DEPARTMENT OF MECHANICAL ENGINEERING
CURTIN UNIVERSITY

Summary of Research Program for PhD Candidacy

Rolling-based Robotic In-hand Manipulation

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Contents

1 ABSTRACT

Background: There has been much research about the establishment of rolling contact to in-hand manipulation dexterity that led to a specific moving frame method from differential geometry. Rolling-based is to be a vitally important capability for robots with in-hand manipulation, which is considered necessary to analyse the moving object in an aspect of rolling contact. Besides, manipulation of the multifingered robot hands via tactile fingertips has been significantly considered to enhance dexterity in term of object manipulation. Nonetheless, the discrete contact theory of discrete differential geometry has not been proposed in in-hand manipulation through rolling contact.

Aim: The target of this project is to eliminate obstacles with in-hand manipulation by using the discrete differential geometry [1, 2] to generate a discrete contact theory. It is also important to consider the curvature theory of smooth surfaces and the Lie group theory [3] in kinematics multifingered robotic hands with rolling contact. To be demonstrated the problem of rolling contact under the discrete space, using Bellman equation [4] for discrete path planning [5] can be an effective method.

Approach: Solving the path planning task is one of crucial stages of the research. From the literature review, there are several methods to tackle Bellman's Equation for discrete path planning problem including policy iteration, value iteration and linear programming. A discrete contact theory between an object and multifingered hand will be also developed by using differential geometry theory in terms of moving frame, curvature, and Lie-group theory.

Significance: Rolling-based contact may improve the dexterous ability of multifingered robot hands to arbitrarily configure or reorient manipulated objects. The developing of the discrete contact theory based on differential geometry will be applied for robot in-hand manipulation that can contribute to the advance of industrial robotic technology.

2 OBJECTIVES

The aim of this research is to generate a new mathematical model of rolling-based robotic in-hand manipulation in discrete space. In addition, rolling contact in the multi-fingered robot hand is further considered. The specific objectives for this research project are as followed by steps below:

(i) Discretized rolling contact model: An initial task of the study is to develop the geometrical framework in discrete space to form differential geometry of the rolling contact between two models (an object and robot fingers) in terms of moving frame, curvature, torsion, and the Lie-group theory.

(ii) Path planning examination: The task is required to examine whether the path exists under solving the motion planning problems. Discrete search algorithms are approached to locate a grid resolution and the discrete contact will be analyzed for the path planning process.

(iii) Discrete path generation: Discrete planning will be investigated by state-space models involving the distinct situation (state) and the set of possible states (space). The connection from an initial configuration to goal configuration in discrete space can generate the discrete path under rolling contact between an object and multifingered robot hands.

(iv) Path optimization: Because path planning between the object and multifingered robot hands faces with the high dimensional environment, the optimal path planning should be considered. I also propose the analysis technique to improve both cost and capacity of the discrete path planning under rolling contact condition in terms of discrete differential geometry.

(v) Experimental validation: At the last stage of the study, Matlab and Robot Operating System (ROS) software will be used to test the system in simulated environment and then ABB robot arm is integrated to BarrettHand to run the system as a physical robot.

3 BACKGROUND

Introduction. Rolling contact has been studied considerably in the literature. Rolling contact is described different types by point contact [6, 7] , line contact [8], and surface contact [9]. Many researches in the field of geometry [10], controllability [11], motion planning [12] or robot manipulation [13] demonstrated that rolling contact in terms of robot manipulation, especially in multifingered robot hands, has played an important role in recent decades. However such simple end-effector through multifingered robot hands can relocate only a few objects and the dexterity of robotic in-hand manipulation still need to study.

Research focus. The purpose of this research firstly focuses on the continuous and discrete path planning generation methods and then employs discrete rolling contact theory in in-hand manipulation and enhances robot hands working dexterously in discrete space. The optimal path planning is also considered to eliminate the cost of the process of path generation. Therefore, developing the discrete contact theory in terms of differential geometry is significantly considered in this research. Experimental validation is the final step to test the whole system including physical robot.

3.1 Rolling Contact

Rolling contact in continuous space. Rolling contact through ball-plate and rolling sphere problems of nonholonomic systems has been intensively investigated in the past by many researchers [9, 14–16]. Later than, Hartmann [17] applied a numerical blending method to develop the classical rolling ball method in terms of constant and variable radius through analyzing the Voronoi surface, Bezier surface and G^2-blending surface. Especially, the rolling sphere model by Brockett [18] has the asymptotic stability problem of the five dimensional nonholonomic systems that can be transformed into a chained form system. A specific geometric formulation in terms of curvature of rolling motion between a sphere and two arbitrarily shape fingers was derived by Montana [10], of which this paper refers to the special case - a rolling sphere and a plate. This rolling contact condition is formulated as contact equation via concepts from differential geometry concept, a well-known nonholonomic constraint.

Kinematic of rolling contact. A simple definition of kinematic chain is a coordinate transformation that demonstrates the relationship between the position and orientation of an object and the fingers [19]. A part of rolling contact is considered in terms of the kinematics which are essentially analyzed from dynamics [20–22], controllability [23–26] and motion planning [6, 27–29]. The majority of study in multifingered robot hands has been involved in differential equations based on kinematic. Cai and Roth [6] used Taylor series expansion to derive the first and second order of kinematics of sliding-rolling. Salisbury and Craig [30] explicitly stated that the contact degree of freedom are virtual joint. These authors also developed the analysis of the contacts between bodies which have the constraints within the effects of friction while [31, 32] discussed the constraints on the fingertips with friction can be arbitrary kinematics constraints. Okamura [33] developed Jacobian relationships in developing dexterous manipulation kinematics. However, the system may be over-constrained or under-constrained that hardly maintains the rolling contact property. Another series of study about kinematics in terms of rolling contact was conducted by Lei [34–40]. The author applied the theory of Darboux moving frames method in differential geometry to demonstrate the contact equation between an object and multifingered robot hands and generate the forward and inverse kinematics of in-hand manipulation.

Contact theory via Cartan's moving frame method. Cartan's moving frame method is essential approach for geometric objects in contact kinematics [41, 42]. The method was widely applied in the computation of symmetry groups, partial differential equations [43, 44], geometrical curves and surfaces [45, 46] or finite dimensional transformation group from Lie algebras [47, 48]; however, there has been little attention to the robotic field. One of the remarkable studies from Lei [49] is to explore differential geometries in terms of curvatures of shapes through the spin motion to establish the contact theory.

