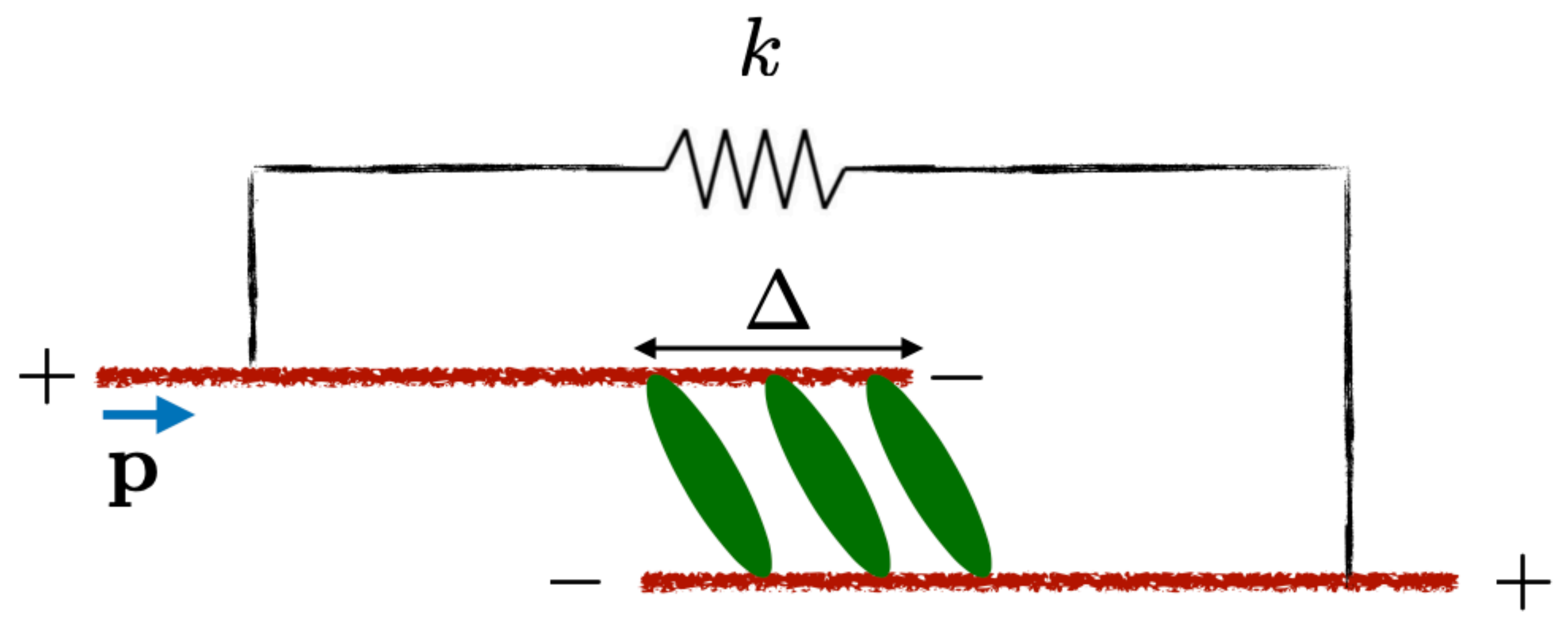
Collective Motion and Pattern Formation in Phase-Synchronizing Active Fluids

Brato Chakrabarti

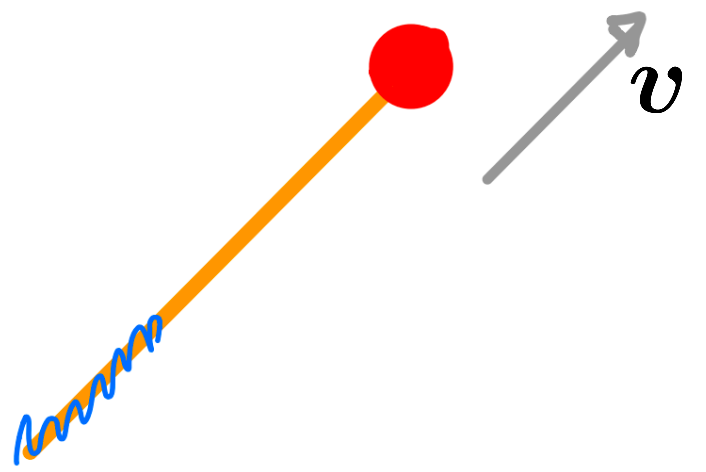
Flatiron Research Fellow
Biophysical Modeling Group, Flatiron Institute

with Sebastian Fürthauer (TU Wien)

