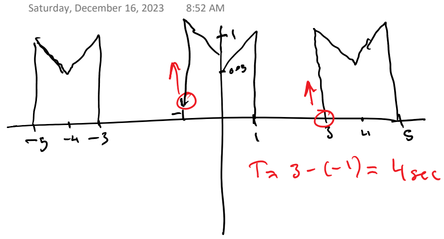
Saturday, December 16, 2023 8:52 AM



$u(t) = 1 \quad t \geq 0$

$r = t \cdot u(t)$

a) $T = \text{period} = \boxed{4 \text{ sec}}$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2} \text{ rad/sec}}$

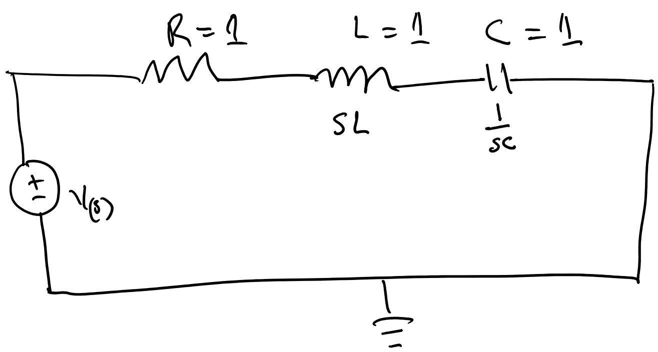
$$\boxed{\text{b) } f(t) = \begin{cases} (0.5t \cdot u(t)) - u(t-1) & 0 \leq t \leq 1 \\ 0 & 1 \leq t \leq 2 \end{cases}}$$

c) $a_0 = \frac{1}{T} \int_0^T f(t)\, dt$

$a_0 = \frac{1}{4} \int_0^4 \left[(0.5t \cdot u(t) - u(t-1) \right] dt$

$$\boxed{a_0 = \frac{1}{4}}$$

Check Matlab for a_n, b_n & fourier series



$$V_{(s)}(t) = V_R(t) + V_L(t) + V_C(t)$$

$$V_C(t) = \frac{1}{C}\int i(t)$$

$$i(t) = \frac{V_{(s)}(t)}{Z}$$

$$z = R + j\left(sL - \frac{1}{s}\right)$$

$$z = 1 + j\left[s - \frac{1}{s}\right]$$

$$Z = 1 + j\left(\frac{s^2 - 1}{s}\right)$$

$$Z = \frac{s + js^2 - j}{s}$$

$$V_C(t) = \frac{1}{sC}\left[\frac{V_s(t)\cdot s}{s + js^2 - j}\right]$$

$$V_C(t) = \frac{V_s(t)}{s + js^2 - j} \quad \text{let } V_{(s)}(t) = 1$$

$$\boxed{V_C(t) = \frac{1}{s + s^2 - 1}}$$

$$V_L(t) = \left(\frac{s}{s^2+s-1}\right) sL \qquad \text{wher } L = 1$$

$$\boxed{V_L(t) = \frac{s^2}{s^2+s-1}}$$

$$V_R(t) = R\left(\frac{s}{s^2+s-1}\right)$$

$$\boxed{V_R(t) = \frac{s}{s^2+s-1}}$$

Check Matlab code for freq response

Z- transform

$x[n] = u[n-1] \Rightarrow$ input

$h[n] = -0.25^n \cdot u[n] \Rightarrow$ Impulse response

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$y[n] = \sum_{k=\infty}^{\infty} u[k-1] \cdot \left[0.25^{n-(k-1)} \cdot u[n-k-1] \right]$$

for $k < 1$ $\quad u[k-1] = 0$

$k \geq 1$ $\quad u[k-1] = 1$

$$y[n] = \sum_{k=1}^{\infty} 1 \cdot \left[0.25^{n-(k-1)} u[n-k] \right]$$

geometric Sum $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$

$$y[n] = 0.25^n \cdot \frac{1-0.25^n}{1-0.25} = 0.25^n \cdot \frac{1-0.25^n}{0.75}$$

$$\boxed{y[n] = \frac{0.25^n \cdot (1-0.25^n)}{0.75}}$$

Saturday, December 16, 2023 8:20 AM

$$2\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^3} + 6\frac{dy}{dt} + 8y = 10\,u(s)$$

$$y\left(2s^3 + 4s^2 + 6s + 8\right) = 10 \cdot U(s)$$

$$\dddot{y} + \underbrace{4\ddot{y}}_{\varphi_3} + \underbrace{6\dot{y}}_{\varphi_2} + \underbrace{8y}_{\varphi_1} = k\,u(s) \xrightarrow{\text{note}}$$

state variables

$s \cdot y \rightarrow \dot{y} \rightarrow \dot{Q}_1$

$s \cdot y \rightarrow \ddot{y} \rightarrow \dot{Q}_2$

$s^3 \cdot y \rightarrow \dddot{y} \rightarrow \dot{Q}_3$

⇒ step 2 get the state variables $\varphi_1(t)$ & $\varphi_2(t)$ & $\varphi_3(t)$

