DEPARTMENT OF MECHANICAL ENGINEERING
CURTIN UNIVERSITY

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Summary of Research Program for PhD Candidacy

# Rolling-based Robotic In-hand Manipulation

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# Contents

# 1 ABSTRACT

Background: There has been much research about the establishment of rolling contact to in-hand manipulation dexterity that led to a specific moving frame method from differential geometry. Rolling-based is to be a vitally important capability for robots with in-hand manipulation, which is considered necessary to analyse the moving object in an aspect of rolling contact. Besides, manipulation of the multifingered robot hands via tactile fingertips are significantly considered to enhance dexterity in term of object manipulation. Nonetheless, the discrete contact theory of discrete differential geometry has not been proposed in in-hand manipulation through rolling contact.

Aim: The target of this project is to eliminate obstacles with in-hand manipulation by using the discrete differential geometry [1, 2] to generate a discrete contact theory. It is also important to consider the curvature theory of smooth surfaces and the Lie group theory [3] in kinematics multifingered robotic hands with rolling contact. To be demonstrated the problem of rolling contact under the discrete space, using Bellman equation [4] for discrete path planning [5] can be an effective method.

Approach: From the literature review, there are several methods to tackle Bellman's Equation for discrete path planning problem including policy iteration, value iteration and linear programming.

*which is significant methods that can be used in the study???*

Significance: *What is the benefit of rolling contact???* Rolling contact may improve the dexterous ability of multifingered robot hands to arbitrarily configure or reorient manipulated objects.

According to [Leicui thesis p.41], the advantages of rolling contacts are including reduction of abrasion wear, simplification of controller, and enlargement of reachable configuration.

# 2 OBJECTIVES

The aim of this research is to generate a new mathematical model of rolling-based robotic in-hand manipulation in discrete space. In addition, rolling contact in the multi-fingered robot hand is further considered. The specific objectives for this research project are as followed by steps below:

(i) to model the rolling contact between two models in discrete space => to form differential geometry.

(ii) to examine if the path exists.

(iii) if the path exists, how can I generate/find the path?

(iv) if I can find the path, how can I know whether the path is optimal or not?

(v) Experiment validation

=> build up a discrete framework to formulate and solve the motion planning problems.

# 3 BACKGROUND

Introduction. Rolling contact has been studied considerably in the literature. Rolling contact is described different types by point contact [6, 7] , line contact [8], and surface contact [9]. Many researches in the field of geometry [10], controllability [11], motion planning [12] or robot manipulation [13] demonstrated that rolling contact in terms of robot manipulation, especially in multifingered robot hands, has played an important role in recent decades. However such simple end-effector through multifingered robot hands can relocate only a few objects and the dexterity of robotic in-hand manipulation still need to study.

Rolling manifolds[]

Research focus. The purpose of this research firstly focuses on path planning generation methods and then employs discrete rolling contact theory in in-hand manipulation and enhances robot hands working dexterously in discrete space.

## 3.1 Rolling Contact

Rolling contact in continuous space. Rolling contact through ball-plate and rolling sphere problems of nonholonomic systems has been intensively investigated in the past by many researchers [9, 14–16]. Later than, Hartmann [17] applied a numerical blending method to develop the classical rolling ball method in terms of constant and variable radius through analyzing the Voronoi surface, Bezier surface and $G^2$-blending surface. Especially, the rolling sphere model by Brockett [18] has the asymptotic stability problem of the five dimensional nonholonomic systems that can be transformed into a chained form system. A specific geometric formulation in terms of curvature of rolling motion between a sphere and two arbitrarily shape fingers was derived by Montana [10], of which this paper refers to the special case - a rolling sphere and a plate. This rolling contact condition is formulated as contact equation via concepts from differential geometry concept, a well-known nonholonomic constraint.

