Contents

1 ABSTRACT

250 words or less, concise summary of research conducted, results obtained, and conclusion reached
Background: Place the question addressed in a broad context and highlight the purpose of the study.

Aim:

Approach: Methods: Describe briefly the main methods or treatments applied;

Significance:Results: Summarize the article's main findings;

Conclusion: Indicate the main conclusions or interpretations. The abstract should be an objective representation of the article, it must not contain results which are not presented and substantiated in the main text and should not exaggerate the main conclusions.

Examples: from "2018 Path Planning of Industrial Robot - RRT"

With the development of modern manufacturing industry,the application scenarios of industrial robot are becoming more and more complex. Manual programming of industrial robot requires a great deal of effort and time. **Therefore**, an autonomous path planning is an important development direction of industrial robot.

Among the path planning methods, the rapidly-exploring random tree (RRT) algorithm based on random sampling has been widely applied for a high-dimensional robotic manipulator because of its probability completeness and outstanding expansion. **However**, especially in the complex scenario, the existing RRT planning algorithms still have a low planning efficiency and some are easily fall into a local minimum.

To tackle these problems, this paper proposes an autonomous path planning method for the robotic manipulator based on an improved RRT algorithm. The method introduces regression mechanism to prevent over-searching configuration space. **In addition**, it adopts an adaptive expansion mechanism to continuously improve reachable spatial information by refining the boundary nodes in joint space, avoiding repeatedly searching for extended nodes. **Furthermore**, it avoids the unnecessary iteration of the robotic manipulator forward kinematics solution and its time-consuming collision detection in Cartesian space. The method can rapidly plan a path to a target point and can be accelerated out of a local minimum area to improve path planning efficiency.

The improved RRT algorithm proposed in this paper is simulated in a complex environment. The results reveal that the proposed algorithm can significantly improve the success rate and efficiency of the planning without losing other performance.

2 INTRODUCTION

- Novelty: Literature review
- Goal: What question you're trying to answer
- Motivation: Why you're asking the question

Guide: *Goal: provide context and encourage reader to read the paper.*
1. Background and motivation (1 paragraph)
2. Overview of the paper and contributions (1-2 paragraphs)
3. More details and summary of the approach
4. Summary of the results and conclusions.
Overview: Q4. Why should the community care?
Related work: Q1. What did the community know before you did whatever you did?
Contribution: Q3. Why exactly did you do?
We focus on....
We propose ABC algorithm...
We prove that ....
We demonstrate the EFG problem through x case studies (Section 3.4). We evaluate the ... (Section 4,5).

In this paper, we present discrete path planning of platonic solids including cube, tetrahedron, octahedron, icosahedron, and dodecahedron. These are types of convex polyhedra with equivalent faces constituted to congruent convex regular polygons....

Not much work has been done in path planning under considering rolling contact. [1] and [2] proposed XYZ method. In their work, they did XYZ (how they did).... However, they did not perform ABC.... => mention Types of rolling contact, and the paper of Z.Li

Literature in the path planning domain describes obstacles avoiding of two general types - continuous and discrete. **Continuous path planning** ....[][] **Discrete path planning** ....[][]. However, bla bla bla ...

Bla bla ....

On the other hand, bla bla bla...

Therefore, in this study, we present three cases of platonic path planning in terms of path finding for the same position and different orientation of initial configuration and goal configuration, direct searching for the long distance between two configuration, and bidirect search within obstacles.

Or: This paper presents a methodology for path planning of platonic solids in known environment. Bla bla ... ref Introduction from "Path planning in multi-scale ocean flows..."

A second contribution of this paper is a technique to compute ....

We explain our algorithms in Section II. We go over experiments and results in simulation in Section III. We verify our algorithms by executing them on a 3D model of the Statue of David and confirming that collision-free trajectories are efficiently generated. Our primary evaluation metric is time taken for the search. We discuss the performance of each individual search, as well as the advantages and shortcomings. Finally, we discuss possible future steps for this work in Section IV.

3 PROBLEM FORMULATION

Five types of Platonic Solids: Platonic solids properties: The platonic solids are also called regular

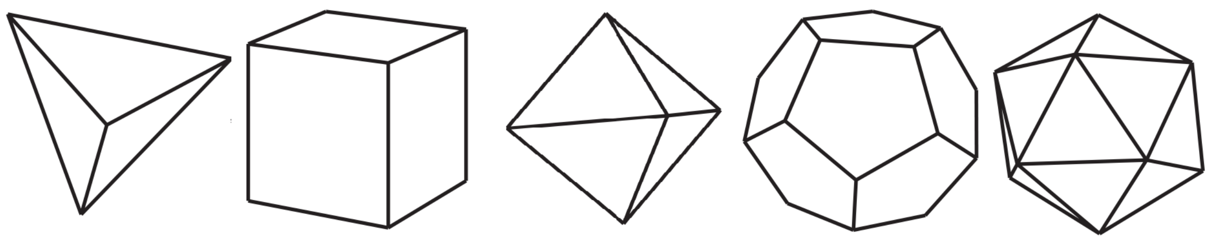


Figure 1: Platonic solids. From left to right: Tetrahedron, Cube, Octahedron, Dodecahedron, and Icosahedron

polyhedra have the convex polyhedra properties. There are only five solids namely cube, tetrahedron, octahedron, dodecahedron and icosahedron. Some of the equivalent statements are used to describe the platonic solids including all the vertices lie on a sphere, all the dihedral angle are equal, and all solid angles are equivalent.

Here is your table 1

Table 1: Properties of polyhedron

	Faces	Edges	Vertices	Edges on each face	Edges meeting at each vertices
Tetrahedron	4	6	4	3	3
Cube	6	12	8	4	3
Octahedron	8	12	6	3	4
Dodecahedron	12	30	20	5	3
Icosahedron	20	30	12	3	5

Here is your table 2

Table 2: Dimensional of platonic solids

	r_d	ρ	R	dihedral angles (β)
Tetrahedron	$\frac{1}{12}\sqrt{6}$	$\frac{1}{4}\sqrt{2}$	$\frac{1}{4}\sqrt{6}$	$\cos^{-1}(\frac{1}{3})$
Cube	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\pi$
Octahedron	$\frac{1}{6}\sqrt{6}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\cos^{-1}(-\frac{1}{3})$
Dodecahedron	$\frac{1}{20}\sqrt{250+110\sqrt{5}}$	$\frac{1}{4}(3+\sqrt{5})$	$\frac{1}{4}(\sqrt{15}+\sqrt{3})$	$\cos^{-1}(-\frac{1}{5}\sqrt{5})$
Icosahedron	$\frac{1}{12}(3\sqrt{3}+\sqrt{15})$	$\frac{1}{4}(1+\sqrt{5})$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\cos^{-1}(-\frac{1}{3}\sqrt{5})$

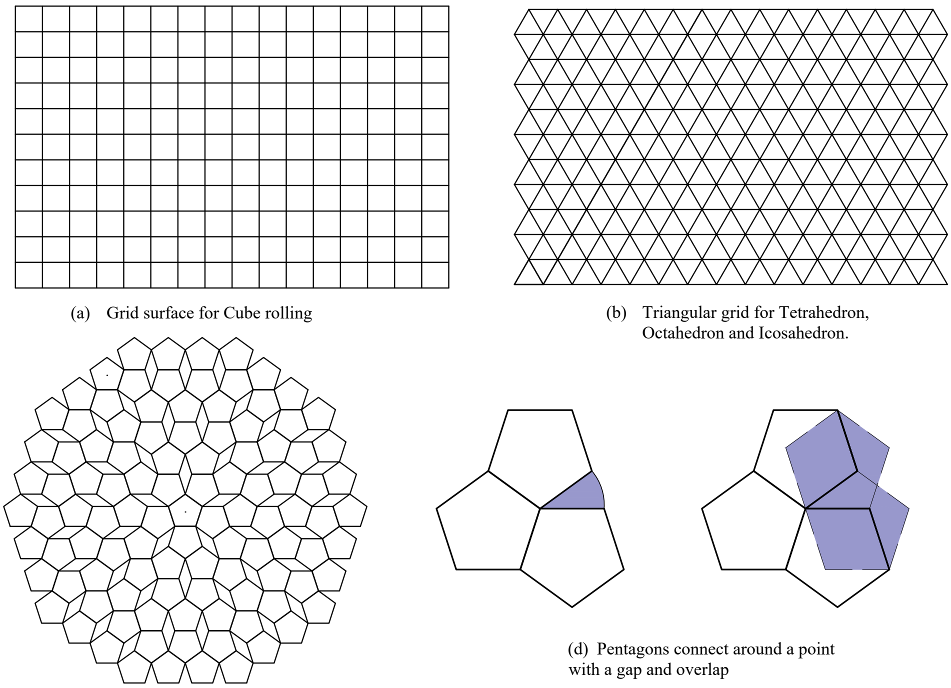
4 ALGORITHM\METHODOLOGY

4.1 Path Planning Based Rolling Contact

Rolling on discretized surfaces: The surface contacts between platonic solids and the plane can be categorized into three types as shown in Figure 2 including square shape for cube, triangle shape for tetrahedron, octahedron, and dodecahedron, pentagon shape for dodecahedron. The bottom surface of a cube occupies each square on the grid when the path planning process is executed. This property is applied for triangular grid with different rotation angle. In physics, the rotation angle of the cube is $\pi/2(rad)$ while the rotation angles of tetrahedron, octahedron, icosahedron and dodecahedron are $\pi - \arctan(2\sqrt{2})$, $\pi - 2\arctan\sqrt{2}$, $\arccos(-\sqrt{5}/3)$, and $\pi - \arccos(-\sqrt{5}/5)$ in radian respectively.

