#LAMIN JAMMEH
#EE597
#HW9

#Pr 13/17.2 (Complex arithmetic)

$z1 := 5 + 12 \cdot I;\ \ z2 := 3 - 7 \cdot I;$

$$z1 := 5 + 12\,\mathrm{I}$$

$$z2 := 3 - 7\,\mathrm{I} \tag{1}$$

$z1 \cdot conjugate(z2);$

$$-69 + 71\,\mathrm{I} \tag{2}$$

$conjugate(z1) \cdot z2;$

$$-69 - 71\,\mathrm{I} \tag{3}$$

$\frac{1}{\mathrm{abs}(z1)};$

$$\frac{1}{13} \tag{4}$$

$\mathrm{abs}(z1) + \mathrm{abs}(z2) - \mathrm{abs}(z1 + z2);$

$$13 + \sqrt{58} - \sqrt{89} \tag{5}$$

$\mathrm{Re}(z1^3);$

$$-2035 \tag{6}$$

$\mathrm{Re}(z1)^3;$

$$125 \tag{7}$$

$\mathrm{Im}\left(\frac{(z1 - z2)}{z1 + z2}\right)$

$$\frac{142}{89} \tag{8}$$

#Pr 13/17.4 (Polar form)

$\mathrm{polar}(1 - I);\ \mathrm{polar}(-3 - 3 \cdot I);\ \ \mathrm{polar}((1 - I) \cdot (-3 - 3 \cdot I));\ \mathrm{polar}\left(\frac{(1 - I)}{1 + I}\right);$

$\mathrm{polar}\left(\left(\frac{(6 + 8 \cdot I)}{4 - 3 \cdot I}\right)^2\right);$

$$\mathrm{polar}\left(\sqrt{2}, -\frac{\pi}{4}\right)$$

$$\mathrm{polar}\left(3\sqrt{2}, -\frac{3\pi}{4}\right)$$

$$\mathrm{polar}(6, \pi)$$

$$\mathrm{polar}\left(1, -\frac{\pi}{2}\right)$$

$$\mathrm{polar}(4, \pi) \tag{9}$$

#Pr 13/17.6 $\left(\textit{Quadratic equation in } z^2\right)$

restart;

$eq := z^4 + z^2 \cdot 6 \cdot I + 3 \cdot z^2 - 8 + 6 \cdot I = 0;$

$$eq := z^4 + 6\,\mathrm{I}\,z^2 + 3\,z^2 - 8 + 6\,\mathrm{I} = 0 \tag{10}$$

$sol := solve(eq, z);$

$$sol := 1 - 2\,\mathrm{I},\ 1 - \mathrm{I},\ -1 + 2\,\mathrm{I},\ -1 + \mathrm{I} \tag{11}$$

#Pr 13/17.8 $\cdot$ (*Roots of unity*)

restart;

$sol := solve(z^{16} = 1, z);$

$$sol := 1, -1, \mathrm{I}, -\mathrm{I}, \frac{\sqrt{2}}{2} + \frac{\mathrm{I}\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - \frac{\mathrm{I}\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - \frac{\mathrm{I}\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + \frac{\mathrm{I}\sqrt{2}}{2}, \frac{\sqrt{-2\,\mathrm{I}\sqrt{2} + 2\sqrt{2}}}{2}, -\frac{\sqrt{-2\,\mathrm{I}\sqrt{2} + 2\sqrt{2}}}{2}, \frac{\sqrt{2\,\mathrm{I}\sqrt{2} + 2\sqrt{2}}}{2}, -\frac{\sqrt{2\,\mathrm{I}\sqrt{2} + 2\sqrt{2}}}{2}, \frac{\sqrt{-2\,\mathrm{I}\sqrt{2} - 2\sqrt{2}}}{2}, -\frac{\sqrt{-2\,\mathrm{I}\sqrt{2} - 2\sqrt{2}}}{2}, \frac{\sqrt{2\,\mathrm{I}\sqrt{2} - 2\sqrt{2}}}{2}, -\frac{\sqrt{2\,\mathrm{I}\sqrt{2} - 2\sqrt{2}}}{2} \tag{12}$$

