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LSTAT2170 - Time series

Final Project

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Contents

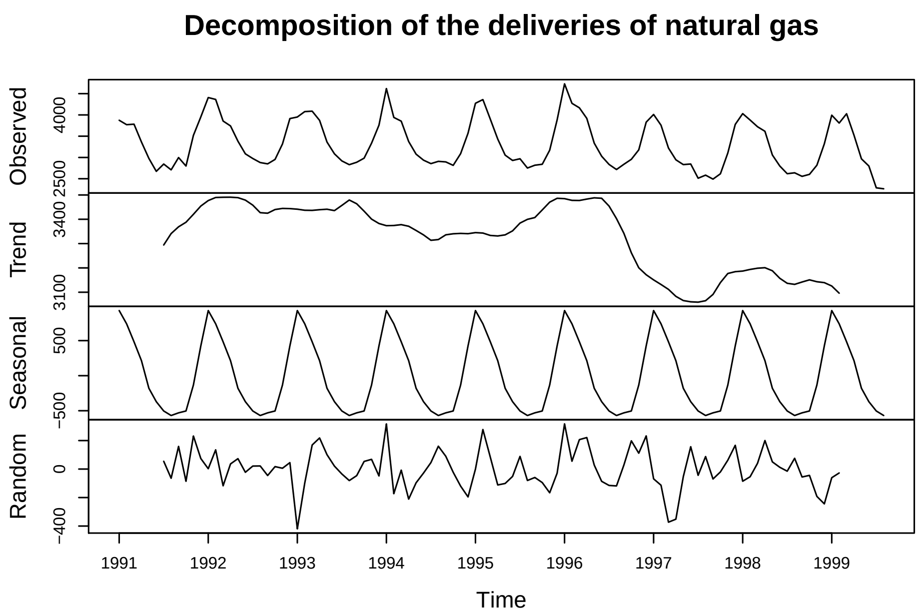
1 Introduction

In this project we will focus on the analysis of real data. These data are the U.S. Natural Gas State Data and concern the monthly quantity of natural gas delivered to residential and commercial consumers (excluding vehicle fuel) in Florida. Aggregated on a monthly basis, the data are presented in millions of cubic feet (MMcf[1]) and cover the period from January 1991 to August 1999.

We will proceed to the beginning of this report with a first visual discovery of the dataset. Then, we will eventually apply a series of transformations to stabilize the variance, remove possible trends and seasonality. Next, we will analyze the autocorrelation and partial autocorrelation functions in order to have a first intuition on the type of model to fit. Following this, we will establish which model would be the most appropriate for our data and verify the insight of our choice by several methods such as a significance test of the coefficients, an analysis of the residuals (by a Portmanteau test) or the evaluation of the predictive capacity "on sample". The final objective is to be able to give a prediction interval for future values over roughly one year.

2 Data discovery

Considering our data to be an additive model $Y_t = T_t + S_t + \epsilon_t$, we can have a first insight by decomposing the series into trend (T), seasonal (S) and random (ϵ) pieces. A plot containing the undecomposed data is available in the appendix. See A.1.

