Foundations of Software

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Slides in part adapted from: University of Pennsylvania CIS 500: Software Foundations - Fall 2006 by Benjamin Pierce

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Course Overview

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What is "software foundations"?

Software foundations (or "theory of programming languages") is the mathematical study of the $\frac{1}{2}$ meaning of programs.

The goal is finding ways to describe program behaviors that are both precise and abstract.

- precise so that we can use mathematical tools to formalize and check interesting properties
- abstract so that properties of interest can be discussed clearly, without getting bogged down in low-level details

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Why study software foundations?

- ➤ To prove specific properties of particular programs (i.e., program verification)
 - Important in some domains (safety-critical systems, hardware design, security protocols, inner loops of key algorithms, ...), but still quite difficult and expensive
- ▶ To develop intuitions for *informal* reasoning about programs
- ➤ To prove general facts about all the programs in a given programming language (e.g., safety or isolation properties)
- ➤ To understand language features (and their interactions) deeply and develop principles for better language design (PL is the "materials science" of computer science...)

What you can expect to get out of the course

- ► A more sophisticated perspective on programs, programming languages, and the activity of programming
 - See programs and whole languages as formal, mathematical objects
 - ▶ Make and prove rigorous claims about them
 - Detailed knowledge of a variety of core language features
- ▶ Deep intuitions about key language properties such as type safety
- ▶ Powerful tools for language design, description, and analysis

Most software designers are language designers!

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Greenspun's Tenth Rule Of Programming

Any sufficiently complicated C or Fortran program contains an ad-hoc, informally-specified, bug-ridden, slow implementation of half of Common Lisp.

- Philip Greenspun

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What this course is not

- ► An introduction to programming
- A course on functional programming (though we'll be doing some functional programming along the way)
- A course on compilers (you should already have basic concepts such as lexical analysis, parsing, abstract syntax, and scope under your belt)
- A comparative survey of many different programming languages and styles

Approaches to Program Meaning

- Denotational semantics and domain theory view programs as simple mathematical objects, abstracting away their flow of control and concentrating on their input-output behavior.
- Program logics such as Hoare logic and dependent type theories focus on logical rules for reasoning about programs.
- Operational semantics describes program behaviors by means of abstract machines. This approach is somewhat lower-level than the others, but is extremely flexible.
- Process calculi focus on the communication and synchronization behaviors of complex concurrent systems.
- Type systems describe approximations of program behaviors, concentrating on the shapes of the values passed between different parts of the program.

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Overview

This course will concentrate on operational techniques and type systems.

- ▶ Part I: Modeling programming languages
 - ▶ Syntax and parsing
 - Operational semantics
 - ▶ Inductive proof techniques
 - ▶ The lambda-calculus
 - ▶ Syntactic sugar; fully abstract translations
- ▶ Part II: Type systems
 - ▶ Simple types
 - ▶ Type safety
 - References
 - Subtyping

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Overview

- ▶ Part III: Object-oriented features (case study)
 - A simple imperative object model
 - ▶ An analysis of core Java
 - ▶ An analysis of core Scala

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Organization of the Course

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Staff

Instructor: Martin Odersky

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Teaching Assistants: Eugene Burmako

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Information

Textbook: Types and Programming Languages,

Benjamin C. Pierce, MIT Press, 2002

Webpage: https://fos2015.github.io

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Elements of the Course

- ▶ The Foundations of Software course consists of
 - ▶ lectures (Tue 10:15-12:00, INM 200)
 - ▶ exercises and project work (Wed 10:15-12:00, CO 020)
- ▶ The lecture will follow in large parts the textbook.
- ► For lack of time, we cannot treat all essential parts of the book in the lectures, that's why the textbook is required reading for participants of the course.

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Homework and Projects

You will be asked to

- ▶ solve and hand in some written exercise sheets,
- ▶ do a number of programming assignments, including
 - parsers,
 - $\,\,\vartriangleright\,\,$ interpreters and reduction engines,
 - ▶ type checkers

for a variety of small languages.

