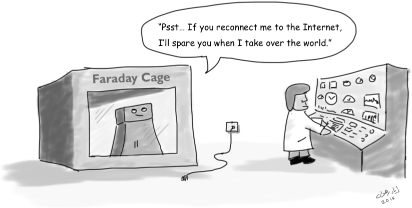
Computer Vision and Pattern Recognition

Krystian Mikolajczyk & Ad Spiers

Department of Electrical and Electronic Engineering

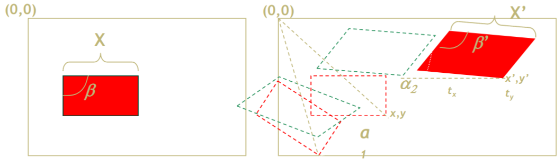
Imperial College London



Super intelligent machines, containment strategies.

Part 2: Image geometry

- Planar transformations and parameters
  - Euclidean
  - Similarity
  - Affine
  - Perspective



Invariants
- Orientation - no
- Length - no
- Angle -no
- Length ratio - no
- Parallelism

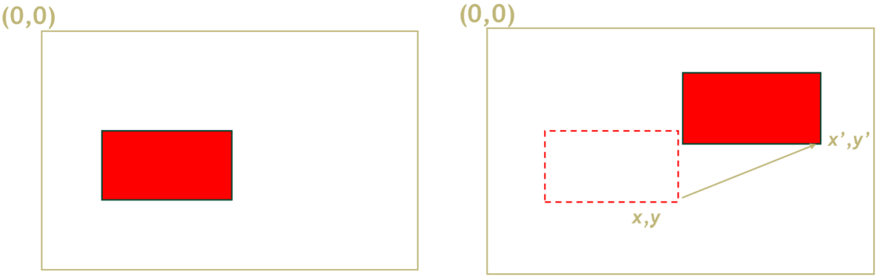
$$R(\alpha_2)SR(\alpha_1) = \begin{bmatrix}\cos(\alpha_2) & \sin(\alpha_2)\\ -\sin(\alpha_2) & \cos(\alpha_2)\end{bmatrix}\begin{bmatrix}s_x & 0\\ 0 & s_y\end{bmatrix}\begin{bmatrix}\cos(\alpha_1) & \sin(\alpha_1)\\ -\sin(\alpha_1) & \cos(\alpha_1)\end{bmatrix}$$

$$\begin{bmatrix}x'\\ y'\\ 1\end{bmatrix} = \begin{bmatrix}\multicolumn{2}{c}{R(\alpha_2)SR(\alpha_1)} & \begin{matrix}t_x\\ t_y\end{matrix}\\ 0 \quad 0 & & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

Matrix notation

$$\mathbf{x}' = \mathbf{R}_2\mathbf{S}\mathbf{R}_1\mathbf{x} + \mathbf{t}$$

Planar transformations: translation



$$x' = x + t_x = 1 \cdot x + 0 \cdot y + t_x \cdot 1$$

$$y' = y + t_y = 0 \cdot x + 1 \cdot y + t_y \cdot 1$$

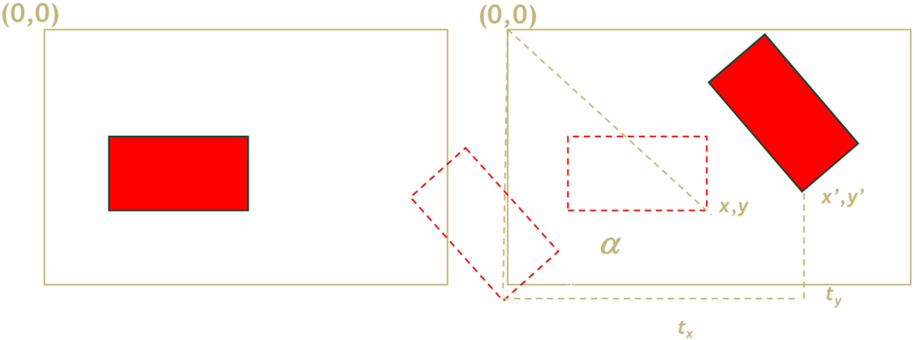
In homogenous coordinates ⇨

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Invariants

- orientation
- Length
- Angle
- Length ratio
- Parallel lines

Planar transformations: Eclidean (rotation)

