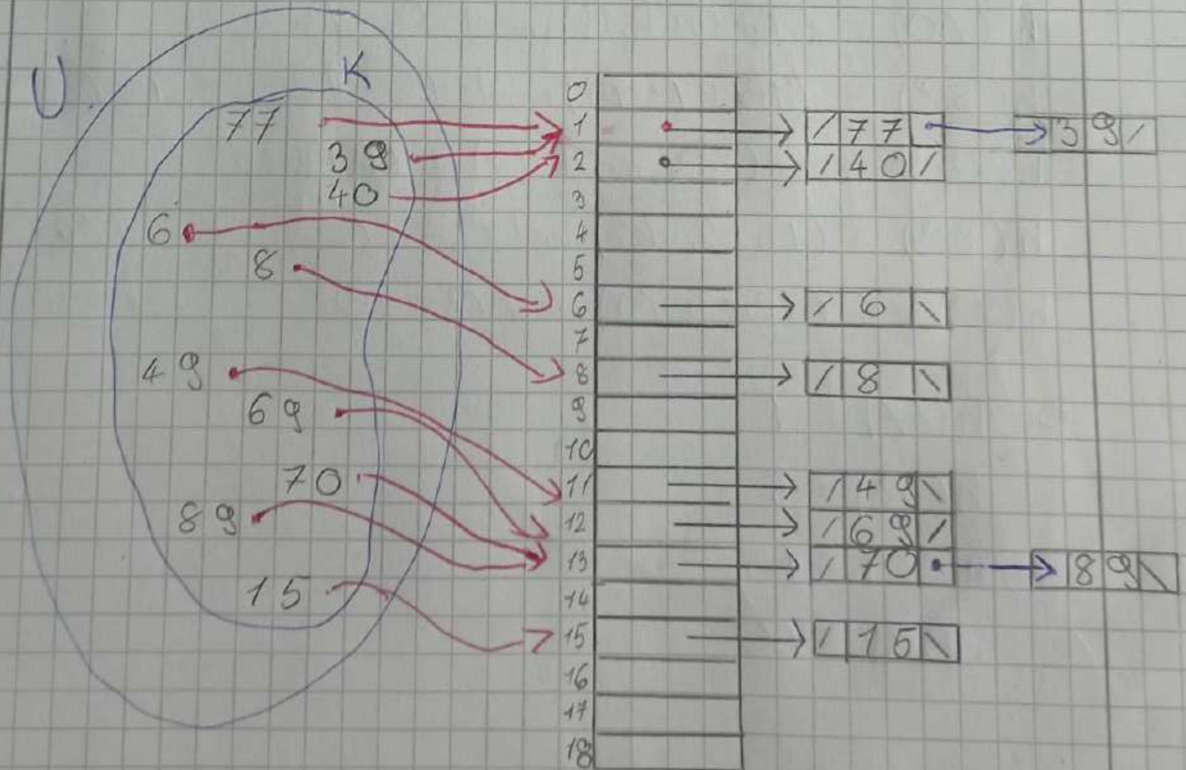
Zadaća 1

(1)

1.) $k \in \{77, 69, 39, 70, 6, 8, 40, 89, 49, 15\}$
$m = 19$

a) $h(k) = k \mod m$

$h(77) = 77 \mod 19 = 1$ ✓
$h(69) = 69 \mod 19 = 12$ ✓
$h(39) = 39 \mod 19 = 1$ ✓
$h(70) = 70 \mod 19 = 13$ ✓
$h(6) = 6 \mod 19 = 6$ ✓
$h(8) = 8 \mod 19 = 8$ ✓
$h(40) = 40 \mod 19 = 2$ ✓
$h(89) = 89 \mod 19 = 13$ ✓
$h(49) = 49 \mod 19 = 11$ ✓
$h(15) = 15 \mod 19 = 15$



