CHEM/PHY 598

Info: You can work in groups, but all solutions must be written up independently. Many of the written problems –which are not assigned directly from the class textbook– are taken from a variety of other textbooks/papers. If any question requires a computational component, provide your written answer on one sheet, then the printout of your Mathematica notebook only for that problem on separate sheets following your write-up for that problem. Then repeat for each problem. i.e. do **not** staple a Mathematica notebook printout for all problems at the end of your problem set. **Only codes that are commented *at every step* and *whose logic can be easily followed* will be graded**.

DUE: **Thursday** April 09th, 3PM. To be handed in within the first five minutes of class.

Problem 1:

In class we discussed a Gibbs sampler for a mixture model of a **Normal**$(\mu_1, 1/\tau)$ and a **Normal**$(\mu_2, 1/\tau)$. In the example, we assumed that the observations $\mathbf{w_{1:N}}$ stem from the first and second components with known probabilities $\omega_1 = \omega$ and $\omega_2 = 1-\omega$, respectively. However, generally the value of ω may be unknown and so we have to estimate it similar to the other variables.

Place a beta prior on ω and develop a Gibbs sampler to draw posterior samples from $p(\omega, \tau, \mu_1, \mu_2, \mathbf{s_{1:N}}|\mathbf{w_{1:N}})$. In this setup, the entire model is

$$\omega \sim \textbf{Beta}(A, B) \tag{1}$$

$$\tau \sim \textbf{Gamma}(\alpha, \beta) \tag{2}$$

$$\mu_1 \sim \textbf{Normal}(\xi, \psi) \tag{3}$$

$$\mu_2 \sim \textbf{Normal}(\xi, \psi) \tag{4}$$

$$s_n|\omega \sim \textbf{Categorical}_{1,2}(\omega, 1-\omega) \tag{5}$$

$$w_n|s_n, \mu_1, \mu_2, \tau \sim \textbf{Normal}\left(\mu_{s_n}, \frac{1}{\tau}\right), \qquad n = 1, \ldots, N \tag{6}$$

Problem 2:

In class we highlighted independent normal-gamma priors to estimate the center and spread of an underlying normal distribution. In doing so, we used known values for the hyperparameters ξ, ψ, α, β; however, in many practical applications specifying values for these hyperparameters might not be easy. In such cases, we may apply a hyper-hyper-model

$$\xi \sim \textbf{Normal}(\eta, \zeta) \tag{7}$$

$$\psi \sim \textbf{Gamma}(\kappa, \lambda) \tag{8}$$

$$\alpha \sim \textbf{Gamma}(\gamma, \omega) \tag{9}$$

$$\beta \sim \textbf{Gamma}(\rho, \phi) \tag{10}$$

$$\mu|\xi, \psi \sim \textbf{Normal}\left(\xi, \psi\right) \tag{11}$$

$$\tau|\alpha, \beta \sim \textbf{Gamma}(\alpha, \beta) \tag{12}$$

$$w_n|\mu, \tau \sim \textbf{Normal}\left(\mu, \frac{1}{\tau}\right), \qquad n = 1, \ldots, N \tag{13}$$

Assume the values of the hyper-hyper-parameters $\eta, \zeta, \kappa, \lambda, \gamma, \omega, \rho, \phi$ are known and develop a Metropolis-within-Gibbs scheme to sample from the joint posterior $p(\mu, \tau, \alpha, \beta, \xi, \psi|\mathbf{w_{1:N}})$. In doing so, use Gibbs updates for $\mu, \tau, \beta, \xi, \psi$ and a Metropolis update for α.