3.2 Motion Planning and Path Planning

Introduction. Motion planning is the most important task of robotics research [50],[51] in static and dynamic environment as an emerging area for a long time. The path planning strategy for robotic research can be categorized into traditional methods and discrete approach or also divided into two folds as the local path planning and the global path planning strategy. However, most of the previous studies focus on mobile robots, unmanned aerial vehicle (UAV) or autonomous sef-driving car while only few research has been implemented on the rolling contact of robotic in-hand manipulation.

3.2.1 Traditional Motion Planning

Reaching the final configuration and reducing the cost of path generation within the minimum distance and time are the most important tasks of robotic motion planning. There are various approaches proposed/implemented by many researchers that are highlighted below.

Roadmap approach. Roadmap is one of the classical techniques has been focused on the precise motion planning where the configuration-free space is withdrawn into the system of 1D lines. The connectivities of the free space F are captured by a network of 1D curves such as Voronoi diagrams, roadmap [52], Star-shaped roadmap (a deterministic sampling approach) [53] and criticality based method [54]. There are some drawbacks on these methods including the computation of free space and less practical algorithms for computing these methods of large environment.

Classical cell decomposition approach. Another classical method of motion planning is called cell decomposition which is divided into full cell, empty cell and mixed cell [55]. The advantages of this method is to compute the planning process incrementally which represent a sequence of cell connecting. The borders between all the cells, which are assigned as the function of the environment, may represent the favourable circumstances of the cell decomposition method. However, the method is still not to compute the free space precisely that can be approached as a dual to the roadmap method.

Potential field category. It is quite different from two previous approaches that the connectivity graph in potential field method is not required to pre-compute in the process of path generation. In stead, searching of a path is guided by a heuristics and constructed by an artificial potential function which is represented the sum of potentials (achieving the goal configuration and avoiding the obstacles) [56]. The potential field is distinct across to avoid obstacles in each step time. Various application from this method onto the movement of nonholonomic mobile robots or human-robot interaction.

Non-holonomic motion planning. Non-holonomic motion planning has received much attention in the past. The simply definition of non-holonomic robot motion planning is the movement of the robot from an initial configuration to a desired configuration [12, 57, 58]. The motion planning of the object to acquire a desired configuration and the grasp planning in terms of contact force optimization are two main categories of dexterous motion planning [33]. Specific motion planning of rolling surfaces in terms of chained-form has been introduced by several authors such as Brockett [59] who introduced sinusoidal inputs then the method was developed in more detail by Murray et. al [13]. However, the article [60] could not transform the triangularized form of the system equation into chained-form while Monaco [61] investigated non-holonomic chained systems through the two constant inputs where they achieved the interactive planning schemes.

3.2.2 Path planning in discrete space

From continuous to discrete. Path planning of robots in complex environments has received quite a lot of attention in the past while only a few studies have focused on discrete space. Interestingly, combining computation frameworks and the discrete algorithms was considered in recent studies to capture the complex environment [62]. This method is combined with the continuous path planning [63] that can generate or model the kinematics, control laws and a path of the robots. Therefore, the computational framework for automatic path planning for robots in unknown environment in discrete space also needs to be considered.

Discrete path planning. Planning techniques are categorized into different aspects. The basic idea of discrete path planning in the most cases is that state-space models will be used to demonstrate the distinct situation in which the task of a planning algorithm solves the sequence actions transforming from a initial state to other states [5]. For example, Thomas [64] applied Delaunay triangulations to discretize the environment, and cubic spline representations are proposed to meet robot kinematic constraints. Considering the continuous curvature on smooth curves has been integrated within the probabilistic approaches in order to compute the piecewise smooth paths for a car-like vehicle as a four-dimensional system [65]. Whereas, dealing with nonholonomic constraints, a sampling-based road map technique was proposed in [66]. In [67], based on decomposing space into cells, a potential field without local minima was assigned with polygonal partitions of planar environments to solve the Laplace's equation problems in each cell exist.

Probability cell decomposition. The simple idea of this method is to determine a path between an initial configuration and the goal configuration in the way of dividing the free space into cells. There are two terms of the method including an approximation cell decomposition and an exact cell decomposition[68, 69]. The former methodology refers to a decomposition in which the cells is bounded approximation by the free space that can allow the robot finalizes the motion planning tasks with complex geometries to achieve connectivity paths. Exact cell decomposition has the first step to decompose the free space into trapezoidal triangular cells then nodes which represent cells in the connectivity graph are adjacent in the configuration space. However, there are intensive time and memory on the computation of decompositions and limited volume of the configuration space, which rise exponentially with the DOFs of system.

Randomized potential field algorithm. Precomputation of a connectivity graph of the global path-planning which contains the guide for grid search in the configuration space is the high cost for computation system. Using the properties of potential function [70] can generate no systematic way to escape the minima at the goal configuration. The technique in the first step is the best-first search which does not require to reach a local minimum of the potential function in hight-dimensional configuration spaces. Then the search algorithm which proceeds along the negated gradient of the potential function until the goal configuration is achieved. The most powerful of this method is to discretize the configuration space and the work space into a hierarchical bitmap grid that can be applied for many DOFs of robots.

Rapidly-exploring Random Trees (RRTs) and RRT-Connect. RRTs is a randomized data structure technique to solve a planning problem[71]. The method does not require any connections of nearby configurations. It can be applied for path planning problems that have the nonholonomic constraints and high degree of freedom. The method still remains on the trajectory optimization problems. Another study in [72] improved the RRTs method called RRT-Connect technique which combines the RRTs and a greedy heuristic to speed up the exploration of configuration (state) space and the connection from an initial configuration to other goal. However, these challenging issues still remain in terms of computational geometry such as the artificial bias which can be given from searching nearest-neighbour to the convergence rate.

Probabilistic Roadmap Planer-PRM. The PRM technique has been successful for path planning problems, which was implemented in different sites [73–75]. The PRM computation consists of two phases: the preprocessing phase and the query phase. Repeating the generating random free configuration space can generate a probabilistic roadmap in the preprocessing phase. The nodes of the graph and the paths are computed through local planner that can create the graph edges. In the query phase, there is starting by connection between the initial and the goal configuration by a Dijkstra's shortest path query [76]. Finding complete edges from connecting nodes in the roadmap to generate a graph search is the feasible path for the planning problem. Nevertheless, Probabilistic Roadmap Planner method should be optimized due to some reasons such as the low quality of searching process - the graph is a tree, not cycle graphs and involving straight-line motion which generates the first order discontinuities at the nodes.

