3.7 Example Configuration Spaces

In most cases, we can model robots as rigid bodies, articulated chains, or combinations of these two. Some common robots and representations of their configuration spaces are given in table 3.1.

When designing a motion planner, it is often important to understand the underlying structure of the robot's configuration space. In particular, we note the following.

- $S^1 \times S^1 \times \cdots \times S^1$ (n times) $= T^n$, the n-dimensional torus
- $S^1 \times S^1 \times \cdots \times S^1$ (n times) $\neq S^n$, the n-dimensional sphere in $\mathbb{R}^{n+1}$
- $S^1 \times S^1 \times S^1 \neq SO(3)$
- $SE(2) \neq \mathbb{R}^3$
- $SE(3) \neq \mathbb{R}^6$

It is sometimes important to know whether a manifold is compact. The manifolds S^n, T^n, and $SO(n)$ are all compact, as are all of their direct products. The manifolds $\mathbb{R}^n$ and $SE(n)$ are not compact, and therefore $\mathbb{R}^n \times \mathcal{M}$ is not compact, regardless of whether or not the manifold $\mathcal{M}$ is compact.

Despite their differences, all of these configuration spaces have an important similarity. When equipped with an atlas, each is a differentiable manifold. In particular,

- $\mathbb{R}^1$ and $SO(2)$ are one-dimensional manifolds;
- $\mathbb{R}^2$, S^2 and T^2 are two-dimensional manifolds;

Type of robot	**Representation of $\mathcal{Q}$**
Mobile robot translating in the plane	$\mathbb{R}^2$
Mobile robot translating and rotating in the plane	$SE(2)$ or $\mathbb{R}^2 \times S^1$
Rigid body translating in the three-space	$\mathbb{R}^3$
A spacecraft	$SE(3)$ or $\mathbb{R}^3 \times SO(3)$
An n-joint revolute arm	T^n
A planar mobile robot with an attached n-joint arm	$SE(2) \times T^n$

Table 3.1 Some common robots and their configuration spaces.