Kinematic of rolling contact. A simple definition of kinematic chain is a coordinate transformation that demonstrates the relationship between the position and orientation of an object and the fingers [19]. A part of rolling contact is considered in terms of the kinematics which are essentially analyzed from dynamics [20–22], controllability [23–26] and motion planning [6, 27–29]. The majority of study in multifingered robot hands has been involved in differential equations based on kinematic. Cai and Roth [6] used Taylor series expansion to derive the first and second order of kinematics of sliding-rolling. Salisbury and Craig [30] explicitly stated the contact degree of freedom are virtual joint and the contacts are enabled the point contacts with friction while [31, 32] discussed the constraints on the fingertips with friction can be arbitrary kinematics constraints. Okamura [33] developed Jacobian relationships in developing dexterous manipulation kinematics. However, the system may be over-constrained or under-constrained that hardly maintains the rolling contact property. Another series of study about kinematics in terms of rolling contact was conducted by Lei [34–40]. The author applied the theory of Darboux moving frames method in differential geometry to demonstrate the contact equation between an object and multifingered robot hands and generate the forward and inverse kinematics of in-hand manipulation.

Contact theory via Cartan's moving frame method. Cartan's moving frame method is essential approach for geometric objects in contact kinematics [41, 42]. The method was widely applied in the computation of symmetry groups, partial differential equations [43, 44], geometrical curves and surfaces [45, 46] or finite dimensional transformation group from Lie algebras [47, 48]; however, there has been little attention to the robotic field. One of the remarkable studies from Lei [49] is to explore differential geometries in terms of curvatures of shapes through the spin motion to establish the contact theory.

Rolling contact in discrete space.

## 3.2 Motion Planning and Path Planning

Introduction. Motion planning is the most important task of robots.[50],[51].

### 3.2.1 Traditional Motion Planning

The topic on motion planning can be divided into three main categories as follows.
Roadmap approach. The first technique focuses on the precise motion planning where the approach based on the roadmap method which is the connectivity of the free space $F$ captured by a network of 1D curves including Voronoi diagrams, roadmap [52], Star-shaped roadmap (a deterministic sampling approach) [53] and criticality based method [54]. There are some drawbacks on these methods including the computation of free space and less practical algorithms for computing these methods of large environment.

Cell decomposition approach. The second one is called approximation cell decomposition which is divided into full cell, empty cell and mixed cell [55]. The advantages of this method is to compute the planning process incrementally which represent a sequence of cell connecting. However, the method is still not to compute the free space precisely that can be approached as a dual to the roadmap method.

Potential filed category. It is quite different from two previous approach that the connectivity graph is not required to pre-compute. In stead, searching of a path is guided by a heuristics and constructed by an artificial potential function which is represented the sum of potentials (achieving the goal configuration and avoiding the obstacles) [56].

Non-holonomic motion planning. Non-holonomic motion planning has received much attention in the past. The simply definition of robot motion planning is the movement of the robot from an initial configuration to a desired configuration [12, 57, 58]. The motion planning of the object to acquire a desired configuration and the grasp planning in terms of contact force optimization are two main categories of dexterous motion planning [33]. Specific motion planning of rolling surfaces in terms of chained-form has been introduced by several authors such as Brockett [59] introduced sinusoidal inputs then were developed in more detail by Murray et. al [13]. However, the article [60] could not transform the triangularized form of the system equation into chained-form while Monaco [61] investigated non-holonomic chained systems through the two constant inputs where they achieved the interactive planning schemes.

### 3.2.2 Path planning in discrete space

From continuous to discrete. A lot of attention has been received in path planning of robots in complex environments with only few studies on discrete space. Interestingly, combining computation frameworks and the discrete algorithms was considered in recent studies to capture the complex environment [62]. This method is combined with the continuous path planning [] that it can generate or model the kinematics, control laws and a path of the robots. Therefore, the computational framework for automatic path planning for robots in unknown environment in discrete space also needs to be considered.

