

### Assignment 3: PCA computation

Suppose the random variables  $X_1, X_2, X_3$  have covariance matrix

$$\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

By hand calculate the eigenvalue-eigenvector pairs & the principal components  $Y_1, Y_2, Y_3$

Calculate eigenvalues

- To do this, value  $\lambda$  which satisfy the characteristic equation of matrix  $\Sigma$

$$\det(\Sigma - \lambda I)$$

\*  $I$  :  $3 \times 3$  identity matrix  $\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- from matrix  $\Sigma - \lambda I$

~~Det~~  $\Rightarrow \det \begin{pmatrix} 1-\lambda & -2 & 0 \\ -2 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix}$

Apply Sarrus  $\Rightarrow (1-\lambda)(5-\lambda)(2-\lambda) + (-2 \times 0 \times 0) +$   
 rule  $(0 \times -2 \times 0) - * (0 \times (5-\lambda) \times 0) -$   
 $(0 \times 0 \times (1-\lambda)) - ((2-\lambda) \times -2 - 2).$