

```
> #Beginning of 2-body orbits program.
```

```
> restart;
```

```
> with(plots) :
```

```
> DEx1 := diff(x1(t), t$2) = 
$$\frac{G \cdot m2 \cdot (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}};$$

```

$$DEx1 := \frac{d^2}{dt^2} x1(t) = \frac{G m2 (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}} \quad (1)$$

```
> DEy1 := diff(y1(t), t$2) = 
$$\frac{G \cdot m2 \cdot (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}};$$

```

$$DEy1 := \frac{d^2}{dt^2} y1(t) = \frac{G m2 (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}} \quad (2)$$

```
> DEx2 := diff(x2(t), t$2) = - 
$$\frac{G \cdot m1 \cdot (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}};$$

```

$$DEx2 := \frac{d^2}{dt^2} x2(t) = - \frac{G m1 (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}} \quad (3)$$

```
> DEy2 := diff(y2(t), t$2) = - 
$$\frac{G \cdot m1 \cdot (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}};$$

```

$$DEy2 := \frac{d^2}{dt^2} y2(t) = - \frac{G m1 (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}} \quad (4)$$

```
> DEs := DEx1, DEy1, DEx2, DEy2;
```

$$DEs := \frac{d^2}{dt^2} x1(t) = \frac{G m2 (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}}, \frac{d^2}{dt^2} y1(t) \quad (5)$$

$$= \frac{G m2 (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}}, \frac{d^2}{dt^2} x2(t) =$$

$$- \frac{G m1 (x2(t) - x1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}}, \frac{d^2}{dt^2} y2(t) =$$

$$- \frac{G m1 (y2(t) - y1(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{3/2}}$$

```
> ics := x1(0) = 0, D(x1)(0) = 0, y1(0) = 0, D(y1)(0) = 0, x2(0) = 2.004600000 1011,  
D(x2)(0) = 0, y2(0) = 0, D(y2)(0) = 23362 \quad (6)
```

```

>
> G := 6.67·10-11;
                                     G := 6.670000000 10-11                                     (7)
> m1 := 0.96·1.989·1030;
                                     m1 := 1.909440000 1030                                     (8)
> m2 := 0.67·1.989·1030;
                                     m2 := 1.332630000 1030                                     (9)
> #std mass:
> mew := G·m1 + G·m2;
                                     mew := 2.162460690 1020                                     (10)
> #eccentricity
> ecc := 0.1
                                     ecc := 0.1                                     (11)
> #semimajor axis
> a := 0.67·1.496·1011
                                     a := 1.002320000 1011                                     (12)
> #Starting distance at perihelion
> r := a·(1 - ecc);
                                     r := 9.020880000 1010                                     (13)
> #Initial velocity
> vel := sqrt(mew·(2/r - 1/a));
                                     vel := 51350.65653                                     (14)
> ics := x1(0) = 0, D(x1)(0) = 0, y1(0) = 0, D(y1)(0) = 0, x2(0) = r, D(x2)(0) = 0, y2(0)
    = 0, D(y2)(0) = vel;
ics := x1(0) = 0, D(x1)(0) = 0, y1(0) = 0, D(y1)(0) = 0, x2(0) = 9.020880000 1010,
    D(x2)(0) = 0, y2(0) = 0, D(y2)(0) = 51350.65653                                     (15)
> solutions := dsolve({DEs, ics}, numeric, output = listprocedure);
solutions := [t = proc(t) ... end proc, x1(t) = proc(t) ... end proc, d/dt x1(t) = proc(t)
...
end proc, x2(t) = proc(t) ... end proc, d/dt x2(t) = proc(t) ... end proc, y1(t) = proc(t)
...
end proc, d/dt y1(t) = proc(t) ... end proc, y2(t) = proc(t) ... end proc, d/dt y2(t) = proc(t)
...

```

```
end proc]
```

```
> xs1 := rhs(solutions[2]);
```

```
xs1 := proc(t) ... end proc
```

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```
> ys1 := rhs(solutions[6]);
```

