Pr 2.2

restart

$Mi := diff(y(x), x, x) - 6 * diff(y(x), x) + y(x);$

$$Mi := \frac{d^2}{dx^2} y(x) - 6 \frac{d}{dx} y(x) + y(x)$$

$r := 0$

$$r := 0$$

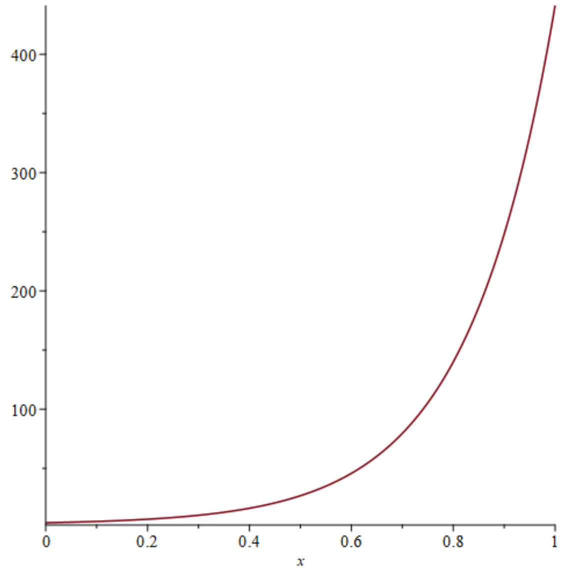
$igen := evalf(dsolve(Mi = r), 3);$

$$igen := y(x) = \_C1\, e^{5.82\, x} + \_C2\, e^{0.18\, x}$$

$igen2 := evalf(dsolve(\{Mi = r, y(0) = 4, D(y)(0) = 8\}), 3);$

$$igen2 := y(x) = 1.30\, e^{5.82\, x} + 2.70\, e^{0.18\, x}$$

$plot(rhs(igen2), x = 0 ..1)$



?plot

Pr.2.6 Type the given ODE as

```
> restart:
> ode := diff(y(t), t, t) + 2*diff(y(t), t) + 145*y(t) = exp(-0.05*t);
```

$$ode := \frac{d^2}{dt^2} y(t) + 2 \left(\frac{d}{dt} y(t) \right) + 145\, y(t) = e^{-0.05\, t}$$

Obtain a general solution by the command [cutting off unnecessary decimals by `evalf[4](...)`]

```
> sol := evalf[4](dsolve(ode));
```

$$sol := y(t) = e^{-1.\, t} \sin(12.\, t)\, \_C2 + e^{-1.\, t} \cos(12.\, t)\, \_C1 + 0.006901\, e^{-0.05000\, t}$$

Obtain the particular solution by the command

```
> yp := evalf[4](dsolve(ode, y(0) = 0, D(y)(0) = 0));
```

$$yp := y(t) = -0.0005463\, e^{-1.\, t} \sin(12.\, t) - 0.006901\, e^{-1.\, t} \cos(12.\, t) + 0.006901\, e^{-0.05000\, t}$$

Pr.2.12

```
> restart:
> ode := diff(y(x), x, x) + 9*y(x) = 0;
```

$$ode := \frac{d^2}{dx^2} y(x) + 9\, y(x) = 0$$

```
> sol := dsolve(ode, y(0) = 2, y(Pi) = -2);
```

$$sol := y(x) = \_C1\, \sin(3x) + 2\cos(3x)$$

but the coefficient of $\sin 3x$ has not been evaluated. Is this, in fact, a solution? We see that (left boundary) $y(0) = 2$ and, for the right boundary condition:

```
> evalf(subs(x = Pi, sol));
```

$$y(\pi) = -1.230620284\, 10^{-9}\, \_C1 - 2.$$

```
> subs(y(x) = rhs(sol), ode);
```

$$\frac{\partial^2}{\partial x^2} (\_C1\, \sin(3\, x) + 2\cos(3\, x)) + 9\, \_C1\, \sin(3\, x) + 18\cos(3\, x) = 0$$

```
> eval(%);                                                    # Resp. 0 = 0
```

Pr.3.16 The three given solutions are

```
> y1 := exp(3*x):  y2 := exp(-5*x):  y3 := exp(6*x):
```