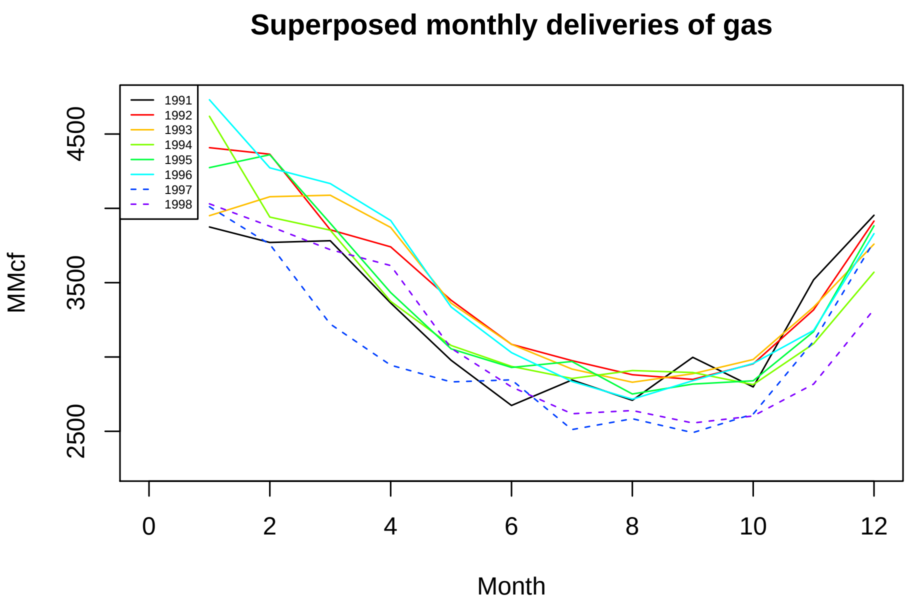


The first obvious observation we can make by looking at just the first line is that we are dealing with seasonal data. Indeed, we can see in the third row an almost perfect seasonality with maximum values at the beginning of each year and minimum values towards the middle. These results are hardly surprising given the nature of the data. Indeed, it seems normal that a greater quantity of gas is used during winter and that gas consumption decreases during summer.

[1] "Mcf" means 1,000 cubic feet of natural gas; "MMcf" means 1,000 Mcf.

Looking at the second line of the graph, we see that the data do not really seem to vary except in 1996-1997 when a decrease is noticeable. A closer look at the first line shows this phenomenon.

Another interesting way to present the data is to display the evolution of the deliveries by stacking the years line by line. The outcome is fortunately the same. We notice a strong seasonality (since the lines all follow the same pattern) as well as a small decrease that seems to start from 1997 (colored dashed lines). Moreover, the lines seems to remain relatively close to each other and there is no drastic change in variance over time.



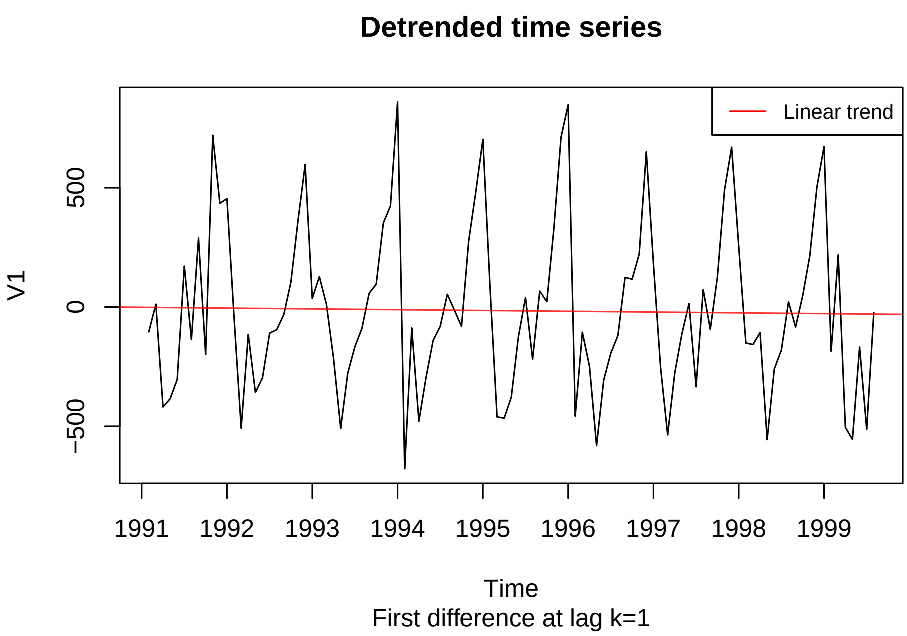
3 Box-Jenkins

The previous statements make us decide not to apply a logarithmic transformation on the data. On the other hand, as we aim to examine the correlation structure of the residuals, we need to achieve stationarity. Meaning that it is necessary to deseasonalize and delinearize the data in order to go further with the visual analysis.

