

#Problem Sets for Chapter1

#Pr1.2 Direction field

restart

#Step1 import the necessary Libraries

with(DEtools)

#Step2 Define the ODE $y' = \frac{-13x^2}{17y}$

$ode1 := \text{diff}(y(x), x) = -\frac{13 \cdot x^3}{17 \cdot y(x)}$

$$ode1 := \frac{d}{dx} y(x) = -\frac{13 x^3}{17 y(x)} \quad (1)$$

#Step3 Define initial conditions [0,1] and [0,1.4]

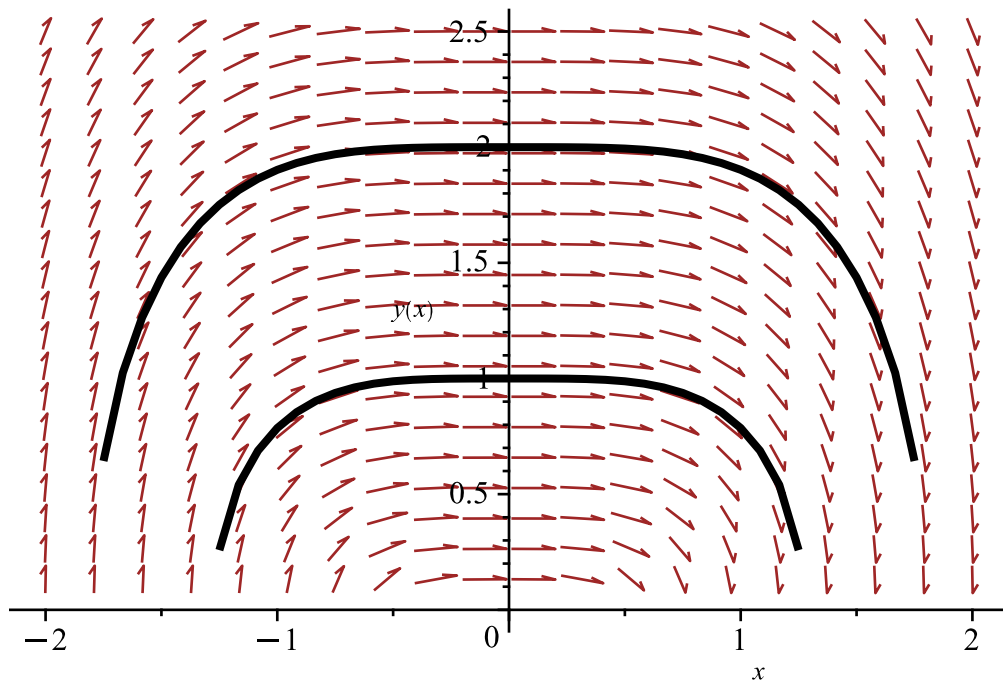
$inits := \{[0, 1], [0, 2]\};$

$inits := \{[0, 1], [0, 2]\} \quad (2)$

#Step3 perform the DE plot of the ODE with the initial condition and scale if necessary

$\text{DEplot}(ode1, y(x), x = -2..2, y = 0..2.5, inits, scaling = constrained, linecolor = black);$

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of 1.2716978, probably a singularity
cannot evaluate the solution further left of -1.2716978, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of 1.7984522, probably a singularity
cannot evaluate the solution further left of -1.7984522, probably a singularity



#Pr1.4 Exponential approach

restart

#Step1 Define the ODE $y' + 0.5y = 1$

ode := diff(*y*(*x*), *x*) + 0.5 *y*(*x*) = 1;

$$\text{ode} := \frac{d}{dx} y(x) + 0.5 y(x) = 1 \quad (3)$$

#Step2: solve the ODE with visualize the generic solution for the ODE

sol1 := dsolve(*ode*);