```
ys1 := proc(t) ... end proc
```

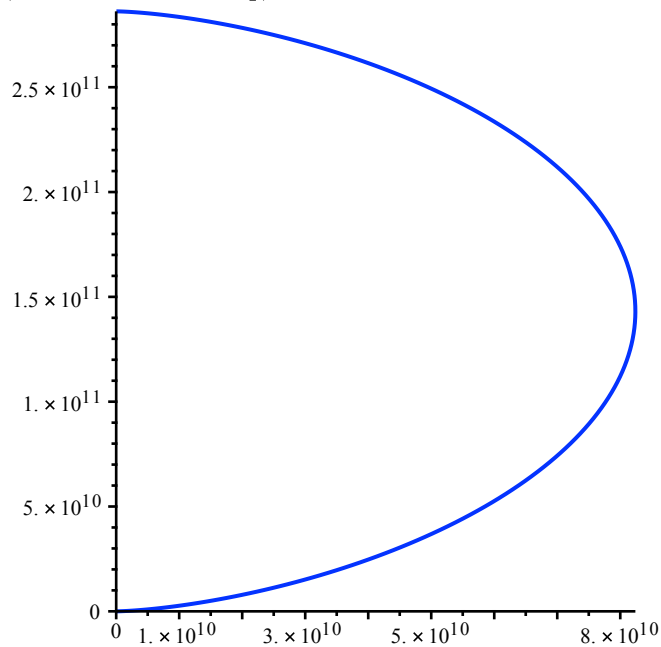
(18)

```
> xs1(4);
```

```
0.0873831297248522
```

(19)

```
> plot([xs1(t), ys1(t), t = 0 .. 13478400])
```



```
> xs2 := rhs(solutions[4]);
```

```
xs2 := proc(t) ... end proc
```

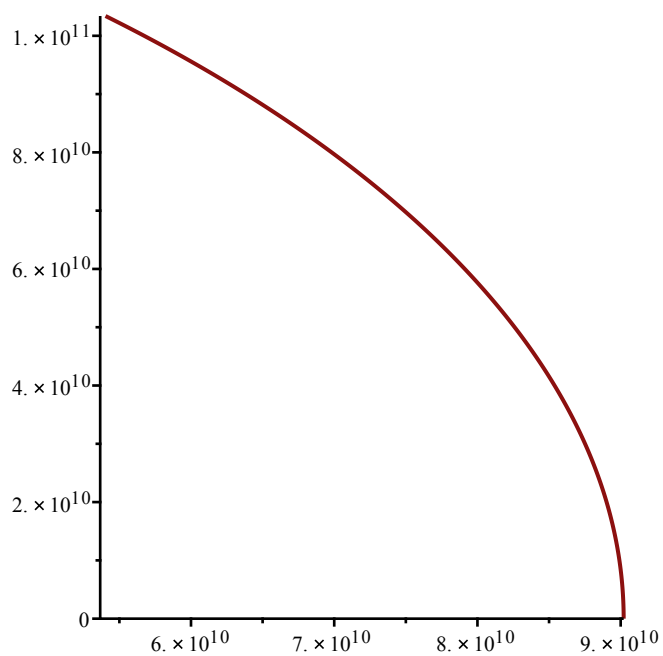
(20)

