3.24 (a) Using the Laplace transform, we have

$$H(s) = \frac{s+1}{(s+1)^2+1}$$
$$X(s) = \frac{Y(s)}{H(s)} = \frac{4}{s(s+1)}$$

The partial fraction expansion of $X(s)$ is

$$X(s) = \frac{4}{s} + \frac{-4}{s+1}$$

which gives

$$x(t) = 4u(t) - 4e^{-t}u(t)$$

3.28 (a) The poles of $H(s)$ are at $s = 0$ and $s = -1 \pm j1$, and the zeros are at $s = \pm j2$. This system is not BIBO stable because of the pole at the origin, the impulse response will not be absolutely integrable.
(b) If $x(t) = 2\cos(2t)u(t)$ then

$$X(s) = 2\frac{s}{s^2 + 4}$$

and thus

$$Y(s) = \frac{2}{(s+1)^2 + 1}$$

i.e., the pole at $s = 0$ and the zeros are cancelled by the transform of the cosine. The remaining poles are in the left-hand s-plane and so the steady state is zero.

This is a very special case because of the cancellation of the pole at $s = 0$ and the numerator due to the used input. The system is not stable, and the frequency response does not exist for all frequencies. Indeed it is infinite whenever the input frequency is 0.

3.42 (a)(b) The following script shows how to do polynomial multiplication using *conv* function.

The multiplication of the polynomials

$$P(s) = s^2 + s + 1 \ \text{and} \ Q(s) = 2s^3 + 3s^2 + s + 1$$

gives

$$Z(s) = P(s)Q(s) = 2s^5 + 5s^4 + 6s^3 + 5s^2 + 2s + 1$$

The script shows how to obtain this result in the Poly Multiplication part of the script.

In the Application part of the script we show how to find the numerator $N_1(s)$ and the denominator $D_1(s)$ of $Y(s)$ by multiplying $X(s)$ and $H(s)$ to give

$$Y(s) = \frac{X(s)N(s)}{D(s)} = \frac{(s+2)}{s^2(s+1)((s+4)^2+9)}$$

so that the poles of $Y(s)$ are $s = 0$ (double), and $s = -4 \pm j3$, and no zeros. The double pole gives a ramp and the other poles a modulated sinusoid by a decaying exponential.

```
% Pr. 3_42
% Poly multiplication
% coefficients of higher to lower orders of s
P=[1 1 1];Q=[2 3 1 1];
Z=conv(P,Q);
% Application
d1=[1 1]; d2=[1 8 25]; D=conv(d1,d2);
d3=[1 0 0]; den=conv(D,d3)
N=length(den)
num=[ zeros(1,N-2) 1 2]
syms s t y
figure(1)
subplot(121)
[r,p]=pfeLaplace(num,den);
disp('>>>>> Inverse Laplace <<<<<')
y=ilaplace((num(N-1)*s+num(N))/(den(1)*s^5+den(2)*s^4+den(3)*s^3+den(4)*s^2+den(5)*s+
subplot(122)
ezplot(y,[0,50])
axis([0 50 -0.1 1.5]); grid
```

```
den =  1     9    33    25     0     0
num = 0     0     0      0      1     2
>>>>>  Zeros <<<<<
z = -2
>>>>>  Poles <<<<<
p = -4.0000 + 3.0000i
      -4.0000 - 3.0000i
      -1.0000
        0
        0
>>>>>  Residues <<<<<
r = 0.0050 - 0.0026i
     0.0050 + 0.0026i
     0.0556
```

```
    -0.0656
     0.0800
>>>>> Inverse Laplace <<<<<
y = -41/625+1/18*exp(-t)+2/25*t+113/11250*exp(-4*t)*cos(3*t)
+59/11250*exp(-4*t)*sin(3*t)
```