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Spatial distribution complexities of traffic congestion and bottlenecks in different network topologies

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ABSTRACT

Recently, urban traffic congestion has become a popular social problem. The generation and the propagation of congestion has close relation with the network topology, the traffic flow, etc. In this study, based on the traffic flow propagation method, we investigate the time and space distribution characteristics of the traffic congestion and bottlenecks in different network topologies (e.g., small world, random and regular network). The simulation results show that the random network is an optimal traffic structure, in which the traffic congestion is smaller than others. Moreover, the regular network is the worst topology which is prone to be congested. Additionally, we also prove the effects of network with community structure on the traffic system and congestion bottlenecks including its generation, propagation and time–space complexities. Results indicate that the strong community structure can improve the network performance and is effective to resist the propagation of the traffic congestion.

1. Introduction

Urban traffic system can be regarded as the combination of different elements and their interactions, which includes three sub-systems: road, flow and management. Uncoordinated behaviors among three systems would induce the traffic problems, such as congestion bottlenecks. A bottleneck is a spatial discontinuity where road capacity is reduced, is the original sources of the grievous traffic problem and has become a key factor in counteracting traffic to be free. A bottleneck will frequently deduce the congestion emergences and queues formation, which will consequently aggravate travel delay, induce traffic congestion, and worsen urban traffic environment and safety. In fact, the effects of a bottleneck on the traffic flow have been studied in the "Lighthill, Whitham and Richards" (LWR) framework [1,2]. Following this work, some approaches are proposed to analyze the traffic flow in the bottleneck with different methods. The entropic solution of the LWR model on both sides of a fixed bottleneck was established by [3,4] and more by Jin and Zhang [5] in a more different way. Newell [6] developed a queuing model at freeway bottlenecks. Daganzo and Laval [7] and Ni and Leonard [8] proposed two models to capture the flow characteristics of the merge bottleneck. Some works focus on the analysis of the congestion bottleneck and the departure time choice.

The cell transmission model (CTM), as a discrete model of the LWR, was firstly presented by [9] which is used for highway traffic simulation. Then, [3] extended CTM to a network situation. As a well simulation tool, it can easily capture the realistic traffic phenomena, such as shock waves, queue formation, dissipation and queue spillback. Recently, various works had

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developed this theory. For example, Lo [21], Lo and Szeto [22] and Szeto and Lo [23] used CTM for dynamic traffic assignment (DTA) to enhance the veracity of estimating dynamic route impedance and improve the application effect of dynamic traffic model. In addition, based on the cell transmission model (CTM), Long et al. [10,11] proposed a congestion propagation model for the urban traffic and applied it to simulate the formation and dissipation of the traffic congestion.

In the previous literatures, however, the related studies mainly focus on the traffic bottleneck by simulation methods. In recent years, the study of topological and dynamical properties of traffic networks attracted much attention. Part of this interest comes from the attempt to understand the macroscopic behavior of traffic networks, i.e., topological behaviors, statistical properties, structure evolution, etc. It is proved that real cities are neither trees nor perfect grids, but a combination of these structures that emerge from the social and constructive processes [12]. Moreover, most of the urban road networks are proved to have the small world effects (a large cluster coefficient and a small average shortest path). For example, [13] concluded that the topological networks of streets in big cities exhibited homogeneous properties but were not heterogeneous networks through analysis of the topological networks in three cities. Gao et al. [14] investigated the urban road network and found a scale-free network with small-world characteristic based on the GIS technology. Lighthill and Whitham [15] studied the transit network of Beijing and found its small world structure.

In order to identify the bottleneck in the traffic network, the propagation of traffic congestion can be considered. Since the dynamic propagation of the traffic flow is extremely complicated which relates to travel behaviors and network structures, it is significant to reveal the effects of network topologies on traffic system performance. Jenelius et al. [16] presented some topological measures of the road network, which can also be used as the guidance to road administrations in their prioritization of maintenance and repair of roads, as well as for avoiding causing unnecessary disturbances in the planning of roadwork. To identify the role of key components of the traffic network, [17] proposed a method in homogenous and heterogeneous topologies which provided an extended identification technology for traffic bottleneck.

However, for the dynamics traffic flow studies, they are limited to a particular network without taking the complexity of the network topology into account, and assumed that the alternative path for the traveler is unique (the shortest path). In reality, not all of the travelers can get the whole link travel information. Generally, they have some alternative path in their travel. However, for the effects of traffic topologies on the traffic system performance, most of them are based on the static user equilibrium in which the dynamic traffic flow characteristics are less considered.

In this paper, the dynamical traffic flow characteristics in different network topologies are investigated based on the link and the node propagation model proposed in our previous works [10,11] by the measures of average journey velocity, total system cost and total delay time. CTM provide the model and simulation method, while the network gives the basic topologies where CTM can be performed. The purpose of this study is to understand the complex behaviors of the traffic flow with respect to the network topology. We focus on examining the performance of the traffic system for three typical topologies, e.g., the regular, random, and small world networks. Another contribution is to explore the effects of the traffic demands on the network performance and to find the optimal network topologies. Besides, the temporal and spatial distribution of the congestion bottleneck by the total duration interval and the largest scale of the congestion bottleneck is analyzed for different topologies.

