

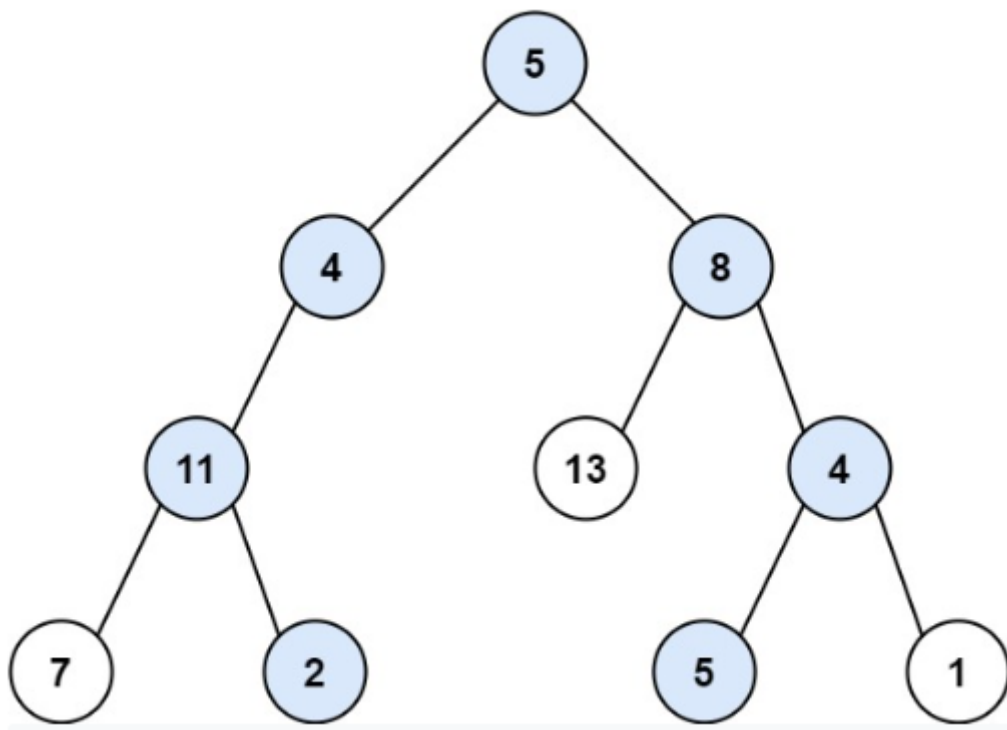
<https://leetcode.com/problems/path-sum-ii/>

Given the root of a binary tree and an integer targetSum, return *all **root-to-leaf** paths where the sum of the node values in the path equals*

*targetSum. Each path should be returned as a list of the node **values**, not node references.*

A **root-to-leaf** path is a path starting from the root and ending at any leaf node. A **leaf** is a node with no children.

Example 1:



Input: root = [5,4,8,11,null,13,4,7,2,null,null,5,1], targetSum = 22

Output: [[5,4,11,2],[5,8,4,5]]

Explanation: There are two paths whose sum equals targetSum:

$$5 + 4 + 11 + 2 = 22$$

$$5 + 8 + 4 + 5 = 22$$

Example 2:

Input: root = [1,2,3], targetSum = 5

Output: []

Example 3:

Input: root = [1,2], targetSum = 0

Output: []

Constraints:

- The number of nodes in the tree is in the range [0, 5000].
- $-1000 \leq \text{Node.val} \leq 1000$
- $-1000 \leq \text{targetSum} \leq 1000$