Heuristic search method. The fundamental robotic path planning problem is to represent the environment as a graph involving the set of possible robot location and a set of edges that can generate the paths. The popular method for determining the least-cost paths is A* as Heuristic based search algorithm in [77–79]. The search algorithm must expand the fewest possible nodes in order to make searching for an admissible path. Then the evaluation of available nodes is needed to determine the next efficient nodes. The initial search approached by A* takes two steps to generate an optimal path in which receiving information from one of the initial cells in free space and replanning from scratch when the environment has changed to expand a new cell. However, the A* computation process needs high configuration processors to successfully reach various nodes. In the real world scenarios, the search operating sometimes may be performed with inaccurate planning graphs.

Dynamic programming. Dynamic programming (DP) is a technique of robot path planning problem solving to calculate the distance of the goal configuration from all the initial configuration in the grid map [4]. The environment in most robot path planning issues is implemented by a topologically organized map where the connections between each grid points and their neighbouring grid points are built. The distances at every iteration are updated to constitute the neighbour cells when the environment is discretized into a grid of points. As proof in [4], the DP method can provide a simple approximation to optimize the trajectory solution that does not suffer from the curse of dimensionality. A criticism of DP that has precluded the practical implementation in path planning problems is somewhat more difficult for executing the DP algorithm due to time consuming [80]. The efficient cost is the property of the DP method by sub-dividing the complex problems into sub-problems and converging various steps of solving sub-problems. However, the expense of the DP algorithm is not as overwhelming nowadays thanks to the faster and stronger computer and the parallel-processing computer which can efficiently execute the expanding nodes.

<u>Genetic algorithm technique.</u> The genetic algorithm was initially proposed by Holland [81] as a non-conventional method. The GAs method has been applied for a wide assortment of problems such as natural genetic operators-reproduction, mutation, and crossover. GAs benefit the great solutions for upgrade problems to find an optimal path of the mobile robots from an initial configuration to the goal configuration in a grid environment. The GA technique currently is used to find the shortest path in terms of less number of generations in different environments such as indoor, moderate or complex scattered environment. However, the GAs tend to find a feasible path without considering the distribution and location of obstacles in the unknown environment.

<u>Optimal path planning.</u> In order to optimal paths which are generated from various path planning techniques, an introduction of a new method based on the randomization and the dynamic programming was proposed in [82]. The method normally is used to optimize paths for mobile robots and articulated robot manipulation. Using only the dynamic programming can lead to cost of calculating the path segments from one cell to other grid cells because of the large size of the search space, especially for curse dimensional issues. To overcome this problem, the use of randomization in the discretized grid within the dynamic programming may reduce the cost path. Considering the orthogonal neighbour relations that are connections between a node and orthogonal one another can decrease the computations. Another study in [83] called Sampling-based algorithm based on the Rapidly-exploring Random Tree (RRT) algorithm by combining the Transition-based RRT (T-RRT) and RRT* can solve complex high-dimensional path planning problems and converge faster to the optimal path.

3.3 Markov Decision Process

<u>Introduction.</u> Markov Decision Process (MDP) is an extension of a Markov chain which is a stochastic model in discrete time. MDP represents a mathematical framework to model decision making process that has been studied in early [84, 85]. For the record, many applied study may have an implicit underlying the MDP framework to determine the expected cost of raw material that the stock can purchase or shortage [86], to control the pest or protect natural resources [87]. In the robotic field, the MDP has been successfully applied for path planning problems such as combination with a quadtree decomposition of the environment to compute the motion plan [88] or association with ideas of deterministic search and dynamic programming techniques to reduce the processing cost and improve performance path planning algorithm [89]. Another promising technique for autonomous trajectory planning is based on the MDP with clothoid tentacles [90]. The study used stochastic transitions [91] that can reduce the presence of distant obstacles in an occupancy grid.

<u>The agent-environment interface.</u> As shown in Figure 1, there are two main objects in the MDP including an agent and the environment that they interact together at each of a sequence of discrete time steps, $t = 0, 1, 2, 3, ...$ The process in some states S_t at each time step t is described that the decision maker chooses any action A_t in the state and the agent will receive the reward R_t from the environment. In [92], the general rule of agent-environment boundary is not the same as the physical boundary of a robot's or animal's body. In some cases, the agent may know everything in the environment, just as the rewards which are computed from the agent's action function or the state received. However, in another case of solving puzzle like Rubik's cube, the agent knows all the environment such as colors, faces and dimensions but hard to solve its work.
MDP is a tuple $< S, A, T, r, \gamma >$ where S is a set of observations when the agent observes a state from the environment; S is a set of actions that the agent can execute from the task to interact with the environment; T is transition probability matrix in which of making the action $a \in A$ from the current state $s \in S$ to the next state s' ($T(s'|s, a) = \mathbb{P}[S_t = s, A_t = a]$); r is the reward model that the agent can receive in the state s when executing an action a ($r(s, a) = \mathbb{E}[R_{t+1}|S_t = s, A_t = a]$), and γ is the

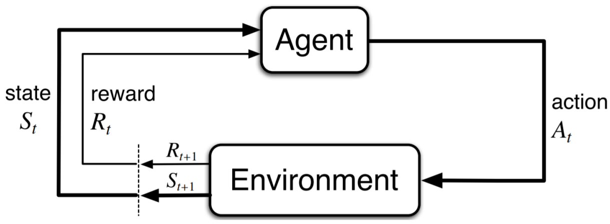

Figure 1: The interaction of agent and environment [92].

discount factor where $0 < \gamma < 1$ that relatives between immediate and future rewards. In path planning problems, the MDP method is applied for finding the path in terms of optimizing the expected sum of discounted rewards and the Bellman equation [93] can be promoted in the study.