```
> ys2 := rhs(solutions[8]);
```

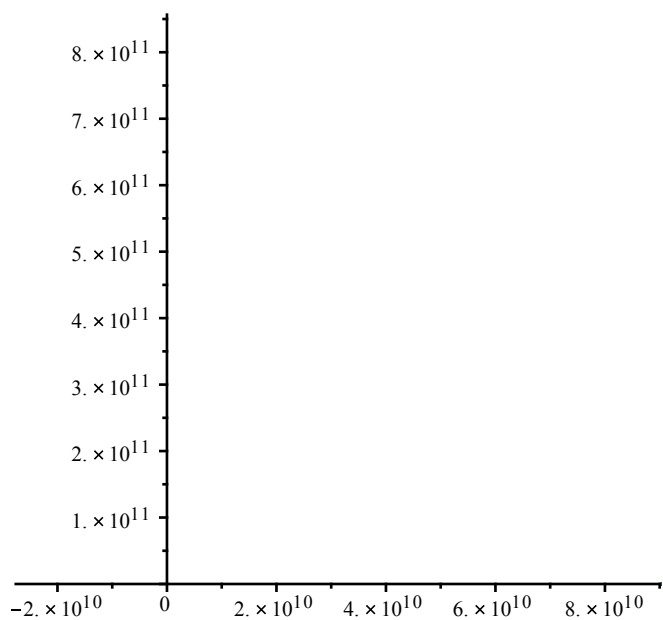
```
ys2 := proc(t) ... end proc
```

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```
> plot([xs2(t), ys2(t), t = 0 .. 2332800]);
```



```
> animate(plot, [[xs1(t), ys1(t), t = 0..A], [xs2(t), ys2(t), t = 0..A]], A = 0..3.13478400);
               A = 0.
```



```
>
>
>
>
>
```

```

> #Beginning of the HD188753 shrinking program.
> restart;
> with(plots) :
> with(DEtools) :

> DEx1 := diff(x1(t), t$2) = - 
$$\frac{G \cdot m2 \cdot (x1(t) - x2(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m3 \cdot (x1(t) - x3(t))}{((x1(t) - x3(t))^2 + (y1(t) - y3(t))^2)^{\frac{3}{2}}} :$$


> DEx2 := diff(x2(t), t$2) = - 
$$\frac{G \cdot m3 \cdot (x2(t) - x3(t))}{((x2(t) - x3(t))^2 + (y2(t) - y3(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m1 \cdot (x2(t) - x1(t))}{((x2(t) - x1(t))^2 + (y2(t) - y1(t))^2)^{\frac{3}{2}}} :$$


> DEx3 := diff(x3(t), t$2) = - 
$$\frac{G \cdot m1 \cdot (x3(t) - x1(t))}{((x3(t) - x1(t))^2 + (y3(t) - y1(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m2 \cdot (x3(t) - x2(t))}{((x3(t) - x2(t))^2 + (y3(t) - y2(t))^2)^{\frac{3}{2}}} :$$


> DEy1 := diff(y1(t), t$2) = - 
$$\frac{G \cdot m2 \cdot (y1(t) - y2(t))}{((x1(t) - x2(t))^2 + (y1(t) - y2(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m3 \cdot (y1(t) - y3(t))}{((x1(t) - x3(t))^2 + (y1(t) - y3(t))^2)^{\frac{3}{2}}} :$$


> DEy2 := diff(y2(t), t$2) = - 
$$\frac{G \cdot m3 \cdot (y2(t) - y3(t))}{((x2(t) - x3(t))^2 + (y2(t) - y3(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m1 \cdot (y2(t) - y1(t))}{((x2(t) - x1(t))^2 + (y2(t) - y1(t))^2)^{\frac{3}{2}}} :$$


> DEy3 := diff(y3(t), t$2) = - 
$$\frac{G \cdot m1 \cdot (y3(t) - y1(t))}{((x3(t) - x1(t))^2 + (y3(t) - y1(t))^2)^{\frac{3}{2}}} - \frac{G \cdot m2 \cdot (y3(t) - y2(t))}{((x3(t) - x2(t))^2 + (y3(t) - y2(t))^2)^{\frac{3}{2}}} :$$


> G := 6.67 · 10-11;

```

$$G := 6.670000000 \cdot 10^{-11} \quad (1)$$

$$\begin{aligned} & \#Star A \\ & m1 := 1.06 \cdot 1.989 \cdot 10^{30}; \\ & m1 := 2.108340000 \cdot 10^{30} \end{aligned} \quad (2)$$

$$\begin{aligned} & \#Star B \\ & m2 := 0.96 \cdot 1.989 \cdot 10^{30}; \\ & m2 := 1.909440000 \cdot 10^{30} \end{aligned} \quad (3)$$

$$\begin{aligned} & \#Star C \\ & m3 := 0.67 \cdot 1.989 \cdot 10^{30}; \\ & m3 := 1.332630000 \cdot 10^{30} \end{aligned} \quad (4)$$

$$\begin{aligned} & \#eccentricity of outer orbit: \\ & ecco := 0.5; \\ & ecco := 0.5 \end{aligned} \quad (5)$$

$$\begin{aligned} & \#outer orbit semi-major axis: <- This is the parameter that we shrink \\ & ao := 11.8 \cdot 1.496 \cdot 10^{11}; \\ & ao := 1.765280000 \cdot 10^{12} \end{aligned} \quad (6)$$

$$\begin{aligned} & \#initial radius of outer orbit \\ & ro := ao \cdot (1 - ecco); \\ & ro := 8.826400000 \cdot 10^{11} \end{aligned} \quad (7)$$

$$\begin{aligned} & \#std gravitational parameter outer \\ & mewo := G \cdot (m1 + m2 + m3); \\ & mewo := 3.568723470 \cdot 10^{20} \end{aligned} \quad (8)$$

$$\begin{aligned} & \#initial outer velocity \\ & velo := \text{sqrt} \left(mewo \cdot \left(\frac{2}{ro} - \frac{1}{ao} \right) \right); \\ & velo := 24626.92997 \end{aligned} \quad (9)$$

$$\begin{aligned} & \#eccentricity of inner orbit \\ & ecci := 0.1; \\ & ecci := 0.1 \end{aligned} \quad (10)$$