The paper will be organized as follows. Section 2 gives the traffic flow propagation model in traffic network based on CTM. Section 3 develops the performance measures of the traffic system and congestion bottleneck. The numerical examples in different network topologies, e.g., regular lattice, random graph and small world network are analyzed in Section 4. In Section 5, the distributions of congestion bottlenecks on community structure are investigated. Section 6 contains the conclusions.

2. Propagation model

2.1. Network traffic flow propagation model

Long et al. [10] established a traffic flow CTM including link and node propagation based on the fundamental work of [3]. They analyzed the congestion propagation and dissipation in two-way rectangular grid networks. We refer to [10] for a full description of the link and node propagation model.

2.1.1. The CTM link model

To simplify the solution scheme of LWR model, [9,3] proposed a famous CTM traffic flow model based on the traffic flow q and density k relationship.

$$q = \min\{vk, q_{\max}, w(k_j - k)\}, \quad 0 \leqslant k \leqslant k_j, \tag{1}$$

where k_j, $q_{\max}$, v, w denote, respectively, jam density, inflow capacity, free-flow speed and the speed of the backward shock wave (or the backward propagation speed of disturbances in congested traffic). Meanwhile, Eq. (1) approximates the fundamental diagram by a piece-wise linear model as shown in Fig. 1.

In the link model, each link a is divided into i homogeneous cells from upstream to downstream whose length is equal to the distance traveled by free-flow traffic in one time interval δ. For any link a, based on the Eq. (1) and the relationship

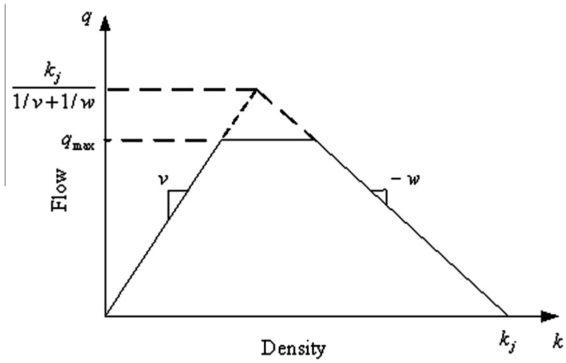

Fig. 1. The equation of state of CTM.

$n_i(t) = k_i(t) \cdot v\delta$ between the density and the number of vehicles presented in cell i, the cell-to-cell flow propagation is given as Eq. (2),

$$y_i(t) = \min\{n_{i-1}(t), Q_i(t), w(N_i(t) - n_i(t))/v\}, \tag{2}$$

where $n_i(t)$ is the number of vehicles that are presented in cell i in time interval t; $N_i(t)$ is the maximum number of vehicles that can be presented in cell i in time interval t; $Q_i(t)$ is the maximum number of vehicles that can flow into cell i in time interval t; $k_i(t)$ is the density of cell i in time interval t, and $y_i(t)$ is the number of vehicles that flow into cell i in time interval t.

According to the flow conversation property, the discrete flow conservation equation can be expressed as,

$$n_i(t+1) = n_i(t) + y_i(t) - y_{i+1}(t). \tag{3}$$

2.1.2. The CTM node model

Extending the CTM link model to node condition, we can obtain the propagation equation of traffic flow from link a to link b,

$$y_{ab}(t) = \min\left\{n_{ab}(t), Q_1^b(t), w(N_1^b(t) - n_1^b(t))/v\right\}, \tag{4}$$

$$y_{r,b}(t) = \min\left\{n_{r,b}(t), Q_1^b(t) - \sum_{a \in A_r} y_{ab}(t)\right\}, \tag{5}$$

where $y_{ab}(t)$ is the number of vehicles that flow from the terminal cell of link a into the first cell of link b in time interval t; $n_{ab}(t)$ is the number of vehicles that will pass by link b and are presented in the terminal cell of link a in time interval t; $n_{r,b}(t)$ is the number of vehicles that are presented at node r (including the waiting vehicles before interval t) and can flow into link b in time interval t; A_r is the set of links leading to node r, and $y_{r,b}(t)$ is the number of vehicles that generate at node r and flow into the first cell of link b in time interval t.

