CHEM/PHY 598

Catch-up Pset

Info: You can work in groups, but all solutions must be written up independently. Many of the written problems–which are not assigned directly from the class textbook–are taken from a variety of other textbooks/papers. If any question requires a computational component, provide your written answer on one page, then a pdf of your Mathematica notebook only for that problem on separate pages following your write-up for that problem. Then repeat for each problem. i.e., do **not** include a Mathematica notebook printouts for all problems combined at the end of your problem set. **Only codes that are commented *at every step* and *whose logic can be easily followed* will be graded**.

DUE: Tuesday January 26th within the first 5 minutes of class.

Problem 1: Coding
There is nothing to hand back in for this problem.
Install Mathematica using `https://myapps.asu.edu`
Watch: `https://www.youtube.com/watch?v=Zp1EV7ytSnA`

Problem 2: Calculus catch-up
Evaluate the following by hand and check your results on Mathematica. These are not easy integrals but we will be seeing them over and over. The goal of this class is “getting things done” not math proofs. Nonetheless, we still need some basic math tricks up our sleeve before we get started. Provide both your calculation and a printout of your Mathematica notebook (integration on Mathematica is simple and just one line. Google “integration Mathematica help"). Pay attention to the names of the integrals if you need to look them up online for help on how to evaluate them. If this is the first time you see them, it is normal to need help. Note that even if you do use online resources to get started the first time though, you are responsible for knowing how to do them independently.

Gaussian integral: $\int_{-\infty}^{\infty} \mathrm{d}x\, e^{-(x-\mu)^2/(2\sigma^2)}$, assume μ is real and σ^2 is positive and real.

Gaussian integral: $\int_{0}^{\infty} \mathrm{d}x\, e^{-(x-\mu)^2/(2\sigma^2)}$, assume μ is real and σ^2 is positive and real.

Gaussian moments: $\int_{-\infty}^{\infty} \mathrm{d}x\, x^2 e^{-(x-\mu)^2/(2\sigma^2)}$, assume μ is real and σ^2 is positive and real.

Gaussian product: Show that the product of N Gaussians is still a Gaussian.

Gamma-function integral: Show that $\int_0^{\infty} \mathrm{d}x\, x^n e^{-x}$ is $n!$, assume n is a positive integer.

Gamma-function integral: Show that $\int_0^{\infty} \mathrm{d}x\, x^n e^{-x/a}$ is $a^{n+1} n!$, assume n is a positive integer and a is positive and real.

Cauchy normalization: $\int_{-\infty}^{\infty} \mathrm{d}x\, \sigma/(\sigma^2 + (x-\mu)^2)$, assume n is a positive integer and a is positive and real.

Dirac-Delta function: Evaluate $\int_0^{\infty} \mathrm{d}x\, (\log x)\, \delta(x-1)$ and describe the behavior of $f(x)$, where $f(x) = \int_{-\infty}^{x} \mathrm{d}x'\, \delta(x')$ (or you can simply name $f(x)$). Check this link in case you do not remember the definition of delta function.

Geometric series: $\sum_{i=0}^{N} x^i$, assume N is a positive integer.

Poisson moments: $\sum_{n=0}^{\infty} n^2 \lambda^n exp(-\lambda)/n!$, μ is real and σ^2 is positive and real.

Taylor expand the following to second order in x (around zero):

$\frac{1}{1-x}$

$\frac{e^x}{1-x}$

$\frac{e^x \log\left(1-x^2\right)}{1-x}$

$\sqrt{1-2x}$

Problem 3: Linear catch-up

Create a 2×2 matrix on Mathematica populated by real numbers (make sure it is invertible). Use Mathematica to find its eigenvectors, eigenvalues, determinant, transpose and inverse. Verify Mathematica's output by hand. If you have not seen these simple matrix operations before, don't worry and look these up online or watch a video on introductory linear algebra. Any linear more complicated than what we do in this problem we will cover in class. For help on doing this on Mathematica, just google something like "Mathematica eigenvector how". All of these operations are one-liners on Mathematica.

Problem 4: Manipulating probabilities

a) Consider two independent random variables R_1 and R_2 with densities $p_1(r_1)$ and $p_2(r_2)$, respectively. Show that the density $p_3(r_3)$ of a random variable R_3, with values $r_3 = r_1 + r_2$, is equal to the convolution $p_3(r_3) = (p_1 * p_2)(r_3)$.

b) Consider two exponential random variables $R_1 \sim \exp(\lambda_1)$ and $R_2 \sim \exp(\lambda_2)$. Show that the random variable R_3, with values $r_3 = \min(r_1, r_2)$, follows an $\exp(\lambda_1 + \lambda_2)$ distribution.