$$\begin{aligned} & \#inner orbit semi-major axis \\ & ai := 0.67 \cdot 1.496 \cdot 10^{11}; \\ & ai := 1.002320000 \cdot 10^{11} \end{aligned} \quad (11)$$

$$\begin{aligned} & \#initial inner radius \\ & ri := ai \cdot (1 - ecci); \\ & ri := 9.020880000 \cdot 10^{10} \end{aligned} \quad (12)$$

$$\begin{aligned} & \#std gravitational parameter inner \\ & mewi := G \cdot (m2 + m3); \\ & mewi := 2.162460690 \cdot 10^{20} \end{aligned} \quad (13)$$

```

> #initial inner velocity
> veli := sqrt( mewi * ( 2 / ri - 1 / ai ) );
veli := 51350.65653

```

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```

> #one year
> T := 365.25 * 24 * 3600;
T := 3.155760000 107

```

(15)

```

> DEs := DEx1, DEy1, DEx2, DEy2, DEx3, DEy3 :
> ics := x1(0) = 0, D(x1)(0) = 0, y1(0) = 0, D(y1)(0) = 0, x2(0) = ro - ri, D(x2)(0) = 0,
y2(0) = 0, D(y2)(0) = velo -  $\frac{veli}{1 + \frac{m3}{m2}}$ , x3(0) = ro, D(x3)(0) = 0, y3(0) = 0, D(y3)(0)
= velo +  $\frac{veli}{1 + \frac{m2}{m3}}$ ;
ics := x1(0) = 0, D(x1)(0) = 0, y1(0) = 0, D(y1)(0) = 0, x2(0) = 7.924312000 1011,
D(x2)(0) = 0, y2(0) = 0, D(y2)(0) = -5616.40148, x3(0) = 8.826400000 1011,
D(x3)(0) = 0, y3(0) = 0, D(y3)(0) = 45734.25505

```

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> solutions := dsolve( {DEs, ics}, numeric, output = listprocedure);
solutions := [ t = proc(t) ... end proc, x1(t) = proc(t) ... end proc,  $\frac{d}{dt} x1(t) = \text{proc}(t)$ 
...
end proc, x2(t) = proc(t) ... end proc,  $\frac{d}{dt} x2(t) = \text{proc}(t)$  ... end proc, x3(t) = proc(t)
...
end proc,  $\frac{d}{dt} x3(t) = \text{proc}(t)$  ... end proc, y1(t) = proc(t) ... end proc,  $\frac{d}{dt} y1(t) = \text{proc}(t)$ 
...
end proc, y2(t) = proc(t) ... end proc,  $\frac{d}{dt} y2(t) = \text{proc}(t)$  ... end proc, y3(t) = proc(t)
...
end proc,  $\frac{d}{dt} y3(t) = \text{proc}(t)$  ... end proc ]