Discrete path planning. Planning techniques are categorized into different aspects. The basic idea of discrete path planning in the most cases is that state-space models will be used to demonstrate the distinct situation in which the task of a planning algorithm solves the sequence actions transforming from a initial state to other states [5]. For example, Thomas [63] applied Delaunay triangulations to discretize the environment, and cubic spline representations are proposed to meet robot kinematic constraints. Considering the continuous curvature on smooth curves has been integrated within the probabilistic approaches in order to compute the piecewise smooth paths for a car-like vehicle as a four-dimensional system [64]. Whereas, dealing with nonholonomic constraints, a sampling-based road map technique was proposed in [65]. In [66], based on decomposing space into cells, a potential field without local minima was assigned with polygonal partitions of planar environments to solve the Laplace's equation problems in each cell exist.

Cell decomposition. The simple idea of this method is to determine a path between an initial configuration and the goal configuration in the way of dividing the free space into cells. There are two terms of the method including an approximation cell decomposition and an exact cell decomposition[67, 68]. The former methodology refers to a decomposition in which the cells is bounded approximation by the free space that can allow the robot finalizes the motion planning tasks with complex geometries to achieve connectivity paths. Exact cell decomposition has the first step to decompose the free space into trapezoidal triangular cells then nodes which represent cells in the connectivity graph are adjacent in the configuration space. However, there are intensive time and memory on the computation of decompositions and limited volume of the configuration space which rises exponentially with the DOFs of system.

Randomized potential field algorithm. Precomputation of a connectivity graph of the global path-planning which contains the guide for grid search in the configuration space is the high cost for computation system. Using the properties of potential function [69] can generate no systematic way to escape the minima at the goal configuration. The technique in the first step is the best-first search which does not require to reach a local minimum of the potential function in hight-dimensional configuration spaces. Then the search algorithm proceeds along the negated gradient of the potential function until the goal configuration is achieved. The most powerful of this method is to discretize the configuration space and the work space into a hierarchical bitmap grid that can be applied for many DOFs of robots.

Rapidly-exploring Random Trees (RRTs) and RRT-Connect. RRTs is a randomized data structure technique to solve a planning problem[70]. The method does not require any connections of nearby configurations. It can be applied for path planning problems that have the nonholonomic constraints and high degree of freedom. The method still remains on the trajectory optimization problems. Another study in [71] improved the RRTs method called RRT-Connect technique which combines the RRTs and a greedy heuristic to speed up the exploration of configuration (state) space and the connection from an initial configuration to other goal. However, there is challenging issue still remains in terms of computational geometry such as the artificial bias which can be given from searching nearest-neighbour to the convergence rate.

Probabilistic Roadmap Planer-PRM. The PRM technique has been successful for path planning problems, which was implemented in different sites [72–74]. The PRM computation consists of two phases: the preprocessing phase and the query phase. Repeating the generating random free configuration space can generate a probabilistic roadmap in the preprocessing phase. The nodes of the graph and the paths are computed through local planner that can create the graph edges. In the query phase, there is starting by connection between the initial and the goal configuration by a Dijkstra's shortest path query [75]. Finding complete edges from connecting nodes in the roadmap to generate a graph search is the feasible path for the planning problem. Nevertheless, Probabilistic Roadmap Planner method should be optimized due to some reasons such as the low quality of searching process - the graph is a tree, not cycle graphs and involving straight-line motion which generates the first order discontinuities at the nodes.

Heuristic search method. The fundamental robotic path planning problem is to represent the environment as a graph involving the set of possible robot location and a set of edges that can generate the paths. The popular method for determining the least-cost paths is A* as Heuristic based search algorithm in [76–78]. The search algorithm must expand the fewest possible nodes in order to make searching for an admissible path. Then the evaluation of available nodes is needed to determine the next efficient nodes. The initial search approached by A* takes two steps to generate an optimal path in which receiving information from one of the initial cells in free space and replanning from scratch when the environment has changed to expand a new cell. However, the A* computation process need high configuration processors to successfully reach various nodes. In the real world scenarios, the search operating sometimes may be performed with inaccurate planning graphs.