$with(plots):$

$plot([sol], style = point, symbolsize = 20, scaling = constrained);$

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Warning, unable to evaluate 14 of the 16 functions to numeric values
in the region; see the plotting command's help page to ensure the
calling sequence is correct
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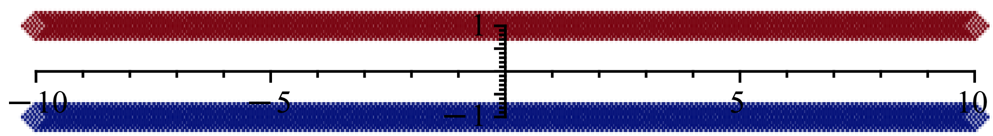

$\#Pr\ 13/17.12 \cdot (Cauchy - Riemann\ equations)$

$restart;$

$f := \operatorname{Re}(z^3) + I \cdot \operatorname{Im}(z^3);$

$$f := \Re(z^3) + \mathrm{I}\,\Im(z^3) \quad \textbf{(13)}$$

$uz := diff(evalc(\operatorname{Re}(f)), z);$

$$uz := 3\,z^2 \quad \textbf{(14)}$$

$vz := diff(evalc(\operatorname{Im}(f)), z);$

$$vz := 0 \quad \textbf{(15)}$$

#The Function is not Analytic

$\#Pr\ 13/17.20 \cdot (General\ powers)$

$restart;$

$f1 := (2 \cdot I)^{2 \cdot I};$

$$f1 := (2\,\mathrm{I})^{2\,\mathrm{I}} \quad \textbf{(16)}$$

$pf1 := \exp(2 \cdot I \cdot \ln(2 \cdot I));$

$$pf1 := \mathrm{e}^{2\,\mathrm{I}\,\ln(2\,\mathrm{I})} \quad \textbf{(17)}$$

$evalc(pf1);$

$$e^{-\pi}\cos(2\ln(2)) + I\,e^{-\pi}\sin(2\ln(2)) \quad \textbf{(18)}$$

$f2 := 4^{3-I};$

$$f2 := 4^{3-I} \quad \textbf{(19)}$$

$pf2 := \exp((3 - I)\cdot\ln(4));$

$$pf2 := e^{(6-2I)\ln(2)} \quad \textbf{(20)}$$

$evalc(pf2);$

$$64\cos(2\ln(2)) - 64\,I\sin(2\ln(2)) \quad \textbf{(21)}$$

$pf3 := \exp(I\cdot\ln(3 + 6*I));$

$$pf3 := e^{I\ln(3+6I)} \quad \textbf{(22)}$$

$evalc(pf3);$

$$e^{-\arctan(2)}\cos\left(\frac{\ln(45)}{2}\right) + I\,e^{-\arctan(2)}\sin\left(\frac{\ln(45)}{2}\right) \quad \textbf{(23)}$$

$simplify(\%)$

$$e^{-\arctan(2)}\left(I\sin\left(\ln(3) + \frac{\ln(5)}{2}\right) + \cos\left(\ln(3) + \frac{\ln(5)}{2}\right)\right) \quad \textbf{(24)}$$

#Pr 14.1 (Use of path)

$restart;$

$z := 1 + t\cdot I;$ *#parameterize z as 1+it as t=1..2*

$$z := 1 + I\,t \quad \textbf{(25)}$$

$z1dot := diff(z, t);$ *#obtain z'(t) as z1dot*

$$z1dot := I \quad \textbf{(26)}$$

$f1 := \mathrm{Im}(z);$ *#f=Im(z) defined* **in** *the question*

$$f1 := \Re(t) \quad \textbf{(27)}$$

$evalc(f1);$

$$t \quad \textbf{(28)}$$

$int(f1\cdot z1dot, t = 1..2);$ *#integral formular of complex numbers*

$$\frac{3\,I}{2} \quad \textbf{(29)}$$

#Pr 14.2 (Contour integral)

$restart;$

$f := \frac{7}{z + I} - \frac{5}{(z + I)^2};$

$$f := \frac{7}{z + I} - \frac{5}{(z + I)^2} \quad \textbf{(30)}$$

#define the circle |z-i|=3

$circle := ComplexCircle(I, 3);$

$$circle := ComplexCircle(I, 3) \quad \textbf{(31)}$$

$Integral\_result := 2 \cdot \text{Pi} \cdot I \cdot residue(f, z = -I)$

$$Integral\_result := 14\, \mathrm{I}\, \pi \quad \mathbf{(32)}$$

$evalf(\%);$

$$44.0\, \mathrm{I} \quad \mathbf{(33)}$$