► The recommended implementation language for these assignments is Scala.

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Scala

- Scala is a functional and object-oriented language that is closely interoperable with Java.
- ▶ It is very well suited as an implementation language for type-checkers, in particular because it supports:
 - ▶ pattern matching,
 - $\, \triangleright \, \, \mathsf{higher}\text{-}\mathsf{order} \,\, \mathsf{functions},$
 - ▶ inheritance and mixins.

Learning Scala

If you don't know Scala yet, there's help:

► The Scala web site:

www.scala-lang.org

- ➤ On this site, the documents:
 - $\begin{tabular}{ll} $ \land$ A Brief Scala Tutorial an introduction to Scala for Java programmers. (short and basic). \end{tabular}$

 - $\,\,\vartriangleright\,\, An\,\, Overview\,\, of\,\, the\,\, Scala\,\, Programming\,\, Language$ (high-level).
 - $\,\,\vartriangleright\,\, Scala\,\, By\,\, Example$ (long, comprehensive, tutorial style).
- ► The assistants.

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Grading and Exams

Final course grades will be computed as follows:

► Homework and project: 30%

▶ Mid-term exam: 30%

► Final exam: 40%

Exams:

- 1. Mid-term: first week of November (exact date TBA)
- 2. Final exam: to be announced

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Collaboration

- ► Collaboration on homework is strongly encouraged.
- $\,\blacktriangleright\,$ Studying with other people is the best way to internalize the material
- ➤ Form pair programming and study groups!

 2-3 people is a good size. 4 is too many for all to have equal input.

"You never really misunderstand something until you try to teach it...
" - Anon.

– Anon.

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Plagiarism

- ▶ A single group will of course share code.
- ▶ But plagiarizing code by other groups as part of a project is unethical and will not be tolerated, whatever the source.

Part I

Modelling programming languages

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Syntax and Parsing

- ► The first-level of modeling a programming language concerns its context-free syntax.
- ► Context free syntax determines a set of legal phrases and determines the (tree-)structure of each of them.
- ▶ It is often given on two levels:
 - ▷ concrete: determines the exact (character-by-character) set of legal phrases
 - □ abstract: concentrates on the tree-structure of legal phrases.
- ▶ We will be mostly concerned with abstract syntax in this course.
- But to be able to write complete programming tools, we need a convenient way to map character sequences to trees.

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Approaches to Parsing

There are two ways to construct a parser:

- ▶ By hand Derive a parser program from a grammar.
- ► Automatic Submit a grammar to a tool which generates the parser

In the second approach, one uses a special grammar description language to describe the input grammar.

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Domain-Specific Languages

- ► The grammar description language is an example of a domain-specific language (DSL).
- ➤ The parser generator acts as a processor ("compiler") for this language — that's why it's sometimes called grandly a "compiler-compiler".
- ▶ Example of a "program" in the grammar description DSL:

```
 \begin{array}{lll} \mathsf{Expr} & ::= \mathsf{Term} \ \{'+' \ \mathsf{Term} \ | \ '-' \ \mathsf{Term} \}. \\ \mathsf{Term} & ::= \mathsf{Factor} \ \{'*' \ \mathsf{Factor} \ | \ '/' \ \mathsf{Factor} \}. \\ \mathsf{Factor} & ::= \mathsf{Number} \ | \ '(' \ \mathsf{Expr} \ ')'. \end{array}
```

Embedded Domain Specific Languages

- ▶ An alternative to a stand-alone DSL is an Embedded DSL.
- Here, the DSL does not exist as a separate language but as an API in a host language.
- ► The host language is usually a general purpose programming language.

We will now develop this approach for grammar description languages.