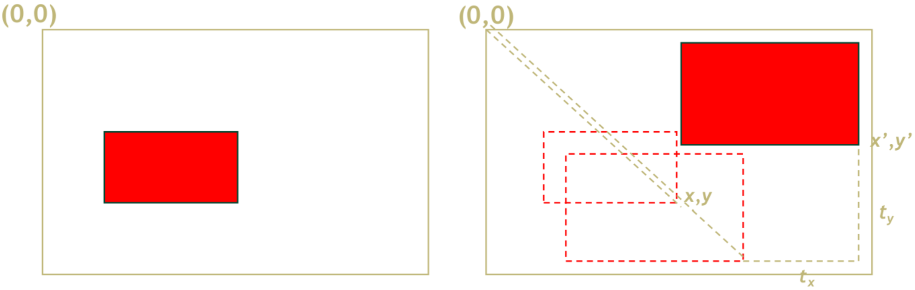


$$x' = \cos(\alpha)\cdot x + \sin(\alpha)\cdot y + t_x$$
$$y' = -\sin(\alpha)\cdot x + \cos(\alpha)\cdot y + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & t_x \\ -\sin(\alpha) & \cos(\alpha) & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Invariants
- Orientation - no
- Length
- Angle
- Length ratio
- Parallelism

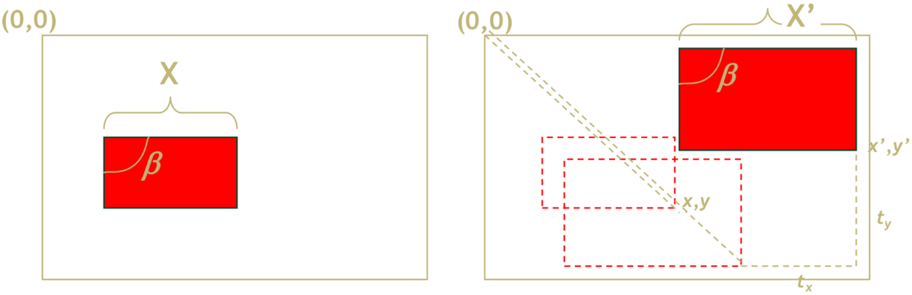
Planar transformations: similarity



$$x' = s \cdot x + 0 \cdot y + t_x$$
$$y' = 0 \cdot x + s \cdot y + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Planar transformations: similarity



Invariants

- Orientation - no
- Length - no
- Angle
- Length ratio
- Parallelism

$$x' = \cos(\alpha)\cdot s\cdot x + \sin(\alpha)\cdot s\cdot y + t_x$$
$$y' = -\sin(\alpha)\cdot s\cdot x + \cos(\alpha)\cdot s\cdot y + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \left\{ \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} \right\} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s\cdot\cos(\alpha) & s\cdot\sin(\alpha) & t_x \\ -s\cdot\sin(\alpha) & s\cdot\cos(\alpha) & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

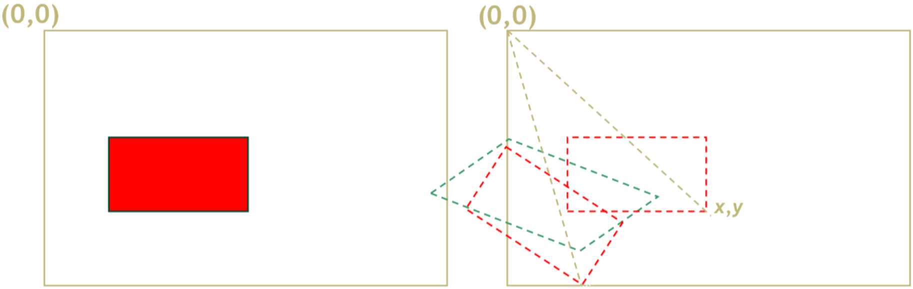
Planar transformations: example

Planar transformations: example

- Derive a matrix w to rotate a point by $\pi/6$, scale by factor 2 and translate by vector $[2, 1]$

$$\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Planar transformations: affine

