→ Rewrite matrix

$$\begin{cases} x_1 + (1+\sqrt{2})x_2 = 0 \Rightarrow x_1 = -(1+\sqrt{2})x_2. \\ 4x_2 = 0 \end{cases}$$

$x_2 = 0$

→ eigenvetce x will be:

$$x = \begin{pmatrix} -1+\sqrt{2}\,x_2 \\ x_2 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

⇒ any real number with $x_3 \neq 0$ → eigenvetors of C with eigenvalue $= 3+2\sqrt{2}$.

• $\boxed{\lambda = 3 - 2\sqrt{2}}$

+ current matrix $C - \lambda I$

$$\Leftrightarrow \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 3-2\sqrt{2} & 0 & 0 \\ 0 & 3-2\sqrt{2} & 0 \\ 0 & 0 & 3-2\sqrt{2} \end{pmatrix}$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2+2\sqrt{2} & -2 & 0 & 0 \to R_1 \\ -2 & 2+2\sqrt{2} & 0 & 0 \to R_2 \\ 0 & 0 & -1+2\sqrt{2} & 0 \to R_3 \end{array}\right)$$

$[R_1 \to (-2+2\sqrt{2})/R_1]$

$$\Leftrightarrow \begin{pmatrix} 1 & 1-\sqrt{2} & 0 \\ -2 & 2+2\sqrt{2} & 0 \\ 0 & 0 & -1+2\sqrt{2} \end{pmatrix}$$

$[R_2 \leftarrow R_2 + 2R_1$