Geometric Deep Learning
Beyond Euclidean Domains

1 Geometric Priors

Definition 1.1. *Our compact euclidean domain Ω*
$\Omega := \prod_{i\in I}[0,1]$.

Definition 1.2. *Classification*
Let $x \in L^2 := L^2(\Omega)$ then $f : L^2 \to \mathscr{C}$ surjective is said to be a classification of L^2 on the set $\mathscr{C}$.

Definition 1.3. *Training set*
Let f be a classification of L^2 on $\mathscr{C}$ and $\{x_i\}_{i\in I} \subset L^2$ then the set $\{(x_i, f(x_i))\}_{i\in I}$ is called a training set for f.

Proposition 1.1. *The classification f is not injective*
Let f be a classification of L^2 on $\mathscr{C}$ then, given the inevitable noise acting on data, there exists a real positive ε such that $\forall (x, x_\varepsilon) \in L^2 \times L^2 : \int\limits_{\Omega} |x - x_\varepsilon|^2 < \varepsilon$ we have that $f(x) = f(x_\varepsilon)$.

Given ideal data classification we can define two functions f-equivalent if and only if their images via the classification f are equal according to an equivalence on $\mathscr{C}$ which so far can be any set.

Proposition 1.2. *The relation $\simeq$ is an equivalence relation*
Let $x, y, z \in L^2$ we define $x \simeq y \iff f(x) = f(y)$ where f is a classification of L^2 on $\mathscr{C}$, then:
(i) $x \simeq x$
(ii) $x \simeq y \iff y \simeq x$
(iii) $x \simeq y, y \simeq z \implies x \simeq z$

Proof. (i),(ii) and (iii) follow from the the equivalence on $\mathscr{C}$ by which they are defined. □

Definition 1.4. *Translation operator*
Let $x \in L^2$ and $\nu \in \Omega$ then $T_\nu : L^2 \to L^2$ such that $x(\xi) \mapsto x(\xi - \nu)$ is said to be a translation operator.

Definition 1.5. *Local deformation operator*
Let $x \in L^2$ and $\tau \in C^\infty(\Omega, \Omega)$ then $L_\tau : L^2 \to L^2$ such that $x(\xi) \mapsto x(\xi - \tau(\xi))$ is said to be a local deformation operator according to the smooth vector field τ.

Definition 1.6. *Invariance*
A classification f of L^2 on $\mathscr{C}$ is said to be A-invariant, where $A : L^2 \to L^2$, if and only if $f(A(x)) = f(x)\ \forall x \in L^2$.

Definition 1.7. *Equivariance*
A classification f of L^2 on $\mathscr{C}$ is said to be A-equivariant, where $A : L^2 \to L^2$, if and only if $f(A(x)) = A(f(x))\ \forall x \in L^2$. This is well defined only if A is defined to act on $\mathscr{C}$

Proposition 1.3. *If f is translation invariant then it is stable under local deformations*
Let f be a translation invariant classification of L^2 on $\mathscr{C}$ then $|f(L_\tau(x)) - f(x)| \approx |J_\tau|$ where $(J_\tau)_{ij} = (\frac{\partial \tau_i}{\partial \xi_j})$ under some misterious norm.

Proof. To be found... □