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#EE597
#HW2

#Problem Sets for Chapter2

#Pr2.2 Maximum of Solution

restart;
#find max of the solution [y"-6y'+y=0 with initial conditions y(0)=4 and y'(0)=8

#Step1 Define the ODE

ODE1 := (D@@2)(y)(x) − 6· D(y)(x) + y(x) = 0;

$$ODE1 := \mathrm{D}^{(2)}(y)(x) - 6\,\mathrm{D}(y)(x) + y(x) = 0 \tag{1}$$

#Step2 solve the ODE to visualize the general form

Sol1 := *dsolve*(*ODE1*);

$$Sol1 := y(x) = c_1\, \mathrm{e}^{(3+2\sqrt{2})\,x} + c_2\, \mathrm{e}^{-(-3+2\sqrt{2})\,x} \tag{2}$$

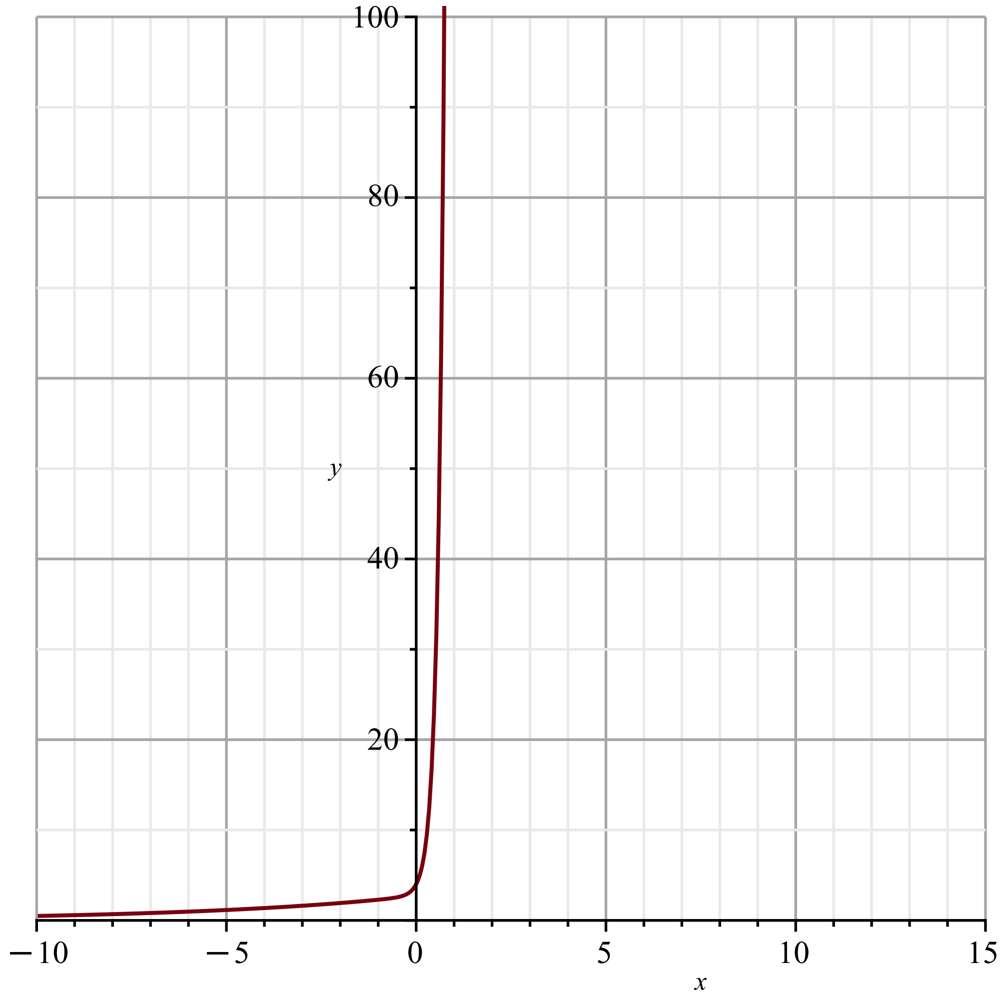
#Step3 apply the initial conditions to find yp

yp := *dsolve*({*ODE1*, y(0) = 4, D(y)(0) = 8}, y(x));

$$yp := y(x) = \left(2 - \frac{\sqrt{2}}{2}\right) \mathrm{e}^{(3+2\sqrt{2})\,x} + \left(2 + \frac{\sqrt{2}}{2}\right) \mathrm{e}^{-(-3+2\sqrt{2})\,x} \tag{3}$$

