Alma Mater Studiorum · Università di Bologna

Scuola di Scienze
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Corso di Laurea in Fisica

GEOMETRIC DEEP LEARNING

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Abstract in italiano...

Abstract in english...

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1 Introduction

1.1 Abstract simplicial complexes

Definition 1.1.1. ***Abstract simplicial complex (finite)***
Let $\mathcal{F}$ be a family of sets we then define an abstract simplicial complex $\mathcal{A}$ to be

$$\mathcal{A} := \{\sigma = \{A_i\}_{i \in I_\sigma} \subset \mathcal{F} : \tau \subset \sigma \Rightarrow \tau \in \mathcal{A}\}$$

where I_σ is a finite set of indexes, we shall call σ abstract simplexes of $\mathcal{A}$.

Definition 1.1.2. ***Dimension of an abstract simplicial complex***
Let $\mathcal{A}$ be an abstract simplicial complex we define its dimension to be

$$dim\mathcal{A} := max_{\sigma \in \mathcal{A}} dim(\sigma),$$

where $dim(\sigma) := |\sigma| - 1$.

Definition 1.1.3. ***Abstract graph***
An abstract graph $\mathcal{G}$ is a 1-dimensional abstract simplicial complex whose vertexes and edges are respectively

$$\mathcal{V} := \{\sigma \in \mathcal{G} : dim(\sigma) = 0\} \text{ and}$$
$$\mathcal{E} := \{\sigma \in \mathcal{G} : dim(\sigma) = 1\} .$$

In Definition 1.1.1. we tacitly assumed the definition of the abstract simplex σ invariant with respect to permutations of the indexes I_σ, this assumption establishes the difference between directed and undirected graphs.

Definition 1.1.4. ***Convex envelop of points in*** $\mathbb{R}^n$
Let I be a finite set of indexes, we define the convex envelope of $\{x_i\}_{i \in I} \subset \mathbb{R}^n$ to be

$$\langle x_i \rangle_{i \in I} := \{a = \sum_{i \in I} \lambda_i x_i : \lambda_i \in \mathbb{R},\ \lambda_i > 0,\ \sum_{i \in I} \lambda_i = 1\},$$

which is the smallest convex set containing $\{x_i\}_{i \in I}$.

Definition 1.1.5. ***Affine independency of points in*** $\mathbb{R}^n$
Let $\{x_i\}_{i \in I} \subset \mathbb{R}^n$ we define $\{x_i\}_{i \in I}$ to be affinely independent if and only if

$$\sum_{i \in I} \lambda_i x_i = \sum_{i \in I} \mu_i x_i \quad \Rightarrow \quad \lambda_i = \mu_i \ \forall i \in I,$$

where $\sum_{i \in I} \lambda_i = \sum_{i \in I} \mu_i = 1$.

Definition 1.1.6. ***Geometric k-simplexes***
We define a geometric k-simplex to be a convex envelop $\langle x_i \rangle_{i \in I}$ where $\{x_i\}_{i \in I} \subset \mathbb{R}^n$ are affinely independent and $|I| = k + 1$.

Definition 1.1.7. ***Faces and cofaces of geometric k-simplexes***

Definition 1.1.8. ***Geometric Simplicial Complex***
We define a geometric simplicial complex $\mathcal{K}$ to be a collection of geometric simplexes such that

(i) $\tau \leq \sigma \in \mathcal{K} \Rightarrow \tau \in \mathcal{K}$

(ii) $\sigma, \tau \in \mathcal{K} \Rightarrow \sigma \cup \tau \in \mathcal{K}$

Theorem 1.1.1. ***Geometric realization of an abstract simplicial complex***