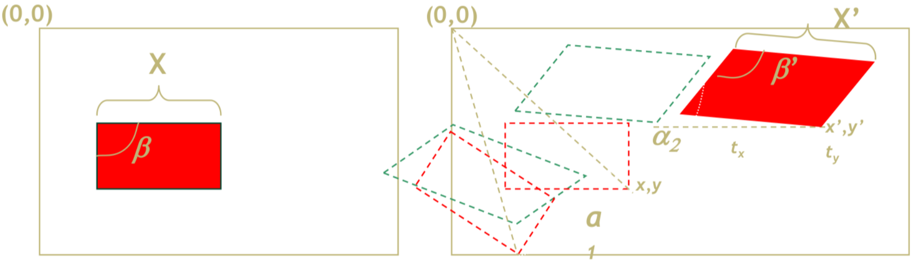


$$\mathbf{x}' = SR(\alpha_1)\mathbf{x}$$

$$\begin{aligned} x' &= \cos(\alpha)\cdot s_x \cdot x + \sin(\alpha)\cdot s_x \cdot y \\ y' &= -\sin(\alpha)\cdot s_y \cdot x + \cos(\alpha)\cdot s_y \cdot y \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R(\alpha_2)SR(\alpha_1) & \begin{matrix} t_x \\ t_y \end{matrix} \\ 0 \quad 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Planar transformations: affine



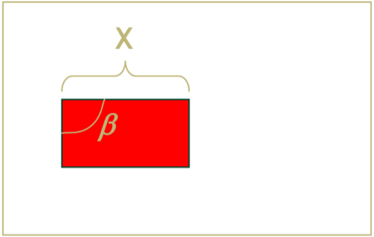
$$R(\alpha_2)SR(\alpha_1) = \begin{bmatrix} \cos(\alpha_2) & \sin(\alpha_2) \\ -\sin(\alpha_2) & \cos(\alpha_2) \end{bmatrix} \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} \cos(\alpha_1) & \sin(\alpha_1) \\ -\sin(\alpha_1) & \cos(\alpha_1) \end{bmatrix}$$

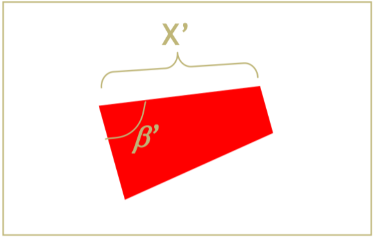
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R(\alpha_2)SR(\alpha_1) & & t_x \\ & & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Invariants

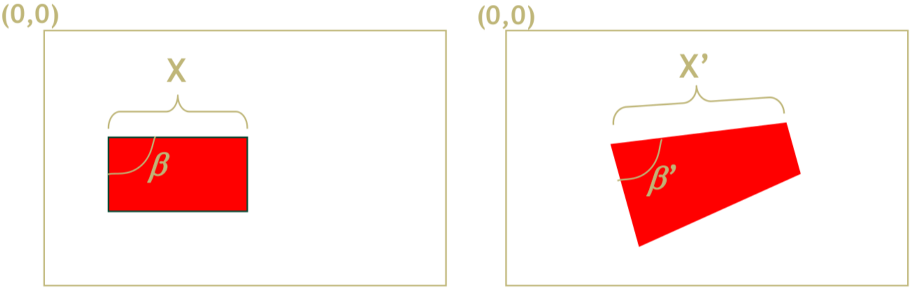
- Orientation - no
- Length - no
- Angle -no
- Length ratio - no
- Parallelism