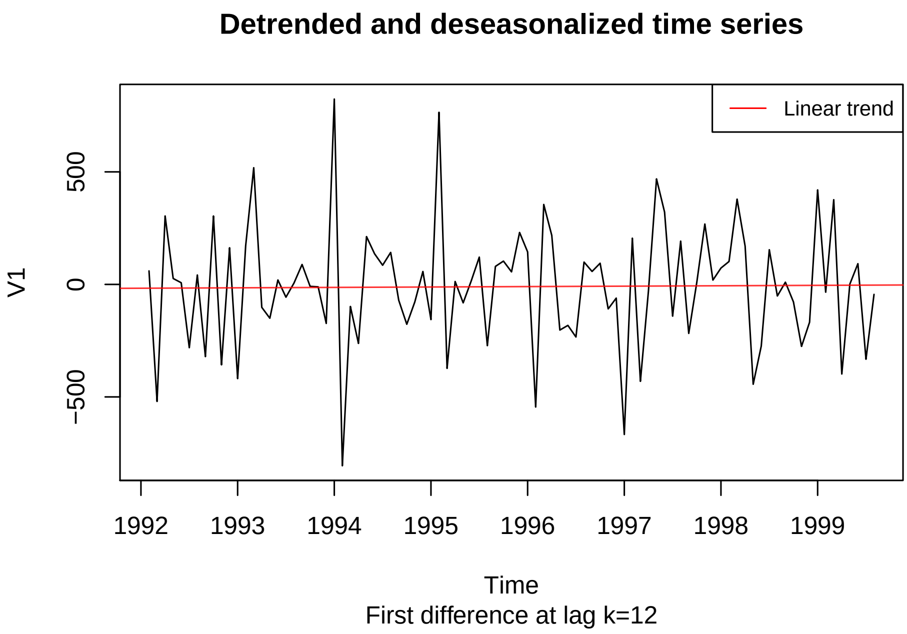
3.1 Deseasonalize and detrend

For this purpose, we will use the method of (iterated) differences. Considering that Y_t is the time series at period t, then the first difference at lag k is $Y_t - Y_{t-k}$. As a first step, we apply the latter on the data at lag 1 to remove the linear trend. Next, we repeat this process at a lag equal to the periodicity. That is, we use the value 12 since the data are collected monthly and that the seasonality is annual.

Note that the resulting time series is therefore shifted by k periods.



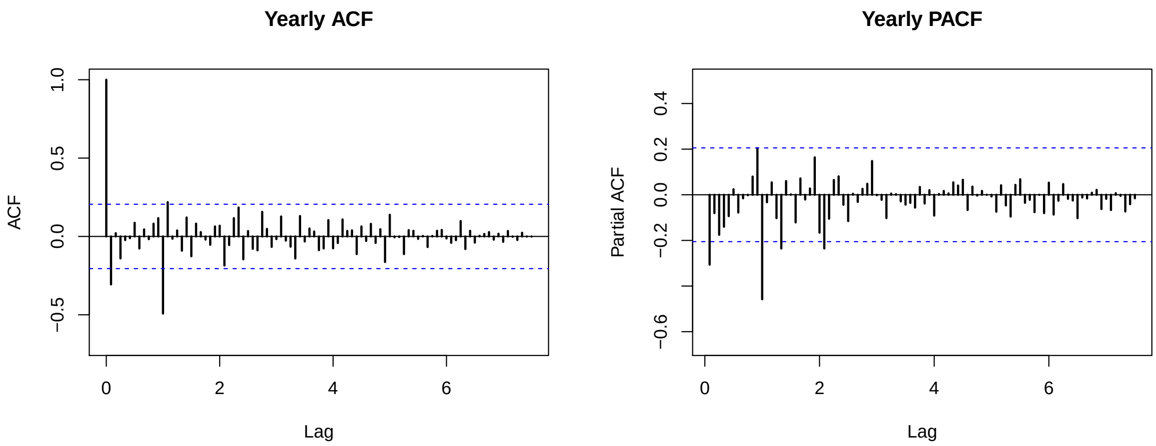
In the first figure above, we observe that the linear trend is well removed since the data are now centered around zero, but we still have a visible periodicity. This particularity seems to be corrected on the second graph below as we now have a time series that appears to be random.



3.2 ACF and PACF

Now that our data is differenced and assumed to be stationary, we can examine the correlation structure and have an intuition of the values to feed into the model. We will use the auto-correlation and partial auto-correlation functions for this purpose.

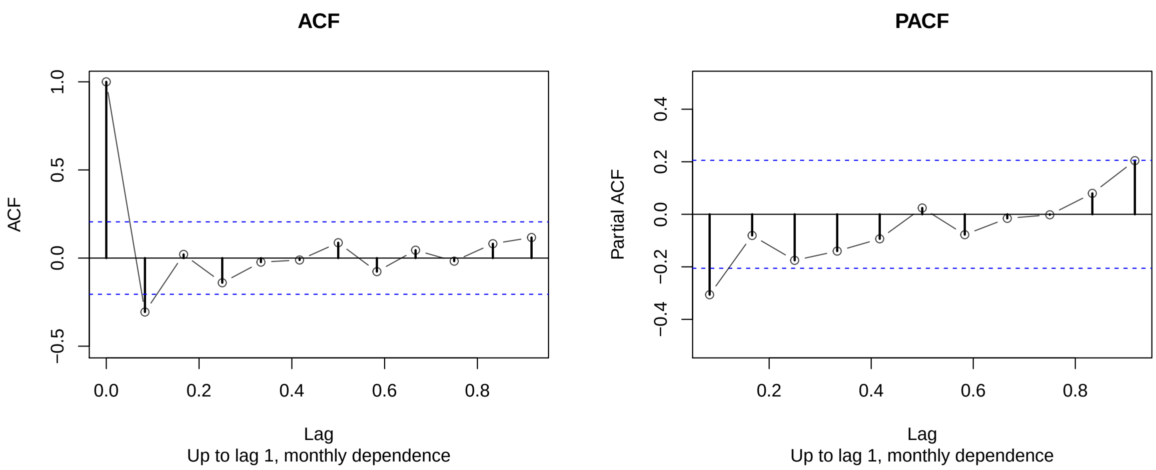
To recall, the former describes how well the present value is correlated with its past values while the latter does the same but removing the linear dependence at intermediate lags (so basically with the residuals).