The square grid has $\pi/2$ at all corners while the triangular grid has $\pi/3$ between two arbitrary edges at a vertex. In the case of dodecahedron rolling contact, the Figure 2d shows the two types of connections between pentagons where the first case has a gap (Figure 2c) and the other has overlap pentagon connection. A regular pentagon has five interior angles of $108°$ which generate a gap between three pentagons surrounding because of $3 * 108° = 324°$, which is different $360°$ of the full circle. Another case of four overlap pentagons with $4 * 108° = 432°$ is greater than the circle of $360°$. The path planning through rolling of the dodecahedron solid can be categorized into two these cases. It would be found the possible paths in the first case of dodecahedron without overlap rolling while the second case with overlap rolling cannot guarantee the paths.

The Algorithm 1 shows that path planning for cube rolling based on tree graph search has some important steps. The first step is to initial the coordinates and the orientations of the initial cube and the target cube which is stored as the initial path. The same as tree expansion, cube will roll in four different directions including the right, left, up and down is the next step. From these new positions and orientations, the cubes will continue expand with only three directions to avoid return the previous positions. From these new positions and orientations, the cubes will continue expand with only three directions to avoid return the previous positions.



(a) Grid surface for Cube rolling

(b) Triangular grid for Tetrahedron, Octahedron and Icosahedron.

(c) The radial arrangement from a single central decogon

(d) Pentagons connect around a point with a gap and overlap

Figure 2: Grid of platonic solids

Algorithms: Due to the different surface contacts, there are three types of direction for the rolling of platonic solids. As shown in the Figure 3, the cube has four directions with the square surface contact while tetrahedron, octahedron and icosahedron have three rolling directions with the triangular surface contact. The dodecahedron with pentagon surface contact has five rolling directions. In the case of rolling cube, the surface contact is surrounded by four edges which means there are four possible directions through the edges. In this work, the proposed path planning algorithm deals with rolling from initial configuration within the position and orientation to the goal within the same position but different orientation. While rolling on the smooth plane, the platonic solid models will contact to the plane though their edges.

The Algorithm 1 shows that path planning for cube rolling based on tree graph search has some important steps. The first step is to initial the coordinates and the orientations of the initial cube and the target cube which is stored as the initial path. The same as tree expansion, cube will roll in four different directions including the right, left, up and down is the next step. From these new positions and orientations, the cubes will continue expand with only three directions to avoid return the previous positions. An example for this step is that from the initial coordinate the cube achieves a new position after doing rolling for right direction, the new three positions of the cube by rolling through right, up, and down direction. After implementing the expansion steps through rolling, the function of checking whether updated models reach the goal is called through the loop. By that means, the loop will stop when reaching the goal whereas the loop will continue to execute and store new models to the initial path. While the searching algorithm is executing, the data structure is used to store the positions and orientations from the start to the current. This process runs in time $O(|E|^3)$ (where $|E|$ is the number of updated cubes) which causes the longer the running time of the searching technique.

The Algorithm 1 shows that path planning for cube rolling based on tree graph search has some important steps. The first step is to initial the coordinates and the orientations of the initial cube and the target cube which is stored as the initial path. The same as tree expansion, cube will roll in four different directions including the right, left, up and down is the next step. From these new positions and orientations, the cubes will continue expand with only three directions to avoid return the previous positions.

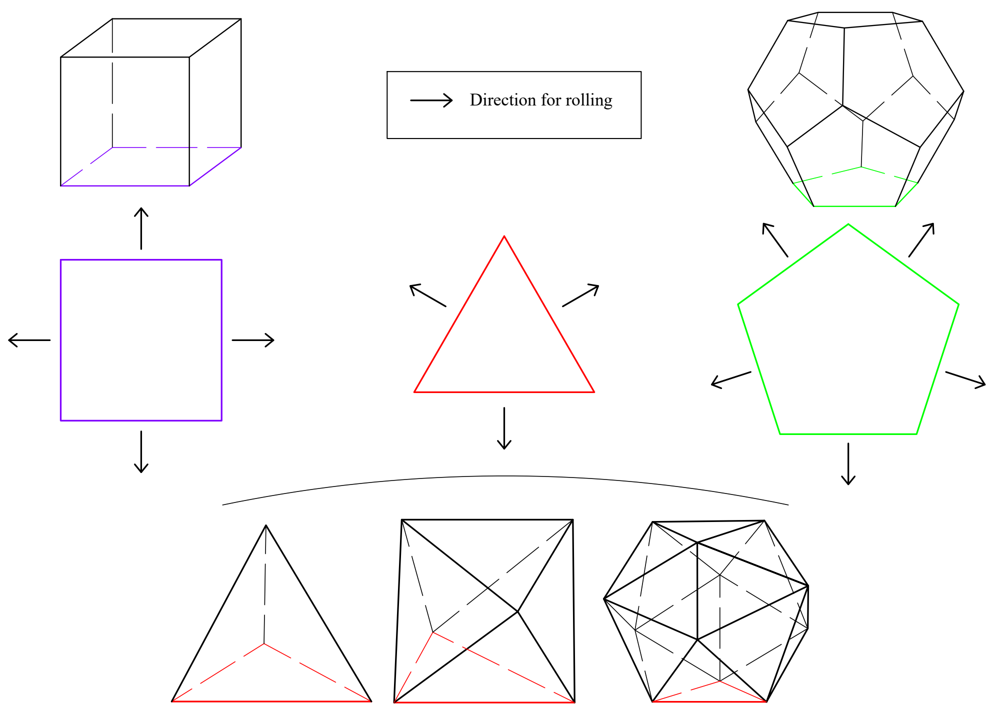


Figure 3: Rolling direction for each types of platonic solids

4.2 Tree Exploration Algorithm

The node tree exploration for searching algorithm described in Algorithm 1 is similar to non-recursive depth-first-search algorithm. The graph search in the Figure 4 shows the expansion from the *root* with node S to multi-level from $level^1 ... level^n$. Each nodes indicates the position of the cube's center and the orientation of the cube. The node S means Start-Point while R, L, U, D are labelled for four different directions including right, left, up and down respectively.