Dynamic programming. Dynamic programming (DP) is a technique of robot path planning problem solving to calculate the distance of the goal configuration from all the initial configuration in the grid map [4]. The environment in most robot path planning issues are implemented by a topologically organized map where the connections between each grid points and their neighbouring grid points are built. The distance at every iteration is updated to constitute the neighbour cells when the environment is discretized into a grid of points. As proof in [4], the DP method can provide a simple approximation to optimize the trajectory solution that does not suffer from the curse of dimensionality. A criticism of DP that has precluded the practical implementation in path planning problems is somewhat more difficult for executing the DP algorithm due to time consuming [79]. The efficient cost is the property of the DP method by sub-dividing the complex problems into sub-problems and converging various steps of solving sub-problems. However, the expense of the DP algorithm is not as overwhelming nowadays thanks to the faster and stronger computer and the parallel-processing computer which can efficiently execute the expanding nodes.

Other general search methods. Breadth first, Depth first, Dijkstra's algorithm, Backward search, Bidirectional search. Monte Carlo methods. Temporal-difference learning.

Optimal path planning. In order to optimal paths which are generated from various path planning techniques, an introduction of a new method based on the randomization and the dynamic programming was proposed in [80]. The method normally is used to optimize paths for mobile robots and articulated robot manipulation. Using only the dynamic programming can lead to cost of calculating the path segments from one cell to other grid cells because of the large size of the search space, especially for curse dimensional issues. To overcome this problem, the use of randomization in the discretized grid within the dynamic programming may reduce the cost path. Considering the orthogonal neighbour relations that are connections between a node and orthogonal one another can decrease the computations. Another study in [81] called Sampling-based algorithm based on the Rapidly-exploring Random Tree (RRT) algorithm by combining the Transition-based RRT (T-RRT) and RRT* can solve complex high-dimensional path planning problems and converge faster to the optimal path.

## 3.3 Markov Decision Process

Introduction. Markov Decision Process (MDP) is an extension of a Markov chain which is a stochastic model in discrete time. MDP represents a mathematical framework to model decision making process that has been studied in early [82, 83]. For the record, many applied study may have an implicit underlying the MDP framework to determine the expected cost of raw material that the stock can purchase or shortage [84], to control the pest or protect natural resources [85]. In the robotic field, the MDP has been successfully applied for path planning problems such as combination with a quadtree decomposition of the environment to compute the motion plan [86] or association with ideas of deterministic search and dynamic programming techniques to reduce the processing cost and improve performance path planning algorithm [87]. Another promising technique for autonomous trajectory planning is based on the MDP with clothoid tentacles [88]. The study used stochastic transitions [89] that can reduce the presence of distant obstacles in an occupancy grid.

The agent-environment interface. As shown in Figure 1, there are two main objects in the MDP including an agent and the environment that they interact together at each of a sequence of discrete time steps, $t = 0, 1, 2, 3, ....$ The process in some states $S_t$ at each time step $t$ is described that the decision maker chooses any action $A_t$ in the state and the agent will receive the reward $R_t$ from the environment. In [90], the general rule of agent-environment boundary is not the same as the physical boundary of a robot's or animal's body. In some cases, the agent may know everything in the environment, just as the rewards which are computed from the agent's action function or the state received. However, in another case of solving puzzle like Rubik's cube, the agent knows all the environment such as colors, faces and dimensions but hard to solve its work.

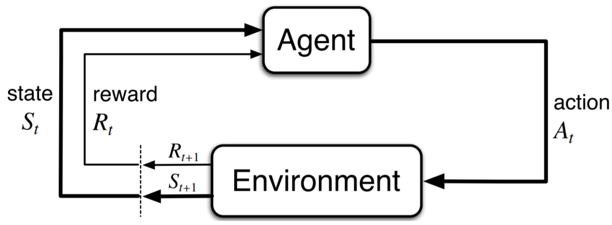


Figure 1: The interaction of agent and environment [90].

