Ting Fung Lam 2924629375

March 29, 2020

Programming HW#9 (Due: Apr 5, 11:59 PM):
Implement the Algorithm for Divided Differences. Run your own codes to solve Problem 19 on the page 304.

[2]:
```
from IPython.display import display, Image
i = Image(filename='image.jpg')
i
```

[2]:

Algorithm for Divided Differences

An algorithm for computing a divided difference table can be very efficient and is recommended as the best means for producing an interpolating polynomial. Let us change the notation so that our divided difference table has the entries as shown here:

$$\begin{array}{ll|llllll} x_0 & c_{00} & c_{01} & c_{02} & c_{03} & \cdots & c_{0,n-1} & c_{0,n} \\ x_1 & c_{10} & c_{11} & c_{12} & c_{13} & \cdots & c_{1,n-1} & \\ x_2 & c_{20} & c_{21} & c_{22} & c_{23} & \iddots & & \\ \vdots & \vdots & \vdots & \vdots & \iddots & & & \\ \vdots & \vdots & \vdots & \iddots & & & & \\ x_{n-1} & c_{n-1,0} & c_{n-1,1} & & & & & \\ x_n & c_{n0} & & & & & & \end{array}$$

Again, the vertical line separates the data (on the left) from the entries to be computed. It is clear that we have set

$$c_{ij} = f[x_i, x_{i+1}, \ldots, x_{i+j}]$$

An algorithm is obtained from a direct translation of Equation (13), and goes as follows:

for $j = 1, 2, \ldots, n$ **do**
  for $i = 0, 1, \ldots, n - j$ **do**
    $c_{ij} \leftarrow (c_{i+1,j-1} - c_{i,j-1})/(x_{i+j} - x_i)$
  end
end

[c]19. For $n = 5, 10$ and 15, find the Newton interpolating polynomial p_n for the function $f(x) = 1/(1 + x^2)$ on the interval $[-5, 5]$. Use equally spaced nodes. In each case, compute $f(x) - p_n(x)$ for 30 equally spaced points in $[-5, 5]$ in order to see the divergence of p_n from f.