$$\text{sol1} := y(x) = 2 + e^{-\frac{x}{2}} c_1 \quad (4)$$

#Step3: apply the initial conditions $x=0$, and $y=0$ [$y(0)=0$] to the ode

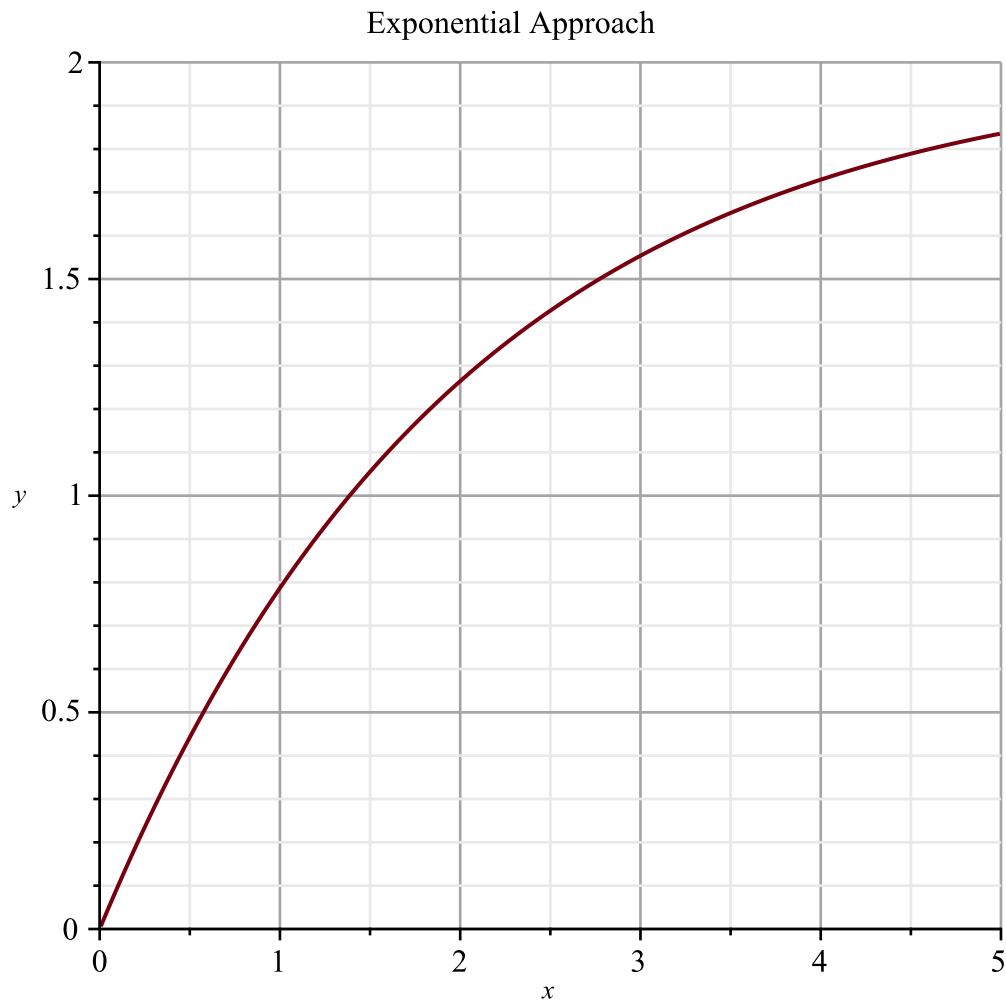
sol2 := dsolve({*ode*, *y*(0) = 0});

$$\text{sol2} := y(x) = 2 - 2e^{-\frac{x}{2}} \quad (5)$$

sol2 indicates that the initial condition is true @ $x=0$ $y(0)=2-2e^0=0$

#Step4: Plot the graph to visualize the model

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plot(rhs(sol2), x=0..5, y=0..2, labels=[x, y], title="Exponential Approach", gridlines=true);
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#Pr1.12 Beat

restart;

#Step1: define P and Q of the ODE

$P := \csc(x);$

$$P := \csc(x) \quad (6)$$

$Q := y \cdot \cot(x) \cdot \csc(x) + 100 \cdot \cos(30 \cdot x);$

$$Q := y \cot(x) \csc(x) + 100 \cos(30 x) \quad (7)$$

#Step1 obtain partial diff of P and Q

$P_diff := \text{diff}(P, x);$

$$P_diff := -\cot(x) \csc(x) \quad (8)$$

$Q_diff := \text{diff}(Q, y);$

$$Q_diff := \cot(x) \csc(x) \quad (9)$$

#Therefore the Two equations are not exact P_diff is not equal to Q_diff

#Step3: use $F(x)$ as integration factor

$eq1 := \text{diff}(F(x) \cdot P, x) - \text{diff}(F(x) \cdot Q, y) = 0;$

$$eq1 := \left(\frac{d}{dx} F(x) \right) \csc(x) - 2 F(x) \cot(x) \csc(x) = 0 \quad (10)$$

$\text{simplify}\left(\frac{eq1}{\csc(x)}\right);$

$$-2 F(x) \cot(x) + \frac{d}{dx} F(x) = 0 \quad (11)$$

$sol := \text{dsolve}(\%);$

$$sol := F(x) = - \frac{c_1 (-1 + \cos(2x))}{2} \quad (12)$$

#Pr1.18 RL-Circuit

restart;

Step1 Obtain mathematical Model for the given RL circuit

#Use this equation $L \frac{d}{dt} i(t) + R \cdot i(t) = v(t)$ where $L = 0.5 \text{ H}$, $R = 7 \text{ ohms}$, and $V = 5 \text{ V}$

$ode := 0.5 * \text{diff}(i(t), t) + 7 * i(t) = 5;$

$$ode := 0.5 \frac{d}{dt} i(t) + 7 i(t) = 5 \quad (13)$$

#Step2: define $i(0)$ at the different given values

#use $i(0) = 5 \text{ amps}$

$sol1 := \text{dsolve}(\{ode, i(0) = 5\}, i(t));$

$$sol1 := i(t) = \frac{5}{7} + \frac{30 e^{-14t}}{7} \quad (14)$$

#use $i(0) = 2.5 \text{ amps}$

$sol2 := \text{dsolve}(\{ode, i(0) = 2.5\}, i(t));$

$$sol2 := i(t) = \frac{5}{7} + \frac{25 e^{-14t}}{14} \quad (15)$$

#use $i(0) = 1 \text{ amps}$

$sol3 := \text{dsolve}(\{ode, i(0) = 1\}, i(t));$

$$sol3 := i(t) = \frac{5}{7} + \frac{2 e^{-14 t}}{7} \quad (16)$$

#use $i(0) = 0$ amps

$sol4 := dsolve(\{ode, i(0) = 0\}, i(t));$

$$sol4 := i(t) = \frac{5}{7} - \frac{5 e^{-14 t}}{7} \quad (17)$$

Plot the functions

$plot(\{rhs(sol1), rhs(sol2), rhs(sol3), rhs(sol4)\}, t = 0 .. 2, y = 0 .. 5, labels = [t, i(t)], title = "RL-Circuit", gridlines = true);$

