

$$H(s) = \frac{1}{s^3 + 0.4s^2 + 1.14s + 0.22}$$

Write the phase variable state model

$$H(s) = \frac{y(s)}{u(s)}$$

⇒ split the system into Output = System • Input  
 $y(s) = H(s) \cdot u(s)$

$$Y(s)(s^3 + 0.4s^2 + 1.14s + 0.22) = U(s)$$

$$s^3 y(s) + 0.4s^2 y(s) + 1.14s y(s) + 0.22 y(s) = U(s)$$

⇒ change to differential equation  
 $\ddot{y} + 0.4\dot{y} + 1.14\dot{y} + 0.22y = u$

⇒ Define state variables ( $q_1, \dots, q_n$ )  $n$  = highest order of  $s$  in a single input & single output system

$$s^3 \Rightarrow q_1, q_2, q_3$$

$$\begin{cases} q_1 = y \\ q_2 = \dot{y} \\ q_3 = \ddot{y} \end{cases} \text{ plant matrix}$$

$$\dot{q}_1 = q_2$$

$$\dot{q}_2 = q_3$$

$$\dot{q}_3 = \ddot{y} = -0.4\dot{y} - 1.14\dot{y} - 0.22y + u$$

⇒ Arrange  $\dot{q}_1, \dot{q}_2, \dot{q}_3$  in terms state-space variable

$$\dot{q}_1 = 0 + q_2 + 0 + u$$

$$\dot{q}_2 = 0 + 0 + q_3 + u$$

$$\dot{q}_3 = 0.22q_1 - 1.14q_2 - 0.4q_3 + u$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.22 & -1.14 & -0.4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot u$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

```
>> syms z;  
%Step defien the two transfer functions  
H_1 = 0.2 / (1 + 0.5*z^(-1));  
H_2 = (0.8 - 0.2*z^(-1)) / (1 - z^(-1) + 0.5*z^(-2));  
%Step2 since the 2 TF are in parallel add the TF  
H_z = H_1 + H_2  
H = simplifyFraction(H_z)  
  
H_z =  
  

$$\frac{1}{5} \left( \frac{1}{2z} + 1 \right) - \left( \frac{1}{5z} - \frac{4}{5} \right) / \left( \frac{1}{2z^2} - \frac{1}{z} + 1 \right)$$
  
  
H =  
  

$$\frac{4z^3}{(2z + 1)(2z^2 - 2z + 1)}$$
  
  
>>
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% Name: Lamin Jammeh
% Class: EE480 Online
% Semester: Fall 2023
% HW_13

% Basic Problems
%% ***** 10.36c *****
clear;
clc;
syms z;
%Step defien the two transfer functions
H_1 = 0.2 / (1 + 0.5*z^(-1));
H_2 = (0.8 - 0.2*z^(-1)) / (1 - z^(-1) + 0.5*z^(-2));
%Step2 since the 2 TF are in parallel add the TF
H_z = H_1 + H_2
H = simplifyFraction(H_z)
```