In the figures above, we notice that both ACF and PACF tend towards zero. The values fall below the confidence interval after lag 1 for the ACF and lag 2 (or 1 according to the degree of tolerance) for the PACF. The simultaneous look on the ACF and PACF might suggest an ARMA(0,1) or ARMA(1,1) on a yearly basis.

Although kind of predictable, the presence of significant correlations at lag 1 is nevertheless intriguing. The transformations performed have only removed a deterministic element of seasonality and a stochastic component could indeed remain. This consolidates the idea of using an S-ARIMA model.

Therefore, let analyze the ACF and PACF on a monthly basis, i.e. between lag [0 and 1[. Both show a significant value at first lag. Note that in the case of ACF, the value at time 0 is always 1. On a monthly basis, we might thus move with an ARMA(1,1).



4 Model selection

In consideration of the aforementioned elements, we decided to proceed with an S-ARIMA model. Before continuing, we should yet provide a small definition. Basically, an S-ARIMA is an Autoregressive Integrated Moving Average that supports the direct modeling of a Seasonal component (S). Meaning we have 4 more elements (P, D, Q) and s, where P is the seasonal AR order, D the seasonal difference order, Q the seasonal MA order and s the seasonal period.

Putting them together, we obtain a model called SARIMA(p, d, q)$\times$(P, D, Q)$_s$ if the time series X_t can be transformed into a stationary series without trend or seasonality.

$$Y_t = \bigtriangledown^d \bigtriangledown_s^D X_t = (1-B)^d \, (1-B^s)^D \, X_t$$

And where the latter can be modelised in the form of a stationary ARMA process

$$\phi(B) \, \Phi(B^s) \, Y_t = \theta(B) \, \Theta(B^s) \, \epsilon_t$$

where B is the backshift operator and where $\phi(z)$, $\Phi(z)$, $\theta(z)$ and $\Theta(z)$ are the generating polynomial of respectively, an AR(p), AR(P), MA(q) and MA(Q) process.

4.1 Selection

Now it's time to figure out which model would be the best fit to the data. To do this, we will iterate over all possible S-ARIMA models with parameters value up to 2. Nevertheless, following the visual analysis of the previous section, we expect to get models going only up to 1.

In order to compare the effectiveness of the different models, we will primarily use the Akaike information criterion (AIC). However, since the AIC tends to overestimate the number of parameters needed, we will also use the Bayesian information criterion (BIC) as a secondary information.

The results below describe the top 3 models found, from best to worst. First, we notice via the AICR[2] that we get models with very close results. The winning model has an AIC of 1229, a BIC of 1241 and is composed of 4 parameters. The second one has a higher AIC of only 0.064 and a lower BIC of 2.447 (relative to the first one) but has only 3 parameters. The last model is somewhat less interesting since the increase in AIC is more important and it has as much parameters as the best one.

```
TOP 3 AIC || MODEL : (p,d,q)x(P,D,Q)[s]

(1,1,1)x(1,1,1)[12] || AIC: 1228.804 | AICR: 0.000 | BIC: 1241.359 || P: 4
(1,1,1)x(0,1,1)[12] || AIC: 1228.868 | AICR: 0.064 | BIC: 1238.912 || P: 3
(2,1,1)x(0,1,1)[12] || AIC: 1230.100 | AICR: 1.296 | BIC: 1242.654 || P: 4
```

As an additional information, the log-likelihood of the models are respectively :

-609.4021406, -610.4341948 and -610.0498036.

[2]Akaike information criterion relative to the best model.