3. Performance measures of traffic system and congestion bottleneck

3.1. Performance measures of the traffic system

3.1.1. Average journey velocity (AJV)

Average journey velocity is a normal measure to evaluate the performance of traffic system. Obviously, the travel time of link a in time interval t can be calculated with [10],

$$\tau_a(t) = \sum_{rs} \sum_{z \in M_{rs}} \tau_z^a \xi_z^a(t) \Bigg/ \sum_{rs} \sum_{z \in M_{rs}} \xi_z^a(t), \tag{6}$$

where M_{rs} is the set of vehicles where the origin node is r and the destination node is s; $\xi_z^a(t)$ is a 0–1 variable, if vehicle z flows into link a in time interval t, then $\xi_z^a(t) = 1$, otherwise $\xi_z^a(t) = 0$; τ_z^a is the time that vehicle z travels through link a, if $a \notin p_z$, $\tau_z^a = 0$; p_z is the travel route of vehicle z; $\tau_a(t)$ is the travel time of vehicles that flow into link a in time interval t; $\overline{\tau}_a$ is the average journey time of link a during period T; T is the period of time that is used to evaluate capability of link or network.

If $\sum_{rs}\sum_{z \in M_{rs}} \xi_z^a(t) = 0$, let

$$\tau_a(t) = \max\{\tilde{\tau}_a, \tau_a(t-1) - 1\}, \tag{7}$$

where $\tilde{\tau}_a$ is the travel time of link a in free-flow condition.

As we know, $\tau_a(t) \geqslant \tilde{\tau}_a$, according to the rule of first in first out (FIFO), we can get $(t-1) + \tau_a(t-1) \leqslant t + \tau_a(t)$ and $\tau_a(t) \geqslant \tau_a(t-1) - 1$.

Then, the average journey time $\overline{\tau}_a$ and AJV $\overline{u}_a$ during period T can be estimated as follows,

$$\overline{\tau}_a = \sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}\tau_z^a\xi_z^a(t) \Bigg/ \sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}\xi_z^a(t), \tag{8}$$

$$\overline{u}_a = \sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}L_a\xi_z^a(t) \Bigg/ \sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}\tau_z^a\xi_z^a(t), \tag{9}$$

where L_a is the length of link a; $\overline{u}_a$ is the average journey velocity of link a during period T.

Similarly, the AJV of a network can be formulated as,

$$\overline{u} = \sum_{a\in A}\sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}L_a\xi_z^a(t) \Bigg/ \sum_{a\in A}\sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}\tau_z^a\xi_z^a(t), \tag{10}$$

where A is the link set of network; $\overline{u}$ is the average journey velocity of network.

3.1.2. Total system cost (TSC)

According to the formulations mentioned above, the total system cost is read,

$$TSC = \sum_{a\in A}\sum_{rs}\sum_{z\in M_{rs}}\sum_{t\in T}\tau_z^a\xi_z^a(t) \tag{11}$$

3.1.3. Total delay time (TDT)

Once a link is congestion, the travel time of vehicle will exceed the free-flow time. Generally, the difference between the actual travel time and the free-flow time is defined as the congestion delay time [21]. Therefore, the total congestion delay $d_i^a(t)$ of vehicles contained in cell i of link a at time interval t can be determined by

$$d_i^a(t) = n_i^a(t) - y_{i+1}^a(t), \quad d_i^a(t) \geqslant 0, \tag{12}$$

where $n_i^a(t)$ is the number of vehicles in cell i of link a at the start time t. $y_{i+1}^a(t)$ is the outflow of cell i on link a at time interval t. Eq. (12) ensures that $n_i^a(t) \geqslant y_{i+1}^a(t)$. It follows that if the exit flow from cell i at time interval t is less than its current occupancy due to congestion, then the vehicles who cannot leave the cell will incur a delay of one time step. Once the delay has been determined at the cell level, it can easily be aggregated to the network level during period T. Therefore, the total delay time of traffic system during period T is,

$$TDT = \sum_t\sum_a\sum_i d_i^a(t). \tag{13}$$

3.2. Performance measures of congestion bottleneck

Four congestion degrees are given here according to the AJV according to the document of the Ministry of Public Security of the People's Republic of China (see website: www.cein.gov.cn). The standard is described as follows [10]:

Freely: The AJV of motor vehicles on the main road is not less than 30 km/h in metropolitan areas.

Light congestion: The AJV of motor vehicles on the main road is less than 30 km/h but not less than 20 km/h in metropolitan areas.

Congestion: The AJV of motor vehicles on the main road is less than 20 km/h but not less than 10 km/h in metropolitan areas.

Serious congestion (jam): The AJV of motor vehicles on the main road is less than 10 km/h in metropolitan areas.

Here, we denote that a link will become a congestion bottleneck in the network if its AJV is less than 10 km/h. To measure the congestion characteristics of time and spatial distribution, we present the distribution of time (Total Duration Interval of Congestion Bottleneck, TDI) and space (largest scale of congestion bottleneck, LS) of the traffic bottleneck.

3.2.1. Total Duration Interval of Congestion Bottleneck (TDI)

TDI is defined as the total time intervals from the appearance of a bottleneck link to its disappearance. It can be recorded in the simulation process.

3.2.2. Scale of congestion bottleneck (LS)

The scale of congestion bottleneck (S) refers to the ratio of the number of bottleneck links in the network to the total number of links. It can be calculated by,

$$S = \frac{\text{the maximum number of bottleneck links}}{\text{the total number of links}}.$$

Similarly, the largest scale of congestion bottleneck is defined as the maximum value of S. Therefore, it indicates the worst traffic performance of traffic network.