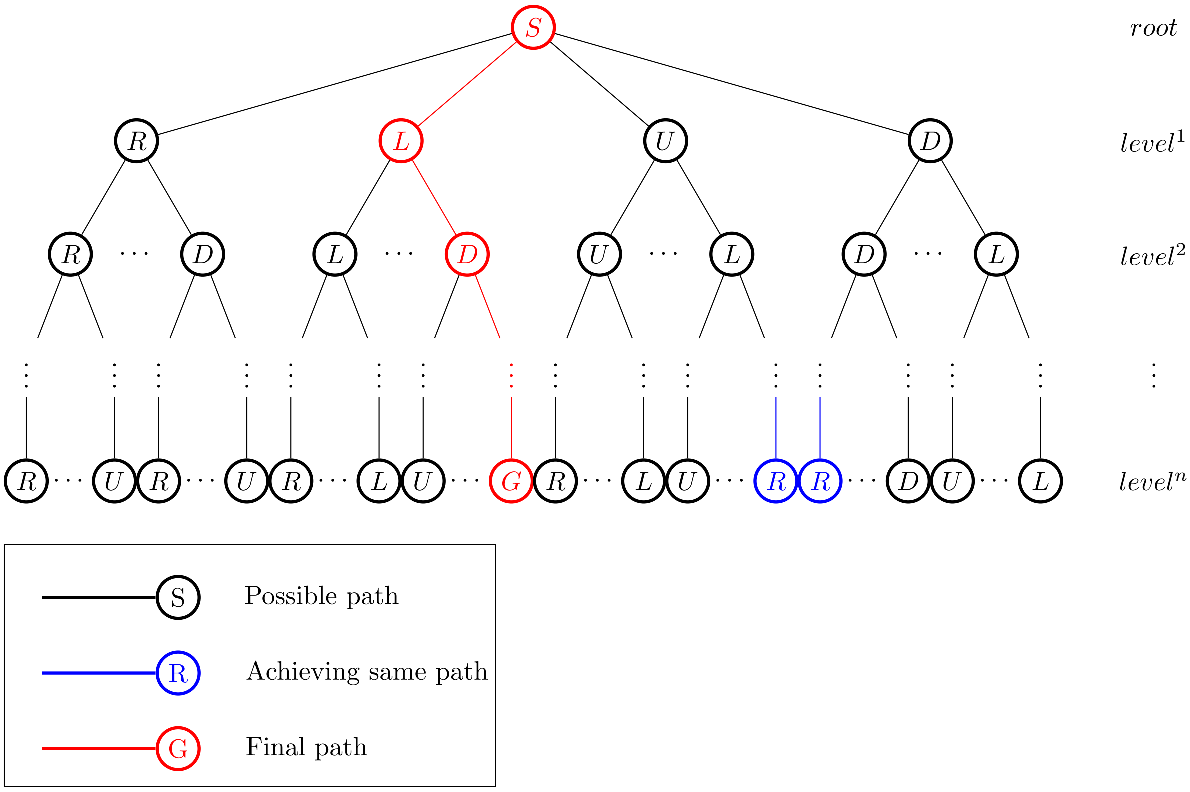


Figure 4: Tree Exploration of Cube Rolling

Starting form the root S, path planning based rolling of the cube model at the first level of expansion will generate to four different direction R, L, U, D. In the next level, the cube can only roll with three directions without rolling back to the previous position. An example of the second level is that node R will roll to right, up, and down directions.

Algorithm 1 Path planning based rolling contact for Cube model.

```
procedure CubePathPlanning(S_p, G_p)  ▷ Find the shortest path from start to goal position with different orientation
    flag ← false
    Path[S_p] ← S_p
    newPoints ← rolling4Directions(S_p)  ▷ Generate first four updated points
    while newPoints != G_p do
        updatedPoints ← treeExploration(newPoints)  ▷ Update new three right rolling models
        n ← size(updatedPoints)
        for i ← 0, n do
            for j ← 1, n do
                if updatedPoints[i] = updatedPoints[j] then
                    remove(updatedPoints[i])
                end if
            end for
        end for
        flag ← checkingTargetPoint(updatedPoints)  ▷ Compare updated points with goal point
        if flag = true then
            return Path[S_p, G_p]  ▷ Store new point to Path
        end if
        newPoints = updatedPoints
    end while
    return "no path found"
end procedure
procedure rolling4Directions(S_p)  ▷ Generate new points in different direction of rolling
    (newRightPoint, newLeftPoint, newUpPoint, newDownPoint) ← rollingContact(S_p)
    return newPoints ← (newRightPoint, newLeftPoint, newUpPoint, newDownPoint)
end procedure
procedure treeExploration(newPoints)
    if dir = right then
        updatedPoints ← (newRightPoint, newUpPoint, newDownPoint)
    else if dir = left then
        updatedPoints ← (newLeftPoint, newUpPoint, newDownPoint)
    else if dir = up then
        updatedPoints ← (newRightPoint, newLeftPoint, newUpPoint)
    else
        updatedPoints ← (newRightPoint, newLeftPoint, newDownPoint)
    end if
    return updatedPoints
end procedure
procedure checkingTargetPoints(updatedPoints,G_p)
    if updatedPoints = G_p then  ▷ Consider both position and orientation
        flag ← true
    end if
    return flag
end procedure
```

5 EVALUATION

The proposed algorithm for platonic solids path planning by rolling through edge contact was implemented in MATLAB environment. In general of path planning, there are three case studies including same location and different orientation between initial configuration and goal configuration, long distance between two configurations, and bi-direction path finding. To validate the proposed algorithm, this study only considers the first case study of path planning that both initial and goal configuration have the same positions and different orientations.

5.1 Cube solid

Properties: The cube has a length which is the same as the length of side of each grid square. The only way to move from initial position to goal position is by rolling from square to square without moving diagonal. The Figure 5 shows the first three layers of cube path finding on the grid. The algorithm implements in $O(|E|^3)$ running time from the second layer. The executed time can be reduced when the updated cubes achieved the same configurations. The $*$ position in the grid is occupied by the two updated cubes with different orientations. To be more visualized, the Figure 6(a) indicates the first step of rolling of the cube with its coordinates in three arrows red, green and blue colors. Four paths of cube rolling are shown in the Figure 6(b).

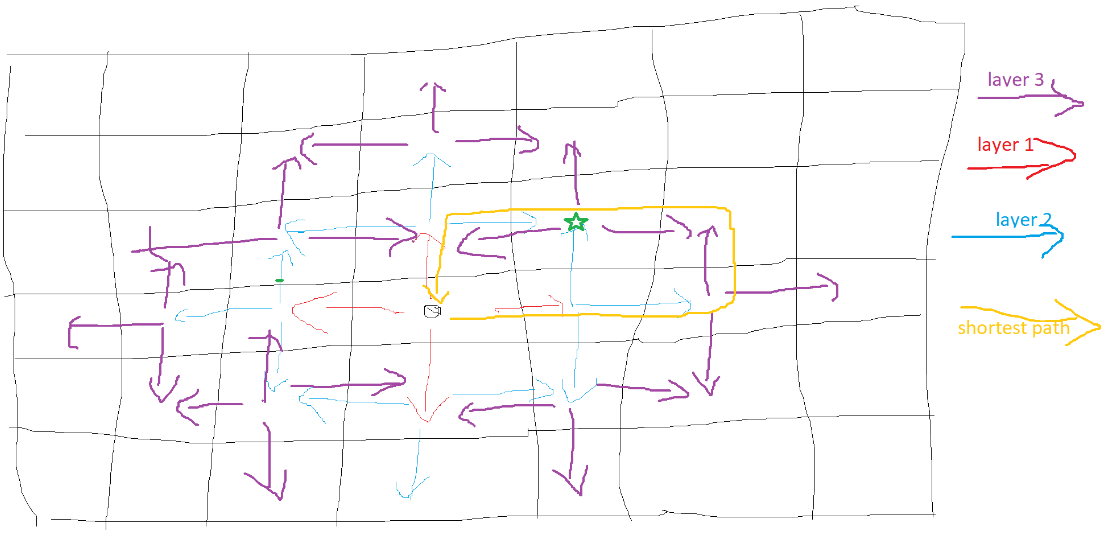
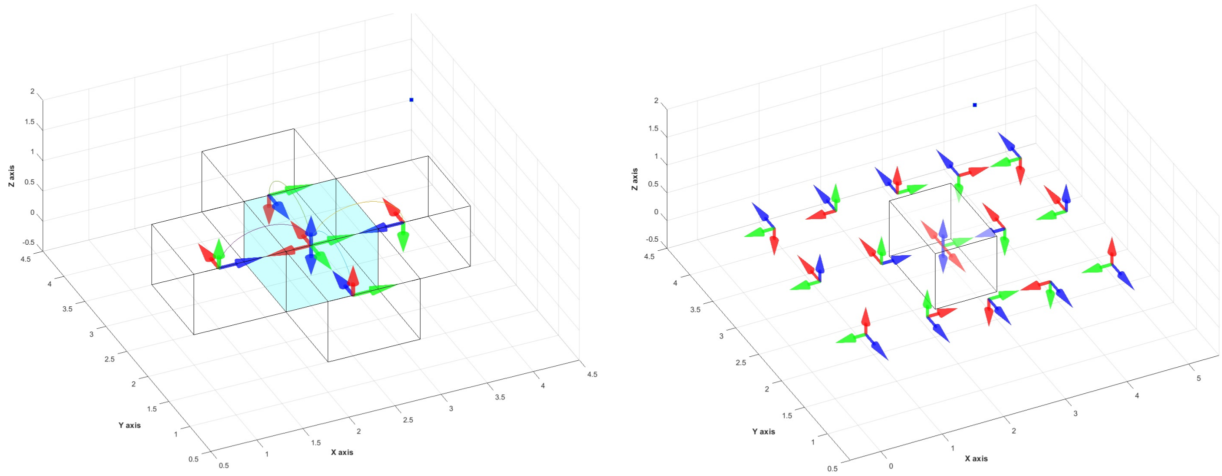


Figure 5: The first three layers of cube rolling

Path planning: ABC here.

