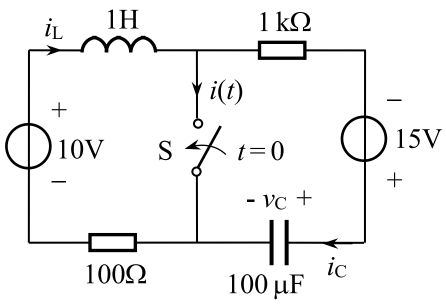
Quiz 2

Student Number ______________________ Name ________________________

1. Having been open for a long time, the switch is closed at t =0. Find $i_L(t)$ and $v_C(t)$, and $i(t)$



$v_C(0_+) = v_C(0_-) = 10 + 15 = 25$ V

$i_L(0_+) = i_L(0_-) = 0$

$v_C(\infty) = 15$ V

$i_L(\infty) = 10 / 100 = 0.1$ A

$\tau_1 = L / R = 1 / 100 = 0.01$ s

$\tau_2 = RC = 10^3 \times 10^{-4} = 0.1$ s

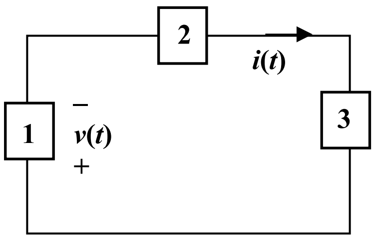
$i_L(t) = 0.1\,(1 - e^{-t/\tau_1})$ A $= 0.1 - 0.1\,e^{-100t}$ A $t \geq 0$

$v_C(t) = 15 + (25 - 15)\,e^{-t/\tau_2} = 15 + 10\,e^{-10t}$ V $t \geq 0$

$$i_C(t) = C\frac{dv_C}{dt} = 10^{-4} \times (-100e^{-10t}) = -0.01e^{-10t}\ \text{A}$$

$i(t) = i_L(t) - i_C(t) = 0.1 - 0.1\,e^{-100t} + 0.01\,e^{-10t}$ A $t > 0$

2. The circuit consists of the three elements R, L and C. The total energy stored in the circuit at t=0 is 5.5J. The current $i(t) = (-e^{-t} + 4e^{-2t})\text{A}, t \geq 0$, $v(t) = (2e^{-t} - 4e^{-2t})\text{V}, t \geq 0$. Determine which one is R, L and C, respectively, and find the values of R, L and C.



$$\frac{dv(t)}{dt} = -2e^{-t} + 8e^{-2t} = 2i(t)$$

Element 1 is the capacitor, and C=0.5F

$$5.5 = \frac{1}{2}L[i(0)]^2 + \frac{1}{2}C[v(0)]^2$$
$$= \frac{1}{2}L(-1+4)^2 + \frac{1}{2} \times \frac{1}{2} \times (2-4)^2$$
$$= \frac{9}{2}L + 1$$
$$\therefore\ L = 1\text{H}$$

$$v(0) + v_R(0) + v_L(0) = 0 \Rightarrow -2 + 3R + \left(e^{-t} - 8e^{-2t}\right)\Big|_{t=0} = 0$$

$$\therefore\ R = 3\Omega$$

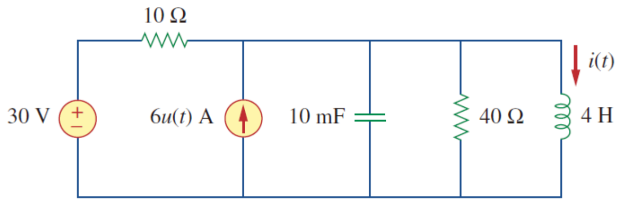
Element 2 and 3 are R and L, or vice versa.

or

$$s_1 = -1,\ s_2 = -2 \Rightarrow s^2 + 3s + 2 = 0$$

$$LCs^2 + RCs + 1 = 0 \Rightarrow \begin{cases} 2LC = 1 \Rightarrow L = 1\text{H} \\ 2RC = 3 \Rightarrow R = 3\Omega \end{cases}$$

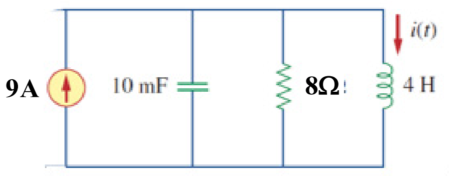
3. For the circuit as below, find $i(t)$ for $t>0$.



$$v_C(0^-)=v_C(0^+)=0,\ i(0^-)=i(0^+)=\frac{30}{10}=3\text{A},\ \left.\frac{di}{dt}\right|_{t=0^+}=\frac{v_C(0^+)}{L}=0$$

$$i_{SS}(t)=9\text{A}$$

For $t>0$, the circuit is equivalent to



R_{eq}=10 Ω ||40 Ω =8Ω

$$I_S=\frac{30}{10}+6=9\text{A}$$

$$LC\frac{d^2i_L}{dt^2}+\frac{L}{R}\frac{di_L}{dt}+i_L=I_S$$

$$0.04\frac{d^2i_L}{dt^2}+0.5\frac{di_L}{dt}+i_L=9$$

$$2s^2+25s+50=0$$

$$s_1=-10,\ \ s_2=-2.5$$

$$i_L(t)=9+K_1e^{-10t}+K_2e^{-2.5t}\qquad \text{overdamped}$$

$$\begin{cases} i_L(0)=3\text{A}\Rightarrow 9+K_1+K_2=3 \\ \left.\frac{di_L}{dt}\right|_{t=0}=0\Rightarrow -10K_1-2.5K_2=0 \end{cases}\Rightarrow\begin{cases} K_1=2 \\ K_2=-8 \end{cases}$$

$$\therefore i_L(t)=9+2e^{-10t}-8e^{-2.5t}\text{A},\ \ t>0$$