Math 131A Homework 9
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Problem 1.
$\boxed{\text{Ross 20.16}}$ Suppose the limits $L_1 = \lim_{x \to a+} f_1(x)$ and $L_2 = \lim_{x \to a+} f_2(x)$ exist.

(a) We want to show if $f_1(x) \leq f_2(x)$ for all x in some interval (a, b), then $L_1 \leq L_2$. Let $\{t_n\}_{n \in N} \subseteq (a, b)$ with $t_n \to a$. Then $\{f_1(t_n)\}_{n \in N} \to L_1$ and $\{f_2(t_n)\}_{n \in N} \to L_2$. Since $f_1(t_n) \leq f_2(t_n)$ for all $n \in N$, we have $\lim_{n \to \infty} f_1(t_n) \leq \lim_{n \to \infty} f_1(t_n)$. We can express this as

$$L_1 = \lim_{x \to a+} f_1(x) = \lim_{n \to \infty} f_1(t_n) \leq \lim_{n \to \infty} f_1(t_n) = \lim_{x \to a+} f_2(x) = L_2$$
$$L_1 \leq L_2.$$

(b) Suppose $f_1(x) < f_2(x)$ for all x in some interval (a, b). We want to conclude that $L_1 < L_2$. But we provide an easy counter example: Consider $f_1(x) = 0$ and $f_2(x) = x$ on the interval $(0, 1)$.

Problem 2.
$\boxed{\text{Ross 28.2(a)}}$

$$\begin{aligned} f'(2) &= \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{x \to 2} \frac{x^3 - 8}{x - 2} \\ &= \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{x - 2} \\ &= \lim_{x \to 2} (x^2 + 2x + 4) \\ &= 2^2 + 2(2) + 4 \\ &= 12. \end{aligned}$$

$\boxed{\text{Ross 28.6(b)}}$ Let $f(x) = \sin \frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Consider

$$\begin{aligned} &\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{x \to 0} \frac{x \sin \frac{1}{x}}{x} \\ &= \lim_{x \to 0} \sin \frac{1}{x}. \end{aligned}$$

The limit does not exist. Therefore f is not differentiable at $x = 0$.