1.)b) $i = 0, 1, 2, \ldots, m-1$ $m = 19$ $m-1 = 18$

$h(k, i) = (h_1(k) + i * h_2(k)) \bmod m$

$h_1(k) = k \bmod m$

$h_2(k) = 1 + (k \bmod (m-1))$

$h(77, 0) = (1 + 0 \cdot h_2(k)) \bmod 19 = 1$

$h(69, 0) = (12 + 0 \cdot h_2(k)) \bmod 19 = 12$

$h(39, 0) = (1 + 0 \cdot h_2(k)) \bmod 19 = 1$ X

$h(39, 1) = (1 + 1 \cdot (1 + (39 \bmod 18)) \bmod 19 = 5$

$h(70, 0) = (13 + 0 \cdot h_2(k)) \bmod 19 = 13$

$h(6, 0) = (6 + 0\, h_2(k)) \bmod 19 = 6$

$h(8, 0) = (8 + 0 \cdot h_2(k)) \bmod 19 = 8$

$h(40, 0) = (2 + 0 \cdot h_2(k)) \bmod 19 = 2$

$h(89, 0) = (13 + 0\, h_2(k)) \bmod 19 = 13$ X

$h(89, 1) = (13 + 1 \cdot (1 + (89 \bmod 18)) \bmod 19 = 12$ X

$h(89, 2) = (13 + 2 \cdot 18) \bmod 19 = 11$

$h(49,0) = (11 + 0 \cdot h_2(k)) \bmod 19 = 11$ ×

$h(49,1) = (11 + 1 \cdot (1 + 13)) \bmod 19 = 6$ ×

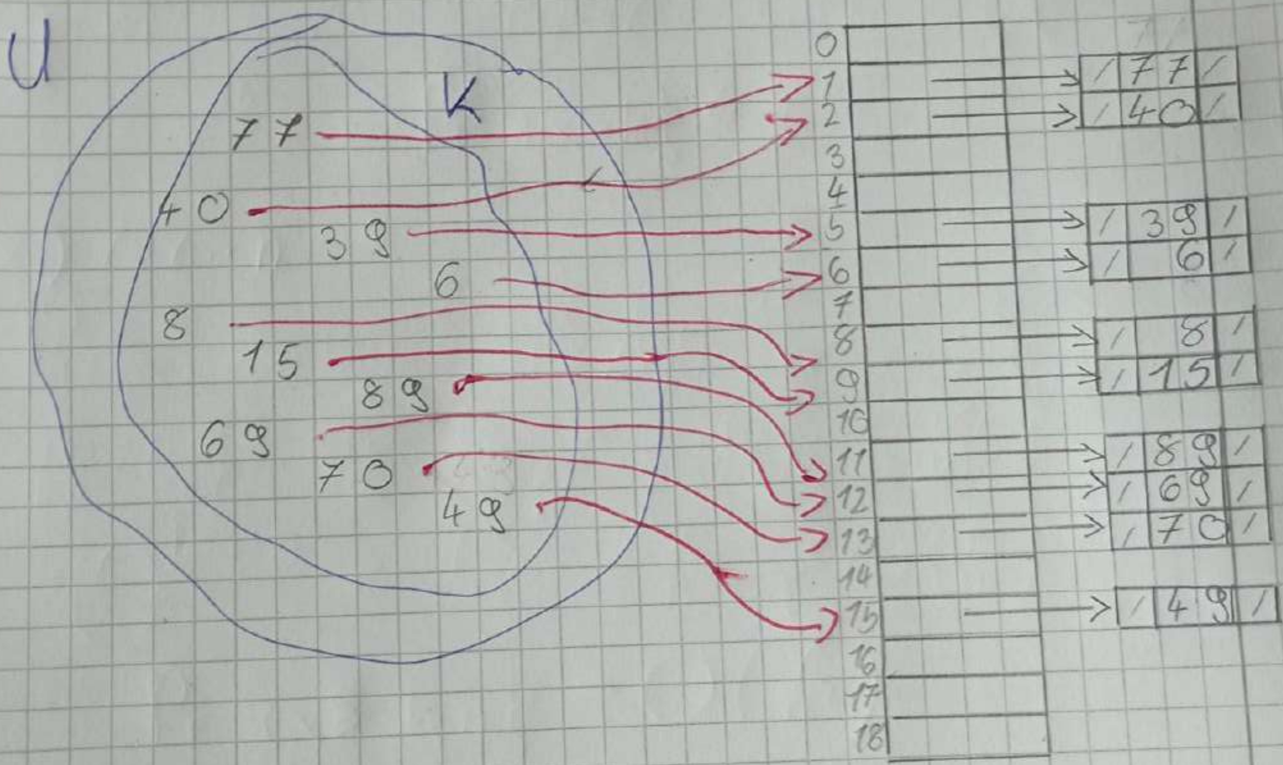
$h(49,2) = (11 + 2 \cdot 14) \bmod 19 = 1$ ×

$h(49,3) = (\overbrace{11 + 3 \cdot 14}^{53}) \bmod 19 = 15$

$h(15,0) = (15 + 0 \cdot h_2(k)) \bmod 19 = 15$ ×

$h(15,1) = (15 + 1(1 + 15)) \bmod 19 = 12$ ×

$h(15,2) = (15 + 2 \cdot 16) \bmod 19 = 9$



2.) $x_1 x_2 \ldots x_n$ $(x_i \in \{0, 1, \ldots, 9\})$

$f(x) = \sum_{i=1}^{n} a_i x_i \pmod 8$ univerzalna ?

Nije univerzalna.