5 Models comparison and validation

Now that we have a selection of models, we need to check if they are valid and correctly fitted. To do so, we will first perform an univariate and two-sided significance test (based on normal approximations) of the coefficients. Then, we will perform an evaluation of the predictive ability "on sample" and finally, after the choice of the final model, we will do an analysis of the residuals by a Portmanteau (Ljung-box) test.

5.1 Coefficients

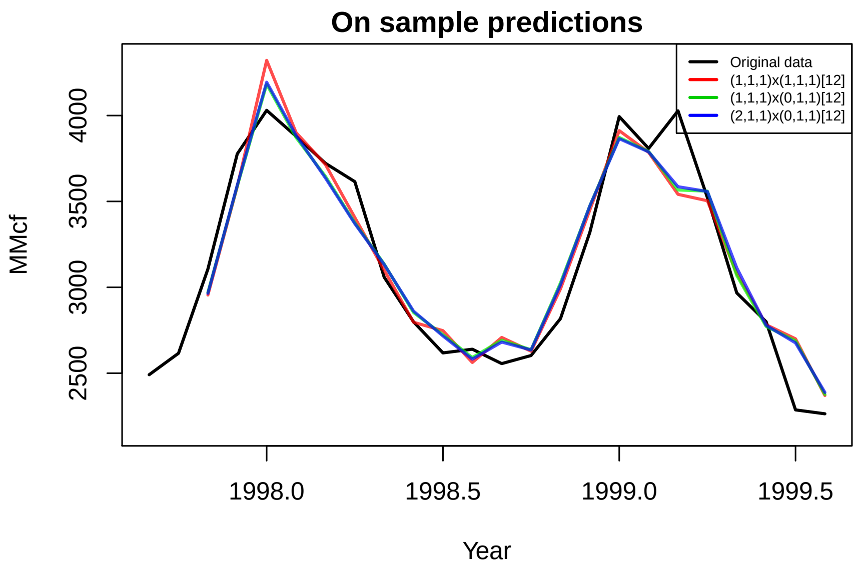
In the same order as displayed in the previous section:

```
           ar1             ma1             sar1             sma1
"(**) 0.0146" "(***) 0.0000"   "(x) 0.1531"   "(**) 0.0018"

           ar1             ma1             sma1
"(**) 0.0333" "(***) 0.0000"  "(***) 2e-04"

           ar1             ar2              ma1             sma1
"(***) 6e-04"   "(x) 0.3609" "(***) 0.0000"  "(***) 1e-04"
```

Thanks to the indicators in parentheses, it is easy to see which coefficients are significant and to what degree. First, we notice that for the model with the lowest AIC (1), we cannot reject the hypothesis that the coefficient sar1 (Φ_1) is statistically different from zero, even with an $\alpha = 0.10$. The same is observed for the third model with ar2 (ϕ_2).

5.2 Predictive power

Let us now continue with a comparison of the predictive power of each model. To do this, we will build models based on the first 80% of data, predict a step ahead and repeat the operation by integrating an additional data into the model until we reach the end of the time series. We thus obtain predictions on the last 20% of data, which we can compare with the real data and also calculate the MSE of the models.



We observe that globally, although not perfect, the models manage to follow the trend, but that models 2 (green) and 3 (blue) seem to perform slightly better than the first one (red), and in fact, their MSE are lower with respectively 30142 and 29607, what corresponds to a decrease of more than 8% compared to 32830.

Model 1	**Model 2**	**Model 3**
32 830	30 142	29 607

Those results lead us to choose the second model. This one being the most parsimonious, it only loses 0.064 points of AIC, has only 1.8% more MSE than the third model and in view of the significance test seems to be the most relevant. The latter can mathematically be written as :