[1]:
```
import numpy as np


def f(x: float):
    """
    function
    :param x: input
    :return: output
    """
    return 1/(1 + x ** 2)


def initial(n, f):
    """
    Initialize x and c
    :param n: number of nodes
    :param f: function
    :return: x and c
    """
    c = np.zeros([n + 1, n + 1])
    x = np.arange(-5, 5.01, 10/n)
    c[:, 0] = f(x)
    return x, c


def divideddiff(x, c):
    """
    Divided difference algorithm
    :param x:
    :param c:
    :return: matrix c finished
    """
    n = np.size(x) - 1
    for j in range(1, n + 1):
        for i in range(0, n - j + 1):
            c[i, j] = (c[i + 1, j - 1] - c[i, j - 1]) / (x[i + j] - x[i])
    return c


def poly(inputx, x, c):
    """
    Newton interpolating polynomial
    :param inputx: input value x
    :param x: x from table
    :param c: c from table
    :return: result
    """
    n = np.size(x)
    p = 0
    result = 0
    for j in range(0, n):
```

```
        temp = c[0, j]
        for i in range(0, j):
            temp = temp * (inputx - x[i])
        result += temp
    return result


def main():
    """
    Testing method
    :return:
    """
    x, c = initial(n=5, f=f)
    c = divideddiff(x, c)
    print("========== n = 5 ==========")
    print("Divided difference table for x and c")
    print(x)
    print(c)
    xtest = np.arange(-5, 5.0001, 10 / 29)
    ytest = np.array(poly(xtest, x, c))
    ftest = np.array(f(xtest))
    print()
    print("f(x) - p_n(x) for 30 points: ")
    print(ftest - ytest)
    print()
    x, c = initial(n=10, f=f)
    c = divideddiff(x, c)
    print("========== n = 10 ==========")
    print("Divided difference table for x and c")
    print(x)
    print(c)
    xtest = np.arange(-5, 5.0001, 10 / 29)
    ytest = np.array(poly(xtest, x, c))
    ftest = np.array(f(xtest))
    print()
    print("f(x) - p_n(x) for 30 points: ")
    print(ftest - ytest)
    print()
    x, c = initial(n=15, f=f)
    c = divideddiff(x, c)
    print("========== n = 15 ==========")
    print("Divided difference table for x and c")
    print(x)
    print(c)
    xtest = np.arange(-5, 5.0001, 10 / 29)
    ytest = np.array(poly(xtest, x, c))
    ftest = np.array(f(xtest))
    print()
    print("f(x) - p_n(x) for 30 points: ")
    print(ftest - ytest)
    print()
```

```python
if __name__ == '__main__':
    main()
```

```
=========== n = 5 ===========
Divided difference table for x and c
[-5. -3. -1.  1.  3.  5.]
[[ 0.03846154  0.03076923  0.04230769 -0.01538462  0.00192308  0.        ]
 [ 0.1         0.2        -0.05        0.          0.00192308  0.        ]
 [ 0.5         0.         -0.05        0.01538462  0.          0.        ]
 [ 0.5        -0.2         0.04230769  0.          0.          0.        ]
 [ 0.1        -0.03076923  0.          0.          0.          0.        ]
 [ 0.03846154  0.          0.          0.          0.          0.        ]]

f(x) - p_n(x) for 30 points:
[ 0.00000000e+00  7.39685376e-02  1.06199317e-01  1.05608530e-01
  8.06477852e-02  3.94068568e-02 -1.02159052e-02 -6.02250427e-02
 -1.02034111e-01 -1.25633737e-01 -1.18319685e-01 -6.34795642e-02
  5.67497987e-02  2.40007148e-01  4.05880238e-01  4.05880238e-01
  2.40007148e-01  5.67497987e-02 -6.34795642e-02 -1.18319685e-01
 -1.25633737e-01 -1.02034111e-01 -6.02250427e-02 -1.02159052e-02
  3.94068568e-02  8.06477852e-02  1.05608530e-01  1.06199317e-01
  7.39685376e-02 -2.18575158e-15]

=========== n = 10 ===========
Divided difference table for x and c
[-5. -4. -3. -2. -1.  0.  1.  2.  3.  4.  5.]
[[ 3.84615385e-02  2.03619910e-02  1.04072398e-02  6.33484163e-03
   4.29864253e-03 -2.03619910e-03 -1.13122172e-03  1.08597285e-03
  -4.29864253e-04  1.13122172e-04 -2.26244344e-05]
 [ 5.88235294e-02  4.11764706e-02  2.94117647e-02  2.35294118e-02
  -5.88235294e-03 -8.82352941e-03  6.47058824e-03 -2.35294118e-03
   5.88235294e-04 -1.13122172e-04  0.00000000e+00]
 [ 1.00000000e-01  1.00000000e-01  1.00000000e-01  4.62592927e-18
  -5.00000000e-02  3.00000000e-02 -1.00000000e-02  2.35294118e-03
  -4.29864253e-04  0.00000000e+00  0.00000000e+00]
 [ 2.00000000e-01  3.00000000e-01  1.00000000e-01 -2.00000000e-01
   1.00000000e-01 -3.00000000e-02  6.47058824e-03 -1.08597285e-03
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.00000000e-01  5.00000000e-01 -5.00000000e-01  2.00000000e-01
  -5.00000000e-02  8.82352941e-03 -1.13122172e-03  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 1.00000000e+00 -5.00000000e-01  1.00000000e-01 -4.62592927e-18
  -5.88235294e-03  2.03619910e-03  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.00000000e-01 -3.00000000e-01  1.00000000e-01 -2.35294118e-02
   4.29864253e-03  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 2.00000000e-01 -1.00000000e-01  2.94117647e-02 -6.33484163e-03
