Title

Computer Science 604

Advanced Algorithms

Lecture 3a: Subset Sum

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SubSet Sum

Recall that the Subset Sum problem is stated as follows.

Given a set $S = \{s_1, \ldots, s_n\} \subseteq \mathbb{Z}^+$ and an integer $t \in \mathbb{Z}^+$, determine whether there exists a subset $S' \subseteq S$ such that

$$\sum_{s \in S'} s = t.$$

This problem can be solved in pseudo-polynomial time via dynamic programming.

Cont'd

If we assume an ordering $(s_1, s_2, \ldots, s_n)$ on the the elements of S (any will do), then we can define the following sub problem that will help define the dynamic programming solution.

Given $S = \{s_1, \ldots, s_n\}$, t, $1 \leq i \leq n$, and $0 \leq t' \leq t$, define a boolean array

$$C[i, t']$$

to be true whenever there exists a subset $S' \subseteq \{s_1, \ldots, s_i\}$ such that

$$\sum_{s \in S'} s = t'$$

Cont'd

Notice that C has a recursive definition since $C[i,t] = C[i-1,t]$ or $C[i-1,t-s_i]$.

To see this, notice that S' either contains s_i or it does not. If it does, then, for such an S' to exist, it must be the case that $C[i-1,t-s_i]$ is true. Similarly, if S' does not contain s_i, then it must be the case that $C[i-1,t]$ is true.

Base Cases

Notice that (i) if $t' < 0$, $C[i, t']$ is false, (ii) if $t = 0$, then $C[i, t]$ is true for all i, and (iii) if $i = 1$, then $C[i, t]$ is true only if $t = 0$ or $t = s_1$.

Now, this gives the following iterative algorithm to compute the array C.

Cont'd

Input: $S = \{s_1, \ldots, s_n\}, t$
for $i = 1$ to n **do**
  $C[i, 0] = false$
end for
for $t' = 1$ to t **do**
  if $t' = s_1$ **then**
    $C[1, t'] = true$
  else
    $C[1, t'] = false$
  end if
end for
for $i = 2$ to n **do**
  for $t' = 1$ to t **do**
    if $t' - s_i < 0$ **then**
      $C[i, t'] = C[i - 1, t']$
    else
      $C[i, t'] = C[i - 1, t']$ or $C[i - 1, t' - s_i]$
    end if
  end for
end for

Running Time

What's the running time of this algorithm for Subset Sum?