Alma Mater Studiorum · Università di Bologna

Scuola di Scienze
Dipartimento di Fisica e Astronomia
Corso di Laurea in Fisica

GEOMETRIC DEEP LEARNING

Relatore:
Prof.ssa. Rita Fioresi

Presentata da:
Tommaso Lamma

Anno Accademico 2020/2021

Abstract in italiano...

Abstract in english...

Contents

1 Introduction

1.1 Abstract simplicial complexes

Definition 1.1.1. ***Abstract simplicial complex (finite)***
Let $\mathcal{F}$ be a family of sets we then define an abstract simplicial complex $\mathcal{A}$ to be

$$\mathcal{A} := \{\sigma = \{A_i\}_{i\in I_\sigma} \subset \mathcal{F} : \tau \subset \sigma \Rightarrow \tau \in \mathcal{A}\}$$

where I_σ is a finite set of indexes, we shall call σ abstract simplexes of $\mathcal{A}$.

Definition 1.1.2. ***Dimension of an abstract simplicial complex***
Let $\mathcal{A} = \{\sigma_j\}_{j\in J}$ be an abstract simplicial complex we define its dimension to be

$$dim\mathcal{A} := max_{\sigma_j\in\mathcal{A}} dim(\sigma_j),$$

where $dim(\sigma_j) := |\sigma_j| - 1$.

Definition 1.1.3. ***Abstract graph***
An abstract graph $\mathcal{G} = \{\sigma_j\}_{j\in J}$ is a 1-dimensional abstract simplicial complex whose vertexes and edges are respectively

$$\mathcal{V} := \{\sigma_j \in \mathcal{G} : dim(\sigma_j) = 0\} \text{ and}$$
$$\mathcal{E} := \{\sigma_j \in \mathcal{G} : dim(\sigma_j) = 1\} .$$

In Definition 1.1.1. we tacitly assumed the definition of the abstract simplex σ_j invariant with respect to permutations of the indexes I_j, this assumption establishes the difference between directed and undirected graphs.

Definition 1.1.4. ***Convex envelop of points in*** $\mathbb{R}^n$
Let $\{x_i\}_{i\in I} \subset \mathbb{R}^n$ we define the convex envelope of $\{x_i\}_{i\in I}$ to be

$$\langle x_i\rangle_{i\in I} := \{a = \sum_{i\in I} \lambda_i x_i : \lambda_i \in \mathbb{R},\ \lambda_i > 0,\ \sum_{i\in I} \lambda_i = 1\},$$

which is the smallest convex set containing $\{x_i\}_{i\in I}$.

Definition 1.1.5. ***Affine independency of points in*** $\mathbb{R}^n$

Definition 1.1.6. ***Geometric k-simplexes***

Definition 1.1.7. ***Geometric Simplicial Complex***

Theorem 1.1.1. ***Geometric realization of an abstract simplicial complex***