0.1

Saturday, September 2, 2023 6:15 AM

a) $z = 8 + j3 \quad v = 9 - j2$

i) $Re(z) + Im(v)$

$Re(z) = 8 \;,\; Im(v) = -2$

$$\boxed{Re(z) + Im(v) = 8 - 2 = 6}$$

ii) $|z+v|$

Step 1 $\Rightarrow$ $z+v = \begin{array}{r} 8 + j3 \\ + \; 9 - j2 \\ \hline 17 + j1 \end{array}$

step 2 $|z+v| = \sqrt{17^2 + 1^2} = \sqrt{289+1}$

$= \sqrt{290}$

$$\boxed{|z+v| = 17.0294}$$

iii) $|zv|$

Step 1 $\Rightarrow$ $zv = (8 + j3)\cdot(9 - j2)$

$= 72 - j16 + j27 - j^2 6$

note $j^2 = -1$

$\therefore zv = 72 + 6 + j\,\underline{11}$

$= 78 + j11$

Step 2 $|ZV| = \sqrt{78^2 + 11^2} = \sqrt{6205}$

$$\boxed{|ZV| = 78.77}$$

iv) $\angle Z + \angle V$

$\angle Z = \left(\frac{180^\circ}{\pi}\right) \cdot \tan^{-1}\left(\frac{3}{8}\right)$

$\angle Z = 20.56^\circ$

$\angle V = \left(\frac{180}{\pi}\right) \cdot \tan^{-1}\left(\frac{-2}{9}\right)$

$\angle V = -12.53^\circ$

$\angle Z + \angle V = 20.56^\circ - 12.53$

$$\boxed{\angle Z + \angle V = 8.03^\circ}$$

v) $\left|\frac{V}{Z}\right|$

$\frac{V}{Z} = \frac{V \times Z \text{ Conjugate}}{Z \times Z \text{ Conjugate}}$

$$Z \times Z_{conj}$$

$$\frac{V}{Z} = \frac{(9-j2)(8-j3)}{(8+j3)(8-j3)}$$

$8^2 - \cancel{24j} + \cancel{24j} - 9j^2$
$+9$

$$\frac{V}{Z} = \frac{(72-6)+j(-16-27)}{64+9}$$

$$\frac{V}{Z} = \frac{66-j43}{73} = \frac{66}{73} - j\frac{43}{73}$$

$$\left|\frac{V}{Z}\right| = \sqrt{\left(\frac{66}{73}\right)^2 + \left(\frac{43}{73}\right)^2} = \sqrt{1.164}$$

$$\boxed{\left|\frac{V}{Z}\right| = 1.08}$$

vii) $\angle\left(\frac{V}{Z}\right)$

$$\frac{V}{Z} = \frac{66}{73} - j\frac{43}{73}$$

$$\angle\left(\frac{V}{Z}\right) = \frac{180}{\pi}\tan^{-1}\left(\frac{43}{\cancel{73}} \times \frac{\cancel{73}}{66}\right)$$

$$\angle\left(\frac{V}{Z}\right) = \frac{180}{\pi}\tan^{-1}\left(\frac{43}{\cancel{73}} \times \frac{73}{66}\right)$$

$$= \frac{180}{\pi}\tan^{-1}\left(\frac{43}{66}\right)$$

$$\boxed{\angle\left(\frac{V}{Z}\right) = -33.08^\circ}$$

b)

i) $Z + V = \begin{array}{r} 8 + j3 \\ + \; 9 - j2 \\ \hline 17 + j \end{array}$

$$\boxed{Z + V = 17 + j}$$

ii) $ZV = (8 + j3)\cdot(9 - j2)$

$$\boxed{ZV = 78 + j11}$$

iii)