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
```

```
 [ 1.00000000e-01 -4.11764706e-02  1.04072398e-02  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.88235294e-02 -2.03619910e-02  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 3.84615385e-02  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00]]

f(x) - p_n(x) for 30 points:
[ 0.00000000e+00 -1.88965714e+00 -8.72811143e-01  6.29246350e-02
  3.27517531e-01  1.76444042e-01 -3.51208765e-02 -1.18634367e-01
 -6.77668169e-02  2.65797191e-02  7.25337147e-02  4.02379515e-02
 -2.46541462e-02 -4.43592062e-02 -9.00027480e-03 -9.00027480e-03
 -4.43592062e-02 -2.46541462e-02  4.02379515e-02  7.25337147e-02
  2.65797191e-02 -6.77668169e-02 -1.18634367e-01 -3.51208765e-02
  1.76444042e-01  3.27517531e-01  6.29246350e-02 -8.72811143e-01
 -1.88965714e+00  2.18526586e-13]

========== n = 15 ==========
Divided difference table for x and c
[-5.         -4.33333333 -3.66666667 -3.         -2.33333333 -1.66666667
 -1.         -0.33333333  0.33333333  1.          1.66666667  2.33333333
  3.          3.66666667  4.33333333  5.        ]
[[ 3.84615385e-02  1.81503889e-02  7.38980121e-03  3.11149525e-03
   1.42833726e-03  6.88726002e-04  2.04132707e-04 -3.74894153e-04
  -1.83719436e-04  3.09926701e-04 -1.70140173e-04  5.75121713e-05
  -1.36591407e-05  2.32260692e-06 -2.48850741e-07  2.14934610e-21]
 [ 5.05617978e-02  2.80034572e-02  1.36127917e-02  6.92039460e-03
   3.72409060e-03  1.50525683e-03 -1.54537334e-03 -1.35473115e-03
   1.67584077e-03 -8.24341121e-04  2.51615749e-04 -5.17609541e-05
   6.47011927e-06 -5.52200325e-20 -2.48850741e-07  0.00000000e+00]
 [ 6.92307692e-02  4.61538462e-02  2.74535809e-02  1.68513029e-02
   8.74161336e-03 -4.67623654e-03 -7.86745202e-03  7.58308628e-03
  -3.27020596e-03  8.53097207e-04 -1.27964581e-04  1.06448941e-18
   6.47011927e-06 -2.32260692e-06  0.00000000e+00  0.00000000e+00]
 [ 1.00000000e-01  8.27586207e-02  6.11561866e-02  4.01622718e-02
  -6.84584178e-03 -3.61460446e-02  2.75202840e-02 -9.85801217e-03
   1.84837728e-03 -1.47812896e-17 -1.27964581e-04  5.17609541e-05
  -1.36591407e-05  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 1.55172414e-01  1.64300203e-01  1.41480730e-01  2.19066937e-02
  -1.27332657e-01  7.39350913e-02 -1.84837728e-02  1.40512602e-16
   1.84837728e-03 -8.53097207e-04  2.51615749e-04 -5.75121713e-05
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 2.64705882e-01  3.52941176e-01  1.85294118e-01 -3.17647059e-01
   1.19117647e-01 -8.32667268e-16 -1.84837728e-02  9.85801217e-03
  -3.27020596e-03  8.24341121e-04 -1.70140173e-04  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.00000000e-01  6.00000000e-01 -4.50000000e-01  2.66453526e-15
   1.19117647e-01 -7.39350913e-02  2.75202840e-02 -7.58308628e-03
   1.67584077e-03 -3.09926701e-04  0.00000000e+00  0.00000000e+00
```

```
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 9.00000000e-01 -3.49720253e-15 -4.50000000e-01  3.17647059e-01
  -1.27332657e-01  3.61460446e-02 -7.86745202e-03  1.35473115e-03
  -1.83719436e-04  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 9.00000000e-01 -6.00000000e-01  1.85294118e-01 -2.19066937e-02
  -6.84584178e-03  4.67623654e-03 -1.54537334e-03  3.74894153e-04
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.00000000e-01 -3.52941176e-01  1.41480730e-01 -4.01622718e-02
   8.74161336e-03 -1.50525683e-03  2.04132707e-04  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 2.64705882e-01 -1.64300203e-01  6.11561866e-02 -1.68513029e-02
   3.72409060e-03 -6.88726002e-04  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 1.55172414e-01 -8.27586207e-02  2.74535809e-02 -6.92039460e-03
   1.42833726e-03  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 1.00000000e-01 -4.61538462e-02  1.36127917e-02 -3.11149525e-03
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 6.92307692e-02 -2.80034572e-02  7.38980121e-03  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 5.05617978e-02 -1.81503889e-02  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 3.84615385e-02  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]]

f(x) - p_n(x) for 30 points:
[ 0.00000000e+00 -1.50545282e+00  5.37417425e-02  1.97756616e-01
 -2.04017887e-02 -4.97144916e-02  1.01377987e-02  2.03676843e-02
 -7.09782762e-03 -1.28209736e-02  7.13672759e-03  1.20913404e-02
 -1.01697942e-02 -1.53989278e-02  1.59963442e-02  1.59963442e-02
 -1.53989278e-02 -1.01697942e-02  1.20913404e-02  7.13672759e-03
 -1.28209736e-02 -7.09782762e-03  2.03676843e-02  1.01377987e-02
 -4.97144916e-02 -2.04017887e-02  1.97756616e-01  5.37417425e-02
 -1.50545282e+00  6.64658606e-12]
```