RIO 2025 Summer School: Isabelle Course

Peter Lammich

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March 2025 @ Rio Cuarto

About the Instructor

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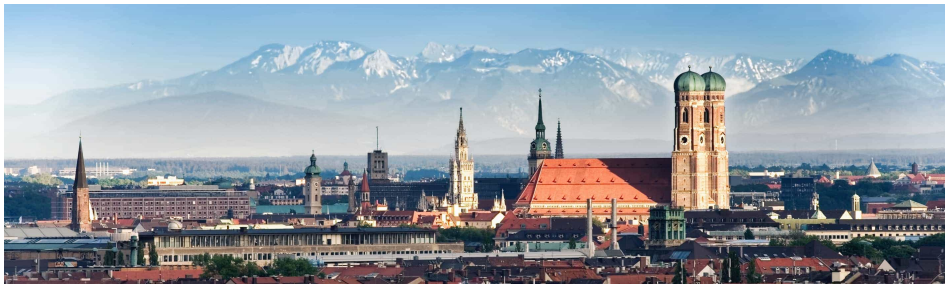

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Made my PhD in Münster, Germany
Then Munich, VirginiaTech, Manchester
At UTwente since March 2021

Email p.lammich@utwente.nl

About the Course

- Goals
  - you'll be able to use Isabelle/HOL
  - verify simple functional programs
  - learn the basics to dive deeper
- 4 lessons, 2:30h each
- Starts with lecture, then hands-on exercises
- Bram (and maybe Edo) will help you
- Books (freely available online)
  - Nipkow, T. (Ed). (2024). *Functional Data Structures and Algorithms: A Proof Assistant Approach* https://functional-algorithms-verified.org/
  - Nipkow, T. and Klein, G. (2014). *Concrete Semantics*. Springer. https://www.concrete-semantics.org/
  - Nipkow, T. *Programming and Proving in Isabelle/HOL* https://isabelle.in.tum.de/dist/Isabelle2024/doc/prog-prove.pdf

Software Errors

Software is buggy!

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Consequences:

Software Errors

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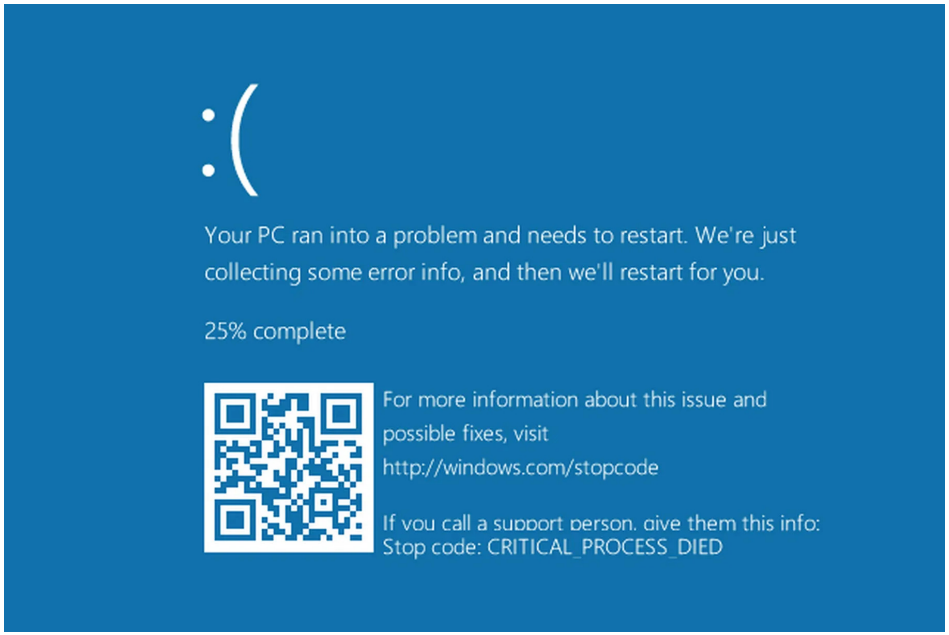
Consequences:annoying



Software Errors

Software is buggy!

Consequences:annoying, expensive

Software Errors

Software is buggy!