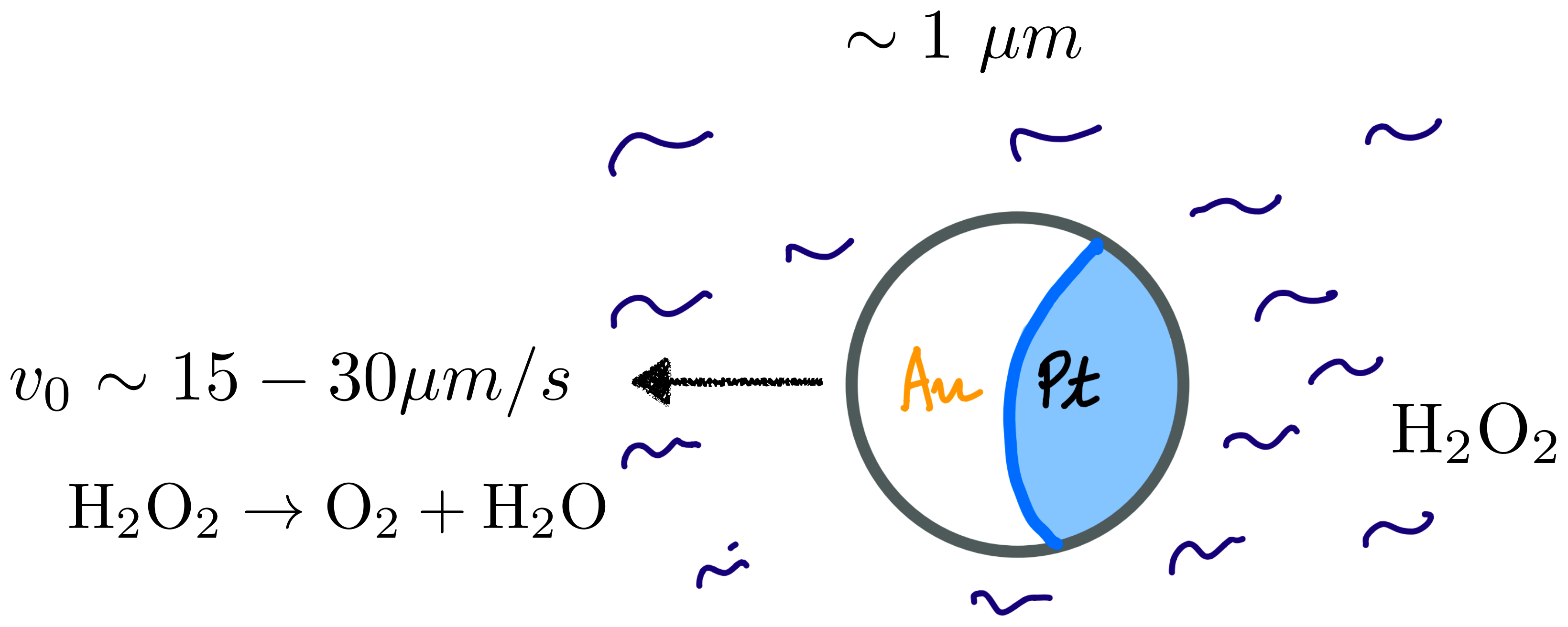


Physical Review Letters (2023)

Active matter - a quick overview



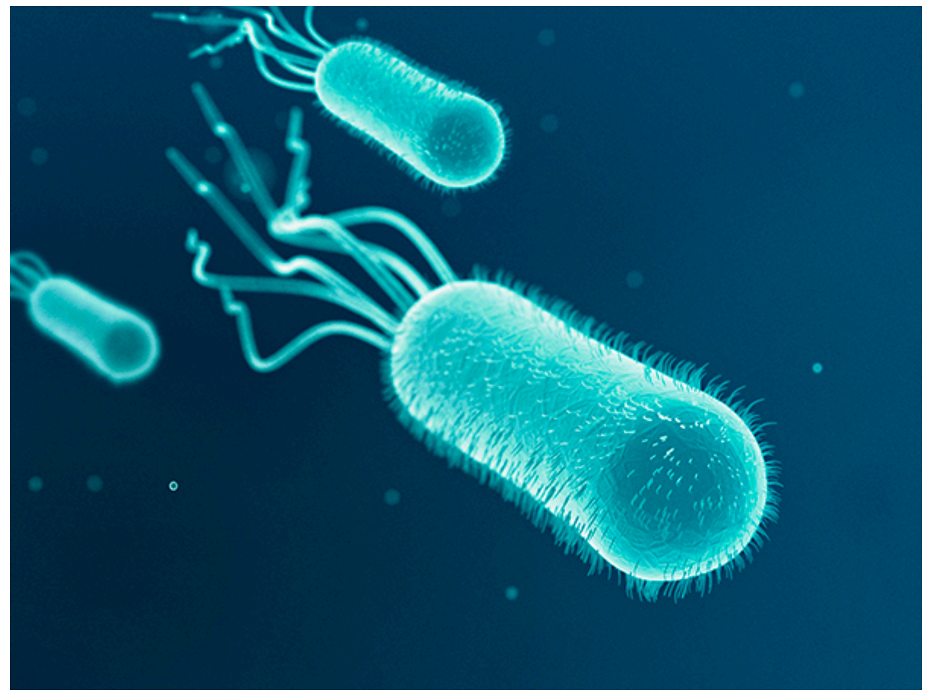
Active mater is collection of active particles



Matchstick as self-propelled-particle (SPP)

Janus particle

Active particle is any entity that is internally driven (with continuous supply of energy). It generates some motion by dissipating that energy. Continuous injection of energy drive them out of equilibrium.