#plot yp vs x (expand x_range and y_range until yp_max is visible)

plot(*rhs*(*yp*), x = −10..15, y = 0..100);



yp will keep growing since there are two distinct roots with an $\exp(e^{+-x})$ *therefor yp_max is* infinity

#Pr2.6 Non-Homogeneous Equation, complex roots

restart;

#find the General Solution of $[y'' + 2\,y' + 145\,y = e^{-0.05\,t}]$ **and use** *the general solution* **to** *find* **and** *plot the particular solution ypartic* **or** *yp*

#Step1 find the homogeneous solution by letting y''+2y'+145y=0

$ODE := (D@@2)(y)(t) + 2\cdot D(y)(t) + 145\cdot y(t) = \exp(-0.05\cdot t);$

$$ODE := D^{(2)}(y)(t) + 2\,D(y)(t) + 145\,y(t) = e^{-0.05\,t} \qquad (4)$$

#Step2 dsolve the ODE to find the general solution y(t)

$dsolve(ODE);$

$$y(t) = e^{-t}\sin(12\,t)\,c_2 + e^{-t}\cos(12\,t)\,c_1 + \frac{400\,e^{-\frac{t}{20}}}{57961} \qquad (5)$$

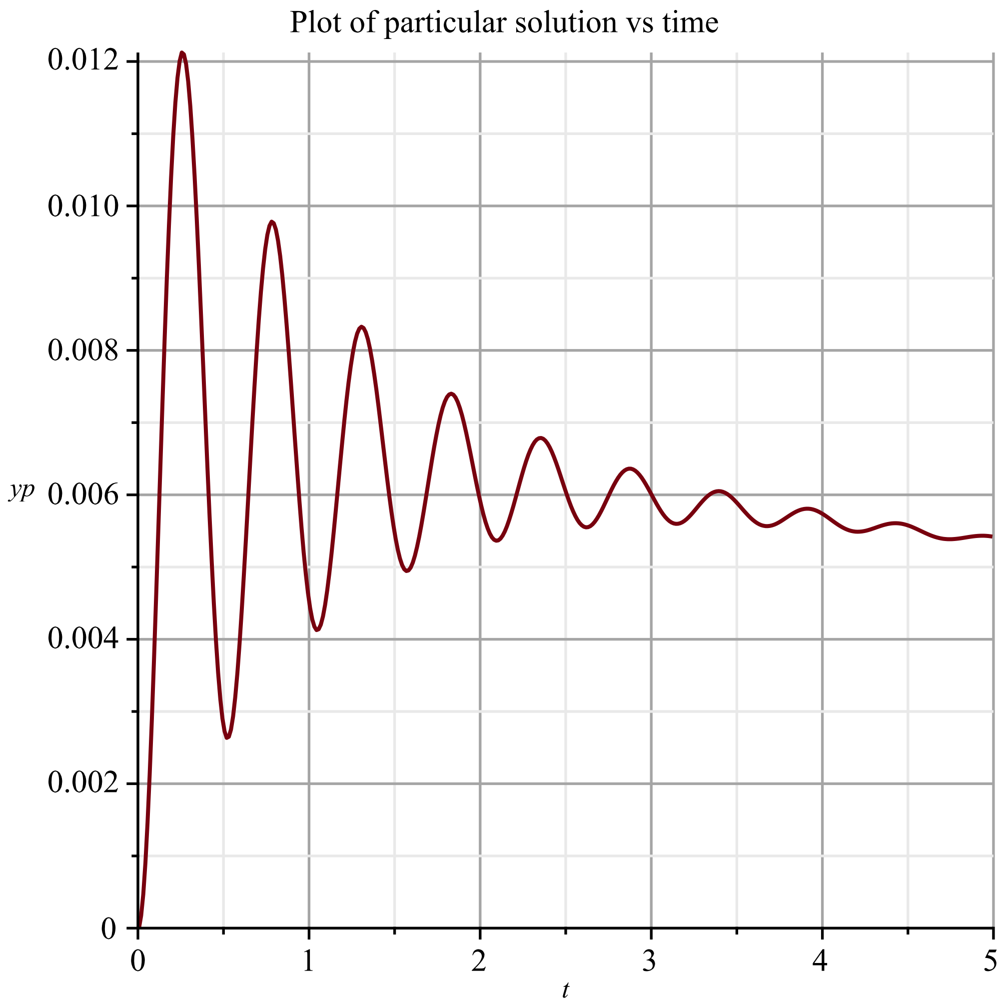
#Step3 determine the particular solution of the ODE