<u>Bellman equation.</u> Before discussing the Bellman equation, there are some of principal components of the Reinforcement Learning framework including Reward and Return, Policies, and Value functions which are briefly introduced as following.
In reinforcement learning, there are two types of value function used to optimize the policy including the state value function $V^\pi(s) = \mathbb{E}_\pi[R_t|s_t = s]$ and the action value function $Q^\pi(s,a) = \mathbb{E}_\pi[s_t = s, a_t = a]$. They are the expected returns generated from the state and action under the policy π. The value function changes dependently on the policy for the same environment due to that fact that the value of the state changes dependently and expected rewards will be received. The action value function represents the value of taking an action in some state s.

If we call $\mathcal{P}$ is the transition probability from starting state s, taking action a, then ending up in state s', we have $\mathcal{P}^a_{ss'} = pr(s_{t+1} = s'|s_t = s, a_t = a)$. In addition, $\mathcal{R}^a_{ss'}$ is called the expected reward received from starting state s, taking action a, and moving into state s', we have $\mathcal{R}^a_{ss'} = \mathbb{E}[r_{t+1}|s_t = s, s_{t+1} = s', a_t = a]$ Finally, the Bellman equation [93] can be derived for the on-policy state value function with the results is as follows:

$$V^\pi(s) = \sum_a \pi(s,a) \sum_{s'} \mathcal{P}^a_{ss'}[\mathcal{R}^a_{ss'} + \gamma V^\pi(s')] \tag{1}$$

and for the on-policy action value function as:

$$Q^\pi(s,a) = \sum_{s'} \mathcal{P}^a_{ss'}[\mathcal{R}^a_{ss'} + \gamma \sum_{a'} \pi(s',a') Q^\pi(s',a')] \tag{2}$$

for all $s \in S, a \in A(s)$.
It is noticed that the Bellman equation can represent the values of states as values of other states like easily calculating the value of state s_t when knowing the value of state s_{t+1}. However, to apply for the stochastic shortest path problems by using the Bellman equation, there are techniques such as value iteration, policy iteration, and linear programming should be taken into consideration to solve the Bellman equation.

<u>Value iteration.</u> This is one of the method to find an optimal policy is to determine the optimal value function. The method in detail to converge to the correct V^* values that can be performed by an iterative algorithm [4, 94]. The Bellman equation for the optimal value function is as follows:

$$Q^*(s,a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q^*(S_{t+1}, a')|S_t = s, A_t = a] = \sum_{s',r} p(s',r|s,a)[r + \gamma \max_{a'} Q^*(s',a')] \tag{3}$$

The algorithm will stop when the value function changes in a small amount of one update of each state (sweep). The combination of one sweep of policy evaluation and one sweep of policy improvement in the value iteration algorithm can make the convergence faster. It is also noted that the value iteration can be obtained when the step of updating the Bellman optimality equation happens and the updated value has to be required maximum over all actions.

<u>Policy iteration.</u> Policy iteration algorithm can be defined as another way of determining an optimal policy in a finite number iteration. The iteration starts with the value function $V(s)$ of the previous

policy π in each policy iteration as $\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} ...\pi_* \xrightarrow{E} v_*$, where $\xrightarrow{E}$ denotes a policy evaluation and $\xrightarrow{I}$ indicates a policy improvement. The Bellman equation for the optimal value function as the policy iteration is:

$$V^*(s) = \max_a \mathbb{E}[R_{t+1} + \gamma V^*(S_{t+1})|S_t = s, A_t = a] = \max_a \sum_{s',r} p(s', r|s, a)[r + \gamma V^*(s')] \tag{4}$$

The policy iteration algorithm may converge only few iterations that will be gained the expected infinite discounted reward by executing the policy [92]:

$$\pi'(s) = argmax_a(R(s, a) + \gamma \sum_{s' \in S} p(s', r|s, a)V^\pi(s')) \tag{5}$$

Linear programming. Linear programming methods can be used to solve the Bellman equation and they are better than dynamic programming in some cases as less number of states and a potential method in the curse of dimensionality. The significant application of the linear programming based path planning algorithm is to derive optimal paths for robotics in various kinds of environment [95]. The article proposed a linear formulation for resource allocation of the path planning issue. Integrating linear programming to a receding horizon implementation for multi-vehicle path planning is introduced as a model simplification. To describe the feasible path planning, the paper also used stochastic models which can propose the current position and predict the future trajectories. Another implementation of linear programming from [96] for the path planning problems has been focus on obstacle avoidance. The most advantage of this improvement is to reduce the computation time cost in the real time computation for the path planning process.

3.4 Robotics In-hand Manipulation

Dexterous manipulation. Simple definition of dexterous manipulation is to manipulate motions of an objects and to move the objects from an initial configuration to a desired configuration via a given trajectory [33, 97, 98]. The dexterous manipulation of an object using multi-fingered robot hand is one of the problems. Li et. al [27] proposed grasp planning algorithm in terms of stability and manipulability and the control algorithm for the coordinated manipulation by a multi-fingered hand. Bicchi et. al [60, 99] demonstrated the technique to achieve the dexterous manipulation via rolling contacts. The author used a continuous method proposed by Sussmann [100] which implemented the dexterous manipulation of an object of arbitrary shape. Developing the technique for dexterous manipulation by integrating the theory of kinematics and nonholonomic motion planing, Han [101] conducted an experiment on dexterous manipulation with multifingered robotic hands with rolling contact.

Tactile feedback in in-hand manipulation. Tactile sensing in robot hands is mostly conducted as the continuous sensing to enhance the dexterity and ability of object manipulation [102]. Tactile feedback plays an essential role for dexterous multifingered-robot hand manipulation tasks. With additional information from tactile sensors, the robustness and the ability to react can be improved by detecting instabilities, disturbances or slippage [103, 104]. However, there are some tools such as the moving frame and curvatures from the geometric differential properties has not been discussed in rolling contact in in-hand manipulation.

Manipulation by rolling contact. Since rolling contact is nonholonomic constraint, the multifingered robot manipulation by rolling contact plays an important role in the improvement of dexterous manipulation. Generally, the manipulation by rolling contact has been considered in different view points including manipulation of objects by multifingered robot hands or consideration of contact points for nonholonomic systems. The study of a three-fingered robot hand manipulating an object has been considered by Cole et al [105] that was extended by Sarkar et al [20] in terms of manipulating an object under the pure rolling contact. However, these studies have not been utilized and the study of the contact point has not been demonstrated enough.