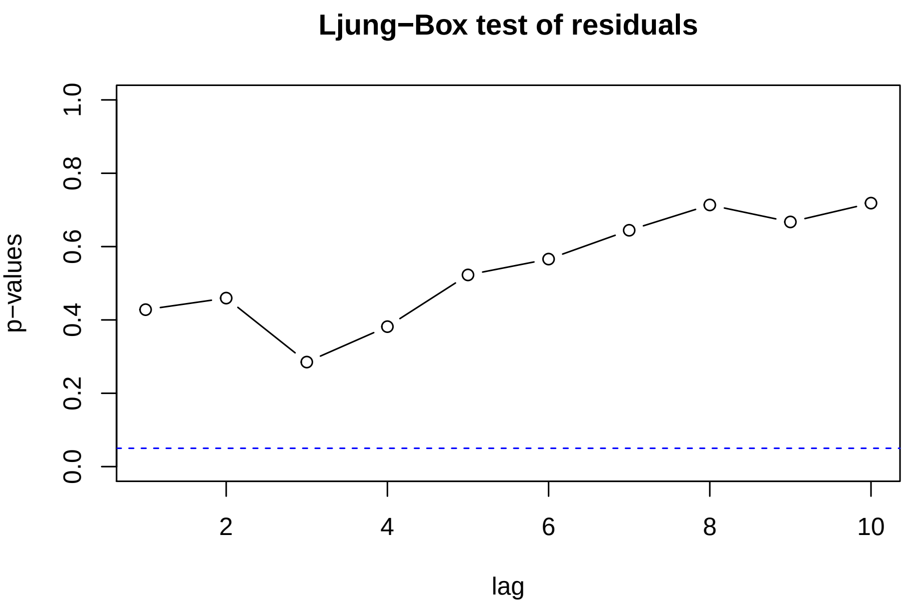
$$\text{S-ARIMA}(1,1,1)\times(0,1,1)_{12}$$
$$(1-\phi_1 B)(1-B)^1(1-B^{12})^1\ Y_t = (1+\theta_1 B)(1+\Theta_1 B^{12})\ \epsilon_t$$

with coefficients, $\phi_1 = 0.453$, $\theta_1 = -0.832$, and $\Theta_1 = -1.000$.

5.3 Ljung-Box

As a last test, we will check the autocorrelation of the residuals of the model via a portmanteau test, more specifically Ljung-box. We test the null hypothesis that the residuals are not different from white noise, up to lag $K = \sqrt{T}$ where T is the length of our data.

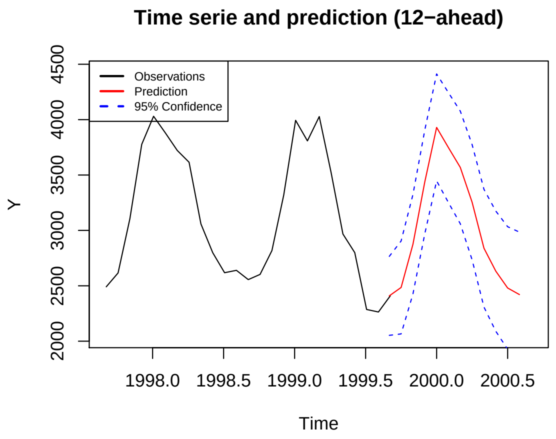
$$H_0 : \rho_\epsilon(1) = ... = \rho_\epsilon(K) = 0$$
$$H_1 : \rho_\epsilon(1) = ... = \rho_\epsilon(K) \neq 0$$

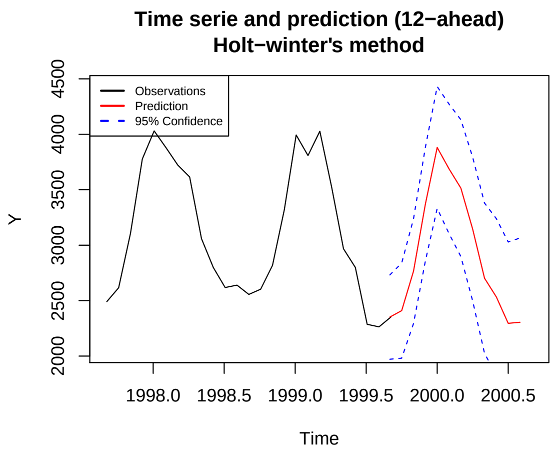


As the p-values are all well above the threshold, we cannot reject the null hypothesis. This is convenient for us because it means that there is no correlation in the residuals.

A complete figure including the ACF of the residuals is available in appendix.