4. Traffic performance of typical network topologies

4.1. Generation of network topologies

The urban traffic network is a complex and huge system, in which nodes represent the infrastructure (i.e., crossroad, on-ramp, off-ramp, bus stops) and edges are the links connecting two nodes. Therefore, the traffic network can be represented as a graph $G = (V, E)$ where V is the set of nodes, and E is the set of edges. G can be described by an adjacency matrix $\{e_{Node\_s,Node\_d}\}$. Define N as the network size. If there is an edge between two nodes, the entry $e_{Node\_s,Node\_d}$ is set 1; otherwise $e_{Node\_s,Node\_d} = 0$. We start by constructing networks according to WS [18] algorithms. The model is based on a rewiring procedure of the edges implemented with a probability p. Then, for each node, each link connected to a neighbor is rewired to a randomly chosen node with a probability p, and preserved with a probability $(1 - p)$. Notice that for $p = 0$, we have a regular network, while for $p = 1$, the model will produce a random graph.

4.2. Simulation framework and model parameters

The simulation steps of traffic network based on CTM model are shown in Fig. 2.

4.2.1. Simulation parameters

Similar to [10], the parameters used in the simulation are given as $t = 10$ s, $k_j = 133$ vehicles/km, $v = 54$ km/h, $w = 21.6$ km/h, $L = 150$ m. In addition, the flow capacity is 1800 vehicles/h/lane, the number of lanes is set 2, the holding capacity of each cell is 20 vehicles/interval, the number of cells of each link is 6 and the OD demands loading intervals is 100.

4.2.2. K-th shortest algorithm

In our CTM simulation, k-th shortest algorithm is used. Among the algorithms to search k-th shortest paths, 'link elimination method' is a simple one, which is based on the shortest path algorithm. Taking the practical application into consideration, we use the shortest and the second shortest paths in this study. The basic steps of algorithm are given as follows:

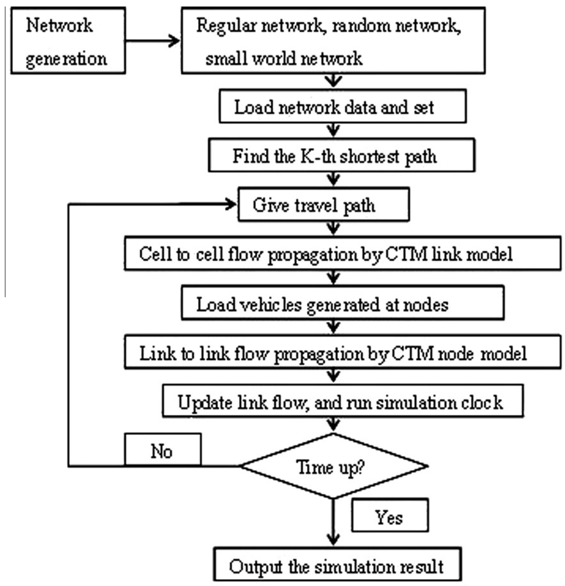

Fig. 2. Simulation steps of traffic network based on CTM.

Step 1. Generating the shortest path. Obtain the shortest paths of each OD pair by using the Dijkstra algorithm;
Step 2. Generating the second shortest path. Repeat the following for each OD pair:
  Step 2.1. Generate a set of sub-networks by removing one link in the shortest path from the original network.
  Step 2.2. Obtain a path set that consists of the shortest paths of the OD pair in all sub-networks.
  Step 2.3. Find the shortest path in the path set as the second shortest path of the original network.
Step 3. Output the k-th shortest path.

4.3. The performance of traffic system for three network topologies

In this section, a two-way network with 50 nodes and 200 links is used to illustrate the application of our model. By adjusting p, we can obtain three classical networks, where $p = 0$, $p = 0.5$ and $p = 1$ correspond to the regular, small world, and random network, respectively.

For the different demand coefficient ϕ, we can obtain the performance of the traffic system for three classical networks in different demand levels. The main measures of the network performance (AJV, TSC and TDT) are shown in Fig. 3.

The changes of AJV, TSC and TDT of three networks with the different demand are plotted in Fig. 3. V_{ER}, V_{SW} and V_{RN} represent the AJV of the regular, small world and random network, respectively. Fig. 3 displays that the increase in total demand significantly shifts down the AJV of networks, but V_{ER} is always smaller than V_{SW} and V_{RN}. V_{ER} is less than 30 km/h when ϕ approximately equals to 0.075, which shows that the regular network is in light congestion. However, at this demand level, the small-world and random network are still in a free state. In the demand interval [0.125, 0.15], V_{ER} is less than 10 km/h, the regular network is in serious congestion, but the other two networks are only in a slight congestion state.