Attempt 1: 2022-11-04

Solution 1: Recursive traversal with Deep Copy on passed in ArrayList to find and store paths first and calculate target sum, fully based on L257.Binary Tree Paths (10min)

```
1  /**
2   * Definition for a binary tree node.
3   * public class TreeNode {
4   *     int val;
5   *     TreeNode left;
6   *     TreeNode right;
7   *     TreeNode(int x) { val = x; }
8   * }
9   */
10 class Solution {
11     public List<List<Integer>> pathSum(TreeNode root, int sum) {
12         List<List<Integer>> result = new ArrayList<List<Integer>>();
13         helper(root, result, sum, new ArrayList<Integer>());
14         return result;
15     }
16
17     private void helper(TreeNode root, List<List<Integer>> result, int sum,
18         List<Integer> list) {
19         if(root == null) {
20             return;
21         }
22         List<Integer> tmp = new ArrayList<Integer>(list);
23         tmp.add(root.val);
24         if(root.left == null && root.right == null) {
25             if(sum == root.val) {
26                 result.add(new ArrayList<Integer>(tmp));
27             }
28         }
29     }
30 }
```

```

28     helper(root.left, result, sum - root.val, tmp);
29     helper(root.right, result, sum - root.val, tmp);
30     // How we remove the last element on 'tmp' list without explicit backtrack ?
31     // Because DFS naturally a type of backtrack but implicit only backtrack
32     // when pass over the leaf nodes during a tree traversal, such as example
33     // here, after pass over the leaf nodes, it will encounter 'null' and return
34     // to previous recursion level which also "auto remove" the last element like a
35     // backtrack but implicitly, and to explain the "auto remove" is because the
36     // input parameter as 'list' in each recursion level never changed, the
changed
37     // object is not 'list' but a deep copy of this 'list' as 'tmp', the impact
38     // range of 'tmp' is limited in current recursion level, so when return to
39     // previous recursion level, the 'tmp' will gone, the only remain we will find
40     // is our unchanged 'list' object
41 }
42 }
43
44 Time Complexity:  $O(n^2)$ , where n is number of nodes in the Binary Tree
45 Space Complexity:  $O(n)$ 

```

Solution 2: Recursive traversal without Deep Copy on passed in ArrayList but use

Backtracking to find and store paths first and calculate target sum, fully based on L257.Binary

Tree Paths (10min)

Style 1: 2ms beats 85.58%

```

1  /**
2   * Definition for a binary tree node.
3   * public class TreeNode {
4   *     int val;
5   *     TreeNode left;
6   *     TreeNode right;
7   *     TreeNode(int x) { val = x; }
8   * }
9   */
10 class Solution {
11     public List<List<Integer>> pathSum(TreeNode root, int sum) {
12         List<List<Integer>> result = new ArrayList<List<Integer>>();

```

```

13         helper(root, result, sum, new ArrayList<Integer>());
14         return result;
15     }
16
17     private void helper(TreeNode root, List<List<Integer>> result, int sum,
18 List<Integer> list) {
19         if(root == null) {
20             return;
21         }
22         // No deep copy of input 'list' here such as 'List<Integer> tmp = new
23 ArrayList<Integer>(list)',
24         // instead the change directly happen on input 'list' as adding new value on
25 it, which change
26         // the 'list' object and will pass into onwards recursion, to remove the
27 change of 'list' object,
28         // we have to use backtrack technic
29         list.add(root.val);
30         if(root.left == null && root.right == null) {
31             if(sum == root.val) {
32                 result.add(new ArrayList<Integer>(list));
33             }
34         }
35         helper(root.left, result, sum - root.val, list);
36         // Do not backtrack here(before the right branch recursion), since we suppose
37 to change
38         // on 'list' should reflect in both left and right branch, if add backtrack
39 here will
40         // make right branch onwards recursion based on wrong version of 'list' that
41 without change
42         helper(root.right, result, sum - root.val, list);
43         // Backtrack: Remove the last element on list for next recursion
44         // We have to add explicit backtrack on 'list' because there is no deep copy
45 as 'tmp'
46         // in this solution, the change directly happen on input 'list' and if no
47 rollback on
48         // that change, the change will pass through all recursion levels, if we have
49 a deep
50         // copy 'tmp', the change will only happen on 'tmp' and impact current
51 recursion level
52         // which when return to previous recursion level, the 'tmp' impact will gone,
53 we will
54         // find our unchanged 'list' back in previous recursion level, that's implicit
55 backtrack

