# 2.1(b)

Friday, September 15, 2023 10:10 PM

Time Invariant

this condition is true when $\boxed{y(t-\tau) = 100\,x(t-\tau)}$ where $\tau = 2$

$$x(t) = 20\cos(2\pi t)\,u(t)$$

$$x(t-\tau) = 20\cos[2\pi(t-\tau)]\,u(t-\tau)$$

$$100\,x(t-\tau) = 100[20\cos[2\pi(t-\tau)]u(t-\tau)]$$

$$100\,x(t-\tau) = 2000\cos[2\pi(t-\tau)u(t-\tau)]$$

$$y(t) = 100\,x(t)$$

$$y(t-\tau) = 100\cdot[20\cos(2\pi(t-\tau))u(t-\tau)]$$

$$y(t-\tau) = 2000\cos[2\pi(t-\tau)]\,u(t-\tau)]$$

$\therefore$ The system is time Invariant

2.5

Saturday, September 16, 2023 5:03 PM

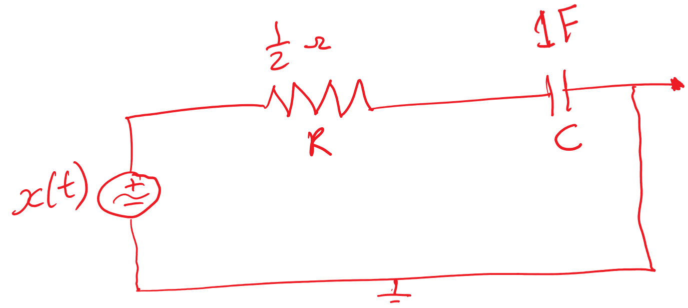
ODE of RC circuit $\Rightarrow \frac{dv(t)}{dt} + \frac{1}{RC} v(t) = 0$

for the said circuit ODE $\Rightarrow \frac{dy(t)}{dt} + 2\, y(t) = 2x(t)$

$\frac{1}{RC} = 2$

$R = \frac{1}{2C}$ when $C = 1F$

$\boxed{R = \frac{1}{2}\Omega}$




$\boxed{2.5b}$

initial condition $x(t) = u(t)$

output $y(t) = e^{-2t}\int_0^t e^{2\tau}\, d\tau = \frac{1}{e^{2t}} \boxed{\int_0^t e^{2\tau}\, d\tau}$

$\int_0^t e^{2\tau}\, d\tau = \left[\frac{1}{2}e^{2\tau} + C\right]_0^t$ let $u = 2\tau$ $\frac{du}{d\tau} = 2$ $\therefore d\tau = \frac{1}{2}du$

$= \frac{1}{2}e^{2t} + C - \frac{1}{2}e^{0} - C$

$= \frac{1}{2}\left[e^{2t} - 1\right]$

$y(t) = \frac{1}{e^{2t}} \cdot \frac{1}{2}\left[e^{2t} - 1\right]$

$y(t) = \frac{1}{2}\left[\frac{e^{2t}}{e^{2t}} - \frac{1}{e^{2t}}\right]$

$y(t) = \frac{1}{2}\left[1 - \frac{1}{e^{2t}}\right]$

Since the initial condition entails a unit step function

$\boxed{y(t) = \frac{1}{2}\left[1 - e^{-2t}\right]u(t)}$

# 2.7

Sunday, September 17, 2023 9:38 PM

a]
$$q(t) = c(t)\,v(t)$$
$$i(t) = \frac{dq(t)}{dt} \Rightarrow \int i(t)\,dt = \int \frac{dq(t)}{\cancel{dt}}\,\cancel{dt}$$
$$\int i(t)\,dt = q(t)$$

Since $q(t) = c(t)\,v(t)$ & $q(t) = \int i(t)\,dt$

$$\therefore\ c(t)\,v(t) = \int i(t)\,dt$$
$$\boxed{v(t) = \frac{1}{c(t)}\int i(t)\,dt}$$

OR

$$\boxed{i(t) = \frac{d}{dt}\,c(t)\,v(t) \quad \text{product Rule} \quad i(t) = c(t)\frac{dv(t)}{dt} + v(t)\frac{dc(t)}{dt}}$$

b]
$$c(t) = 1 + \cos(2\pi t) \qquad v(t) = \cos(2\pi t)$$
$$i(t) = [1 + \cos(2\pi t)]\frac{d}{dt}[\cos(2\pi t)] + \cos(2\pi t)\frac{d}{dt}[1 + \cos(2\pi t)]$$
$$i(t) = -2\pi\sin(2\pi t)[1 + \cos(2\pi t)] + \cos(2\pi t)[0 - 2\pi\sin(2\pi t)]$$
$$i(t) = -2\pi\sin(2\pi t) - 2\pi\sin(2\pi t)\cos(2\pi t) - 2\pi\cos(2\pi t)\sin(2\pi t)$$
$$i(t) = -2\pi\sin(2\pi t) - 4\pi\sin(2\pi t)\cos(2\pi t)$$
$$\boxed{i(t) = -2\pi\sin(2\pi t)\,[1 + 2\cos(2\pi t)]}$$

c) when $c(t)$ stays the same and $v(t)$ is delayed by 0.25sec

$$i_2(t) = c(t)\frac{d}{dt}v(t - \tfrac{1}{4}) + v(t - \tfrac{1}{4})\frac{d}{dt}c(t)$$
$$i_2(t) = [1 + \cos(2\pi t)]\frac{d}{dt}\left[\cos\left(2\pi(t - \tfrac{1}{4})\right)\right] + \cos\left(2\pi(t - \tfrac{1}{4})\right)\frac{d}{dt}[1 + \cos(2\pi t)]$$
$$i_2(t) = [1 + \cos(2\pi t)\frac{d}{dt}\left[\cos(2\pi t - \tfrac{\pi}{2})\right] + \cos(2\pi t - \tfrac{\pi}{2})\frac{d}{dt}[1 + \cos(2\pi t)]$$

$$i_2(t) = -2\pi \sin\left(2\pi t - \frac{\pi}{2}\right)\left[1 + \cos(2\pi t)\right] + \cos\left(2\pi t - \frac{\pi}{2}\right)\left[0 - 2\pi \sin(2\pi t)\right]$$

note $\cos(2\pi \cdot n) = 1$ where $-\infty < n < \infty$

$$i_2(t) = 2\pi \sin\left(2\pi t - \frac{\pi}{2}\right)\left[1 + 1\right] + \cos\left(2\pi t - \frac{\pi}{2}\right)\left[-2\pi \sin(2\pi t)\right]$$

$$i_2(t) = 4\pi \sin\left(2\pi t - \frac{\pi}{2}\right) - 2\pi \sin(2\pi t)\cos\left(2\pi t - \frac{\pi}{2}\right)$$

$$\boxed{i_2(t) = 2\pi\left[2 \sin\left(2\pi t - \frac{\pi}{2}\right) - \sin(2\pi t)\cos\left(2\pi t - \frac{\pi}{2}\right)\right]}$$