MDP is a tuple $< S, A, T, r, \gamma >$ where $S$ is a set of observations when the agent observes a state from the environment; $S$ is a set of actions that the agent can execute from the task to interact with the environment; $T$ is transition probability matrix in which of making the action $a \in A$ from the current state $s \in S$ to the next state $s'$ $(T(s'|s, a) = \mathbb{P}[S_t = s, A_t = a])$; $r$ is the reward model that the agent can receive in the state $s$ when executing an action $a$ $(r(s, a) = \mathbb{E}[R_{t+1}|S_t = s, A_t = a])$, and $\gamma$ is the discount factor where $0 < \gamma < 1$ that relatives between immediate and future rewards. In path planning problems, the MDP method is applied for finding the path in terms of optimizing the expected sum of discounted rewards and the Bellman equation [91] can be promoted in the study.

Bellman equation. Before discussing the Bellman equation, there are some of principal components of the Reinforcement Learning framework including Reward and Return, Policies, and Value functions which are briefly introduced as following.
In reinforcement learning, there are two types of value function used to optimize the policy including the state value function $V^\pi(s) = \mathbb{E}_\pi[R_t|s_t = s]$ and the action value function $Q^\pi(s, a) = \mathbb{E}_\pi[s_t = s, a_t = a]$. They are the expected returns generated from the state and action under the policy $\pi$. The value function changes dependently on the policy for the same environment due to that fact that the value of the state changes dependently and expected rewards will be received. The action value function represents the value of taking an action in some state $s$.

If we call $\mathcal{P}$ is the transition probability from starting state $s$, taking action $a$, then ending up in state $s'$, we have $\mathcal{P}^a_{ss'} = pr(s_{t+1} = s'|s_t = s, a_t = a)$. In addition, $\mathcal{R}^a_{ss'}$ is called the expected reward received from starting state $s$, taking action $a$, and moving into state $s'$, we have $\mathcal{R}^a_{ss'} = \mathbb{E}[r_{t+1}|s_t = s, s_{t+1} = s', a_t = a]$ Finally, the Bellman equation [91] can be derived for the on-policy state value function with the results is as follows:

$$V^\pi(s) = \sum_a \pi(s, a) \sum_{s'} \mathcal{P}^a_{ss'}[\mathcal{R}^a_{ss'} + \gamma V^\pi(s')] \quad (1)$$

and for the on-policy action value function as:

$$Q^{\pi}(s,a) = \sum_{s'} \mathcal{P}^{a}_{ss'}[\mathcal{R}^{a}_{ss'} + \gamma \sum_{a'} \pi(s',a') Q^{\pi}(s',a')] \tag{2}$$

for all $s \in S, a \in A(s)$.
It is noticed that the Bellman equation can represent the values of states as values of other states like easily calculating the value of state $s_t$ when knowing the value of state $s_{t+1}$. However, to apply for the stochastic shortest path problems by using the Bellman equation, there are techniques such as value iteration, policy iteration, and linear programming should be taken into consideration to solve the Bellman equation.

Value iteration. This is one of the method to find an optimal policy is to determine the optimal value function. The method in detail to converge to the correct $V^*$ values that can be performed by an iterative algorithm [4, 92]. The Bellman equation for the optimal value function is as follows:

$$Q^*(s,a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q^*(S_{t+1},a')|S_t = s, A_t = a] = \sum_{s',r} p(s',r|s,a)[r + \gamma \max_{a'} Q^*(s',a')] \tag{3}$$

The algorithm will stop when the value function changes in a small amount of one update of each state (sweep). The combination of one sweep of policy evaluation and one sweep of policy improvement in the value iteration algorithm can make the convergence faster. It is also noted that the value iteration can be obtained when the step of updating the Bellman optimality equation happens and the updated value has to be required maximum over all actions.