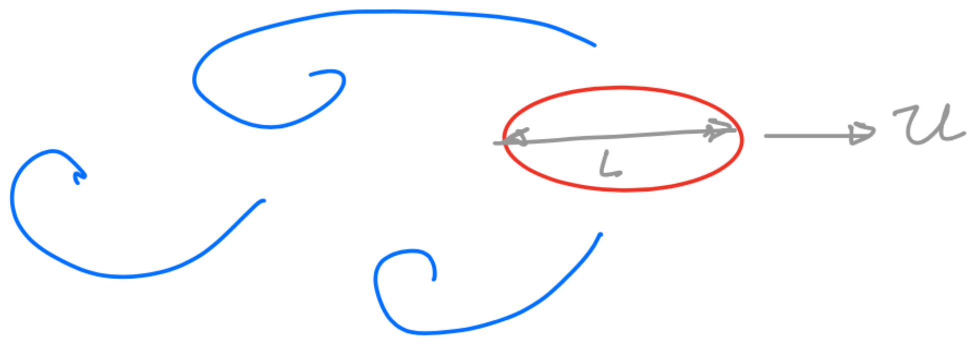


$Re \sim \mathcal{O}(1 - 10 \times 10^3)$

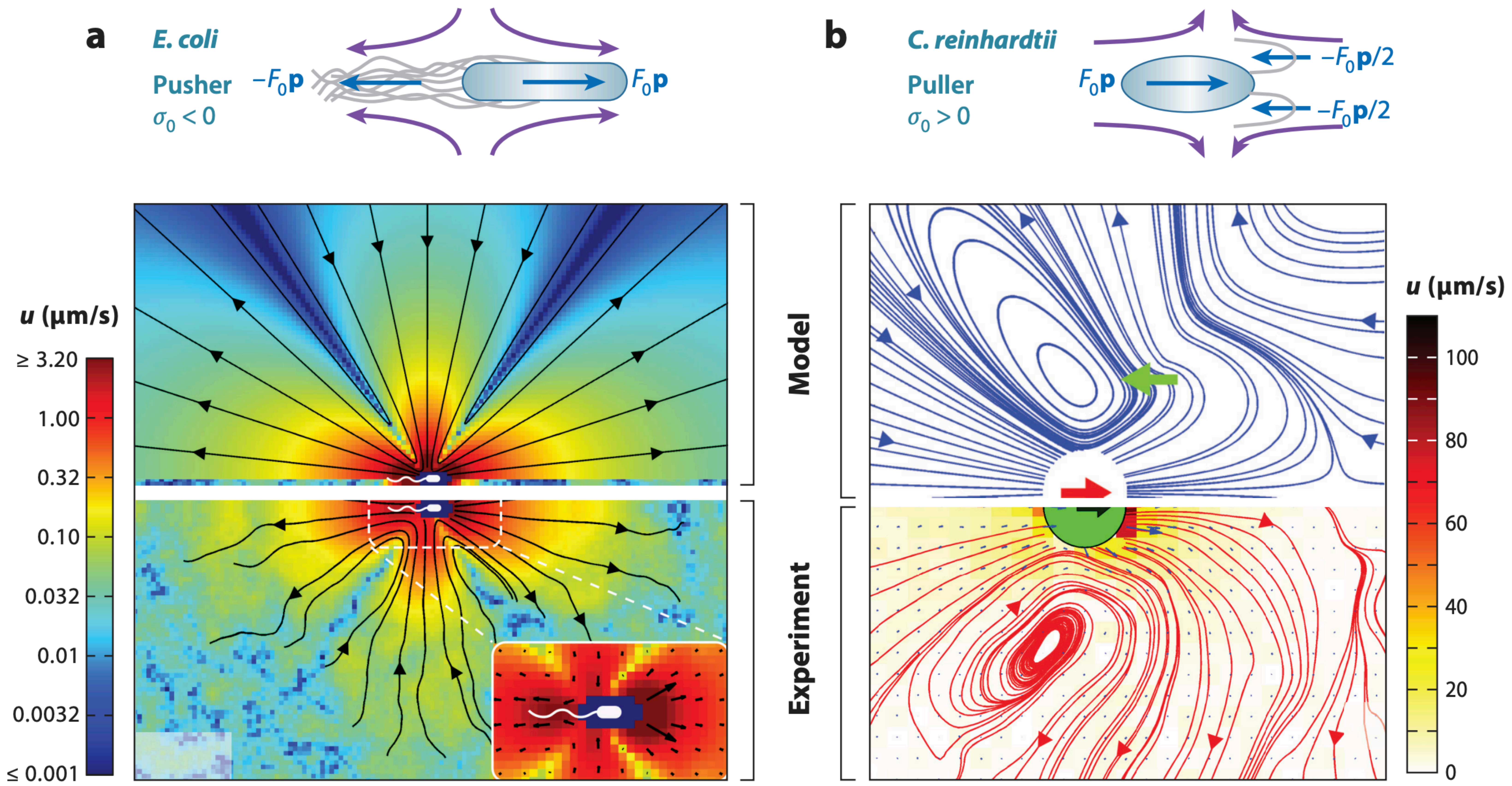
$Re \sim \mathcal{O}(10^{-5})$

$$Re \sim \frac{\text{inertial forces}}{\text{viscous forces}} \equiv \frac{\rho U L}{\mu} \sim 10^{-2} - 10^{-6}$$

$$U \sim \mathcal{O}(1 - 50\ \mu\text{m/s}), \quad L \sim \mathcal{O}(10 - 100\ \mu\text{m}), \quad \mu \sim \left(10 - 10^3\right) \times \mu_{\text{water}}$$

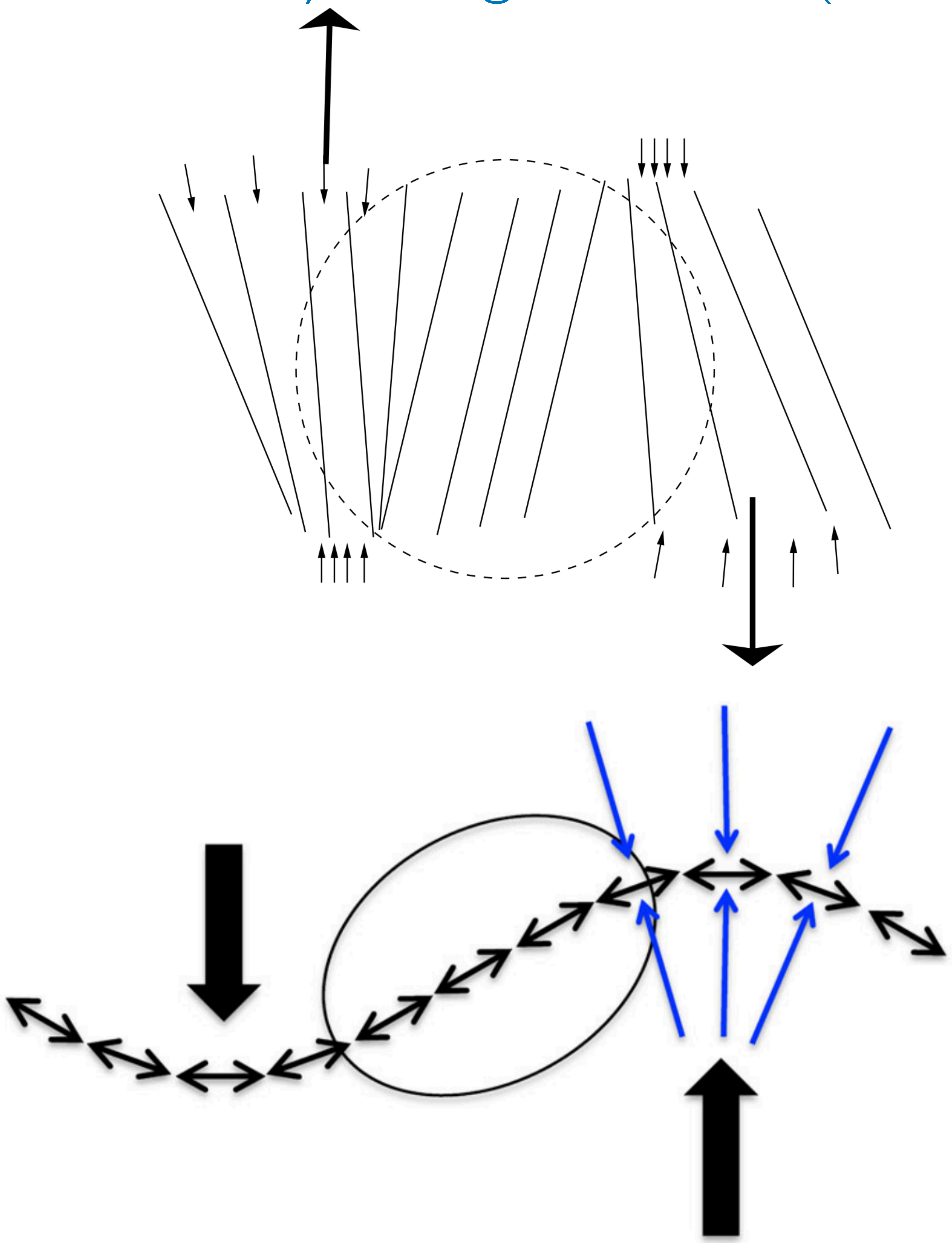


The *generic* instability of aligned states (in the over-damped limit)