```

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```

> xs1 := rhs(solutions[2]);

```

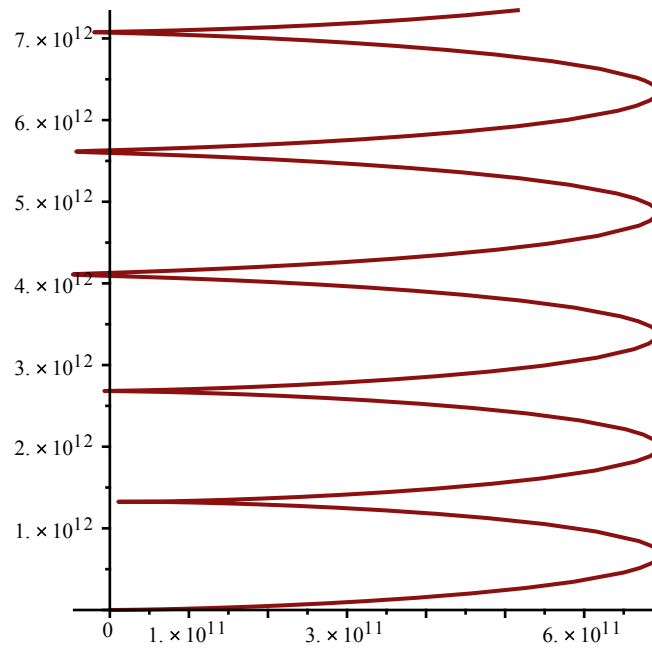
(18)

`xs1 := proc(t) ... end proc` (18)

`> ys1 := rhs(solutions[8]);`

`ys1 := proc(t) ... end proc` (19)

`> plot([xs1(t), ys1(t), t = 0 .. 811030320]);`



`> xs2 := rhs(solutions[4]);`

`xs2 := proc(t) ... end proc` (20)

`> ys2 := rhs(solutions[10]);`

`ys2 := proc(t) ... end proc` (21)

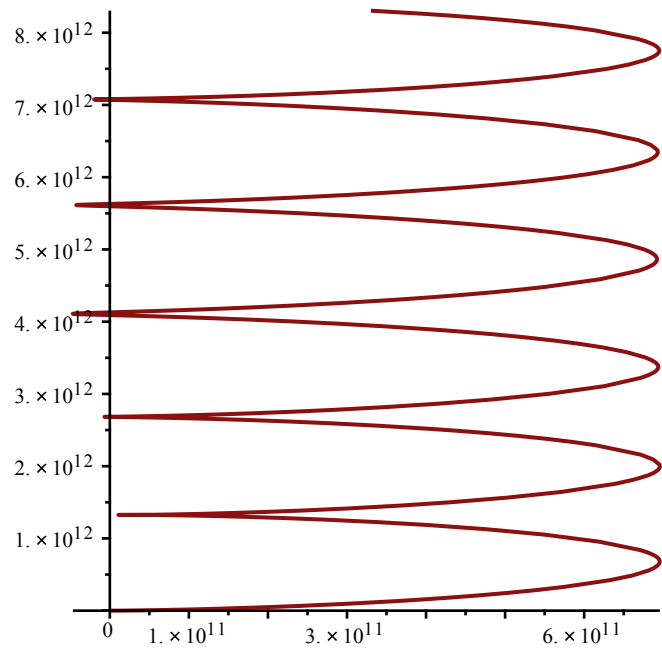
`> xs3 := rhs(solutions[6]);`

`xs3 := proc(t) ... end proc` (22)

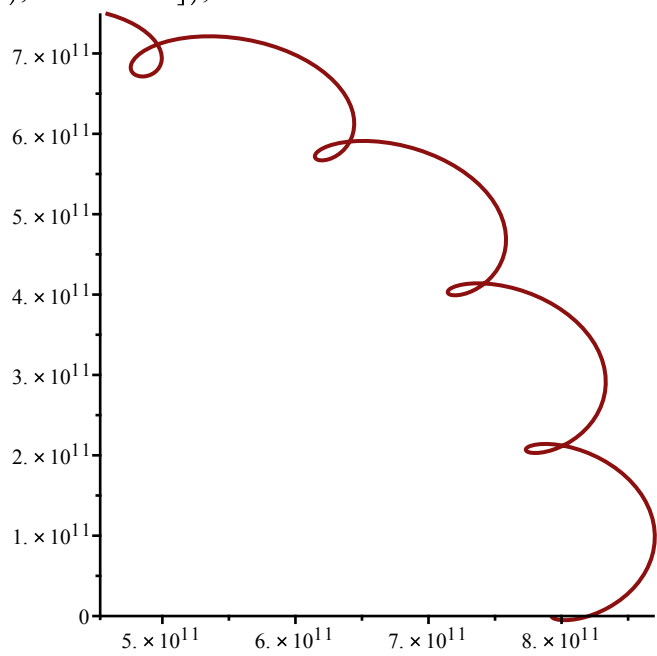
`> ys3 := rhs(solutions[12]);`

`ys3 := proc(t) ... end proc` (23)

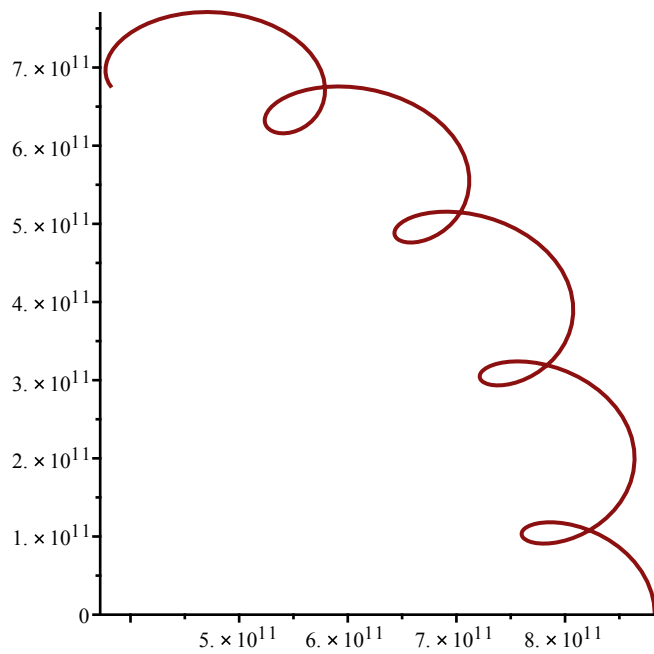
