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% Name: Lamin Jammeh
% Class: EE480 Online
% Semester: Fall 2023
% HW_5

% Basic Problems
%% ***** 3.5b *****
%finding x1(t)
clear;
clc;
syms t s;
X = (s+2)/(1+(2+s^2));
x = ilaplace(X)

%% ***** question 3.5(c) *****
clear;
clc;
syms t;
z = diff(exp(-t)*heaviside(t))
Z = laplace(z)
Z_simplify = simplifyFraction(Z)

%% ***** question 3.8 *****
clear;
clc;
% ***** Part a *****
% Step 1 define the syms variables
syms t s;
% Step 2 define the x(t)
x = heaviside(t) - 2*heaviside(t-1) + 2*heaviside(t-3) - heaviside(t-4);
% Step 3 plot x(t)
subplot(3,1,1);
fplot(x,[0,10],'r','LineWidth',2);
xlabel('Time (sec)');
ylabel('x(t)');
title('x(t) vs Time');
grid on

% Step 4 differentiate x
x_diff1 = diff(x);
subplot(3,1,2);
fplot(x_diff1,[0,10],'b','LineWidth',2);
xlabel('Time (sec)');
ylabel('d/dt(x(t))');
title('d/dt(x(t)) vs Time');
grid on

% Step 4 differentiate x_diff1
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x_diff2 = diff(x_diff1);
subplot(3,1,3);
fplot(x_diff2,[0,10], 'g', 'LineWidth',2);
xlabel('Time (sec)');
ylabel("d2/dt2(x(t))");
title("d2/dt2(x(t)) vs Time");
grid on

% ***** Part b *****
X_s = laplace(x_diff2) % define Laplace transform of d2/dt2 (x(t))

%% ***** 3.9(a) *****
clear;
clc;
% Step 1 define Syms function as s
syms s;
% define Y(s)
Y = [exp(-2*s)/(s^2 +1)] + [(s+2)^2+2)/(s+2)^3];
y = ilaplace(Y);

%% ***** 3.9(c) *****
% finding the steady state and transient given the out laplace transform
% Y(s)

%find the steady state laplace transform
clear;
clc;
syms t s;
%define the output lapce transform Y(s)
Y = 1/(s*((s+1)^(2) + 4));
%define the inverse laplace transform of Y(s)
y = ilaplace(Y,s,t);
%define the steady steady using the lim as t approaches inf
Y_ss = limit(y,t,inf);

%find the transient by subtracting output from steady state response
Y_transient = y - Y_ss;

%% ***** 3.13b *****
clear;
clc;
syms t s;
%define the H(s)
H = (s^2 + 4)/(s*((s+1)^2 + 1));
%define input x(t)
x = cos(2*t)*heaviside(t);
%define X(s)
X = laplace(x);
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% Y(s)=H(s)*X(s)
Y = H * X;
y = ilaplace(Y);
%define steady state by using the lim as t approaches inf
y_ss = limit(y,t,inf);

%% ***** 3.13c *****
clear;
clc;
syms t s;
%define the H(s)
H = (s^2 + 4)/(s*((s+1)^2 + 1));
%define input x(t)
x = sin(2*t)*heaviside(t);
%define X(s)
X = laplace(x);
% Y(s)=H(s)*X(s)
Y = H * X;
y = ilaplace(Y);
%define steady state by using the lim as t approaches inf
y_ss = limit(y,t,inf);

%% ***** 3.13d *****
clear;
clc;
syms t s;
%define the H(s)
H = (s^2 + 4)/(s*((s+1)^2 + 1));
%define input x(t)
x = heaviside(t);
%define X(s)
X = laplace(x);
% Y(s)=H(s)*X(s)
Y = H * X;
y = ilaplace(Y);
%define steady state by using the lim as t approaches inf
y_ss = limit(y,t,inf);

%% ***** 3.15a *****
clear;
clc;
% Note the H(s) =output(s) / input(s)
syms t s;

% Step find the output laplace S(s)
s_t = (0.5-exp(-t) + 0.5*exp(-2*t))*heaviside(t); % ouput in time domian
S = laplace(s_t);