In order to understand the traffic performance in different networks, we further analyze TSC and TDT of three classical networks (C_{ER} and D_{ER}, C_{SW} and D_{SW}, C_{RN} and D_{RN} represent the TSC and TDT of the regular, small world and random network, respectively). As shown in Fig. 3, we can easily found that, the trends of TSC and TDT are similar. With the increase of the demand, the TSC and TDT gradually increase, but the growth of C_{ER} and D_{ER} is relatively fast and far greater than that of C_{SW}, D_{SW} and C_{RN}, D_{RN}. C_{ER} and D_{ER} increase rapidly and V_{ER} is less than 10 km/h when ϕ is greater than to 0.125. The network is in serious congestion, while C_{SW}, D_{SW} and C_{RN}, D_{RN} increase slowly.

These phenomena are due to the characteristics of network structures. The average shortest path length of the regular network is larger than that of the other two networks. In the regular network, the routes for travelers to choose are relatively long, so it will take much more time in the trip. As the demand continuously loading, more and more vehicles are accumulated in the network, and it easily leads to traffic congestion. The random graph has a smaller average shortest path length and larger clustering coefficient, so it has relatively shorter routes for travelers. At the same time, the random network is conductive to the dissipation of traffic congestion.

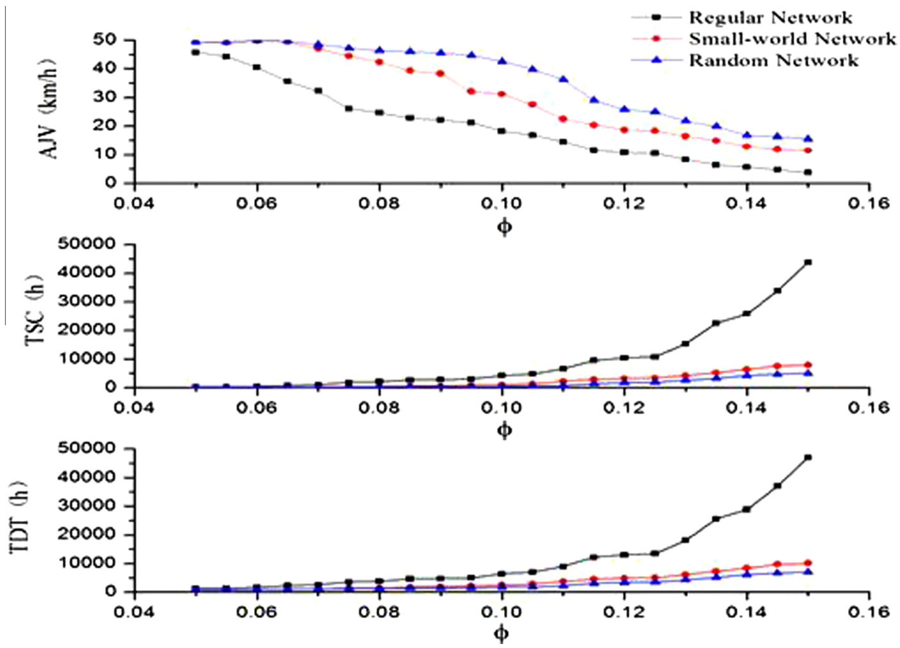

Fig. 3. AJV, TSC and TDT of three networks.

Hence, we can conclude that the random network is optimal by the reason that it can bear much demand. And the regular network structure is the worse one for a city road network, especially for one with a larger traffic demand. Therefore, it is significant to choose an appropriate network topology to get an efficient urban traffic.

4.4. Time and space distributions of traffic congestion bottlenecks

In this section, the traffic performance of three classical networks with different ϕ is simulated. Model parameters used in this study are the same as in Section 4.1. Fig. 4 plots the temporal and spatial distributions of the traffic congestion bottlenecks with different networks. It can be observed that with the growth of demand ϕ, TDI of the regular network increases rapidly and much larger than that of the other two networks. There has not bottleneck link in the random network when ϕ equals to 0.10, but the TDI of the small-world and regular network are 110 and 255, respectively. With the increase of the demand, the growth of TDI of the regular network becomes faster. However, TDI of the other two networks change slowly with the demand range of the simulation. Because of its large shortest path length, the regular network is prone to being congested and requires a longer time to dissipate when congestion occurs. Therefore, in all cases, the random network structure is the optimal one. The LS of three networks has the same variation with TDI.

5. Performance of traffic network with community structure

Among complex characteristics, an important common property of many networks is the presence of community structure. It means that many networks can be divided into some groups such that the connections within each group are dense, while connections between groups are sparse, such as the traffic networks of the main city and satellite towns.

Here, the link whose two nodes are in a same community is defined as the local link and the bridge link represents the edge whose two nodes belong to different communities. The finding of community structure provides a powerful tool for us to understand the complexities of real networks. The bridge links in a traffic network are prone to become congestion bottlenecks, which mean that community structure would affect the congestion scale and even the whole system. How does the congestion scale vary with different community structures? Is there a general law of the bottleneck distribution in the network with the community structure? The present study aims to address precisely these questions.

