Reinforcement Learning 1

MDPs, policies, values, TD learning, and Q-learning

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Psych 209, Winter 2018

Introduction

Plan for these lectures

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- … potentially many features in common:
  - Similar structure: states, actions, occasional rewards. We'll discuss a unified formal framework for many tasks like this (**MDPs**).
  - They're not directly supervised – nobody tells you *exactly* the right answer. We'll discuss how to learn tasks like this (**Reinforcement Learning**).

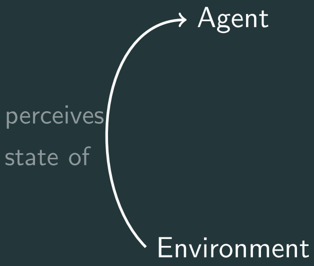
Formalizing tasks: MDPs

Markov Decision Processes (MDPs)

Agent

Environment

Markov Decision Processes (MDPs)



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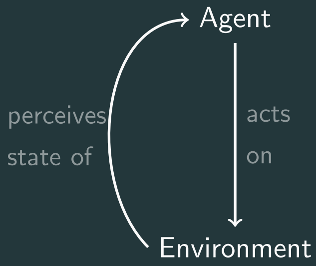

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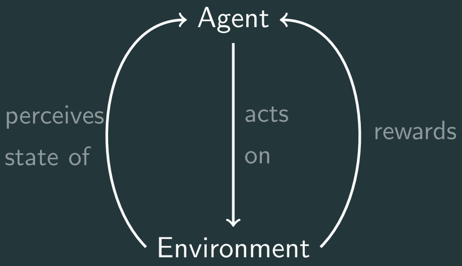

Markov Decision Processes (MDPs)

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$(t = 0)$

Environment
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Markov Decision Processes (MDPs)

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state

Environment

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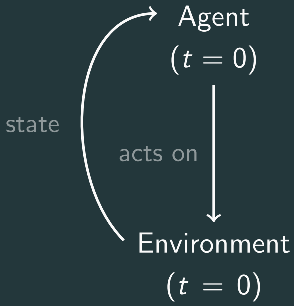

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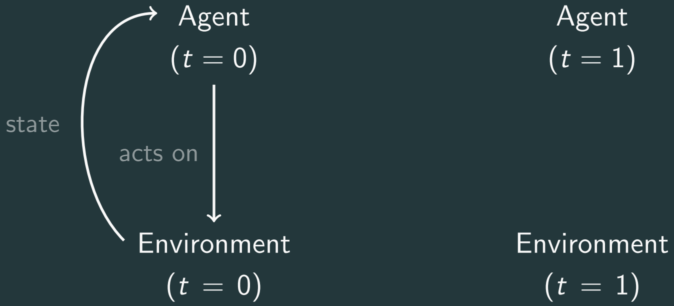

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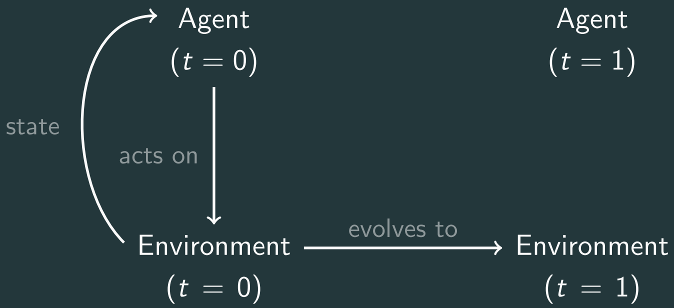

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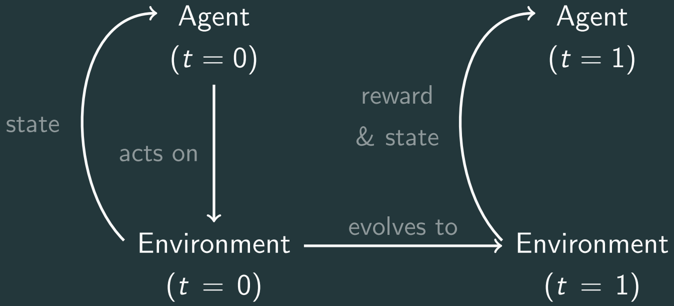

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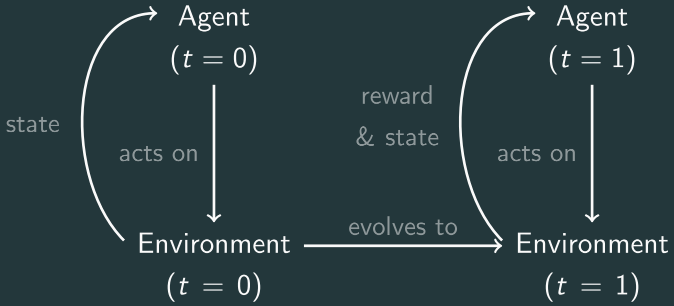

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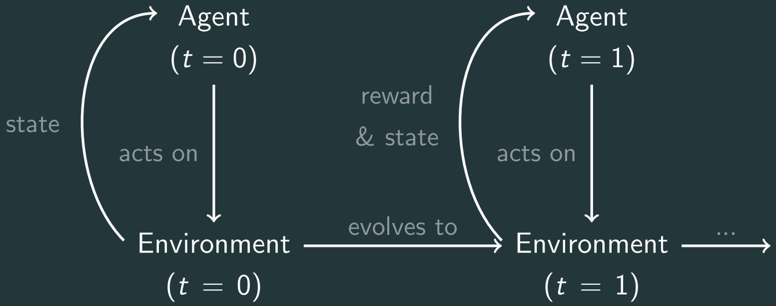

Agents & actions

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- At each time step t,
  perceives the **state**, s_t
  decides on an **action**, a_t
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- E.g. press gas, brake, turn wheel left 0.57 radians, press gas + turn right 2.2 radians, shift to 4th gear, ...