```

```

43         // in deep copy DFS style, since no deep copy here requires explicit backtrack
    on 'list'
44         list.remove(list.size() - 1);
45     }
46 }
47
48 Time Complexity:  $O(n^2)$ , where n is number of nodes in the Binary Tree
49 Space Complexity:  $O(n)$ 

```

Style 2: 1ms beats 100%, the promotion comes from direct Backtrack and Return on leaf node, it will save two more next recursion calls which will eventually return when root == null

```

1  /**
2   * Definition for a binary tree node.
3   * public class TreeNode {
4   *     int val;
5   *     TreeNode left;
6   *     TreeNode right;
7   *     TreeNode(int x) { val = x; }
8   * }
9   */
10 class Solution {
11     public List<List<Integer>> pathSum(TreeNode root, int sum) {
12         List<List<Integer>> result = new ArrayList<List<Integer>>();
13         helper(root, result, sum, new ArrayList<Integer>());
14         return result;
15     }
16
17     private void helper(TreeNode root, List<List<Integer>> result, int sum,
18         List<Integer> list) {
19         if(root == null) {
20             return;
21         }
22         list.add(root.val);
23         if(root.left == null && root.right == null) {
24             if(sum == root.val) {
25                 result.add(new ArrayList<Integer>(list));
26                 // The promotion comes from direct Backtrack and Return on leaf node,

```

```

26         // it will save two more next recursion calls which will eventually
27         // return when root == null
28         list.remove(list.size() - 1);
29         return;
30     }
31 }
32 helper(root.left, result, sum - root.val, list);
33 // Do not backtrack here(before the right branch recursion), since we suppose
to change
34 // on 'list' should reflect in both left and right branch, if add backtrack
here will
35 // make right branch onwards recursion based on wrong version of 'list' that
without change
36 helper(root.right, result, sum - root.val, list);
37 // Backtrack: Remove the last element on list for next recursion
38 list.remove(list.size() - 1);
39 }
40 }
41
42 Time Complexity:  $O(n^2)$ , where n is number of nodes in the Binary Tree
43 Space Complexity:  $O(n)$ 

```

Solution 3: Iterative Inorder traversal with One Stack (360 min, based on L94.Binary Tree Inorder Traversal)

```

1 /**
2  * Definition for a binary tree node.
3  * public class TreeNode {
4  *     int val;
5  *     TreeNode left;
6  *     TreeNode right;
7  *     TreeNode() {}
8  *     TreeNode(int val) { this.val = val; }
9  *     TreeNode(int val, TreeNode left, TreeNode right) {
10 *         this.val = val;
11 *         this.left = left;
12 *         this.right = right;
13 *     }

```

```

14     * }
15     */
16     class Solution {
17     public List<List<Integer>> pathSum(TreeNode root, int targetSum) {
18         List<List<Integer>> result = new ArrayList<List<Integer>>();
19         if(root == null) {
20             return result;
21         }
22         TreeNode prev = null;
23         int pathSum = 0;
24         List<Integer> path = new ArrayList<Integer>();
25         Stack<TreeNode> stack = new Stack<TreeNode>();
26         // No modification on tree structure, can use original object 'root' to
traverse
27         // Similar style as L94.Binary Tree Inorder Traversal
28         while(root != null || !stack.isEmpty()) {
29             // Find as left as possible from root to leaf
30             while(root != null) {
31                 stack.push(root);
32                 path.add(root.val);
33                 pathSum += root.val;
34                 root = root.left;
35             }
36             root = stack.peek();
37             // Check if current node has right subtree and not a duplicate go through,
38             // only when its first visit the right subtree we go to right subtree
root,
39             // for a new right subtree we should start over from the outside while
loop
40             // all find as left as possible steps
41             if(root.right != null && root.right != prev) {
42                 root = root.right;
43                 continue;
44             }
45             // Check leaf node for potential path
46             if(root.left == null && root.right == null && pathSum == targetSum) {
47                 result.add(new ArrayList<Integer>(path));
48             }
49             // Remove current node
50             stack.pop();