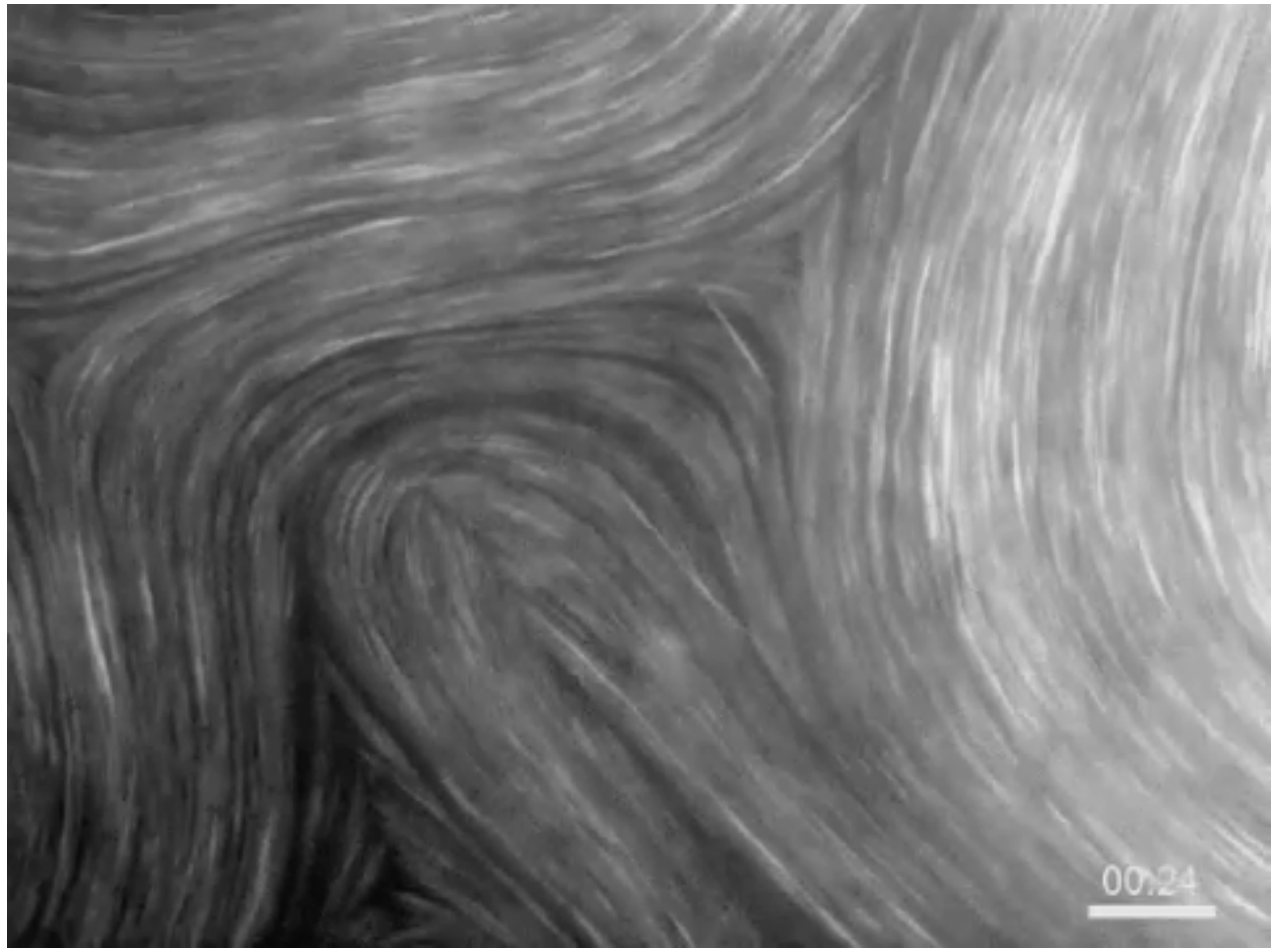


$$\Sigma_{\alpha\beta} = \sigma_0 \; p_\alpha p_\beta$$

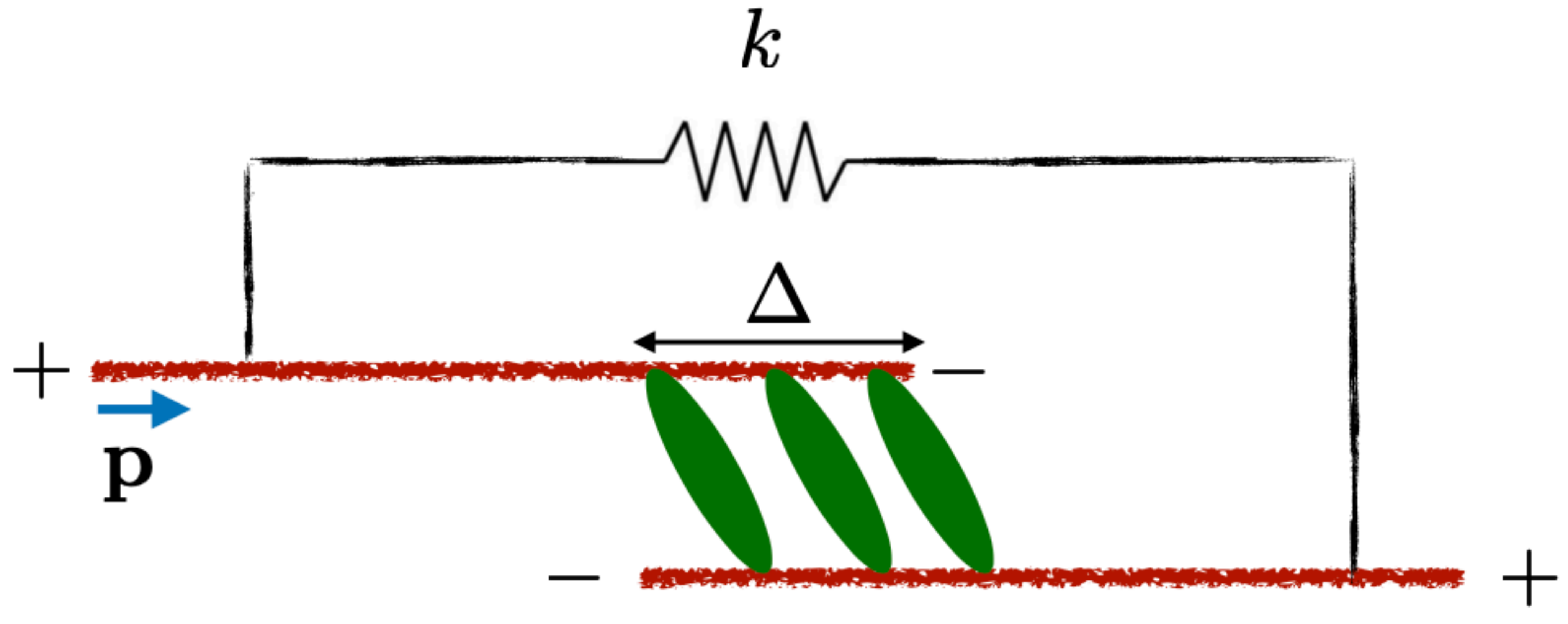
The *generic* instability of aligned states (in the over-damped limit)

