1.3

(b) $x(t) = \cos(t) + j\sin(t)$ is a complex signal, $x_e(t) = 0.5[e^{jt} + e^{-jt}] = \cos(t)$ so $x_o(t) = j\sin(t)$.

1.6 (a) Using $\Omega_0 = 2\pi f_0 = 2\pi/T_0$ for

i. $\cos(2\pi t)$: $\Omega_0 = 2\pi$ rad/sec, $f_0 = 1$ Hz and $T_0 = 1$ sec.

ii. $\sin(t - \pi/4)$: $\Omega_0 = 1$ rad/sec, $f_0 = 1/(2\pi)$ Hz and $T_0 = 2\pi$ sec.

iii. $\tan(\pi t) = \sin(\pi t)/\cos(\pi t)$: $\Omega_0 = \pi$ rad/sec, $f_0 = 1/2$ Hz and $T_0 = 2$ sec.

1.9 This problem can be done in the time domain or in the phasor domain. The series connection of the source $v_s(t) = \cos(t)$, the resistor R and the inductor L is equivalent to the connection of a phasor source $V_s = 1e^{j0}$, and impedances R and $j\Omega L = jL$ (the frequency of the source is $\Omega = 1$). The corresponding to the current across the resistor and the inductor, in steady state, is

$$I = \frac{V_s}{R + jL}$$

(a) $L = 1$, $R = 0$ —intuitively, the power used by the inductor is zero since only the resistor uses power.

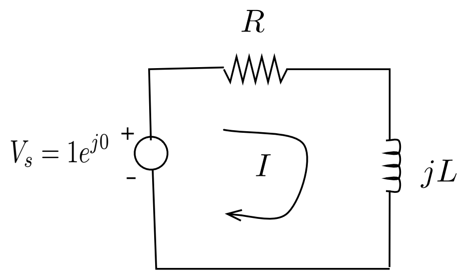

Figure 1.6: Problem 9: Phasor circuit.

In this case, the current $i(t)$ has a phasor

$$I = \frac{1}{j} = -j = 1e^{-j\pi/2}$$

so that the current across the inductor in steady state is given by

$$i(t) = \cos(t - \pi/2)$$

We can compute the average power P_a in time by finding the instantaneous power as

$$p(t) = i(t)v_s(t) = \cos(t - \pi/2)\cos(t) = \frac{1}{2}(\cos(\pi/2) + \cos(2t - \pi/2))$$

so that

$$\begin{aligned} P_a &= \frac{1}{T_0}\int_0^{T_0} p(t)dt \\ &= \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}[\cos(\pi/2) + \cos(2t - \pi/2)]dt = 0 \end{aligned}$$

since $\cos(\pi/2) = 0$ and the area under $\cos(2t - \pi/2)$ in a period is zero.

You probably remember from Circuits that the average power is computed using the equivalent expression

$$P_a = \frac{V_{sm} I_m}{2}\cos(\theta)$$

where V_{sm} and I_m are the peak-to-peak values of the phasors corresponding to V_s and I, and θ is the angle in the impedance of the inductor, i.e, $j1 = e^{j\pi/2}$ or $\theta = \pi/2$, and the average power is then

$$P_a = 0.5\cos(\pi/2) = 0$$

Confirming our intuition!

1.18 (a) $\Omega_0 = 2\pi = 2\pi f_0$ (rad/sec), so $f_0 = 1/T_0 = 1$ (Hz) and $T_0 = 1$ sec.
The sum

$$\begin{aligned} z(t) &= x(t) + y(t) \\ &= (2\cos(2\pi t) + \cos(\pi t)) + j(2\sin(2\pi t) + \sin(\pi t) \end{aligned}$$

is also periodic of period $T_1 = 2$.
(b) $v(t) = x(t)y(t) = 2e^{j3\pi t}$ with frequency $\Omega_3 = 3\pi$ so that

$$T_3 = 2\pi/\Omega_3 = 2/3$$

1.24 (a) The expanded signal $x(t/2)$ is periodic. The first period of $x(t)$ is $x_1(t)$ for $0 \leq t \leq 2$, and so the period of $x(t/2)$ is $x_1(t/2)$ which is supported in $0 \leq t/2 \leq 2$ or $0 \leq t \leq 4$, so the period of $x(t/2)$ is 4.
(b) The compressed signal $x(2t)$ is periodic. The first period of $x(t)$, $x_1(t)$ for $0 \leq t \leq 2$, becomes $x_1(2t)$ for $0 \leq 2t \leq 2$ or $0 \leq t \leq 1$, its support is halved. So the period of $x(2t)$ is 1.

```
% Pr. 1_24, part(b)
clear all; clf
t=0:0.002:8;
t1=0:0.001:8; t2=0:0.004:8;
x=cos(pi*t);
x1=cos(pi*t1/2);
x2=cos(pi*2*t2);
figure(7)
 subplot(211)
 plot(t1,x1)
 hold on
 plot(t,x,'r')
 xlabel('t (sec)')
 ylabel('x(t/2), x(t)')
 legend('expanded signal', 'original signal')
 subplot(212)
 plot(t2,x2)
 hold on
 plot(t,x,'r')
 xlabel('t (sec) ')
 ylabel('x(2t), x(t)')
 hold off
 legend('compressed signal', 'original signal')
```