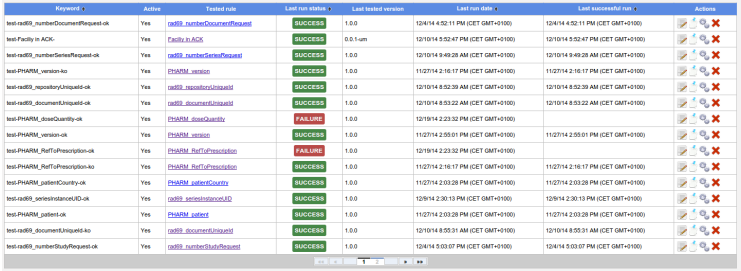
Consequences:annoying, expensive, **deadly**!

Software Verification

Testing?

Software Verification

Testing? Won't find all bugs!



Keyword	Active	Tested rule	Last run status	Last tested version	Last run date	Last successful run	Actions
test-rad69_numberDocumentRequest-ok	Yes	rad69_numberDocumentRequest	SUCCESS	1.0.0	12/4/14 4:52:11 PM (CET GMT+0100)	12/4/14 4:52:11 PM (CET GMT+0100)	
test-Facility in ACK-	Yes	Facility in ACK	SUCCESS	0.0.1-um	12/10/14 5:52:47 PM (CET GMT+0100)	12/10/14 5:52:47 PM (CET GMT+0100)	
test-rad69_numberSeriesRequest-ok	Yes	rad69_numberSeriesRequest	SUCCESS	1.0.0	12/10/14 9:49:28 AM (CET GMT+0100)	12/10/14 9:49:28 AM (CET GMT+0100)	
test-PHARM_version-ko	Yes	PHARM_version	SUCCESS	1.0.0	11/27/14 2:16:17 PM (CET GMT+0100)	11/27/14 2:16:17 PM (CET GMT+0100)	
test-rad69_repositoryUniqueId-ok	Yes	rad69_repositoryUniqueId	SUCCESS	1.0.0	12/10/14 8:52:39 AM (CET GMT+0100)	12/10/14 8:52:39 AM (CET GMT+0100)	
test-rad69_documentUniqueId-ok	Yes	rad69_documentUniqueId	SUCCESS	1.0.0	12/10/14 8:53:22 AM (CET GMT+0100)	12/10/14 8:53:22 AM (CET GMT+0100)	
test-PHARM_doseQuantity-ok	Yes	PHARM_doseQuantity	FAILURE	1.0.0	12/19/14 2:23:32 PM (CET GMT+0100)		
test-PHARM_version-ok	Yes	PHARM_version	SUCCESS	1.0.0	11/27/14 2:55:01 PM (CET GMT+0100)	11/27/14 2:55:01 PM (CET GMT+0100)	
test-PHARM_RefToPrescription-ok	Yes	PHARM_RefToPrescription	FAILURE	1.0.0	12/19/14 2:23:32 PM (CET GMT+0100)		
test-PHARM_RefToPrescription-ko	Yes	PHARM_RefToPrescription	SUCCESS	1.0.0	11/27/14 2:16:17 PM (CET GMT+0100)	11/27/14 2:16:17 PM (CET GMT+0100)	
test-PHARM_patientCountry-ok	Yes	PHARM_patientCountry	SUCCESS	1.0.0	11/27/14 2:03:28 PM (CET GMT+0100)	11/27/14 2:03:28 PM (CET GMT+0100)	
test-rad69_seriesInstanceUID-ok	Yes	rad69_seriesInstanceUID	SUCCESS	1.0.0	12/9/14 2:30:13 PM (CET GMT+0100)	12/9/14 2:30:13 PM (CET GMT+0100)	
test-PHARM_patient-ok	Yes	PHARM_patient	SUCCESS	1.0.0	11/27/14 2:03:28 PM (CET GMT+0100)	11/27/14 2:03:28 PM (CET GMT+0100)	
test-rad69_documentUniqueId-ko	Yes	rad69_documentUniqueId	SUCCESS	1.0.0	12/10/14 8:55:31 AM (CET GMT+0100)	12/10/14 8:55:31 AM (CET GMT+0100)	
test-rad69_numberStudyRequest-ok	Yes	rad69_numberStudyRequest	SUCCESS	1.0.0	12/4/14 5:03:07 PM (CET GMT+0100)	12/4/14 5:03:07 PM (CET GMT+0100)	

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Mathematical proof that SW is correct?

Q.E.D.