Policy iteration. Policy iteration algorithm can be defined as another way of determining an optimal policy in a finite number iteration. The iteration starts with the value function $V(s)$ of the previous policy $\pi$ in each policy iteration as $\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} ...\pi_* \xrightarrow{E} v_*$, where $\xrightarrow{E}$ denotes a policy evaluation and $\xrightarrow{I}$ indicates a policy improvement. The Bellman equation for the optimal value function as the policy iteration is:

$$V^*(s) = \max_a \mathbb{E}[R_{t+1} + \gamma V^*(S_{t+1})|S_t = s, A_t = a] = \max_a \sum_{s',r} p(s',r|s,a)[r + \gamma V^*(s')] \tag{4}$$

The policy iteration algorithm may converge only few iterations that will be gained the expected infinite discounted reward by executing the policy [90]:

$$\pi'(s) = argmax_a(R(s,a) + \gamma \sum_{s' \in S} p(s',r|s,a) V^{\pi}(s')) \tag{5}$$

Linear programming. Linear programming methods can be used to solve the Bellman equation and they are better than dynamic programming in some cases as less number of states and a potential method in the curse of dimensionality.

## 3.4 Robotics In-hand Manipulation

Dexterous manipulation. Simple definition of dexterous manipulation is to manipulate motions of an objects and to move the objects from an initial configuration to a desired configuration via a given trajectory [33, 93, 94]. The dexterous manipulation of an object using multi-fingered robot hand is one of the problems. Li et. al [27] proposed grasp planning algorithm in terms of stability and manipulability and the control algorithm for the coordinated manipulation by a multi-fingered hand. Bicchi et. al [60, 95] demonstrated the technique to achieve the dexterous manipulation via rolling contacts. The author used a continuous method proposed by Sussmann [96] which implemented the dexterous manipulation of an object of arbitrary shape. Developing the technique for dexterous manipulation by integrating the theory of kinematics and nonholonomic motion planing, Han [97] conducted an experiment on dexterous manipulation with multifingered robotic hands with rolling contact.

Tactile feedback in in-hand manipulation. Tactile sensing in robot hands is mostly conducted as the continuous sensing to enhance the dexterity and ability of object manipulation [98]. Tactile feedback plays an essential role for dexterous multifingered-robot hand manipulation tasks. With additional information from tactile sensors, the robustness and the ability to react can be improved by detecting instabilities, disturbances or slippage [99, 100]. However, there are some tools such as the moving frame and curvatures from the geometric differential properties has not been discussed in rolling contact in in-hand manipulation.

Manipulation by rolling contact. Since rolling contact is nonholonomic constraint, the multifingered robot manipulation by rolling contact plays an important role in the improvement of dexterous manipulation. Generally, the manipulation by rolling contact has been considered in different view points including manipulation of objects by multifingered robot hands or consideration of contact points for nonholonomic systems. The study of a three-fingered robot hand manipulating an object has been considered by Cole et al [101] that was extended by Sarkar et al [20] in terms of manipulating an object under the pure rolling contact. However, these studies has not been utilized and the study of the contact point has not been demonstrated enough.

# 4 SIGNIFICANCE

Research gap. As deeply searching on the literature review, integrating tactile sensors into robot hands may improve the dexterity of robot in-hand manipulation [102] [103] that may emulate human sensing. Besides, the important tools in differential geometry including curvature theory and Lie group theory are formed to approximate geometry attributes [34, 37, 38, 49, 104, 105]. However, there were not significant investigations into discrete differential geometry and rolling-based multifingered robotic in-hand manipulation.

Research outcome. During the PhD program, I propose to develop a discrete contact theory based on the discrete differential geometry; then apply this tool to implement the dexterity of rolling contact into the robotic hands. The research will apply for robot manipulation that contribute to the development of robot hands to achieve human-like capacity for in-hand manipulation. Not only in the academic sector, the industrial applications also benefit from the research.