4 SIGNIFICANCE

Research gap. As deeply searching on the literature review, integrating tactile sensors into robot hands may improve the dexterity of robot in-hand manipulation [106] [107] that may emulate human sensing.

Besides, the important tools in differential geometry including curvature theory and Lie group theory are formed to approximate geometry attributes [34, 37, 38, 49, 108, 109]. However, there were not significant investigations into discrete differential geometry and rolling-based multifingered robotic in-hand manipulation.

Research outcome. During the PhD program, I propose to develop a discrete contact theory based on the discrete differential geometry; then apply this tool to implement the dexterity of rolling contact into the robotic hands. The research will apply for robot manipulation that contribute to the development of robot hands to achieve human-like capacity for in-hand manipulation. Not only in the academic sector, the industrial applications are also benefited from the research.

5 RESEARCH METHOD

Proposed methodology. The research aims to propose a new discrete contact theory which will develop the capacity of tactile sensing in robot hands. To begin, the deep exploration of literature review on the theory of rolling contact in multifingered robot hands in both continuous and continuous spaces. Then, it is also important to build up new mathematical model of discrete rolling contact which can apply on the object manipulation by robot hands. To complete the proposed study, the process should be separated into three main steps: Theoretical approach, Simulated system, and Experimental validation.

Theoretical approach. The study mainly focus on the rolling contact between robot hands and objects in discrete space. Firstly, it is crucial to explore the previous theory from continuous to discrete differential geometry as the tools to describe the rolling contact. It can be referenced from previous studies in [37, 38, 49, 109, 110] which described the contact theory in terms of the moving frame, normal curvature, geodesic curvature, and geodesic torsion. In the next stage, specifically reviewing the path planning of objects in in-hand manipulation in discrete space is also proposed in the study.

Simulated system. In discrete path planning stage, using Bellman equation is also the key element to build up a mathematical model. A simple task would be solving the probelm of the rolling contact between a ball and a plane in discrete space. Then the developed tasks can be applied for other objects within different geometries such as a cube or a soccer ball. The successful of these problem solving will be analysed for point contacts between an object and the robot fingers. The new platform in a simulated environment will be designed and tested by using Matlab programming (solving maths and evaluating mathematical formula) and the ROS software as a great simulated environment. Once the successful application of the system is achieved, it can be moved on to the next stage to validate the full operation in real multifingered robot hands.

Experimental validation. The final stage of this study is to validate the proposed theory in terms of rolling contact in multifingered robot hands. The BarrettHand is used with the tactile sensing that can provide the tactile-array data from robot fingers and palm. The ABB IRB 120 robotic arm will be integrated with the BarrettHand to manipulate the object in discrete space.

6 ETHICAL ISSUES

This research proposal will be conducted by the Ph.D. candidate under guide of supervisor without any ethical issues. No any sensitive data and no harmful chemicals will be used in this research.

7 DATA STORAGE

Refer to the Data Management Plan in Appendix A.

8 FACILITY AND RESOURCES

Refer to the Table 1, the facility and resources base on Curtin University and Open Source.

Table 1: Exemplary table

Resources	**Provider**
Robot Operating Software	Open Source
MatLab	Curtin University
ABB Robotic Arm	Curtin University
Computer and Printing	Curtin University

9 TIME LINE

Year		2018				2019				2020			2021	
Activity	Sep	Oct	Dec	Feb	May	Aug	Nov	Feb	May	Aug	Nov	Feb	May	Aug
Candidacy proposal														
Literature review														
Mathematical model														
Simulated approach														
Result & Validation														
Thesis preparation														

References

[1] M. Lipshutz, Shaum's Outline of Theory and Problems of Differential Geometry. 1969.

[2] M. P. d. Carmo, Differential Geometry of Curves and Surfaces. 1976.

[3] J. M. Selig, Geometric Fundamentals of Robotics. Monographs in Computer Science, Springer-Verlag New York, 2005.

[4] R. Bellman, Dynamic Programming. Princeton, NJ, USA: Princeton University Press, 1 ed., 1957.

[5] S. M. LaValle, Planning Algorithms. New York, NY, USA: Cambridge University Press, 2006.

[6] C. Chun Sheng and B. Roth, "On the planar motion of rigid bodies with point contact," Mechanism and Machine Theory, vol. 21, no. 6, pp. 453–466, 1986.

[7] C. Chun Sheng and B. Roth, "On the spatial motion of a rigid body with point contact," in Proceedings. 1987 IEEE International Conference on Robotics and Automation, vol. 4, pp. 686–695, 1987.

[8] C. Cai and B. Roth, "On the spatial motion of a rigid body with line contact," in Proceedings. 1988 IEEE International Conference on Robotics and Automation, pp. 1036–1041 Vol.2, 1988.

[9] A. V. Borisov, Y. N. Fedorov, and I. S. Mamaev, "Chaplygin ball over a fixed sphere: an explicit integration," Regular and Chaotic Dynamics, vol. 13, no. 6, pp. 557–571, 2008.

[10] D. J. Montana, "The kinematics of contact and grasp," The International Journal of Robotics Research, vol. 7, no. 3, pp. 17–32, 1988.

[11] A. Marigo and A. Bicchi, "Rolling bodies with regular surface: controllability theory and applications," IEEE Transactions on Automatic Control, vol. 45, no. 9, pp. 1586–1599, 2000.

[12] Z. Li and J. Canny, "Nonholonomic motion planning," 1991 IEEE International Conference on Robotics and Automation, vol. 192, pp. XV, 448, 1991.

[13] R. M. Murray, S. S. Sastry, and L. Zexiang, A Mathematical Introduction to Robotic Manipulation. Boca Raton, FL, USA: CRC Press, Inc., 1994.

[14] R. Muszyński and K. Tchoń, "Singularities and mobility of nonholonomic systems: The ball rolling on a plane," IFAC Proceedings Volumes, vol. 33, no. 27, pp. 593–598, 2000.

[15] A. V. Borisov and I. S. Mamaev, "Conservation laws, hierarchy of dynamics and explicit integration of nonholonomic systems," Regular and Chaotic Dynamics, vol. 13, no. 5, pp. 443–490, 2008.

[16] A. V. Borisov, I. S. Mamaev, and V. G. Marikhin, "Explicit integration of one problem in nonholonomic mechanics," Doklady Physics, vol. 53, no. 10, pp. 525–528, 2008.

