

$$\Leftrightarrow \begin{pmatrix} 1 & 1-\sqrt{2} & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1+2\sqrt{2} \end{pmatrix} \quad [R_2 \rightarrow R_2 - 4]$$

$$\begin{pmatrix} 1 & 1-\sqrt{2} & 0 \\ 0 & \cancel{4} & \cancel{0} \\ 0 & \cancel{0} & \cancel{-1+2\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1-\sqrt{2} & 0 \\ 0 & \cancel{c} & \cancel{0} \\ 0 & \cancel{c} & -1+2\sqrt{2} \end{pmatrix} \quad [R_2 \leftrightarrow R_3]$$

$$\begin{pmatrix} 1 & 1-\sqrt{2} & 0 \\ 0 & \cancel{c} & 2\sqrt{2}-1 \\ 0 & \cancel{c} & 0 \end{pmatrix}$$

\Rightarrow Rewrite augmented matrix

$$\begin{cases} x_1 + (1-\sqrt{2})x_2 = 0 \Rightarrow x_1 = (\sqrt{2}-1)x_2 \\ (2\sqrt{2}-1)x_3 = 0 \Rightarrow x_3 = 0 \end{cases}$$

\Rightarrow eigenvector x will be:

$$x = \begin{pmatrix} (\sqrt{2}-1)x_2 \\ x_2 \\ x_3 = 0 \end{pmatrix} = x_2 \begin{pmatrix} \sqrt{2}-1 \\ 1 \\ 0 \end{pmatrix}$$

\Rightarrow any real number with $x_2 \neq 0 \rightarrow$
eigenvector of C with
eigenvalue $= 3 - e\sqrt{2}$.