LSTAT 2180

Resampling methods with applications

Projet 2020-2021

1 Project description

The problem below has been randomly allocated to you. We are interested in a parameter θ. You have to:

1. propose an estimator for θ and
2. for a particular sample of size $n = 30$:
   - evaluate using a nonparametric bootstrap method the bias and the variance of the proposed estimator
   - give a confidence interval for θ using a traditional asymptotic method (if it is available for the allocated theme), the basic bootstrap method, the percentile bootstrap method and the $t-$ bootstrap method.
   - (*optional*) use an iterated bootstrap procedure: for example to allow for a $t-$ bootstrap method.
3. For the same sample of size $n = 30$, perform a hypothesis test based on a bootstrap procedure for the parameter of interest. Choose (i) a null hypothesis that is satisfied for the model and (ii) a different null hypothesis that is not satisfied. Compare the power of different bootstrap procedures that were seen in class.
4. Study by Monte-Carlo the performance of the methods used to obtain confidence intervals. Using samples of size n drawn B times, study the performance for the methods available for your theme, used above. Choose different sample sizes and use the coverage probability and the width of the interval as measures of evaluation.

Your report has to be clear and concise and has to contain a presentation of the problem and the used procedures (explain for example why a certain method is not available for your theme), as well as a discussion on the obtained results.

The report has to be at most 8 pages long. You have to include the programs in an Annex (which does not count for the limit of 8 pages). Attach the code as well in a file on email so that I can reproduce the results, if needed.

Limit date to hand in the project : 7 days before the exam at 13:30pm (to be deposited preferably in my mailbox). During the exam, you present the project for 15-20 minutes and answer questions about the materials seen throughout the course.

 Projects arriving after the deadline are not taken into account!

2 Problem

1. Let $X_1, \ldots, X_n$ be an i.i.d. sample $X_i \sim Geometric(p)$, i.e. $P(X_i = k) = p(1-p)^k$, $k = 0, 1, \cdots$. We want to estimate $\theta = P(X \geq 3)$. For the simulations take $p = 0.5$.