3.5(b)

Wednesday, September 27, 2023 11:14 PM

b) $x_1(t)$ has Laplace Transform $X_1(s) = \dfrac{s+2}{(s+2)^2+1}$

$$X_1(s) = \frac{N(s)}{D(s)}$$

for zero $N(s) = 0 = s+2$

$s+2=0$

$\boxed{s=-2 \quad \text{OR} \quad s+2=0}$ zero

for poles $D(s) = 0 = (s+2)^2+1$

$(s+2)^2+1=0$

$(s+2)^2 = -1$

$s+2 = \pm\sqrt{-1}$

$s = -2 \pm \sqrt{-1}$

$s = -2 \pm j$

$\boxed{\begin{array}{l} \therefore\ s=-2+j \ ,\ s=-2-j \\ s+2-j=0 \ ,\ s+2+j=0 \end{array}}$ poles

3.5©

Thursday, September 28, 2023 7:18 PM

1) $z(t) = \frac{d}{dt} e^{-t} u(t)$

let $a = e^{-t}$ & $b = u(t)$

Apply product Rule

$z'(t) = a'b + ab'$

$z'(t) = u(t) \frac{d}{dt} e^{-t} + e^{-t} \frac{d}{dt} u(t)$

$z'(t) = u(t) \left[-e^{-t} \right] + e^{-t} \left[\delta(t) \right]$

$z'(t) = -e^{-t} u(t) + \delta(t) e^{-t}$

$z'(t) = e^{-t} \delta(t) - e^{-t} u(t)$

$Z(s) = 1 - \frac{1}{s+1}$

$Z(s) = \frac{s+1-1}{s+1}$

$$\boxed{Z(s) = \frac{s}{s+1}}$$

3.13a

Saturday, September 30, 2023 3:46 PM

$$H(s) = \frac{s^2+4}{s((s+1)^2+1)}$$

BIBO stable ⇒ Bounded input of Bounded output

⇒ One condition is to have all poles of H(s) to have real negative real parts

$$s[(s+1)^2+1] = 0$$

$$s(s^2+2s+2) = 0 \Rightarrow \begin{matrix} s=0 \\ s^2+2s+2=0 \end{matrix}$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm j2}{2} = -1+j$$

$$s = \begin{cases} 0 \Rightarrow H(s) \text{ is undefine} \\ -1 \pm j \Rightarrow H(s) \text{ is BIBO stable} \end{cases}$$

$\therefore$ System is not BIBO stable because one of the poles is zero

3.17a

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$$H(s) = \frac{1}{s^2+4} = \frac{Y(s)}{X(s)}$$

$$X(s) = \frac{Y_s}{H_s} = \frac{Y_s}{\frac{1}{s^2+4}} = Y(s)\left(s^2+4\right)$$

$$X(s) = Y(s)\left(s^2+4\right) = s^2 Y(s) + 4\,Y(s)$$

$$X(s) = s^2 Y(s) + 4\,Y(s)$$

note $s^n\left(F(s)\right) = \frac{d^n}{dt^n} f(t)$

$$\boxed{x(t) = \frac{d^2}{dt^2} y(t) + 4\,y(t)}$$

3.17b

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$$x(t) = \frac{d^2}{dt^2} y(t) + 4y(t)$$

$$x(t) = \frac{d^2}{dt^2} y(0) + 4y(0)$$

y $y(0) = 0$

$$x(t) = \frac{d^2}{dt^2} y(0) + 4(0)$$

$$x(t) = \frac{d^2}{dt^2} y(0)$$ but $\frac{d}{dt} y(0) = 1$

$$x(t) = \frac{d^2}{dt^2}(1)$$

Relation $r(t)$ & $u(t)$ & $\delta(t)$

$$\frac{d\,r(t)}{dt} = u(t)$$

$$\frac{d^2 r(t)}{dt} = \delta(t)$$

$$x(t) = \delta(t)$$