}\,x^3 \tag{37}$$

$points := [seq([data[1,j], data[2,j]], j = 1..4)];$

$$points := [[300, 470], [400, 580], [500, 1030], [600, 1420]] \tag{38}$$

$P1 := plot(p1, x = 0..600, linestyle = dot) :$
$P2 := plot(p2, x = 0..600, linestyle = dash) :$
$P3 := plot(p3, x = 0..600, y = 250..1500) :$
$P4 := plot(points, x = 0..600, style = point, symbol = solidcircle, symbolsize = 20) :$
with(*plots*) :
display(*P1*, *P2*, *P3*, *P4*,);



#Pr.20.20 (QR-factorization)

restart;
with(*Student*[*LinearAlgebra*]) :
$A := Matrix([[14.2, -0.1, 0.0], [-0.1, -6.3, 0.2], [0.0, 0.2, 2.1]]);$

$$A := \begin{bmatrix} 14.2 & -0.1 & 0. \\ -0.1 & -6.3 & 0.2 \\ 0. & 0.2 & 2.1 \end{bmatrix} \tag{39}$$

for s **from** 1 **to** 5 **do**
$(Q, R) := QRDecomposition(A)$:
$A := R.\,Q$;
end;

$$Q, R := \begin{bmatrix} -0.999975205 & -0.007038533665 & 0.0002234260788 \\ 0.007042078909 & -0.9994717797 & 0.03172650316 \\ -0. & 0.03172728986 & 0.9994965628 \end{bmatrix},$$

$$\begin{bmatrix} -14.20035210 & 0.0556324234 & 0.001408415782 \\ 0. & 6.303721524 & -0.1332670473 \\ 0. & 0. & 2.105288083 \end{bmatrix}$$

$$A := \begin{bmatrix} 14.2003917701852 & 0.0443913043018404 & 2.61848637390927 \times 10^{-12} \\ 0.0443913043923697 & -6.30461997256395 & 0.0667950851400910 \\ 0. & 0.0667950852481447 & 2.10422820266230 \end{bmatrix}$$

$$Q, R := \begin{bmatrix} -0.999995114 & 0.003125871255 & -0.00003311686706 \\ -0.003126046677 & -0.9999389982 & 0.01059379743 \\ -0. & 0.01059384919 & 0.9999438836 \end{bmatrix},$$

$$\begin{bmatrix} -14.20046115 & -0.02468255109 & -0.0002088045565 \\ 0. & 6.305081752 & -0.04449913423 \\ 0. & 0. & 2.104817735 \end{bmatrix}$$

$$A := \begin{bmatrix} 14.2004689253536 & -0.0197099799505561 & -1.34782658124666 \times 10^{-12} \\ -0.0197099798590529 & -6.30516854778110 & 0.0222981217614936 \\ 0. & 0.0222981216570274 & 2.10469962020606 \end{bmatrix}$$

$$Q, R := \begin{bmatrix} -0.999999037 & -0.001387970899 & 4.908518981 \times 10^{-6} \\ 0.001387979578 & -0.9999927838 & 0.003536445587 \\ -0. & 0.003536448993 & 0.9999937467 \end{bmatrix},$$

$$\begin{bmatrix} -14.20048261 & 0.01095851578 & 0.00003094933899 \\ 0. & 6.305229258 & -0.01485479800 \\ 0. & 0. & 2.104765315 \end{bmatrix}$$

$$A := \begin{bmatrix} 14.2004841451314 & 0.00875152936403584 & 1.79420393879538 \times 10^{-12} \\ 0.00875152944471209 & -6.30523629144006 & 0.00744339507598572 \\ 0. & 0.00744339517873308 & 2.10475215327106 \end{bmatrix}$$

$$Q, R := \begin{bmatrix} -0.999999810 & 0.0006162833315 & -7.275282183 \times 10^{-7} \\ -0.0006162837611 & -0.9999991131 & 0.001180508276 \\ -0. & 0.001180508500 & 0.9999993032 \end{bmatrix},$$

$$\begin{bmatrix} -14.20048685 & -0.004865712966 & -4.587245306 \times 10^{-6} \\ 0. & 6.305244882 & -0.004958710668 \\ 0. & 0. & 2.104759473 \end{bmatrix}$$