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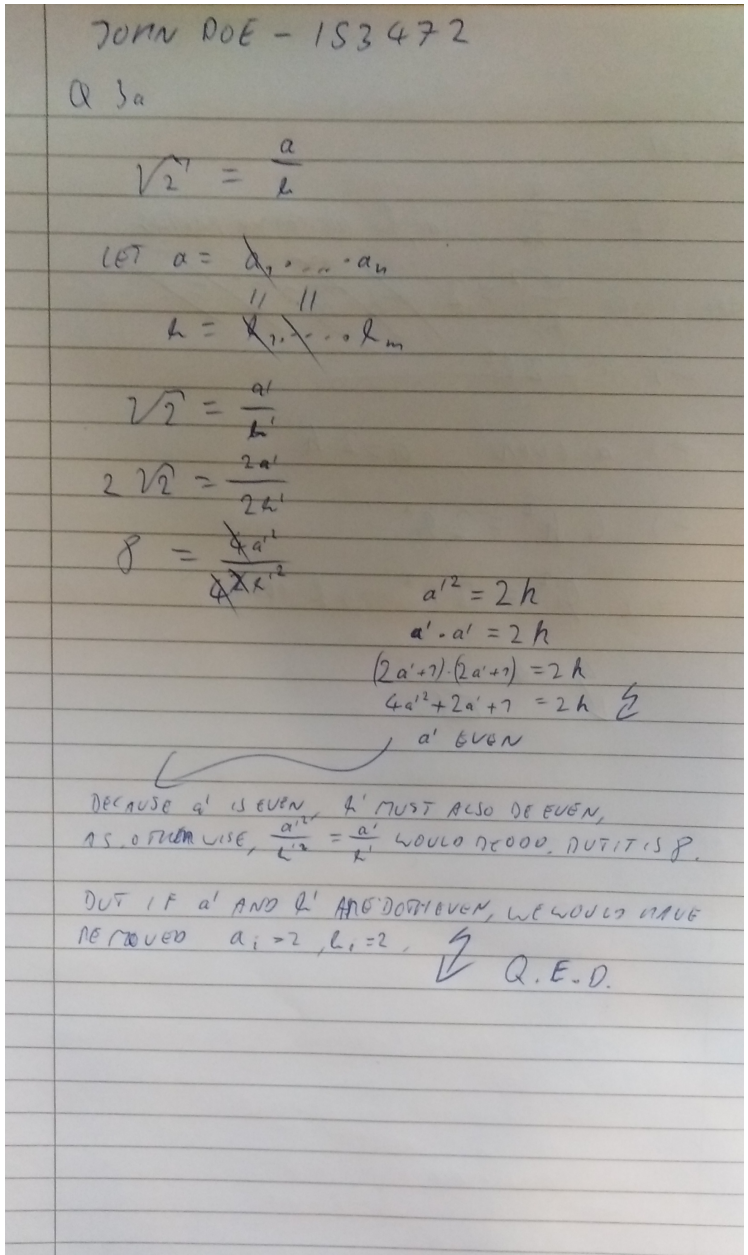
Proofs (pen&paper) are error prone!



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In particular those about software!

Line number	Formal proof	Justification
1	$\mathbf{true} \supset x = x + y \times 0$	Lemma 1
2	$x = x + y \times 0\{r := x\}x = r + y \times 0$	D0
3	$x = r + y \times 0\ \{q := 0\}\ x = r + y \times q$	D0
4	$\mathbf{true}\ \{r := x\}\ x = r + y \times 0$	D1 (1, 2)
5	$\mathbf{true}\ \{r := x;\ \ q := 0\}\ x = r + y \times q$	D2 (4, 3)
6	$x = r + y \times q \wedge y \leqslant r \supset x = (r-y) + y \times (1+q)$	Lemma 2
7	$x = (r-y) + y \times (1+q)\{r := r-y\}x = r + y \times (1+q)$	D0
8	$x = r + y \times (1+q)\{q := 1+q\}x = r + y \times q$	D0
9	$x = (r-y) + y \times (1+q)\{r := r-y;\ q := 1+q\}\ x = r + y \times q$	D2 (7, 8)
10	$x = r + y \times q \wedge y \leqslant r\ \{r := r-y;\ q := 1+q\}\ x = r + y \times q$	D1 (6, 9)
11	$x = r + y \times q\ \{\mathbf{while}\ y \leqslant r\ \mathbf{do}\ (r := r-y;\ \ q := 1+q)\}$ $\neg y \leqslant r \wedge x = r + y \times q$	D3 (10)
12	$\mathbf{true}\ \{((r := x;\ \ q := 0);\ \ \mathbf{while}\ y \leqslant r\ \mathbf{do}\ (r := r-y;\ \ q := 1+q))\}\ \neg y \leqslant r \wedge x = r + y \times q$	D2 (5, 11)