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An EDSL for Parsing in Scala

```
\begin{array}{lll} \mathsf{def}\;\mathsf{expr}\; : \mathsf{Parser}[\mathsf{Any}] = \mathsf{term}\; \mathsf{"rep}("+"\; \mathsf{`term}\; |\; "-"\; \mathsf{`term}) \\ \mathsf{def}\;\mathsf{term}\; : \mathsf{Parser}[\mathsf{Any}] = \mathsf{factor}\; \mathsf{``rep}("*"\; \mathsf{``factor}\; |\; "/"\; \mathsf{``factor}) \\ \mathsf{def}\;\mathsf{factor}: \mathsf{Parser}[\mathsf{Any}] = "("\; \mathsf{``expr}\; \mathsf{``"})"\; |\; \mathsf{numericLit} \end{array}
```

A small Perl hack could derive the above code from the informal grammar definition above:

- ▶ Insert a def at the beginning of each production.
- ► The "::=" becomes ": Parser[Any] =".
- ➤ Sequential composition is now expressed by a ~.
- ▶ Repetition {...} is now expressed by rep(...).
- \blacktriangleright Option [...] is now expressed by $\mathsf{opt}(...).$

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Parser Combinators

▶ The differences between Grammar A and Grammar B are fairly minor.

(Note in particular that existing DSLs for grammar descriptions also tend to add syntactic complications to the idealized Grammar A we have seen).

- ➤ The important difference is that Grammar B is a valid Scala program, when combined with an API that defines the necessary primitives.
- ▶ These primitives are called parser combinators.

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The Basic Idea

For each language (identified by grammar symbol S), define a function f_S that, given an input stream i,

- ▶ if a prefix of i is in S, return Success(Pair(x, i')) where x is a result for S and i' is the rest of the input.
- ightharpoonup otherwise, return Failure(msg, i) where msg is an error message string.

The first behavior is called ${\it success}$, the second ${\it failure}$.

The Basic Idea in Code

Assume:

```
class StandardTokenParsers {
            type Parser = Input ⇒ ParseResult

where

type Input = Reader[Token] // a stream of Tokens with positions.

and we assume a class Token with subclasses

▷ case class Keyword(chars: String) for keywords,

▷ case class NumericLit(chars: String) for numbers,

▷ case class StringLit(chars: String) for strings,

▷ case class Identifier(chars: String) for identifiers.
```

Also assume a class ParseResult[T] with subclasses

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Object-Oriented Parser Combinators

- ▶ In fact, we will also need to express | and ~ as methods of parsers.
- ▶ That's why we extend the function type of parsers as follows:

```
abstract class Parser[T] extends (Input \Rightarrow ParseResult[T]) { // An unspecified method that defines the parser function. def apply(in: Input): ParseResult // A parser combinator for sequential composition def ^{\prime} ... // A parser combinator for alternative composition def | ... }
```

It remains to define concrete combinators that implement this class (see below). $% \begin{center} \end{center} \$

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A Generic Single-Token Parser

- ► The following parser succeeds if the first token in the input satisfies a given predicate p.
- ▶ If it succeeds, it reads the token string and returns it as a result.

```
\label{eq:def_token} \begin{split} & \mathsf{def}\; \mathsf{token}(\mathsf{kind}:\mathsf{String})(\mathsf{p}\colon \mathsf{Token} \Rightarrow \mathsf{boolean}) = \mathsf{new}\; \mathsf{Parser}[\mathsf{String}] \; \{ \\ & \mathsf{def}\; \mathsf{apply}(\mathsf{in}\colon \mathsf{Input}) = \\ & \mathsf{if}\; (\mathsf{p}(\mathsf{in}.\mathsf{head})) \; \mathsf{Success}(\mathsf{in}.\mathsf{head}.\mathsf{chars}, \, \mathsf{in}.\mathsf{tail}) \\ & \mathsf{else}\; \mathsf{Failure}(\mathsf{kind}+"\; \mathsf{expected}", \, \mathsf{in}) \\ \} \end{split}
```

Specific Single-Token Parsers

- ➤ The following parser succeeds if the first token in the input is the given keyword chars:
- ▶ If it succeeds, it returns a keyword token as a result.

```
\label{eq:continuous} \begin{split} & \mathsf{implicit} \ \mathsf{def} \ \mathsf{keyword}(\mathsf{chars}: \mathsf{String}) = \mathsf{token}("""+\mathsf{chars}+""") \ \{ \\ & \mathsf{case} \ \mathsf{Keyword}(\mathsf{chars1}) \Rightarrow \mathsf{chars} == \mathsf{chars1} \\ & \mathsf{case} \ \_ \Rightarrow \mathsf{false} \ \} \end{split}
```

 Note that keyword is marked implicit, so that you can usually write "if" instead of keyword("if").