Kontraprimjer :

Npr. $n=3$, $a_1 = a_2 = a_3 = 1$ i za

$x = 341$

$y = 844$

$$\left.\begin{array}{l} \Rightarrow 3 \cdot 1 + 4 \cdot 1 + 1 \cdot 1 = 8 \bmod 8 = 0 \\ 8 \cdot 1 + 4 \cdot 1 + 4 \cdot 1 = 16 \bmod 8 = 0 \end{array}\right\} =$$

Vj kolizije je 1 što je veće od $\frac{1}{8}$.

② $h \in H$ raspršuje n različitih ključeva u tablicu T duljine m.

f_k - sl. var. koja označava ukupni br. kolizija s k u tablici T. $k \neq l$

$$f_{k,l} = \begin{cases} 1, & h(k)=h(l) \\ 0, & \text{inače} \end{cases}, \quad k \neq l$$

$$\Rightarrow f_k = \sum_{l \in T/\{k\}} f_{k,l}$$

Očekivanje

$$\Rightarrow E[f_k] = E\Big[\sum_{l \in T/\{k\}} f_{k,l}\Big] = \sum_{l \in T/\{k\}} E[f_{k,l}]$$

$$= \sum_{l \in T\{k\}} \big(1 \cdot P_r\{f_{k,l} = 1\} + \overbrace{0 \cdot P_r\{f_{k,l} = 0\}}^{0}\big)$$

Lemma 5.1

$$= \sum_{l \in T\{k\}} \big(\underbrace{P_r\{f_{kl} = 1\}}_{\le \frac{1}{m}}\big) \le \sum_{l \in T/\{k\}} \frac{1}{m} = \frac{n-1}{m} < \frac{n}{m}$$

Očekivani br. kolizija tj. očekivani kardinalitet $\{\{k,l\} : k \neq l, h(k) = h(l)\}$ je

$$E[\#\ \text{kolizija s } k] < \frac{n}{m}$$

③ Tablica veličine m koristi se za spremanje n ključeva, gdje je $n \le m/2$. Korišteno je otvoreno adresiranje s probiranjem za rezolviranje kolizija: X - br. prebiranja

1) $i = 1, 2 \dots, n$

$$P_r\{X > k\} = P_r\{A_1 \cap \dots \cap A_k\} = \frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \dots \cdot \frac{n-k+1}{m-k+1} \le \left(\frac{n}{m}\right)^k \overset{n \le m/2}{<} \left(\frac{\cancel{m}/2}{\cancel{m}}\right)^k$$

$$= 2^{-k} \Rightarrow P_r\{X > k\} \le 2^{-k}$$

2.) $i = 1 \dots n$

$$P_r\{X > 2\lg n\} = P_r\{A_1 \cap \dots A_{2\log n}\}$$

$$= \frac{n}{m} \cdot \frac{n-1}{m-1} \dots \frac{n - 2\lg n + 1}{m - 2\lg n + 1} \le \left(\frac{n}{m}\right)^{2\lg n} < \left(\frac{\cancel{m}/2}{\cancel{m}}\right)^{2\log_2 n}$$

$$= \left(\frac{1}{2}\right)^{\log_2 n^2} = \frac{1}{2^{\log_2 n^2}} = \frac{1}{n^2} \Rightarrow O\left(\frac{1}{n^2}\right)$$

$$P_r\{X > 2\lg n\} = O\left(\frac{1}{n^2}\right)$$

X_i - br. probiranja potrebnih za i-to ubacivanje

$X = \max_{1 \le i \le n} X_i$ - najveći br. probiranja potrebnih za bilo koje od n ubacivanja

3.) $P_r\{X > 2\lg n\} = O(1/n)$

$$P_r\{X > 2\lg n\} = \sum_{i=1}^{n} P_r\{X_i > 2\lg n\}$$

$$\le \sum_{i=1}^{n} \frac{1}{n^2} = \frac{1}{n^2} \underbrace{\sum_{i=1}^{n} 1}_{n} = \frac{1}{n^2} \cdot n = \frac{\cancel{n}}{n^{\cancel{2}}} = \frac{1}{n}$$

$$\Rightarrow P_r\{X > 2\lg n\} \le \frac{1}{n}$$

$$= P_r\{X > 2\lg n\} = O\left(\frac{1}{n}\right)$$

4) $E[X] = O(\lg n)$

Očekivana dužina najdužeg niza probiranja

$$E[X] = \sum_{i=1}^{n} i \cdot P_r\{X = i\}$$

$$\le \overset{\frac{n-1}{n}}{P_r\{X \le 2\lg n\}} \cdot 2\lg n + \overset{\frac{1}{n}}{P_r\{X > 2\lg n\}} \cdot n$$

$$\le \frac{n-1}{n} \cdot 2\lg n + \frac{1}{\cancel{n}} \cdot \cancel{n} = \frac{n 2\lg n - 2\lg n}{n} + 1$$

$$= 2\lg n - \frac{2\lg n}{n} + 1 \Rightarrow O(\lg n)$$

$$\Rightarrow E[X] = O(\lg n)$$