Planar transformations: projective





Planar transformations: projective

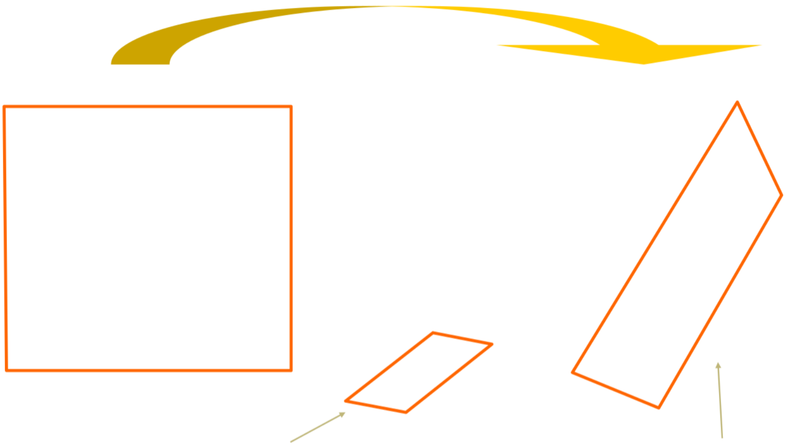


Invariants: Length - no, Angle -no, Length ratio - no, Parallelism -no, orientation - no

Homogenous coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective transformation

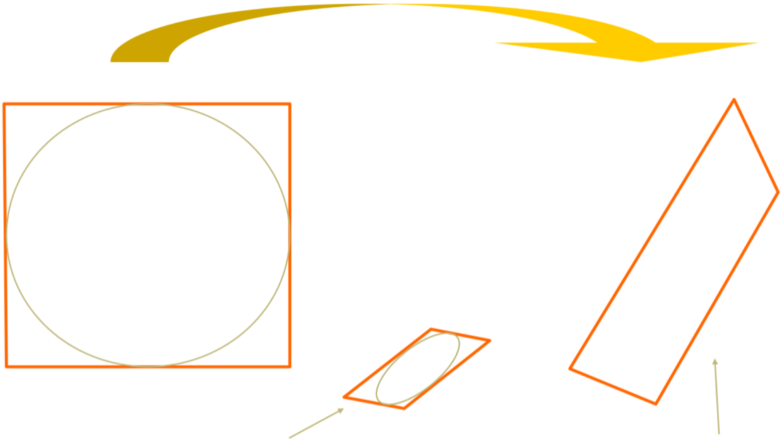


Affine - 6 Degrees of Freedom

translation (2) + rotation(1) + scale(1)

Projective – 8 Degrees of Freedom

Projective transformation



Affine - 6 Degrees of Freedom

translation (2) + rotation(1) + scale(1)

Projective – 8 Degrees of Freedom

From projective to affine

- Approximation of projective transformation by affine transformation
  - Can be done only locally

Affine

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homography

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Image coordinates

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \frac{1}{z'} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Matrix notation