[17] E. Hartmann, "G/sup n/-blending with rolling ball contact curves," in Proceedings Geometric Modeling and Processing 2000. Theory and Applications, pp. 385–389.

[18] R. W. Brockett and L. Dai, Non-holonomic Kinematics and the Role of Elliptic Functions in Constructive Controllability, pp. 1–21. Boston, MA: Springer US, 1993.

[19] D. J. Montana, "The kinematics of multi-fingered manipulation," Robotics and Automation, IEEE Transactions on, vol. 11, no. 4, pp. 491–503, 1995.

[20] N. Sarkar, Y. Xiaoping, and V. Kumar, "Dynamic control of 3-d rolling contacts in two-arm manipulation," IEEE Transactions on Robotics and Automation, vol. 13, no. 3, pp. 364–376, 1997.

[21] S. Arimoto, M. Yoshida, and J. Bae, "Dynamic force/torque closure for 2d and 3d objects by means of rolling contacts with robot fingers," in IEEE International Conference on Robotics, Intelligent Systems and Signal Processing, 2003. Proceedings. 2003, vol. 1, pp. 178–183 vol.1.

[22] M. Svinin and H. Shigeyuki, On the dynamics and motion planning for a rolling system With variable inertia. 2007.

[23] X. Yun, V. Kumar, N. Sarkar, and E. Paljug, "Control of multiple arms with rolling constraints," in Proceedings 1992 IEEE International Conference on Robotics and Automation, pp. 2193–2198 vol.3, 1992.

[24] M. Zribi, J. Chen, and M. S. Mahmoud, "Coordination and control of multi-fingered robot hands with rolling and sliding contacts," Journal of Intelligent and Robotic Systems, vol. 24, no. 2, pp. 125–149, 1999.

[25] A. Marigo and A. Bicchi, "Rolling bodies with regular surface: controllability theory and applications," IEEE Transactions on Automatic Control, vol. 45, no. 9, pp. 1586–1599, 2000.

[26] A. Nakashima, K. Nagase, and Y. Hayakawa, "Simultaneous control of grasp/manipulation and contact points with rolling contact," IFAC Proceedings Volumes, vol. 38, no. 1, pp. 415–420, 2005.

[27] Z. Li, J. F. Canny, and S. S. Sastry, "On motion planning for dexterous manipulation. i. the problem formulation," in Proceedings, 1989 International Conference on Robotics and Automation, pp. 775–780 vol.2, 1989.

[28] A. Chelouah and Y. Chitour, "Motion planning of rolling surfaces," in Proceedings of the 40th IEEE Conference on Decision and Control (Cat. No.01CH37228), vol. 1, pp. 502–507 vol.1, 2001.

[29] M. Svinin and S. Hosoe, "Motion planning algorithms for a rolling sphere with limited contact area," IEEE Transactions on Robotics, vol. 24, no. 3, pp. 612–625, 2008.

[30] J. K. Salisbury and B. Roth, "Kinematic and force analysis of articulated mechanical hands," Journal of Mechanisms, Transmissions, and Automation in Design, vol. 105, no. 1, pp. 35–41, 1983. 10.1115/1.3267342.

[31] Z. Li and S. Sastry, "A unified approach for the control of multifingered robot hands," Contemporary Mathematics, vol. 97, pp. 217–239, 1989.

[32] R. Murray and S. S. Sastry, "Grasping and manipulation using multifingered robot hands," Tech. Rep. UCB/ERL M90/24, EECS Department, University of California, Berkeley, 1990.

[33] A. M. Okamura, N. Smaby, and M. R. Cutkosky, "An overview of dexterous manipulation," in Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference on, vol. 1, pp. 255–262, IEEE, 2000.

[34] L. Cui and J. S. Dai, "Geometric kinematics of rigid bodies with point contact," in Advances in Robot Kinematics: Motion in Man and Machine (J. Lenarcic and M. M. Stanisic, eds.), pp. 429–436, Springer Netherlands.

[35] L. Cui and J. S. Dai, "A polynomial approach to inverse kinematics of rolling contact," no. 45035, pp. 1563–1570, 2012. 10.1115/DETC2012-70852.

[36] L. Cui and J. S. Dai, "Reciprocity-based singular value decomposition for inverse kinematic analysis of the metamorphic multifingered hand," Transactions of the ASME Journal of Mechanisms and Robotics, vol. 4, no. 3, pp. –, 2012. 10.1115/1.4006187.

[37] L. Cui and J. S. Dai, "From sliding–rolling loci to instantaneous kinematics: An adjoint approach," Mechanism and Machine Theory, vol. 85, pp. 161–171, 2015.

[38] L. Cui and J. S. Dai, "A polynomial formulation of inverse kinematics of rolling contact," Journal of Mechanisms and Robotics, vol. 7, no. 4, pp. 041003–041003–9, 2015. 10.1115/1.4029498.

[39] L. Cui, J. Sun, and J. S. Dai, "In-hand forward and inverse kinematics with rolling contact," Robotica, vol. 35, no. 12, pp. 2381–2399, 2017.

[40] L. Cui and J. S. Dai, "Rolling contact in kinematics of multifingered robotic hands," in Advances in Robot Kinematics 2016 (J. Lenarčič and J.-P. Merlet, eds.), pp. 217–224, Cham: Springer International Publishing, 2018.

[41] H. Cartan, Differential Forms. New York: Dover Publisher, 1996.

[42] E. J. Cartan, Riemannian Geometry in an Orthogonal Frame. Singapore : World Scientific, 2002.

[43] E. L. Mansfield, "Algorithms for symmetric differential systems," Foundations of Computational Mathematics, vol. 1, no. 4, pp. 335–383, 2001.

[44] M. Oleg, "Moving coframes and symmetries of differential equations," Journal of Physics A: Mathematical and General, vol. 35, no. 12, p. 2965, 2002.

[45] G. M. Beffa, "Relative and absolute differential invariants for conformal curves," Journal of Lie Theory, vol. 13, no. 1, pp. 231–245, 2003.

[46] G. M. Beffa and xed, "Poisson geometry of differential invariants of curves in some nonsemisimple homogeneous spaces," Proceedings of the American Mathematical Society, vol. 134, no. 3, pp. 779–791, 2006.