6 Predictions

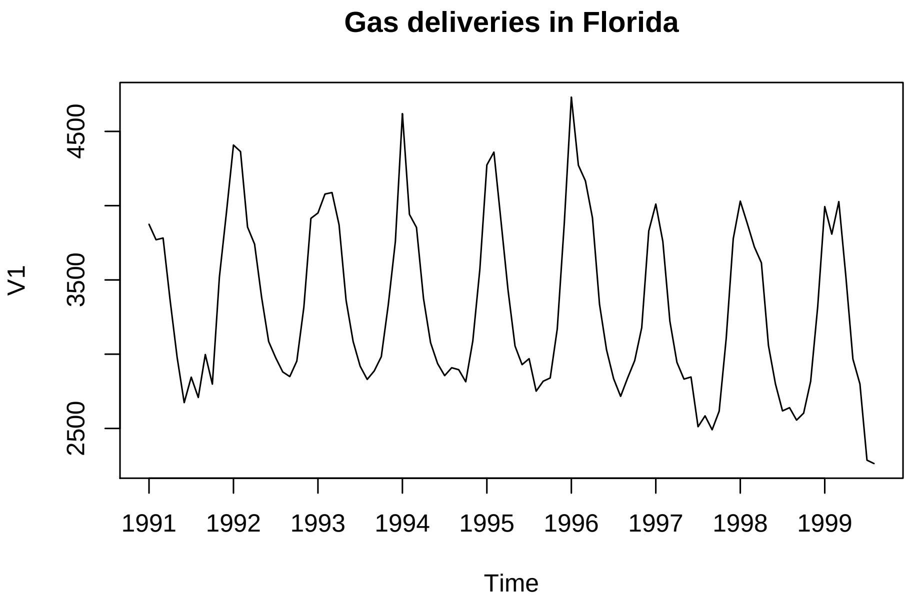


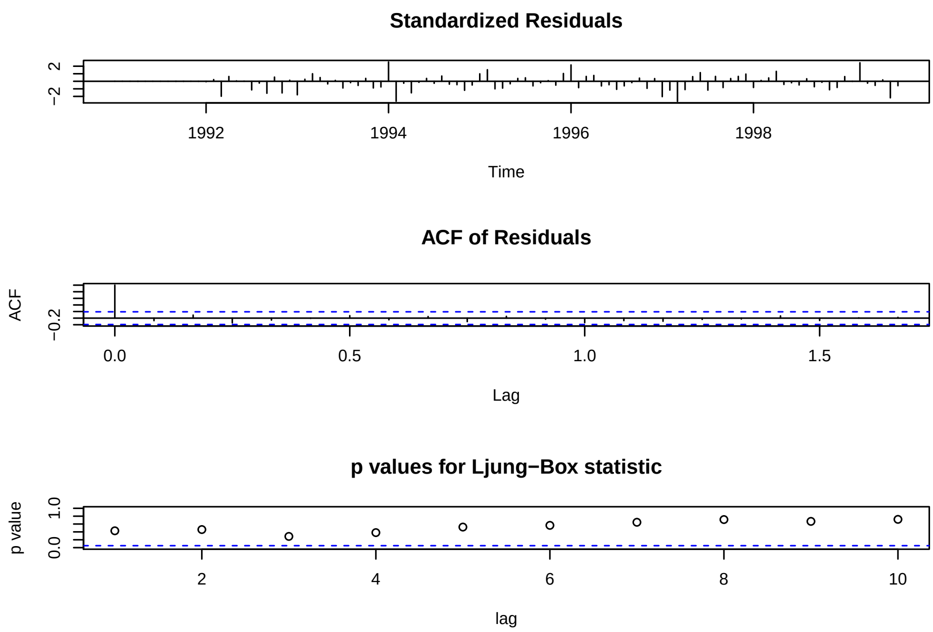


7 Conclusion

A Appendix

A.1 Figures





A.2 Code

Note

For reproducibility purposes, the complete project containing the source code and the results is available on https://github.com/lamylio/LSTAT2170-Project.