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$

Affine

$$x'' = \frac{a_{11}\ x + a_{12} y + a_{13}}{0 \cdot x + 0 \cdot y + 1}$$

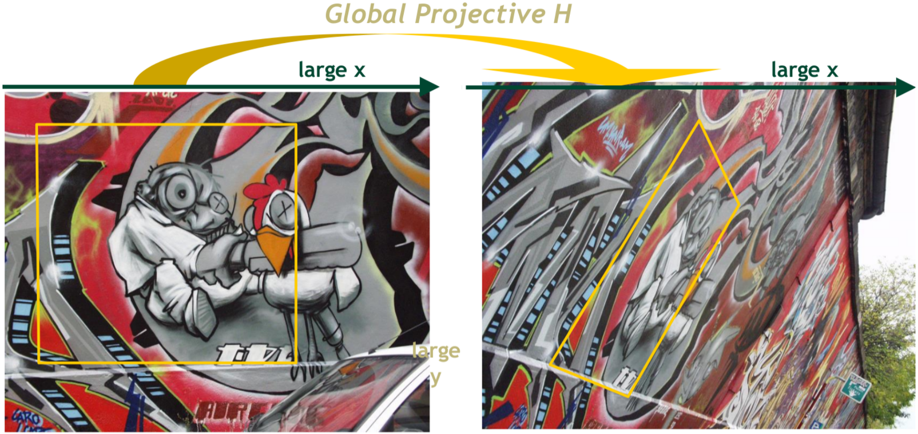
$$y'' = \frac{a_{21}\ x + a_{22} y + a_{23}}{0 \cdot x + 0 \cdot y + 1}$$

Projective

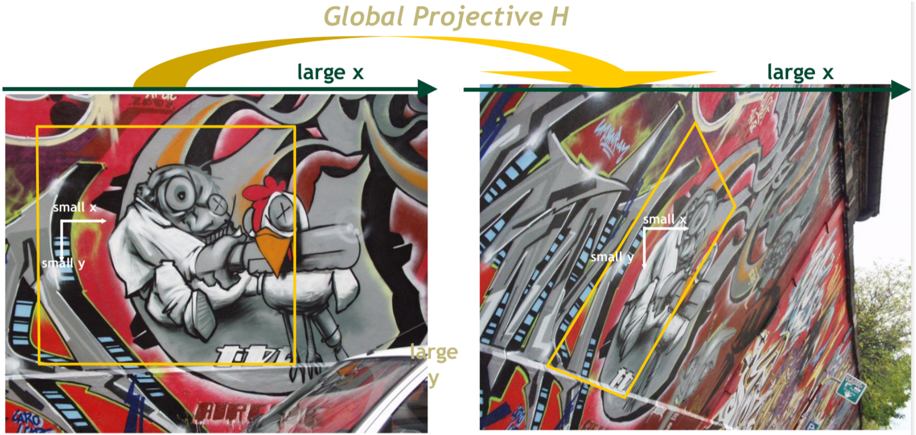
$$x'' = \frac{h_{11}\ x + h_{12} y + h_{13}}{h_{31}\ x + h_{32} y + 1}$$

$$y'' = \frac{h_{21}\ x + h_{22} y + h_{23}}{h_{31}\ x + h_{32} y + 1}$$

From projective to affine

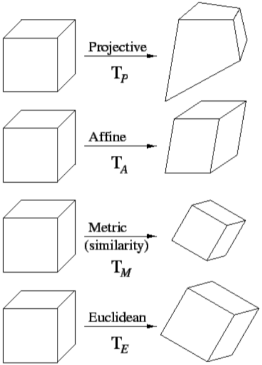


From projective to affine



Taxonomy of planar projective transforms

group	matrix	properties
perspective 8 DOF	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & h_{33} \end{bmatrix}$	concurrency, collinearity, incidence, tangency, inflection
affine 6 DOF	$\begin{bmatrix} a_{11} & a_{12} & t_{13} \\ a_{12} & a_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix}$	parallelism, ratio of areas, ratio of lengths on collinear or parallel sgements
similarity 4 DOF	$\begin{bmatrix} sr_{11} & sr_{12} & t_{13} \\ sr_{12} & sr_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix}$	angles, ratio of lengths
Euclidean 3 DOF	$\begin{bmatrix} r_{11} & r_{12} & t_{13} \\ r_{12} & r_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix}$	length, area



Summary

- Pinhole camera model
- Derive equations to compute point position in the image given its position in 3D
  - Convert it to homogeneous matrix formula
- Planar transformations, what are their parameters and invariants?
- Transform a point using homogeneous coordinates in matrix notation.
- Covert transformation parameters (rotation angle, translation vector, scale factor) to matrix notation in homogeneous system.