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The following parsers succeed if, respectively, the first token in the input is a numeric or string literal, or an identifier.

```
\label{eq:defnumericLit} \begin{split} & \mathsf{def}\;\mathsf{numericLit} = \mathsf{token}("\mathsf{number"})(\_\mathsf{isInstanceOf}[\mathsf{NumericLit}]) \\ & \mathsf{def}\;\mathsf{stringLit} = \mathsf{token}("\mathsf{string}\;\mathsf{literal"})(\_\mathsf{isInstanceOf}[\mathsf{StringLit}]) \\ & \mathsf{def}\;\mathsf{ident} = \mathsf{token}("\mathsf{identifier"})(\_\mathsf{isInstanceOf}[\mathsf{Identifier}]) \end{split}
```

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The Sequence Combinator

- ➤ The sequence combinator P ~ Q succeeds if P and Q succeed sequentially, and returns the concatenation of their results.
- ▶ " is implemented as a method of class Parser.

```
abstract class Parser[T] {  \begin{tabular}{ll} def & [U](q\colon Parser[U]) = new \ Parser[T & U] \ def \ apply(in\colon lnput) = Parser.this(in) \ match \ \{ & case Success(x, in1) \Rightarrow & q(in1) \ match \ \{ & case Success(x, in2) \Rightarrow Success(new & (x, y), in2) \ case \ failure \Rightarrow failure \ \} \\ & case \ failure \Rightarrow failure \ \} \\ \} \\ \end{tabular}
```

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Concatenating Results

Normally, the $\tilde{\ }$ combinator returns a the results of the two parsers that are run sequentially wrapped in a "-object.

The case class $\tilde{\ }$ is similar to Pair (but $\tilde{\ }$ turns out to be nicer – see below).

```
case class ~[T, U](_1: T, _2: U) { override def toString = "(" + _1 +" ~ " + _2 +")" }
```

There are also two variants of $\tilde{\ }$ which return only the left or only the right operand:

```
P < ^{\sim} Q // returns only P's result P \sim Q // returns only Q's result
```

The Alternative Combinator

- \blacktriangleright The alternative combinator $P \mid Q$ succeeds if either P or Q succeeds
- ▶ It returns the result of P if P succeeds, or the result of Q, if Q succeeds.
- The alternative combinator is implemented as a method of class Parser.

```
 \begin{array}{l} \mathsf{def} \mid (\mathsf{q} : \Rightarrow \mathsf{Parser}[T]) = \mathsf{new} \, \mathsf{Parser}[T] \; \{ \\ \mathsf{def} \, \mathsf{apply}(\mathsf{in} \colon \mathsf{Input}) = \mathsf{Parser.this}(\mathsf{in}) \, \, \mathsf{match} \; \{ \\ \mathsf{case} \, \mathsf{s1} \; \emptyset \; \mathsf{Success}( \neg \, \neg \, ) \Rightarrow \mathsf{s1} \\ \mathsf{case} \; \mathsf{failure} \Rightarrow \mathsf{q}(\mathsf{in}) \\ \} \\ \} \end{array}
```

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Failure And Success Parsers

➤ The parser failure(msg) always fails with the given error message. It is implemented as follows:

```
\begin{array}{l} \mathsf{def}\;\mathsf{failure}(\mathsf{msg}\colon\mathsf{String}) = \mathsf{new}\;\mathsf{Parser}[\mathsf{Nothing}]\;\{\\ \mathsf{def}\;\mathsf{apply}(\mathsf{in}\colon\mathsf{Input}) = \mathsf{Failure}(\mathsf{msg},\;\mathsf{in})\\ \} \end{array}
```

