

The additivity of ignorance

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Proposition. ADDITIVITY OF IGNORANCE

Nulla di tutto ciò è legale ma ora so usare latex

Let $\sigma : \mathbb{N} \rightarrow \mathbb{R}$ be a possible ignorance function, if $\sigma(1) = 0$ and $\sigma(xy) = f(\sigma(x), \sigma(y))$, where $f : \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function, then $\sigma(xy) = \sigma(x) + \sigma(y)$.

Proof.

$$\sigma(xy) = f(\sigma(x), \sigma(y)) \quad (1)$$

$$\frac{\partial f}{\partial x_1} = g(x_1) \quad (2)$$

$$\frac{\partial f}{\partial x_2} = g(x_2) \quad (3)$$

$$f = G(x_1) + c(x_2) + d_1 \quad (4)$$

$$f = G(x_2) + c(x_1) + d_2 \quad (5)$$

$$f = G(x_1) + G(x_2) + d \quad (6)$$

$$\sigma(xy) = G(\sigma(x)) + G(\sigma(y)) + d \quad (7)$$

$$\left. \frac{\partial \sigma(xy)}{\partial x} \right|_{y=1} = y \Big|_{y=1} \frac{d\sigma(x)}{dx} = \frac{dG(x_1)}{dx_1} \frac{dx_1}{dx} = \frac{dG(\sigma(x))}{d\sigma} \frac{d\sigma}{dx} \quad (8)$$

$$\frac{dG(\sigma)}{d\sigma} = 1 \quad (9)$$

$$G(\sigma) = \sigma + c \quad (10)$$

$$\sigma(xy) = \sigma(x) + \sigma(y) + 2c + d \quad (11)$$

$$\sigma(xy) = \sigma(x) + \sigma(y) + C \quad (12)$$

$$\sigma(x) = \sigma(1) + \sigma(x) + C = \sigma(x) + C \quad (13)$$

$$\sigma(1) = 0 \rightarrow C = 0 \quad (14)$$

$$(15)$$

□