

current matrix

$$= \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \left( \begin{array}{ccc|c} -1 & -2 & 0 & 0 \\ -2 & 3 & a & 0 \\ 0 & 0 & a & 0 \end{array} \right) \rightarrow \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

Convert to echelon form

$$\rightarrow [R_1 \rightarrow -1R_1] \left( \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ -2 & 3 & a & 0 \\ 0 & 0 & a & 0 \end{array} \right) \rightarrow [R_2 \rightarrow 2R_1 + R_2]$$

$$\left( \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & a & 0 \end{array} \right) \rightarrow [R_2 \rightarrow R_2]$$

$$\left( \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \rightarrow [R_1 \rightarrow R_1 + 2R_2]$$

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

Rewrite matrix as linear system

$$x_1 = 0$$

$$x_2 = 0$$