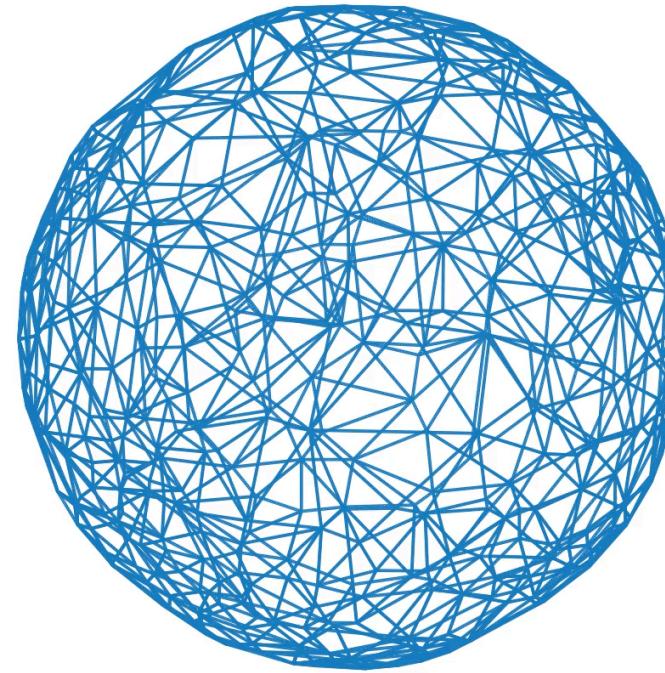


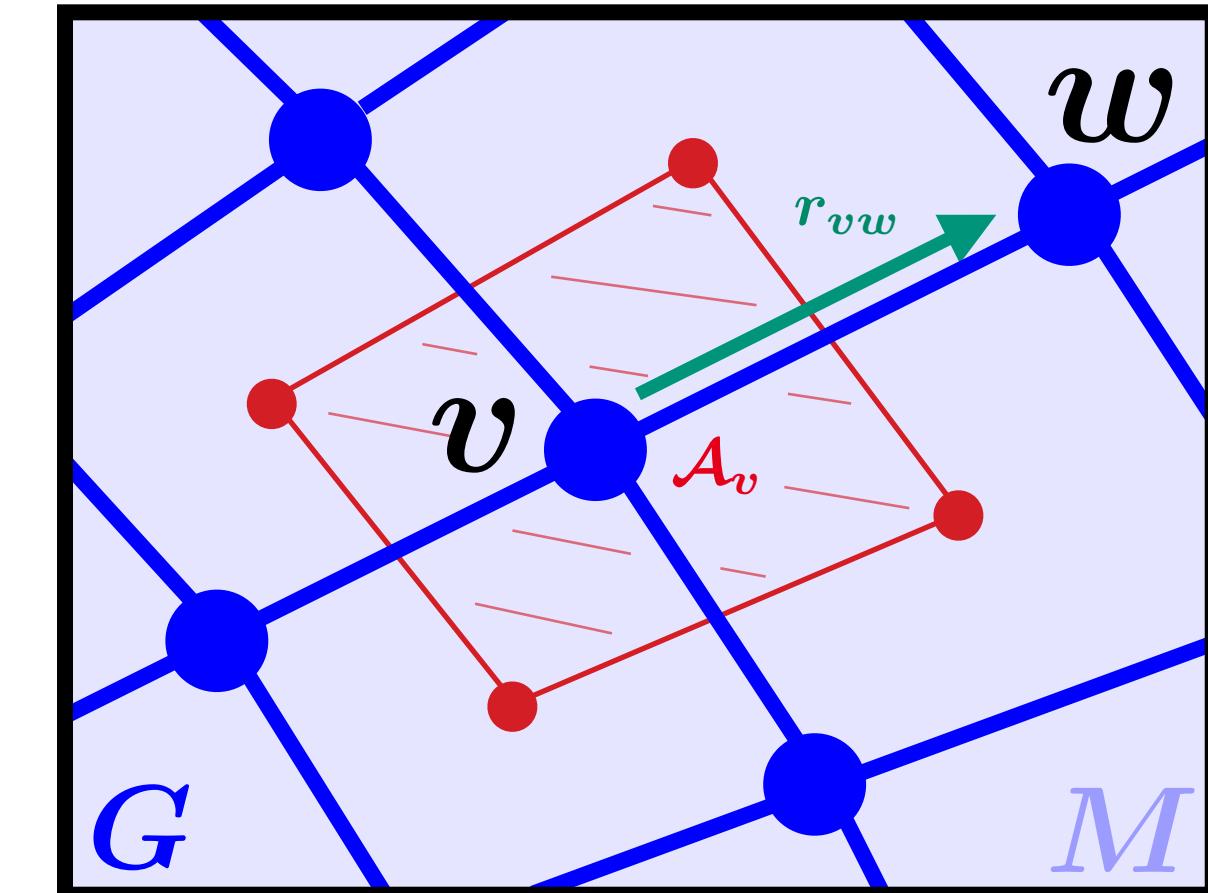
Edges:
wave equation



Vertices:
continuity &
Kirchhoff's current law

Q: Is there a continuum model of
the large-scale behaviour?