5.1. Generation of networks with community structure

Newman and Girvan proposed the concept of modality to measure the quality of a particular division of a network with community structure as follows [19]:

$$QS = \sum_r \left(h_{rr} - a_r^2\right),$$

where $a_r = \sum_w h_{rw}$ denotes the row (or column) sums which represent the fraction of edges that connect to nodes in community r and h_{rw} is the fraction of edges in the original network that connect nodes in subset r with nodes in subset w. In a given network in which edges fall between nodes without regard for the communities they belong to, $h_{rw} = a_r a_w$ can be obtained. The larger the value of QS is, the more accurate the community partition will be. Especially, $QS = 1$ Z indicates strong community structure [19].

Here, we generate three scale-free network topologies with community structure and the modular QS = 0.1516, 0.1517 and 0.1518 based on our generation algorithm in Ref. [20], respectively. All the networks have 45 nodes, 200 edges and 5 community structures.

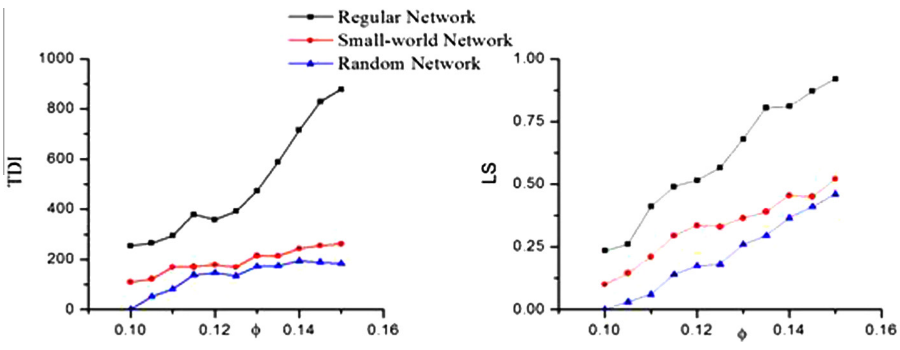

Fig. 4. TDI and LS of three networks.

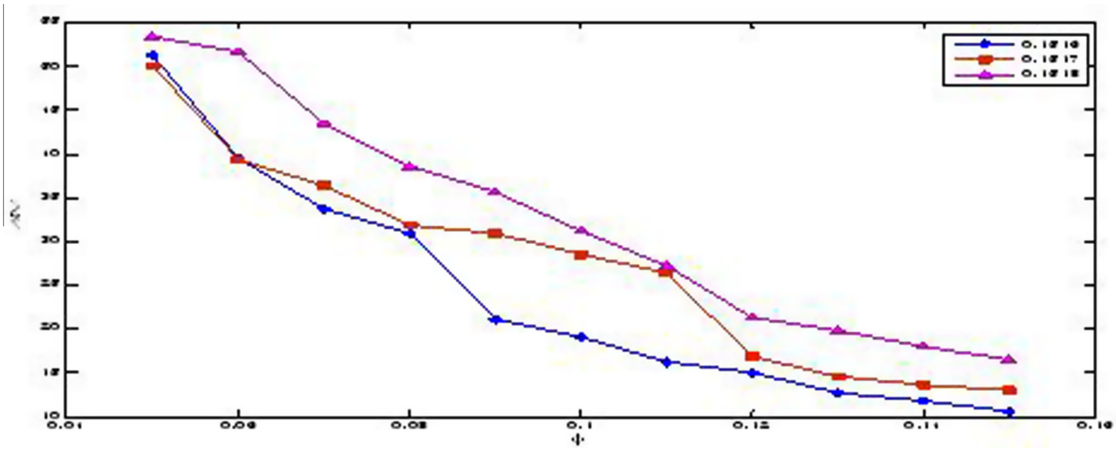

Fig. 5. Effects of demand parameter on AJV of three networks.

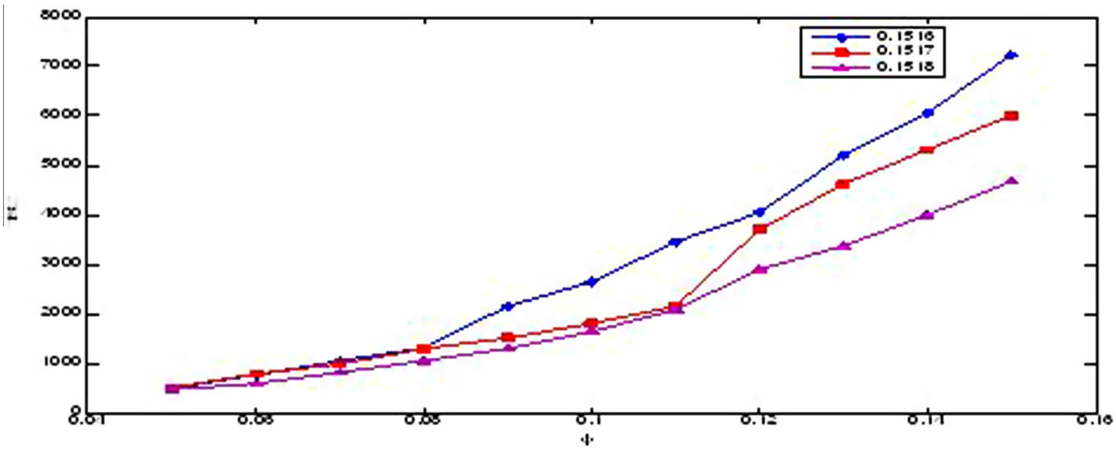

Fig. 6. Effects of demand parameter on TSC of three networks.