Result:



(a) The initial configuration is the same position but different orientation with goal configuration

(b) First four paths of the cube rolling

Figure 6: Blah Blah

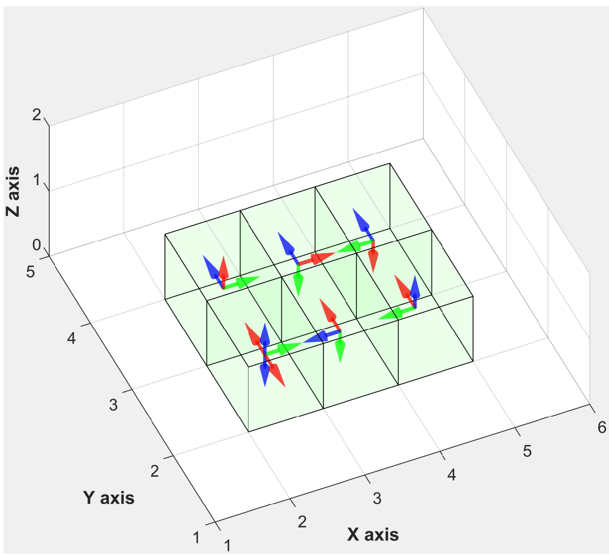
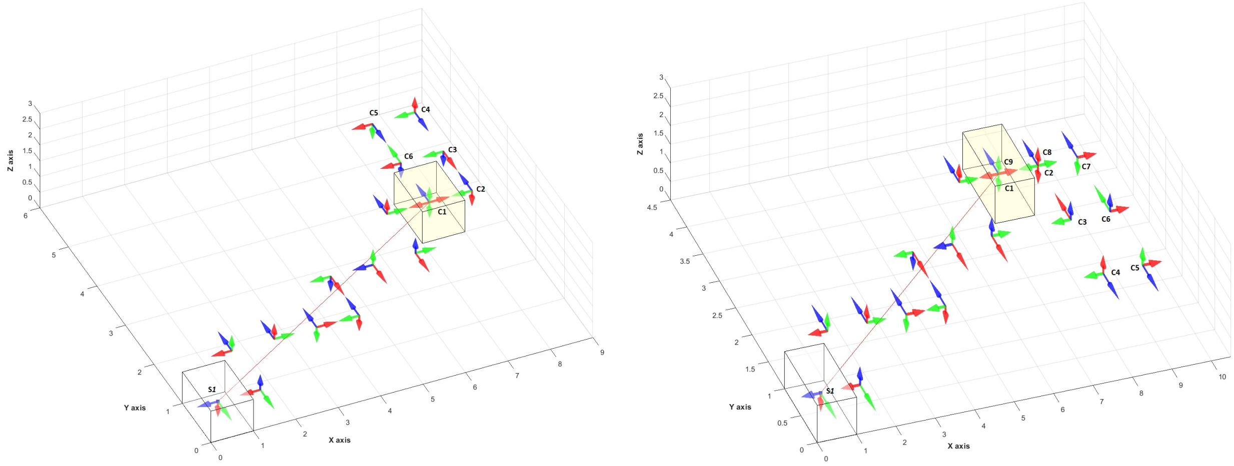


Figure 7: Shortest path of cube rolling

Although considering the case study within

The proposed algorithm in this study can be applied for the case study of long distance between the initial and goal configurations. Figure 8 shows an example with two different paths. Shortest path-finding algorithm is added to the original algorithm to find the shortest path from start point to the goal point. After finishing this step, the cube updated to a new orientation with different orientation at the goal configuration. Then, the original algorithm will be implemented from the updated cube to the goal configuration. Assume that the initial configuration is at S_1 and the goal configuration is at $S2$. In the first step, the red segment S_1S_2 shows the shortest distance from start position to goal position. The cube will roll through this segment and achieve the updated orientation at the goal position called shortest path for the case of long distance.



(a) Path1 (b) Path2

Figure 8: The case study of long distance between the initial and goal configurations

5.2 Tetrahedron solid

Properties As can be seen from the Figure 9, the Tetrahedron has constructed by four faces of the equilateral triangles. Then the height of triangle ABC is AM and $AM = DM = a\sqrt{3}/2$. Because of $r = OH = a\sqrt{6}/12$ (the radius of insphere) and $R = OA = a\sqrt{6}/4$ (the radius of circumsphere), the height of tetrahedron is $AH = OA + OH = a\sqrt{6}/3$. The rotation angle is α determined by supplementary angles $\beta = \arctan(2\sqrt{2})$ or $\alpha = \pi - \beta = \pi - \arctan(2\sqrt{2})$.

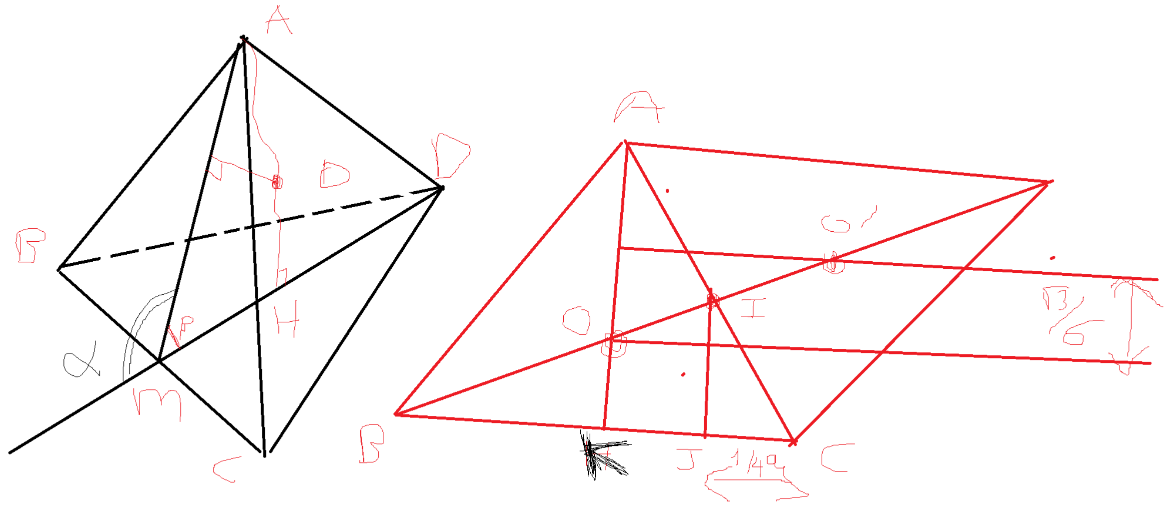
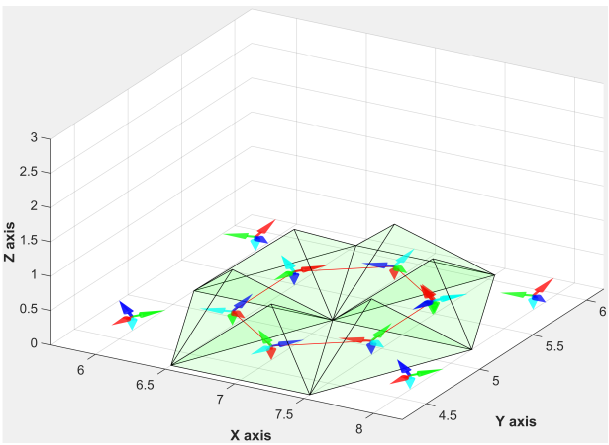
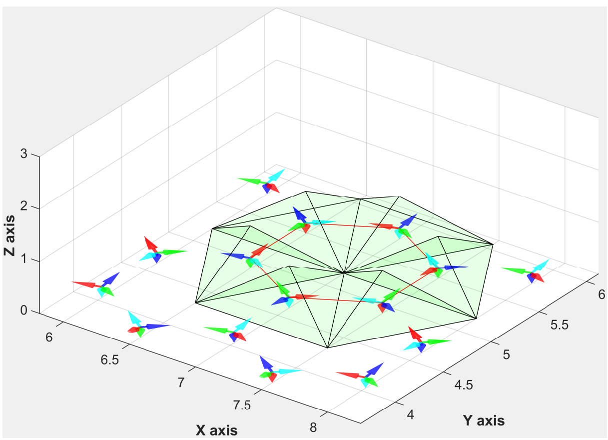


Figure 9: Tetrahedron geometrical properties

Path Planning: The case study of tetrahedron in this study is the same of the rolling cube path finding which only considering the path planning through rolling from initial configuration to origin coordinate with different orientation. The Figure 10 illustrates two of four cases of the tetrahedron path-finding within rolling (red and cyan arrows are pointing down to plane respectively). Tetrahedron has symmetry properties with indistinguishable for any two faces, edges and vertices. To be more specific, dihedral triangles have same three angles within 60°. In one cycle of rolling a tetrahedron around any vertices, the tetrahedron always achieve the initial configuration due to six times of rolling ($6*60° = 360°$, a full circle).