# 5 RESEARCH METHOD

Proposed methodology. The research aims to propose a new discrete contact theory which will develop the capacity of tactile sensing in robot hands. To begin, the deep exploration of literature review on the theory of rolling contact in multifingered robot hands in both continuous and continuous spaces. Then, it is also important to build up new mathematical model of discrete rolling contact which can apply on the object manipulation by robot hands.

Theoretical approach.

Simulated system. Matlab programming:....solving maths, evaluate math... ROS software:....

Experimental validation. Barrete hand to explore the proposed theory.

[62] (Discrete abstractions for robot motion planning and control in polygonal environments)

**4-Research Design and Method**

**Overview** Further, I will develop a new discrete contact theory by extending my pioneering work and applying the recent progress in discrete differential geometry.

**Aim 2: To develop a discrete contact theory.**

**Rationale**. The grid-based tactile sensors naturally discretise the fingertip surfaces into quadrilateral faces, which can be converted into triangle meshes that have been extensively used in Computer Graphics [106]. Contact theory requires an approximation of the first and second order properties, such as curvatures and principle directions [10, 49]. The current patch-based contact theory needs analytical evaluation, which often introduces overshooting or unexpected surface behaviour.

On the other hand, moving frame and curvatures can be consistently derived by the rich tool set developed in discrete differential geometry, and these appropriations are guaranteed to be an extension from the continuous to the discrete setting [107].

**Theoretical approach.** I propose to apply the tool set developed in **discrete differential geometry to establish a discrete contact theory**. My contact theory for smooth surfaces is formulated in terms of the moving frame, normal curvature, geodesic curvature, and geodesic torsion [[37, 38, 49, 105, 108]. Each of these geometric entities has its counterpart in discrete differential geometry, as in Fig. 5 (Lei's PhD thesis [108]). When I was deriving the contact theory, the velocity constraint was embedded in the arc length, eliminating the need of nonlinear differential equations and facilitating grid-based path planning, as in Fig. 6 (in [105]). I will apply the same approach in the discrete case **to establish a discrete contact theory** that is coordinate-independent and does not involve difference terms.

**Empirical validation.** The BarrettHand is equipped with tactile sensors capable of providing 96 cells of tactile-array data spread across all three fingers and the palm. The density of cells becomes higher towards the very tips of the fingers where finer spatial resolution is desirable [106]. The embedded signal processing of raw data from the tactile sensors can determine point of contact, and this will facilitate validation of the proposed discrete contact theory.

**Reference**

- 1 From Dr.LeiCui research : [34–40, 49, 104, 105, 109–111]
- 2 Text book about dynamic-discrete [112]

# 6 ETHICAL ISSUES

This research proposal will be conducted by the Ph.D. candidate under guide of supervisor without any ethical issues. No any sensitive data and no harmful chemicals will be used in this research.

# 7 DATA STORAGE

Refer to the Data Management Plan in Appendix A.

# 8 FACILITY AND RESOURCES

Refer to the Table 1, the facility and resources base on Curtin University and Open Source.

Table 1: Exemplary table

| Resources | Provider |
|---|---|
| Robot Operating Software | Open Source |
| MatLab | Curtin University |
| ABB Robotic Arm | Curtin University |
| Computer and Printing | Curtin University |

# 9 TIME LINE

| Year | | 2018 | | | 2019 | | | | | 2020 | | | 2021 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Activity** | Sep | Oct | Dec | Feb | May | Aug | Nov | Feb | May | Aug | Nov | Feb | May | Aug |
| Candidacy proposal | | | | | | | | | | | | | | |
| Literature review | | | | | | | | | | | | | | |
| Activity | | | | | | | | | | | | | | |
| Activity | | | | | | | | | | | | | | |
| Activity | | | | | | | | | | | | | | |
| Activity | | | | | | | | | | | | | | |
| Result & Validation | | | | | | | | | | | | | | |
| Thesis preparation | | | | | | | | | | | | | | |

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# A APPENDIX

This is the appendix