HW13

Sunday, December 3, 2023 2:43 AM

$$H(s) = \frac{1}{s^3 + 0.4s^2 + 1.14s + 0.22}$$

Write the phase variable state model

$$H(s) = \frac{Y(s)}{U(s)}$$

Output = System • Input

⇒ split the system into

$Y(s) = H(s) \cdot U(s)$

$$Y(s)\left(s^3 + 0.4s^2 + 1.14s + 0.22\right) = U(s)$$

$$s^3 Y(s) + 0.4s^2 Y(s) + 1.14s\, Y(s) + 0.22\, Y(s) = U(s)$$

⇒ change to differential equation

$$\dddot{y} + 0.4\ddot{y} + 1.14\dot{y} + 0.22y = u$$

⇒ Define state variables $(q_1, \dots q_n)$ n = highest order of S in a single input & single output system

$s^3 \Rightarrow q_1, q_2, \& q_3$

$$\left.\begin{aligned} q_1 &= y \\ q_2 &= \dot{y} \\ q_3 &= \ddot{y} \end{aligned}\right\} \text{plant matrix}$$

$$\dot{q}_1 = q_2$$

$$\dot{q}_2 = q_3$$

$$\dot{q}_3 = \dddot{y} = -0.4\ddot{y} - 1.14\dot{y} - 0.22y + u$$

$$= -0.4q_3 - 1.14q_2 - 0.22q_1 + u$$

in terms state-space variable

⇒ Arrange $\dot{q}_1$, $\dot{q}_2$, & $\dot{q}_3$

$$\dot{q}_1 = 0 + q_2 + 0 + u$$

$$\dot{q}_2 = 0 + 0 + q_3 + u$$

$$\dot{q}_3 = 0.22q_1 - 1.14q_2 - 0.4q_3 + u$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.22 & -1.14 & -0.4 \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$