$$A := \begin{bmatrix} 14.2004871505674 & -0.00388582010952285 & -1.63763789702754 \times 10^{-12} \\ -0.00388582003053549 & -6.30524514367841 & 0.00248468655263724 \\ 0. & 0.00248468644833202 & 2.10475800640360 \end{bmatrix}$$

$$Q, R := \begin{bmatrix} -0.999999963 & -0.0002736398791 & 1.078323196 \times 10^{-7} \\ 0.0002736399001 & -0.9999998856 & 0.0003940664922 \\ -0. & 0.0003940665070 & 0.9999999224 \end{bmatrix},$$

$$\begin{bmatrix} -14.20048768 & 0.002160453314 & 6.799110178 \times 10^{-7} \\ 0. & 6.305246457 & -0.001655271633 \\ 0. & 0. & 2.104758822 \end{bmatrix}$$

$$A := \begin{bmatrix} 14.2004877457682 & 0.00172536693300226 & 1.69806312768999 \times 10^{-12} \\ 0.00172536701059936 & -6.30524638796692 & 0.000829414849215547 \\ 0. & 0.000829414957062975 & 2.10475865867072 \end{bmatrix} \quad \textbf{(40)}$$

Eigenvalues(A);

$$\begin{bmatrix} 14.20048790 \\ -6.305246613 \\ 2.104758741 \end{bmatrix} \quad \textbf{(41)}$$

#Pr.21.10 (Mass-spring system)

restart;

#step 1 change the second order to a first order system y''+2y'+0.75y
#y[1]=y, y[2]=y' therefore y''=y'[2]=-2y[2]-0.75y[1]

$f := (x, y) \rightarrow [y[2], -2 \cdot y[2] - 0.75 \cdot y[1]]$;

$$f := (x, y) \mapsto [y_2, -2 \cdot y_2 - 0.75 \cdot y_1] \quad \textbf{(42)}$$

#define ics as y[0]=[y(0),y'(0)]
$y[0] := [3, -2.5]$;

$$y_0 := [3, -2.5] \quad \textbf{(43)}$$

#define the input of the RKS function
$x[0] := 0 : h := 0.2 : N := 5 :$

#call the RKS function
RKS(f, x, y, h, N) :

#display result

printf(" x   y\n");

```
for n from 0 to N do
        printf("%4.2f %12.8f\n", x[n], Vector(y[n]));
end;
```

```
 x             y
0.00   3.00000000  -2.50000000
0.20   2.55051250  -2.01609375
0.40   2.18630201  -1.64199120
0.60   1.88823825  -1.35072053
0.80   1.64186615  -1.12215864
1.00   1.43622107  -0.94126973
```

#Pr.21.12 (Runge-Kutta-Nystroem method)

restart;

#Step 1 find the system equation

$ODE := diff(y(x), x, x) = -2 \cdot diff(y(x), x) - 0.75 \cdot y(x);$

$$ODE := \frac{d^2}{dx^2}\, y(x) = -2\, \frac{d}{dx}\, y(x) - 0.75\, y(x) \tag{44}$$

$ics := y(0) = 3, D(y)(0) = -2.5$

$$ics := y(0) = 3, D(y)(0) = -2.5 \tag{45}$$

$sys := dsolve(\{ics, ODE\})$

$$sys := y(x) = e^{-\frac{3x}{2}} + 2\, e^{-\frac{x}{2}} \tag{46}$$

#Step2 define the function from Pr21.10 where $y[2] = y'$ **and** $y[1] = y$

$f := (x, y, yp) \rightarrow -2 \cdot D(y)(x) - 0.75 \cdot y(x);$

$$f := (x, y, yp) \rightarrow -2\, D(y)(x) - 0.75\, y(x) \tag{47}$$

#define ics

$x(0) := 0 : \; y(0) := 3 : \; yp(0) := -2.5 :$

#define the input of the RKN

$h := 0.2 : \quad N := 5 :$

#call the RKN function

$RKN(f, x, y, yp, h, N) :$

```
printf("x              y              err  y\n");
for n from 0 to N do
    printf("%4.2f      %10.6f      %11.8f\n", x(n), y(n),
        evalf(evalf(exp(-3/2·x(n)) + 2·exp(-x(n)/2))) - y(n));
end;
```

x	y	err y
0.00	3.000000	0.00000000
0.20	2.457612	0.09288056
0.40	1.841681	0.34459192
0.60	1.170638	0.71756823
0.80	0.464563	1.17727095
1.00	-0.255413	1.69160459

 # Note the y-error is getting wider as n increases