Pr.7.2 The commands are as follows

```
> with(LinearAlgebra):
> A := <<7, 3> | <-4, -5> | <5, 0>>:
  B := Matrix([[5, -7, 8], [-6, 2, 4]]):
> A.Transpose(A);                                # Resp.
```

$$\begin{bmatrix} 90 & 41 \\ 41 & 34 \end{bmatrix}$$

```
> Transpose(A).A;                                # Resp.
```

$$\begin{bmatrix} 58 & -43 & 35 \\ -43 & 41 & -20 \\ 35 & -20 & 25 \end{bmatrix}$$

```
> %^2;                                           # Resp.
```

$$\begin{bmatrix} 6438 & -4957 & 3765 \\ -4957 & 3930 & -2825 \\ 3765 & -2825 & 2250 \end{bmatrix}$$

> $(A + B) \cdot Transpose(A - B);$

$$\begin{bmatrix} -48.00 & 133.00 \\ -27.00 & -22.00 \end{bmatrix}$$

>

Pr.7.8 Show that the left-hand side of the rule minus the right-hand side equals the zero matrix:

```
> with(LinearAlgebra):
> A := Matrix([[a11, a12], [a21, a22]]):
> B := Matrix([[b11, b12], [b21, b22]]):
> Transpose(A.B) - Transpose(B).Transpose(A);    # Resp.
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A := \langle\langle a, b\rangle | \langle c, d\rangle\rangle; B := \langle\langle e, f\rangle | \langle g, h\rangle\rangle$

$$A := \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$B := \begin{bmatrix} e & g \\ f & h \end{bmatrix}$$

$Sol1 := Transpose(A \cdot B);$

$$Sol1 := \begin{bmatrix} a\,e + c\,f & b\,e + d\,f \\ a\,g + c\,h & b\,g + d\,h \end{bmatrix}$$

$Sol2 := Transpose(B) \cdot Transpose(A);$

$$Sol2 := \begin{bmatrix} a\,e + c\,f & b\,e + d\,f \\ a\,g + c\,h & b\,g + d\,h \end{bmatrix}$$

$Sol2 - Sol1;$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Pr.7.12 $\mathbf{x}$ must be orthogonal to each of the three given vectors. Thus, $\mathbf{c} \bullet \mathbf{x} = 0$, $\mathbf{d} \bullet \mathbf{x} = 0$, $\mathbf{e} \bullet \mathbf{x} = 0$. This is a linear system in the unknown components of $\mathbf{x}$, the coefficients of the system being the components of the given vectors. Hence the coefficient matrix $\mathbf{A}$ of the system has the row vectors $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$. You can obtain $\mathbf{A}$ from $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$ by augmenting, which gives you the matrix with those vectors as *column* vectors, and then taking the transpose.

```
> with(LinearAlgebra):
> c := <3 | 2 | -2 | 1 | 0>:
> d := <2 | 0 | 3 | 0 | 4>:
> e := <1 | -3 | -2 |-1 | 1>:
> A := <c, d, e>;
> b := <0, 0, 0>;
```

`linsolve` then gives you the solution, depending on two arbitrary parameters t_2 (the third component of $\mathbf{x}$) and t_1 (the fifth component of $\mathbf{x}$).

Pr.7.14 You can use the determinant of the matrix with the given vectors as row vectors and conclude from its vanishing that the vectors are linearly dependent.

```
> with(LinearAlgebra):
> A := Matrix([[2, -10, 0], [64, -56, -18], [-32, -16, 12]]);
> Determinant(%);                                # Resp. 0
> Rank(A);                                       # Resp. 2
```

Pr.8.6 Note that the eigenvalues are real, as they should be, whereas the eigenvectors are complex (and you may obtain them multiplied by some real or complex factor).

```
> with(LinearAlgebra):
> A := <<2 | 1-I>, <1+I | 3>>;
```

$$A := \begin{bmatrix} 2 & 1-\mathrm{I} \\ 1+\mathrm{I} & 3 \end{bmatrix}$$

```
> HermitianTranspose(A);
```

\# Resp. $\begin{bmatrix} 2 & 1-\mathrm{I} \\ 1+\mathrm{I} & 3 \end{bmatrix}$

```
> Eigenvectors(A);
```

\# Resp. $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}-\frac{1}{2}\mathrm{I} & -1+\mathrm{I} \\ 1 & 1 \end{bmatrix}$

Pr.8.12 You will see that the eigenvalues of **A** are 6 and 1

```
> with(LinearAlgebra):
> A := Matrix([[6, 5], [3, 4]]);
```

\# Resp. $A := \begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix}$

```
> eig := Eigenvectors(A);
```

\# Resp. $eig := \begin{bmatrix} 1 \\ 9 \end{bmatrix}, \begin{bmatrix} -1 & \frac{5}{3} \\ 1 & 1 \end{bmatrix}$

```
> X := eig[2];
```

\# Resp. $X := \begin{bmatrix} -1 & \frac{5}{3} \\ 1 & 1 \end{bmatrix}$

```
> Diag := (MatrixInverse(X).A).X;
```

\# Resp. $Diag := \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$