$\varphi_1(t) = y(t)$

$\varphi_2(t) = \dot{y}(t) = \dot{\varphi}_1(t)$

$\varphi_3(t) = \ddot{y}(t) = \dot{\varphi}_2(t)$

} 1^{st} order equation for the variable

$\dot{\varphi}_1(t)$, $\dot{\varphi}_2(t)$, and $\dot{\varphi}_3(t)$ are given by

$\dot{\varphi}_1 = \varphi_2$

$\dot{\varphi}_2 = \varphi_3$

} step 3

$\dot{\varphi}_3 = \dddot{y} = -2\ddot{y} - 3\dot{y} - 4y + 10\,u(t)$

now put $\dot{\varphi}_3$, $\dot{\varphi}_2$ & $\dot{\varphi}_1$ in terms of state variable

$\dot{\varphi}_3 = -2\varphi_3 - 3\varphi_2 - 4\varphi_1 + 10 \cdot u(t)$

$\dot{\varphi}_2 = \varphi_3 + 0 \cdot \varphi_2 + 0 \cdot \varphi_1 + 0 \cdot u(t)$

$\dot{\varphi}_1 = 0 \cdot \varphi_3 + \varphi_2 + 0 \cdot \varphi_1 + 0 \cdot u(t)$

} Step 4

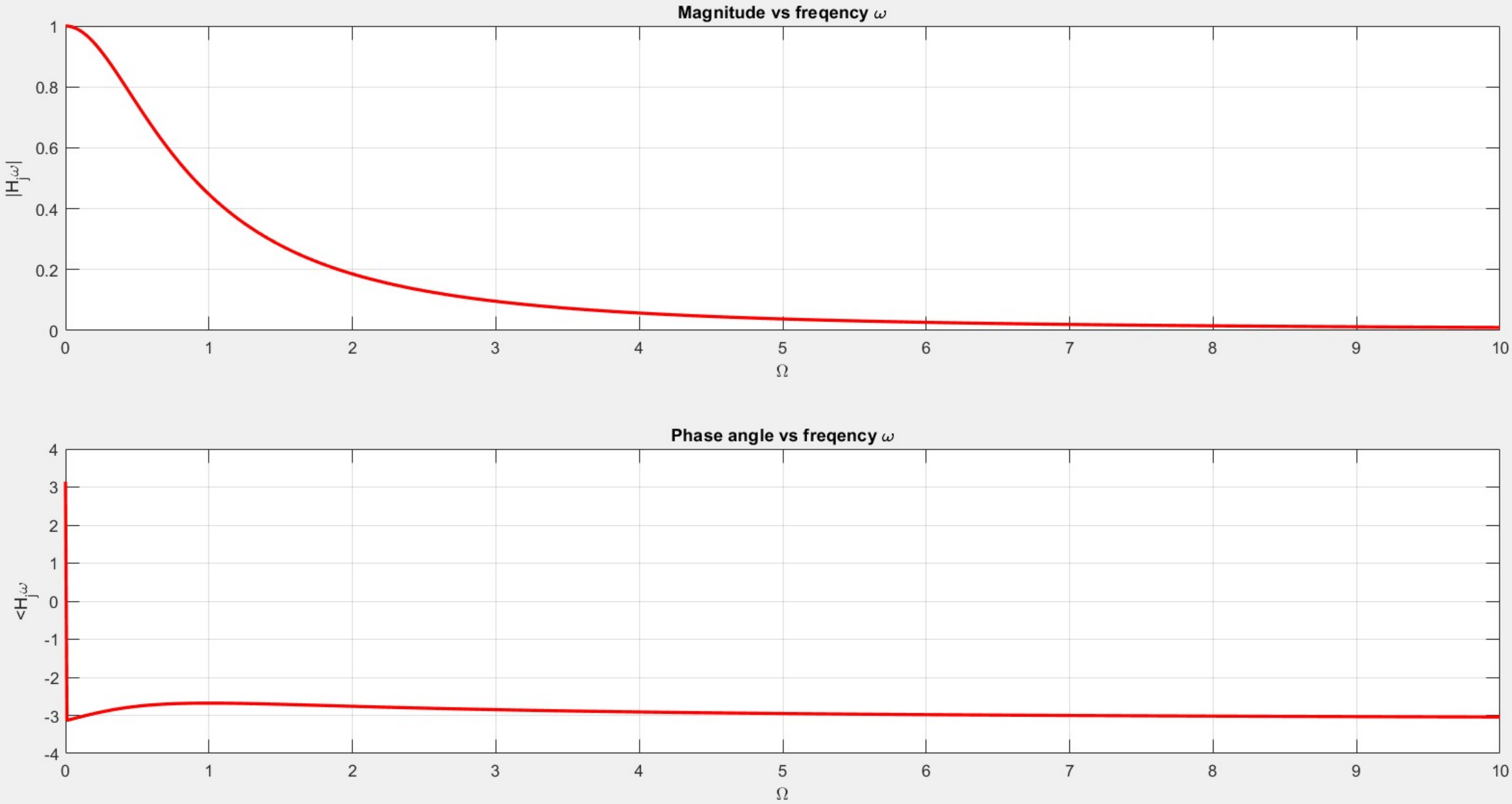
$$\begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & -3 & -4 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \cdot \left[U(t) \right]$$