```

```

51         prev = root;
52         // Subtract current node's val from path sum
53         pathSum -= root.val;
54         // As this current node is done, remove it from the current path
55         path.remove(path.size() - 1);
56         // Reset current node to null, so check the next item from the stack
57         root = null;
58     }
59     return result;
60 }
61 }
62
63 Time Complexity:  $O(n^2)$ , where n is number of nodes in the Binary Tree
64 Space Complexity:  $O(n)$ 

```

Refer to

<https://leetcode.com/problems/path-sum-ii/discuss/36695/Java-Solution:-iterative-and-recursive/34840>

```

1  public List<List<Integer>> pathSum(TreeNode root, int sum) {
2      List<List<Integer>> list = new ArrayList<>();
3      if (root == null) return list;
4      List<Integer> path = new ArrayList<>();
5      Stack<TreeNode> s = new Stack<>();
6      // sum along the current path
7      int pathSum = 0;
8      TreeNode prev = null;
9      TreeNode curr = root;
10     while (curr != null || !s.isEmpty()){
11         // go down all the way to the left leaf node
12         // add all the left nodes to the stack
13         while (curr != null){
14             s.push(curr);
15             // record the current path
16             path.add(curr.val);
17             // record the current sum along the current path
18             pathSum += curr.val;

```



```

19         curr = curr.left;
20     }
21     // check left leaf node's right subtree
22     // or check if it is not from the right subtree
23     // why peek here?
24     // because if it has right subtree, we don't need to push it back
25     curr = s.peek();
26     if (curr.right != null && curr.right != prev){
27         curr = curr.right;
28         continue; // back to the outer while loop
29     }
30     // check leaf
31     if (curr.left == null && curr.right == null && pathSum == sum){
32         list.add(new ArrayList<Integer>(path));
33         // why do we need new arraylist here?
34         // if we are using the same path variable path
35         // path will be cleared after the traversal
36     }
37     // pop out the current value
38     s.pop();
39     prev = curr;
40     // subtract current node's val from path sum
41     pathSum -= curr.val;
42     // as this current node is done, remove it from the current path
43     path.remove(path.size()-1);
44     // reset current node to null, so check the next item from the stack
45     curr = null;
46 }
47 return list;
48 }

```

How iterative Inorder traversal with One Stack step by step ?

1 e.g

```

2         5
3       /  \
4      3    6

```

```

5         /  \   \
6         2   4   7
7 Go down all the way to the left leaf node add all the left nodes to the stack
8
9                                     ===
10                                    2  push 2
11                                ===    ---
12                                3  push 3    3
13                                ---    ---
14                                ===    ---
15                                curr.right=null
16                                curr=5
17                                root=5
18                                prev=null
19                                path={5}
20                                pathSum=5
21                                curr=3
22                                root=5
23                                prev=null
24                                path={5,3}
25                                pathSum=8
26                                curr=2
27                                root=5
28                                prev=null
29                                path={5,3,2}
30                                pathSum=10
31
32                                ===
33                                3
34                                ---
35                                check leaf
36                                curr.left=null ---> 1.pop out current value ---> 5 ---> curr=null -> curr=s.peek()=3
37                                -> curr=curr.right=4,continue ->
38                                curr.right=null    prev=curr=2    ===    curr.right=4!=null,
39                                curr.right=4!=prev=2
40                                pathSum=10!=sum=12  2.subtract current node's    prev=2
41                                val from path sum    pathSum=8
42                                pathSum=10-2=8    path={5,3}
43                                3.reset current node to    curr=null
44                                null, so check the next
45                                item from the stack
46                                curr=null
47
48                                ===
49                                4  push 4
50                                ---
51                                ===
52                                3
53                                3
54                                ---
55                                check leaf
56                                ---