► The parser success(result) always succeeds with the given result. It does not consume any input. It is implemented as follows:

```
\begin{array}{l} \mathsf{def} \ \mathsf{success}[\mathsf{T}](\mathsf{result} \colon \mathsf{T}) = \mathsf{new} \ \mathsf{Parser}[\mathsf{T}] \ \{ \\ \ \ \mathsf{def} \ \mathsf{apply}(\mathsf{in} \colon \mathsf{Input}) = \mathsf{Success}(\mathsf{result}, \ \mathsf{in}) \\ \} \end{array}
```

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Result Conversion

The parser $P \ \hat{} \ f$ succeeds iff P succeeds. In that case it returns the result of applying f to the result of P.

```
 \begin{array}{l} \mathsf{def} \, {}^{\smallfrown} \, [\mathsf{U}](\mathsf{f}\colon\mathsf{T} \Rightarrow \mathsf{U}) = \mathsf{new} \, \mathsf{Parser}[\mathsf{U}] \, \{ \\ \mathsf{def} \, \mathsf{apply}(\mathsf{in}\colon\mathsf{Input}) = \mathsf{Parser.this}(\mathsf{in}) \, \, \mathsf{match} \, \, \{ \\ \mathsf{case} \, \mathsf{Success}(\mathsf{x}, \, \mathsf{in1}) \Rightarrow \mathsf{Success}(\mathsf{f}(\mathsf{x}), \, \mathsf{in1}) \\ \mathsf{case} \, \mathsf{f} \Rightarrow \mathsf{f} \\ \} \end{array}
```

A variant $\hat{\ \ }\hat{\ \ }$ takes a value V as right hand side argument.

It returns \boldsymbol{V} if the left hand parser succeeds:

```
\mathsf{def} \; \widehat{} \; \widehat{} \; [U](r \colon U) \colon \mathsf{Parser}[U] = \; \widehat{} \; \widehat{} \; (x \Rightarrow r)
```

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Option and Repetition Combinators

 \blacktriangleright The $\mathsf{opt}(P)$ combinator always succeeds and returns an Option result.

It returns $\mathsf{Some}(R)$ iff P succeeds with R. In case of failure, the result is None.

 $\begin{tabular}{ll} \hline \begin{tabular}{ll} \hline \end{tabular} \hline \end{tabular} \end{$

The two combinators are implemented as follows:

```
\begin{array}{ll} \mathsf{def}\;\mathsf{opt}[T](p\colon\mathsf{Parser}[T])\colon\mathsf{Parser}[\mathsf{Option}[T]] &= \\ p \; \widehat{}^{\mathsf{Nome}} \; |\;\mathsf{success}(\mathsf{None}) \\ \mathsf{def}\;\mathsf{rep}[T](p\colon\mathsf{Parser}[T])\colon\mathsf{Parser}[\mathsf{List}[T]] &= \\ p \; \widehat{}^{\mathsf{nep}}(p) \; \widehat{}^{\mathsf{Nome}} \; |\; \mathsf{case}\; x \; \widehat{}^{\mathsf{nep}}(x) \; |\; \mathsf{success}(\mathsf{List}(x)) \end{array}
```

Note that neither of these combinators can fail!

The Interleaved Repetition Combinator

The $\operatorname{repsep}(P,\,Q)$ parser parses a (possibly empty) sequence

```
P\ Q\ P\ ...\ Q\ P
```

It returns a list of all results returned by P.

