Geometric Deep Learning
Beyond Euclidean Domains

1 Geometric Priors

In this section we want to define the mathematical foundation of deep learning via CNN'S.

Definition 1.1. *Our compact euclidean domain* Ω
$\Omega := \prod_{i \in I} [0,1]$.

Definition 1.2. *Classification*
Let $x \in L^2 := L^2(\Omega)$ *then* $f : L^2 \to \mathscr{C}$ *surjective is said to be a classification of* L^2 *on the set* $\mathscr{C}$.

Definition 1.3. *Training Set*
Let f *be a classification of* L^2 *on* $\mathscr{C}$ *and* $\{x_i\}_{i \in I} \subset L^2$ *then the set* $\{(x_i, f(x_i))\}_{i \in I}$ *is called a training set for* f.

Proposition 1.1. $card(\mathscr{C}) < card(L^2(\Omega))$
Let f *be a classification of* L^2 *on* $\mathscr{C}$ *then, given the inevitable noise acting on data, there exists a real positive* ε *such that* $\forall (x, x_\varepsilon) \in L^2 \times L^2 : \int_\Omega |x - x_\varepsilon|^2 < \varepsilon$ *we have that* $f(x) = f(x_\varepsilon)$.

Given ideal data classification we can define two function f-equivalent if and only if their images via the classification f are equal according to an equivalence on $\mathscr{C}$.

Theorem 1.2. $\simeq$ *is an equivalence relation*
Let $x, y, z \in L^2$ *we define* $x \simeq y \iff f(x) = f(y)$ *where* f *is a classification of* L^2 *on* $\mathscr{C}$, *then:*
(i) $x \simeq x$
(ii) $x \simeq y \iff y \simeq x$
(iii) $x \simeq y, y \simeq z \implies x \simeq z$

Proof. (i),(ii) and (iii) follow from the the equivalence on $\mathscr{C}$ by which they are defined. □