```

```

39 push 4 -> 5 ---> curr=null -> curr=s.peek()=4 ---> curr.left=null ---> 1. pop out
   current value ---> 5 --->
40      ===      curr.right=null      curr.right=null      prev=curr=4
      ===
41      curr=4      pathSum=12==sum=12 2. subtract
   current node's
42      root=5      result={{5,3,4}} val from
   path sum
43      prev=2      pathSum=12-
   4=8
44      path={5,3,4}      3.reset
   current node to
45      pathSum=12      null, so
   check the next
46      item from
   the stack
47      curr=null
48
49      check leaf
      ===
50 curr=null -> curr=s.peek()=3 -----> curr=3 not a leaf ---> 1. pop out current
   value ---> 5 --->
51 curr.right=4!=null, curr.right=4==prev=4      prev=curr=3
      ===
52 we have visited curr.right=4 node before,      2. subtract current
   node's
53 no need to visit again      val from path sum
54      pathSum=8-3=5
55      3.reset current
   node to
56      null, so check the
   next
57      item from the stack
58      curr=null
59      ===
60      6 push 6
61      ---
62 curr=null -> curr=s.peek()=5 -> curr=curr.right=6,continue -> push 6 -> 5 --->
   curr=null -> curr=s.peek()=6... etc
63 curr.right=6!=null, curr.right=6!=prev=3      ===
64      curr=6
65      root=5
66      prev=3
67      path={5,6}

```

Why do we have a "prev" there? What does "curr.right != prev" exactly do?

<https://leetcode.com/problems/path-sum-ii/discuss/36695/Java-Solution:-iterative-and-recursive/759308>

It ensures that we **do not visit a right subtree again**. Let's say we did not have `curr.right != prev` check before we visit the right subtree. Consider the following case of 3 nodes:

```

parent
 / \
node1 node2

```

1. We start moving left until `curr` becomes null (`curr = node1.left = null`), adding `parent` and `node1` to the stack. Then we set `curr` to `stack.peek()` which is the `node1`. Since `node1.right` is null, we do not need to traverse its right subtree. We then pop `node1` from the stack, set `curr` to null and `pre` to `node1`.
2. In the next iteration of while loop, since `curr` is null, we skip the part where we continually traverse left. We set `curr` to `stack.peek()`, which is `parent`. Now we check if `parent.right` exists, and it does, so we will set `curr` to `parent.right = node2`. Since `node2` has no children, it is exactly the same scenario as `node1` and just like before in step 1), we will pop `node2` from stack after traversing, setting `curr` to null and `pre` to `node2`.
3. This is where the problem will happen without the `curr.right != prev` check. Since `curr` is null, we skip the part where we continually traverse left. We set `curr` to `stack.peek()`, which is `parent`. Now when we want to traverse right, since `parent.right != null` we would have revisited the right subtree again if we did not check if `curr.right != prev`. So you can see how the `prev` variable actually stores the **most recently visited subtree when some nodes have "resolved"** so that in the event it is the right subtree of a parent node, we do not get stuck in an infinite loop revisiting the same right subtree.

Why not "curr=s.pop() early + no need s.pop() later" ?

Failed on test:

```

1 Input: [5,4,8,11,null,13,4,7,2,null,null,5,1], 22
2
3           5
4          / \
5         4   8
6        /   / \
7       11  13  4
8      / \   / \
9     7  2  5  1