$$y(t) = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

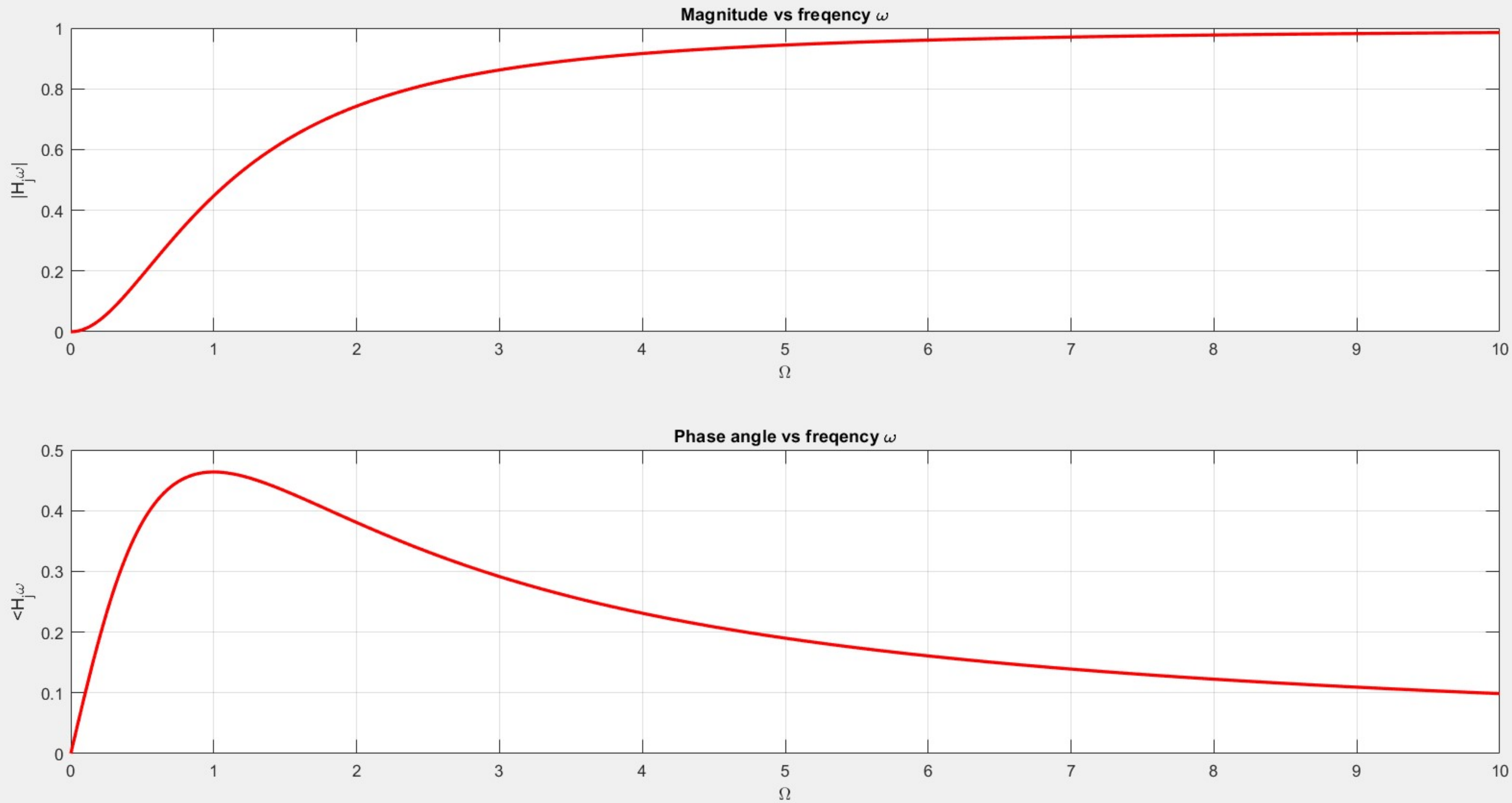
```
% Name: Lamin Jammeh
% Class: EE480 Online
% Semster: Fall 2023
% Final Exam

%% ****************** Question 1 *************
clear;
clc;
syms t n;
T = 4 % Period of the function
w = 2*pi/T

% Define the function
x = (0.5*t*heaviside(t)) - heaviside(t-1);

% Calculate the Fourier coefficients using integration
a0 = (1/T) * int(x, t, 0, T)
an = (2/T) * int(x * cos(w*n*t), t, 0, T)
bn = (2/T) * int(x * sin(w*n*t), t, 0, T)

Fourier_series = a0  + an*cos(w*n*t) + bn*sin(w*n*t)
%% ****************** Question 2 Voltage at capacitior *************
clear;
clc;

syms s

% Define the transfer function
H = 1/(s^2 + s - 1);

% define the w-max
wmax = 10;
w = 0:0.01:wmax;

% Substitute s with jw (j times omega) for frequency domain analysis
H_jw = subs(H, s, 1i*w);

% Calculate magnitude
magnitude = simplify(abs(H_jw));

% Calculate phase
phase = angle(H_jw);

%plot the mag and phase vs freq(w)
subplot(2,1,1)
plot(w,magnitude,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('|H_j\omega|');
title('Magnitude vs freqency \omega')
```

```
grid on

%% ****************** Question 2 Voltage at inductor *************
clear;
clc;

syms s

% Define the transfer function
H = s^2/(s^2 + s - 1);

% define the w-max
wmax = 10;
w = 0:0.01:wmax;

% Substitute s with jw (j times omega) for frequency domain analysis
H_jw = subs(H, s, 1i*w);

% Calculate magnitude
magnitude = simplify(abs(H_jw));

% Calculate phase
phase = angle(H_jw);

%plot the mag and phase vs freq(w)
subplot(2,1,1)
plot(w,magnitude,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('|H_j\omega|');
title('Magnitude vs freqency \omega')
grid on

subplot(2,1,2)
plot(w,phase,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('<H_j\omega');
title('Phase angle vs freqency \omega')
grid on

subplot(2,1,2)
plot(w,phase,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('<H_j\omega');
title('Phase angle vs freqency \omega')
grid on

%% ****************** Question 2 Voltage at resistor *************
clear;
clc;
```