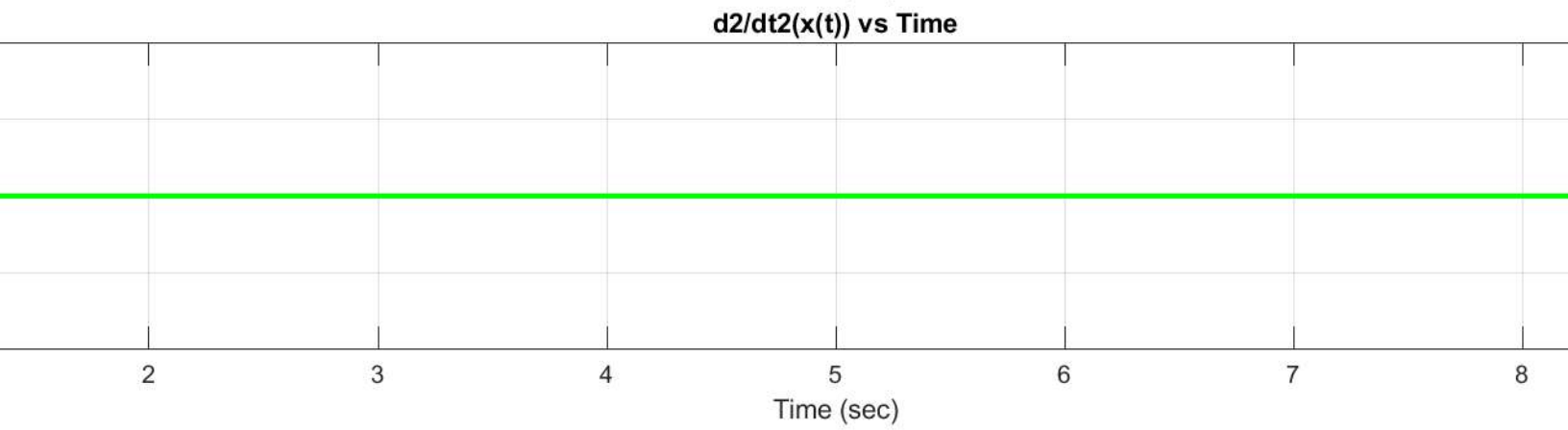
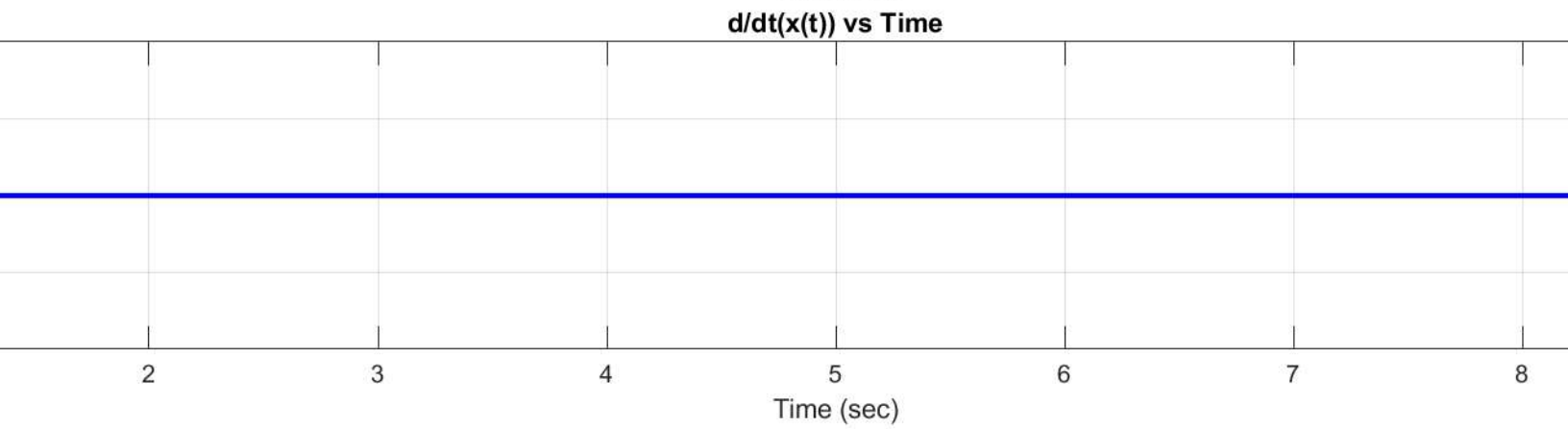
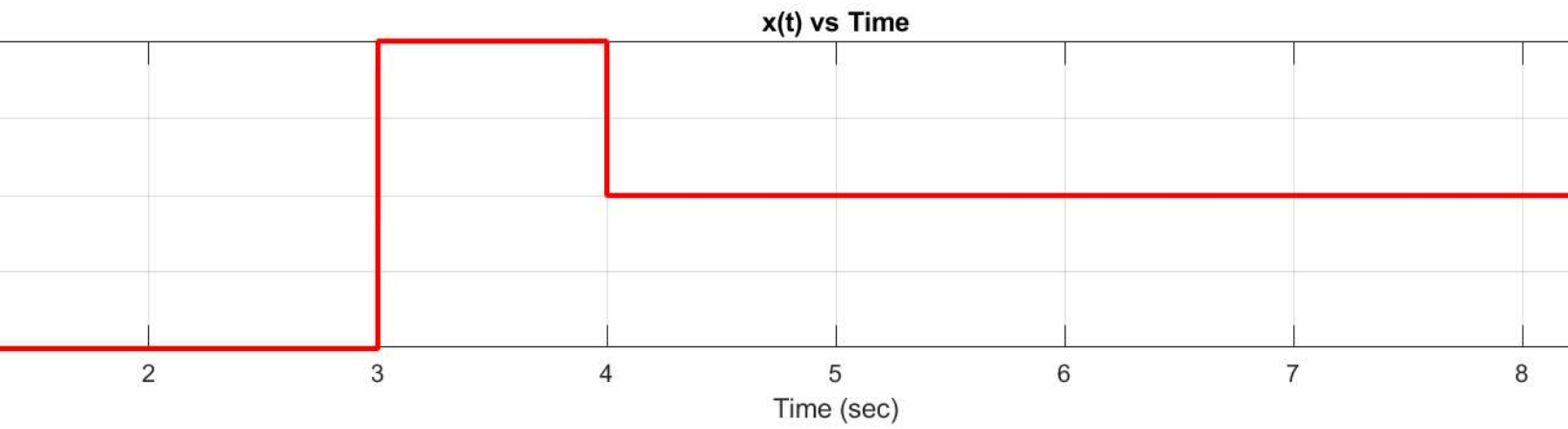
% Step 2 find the input U(s)
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u_t = heaviside(t);  
U = laplace(u_t);
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% Step 3 find  $H(s)=S(s)/U(s)$ 
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H = S / U;
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H_simplify = simplifyFraction(H); % simplified fraction of  $H(s)$ 
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3.5(b)