```

10

11 Output: [[5,4,11,2]]

12 Expected: [[5,4,11,2],[5,8,4,5]]

```
1  /**
2   * Definition for a binary tree node.
3   * public class TreeNode {
4   *     int val;
5   *     TreeNode left;
6   *     TreeNode right;
7   *     TreeNode() {}
8   *     TreeNode(int val) { this.val = val; }
9   *     TreeNode(int val, TreeNode left, TreeNode right) {
10  *         this.val = val;
11  *         this.left = left;
12  *         this.right = right;
13  *     }
14  * }
15  */
16  class Solution {
17      public List<List<Integer>> pathSum(TreeNode root, int sum) {
18          List<List<Integer>> list = new ArrayList<>();
19          if (root == null) return list;
20          List<Integer> path = new ArrayList<>();
21          Stack<TreeNode> s = new Stack<>();
22          // sum along the current path
23          int pathSum = 0;
24          TreeNode prev = null;
25          TreeNode curr = root;
26          while (curr != null || !s.isEmpty()){
27              // go down all the way to the left leaf node
28              // add all the left nodes to the stack
29              while (curr != null){
30                  s.push(curr);
31                  // record the current path
32                  path.add(curr.val);
```

```

33         // record the current sum along the current path
34         pathSum += curr.val;
35         curr = curr.left;
36     }
37     // check left leaf node's right subtree
38     // or check if it is not from the right subtree
39     // why peek here?
40     // because if it has right subtree, we don't need to push it back
41     //curr = s.peek();
42     curr = s.pop();
43     if (curr.right != null && curr.right != prev){
44         curr = curr.right;
45         continue; // back to the outer while loop
46     }
47     // check leaf
48     if (curr.left == null && curr.right == null && pathSum == sum){
49         list.add(new ArrayList<Integer>(path));
50         // why do we need new arraylist here?
51         // if we are using the same path variable path
52         // path will be cleared after the traversal
53     }
54     // pop out the current value
55     //s.pop();
56     prev = curr;
57     // subtract current node's val from path sum
58     pathSum -= curr.val;
59     // as this current node is done, remove it from the current path
60     path.remove(path.size()-1);
61     // reset current node to null, so check the next item from the stack
62     curr = null;
63 }
64 return list;
65 }
66 public static void main(String[] args) {
67     /**
68         5
69         /  \
70        4    8
71       /  /  \
72      11 13  4

```

```

73         /   \       /   \
74        7     2     5     1
75       */
76       Test b = new Test();
77       TreeNode five_a = b.new TreeNode(5);
78       TreeNode four_a = b.new TreeNode(4);
79       TreeNode eight = b.new TreeNode(8);
80       TreeNode eleven = b.new TreeNode(11);
81       TreeNode thirteen = b.new TreeNode(13);
82       TreeNode four_b = b.new TreeNode(4);
83       TreeNode seven = b.new TreeNode(7);
84       TreeNode two = b.new TreeNode(2);
85       TreeNode five_b = b.new TreeNode(5);
86       TreeNode one = b.new TreeNode(1);
87
88       five_a.left = four_a;
89       five_a.right = eight;
90       four_a.left = eleven;
91       eight.left = thirteen;
92       eight.right = four_b;
93       eleven.left = seven;
94       eleven.right = two;
95       four_b.left = five_b;
96       four_b.right = one;
97       List < List < Integer >> result = b.pathSum(five_a, 22);
98       System.out.println(result);
99   }
100
101   private class TreeNode {
102       public int val;
103       public TreeNode left, right;
104       public TreeNode(int val) {
105           this.val = val;
106           this.left = this.right = null;
107       }
108   }
109 }

```

Wrong code version with "curr=s.pop() + no need s.pop() later"

Take the above example, the issue is happen if we pop out current node early as 'curr=s.pop()' before check right subtree, in our example, after first round as left as possible traversal, stack s={5, 4, 11, 7}, and following pop out current node early logic it will pop 7 out, s={5,4,11}, then since 7 is leaf node no right subtree after it and not match target sum condition, at the end of first round we set 'curr=null' and move ahead to next round which suppose check next element on stack, till now, no difference between correct logic as "curr=s.peek() + s.pop() later" and wrong logic as "curr=s.pop() early", but in second round, if follow wrong logic, it will pop out 11 and s={5,4}, since 11 has right subtree, after reach its right subtree leaf node 2, it will hit 'continue' logic and in third round we will push 2 onto stack, s={5,4,2}, then directly pop 2 out, s={5,4}, yes, then the logic superficially still looks fine since it will hit target sum match logic and result get one path as {5,4,11,2}, but stack status is quite wrong, it will pop out 4 now and s={5}, then pop out 5 and s={}.