$$\mathbf{A:} \quad \frac{1}{\mu(x)\text{Tr}(R(x))} \nabla \cdot (\mu(x)R(x) \cdot \nabla f(x)) = -k^2 f(x)$$

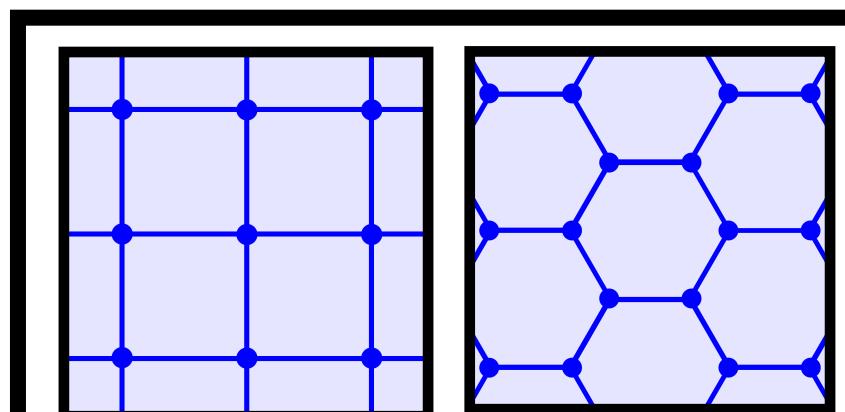


$$R(v) = \sum_{w \sim v} |r_{vw}| \hat{r}_{vw} \otimes \hat{r}_{vw}$$

$$\mu(v) = \frac{1}{A_v}$$

PDE sols
vs.