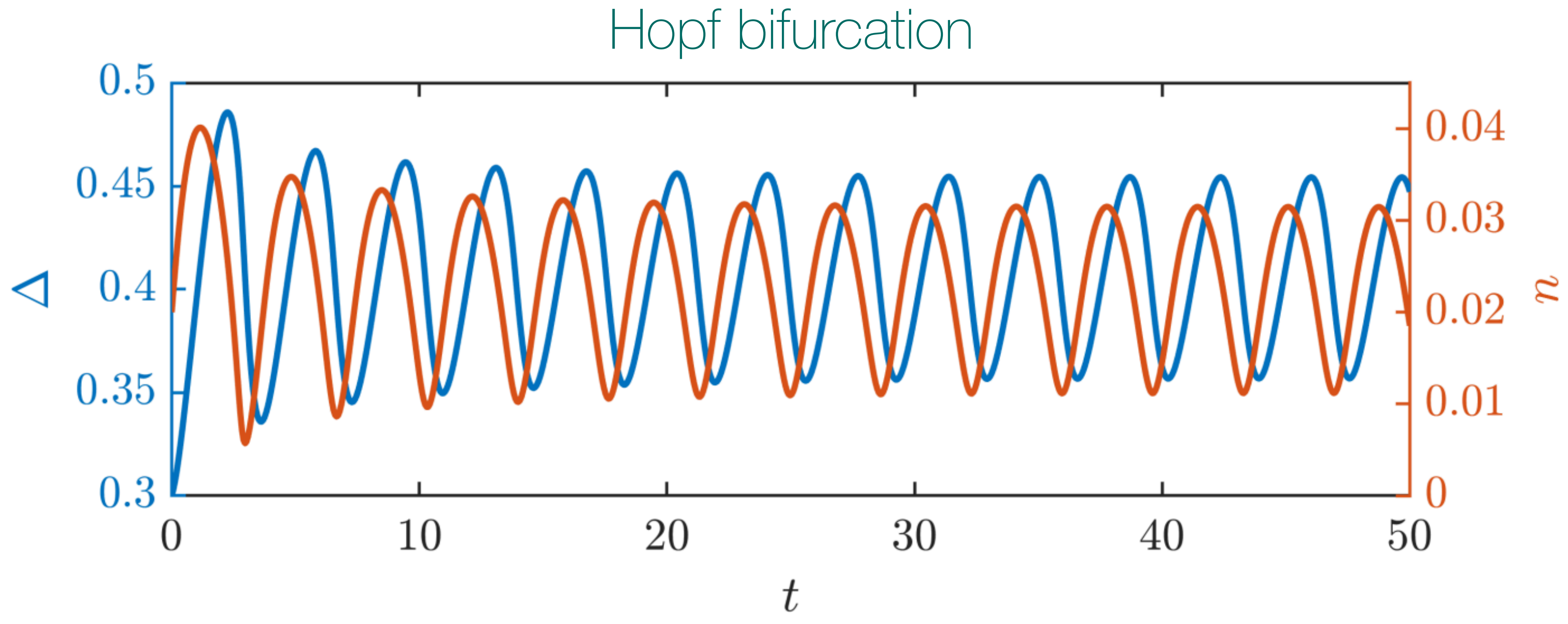


Microtubule suspension: active nematics



What happens when there is no net-dipole?

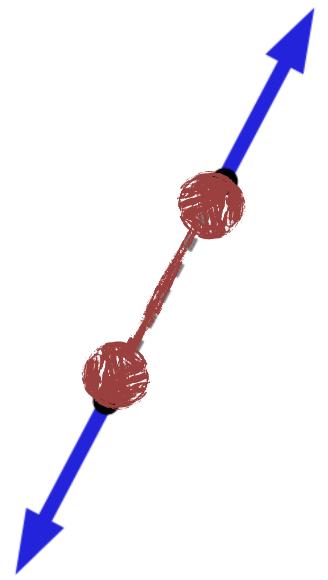




Newton's law: $\tilde{\xi}\left(\dot{\Delta} - v_f\right) = -k\Delta + f_m n$

Force-velocity curve: $f_m = f_0\left(1 - \frac{\dot{\Delta}}{v_0}\right)$

Motor dynamics: $\dot{n} = p_{\text{on}}(1-n) - p_{\text{off}}\, n \exp\left(\frac{f_m}{f_c}\right)$



Stress ⟷ Phase

$\dot{\phi} = \Omega +$ Stuff

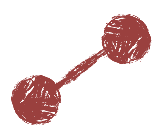
Flows can alter the phase dynamics of oscillators!

Can instantaneous flows through phase synchronization lead to emergent behavior?