$yp := dsolve(\{ODE, y(0) = 0, D(y)(0) = 0\}, y(t));$

$$y(t) = -\frac{95\,e^{-t}\sin(12\,t)}{173883} - \frac{400\,e^{-t}\cos(12\,t)}{57961} + \frac{400\,e^{-\frac{t}{20}}}{57961} \tag{6}$$

#Plot the particular solution yp vs time (t) from t=0 since velocity or y'(0)=0

plot(*rhs*(*yp*), *t* = 0 ..5, *labels* = [*t* ,'*yp*'], *title* = "Plot of particular solution vs time", *gridlines* = *true*);



#Pr3.16 Wronskian

#Detemine if $y=[e^{3x}, e^{-5x}, e^{6x}]$ *form the basis of the solution* **for** *the ODE* $y''' - 4\,y'' - 27\,y' + 90\,y = 0$

restart;

#import the necessary libraries

with(*LinearAlgebra*) : *with*(*VectorCalculus*) :

#define y1,y2 and y3 as the given initial solutions

$y1 := \exp(3\cdot x) : y2 := \exp(-5\cdot x) : y3 := \exp(6\cdot x) :$

#define y =[y1,y2,y3]

$y := [y1, y2, y3];$

$$y := [e^{3x}, e^{-5x}, e^{6x}] \tag{7}$$

#define the Wronskian of y interms of x

$A := Wronskian(y, x);$

$$A := \begin{bmatrix} e^{3x} & e^{-5x} & e^{6x} \\ 3\,e^{3x} & -5\,e^{-5x} & 6\,e^{6x} \\ 9\,e^{3x} & 25\,e^{-5x} & 36\,e^{6x} \end{bmatrix} \tag{8}$$

#find the determinant of the wronskian

$W := Determinant(A);$

$$W := -264\,e^{3x}\,e^{-5x}\,e^{6x} \tag{9}$$

$simplify(\%);$

$$-264\,e^{4x} \tag{10}$$

#Since W is not equal to Zero therfore the 3 solutions form the basis of solutions for the given ODE on any interval

#Pr3.21 RLC-Circuit

#FInd current i(t) in the RLC-Circuit. Note the current i(t) in a RLC_Circuit is a Non-Homogeneous ODE where LHR=r

#R=16ohms, L=16henrys, C=$\frac{1}{4}$farad,* and $E = 260\cos(4t)$ *volts

#Define the circuit elements

$L := 16 : R := 16 : C := \frac{1}{4} : E0 := 260 : \omega := 4 :$

#define the ODE as LHS and r

$LHS := L\cdot diff(i(t), t, t) + R\cdot diff(i(t), t) + \frac{1}{C}\cdot i(t);$

$$LHS := 16\,\frac{d^2}{dt^2}\,i(t) + 16\,\frac{d}{dt}\,i(t) + 4\,i(t) \tag{11}$$

$r := 260\cdot\cos(\omega\cdot t);$

$$r := 260\cos(4t) \tag{12}$$

#determine the general solution ih of the homogeneousODE

$ih := evalf[3](\,dsolve(LHS = 0)\,);$ *#eval as a floating point with 3 signicant figures*

$$ih := i(t) = c_1\,e^{-0.500\,t} + c_2\,e^{-0.500\,t}\,t \tag{13}$$

#determine the general solution ih_n of the non-homogeneousODE

$ih\_n := evalf[3](\,dsolve(LHS = r)\,);$ *#eval as a floating point with 3 signicant figures*

$$ih\_n := i(t) = e^{-0.500\,t}\,c_2 + e^{-0.500\,t}\,t\,c_1 - 0.969\cos(4.\,t) + 0.246\sin(4.\,t) \tag{14}$$

#find and plot the particular solution ip using the initial conditions i(0)=0 and i'(0)=0

$ip := dsolve(\{LHS = r, i(0) = 0, D(i)(0) = 0\}, i(t));$

$$ip := i(t) = \frac{63\,e^{-\frac{t}{2}}}{65} - \frac{e^{-\frac{t}{2}}\,t}{2} - \frac{63\cos(4t)}{65} + \frac{16\sin(4t)}{65} \tag{15}$$

#Plot the particular solution ip vs time (t) from t=0

$plot(rhs(ip), t = 0 ..10, labels = [t, 'ip'], title = \text{"Plot of particular solution ip(t) vs time"}, gridlines = true);$