`> plot([xs1(t), ys1(t), t = 0 .. 27 · T]);`



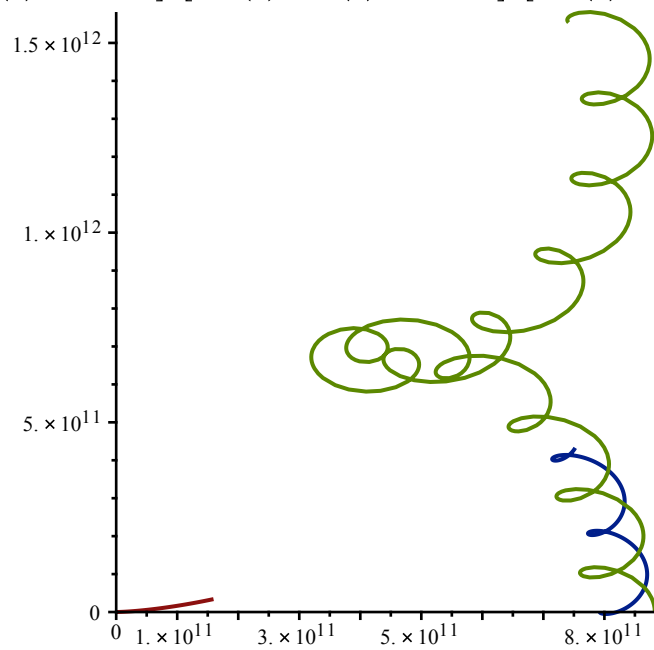
> `plot([xs2(t), ys2(t), t = 0 .. 2 · T]);`



> `plot([xs3(t), ys3(t), t = 0 .. 2 · T]);`

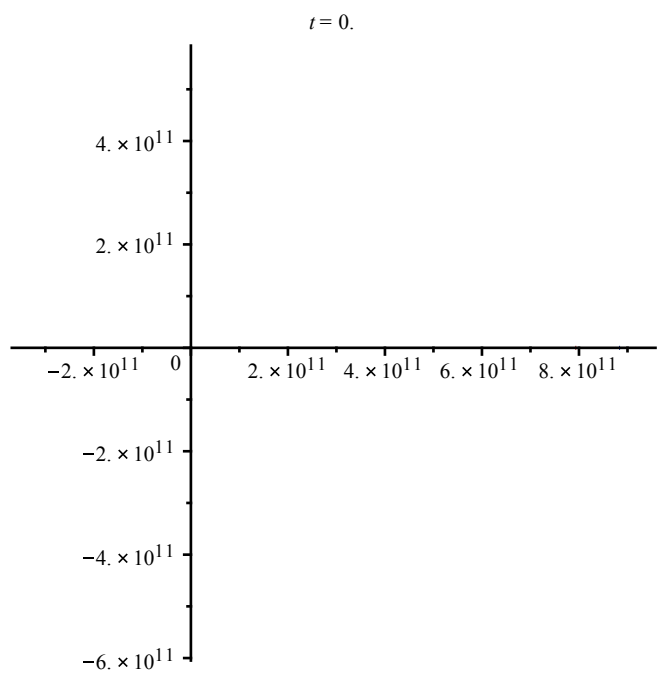


```
> plot([ [xs1(t), ys1(t), t = 0 .. T], [xs2(t), ys2(t), t = 0 .. T], [xs3(t), ys3(t), t = 0 .. 5 · T] ])
```



```
> #Plot with all three bodies for 95% of an earth year
```

```
> #animate(plot, [[ [xs2(s·T) - xs1(s·T), ys2(s·T) - ys1(s·T), s = 0 .. t], [xs3(s·T) - xs1(s·T), ys3(s·T) - ys1(s·T), s = 0 .. t] ], t = 0 .. 9);
```



```

=> #Plot with just the second two systems.
=> #animate(plot, [[xs2(t), ys2(t), t = 0 ..A], [xs3(t), ys3(t), t = 0 ..A]]], A = 0 ..20· T, frames
    = 50);
=>
=>

```