Exercise 4:

4. Generator of the Heston model (3 points) Consider a two-dimensional process X, v with

$$\begin{aligned} dX_t &= \mu X_t dt + \sigma_1 \sqrt{v_t} dB_{t,1} \\ dv_t &= \kappa(\theta - v_t)dt + \sigma_2 \sqrt{v_t} dB_{t,2} \end{aligned}$$

for constants $\mu, \sigma_1, \sigma_2, \kappa, \theta > 0$ and two Brownian motions B_1, B_2 with $[B_1, B_2]_t = \rho t$ for some $-1 \leq \rho \leq 1$. Compute the generator of the process (X, v).

$$Af(t,x) = \sum_{i=1}^{n} \mu_i(t,x) \frac{\partial f}{\partial x_i}(x) + \frac{1}{2} \sum_{i,j=1}^{n} c_{ij}(t,x) \frac{\partial^2 f}{\partial x_i \partial x_j}(t,x)$$