graph/explicit/
numerical sols



$$f(x + N_i e_i) = f(x)$$

$$f(x) = Ae^{ia \cdot x} + c.c.$$

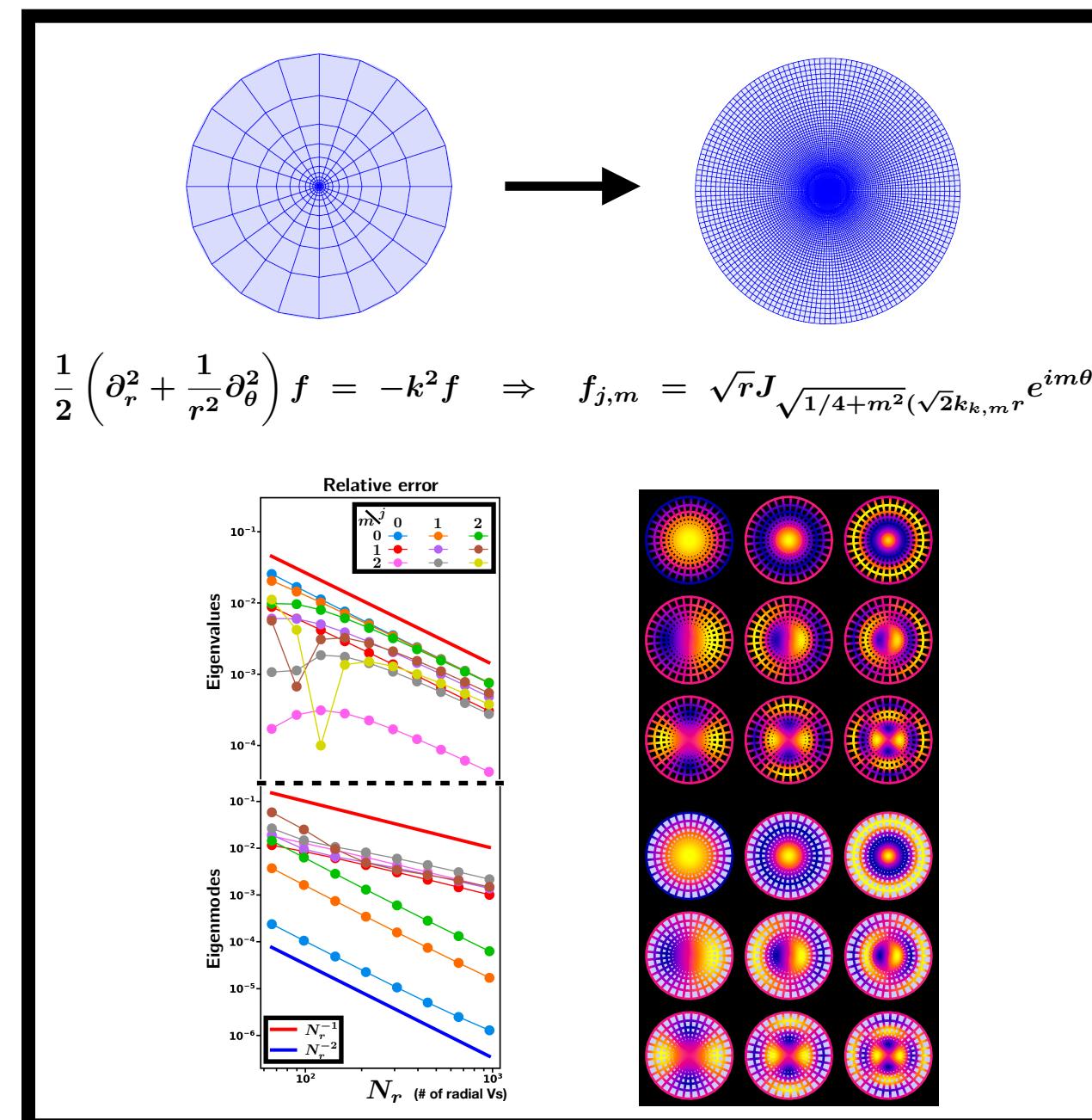
$$k^2 = \frac{a \cdot a}{\dim(M)} + O(\ell^2)$$

A high-density, drum-shaped graph
vibrates at $1/\sqrt{2}$ the frequency of the
Riemannian equivalent!