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Let computer check the proof?

```
theorem sqrt2_not_rational:
  "sqrt 2 ∉ ℚ"
proof
  let ?x = "sqrt 2"
  assume "?x ∈ ℚ"
  then obtain m n :: nat where
    sqrt_rat: "¦?x¦ = m / n" and lowest_terms: "coprime m n"
    by (rule Rats_abs_nat_div_natE)
  hence "m^2 = ?x^2 * n^2" by (auto simp add: power2_eq_square)
  hence eq: "m^2 = 2 * n^2" using of_nat_eq_iff power2_eq_square by fastforce
  hence "2 dvd m^2" by simp
  hence "2 dvd m" by simp
  have "2 dvd n" proof -
    from ‹2 dvd m› obtain k where "m = 2 * k" ..
    with eq have "2 * n^2 = 2^2 * k^2" by simp
    hence "2 dvd n^2" by simp
    thus "2 dvd n" by simp
  qed
  with ‹2 dvd m› have "2 dvd gcd m n" by (rule gcd_greatest)
  with lowest_terms have "2 dvd 1" by simp
  thus False using odd_one by blast
qed
```

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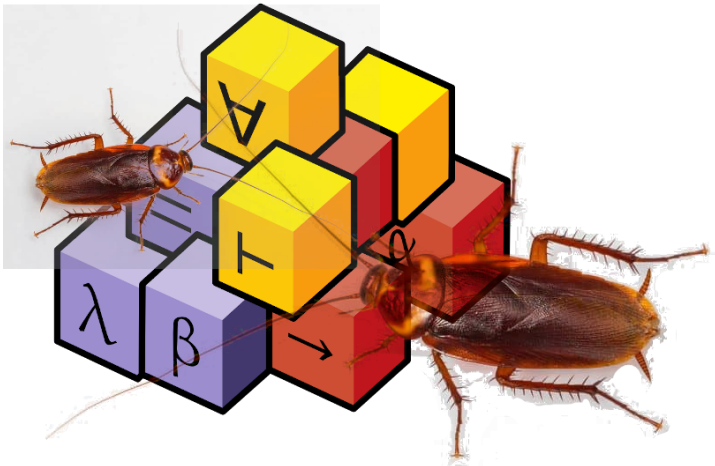
But you have to trust proof checker!



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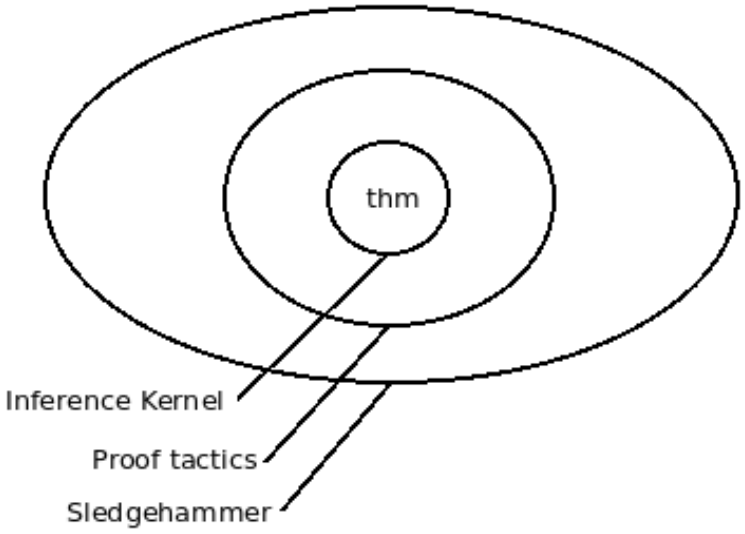
Can be designed with small TCB!



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- Archive of Formal Proofs
- mature, production quality IDE, based on JEdit

What can be done with Isabelle

- Verifying operating systems
  - seL4 https://sel4.systems/
- Proving sophisticated maths
  - Kepler conjecture (Hales' thm) https://github.com/flyspeck/flyspeck
- And many more. See https://www.isa-afp.org/

What I do:

- Verifying algorithms and software
  - From specification down to efficient LLVM IR
  - (Parallel) sorting algorithms
  - Model-Checkers
  - SAT-solvers
  - Numeric algorithms

What we'll do in this course

- Hands-on basics of Isabelle
- Verify algorithms on natural numbers and lists
  - down to (more or less efficient) Haskell
- Give you the basics to dive deeper