1. (a) Let's say the event that Daphne does not have meleagrisphobia as D and the event that the test comes back positive as A.
The probability of a false positive is given as $P(A|D) = \frac{1}{3}$.

$E[A] = P(A|D) \times n = \frac{n}{3}$

$\therefore E[A] = \frac{n}{3}$

(b)

i) Let's say the event that Daphne has meleagrisphobia as M and the event that the test comes back positive as Y. The probability of a true positive is given as $P(Y|M) = \frac{2}{3}$.

$E[Y] = P(Y|M) \times n$

$\therefore E[Y] = \frac{2n}{3}$

ii) $P(Y < (1-\delta)\mu) \le e^{-\frac{\delta^2 \mu}{2+\delta}}$

in this case, $\delta = \frac{1}{2}$, $\mu = \frac{2n}{3}$

$P(Y < \frac{n}{4}) \le e^{-\frac{1}{18}n}$

Now, we want to find odd n such that $P(Y < \frac{n}{4}) \le 0.05$.

$e^{-\frac{1}{18}n} \le 0.05$

$-\frac{1}{18}n \le \ln(0.05)$

$n \ge -18 \ln(0.05) \approx 66.51$

$\therefore$ minimum odd value of $n = 67$

2. (a) The area of the rhombus can be calculated as the square's area minus the four right triangles' areas at the corners

Area of triangle $= \frac{1}{2} \times \left(1 - \frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \frac{\sqrt{2}}{4}$

Area of 4 triangles $= 2 - \sqrt{2}$

Area of the rhombus $= 1 - (2 - \sqrt{2}) = \sqrt{2} - 1$

$\therefore \; p = \frac{\text{area of rhombus}}{\text{area of square}} = \sqrt{2} - 1$

(b) Let Y_i be the variable indicating whether the ith dart lands in the rhombus. Then, $\mathrm{Ex}[Y_i] = p = \sqrt{2} - 1$. Let Y be the number of darts that land in the rhombus, and observe that $Y = \Sigma_{i=1}^{n} Y_i$. Then by linearity of expectation we have $\mathrm{Ex}[1 + Y/n] = 1 + \frac{1}{n}\Sigma_{i=1}^{n} \mathrm{Ex}[Y_n]$

$= 1 + (pn)/n = 1 + p = 1 + (\sqrt{2} - 1) = \sqrt{2}$

(c) By above we have $\tau = 1 + Y/n$, so $|\tau - \sqrt{2}| < 0.1$ if and only if $|Y/n - (\sqrt{2} - 1)| = |Y/n - p| < 0.1$. Because Y is the sum of independent indicator variables Y_i, each having expection $\mathrm{Ex}[Y_i] = p$, by the chernoff bound we have

$$\Pr[\tau - \sqrt{2} \geq \varepsilon] \leq 2e^{-2\varepsilon^2 n}$$

Because we want these probabilities to be at most 0.01, we should choose a large enough value n to make $2e^{-2\varepsilon^2 n} \leq 0.01$. Solving for n, we have

$$e^{-2\varepsilon^2 n} \leq 0.005$$

$$-2\varepsilon^2 n \leq \ln(0.005)$$

$$n \geq \frac{\ln(200)}{2\varepsilon^2} \longrightarrow n = \left\lceil \frac{\ln(200)}{2\varepsilon^2} \right\rceil$$

i) $\varepsilon = 0.1$, so $n = 265$ suffices

ii) $\varepsilon = 0.01$, so $n = 26492$ suffices

iii) $\varepsilon = 0.001$, so $n = 2649159$ suffices

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(a)

The number of recursive calls to RQuickSort containing a particular input element e can be bounded by log base (3/2) of n when considering the Partition(B[1..m], p) calls where e ∈ B[1..m], p is a good pivot, and B[1..m] is a subarray of A. The reason is that for a good pivot, the size of the partitions is at most (2/3)n, leading to a reduction in the problem size by at least 1/3.

(b)

The probability that e is involved in t or more recursive calls can be bounded by the probability of a sum of t independent indicator variables (X1 + ... + Xt) being greater than or equal to t. Using the properties of these indicator variables, the probability can be related to Pr[X ≤ log base (3/2) n].

(c)

Utilizing parts (a) and (b), you can show that Pr[e is involved in ≥t recursive calls] < 1/n^2 for t = c * log n and an appropriately chosen constant c.

(d)

Part (d) uses the analysis from part (b) to argue that with a probability of at least 1 - 1/n, each element e is involved in less than t = c * log n recursive calls simultaneously.

(e)

Finally, by concluding from parts (c) and (d), it can be inferred that with a probability of at least 1 - 1/n, RQuickSort takes O(nlogn) time.