Equations of motion

$\Psi(\boldsymbol{x}, \varphi, \boldsymbol{p}, t) \;\equiv$ Probability density function

$\boldsymbol{x} = \{x, y\}$ 2D description



$\varphi \in [0 \; 2\pi)$ Internal phase

$\boldsymbol{n} \equiv$ Orientation

Sharply aligned state

$\boldsymbol{n} = (\cos\theta, \sin\theta)$ Director field

$Q_{\alpha\beta} = n_\alpha n_\beta$ Nematic tensor

Fokker-Planck equation

$$\partial_t \psi + u_\alpha \partial_\alpha \psi + \partial_\varphi(\dot{\varphi}\psi) - D\partial_\alpha^2 \psi = 0,$$

needs to be specified

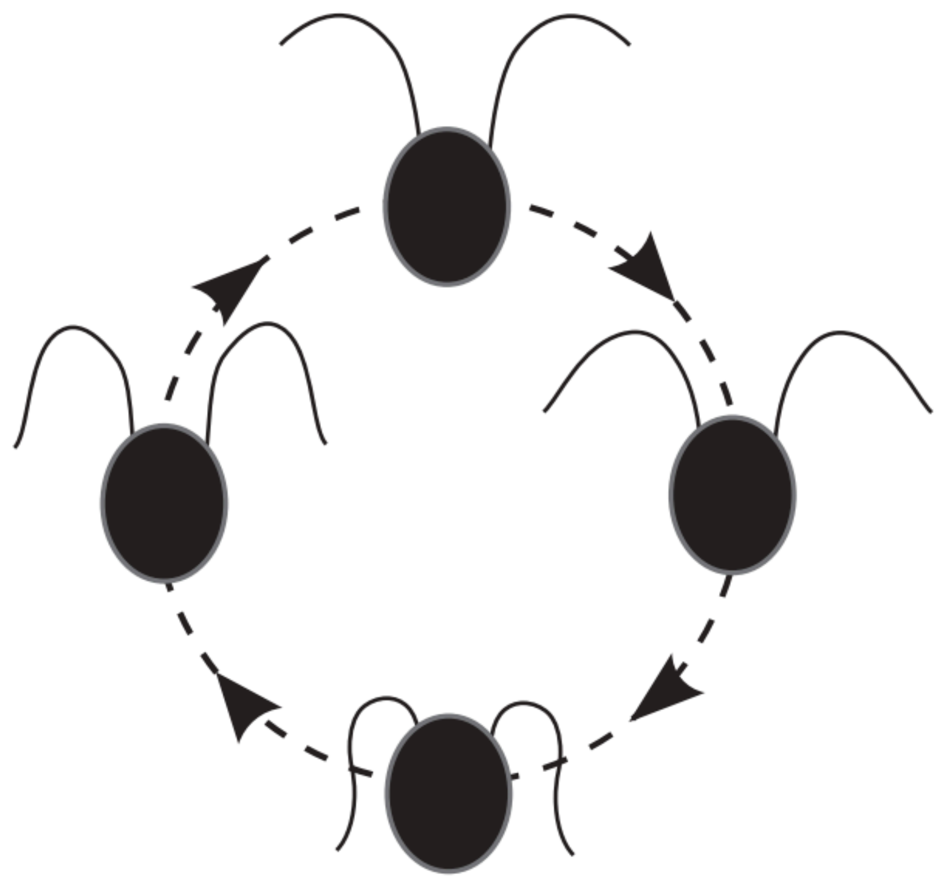
Forced Stokes equation

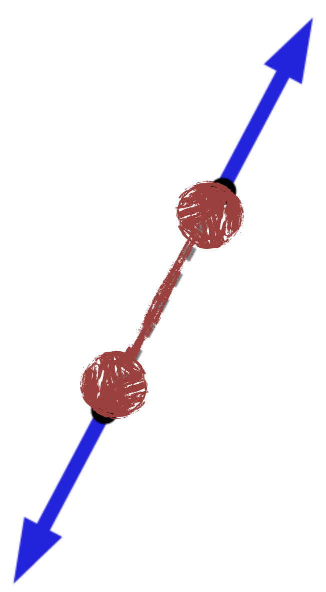
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\mu \Delta \boldsymbol{u} - \nabla q + \nabla \cdot \left(\boldsymbol{\sigma}^{\mathrm{act}} + \boldsymbol{\sigma}^{\mathrm{el}}\right) = 0$$

Active stress

$$\sigma_{\alpha\beta}^{\mathrm{act}}(\boldsymbol{x}, t) = Q_{\alpha\beta} \int_0^{2\pi} \psi(\boldsymbol{x}, \varphi, t) s(\varphi) \dot{\varphi} \, \mathrm{d}\varphi,$$