[47] B. Vyacheslav, P. Jiri, and P. Roman, "Computation of invariants of lie algebras by means of moving frames," Journal of Physics A: Mathematical and General, vol. 39, no. 20, p. 5749, 2006.

[48] V. Boyko, J. Patera, and R. Popovych, "Invariants of solvable lie algebras with triangular nilradicals and diagonal nilindependent elements," Linear Algebra and its Applications, vol. 428, no. 4, pp. 834–854, 2008.

[49] L. Cui and J. S. Dai, "A darboux-frame-based formulation of spin-rolling motion of rigid objects with point contact," IEEE Transactions on Robotics, vol. 26, no. 2, pp. 383–388, 2010.

[50] A. Sudsang, N. Srinivasa, et al., "Grasping and in-hand manipulation: Geometry and algorithms," Algorithmica, vol. 26, no. 3-4, pp. 466–493, 2000.

[51] J. Pajarinen and V. Kyrki, "Robotic manipulation of multiple objects as a pomdp," Artificial Intelligence, vol. 247, pp. 213–228, 2017.

[52] J. F. Canny, Complexity of Robot Motion Planning. PhD thesis, ACM Doctoral Dissertation Award, MIT Press, 1988.

[53] G. Varadhan and D. Manocha, "Star-shaped roadmaps - a deterministic sampling approach for complete motion planning," in Robotics: Science and Systems, 2005.

[54] J. Latombe, "Motion planning: A journey of robots, molecules, digital actors, and other artifacts," I. J. Robotics Res., vol. 18, no. 11, pp. 1119–1128, 1999.

[55] J. Latombe, "A fast path planner for a car-like indoor mobile robot," in Proceedings of the 9th National Conference on Artificial Intelligence, Anaheim, CA, USA, July 14-19, 1991, Volume 2., pp. 659–665, 1991.

[56] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," in Proceedings. 1985 IEEE International Conference on Robotics and Automation, vol. 2, pp. 500–505, March 1985.

[57] S. Sastry and Z. Li, "Robot motion planning with nonholonomic constraints," in Proceedings of the 28th IEEE Conference on Decision and Control, pp. 211–216 vol.1, 1989.

[58] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: steering using sinusoids," IEEE Transactions on Automatic Control, vol. 38, no. 5, pp. 700–716, 1993.

[59] R. W. Brockett, Control Theory and Singular Riemannian Geometry, pp. 11–27. New York, NY: Springer New York, 1982.

[60] A. Bicchi and R. Sorrentino, "Dexterous manipulation through rolling," in Proceedings of 1995 IEEE International Conference on Robotics and Automation, vol. 1, pp. 452–457 vol.1.

[61] S. Monaco and D. Normand-Cyrot, "An introduction to motion planning under multirate digital control," in [1992] Proceedings of the 31st IEEE Conference on Decision and Control, pp. 1780–1785 vol.2.

[62] C. Belta, V. Isler, and G. J. Pappas, "Discrete abstractions for robot motion planning and control in polygonal environments," IEEE Transactions on Robotics, vol. 21, no. 5, pp. 864–874, 2005.

[63] I. M. Mitchell and S. Sastry, "Continuous path planning with multiple constraints," in 42nd IEEE International Conference on Decision and Control (IEEE Cat. No.03CH37475), vol. 5, pp. 5502–5507 Vol.5, 2003.

[64] J. Thomas, A. Blair, and N. Barnes, "Towards an efficient optimal trajectory planner for multiple mobile robots," in Intelligent Robots and Systems, 2003.(IROS 2003). Proceedings. 2003 IEEE/RSJ International Conference on, vol. 3, pp. 2291–2296, IEEE, 2003.

[65] F. Lamiraux and J.-P. Lammond, "Smooth motion planning for car-like vehicles," IEEE Transactions on Robotics and Automation, vol. 17, no. 4, pp. 498–501, 2001.

[66] P. Cheng, Z. Shen, and S. La Valle, "Rrt-based trajectory design for autonomous automobiles and spacecraft," Archives of control sciences, vol. 11, no. 3/4, pp. 167–194, 2001.

[67] D. C. Conner, A. A. Rizzi, and H. Choset, "Composition of local potential functions for global robot control and navigation," in Intelligent Robots and Systems, 2003.(IROS 2003). Proceedings. 2003 IEEE/RSJ International Conference on, vol. 4, pp. 3546–3551, IEEE, 2003.

[68] F. Lingelbach, "Path planning using probabilistic cell decomposition," in IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA '04. 2004, vol. 1, pp. 467–472 Vol.1, 2004.

[69] J. Rosell and P. Iniguez, "Path planning using harmonic functions and probabilistic cell decomposition," in Proceedings of the 2005 IEEE International Conference on Robotics and Automation, pp. 1803–1808, 2005.

[70] J. Barraquand and J.-C. Latombe, "Robot motion planning: A distributed representation approach," The International Journal of Robotics Research, vol. 10, no. 6, pp. 628–649, 1991.

[71] S. M. Lavalle, "Rapidly-exploring random trees: A new tool for path planning," tech. rep., Department of Computer Science; Iowa State University, 1998.

[72] J. J. Kuffner and S. M. LaValle, "Rrt-connect: An efficient approach to single-query path planning," in Proceedings 2000 ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation. Symposia Proceedings (Cat. No.00CH37065), vol. 2, pp. 995–1001 vol.2.

[73] N. M. Amato and Y. Wu, "A randomized roadmap method for path and manipulation planning," in Proceedings of IEEE International Conference on Robotics and Automation, vol. 1, pp. 113–120 vol.1.

[74] L. E. Kavraki, Random Networks in Configuration Space for Fast Path Planning. Phd, 1995.

[75] M. H. Overmars, "A random approach to motion planning," report, Utrecht University, 1992.

[76] R. Geraerts and M. H. Overmars, A Comparative Study of Probabilistic Roadmap Planners, pp. 43–57. Berlin, Heidelberg: Springer Berlin Heidelberg, 2004.

[77] P. H. Raphael, N. Nilsson, and Bertram, "A formal basis for the heuristic determination of minimum cost paths," IEEE Transactions on Systems Science and Cybernetics, vol. 4, pp. 100–107, 1968.

[78] N. J. Nilsson, Principles of Artificial Intelligence. Springer-Verlag Berlin Heidelberg, 1982.