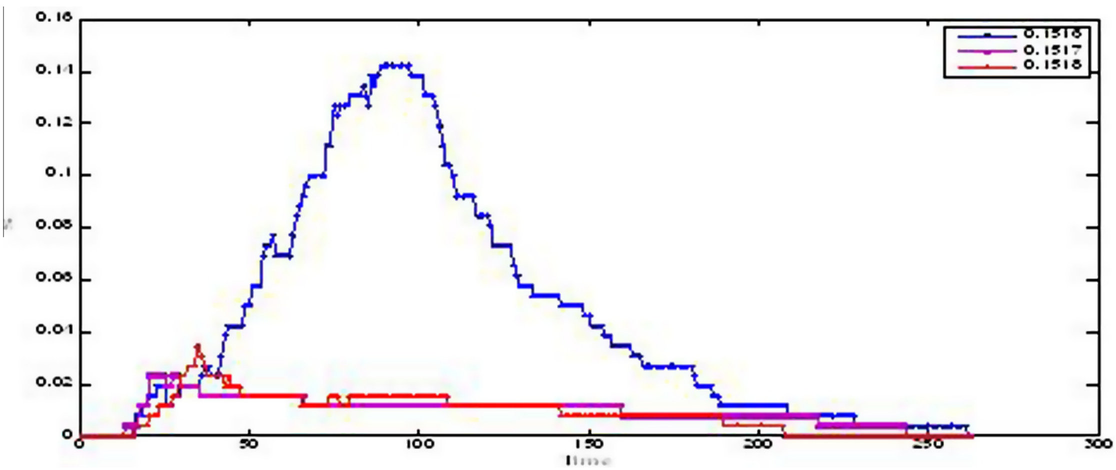

Fig. 7. Effects of demand parameters on S of three networks with $\phi = 0.1$.

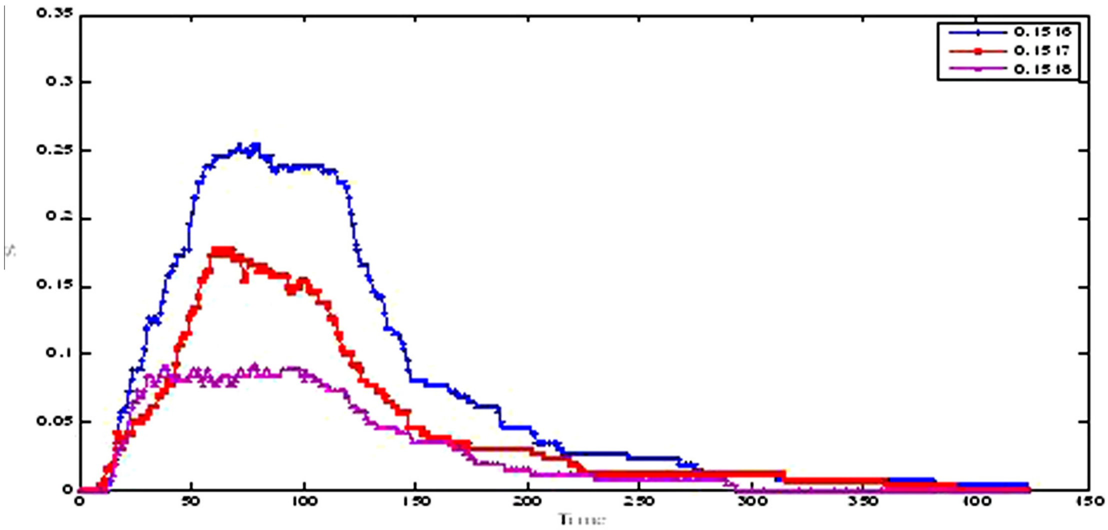

Fig. 8. Effects of demand parameters on *S* of three networks with $\phi = 0.15$.