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b) $x_1(t)$ has Laplace Transform $X_1(s) = \frac{s+2}{(s+2)^2+1}$

$$X_1(s) = \frac{N(s)}{D(s)}$$

for zero $N(s) = 0 = s+2$

$$s+2=0$$

$$s=-2 \text{ OR } s+2=0$$

zero

for poles $D(s) = 0 = (s+2)^2+1$

$$(s+2)^2+1=0$$

$$(s+2)^2 = -1$$

$$s+2 = \pm \sqrt{-1}$$

$$s = -2 \pm \sqrt{-1}$$

$$s = -2 \pm j$$

$$\therefore s = -2+j, s = -2-j$$

$$s+2-j=0, s+2+j=0$$

poles

$$1) \quad z(t) = \frac{d}{dt} e^{-t} u(t)$$

$$\text{let } a = e^{-t} \text{ \& } b = u(t)$$

Apply product Rule

$$z'(t) = a'b + ab'$$

$$z'(t) = u(t) \frac{d}{dt} e^{-t} + e^{-t} \frac{d}{dt} u(t)$$

$$z'(t) = u(t) [-e^{-t}] + e^{-t} [\delta(t)]$$

$$z'(t) = -e^{-t} u(t) + \delta(t) e^{-t}$$

$$z'(t) = e^{-t} \delta(t) - e^{-t} u(t)$$

$$Z(s) = 1 - \frac{1}{s+1}$$

$$Z(s) = \frac{s+1 - 1}{s+1}$$

$$Z(s) = \frac{s}{s+1}$$

$$H(s) = \frac{s^2 + 4}{s((s+1)^2 + 1)}$$

BIBO stable \Rightarrow Bounded input of Bounded output

\Rightarrow One condition is to have all poles of $H(s)$ to have real negative real parts

$$s[(s+1)^2 + 1] = 0$$

$$s(s^2 + 2s + 2) = 0 \Rightarrow \begin{matrix} s = 0 \\ s^2 + 2s + 2 = 0 \end{matrix}$$

$$s = \begin{cases} 0 \Rightarrow H(s) \text{ is undefined} \\ -1 \pm j \Rightarrow H(s) \text{ is BIBO stable} \end{cases} \quad s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm j2}{2} = -1 \pm j$$

\therefore System is not BIBO stable because one of the poles is zero

3.17a

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$$H(s) = \frac{1}{s^2 + 4} = \frac{Y(s)}{X(s)}$$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{Y(s)}{\frac{1}{s^2 + 4}} = Y(s) (s^2 + 4)$$

$$X(s) = Y(s) (s^2 + 4) = s^2 Y(s) + 4 Y(s)$$

$$X(s) = s^2 Y(s) + 4 Y(s)$$

note $s^n(F(s)) = \frac{d^n}{dt^n} f(t)$

$$X(t) = \frac{d^2}{dt^2} y(t) + 4 y(t)$$

3.17b

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$$x(t) = \frac{d^2}{dt^2} y(t) + 4y(t)$$

$$x(t) = \frac{d^2}{dt^2} y(0) + 4y(0)$$

$$\text{If } y(0) = 0$$

$$x(t) = \frac{d^2}{dt^2} y(0) + 4(0)$$

$$x(t) = \frac{d^2}{dt^2} y(0) \quad \text{but} \quad \frac{d}{dt} y(0) = 1$$

$$x(t) = \frac{d^2}{dt^2} (1)$$

Relation $r(t) \nleftrightarrow u(t) \nleftrightarrow s(t)$

$$\frac{dr(t)}{dt} = u(t)$$

$$x(t) = s(t)$$

$$\frac{d^2 r(t)}{dt^2} = s(t)$$