iii) Z^*

note Z^* = Mirror Z on the imaginary axis

$$\boxed{\therefore \quad Z^* = 8 - j3}$$

iv) ZZ^*

$$ZZ^* = (8+j3)\cdot(8-j3)$$
$$= 8^2 - j^2 3^2 = 64+9$$

$$\boxed{ZZ^* = 73}$$

v) $Z - V$

$$\begin{array}{r} 8 + j3 \\ - \quad 9 - j2 \\ \hline -1 + j5 \end{array}$$

$$\boxed{Z - V = -1 + j5}$$

0.2

Sunday, August 27, 2023 5:11 PM

a) $Z = 6e^{j\frac{\pi}{4}}$

note $re^{j\theta} = r[\cos(\theta) + j\sin(\theta)]$ Euler Identity

i) $Re(z) = r\cos(\theta)$

$Re(z) = 6\cos\left(\frac{\pi}{4}\right)$

$\boxed{Re(z) = 4.24}$

ii) $Im(z) = r\sin(\theta)$

$Im(z) = 6\sin\left(\frac{\pi}{4}\right)$

$\boxed{Im(z) = 4.24}$

---

b) $z = 8 + j3$ $v = 9 - j2$

i) $Re(z) = \frac{1}{2}(z + z^*)$

$Re(z) = 8$

$z + z^* = \begin{array}{r} 8 + j3 \\ + 8 - j3 \\ \hline 16 \end{array}$

$$z + z^* = \begin{array}{r} 8 + j3 \\ +\ 8 - j3 \\ \hline 16 \end{array}$$

$$\frac{1}{2}(z + z^*) = \frac{1}{2}(16) = 8$$

$$\boxed{\therefore\ Re(z) = \frac{1}{2}(z + z^*) = 8}$$ True

ii) $Im(v) = -\frac{1}{2}j(v - v^*)$

$$Im(v) = -2$$

$$v - v^* = \begin{array}{r} 9 - j2 \\ -\ 9 + j2 \\ \hline 0 - j4 \end{array}$$

$$-\frac{1}{2}j(v - v^*) = -\frac{1}{2}j(-j4) = j^2 2 = -2$$

$$\boxed{\therefore\ Im(v) = -\frac{1}{2}j(v - v^*) = -2}$$ True

iii) $Re(z + v^*) = Re(z + v)$

$$Re\left[\begin{array}{r} 8 + j3 \\ +\ 9 + j2 \\ \hline 17 + j5 \end{array}\right] = Re\left[\begin{array}{r} 8 + j3 \\ +\ 9 - j2 \\ \hline 17 + j \end{array}\right]$$

$$Re(17 + j5) = Re(17 + j)$$

$$Re(17) = Re(17)$$

$Re(17) = Re(17)$

$$\boxed{\therefore Re(z+v^*) = Re(z+v) = 17}$$

True

iv) $Im(z+v^*) = Im(z-v)$

$$Im\left[\frac{\begin{matrix} 8+j3 \\ +\ 9+j2 \end{matrix}}{17+j5}\right] = Im\left[\frac{\begin{matrix} 8+j3 \\ -\ 9-j2 \end{matrix}}{-1+j5}\right]$$

$Im(5) = Im(5)$

$$\boxed{\therefore Im(z+v^*) = Im(z-v) = 5}$$

True

0.9

Saturday, September 2, 2023 8:52 AM

a) $z = 1 + j1$ $\quad w = e^z$

i) $\log(w) = \log e^z = z$

$\boxed{\therefore \log(w) = z}$

ii) $Re(w)$

Using Euler Identity $re^{j\theta} = r[\cos(\theta) + j\sin(\theta)]$

$w = e^z = e^{1+j1} = e^1 \times e^{j1}$

$\boxed{Re(w) = e^1 \cos(1) = 1.47}$

iii) $Im(w) = e^1 \sin(1)$

$\boxed{Im(w) = 2.29}$

---

b) $w + w^*$

Step 1 change w to Rectangular Complex form

$w = e^1 e^{j1} = e^1[\cos(1) + j\sin(1)]$

$w = 1.47 + j2.28$

$$w + w^* = \left[\begin{array}{r} 1.47 + j\,2.28 \\ + \;\; 1.47 - j\,2.28 \\ \hline 2.94 + 0 \end{array}\right]$$

$\boxed{\therefore w + w^* = 2.94}$

---

c) $|w| = \sqrt{1.47^2 + 2.28^2}$

$\boxed{|w| = \sqrt{7.3593} = 2.71}$

$\angle w = \frac{180}{\pi} \tan^{-1}\left(\frac{2.28}{1.47}\right)$

$\boxed{\angle w = 57.19^\circ}$

$$\boxed{\angle w = 57.19^\circ}$$

$$\left|\log(w)\right|^2$$

step 1 $\Rightarrow \log(w) = \log e^z = z = 1 + j1$

$$\left|\log(w)\right|^2 = |z|^2 = \left(\sqrt{1^2 + 1^2}\right)^2$$

$$= \left(\sqrt{2}\right)^2 = 2^{\cancel{2} \times \frac{1}{\cancel{2}}} = 2$$

$$\boxed{\left|\log(w)\right|^2 = 2}$$

---

d) $\cos(1)$ in terms of w using Euler's Identity

$w = e^z = e^{1+j1} = e^1 \times e^{j1}$

$\theta = 1 \quad r = e^1$

Euler's Identity $\quad r e^{j\theta} = r\left[\cos(\theta) + j\sin(\theta)\right]$