$s(\varphi) \equiv$ Periodic function of phase

$s(\varphi) = s_0 \cos(\varphi)$

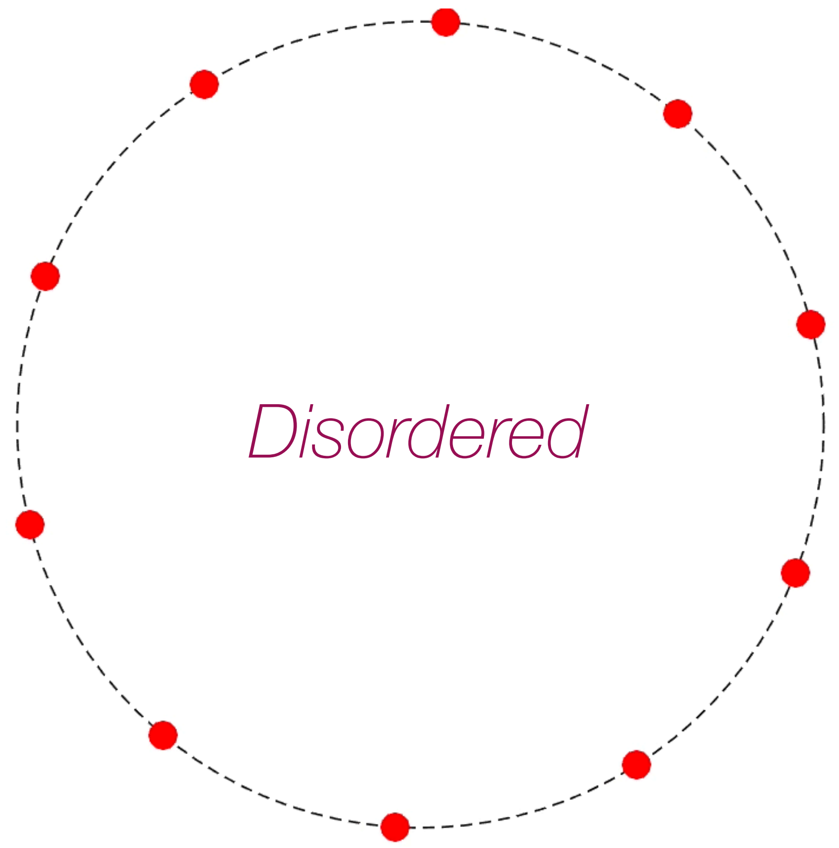
Coupling as a force-velocity relation

$$\dot{\varphi} = \Omega_0 + X(\varphi) p_\alpha p_\beta Q_{\alpha\beta}$$

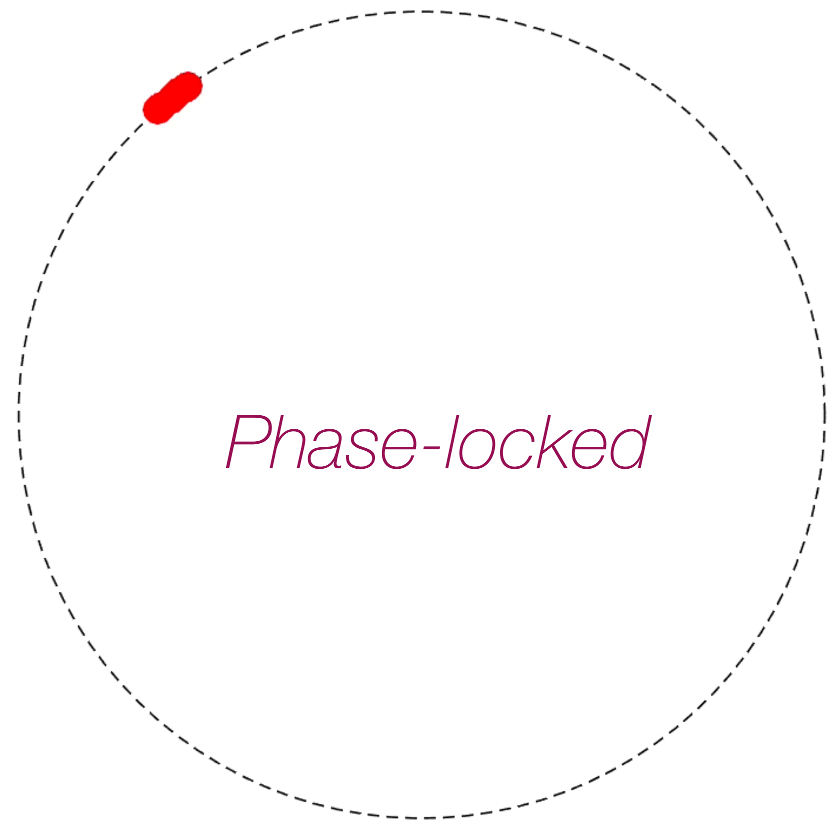
$X(\varphi) \equiv$ Periodic function of phase