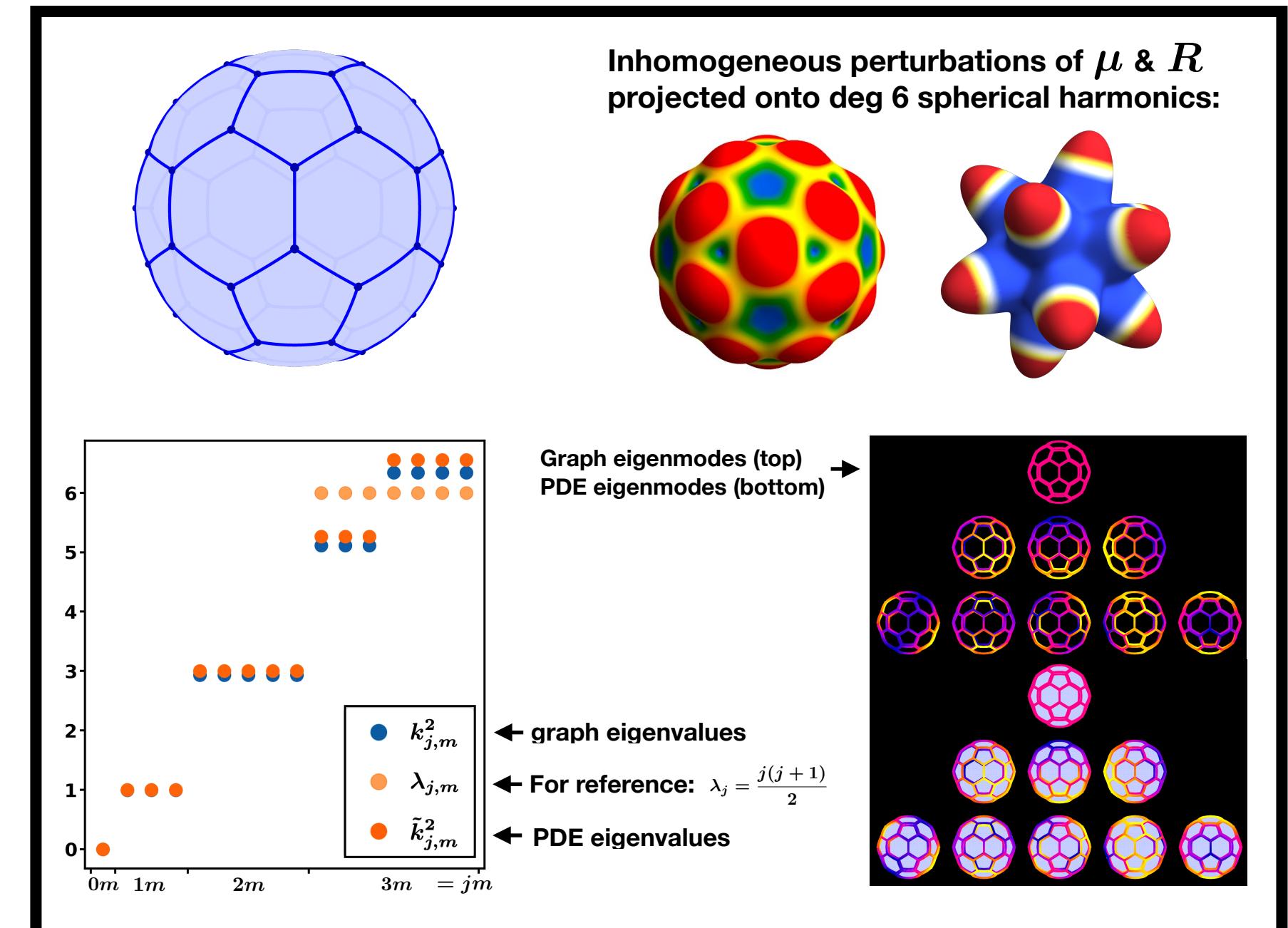
1) Periodic 2d lattices:

2) Spider web:

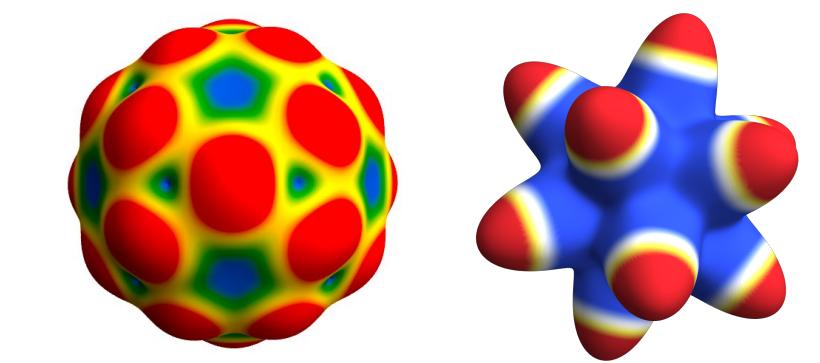
3) Truncated icosahedron: (a.k.a soccer ball / buckminsterfullerene)



$$\frac{1}{2} \left(\partial_r^2 + \frac{1}{r^2} \partial_\theta^2 \right) f = -k^2 f \Rightarrow f_{j,m} = \sqrt{r} J_{\sqrt{1/4+m^2}}(\sqrt{2} k_{j,m} r) e^{im\theta}$$



Inhomogeneous perturbations of μ & R
projected onto deg 6 spherical harmonics:



Graph eigenmodes (top)
PDE eigenmodes (bottom) →

← graph eigenvalues
← For reference: $\lambda_j = \frac{j(j+1)}{2}$
← PDE eigenvalues