(a) Tetrahedron path 1



(b) Tetrahedron path 2

Figure 10: The two of four cases

5.3 Octahedron solid

Properties: Dennis also went his own way and divided the sides of the triangles into equal-angles (as measured from the center of the geodesic), instead of equal-length pieces. This technique is slightly more effective at evenly distributing the triangles across the surface of the sphere. For example, compare an octahedron subdivided with frequency 20, using the linear technique (as outlined by the quiz) versus the angular technique Dennis used in this picture.

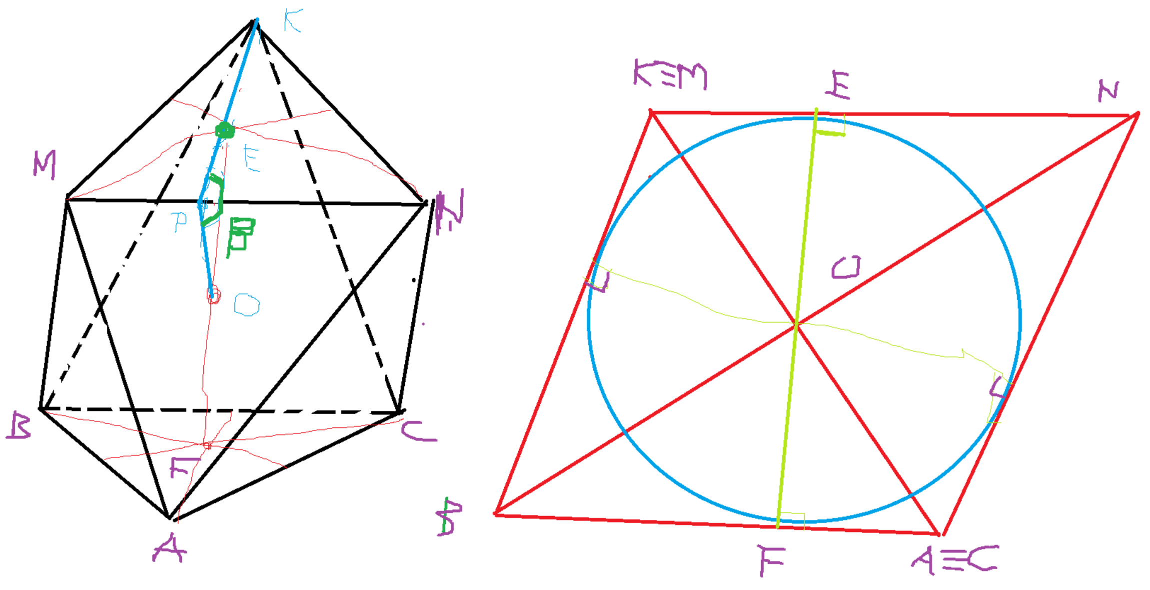
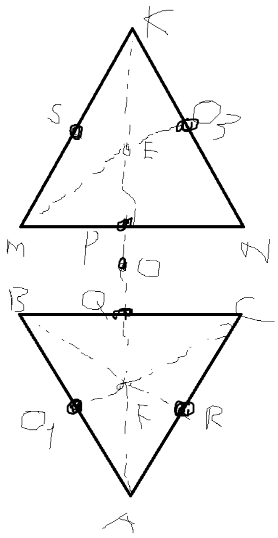


Figure 11: Octahedron geometrical properties

From Figure 11 we have:

$$AK = 2OK = 2\sqrt{(MK^2 - MO^2)} = a\sqrt{2}$$
$$\Rightarrow OK = OM = ON = a\frac{\sqrt{2}}{2}$$

WTF here



$$\alpha = \angle OPK = \arctan\frac{OK}{OP} = \arctan\sqrt{2}$$

Then, the rotation angle has the result as $\beta = \pi - 2\alpha = \pi - 2\arctan\sqrt{2}$,

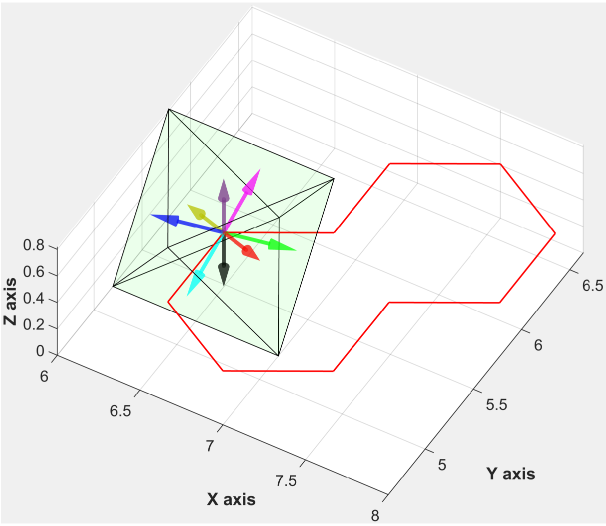
Due to $\angle POK = 90°$, we have

$$\begin{aligned}\frac{1}{OE^2} &= \frac{1}{OP^2} + \frac{1}{OK^2}\\ &= \frac{1}{(\frac{\sqrt{2}}{2})^2} + \frac{1}{(\frac{1}{2})^2}\\ \Rightarrow OE &= \frac{\sqrt{6}}{6}\end{aligned}$$

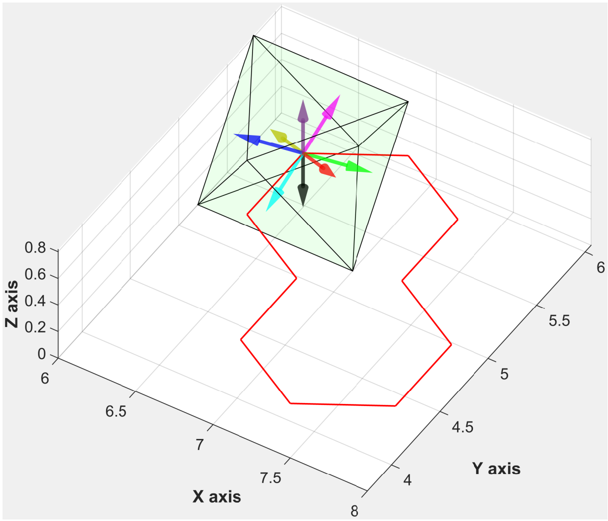
Applying the theory of the equilateral triangle

$$EO = OF = a\frac{\sqrt{6}}{6}$$
$$KE = FA = a\frac{\sqrt{3}}{3}$$
$$EP = FQ = a\frac{\sqrt{3}}{6}$$

Path planning: Based on the classical path planning which is the movement from point to other points, the octahedron path planning within rolling has three directions to move. In the Figure 11, the bottom layer $\triangle ABC$ which contacts to OXY can roll with the directions FQ, FO_1, FR. The distance between two shortest positions is $2FR = 2a\frac{\sqrt{3}}{6} = a\frac{\sqrt{3}}{3}$.
As can be seen from the Figure 12(a), the shortest path within red line includes ten segments for rolling the octahedron. The total length of the path equals $10a\frac{\sqrt{3}}{3}$ (edge length of the octahedron is a).



(a) The first shortest path of Octahedron path rolling



(b) The second shortest path of Octahedron path rolling

Figure 12: Two shortest paths of octahedron based rolling

5.4 Icosahedron solid

Properties: The convex regular icosahedron in the Figure 13 is one of the five regular Platonic solids has 12 vertices, 20 triangular faces, and 30 edges. Assume that the icosahedron has the edge length with a. The crossing surface of the solid at vertex A perpendicular with OD will generate a pentagon $ABCEF$.