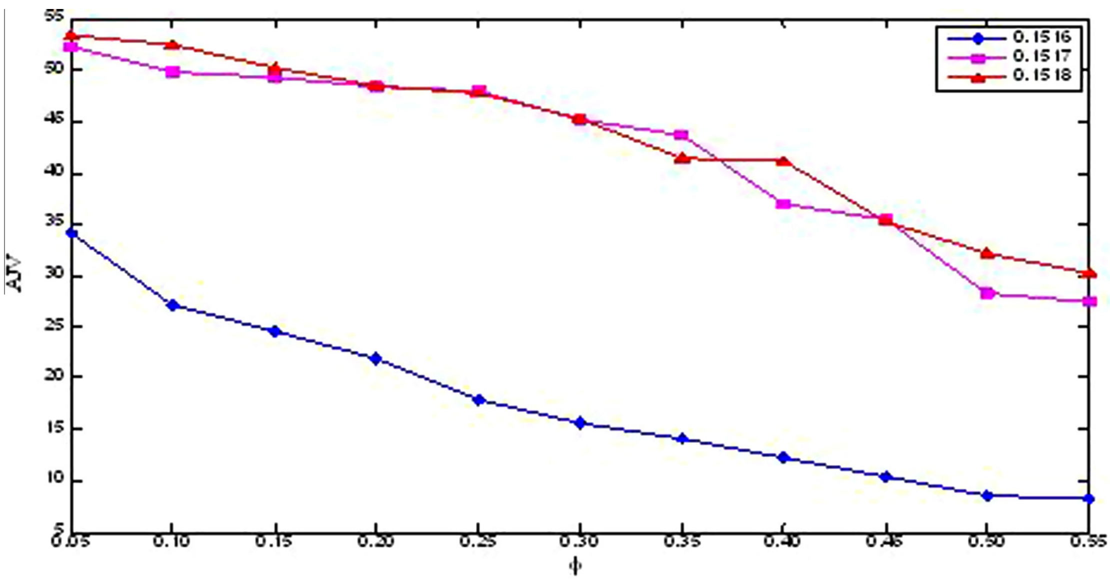

Fig. 9. AJV of bridge links with different network structures.

5.2. Results and analysis

First, we analyze the characteristics of traffic flow in the given three networks. Figs. 5 and 6 give the changes of AJV and TSC of the network against varied values of demand parameters with different community structure, $QS = 0.1516$, 0.1517, 0.1518. It is found that AJV decreases with the increase of demand parameters, and TSC increases vice verse. This is because that with the increase of demand, more and more traffic flows enter the system, which leads to the traffic congestion. However, an interesting result is that a larger *QS* would improve the performance of traffic system which means the community structure can alleviate the traffic congestion effectively, especially for a larger demand parameter. For example, the AJV and TSC are about 18 and 4000 when $QS = 0.1518$ and $\phi = 0.135$, while they are about 12 and 5100 when $QS = 0.1516$ and $\phi = 0.135$. The reason is that, compared with the sparser community structure, most of the traffic flows are transported within the community with the lower total system cost in the stronger ones. In addition, from the structure viewpoint, the average shortest path between arbitrary two nodes in the strong community structure is generally smaller than the sparser one. As we know, the larger average shortest path can cause the increase of travel time for the whole system.

Figs. 7 and 8 present the changes of the traffic congestion scale over time step under demand parameter $\phi = 0.1$ and $\phi = 0.15$. In the two graphs, three curves correspond to different network structures with $QS = 0.1516$, 0.1517 and 0.1518. It is clear that a sparser community structure is easy to form the traffic bottleneck than the stronger one. From Fig. 7, we can see that the largest perception of bottlenecks is about 0.14 with $\phi = 0.1$ and $QS = 0.1516$. It means that the congestion scale is about 0.14. But for the other two networks, only few bottlenecks can be found. With the increase of demand parameter, more and more bottlenecks are generated in the system. However, the duration of bottlenecks is different for three networks. These figures indicate that the congestion will dissipate at time 207, 243 and 261 for three networks with $\phi = 0.1$. But for $\phi = 0.15$, the dissipation time is about 291, 390 and 422. Therefore, the generation and dissipation of the traffic congestion is strongly dependent on the network structure and demand parameters. The network with strong community structure can bearing a larger demand and can help to the dissipation of the traffic congestion.

Fig. 9 depicts the change of AJV of all bridge links for three network structures with different demand parameters. It shows that the speed is higher in the network with larger modular than smaller ones. For example, the bridge links of the network with $QS = 0.1516$ has a larger traffic demand than that of $QS = 0.1517$ and 0.1518. This is because that in a smaller modular network, a large number of bridge links will be prone to further serious traffic congestion. Therefore, only a small improvement of the network, e.g., reducing few of bridge links, might have a great help to alleviate the traffic congestion.

6. Conclusions

In this paper, based on CTM, the dynamical traffic flow in different network topologies is mainly investigated by the measure of AJV, TSC, TDT and TDI. The temporal and spatial distribution for the traffic bottleneck, its largest scale in different topologies and the network with community structure is also discussed. The simulation results indicated that random network is not prone to be congested because of its smaller shortest path length, and the regular network is the worst one. Additionally, the stronger community structure has a good ability to resist the traffic congestion than that of the lower ones, which show that the traffic congestion is dependent on the network structure. The results can help to improve performance in the urban traffic network design.

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