[79] A. L. Rankin and C. D. Crane III, "A multi-purpose off-line path planner based on an a* search algorithm," in Proceedings of the ASME Design Engineering Technical Conferences, pp. 1–10, Citeseer, 1996.

[80] R. Kala, A. Shukla, and R. Tiwari, "Robot path planning using dynamic programming with accelerating nodes," 2012.

[81] J. H. Holland, Adaptation in Natural and Artificial Systems. In University of Michigan Press, Ann Arbor., 1975.

[82] C. S. Sallaberger and G. M. T. D'Eleuterio, "Optimal robotic path planning using dynamic programming and randomization," Acta Astronautica, vol. 35, no. 2, pp. 143–156, 1995.

[83] D. Devaurs, T. Siméon, and J. Cortés, "Optimal path planning in complex cost spaces with sampling-based algorithms," IEEE Transactions on Automation Science and Engineering, vol. 13, no. 2, pp. 415–424, 2016.

[84] R. A. Howard, Dynamic programming and Markov processes. 1964.

[85] B. L. Miller, "Finite state continuous time markov decision processes with an infinite planning horizon," Journal of Mathematical Analysis and Applications, vol. 22, no. 3, pp. 552–569, 1968.

[86] D. J. White, "Real applications of markov decision processes," Interfaces, vol. 15, no. 6, pp. 73–83, 1985.

[87] D. J. White, "A survey of applications of markov decision processes," The Journal of the Operational Research Society, vol. 44, no. 11, pp. 1073–1096, 1993.

[88] J. Burlet, O. Aycard, and T. Fraichard, "Robust motion planning using markov decision processes and quadtree decomposition," in IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA '04. 2004, vol. 3, pp. 2820–2825 Vol.3, 2004.

[89] D. Ferguson and A. Stentz, "Focussed processing of mdps for path planning," in 16th IEEE International Conference on Tools with Artificial Intelligence, pp. 310–317, 2004.

[90] H. Mouhagir, R. Talj, V. Cherfaoui, F. Guillemard, and F. Aioun, "A markov decision process-based approach for trajectory planning with clothoid tentacles," in 2016 IEEE Intelligent Vehicles Symposium (IV), pp. 1254–1259, 2016.

[91] M. L. Puterman, Markov decision processes: discrete stochastic dynamic programming. 2014.

[92] R. S. Sutton and A. G. Barto, Reinforcement learning: An introduction. MIT press, 2018.

[93] R. Bellman, "A markovian decision process," Journal of Mathematics and Mechanics, pp. 679–684, 1957.

[94] D. P. Bertsekas, Dynamic programming: deterministic and stochastic models. Englewood Cliffs, N.J. : Prentice-Hall, 1987.

[95] G. C. Chasparis and J. S. Shamma, "Linear-programming-based multi-vehicle path planning with adversaries," in Proceedings of the 2005, American Control Conference, 2005., pp. 1072–1077 vol. 2, 2005.

[96] L. Yang, J. Qi, and J. Han, "Path planning methods for mobile robots with linear programming," in 2012 Proceedings of International Conference on Modelling, Identification and Control, pp. 641–646, 2012.

[97] I. M. Bullock and A. M. Dollar, "Classifying human manipulation behavior," in 2011 IEEE International Conference on Rehabilitation Robotics, pp. 1–6.

[98] R. R. Ma and A. M. Dollar, "On dexterity and dexterous manipulation," in Advanced Robotics (ICAR), 2011 15th International Conference on, pp. 1–7, IEEE, 2011.

[99] A. Bicchi and A. Marigo, "Rolling contacts and dexterous manipulation," in Robotics and Automation, 2000. Proceedings. ICRA'00. IEEE International Conference on, vol. 1, pp. 282–287, IEEE, 2000.

[100] H. Sussmann, "A continuation method for nonholonomic path-finding problems," in Proceedings of 32nd IEEE Conference on Decision and Control, pp. 2718–2723 vol.3.

[101] L. Han, Y. S. Guan, Z. X. Li, Q. Shi, and J. C. Trinkle, "Dextrous manipulation with rolling contacts," in Proceedings of International Conference on Robotics and Automation, vol. 2, pp. 992–997 vol.2.

[102] H. R. Nicholls and M. H. Lee, "A survey of robot tactile sensing technology," The International Journal of Robotics Research, vol. 8, no. 3, pp. 3–30, 1989.

[103] Y. Bekiroglu, J. Laaksonen, J. A. Jorgensen, V. Kyrki, and D. Kragic, “Assessing grasp stability based on learning and haptic data,” IEEE Transactions on Robotics, vol. 27, no. 3, pp. 616–629, 2011.

[104] M. Li, Y. Bekiroglu, D. Kragic, and A. Billard, “Learning of grasp adaptation through experience and tactile sensing,” in 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 3339–3346, 2014.

[105] A. B. A. Cole, J. E. Hauser, and S. S. Sastry, “Kinematics and control of multifingered hands with rolling contact,” IEEE Transactions on Automatic Control, vol. 34, no. 4, pp. 398–404, 1989.

[106] L. Cui, U. Cupcic, and J. S. Dai, “An optimization approach to teleoperation of the thumb of a humanoid robot hand,” Journal of Mechanical Design, vol. 136, no. 9, 2014. 10.1115/1.4027759.

[107] J. A. Bagnell, F. Cavalcanti, L. Cui, T. Galluzzo, M. Hebert, M. Kazemi, M. Klingensmith, J. Libby, T. Y. Liu, N. Pollard, M. Pivtoraiko, J. Valois, and R. Zhu, “An integrated system for autonomous robotics manipulation,” in 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 2955–2962.

[108] L. Cui, D. Wang, and J. S. Dai, “Kinematic geometry of circular surfaces with a fixed radius based on euclidean invariants,” Journal of Mechanical Design, vol. 131, no. 10, pp. 101009–101009–8, 2009. 10.1115/1.3212679.

[109] L. Cui and J. S. Dai, “A coordinate-free approach to instantaneous kinematics of two rigid objects with rolling contact and its implications for trajectory planning,” in 2009 IEEE International Conference on Robotics and Automation, pp. 612–617, 2009.

[110] L. Cui, Differential Geometry Based Kinematics of Sliding-Rolling Contact and Its Use for Multifingered Hands. PhD Thesis, King’s College London, Centre for Robotics Research, London, UK, 2010.

A APPENDIX

This is the appendix