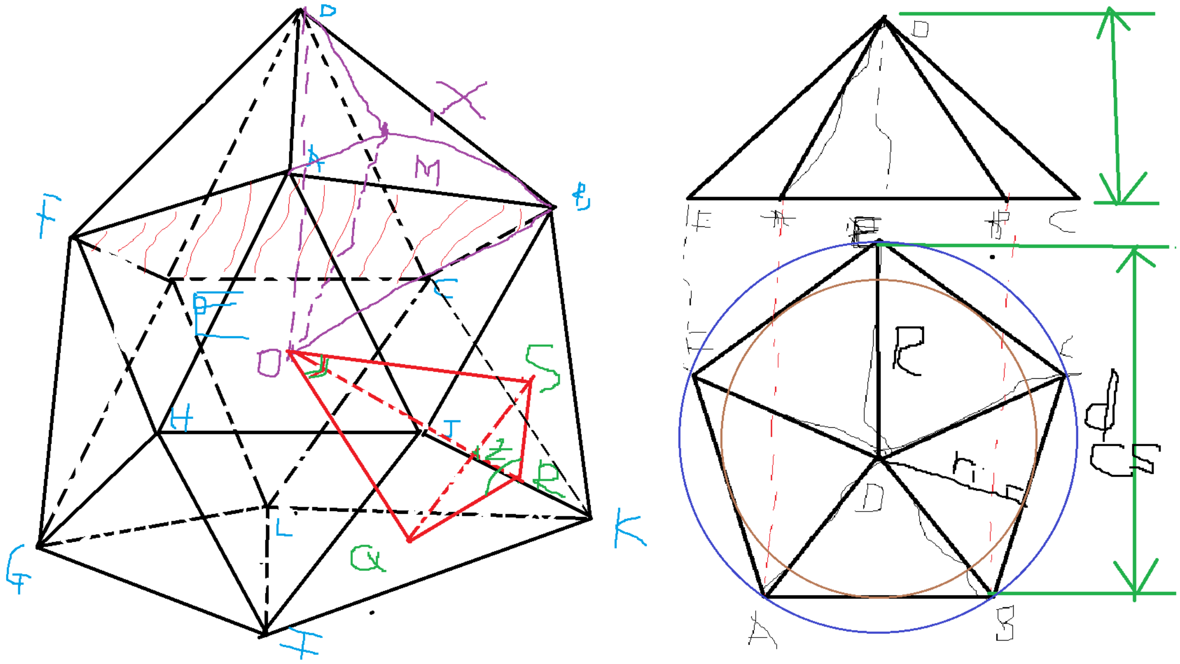


Figure 13: Icosahedron geometrical properties

At the right side of the Figure 13, the pentagon $ABCEF$ has circumcircle radius R, inscribed circle r_i, and the height d_{CS} $(R + r_i)$. Within the golden ratio Φ (irrational number), $\Phi = \frac{1+\sqrt{5}}{2}$, the detail dimensions of the icosahedron to be proved is the following.

$$\Phi = \frac{OX}{XB} = \frac{OX}{\frac{1}{2}a}$$

$$\sin(\angle XOB) = \frac{XB}{OB} = \frac{\frac{1}{2}a}{\frac{\sqrt{\Phi^2+1}}{2}a}$$

$$= \frac{1}{\sqrt{\Phi^2+1}}$$

$$\Rightarrow \angle XOB = \arcsin\frac{1}{\sqrt{\Phi^2+1}}$$

$$\Rightarrow \angle DOB = 2\angle XOB = 2\arcsin\frac{1}{\sqrt{\Phi^2+1}}$$

Same as the case of cube path planning algorithm, the initial step of rolling icosahedron is to find the rotation angle. To be more details in the Figure 14, the rotation angle is a supplementary of the $\angle QRS$. A line OR is perpendicular to QS and $OS = OQ = OM = r_i = \frac{\Phi^2}{2\sqrt{3}}a$. BI is a diagonal of pentagon $BCLIJ$ and $\angle QRS = 2\angle ORS$.

Otherwise, $OR = OX = \frac{1}{2}a\Phi$. And

$$\angle QRS = 2\angle ORS$$

$$\sin\angle ORS = \frac{OS}{OR} = \frac{\frac{\Phi^2}{2\sqrt{3}}a}{\frac{1}{2}a\Phi} = \frac{\Phi}{\sqrt{3}}$$

$$\Rightarrow \angle QRS = 2\arcsin\frac{\Phi}{\sqrt{3}}$$

George Odom has given a remarkably simple construction for jh involving an equilateral triangle: if an equilateral triangle is inscribed in a circle and the line segment joining the midpoints of two sides is produced to intersect the circle in either of two points, then these three points are in golden proportion. This result is a straightforward consequence of the intersecting chords theorem and can be used to construct a regular pentagon, a construction that attracted the attention of the noted Canadian geometer H. S. M. Coxeter who published it in Odom's name as a diagram in the American Mathematical Monthly accompanied by the single word

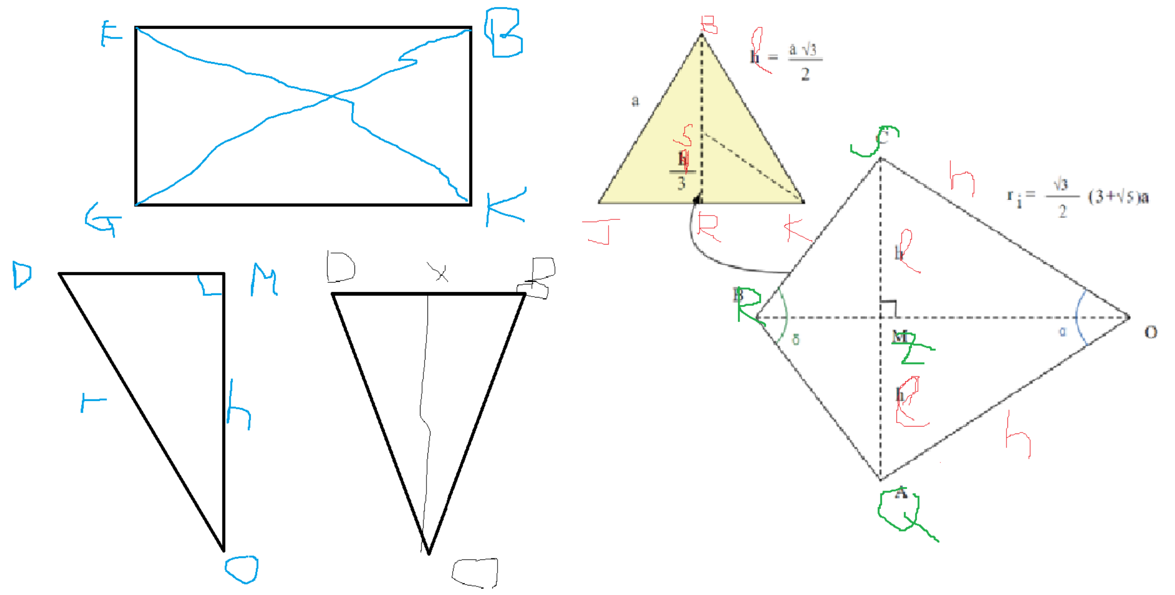
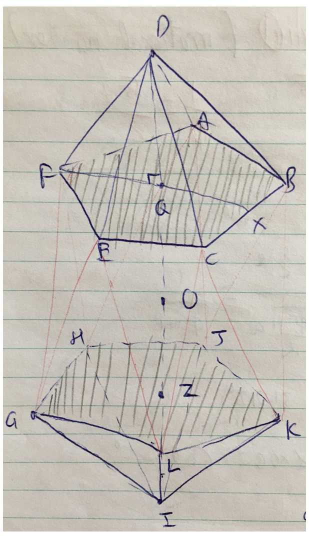


Figure 14: Icosahedron geometrical properties

22222 CHeck book "A geometric analysis of the Platonic solids". ksdhajkdas George Odom has given a remarkably simple construction for jh involving an equilateral triangle: if an equilateral triangle is inscribed in a circle and the line segment joining the midpoints of two sides is produced to intersect the circle in either of two points, then these three points are in golden proportion. This result is a straightforward consequence of the intersecting chords theorem and can be used to construct a regular pentagon, a construction that attracted the attention of the noted Canadian geometer H. S. M. Coxeter who published it in Odom's name as a diagram in the American Mathematical Monthly accompanied by the single word



33333 "A geometric analysis of the Platonic solids". ksdhajkdas George Odom has given a remarkably simple construction for jh involving an equilateral triangle: if an equilateral triangle is inscribed in a circle and the line segment joining the midpoints of two sides is produced to intersect the circle in either of two points, then these three points are in golden proportion. This result is a straightforward consequence of the intersecting chords theorem and can be used to construct a regular pentagon, a construction that attracted the attention of the noted Canadian geometer H. S. M. Coxeter who published it in Odom's name as a diagram in the American Mathematical Monthly accompanied by the single word

From Figure 11 we have:

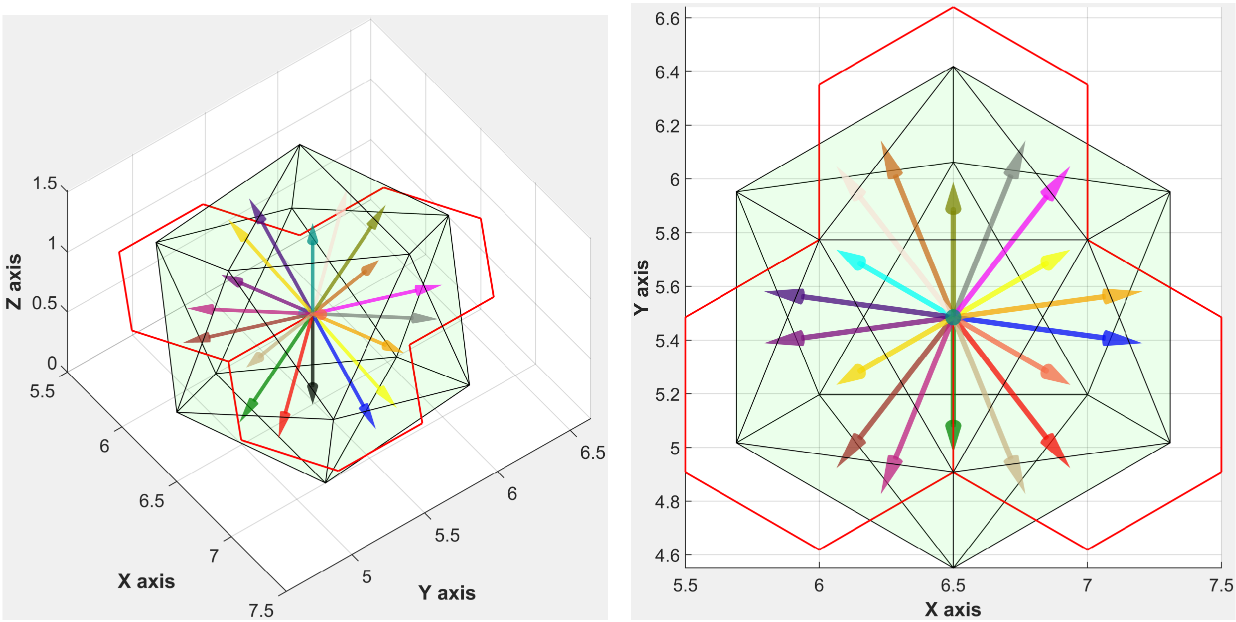
$$AK = 2OK = 2\sqrt{(MK^2 - MO^2)} = a\sqrt{2}$$

$$\Rightarrow OK = OM = ON = a\frac{\sqrt{2}}{2}$$

Note how l face of the octahedron, where the radial technique does a better job of spacing them out.

$$\beta = \angle OPK = \arctan\frac{OK}{OP} = \arctan\frac{2\sqrt{2}}{2}$$

<u>Path planning</u> This is the rolling path of an icosahedron solid (Figure 15).



(a) The first shortest path of Icosahedron path rolling (Red line)

(b) The top view of path planning for Icosahedron

Figure 15: Path of icosahedron rolling though edges

5.5 Dodecahedron solid

Properties: An dodecahedron has 12 faces and 20 vertices of which generates a pentagon as shown in Figure 16. It will be assumed that the coordinates $Oxyz$ lie on $ABCDE$ surface within Oy through A and Oz perpendicular to $ABCDE$. The 30 edges have the same length as a. It should be determined all the vertices' coordinates in the three dimensional system. The Figure 16 indicates the lengths of each vertices from l_1 to l_4 and the angles α_1 to α_4 which correspond to the five sides of a pentagon.

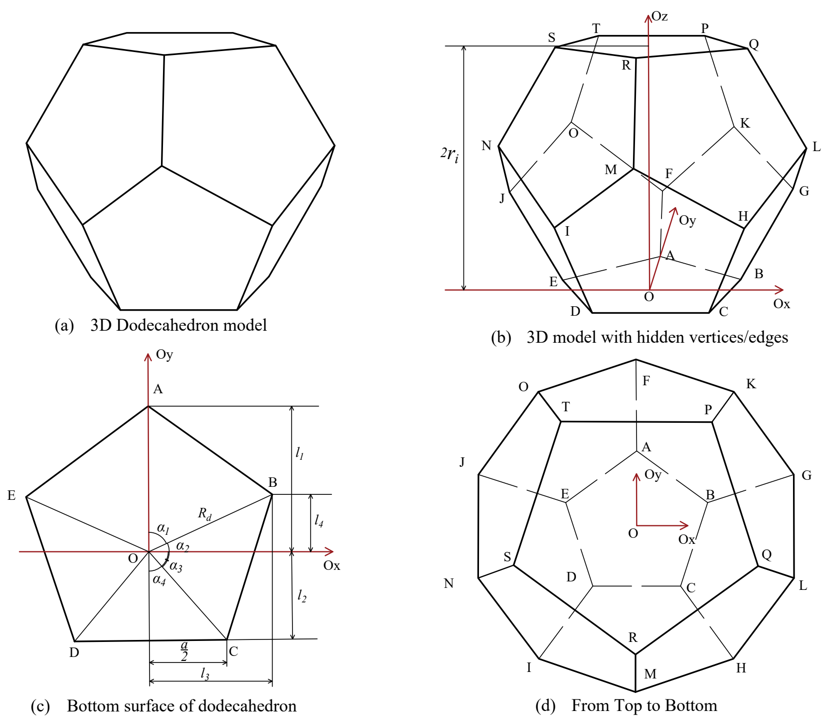


Figure 16: Dodecahedron's vertices.

The path planning will implement on a surface but it will be considered in $3D$ spaces. Then, each of vertices will be determined on $3D$ coordinates such as the vertices A has coordinate with $[A_x A_y A_z]$. Based on the properties of pentagon, the angle $\alpha_1 = \frac{2\pi}{5}$ and $\alpha_4 = \frac{\pi}{5}$. Because the angle between Ox and Oy is $\frac{\pi}{2}$, the sum of α_1 and α_2 is $\alpha_1 + \alpha_2 = \frac{\pi}{2}$. Then the other two angles α_2 and α_3 can be determined by $\alpha_2 = \frac{\pi}{2} - \alpha_1 = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{\pi}{10}$ and $\alpha_3 = \alpha_1 - \alpha_2 = \frac{2\pi}{5} - \frac{pi}{10} = \frac{3\pi}{10}$.

From the Figure 16(c), these labelled dimensions can be calculated as $l_1 = R_d = \frac{a}{2\sin\alpha_4}$ with R_d is the circumradius of dodecahedron, $l_2 = l_1 \cos\alpha_4$, $l_3 = l_1 \cos\alpha_2$, and $l_4 = l_1 \sin\alpha_2$. Referencing to the properties of a dodecahedron with length a, the radius of an inscribed sphere is $r_i = \frac{a}{20}\sqrt{10(25 + 11\sqrt{5})}$ and the circumscribed sphere radius is $r = a\frac{\sqrt{3}}{2}\frac{1+\sqrt{5}}{2}$.

There are total 20 vertices of a dodecahedron. This article focuses on the rolling contact to $2D$ surface, the bottom surface of the dodecahedron integrated to the Oxy which contact to the $2D$ surface. This condition express the Oz dimension of the vertices $ABCDE$ equal to 0 or $A_z = B_z = C_z = D_z = E_z = 0$. Then $P_z = Q_z = R_z = S_z = T_z = 2.r_i = \frac{a}{10}\sqrt{10(25 + 11\sqrt{5})}$.

It can be seen that the distance $|AF|$ is a and the distance $|BF|$ is $2l_3$. Using the distance properties and squaring the results give:

$$AF^2 = a^2 = (A_x - F_x)^2 + (A_y - F_y)^2 + (A_z - F_z)^2$$
$$BF^2 = (2l_3)^2 = (B_x - F_x)^2 + (B_y - F_y)^2 + (B_z - F_z)^2$$

Figure 16(d) shows that $A_x = F_x = 0$, $A_y = l_1$, $B_y = l_4$, $B_x = l_3$, $A_z = B_z$. Define $l_5 = A_z - F_z$, the relations of these equations are:

$$a^2 = (F_y - l_1)^2 + l_5^2$$
$$(2l_3)^2 = (F_y - l_4)^2 + l_5^2 + l_3^2$$

Solving F_y and l_5 gives:

$$F_y = \frac{a^2 - (2l_3)^2 - (l_1^2 - l_3^2 - l_4^2)}{2(l_4 - l_1)}$$
$$l_5 = \frac{1}{\sqrt{2}}\sqrt{a^2 + (2l_3)^2 - (F_y - l_1)^2 - (F_y - l_4)^2 - l_3^2}$$

From these equations 5.5,5.5,5.5, all the vertices will be founded in the three-dimensional space. Path planning based on rolling is the motion of all these vertices through edges' contact.

<u>Path Planning</u>:

Experiments: Writing about cube solid properties

Discussion: Q2 & Q3
- Q2: What are the new things you learned after you did whatever you did?
- Q3: What exactly did you do?