A linear combination

```
> y(x) := c1*y1 + c2*y2 + c3*y3;
```

$$y(x) := c1\, e^{3x} + c2\, e^{-5x} + c3\, e^{6x}$$

is a solution of the given ODE because

```
> ode := diff(y(x), x, x, x) - 4*diff(y(x), x, x) - 27*diff(y(x), x)
   + 90*y(x);
```

$$ode := 0$$

The Wronskian is the determinant of the 3×3 matrix

```
> with(LinearAlgebra):  with(VectorCalculus):
> A := Matrix([[y1, y2, y3], [diff(y1, x), diff(y2, x), diff(y3, x)],
      [diff(y1, x, x), diff(y2, x, x), diff(y3, x, x)]]);
```

$$A := \begin{bmatrix} e^{3x} & e^{-5x} & e^{6x} \\ 3\,e^{3x} & -5\,e^{-5x} & 6\,e^{6x} \\ 9\,e^{3x} & 25\,e^{-5x} & 36\,e^{6x} \end{bmatrix}$$

Thus the Wronskian is

```
> W := Determinant(A);                    # Resp.  W := -264 e^(3x) e^(-5x) e^(6x)
> simplify(%);                                            # Resp.  -264 e^(4x)
```

or

```
> W := Wronskian([y1, y2, y3], x);
```

$$W := \begin{bmatrix} e^{3x} & e^{-5x} & e^{6x} \\ 3\,e^{3x} & -5\,e^{-5x} & 6\,e^{6x} \\ 9\,e^{3x} & 25\,e^{-5x} & 36\,e^{6x} \end{bmatrix}$$

This proves linear independence of the three solutions.

Pr.3.21

```
> Mi := 16·diff(i(t), t, t) + 16·diff(i(t), t) + 4·i(t);
```

$$Mi := 16 \frac{d^2}{dt^2} i(t) + 16 \frac{d}{dt} i(t) + 4\, i(t) \qquad \textbf{(1)}$$

```
> r := -4·260 · sin(4·t);
```

$$r := -1040 \sin(4\,t) \qquad \textbf{(2)}$$

```
> igen := evalf(dsolve(Mi = 0), 5);
```

$$igen := i(t) = \_C1\, e^{-0.50000\, t} + \_C2\, e^{-0.50000\, t}\, t \qquad \textbf{(3)}$$

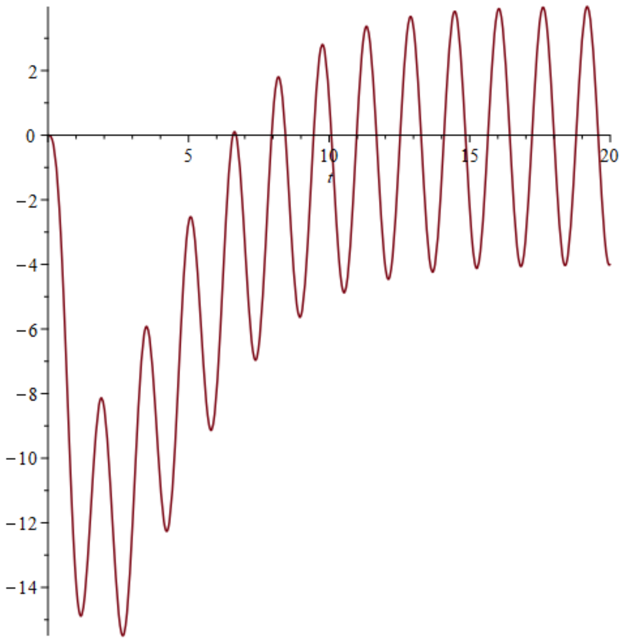
```
> igen2 := evalf(dsolve(Mi = r), 3);
```

$$igen2 := i(t) = e^{-0.500\, t}\, \_C2 + e^{-0.500\, t}\, t\, \_C1 + 0.985 \cos(4.\,t) + 3.88 \sin(4.\,t) \qquad \textbf{(4)}$$

```
> ipart := evalf(dsolve({Mi = r, i(0) = 0, D(i)(0) = 0}), 3);
```

$$ipart := i(t) = -0.985\, e^{-0.500\, t} - 16.\, e^{-0.500\, t}\, t + 0.985 \cos(4.\,t) + 3.88 \sin(4.\,t) \qquad \textbf{(5)}$$

```
>
>
> plot(rhs(ipart), t = 0..20);
```




```
> where:
> r := diff(260·cos(4·t), t);
```

$$r := -1040 \sin(4\,t) \qquad \textbf{(6)}$$