The below section is automatically generated and tidied.

```r
#' Import facility functions in a separate attached environment
#' To keep our global clean

facilities <- new.env()
source("./resources/scripts/fonctionsSeriesChrono.R", local = facilities)

# Please check the github repository
source("./resources/scripts/sarima_model_selection.R", local = facilities)
source("./resources/scripts/ts_custom_plots.R", local = facilities)
source("./resources/scripts/ts_significance_test.R", local = facilities)
source("./resources/scripts/ts_on_sample_prediction.R", local = facilities)

attach(facilities, name = "facilities")

# Import the gas dataset
gas <- read.table("./resources/data/gasflorida.txt", header = F)
gas <- ts(gas, start = 1991, frequency = 12)

# Define some useful variables
gas.start <- tsp(gas)[1]
gas.end <- tsp(gas)[2]
gas.freq <- tsp(gas)[3]
gas.t <- seq(gas.start, to = gas.end, length = length(gas))
gas.xaxp <- c(floor(gas.start), floor(gas.end), floor(gas.end - gas.start))

# Plot the decomposition of the data (see ts_custom_plot.R)
plot.decompose.ts(gas, main = "Decomposition of the deliveries of natural gas",
    cex.lab = 0.9, cex.axis = 0.95, xaxp = gas.xaxp)

# Plot the superposed view of the data (see ts_custom_plot.R)
plot.superposed.ts(gas, title = "Superposed monthly deliveries of gas",
    xlab = "Month", ylab = "MMcf", dashed_thick_from = c(6, 8), xlim = c(0,
        12))

# Remove global trend
gas.1 <- diff(gas, lag = 1, differences = 1)
# Remove the seasonality using lag 12 as we have monthly data
gas.2 <- diff(gas.1, lag = 12, differences = 1)

# Removed trend
plot(gas.1, main = "Detrended time series", sub = "First difference at lag k=1",
    xaxp = gas.xaxp)
abline(reg = lm(gas.1 ~ tail(gas.t, -1)), col = rgb(1, 0, 0, 0.8),
    lty = 1)
legend("topright", legend = "Linear trend", col = 2, cex = 0.8, lty = 1)

# Removed trend and seasonality
plot(gas.2, main = "Detrended and deseasonalized time series", xaxp = gas.xaxp,
    sub = "First difference at lag k=12")
abline(reg = lm(gas.2 ~ tail(gas.t, -12 - 1)), col = rgb(1, 0, 0,
    0.8), lty = 1)
```

```r
legend("topright", legend = "Linear trend", col = 2, cex = 0.8, lty = 1)

# Plot the ACF and PACF, yearly basis
plot.acf.pacf(gas.2, lag.max = length(gas.2), simplify = F, linked_by_line = F,
    titles = c("Yearly ACF", "Yearly PACF"))

# Plot the ACF and PACF, monthly basis
plot.acf.pacf(gas.2, lag.max = gas.freq - 1, simplify = F, linked_by_line = T,
    titles = c("ACF", "PACF"), sub = "Up to lag 1, monthly dependence")

# Model comparison via AIC. (see sarima_model_selection.R)
model.1 = sarima_model_selection(gas, max.pq = c(2, 2), max.PQ = c(1,
    1), d = 1, D = 1, top = 3, return.best = T)

# Second best model (best bic)
model.2 = arima(gas, order = c(1, 1, 1), seasonal = list(order = c(0,
    1, 1), period = gas.freq))
model.3 = arima(gas, order = c(2, 1, 1), seasonal = list(order = c(0,
    1, 1), period = gas.freq))

# Test the coefficients of each model (see ts_significance_test.R)
significance.test(model.1, T)
significance.test(model.2, T)
significance.test(model.3, T)

# MSE and predictions for the last 20% (see
# ts_on_sample_prediction.R)
model.1.osp = on.sample.prediction(gas, order = c(1, 1, 1), seasonal = list(order = c(1,
    1, 1), period = gas.freq))
model.2.osp = on.sample.prediction(gas, order = c(1, 1, 1), seasonal = list(order = c(0,
    1, 1), period = gas.freq))
model.3.osp = on.sample.prediction(gas, order = c(2, 1, 1), seasonal = list(order = c(0,
    1, 1), period = gas.freq))


# Adapt the predictions to the plot of the time series
models.osp.ts = function(values) ts(values, end = gas.end, frequency = gas.freq)

par(mar = c(4, 4, 1.5, 4))
# Plot the predictions
plot(models.osp.ts(tail(gas, 24)), lwd = 2, main = "On sample predictions",
    ylab = "MMcf", xlab = "Year", ylim = c(min(tail(gas, 24)) - 100,
        max(tail(gas, 24)) + 300))
lines(models.osp.ts(model.1.osp$pred), col = rgb(1, 0, 0, 0.7), lwd = 2)
lines(models.osp.ts(model.2.osp$pred), col = rgb(0, 1, 0, 0.7), lwd = 2)
lines(models.osp.ts(model.3.osp$pred), col = rgb(0, 0, 1, 0.7), lwd = 2)
legend("topright", legend = c("Original data", "(1,1,1)x(1,1,1)[12]",
    "(1,1,1)x(0,1,1)[12]", "(2,1,1)x(0,1,1)[12]"), col = 1:4, lty = 1,
    lwd = 2, cex = 0.6)

plot.ljungbox(model.2, floor(sqrt(length(gas))))
```

```
plot.n.ahead.predictions(gas, model.2, n = gas.freq, before = 2 *
    gas.freq)
plot.n.ahead.predictions(gas, model.2, n = gas.freq, before = 2 *
    gas.freq, holtwinters = T)

# ======== #
#'Appendix #

# Plot the complete dataset non decomposed
plot(gas, main = "Gas deliveries in Florida", xaxp = gas.xaxp)

# Ljung-Box test
tsdiag(model.2, gof.lag = floor(sqrt(length(gas))))
```