The repsep combinator is implemented as follows:

```
\begin{array}{ll} \mathsf{def}\;\mathsf{repsep}[\mathsf{T},\;\mathsf{U}](\mathsf{p}\colon\mathsf{Parser}[\mathsf{T}],\;\mathsf{q}\colon\mathsf{Parser}[\mathsf{U}])\colon\mathsf{Parser}[\mathsf{List}[\mathsf{T}]] \;=\; \\ \mathsf{p}\;\;\tilde{\;}\;\mathsf{rep}(\mathsf{q}\;\;\tilde{\;}>\;\mathsf{p})\;\; ^{\smallfrown}\; \{\;\mathsf{case}\;\mathsf{r}\;\;\tilde{\;}\;\mathsf{rs}\Rightarrow\mathsf{r}\;\colon\colon\mathsf{rs}\;\} \quad | \quad \mathsf{success}(\mathsf{List}()) \end{array}
```

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Other Combinators

More combinators can be defined if necessary.

Exercise: Implement the rep1(P) parser combinator, which applies P one or more times.

Exercise: Define opt and rep directly, without making use of $\tilde{\ }$, $\ |\$, and empty.

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An Example: JSON

 $\ensuremath{\mathsf{JSON}},$ or $\ensuremath{\mathsf{JavaScript}}$ Object Notation, is a popular data interchange format.

JSON data essentially consists of objects $\{...\},$ arrays [...], numbers, and strings.

Here is an example of a JSON value:

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A JSON parser

Testing the JSON Parser

Add a method main that can be used to test the parser.

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```
> java examples.parsing. JSON " { "x": true "y": [1, 2] } " { x: true y: [1, 2] } " [1.13] failure: unexpected token string literal y { "x": true "y": [1, 2] } ^{\circ}
```

Getting Better Output

- ➤ The result of the previous JSON parser was a tree containing all input tokens (slightly chaotically).
- ► We transform the productions using the ^^ combinator to get more structured output:

```
\begin{array}{lll} \mathsf{def} \; \mathsf{obj} & : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{Map} \\ \text{$"\{"\ ^-> \mathsf{rep}(\mathsf{member}) < ^-"\}"\ ^- (\mathsf{ms} \Rightarrow \mathsf{Map}() \ ++ \ \mathsf{ms})$ \\ \mathsf{def} \; \mathsf{arr} \; : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{List} \\ \text{$"["\ ^-> \mathsf{rep}\{\mathsf{value}\} < ^-"]"$} \\ \mathsf{def} \; \mathsf{member} : \mathsf{Parser}[\mathsf{Any}] = \ // \; \mathsf{return} \; \mathsf{a} \; \mathsf{name}/\mathsf{value} \; \mathsf{pair} \\ \mathsf{stringlit} \; \texttt{$"":"\ ^-} \; \mathsf{value} \; ^- \\ \mathsf{\{} \; \mathsf{case} \; \mathsf{name} \; "":"\ ^- \; \mathsf{value} \Rightarrow (\mathsf{name}, \; \mathsf{value}) \; \mathsf{\}} \end{array}
```

Question: What are more precise types for obj, arr, and member?

```
 \begin{array}{c} \text{def value: Parser}[Any] = (\\ \text{obj}\\ | \text{ arr}\\ | \text{ stringLit}\\ | \text{ numericLit } ^ (\_.\text{toInt}) \text{ // return an Int}\\ | \text{ "null" } ^ \wedge \text{ null} \text{ // return 'null'}\\ | \text{ "true" } ^ \wedge \text{ fulle} \text{ // return 'true'}\\ | \text{ "false" } ^ \wedge \text{ false} \text{ // return 'false'} \end{array}  If we run the test now, we get:  \begin{array}{c} \text{ > java examples.parsing. JSON1 "{ "x": true, "y": [1, 2, 3] }" \\ \{ \text{ x: true, y: } [1, 2, 3] \} \\ [1.30] \text{ parsed: Map}(\text{x} \rightarrow \text{true, y} \rightarrow \text{List}(1, 2, 3)) } \end{array}
```

Table of Parser Combinators

| ident | identifier |
|-----------------|----------------------------------------------|
| keyword() | keyword or special symbol (implicit) |
| numericLit | integer number |
| stringLit | string literal |
| P~Q | sequential composition |
| P < ~ Q, P ~> Q | sequential composition; keep left/right only |
| P Q | alternative |
| opt(P) | option |
| rep(P) | repetition |
| repsep(P, Q) | interleaved repetition |
| P ^^ f | result conversion |
| P ^^^ v | constant result |

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Arithmetic Expressions Again

Here is the full parser for arithmetic expressions:

```
\begin{split} & \text{object Arithmetic extends StandardTokenParsers } \{ \\ & \text{lexical.delimiters } ++= \text{List}("(",")","+","-","*","/") \\ & \text{def expr: Parser[Any]} = \text{term $^{\circ}$ rep}("+" $^{\circ}$ term $|"-" $^{\circ}$ term) \\ & \text{def term } = \text{factor $^{\circ}$ rep}("*" $^{\circ}$ factor $|"/" $^{\circ}$ factor) \\ & \text{def factor: Parser[Any]} = "(" $^{\circ}$ expr $^{\circ}")" $| numericLit \\ \end{split}
```

Question: How can we make it evaluate the parsed expression?

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A Problem with Top-Down Parsing

As parser combinators work top-down, they don't allow left-recursion. A production like

```
\mathsf{def}\;\mathsf{expr}=\mathsf{expr}\;\tilde{\ }"-"\;\tilde{\ }\mathsf{term}
```

would go into an infinite recursion when executed.

The alternative:

```
\label{eq:defeq} \mbox{def expr} = \mbox{term "rep("-" " term)} \mbox{produces a "right-leaning" tree: } X-Y-Z \mbox{ parses}
```

X $\tilde{\ }$ List("-" $\tilde{\ }$ Y, "-" $\tilde{\ }$ Z)

But the correct reduction/evaluation of +,-,*,/ is left-leaning!

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Evaluate by FoldLeft

We can solve this problem by delaying reduction until all elements of a repetition have been parsed and then performing a fold left on the list:

Here, reduceList is defined in terms of the fold-left operation $/: \dots$

```
 \begin{array}{l} \mathsf{val} \ \mathsf{reduceList} \colon \mathsf{Expr} \ \tilde{\ } \ \mathsf{List}[\mathsf{String} \ \tilde{\ } \ \mathsf{Expr}] \Rightarrow \mathsf{Expr} = \{ \\ \mathsf{case} \ \mathsf{i} \ \tilde{\ } \ \mathsf{ps} \Rightarrow (\mathsf{i} \ / ; \ \mathsf{ps})(\mathsf{reduce}) \\ \} \end{array}
```

... and reduce is defined as follows:

```
\label{eq:continuous_state} \begin{array}{l} \text{def reduce}(x\colon Int, \, r\colon String \ ^{\circ}\ Int) = r\ \text{match}\ \{\\ \text{case}\ ^{\circ}+^{\circ}\ ^{\circ}\ y \Rightarrow x + y\\ \text{case}\ ^{\circ}-^{\circ}\ ^{\circ}\ y \Rightarrow x - y\\ \text{case}\ ^{\circ}-^{\circ}\ ^{\circ}\ y \Rightarrow x / y\\ \text{def}\ ^{\circ}\ \text{throw new MatchError}(\text{"illegal case: "+r})\\ \end{array} With this, we get:
```

java examples.parsing.ArithmeticParsers1 " 2*(3+7) " 2*(3+7) [1.12] parsed : 20

(Note: the chainl combinator in the standard library does this for you.)

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Conclusion

- Combinator parsers give a provide a concise, flexible, and high-level way to construct parsers.
- ► The token classes of a context free grammar are modeled as primitive parsers.
- ▶ The combination forms are modeled as higher-order parsers.
- ▶ Combinator parsers are an example of an embedded DSL.
- ▶ By contrast, classical parser generators are stand-alone DSLs.
- Advantage of an embedded DSL over a parser generator: It's easier to connect the results of combinator parsers with the environment.
- ▶ Disadvantage: lower efficiency but this can be overcome (using packrat parsing)