$$e^1 \times e^{j1} = e^1\left[\cos(1) + j\sin(1)\right]$$

$$\cancel{2.72}\, e^{j1} = \cancel{2.72}\left[\cos(1) + j\sin(1)\right]$$

$$e^{j1} = \cos(1) + j\sin(1)$$

$$\boxed{\cos(1) = e^{j1} - j\sin(1)}$$

0.10

Saturday, September 2, 2023 9:44 AM

e) find Phasors

i) $4\cos\left(2t+\frac{\pi}{3}\right) \Rightarrow A\cos(\Omega t+\theta)$

$\boxed{\text{In phasor} = 4\angle\frac{\pi}{3}}$

ii) $-4\cos\left(2t+\frac{\pi}{3}\right)$

$\boxed{\text{In phasor} = -4\angle\frac{\pi}{3}}$

iii) $4\cos\left(2t+\frac{\pi}{3}\right) - 4\sin\left(2t+\frac{\pi}{3}\right)$

step 1 Change $-4\sin\left(2t+\frac{\pi}{3}\right)$ to the Cosine Signal

$$-4\sin\left(2t+\frac{\pi}{3}\right) = +4\cos\left[2t+\left(\frac{\pi}{3}+\pi-\frac{\pi}{2}\right)\right]$$

$$= +4\cos\left[2t+\frac{2\pi+6\pi-3\pi}{6}\right]$$

$$= +4\cos\left(2t+\frac{\pi}{6}\right)$$

$$4\cos\left(2t+\frac{\pi}{3}\right) + 4\cos\left(2t+\frac{\pi}{6}\right)$$

$$\Rightarrow \underset{\text{Amp}}{4}\left[\cos\left(\underset{\text{freq}}{2t}+\underset{\theta}{\frac{\pi}{3}}\right) + \cos\left(\underset{\text{freq}}{2t}+\underset{\theta}{\frac{\pi}{6}}\right)\right]$$

$$\text{Phasor} \Rightarrow 4\angle\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$$

$$\frac{2\pi+\pi}{6} = \frac{\cancel{3}^{1}\pi}{\cancel{6}_{2}} = \frac{\pi}{2}$$

$$\boxed{\text{Phasor} \Rightarrow 4\angle\frac{\pi}{2}}$$

0.13

Saturday, September 2, 2023 10:21 AM

$x(t) = 4\cos(2\pi t)$

using nyquist theorem $f_s \geq 2f$

Signal $= A\cos(\omega t)$

$\omega = 2\pi f =$

$x(t) = 4\cos(2\pi t)$

$2\pi f = 2\pi$

$f = 1\ Hz$

$\therefore$ for signal to exist or not lost $f_s \geq 2\ Hz$

ⓐ $T_s = 0.1$

$f_s = \frac{1}{T_s} = \frac{1}{0.1} = 10\ Hz$ signal not lost

ⓐ $T_s = 0.5$

$f_s = \frac{1}{0.5} = 2\ Hz$ signal not lost

ⓐ $T_s = 1$

$f_s = \frac{1}{1} = 1\ Hz$ signal lost

Check matlab Code for 2^{nd} part of questn 0.13

0.19

Saturday, September 2, 2023 12:44 PM

$$\int_0^1 t\,dt = \int_0^{1/2} t\,dt + \int_{1/2}^1 t\,dt$$

$$\left.\frac{t^2}{2}+c\right]_0^1 = \left.\frac{t^2}{2}+c\right]_0^{1/2} + \left.\frac{t^2}{2}+c\right]_{1/2}^1$$

$$\frac{1^2}{2} - \underbrace{\frac{0^2}{2}+(c-c)}_{0} = \left[\frac{(1/2)^2}{2} - \underbrace{\frac{0^2}{2}+(c-c)}_{0}\right] + \left[\frac{1^2}{2} - \frac{(1/2)^2}{2} + \underbrace{(c-c)}_{0}\right]$$

$$\frac{1}{2} = \underbrace{\frac{\left(\frac{1}{2}\right)^2}{2} + \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^2}{2}}_{0}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\boxed{\therefore \int_0^1 t\,dt = \int_0^{1/2} t\,dt + \int_{1/2}^1 t\,dt = 1/2}$$

Sumation

$$S = \sum_{n=0}^{100} n = \frac{n}{2}\left(a_1 + a_n\right)$$

$$a_1 = 1 \qquad a_n = 100 \qquad n = 100$$

$$S = \frac{100}{2}\left(1 + 100\right)$$

$$= 50(101)$$

$= 50(101)$

$$\boxed{S = 5050}$$

i) $S_1 = \sum_{n=0}^{50} n + \sum_{n=50}^{100}$

$$S_1 = \frac{50}{2}(1+50) + \frac{50}{2}(51+100)$$

$$S_1 = 25(51) + 25(151)$$

$$S_1 = 1275 + 3775$$

$$S_1 = 5050$$

$$\boxed{S_1 = S = 5050}$$

ii) $S_2 = \sum_{n=0}^{50} n + \sum_{n=51}^{100} n$

$$S_2 = \frac{50}{2}(1+50) + \frac{49}{2}(52+100)$$

$$S_2 = 25(51) + \frac{49}{\cancel{2}_1}(\cancel{152}^{76})$$

$$= 1275 + (49 \times 76)$$

$S_2 = 1275 + (49 \times 76)$

$S_2 = 4999$

$$\boxed{\therefore \quad S_2 \neq S}$$

Question 0.21 Asked to use Matlab