Order parameter

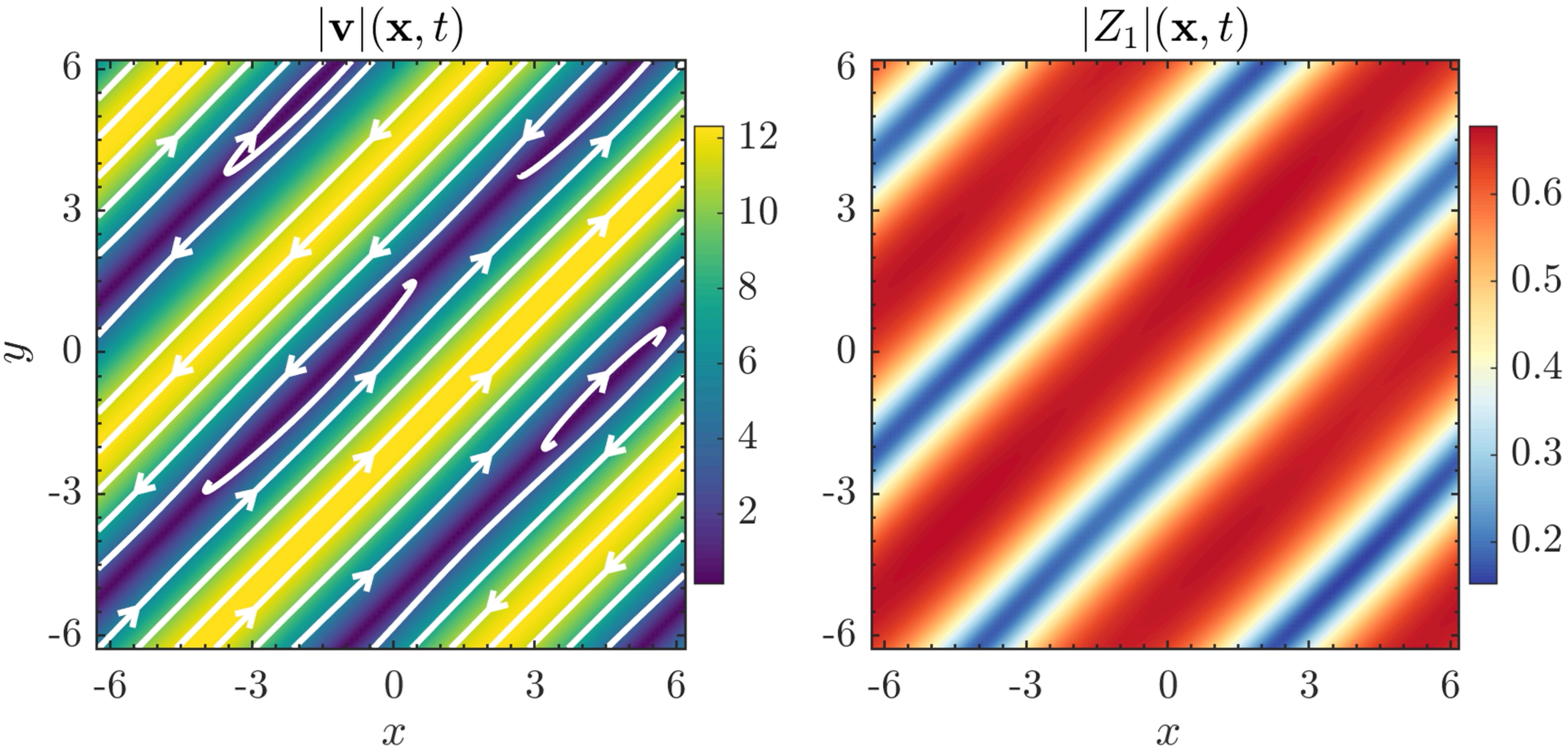
$|Z_1| \sim 0$



$|Z_1| \sim 1$

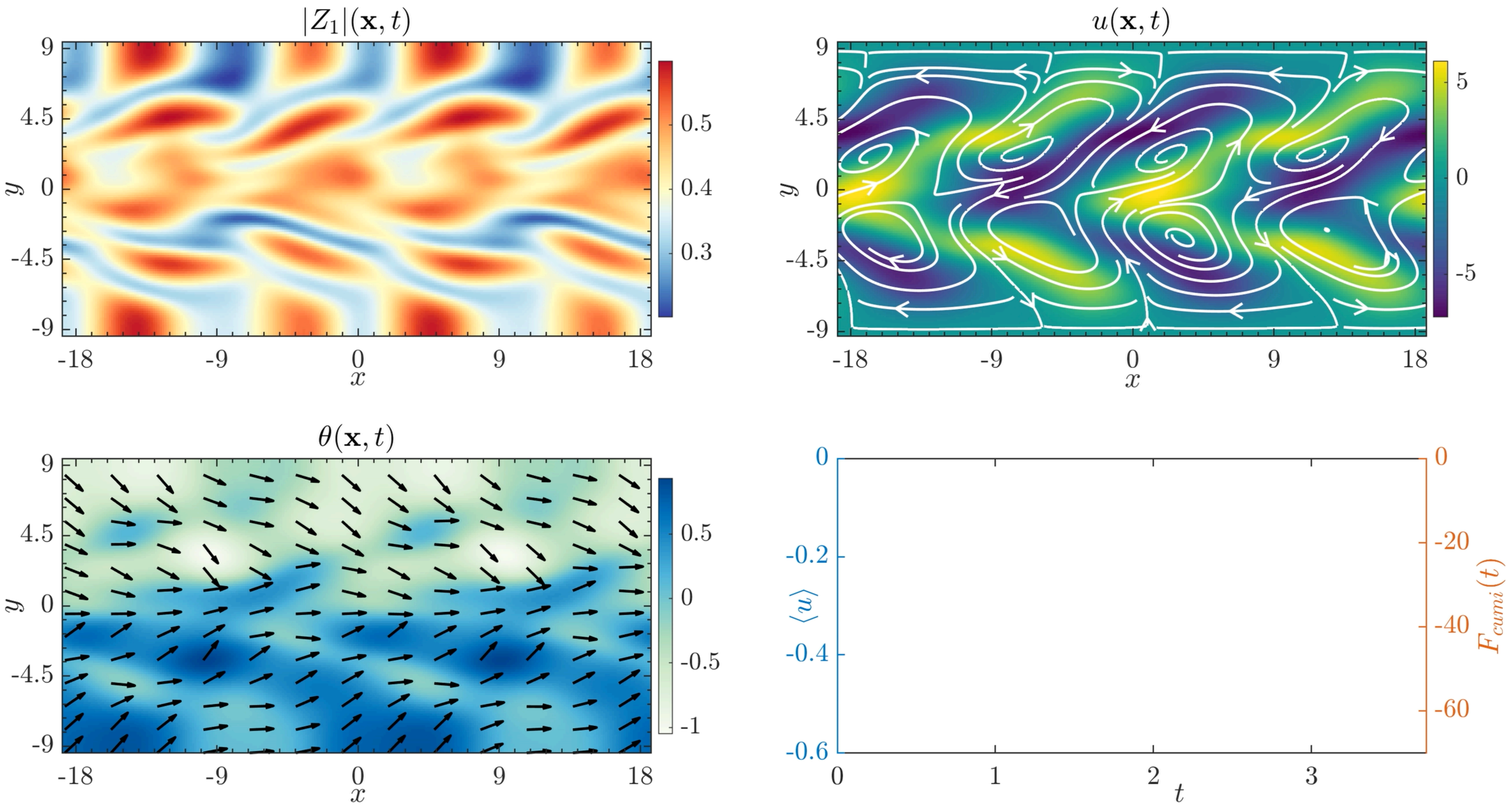


A new class of emergent states: *chimera* and spontaneous flows



Fourier in both directions (*Dedalus* V2)

Active pumps and other properties



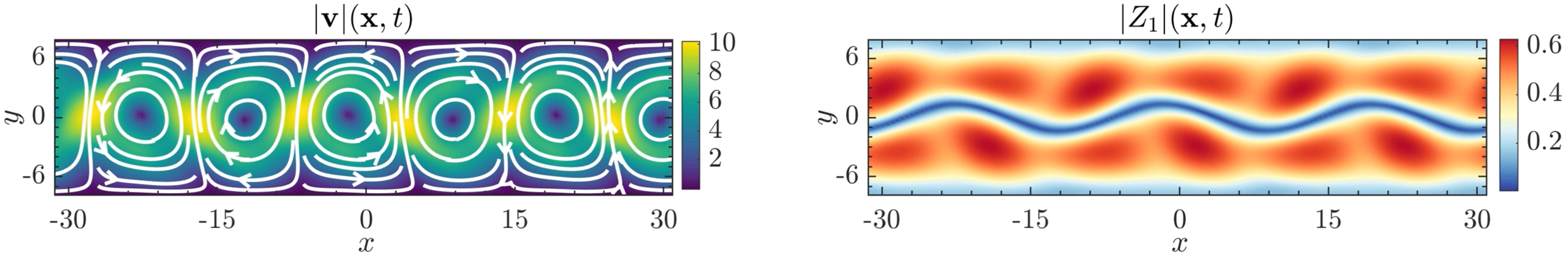
Fourier in horizontal direction + Chebyshev in vertical direction (*Dedalus* V2)

Takeaways for *Dedalus* (this slide was prepared before Keaton's presentation)

1. Simulations in *nice* geometries:
    - Periodic box
    - Channel
    - Disks
    - Sphere

2. Stability analysis of PDEs (not in periodic box)

3. Type in your PDEs with V3!! (Sneak peak on last slide)



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