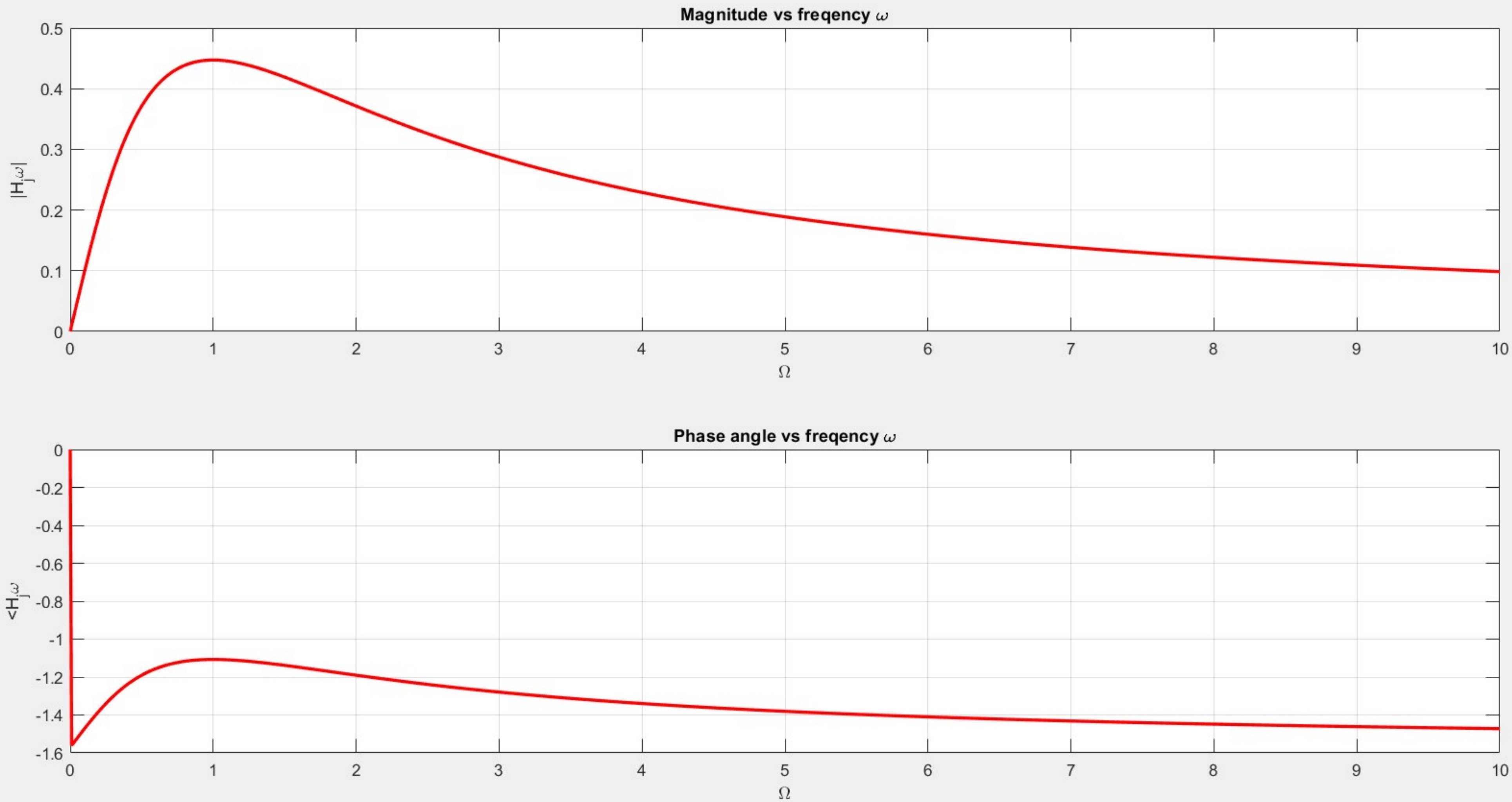
```
syms s

% Define the transfer function
H = s/(s^2 + s - 1);

% define the w-max
wmax = 10;
w = 0:0.01:wmax;

% Substitute s with jw (j times omega) for frequency domain analysis
H_jw = subs(H, s, 1i*w);

% Calculate magnitude
magnitude = simplify(abs(H_jw));

% Calculate phase
phase = angle(H_jw);

%plot the mag and phase vs freq(w)
subplot(2,1,1)
plot(w,magnitude,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('|H_j\omega|');
title('Magnitude vs freqency \omega')
grid on

subplot(2,1,2)
plot(w,phase,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('<H_j\omega');
title('Phase angle vs freqency \omega')
grid on

subplot(2,1,2)
plot(w,phase,'r',"LineWidth",2)
xlabel('\Omega');
ylabel('<H_j\omega');
title('Phase angle vs freqency \omega')
grid on
```