- **Discussion**
- *What your results mean*
- *Why it makes a difference*
- **Conclusion**
- *Broader implications*
- *Areas for further study*

6 CONCLUSION AND FUTURE PROSPECTS

Questions: Q4. Why should the community care?
Should do: - Overview of Q1, Q2, and Q3; plus
- What does the community still not know?

Examples: - We have introduced a method of ....
- Most of our effort has focused on .... The results of our method often contain .... We believe that there is significant room for improvement by applying ABC methods to the XYZ problem.
- What do we not do?

In this study, we established a method for ... Although we focused on discrete path planning of platonic solid - regular convex polyhedra in known environment, as illustrated using ABC model and EFG example, the developed method/algorithm can be easily implemented to the complex convex polyhedra such as elipsoil ???. The contributions of this study can be summarized as follows:

SECTION IX.
CONCLUDING REMARKS

In this paper, we have introduced Bayesian optimization from a modeling perspective. Beginning with the beta-Bernoulli and linear models, and extending them to nonparametric models, we recover a wide range of approaches to Bayesian optimization that have been introduced in the literature. There has been a great deal of work that has focused heavily on designing acquisition functions; however, we have taken the perspective that the importance of this plays a secondary role to the choice of the underlying surrogate model.

In addition to outlining different modeling choices, we have considered many of the design decisions that are used to build Bayesian optimization systems. We further highlighted relevant theory as well as practical considerations that are used when applying these techniques to real-world problems. We provided a history of Bayesian optimization and related fields and surveyed some of the many successful applications of these methods. We finally discussed extensions of the basic framework to new problem domains, which often require new kinds of surrogate models.

Although the underpinnings of Bayesian optimization are quite old, the field itself is undergoing a resurgence, aided by new problems, models, theory, and software implementations. In this paper, we have attempted to summarize the current state of Bayesian optimization methods; however, it is clear that the field itself has only scratched the surface and that there will surely be many new problems, discoveries, and insights in the future.

Full Text
Abstract
Authors
Figures
References
Citations
Keywords
Footnotes
Back to Top

Figure 17: First four paths of the cube rolling

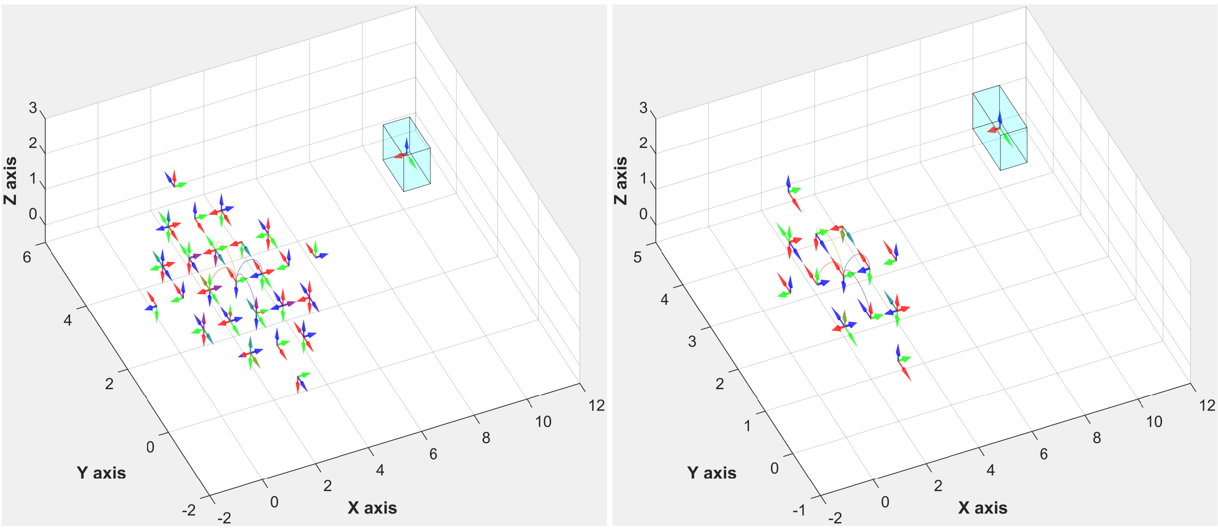
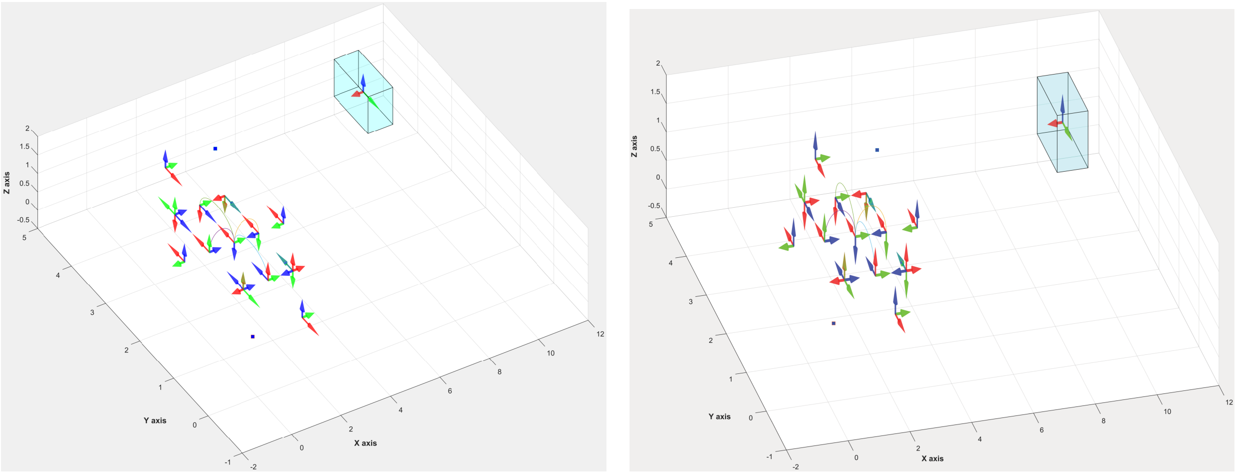


Figure 18: Test



(a) Cube1

(b) First four paths of the cube rolling

Figure 19: Blah Blah

Refer to the Data Management Plan in Appendix A.
[1]

7 Reviews from Prof. Jonathan Paxman for PhD Candidacy Proposal

• Include a discussion of the motivation and advantages for rolling contact for in-hand manipulation
• Reduce the length of the discussion on modelling the kinematics of rolling motion
• Add a brief review of path planning for two general objects under nonholonomic constraints
• Simply the aims: remove specific techniques and algorithms, and describe the broad aim of the project general terms, and in one or two sentences. Ensure that specific objectives are framed so that the aim can be achieved.
• Include a section which describes how a discretised model will be produced such that the discrete planning algorithms described can be applied. How is this discrete model to be obtained from the continuous-time models discussed?
• If optimal planning is discussed, ensure you are specific about in what sense the solution is optimal. In some cases, optimality is not required, only a satisfactory or satisficing solution in the sense of a cost function being below some bound. In such cases, sampling-based solutions (such as RRT) are appropriate.
• Please also review the writing for grammatical correctness (seek some assistance on this if needed).
• Note Robot Operating System (not Software) in Table 1.

[2], [3],[4],[5], [6].

References

1. Barraquand, J. & Latombe, J.-C. Robot Motion Planning: A Distributed Representation Approach. *The International Journal of Robotics Research* **10,** 628–649. ISSN: 0278-3649. `https://doi.org/10.1177/027836499101000604` (1991).
2. Benitez, A. & Vallejo, D. *New Technique to Improve Probabilistic Roadmap Methods* in *Advances in Artificial Intelligence – IBERAMIA 2004* (eds Lemaître, C., Reyes, C. A. & González, J. A.) (Springer Berlin Heidelberg), 514–523. ISBN: 978-3-540-30498-2.
3. Kavraki, L. & Latombe, J. *Randomized preprocessing of configuration space for path planning: articulated robots* in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'94)* **3** (1994), 1764–1771 vol.3.
4. Chun Sheng, C. & Roth, B. *On the spatial motion of a rigid body with point contact* in *Proceedings. 1987 IEEE International Conference on Robotics and Automation* **4** (1987), 686–695.
5. Holland, J. H. *Adaptation in Natural and Artificial Systems* (In University of Michigan Press, Ann Arbor., 1975).
6. Alouges, F., Chitour, Y. & Long, R. A Motion-Planning Algorithm for the Rolling-Body Problem. *IEEE Transactions on Robotics* **26,** 827–836. ISSN: 1552-3098 (2010).

A APPENDIX