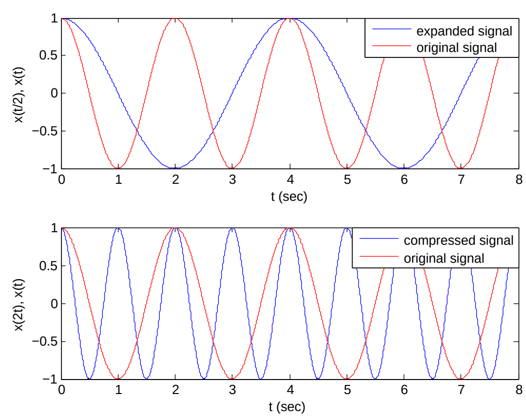

Figure expanded and compressed sinusoids vs original sinusoid.

1.29 (a)(b) The following script generates the chirps

```
% Pr. 1_29
clear all;clf
t=0:0.05:40;
% chirps
y=cos(2*t+t.^2/4);
y1=cos(2*t- 2*sin(t));
figure(14)
 subplot(211)
 plot(t,y); title('linear chirp')
 axis([0 20 1.1*min(y) 1.1*max(y)]);grid
 subplot(212)
 plot(t,y1);title('sinusoidal chirp');xlabel('t')
 axis([0 20 1.1*min(y1) 1.1*max(y1)]);grid
% instantaneous frequencies
IF=2+2*t/4;
IF1=2-2*cos(2*t);
figure(15)
 subplot(211)
 plot(t,IF);title('IF of linear chirp')
 ylabel('frequency'); xlabel('t');grid
 subplot(212
 plot(t,IF1);title('IF of sinusoidal chirp')
 ylabel('frequency');xlabel('t');grid
```

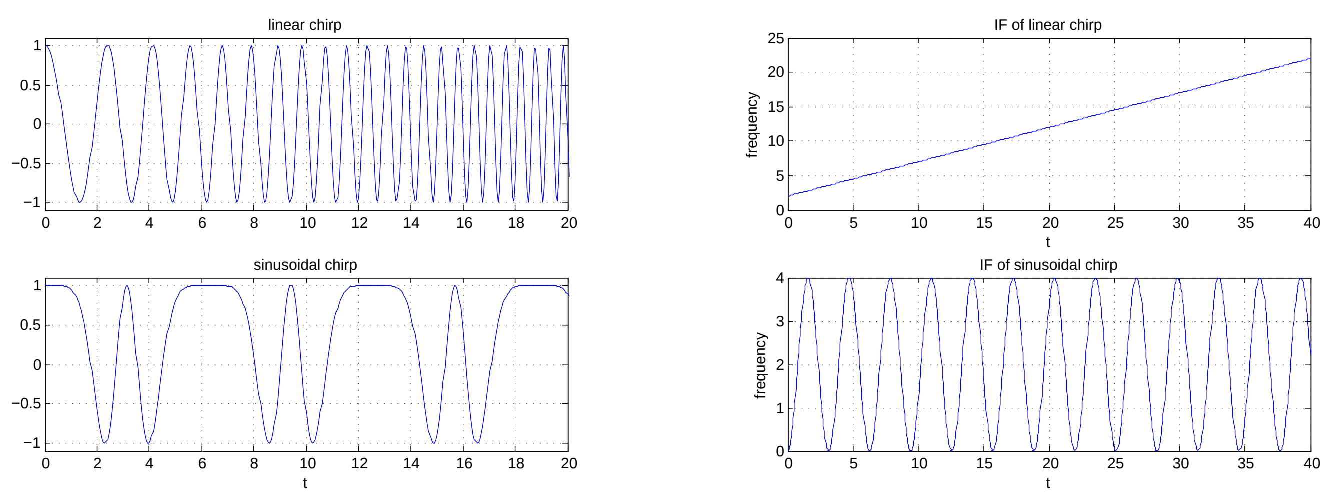

Figure linear and sinusoidal chirps (left) and their corresponding instantaneous frequencies (right).