Magnitude vs freqency ω
|H_jω|
Ω
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6
7
8
9
10
Phase angle vs freqency ω
<H_jω
Ω
-4
-3
-2
-1
0
1
2
3
4



Magnitude vs freqency ω
$|H_{j\omega}|$
Ω
Phase angle vs freqency ω
$<H_{j\omega}$
Ω


Magnitude vs freqency ω
|Hjω|
Ω
Phase angle vs freqency ω
<Hjω
Ω

```
T =

     4


w =

    1.5708


a0 =

1/4


an =

-(4*sin(pi*n)^2 - n*pi*(2*sin(2*pi*n) + 2*sin((pi*n)/2)))/(2*n^2*pi^2)


bn =

(2*sin(2*pi*n) - n*(2*pi*cos(2*pi*n) + 2*pi*cos((pi*n)/2)))/(2*n^2*pi^2)


Fourier_series =

(sin((pi*n*t)/2)*(2*sin(2*pi*n) - n*(2*pi*cos(2*pi*n) + 2*pi*cos((pi*n)/2))))/
(2*n^2*pi^2) - (cos((pi*n*t)/2)*(4*sin(pi*n)^2 - n*pi*(2*sin(2*pi*n) + 2*sin((pi*n)
/2))))/(2*n^2*pi^2) + 1/4

>>
```