In conclusion, wrong stack status flow:

s={5, 4, 11, 7}

s={5, 4, 11}

s={5, 4} --> wrong operation as s.pop() early

s={5, 4, 2}

s={5, 4}

s={5}

s={} --> wrong operation as s.pop() early

Which eventually result into missing of second combination as {5,8,4,5} majorly locate on right subtree.

To compare, the correct stack status flow:

s={5, 4, 11, 7}

s={5, 4, 11}

s={5, 4, 11, 2} --> correct operation as s.peek()

s={5, 4, 11}

s={5, 4}

s={5}

s={5, 8} --> wrong operation as s.pop() early

.... etc

So it suppose not pop out current node early, have to reserve the current node but check only by peek() function in case current node has right subtree and requires direct continue to next round, otherwise when pop out current node early and move on to next round with 'continue'

we will wrongly pop out same path parent nodes stored on stack and miss other branch check.

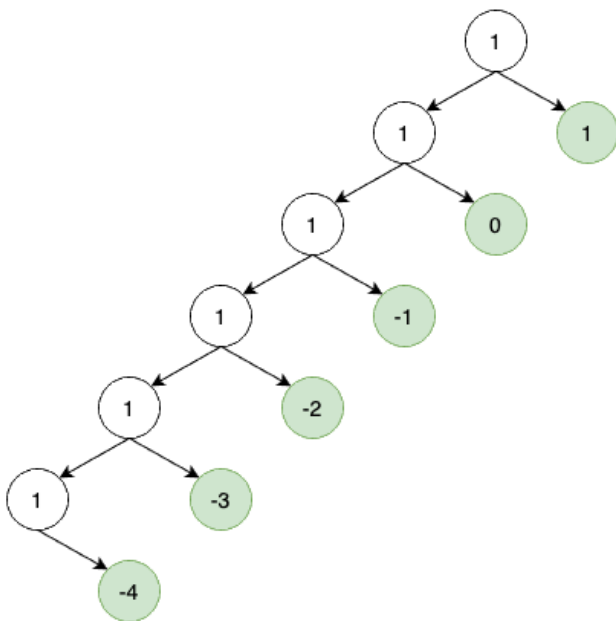
In conclusion: Reserve current node but check its status with peek() function before identify if right subtree exist or not, then after handling leaf node we are able to pop out current node and prepare for next round.

Complexity Analysis

<https://leetcode.com/problems/path-sum-ii/discuss/1382332/C%2B%2BPython-DFS-Clean-and-Concise-Time-complexity-explained>

Time: $O(N^2)$, where $N \leq 5000$ is the number of elements in the binary tree.

- First, we think the time complexity is $O(N)$ because we only visit each node once.
- But we forgot to calculate the cost to copy the current path when we found a valid path, which in the worst case can cost $O(N^2)$, let see the following example for more clear.



Worst case example

Let's consider the **binary tree** with **N** nodes in the left picture, **targetSum=2**

- There are $N/2$ leaf nodes.
- The maximum depth is $d=N/2$

The cost to copy paths:

$$\begin{aligned} 2 + 3 + 4 + \dots + d \\ \approx \frac{d*(d+1)}{2} \\ = \frac{d^2}{2} + \frac{d}{2} \\ = \frac{N^2}{8} + \frac{N}{4} \end{aligned}$$

- Extra Space (without counting output as space): $O(H)$, where H is height of the binary tree. This is the space for stack recursion or keeping path so far.

Refer to

 L94.Binary Tree Inorder Traversal (Ref.L98,L230,L144,L145)

 L112.P9.1.Path Sum (Ref.L257,L113)

 L257.Binary Tree Paths (Ref.L1430,L549,L124)