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- **Markov:** transition probabilities depend *only* on s_t, a_t, not history.

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- **Discount factor** γ tells how much we prioritize the present over the future.

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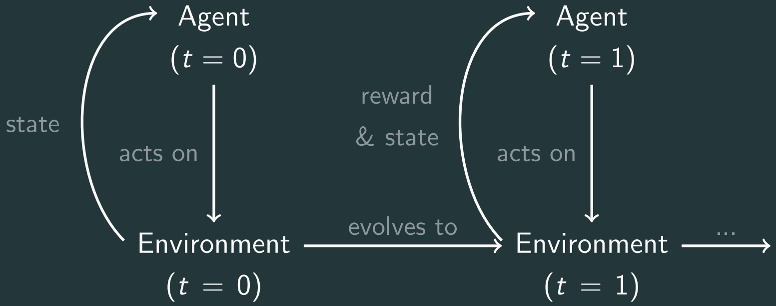

Questions?

Learning in MDPs

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- By using a **policy** π which maps states to actions.
- This policy could take many forms: picking randomly, a set of rules, a table, a neural network, or some combination thereof.
- Ideally, you would want your policy to pick the best action in every state, but what does "best" mean?

Value functions

- A natural way to define "best" states is based on expected return (which we call **value**).

$$V^\pi(s) = \mathbb{E}\left[\text{Return} \mid s, \pi\right] = \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^t r_t \,\middle|\, s, \pi\right]$$

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- But this also depends on the policy! (That's why it's V^{π}.)

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- But how can we try to find π^* or V^*?

TD Learning and iteration

Chess

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- ... but then he makes a move you don't expect, and you realize you're doing worse than you thought, so $V^{\pi}(s_{t+1})$ is smaller.
- Can we learn from this difference?

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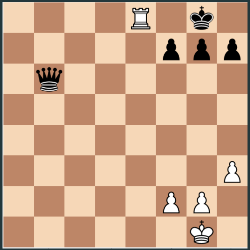
- Example: grid world.

TD Learning (applied to chess)



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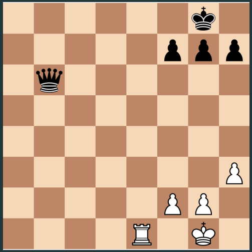

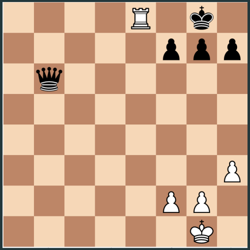

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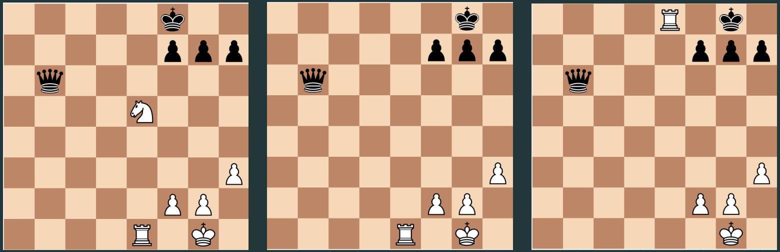
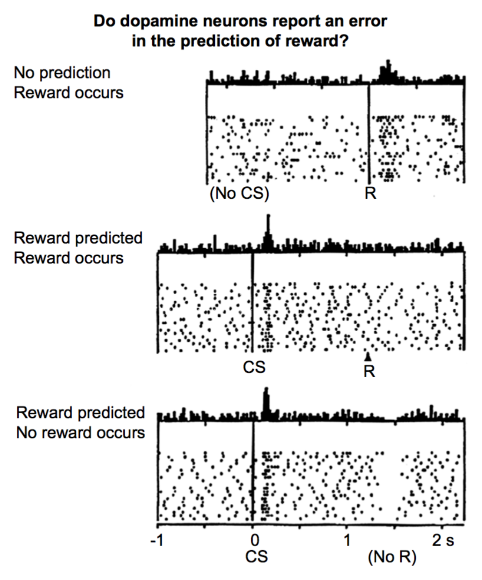


TD Learning (in the brain)



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- In chess, it's easy – just look at the next position after you move, and figure out which one has the max value.
- But what about in more complicated situations, where we don't know how the environment will change after our actions?

Q-learning

Q-values

- The solution we'll explore incorporates the action into our values by estimating the *value of an action in a state*, which we'll denote by $Q(s, a)$:

$$Q^{\pi}(s, a) = \mathbb{E}\left[\text{Return} \mid s, \pi, a\right] = \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^t r_t \,\middle|\, s, \pi, a\right]$$

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- But it also gives us ways of picking policies, e.g. pick the action with the highest Q.

Learning Q-values

- Using TD learning, we have another Bellman equation:

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- Are there potential problems with what we've learned so far?