The additivity of ignorance

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Proposition. ***Additivity of ignorance***
Let $\sigma : \mathbb{N} \to [0, +\infty[$ *be a possible ignorance function, if* $\sigma(1) = 0$ *and* $\sigma(xy)$ *of type* $f(\sigma(x), \sigma(y))$*, where* $f : [0, +\infty[\to [0, +\infty[$ *is a differentiable function then* $\sigma(xy) = \sigma(x) + \sigma(y)$*.*

Proof.

$$\sigma(xy) = f(\sigma(x), \sigma(y)) \tag{1}$$

$$\frac{\partial f}{\partial x_1} = g(x_1) \tag{2}$$

$$\frac{\partial f}{\partial x_2} = g(x_2) \tag{3}$$

$$f = G(x_1) + c(x_2) + d_1 \tag{4}$$

$$f = G(x_2) + c(x_1) + d_2 \tag{5}$$

$$f = G(x_1) + G(x_2) + d \tag{6}$$

$$\sigma(xy) = G(\sigma(x)) + G(\sigma(y)) + d \tag{7}$$

$$\frac{\partial \sigma(xy)}{\partial x}\bigg|_{y=1} = y\bigg|_{y=1} \frac{d\sigma(x)}{dx} = \frac{dG(x_1)}{dx_1}\frac{dx_1}{dx} = \frac{dG(\sigma(x))}{d\sigma}\frac{d\sigma}{dx} \tag{8}$$

$$\frac{dG(\sigma)}{d\sigma} = 1 \tag{9}$$

$$G(\sigma) = \sigma + c \tag{10}$$

$$\sigma(xy) = \sigma(x) + \sigma(y) + 2c + d \tag{11}$$

$$\sigma(xy) = \sigma(x) + \sigma(y) + C \tag{12}$$

$$\sigma(x) = \sigma(1) + \sigma(x) + C = \sigma(x) + C \tag{13}$$

$$\sigma(1) = 0 \to C = 0 \tag{14}$$

$$\tag{15}$$

□