MICHIGAN ENGINEERING
UNIVERSITY OF MICHIGAN

EECS 376 Discussion

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Week 4: Greedy, DFAs, Languages
9/22/23
Friday 11:30am @ NAME 138

Today + announcements

- Greedy algorithms
- DFAs
- Formal languages

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Greedy algorithms

- Compared to the optimal algorithm…
- What do we want to show?

Greedy algorithms

- Take the “locally optimal” step → what does this mean?
- Compared to optimal algorithm…
- Seen in heuristics (go to 477!)

Due to high demand, the EECS Department has finally decided to install a vending machine, called the Bob and Betty Beyster Building Beverage Bestower. Like any decent vending machine, the B^6 must provide change to customers after purchases. Because this is a building teeming with computer scientists, the machine should return the smallest number of coins possible for any given amount of change. Due to design constraints, the machine only has space for three types of coins: 1¢, 2¢, and 5¢. The student designing the machine (who did not take EECS 376) decides that it should use a greedy algorithm to dispense change. Specifically, it first returns as many 5¢ coins as it can (without overpaying), then as many 2¢ coins as it can, then the appropriate number of 1¢ coins. The student hopes that this strategy will always return an optimal (smallest) number of coins; in this question, you (an EECS 376 student) will prove that this is indeed the case.

Let c denote the amount of change and let n_1, n_2 and n_5 respectively denote the number of 1¢, 2¢, and 5¢ coins returned by the greedy algorithm. Likewise, let n_1^*, n_2^* and n_5^* denote the number of 1¢, 2¢, and 5¢ coins in an optimal solution. (That is, the sum $n_1^* + n_2^* + n_5^*$ is minimal.)

i. Show that $n_1^* \leq 1$.

Show that $2n_2^* + n_1^* \leq 4$. In other words, the *total amount* given in 1¢ and 2¢ coins at most 4.

You are copying a list of n problems from your textbook to the paper you do your homework on. You need to copy all the problems over, and you want to have them in order. Formally, the ith problem is l_i lines long, and you're copying them onto an ordered set of sheets of paper which are each m lines long.

- The problems must be copied in the assigned order.
- All problems must be copied.
- Problems must be kept on one sheet of paper and can't be split across them.

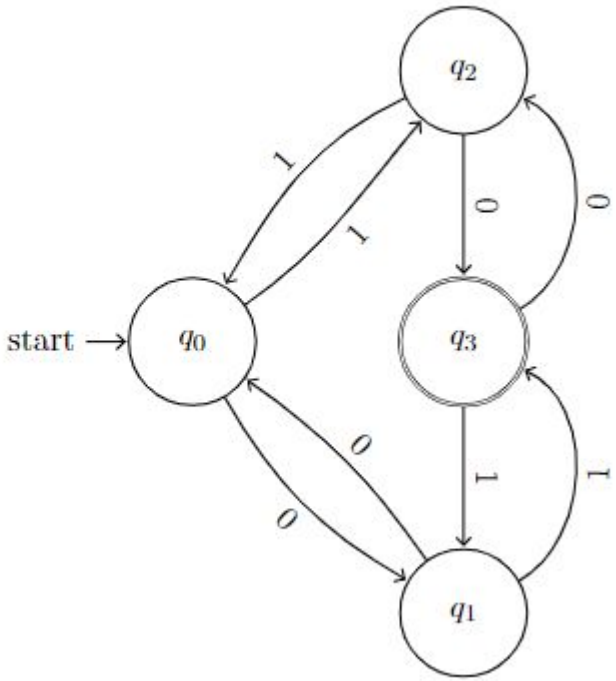
Describe a greedy algorithm to minimize the number of pages you need.

Prove that the proposed greedy algorithm is correct. In other words, prove that the proposed algorithm satisfies each of the three constraints given in the problem.

DFAs

- Outputs: accept, reject
- 5-tuple
- Build DFAs to decide languages

What language does the following DFA decide?



Draw a DFA that decides the language 1110^*.

Languages

- “yes” or “no” answer
- Examples:
  - Can Daphne walk from the NAME building to the BBB in 10 minutes?
  - Can Daphne fit all of her 3 laptops in her tote bag?
    - (unfortunately no.)
- NOT languages:
  - How many miles does Daphne walk in a day?
  - How many hours of sleep did Daphne get last night?

The language which represents all positive integers divisible by 4 can be written as:

(a) $L = \{n \in \mathbb{Z}^+ : n \mod 4 = 0\}$

(b) $L = \{n, k \in \mathbb{Z} : n/4 = k\}$

(c) $L = \{n \in \mathbb{Z}^+ : \gcd(n, 4) = 1\}$

(d) $L = \{n, k \in \mathbb{Z}^+ : n \mod 4 = 2\}$

(e) None of the above