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Remaining Useful Life Prediction of Gearbox Based on A Nonlinear State Space Model

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***Abstract*—To solve remaining useful life prediction problems of nonlinear and non-stationary process of components, a data-driven approach is presented. The approach constructs a state space model (SSM) to describe degradation evolution process; uses extend Kalman filter to estimate state distribution in SSM and take the Expectation-Maximization (EM) algorithm to update parameters. Based on the measured data, the time to reach the critical value is determined by estimating the distribution of the remaining useful life by using the estimated nonlinear model. Finally taking gearbox as an example, the results show the approach accurately estimating remaining useful life (RUL) of a gearbox.**

Keywords-state space model; EM algorithm; extend kalman filtering; remaining useful life prediction

I. INTRODUCTION

To support critical decision-making processes such as maintenance replacement, renew and operation, activities of remaining useful life prediction are of great importance to high-risk engineered systems such as aerospace systems, advanced military systems and so on [1-3]. Since the system operation process is nonlinear and non-stationary and an feature (Feature is represented by a time series of samples.) evolution over time is non-stationary stochastic process. General, State space model is used to describe the dynamic behavior of components and construct the relationship between system condition and feature signal [4]. Hence, the distribution of the remaining useful life can be estimated by propagating the distribution of the current system state and using a state space model of the damage evolution process. Therefore, with the appropriate model selection and estimation techniques, the approach can be made computationally tractable, while still achieving an accurate prediction [4-5]. Traditional kalman filter is well application for dynamic state estimation problems. But for nonlinear systems or non-Gaussian noise, kalman filter is no appreciating [6]. Simultaneity, Monte Carlo methods, especially particle filters are more computationally expensive for any moderately dimensioned state-space [7]. The extended kalman filter (EKF) is the most popular solution to recursive nonlinear state estimation problem. For the system model described below is nonlinear and non-stationary. EKF can more quickly predict results. And obtain accuracy results [8-10]. Then the EM algorithm is used to estimate the parameters of the state space model. Finally, gearbox experimental results show that the model obtained in this way can effectively predict the distribution of the remaining useful life of the gearbox.

The paper is organized as follows. The state space model will be presented to model component dynamic behaviors in section II. In section III, first, EKF is used to estimate the model condition distribution; second, EM Algorithm is presented to estimate the parameters of the state space model; third， the process of remaining useful life prediction is proposed. The results of the prediction using data from the gearbox experiment are presented in section IV. The final section extracts the main conclusions.

II. STATE SPACE MODEL

The SSM provides a tool to model the outputs of a dynamic system perturbed by disturbances as a unvaried or multivariate time series in the presence of non-stationary, structural changes, and irregular patterns [4].

Assume that there are an unobservable state process $\{x_t\}_{t\geq 0}$ and an observation series $\{y_t\}_{t>0}$ that satisfy the assumptions:

(1) $\{x_t\}_{t\geq 0}$ is a Markov process.

(2) Conditional on $\{x_t\}_{t\geq 0}$, the y_t's are independent and y_t depends on x_t only.

$$x_{t+1} = f(x_t, w_t, \theta) \tag{1}$$

$$y_{t+1} = h(x_t, e_t, \theta) \tag{2}$$

where the first part of the SSM is called the observation equation and the second part is the state equation; x_t is the unobserved state of the system at time t, which can be real time or another measure such as cycles or miles; y_t is the observation at time t, which is statistically dependent on the latent state governed by the observation equation; w_t and e_t are the process and measurement noises, respectively, and independent of each other, especially in this paper for Gaussian noise, θ is model parameter.

III. PARAMETERS ESTIMATION

A. One Order Extended Kalman Filter For State Estimate

EKF is appropriate method to estimate model condition for the system dynamics Eq. (1) and Eq. (2) were nonlinear, and that the measurement noise was Gaussian. SSM can be modified as

$$x_{t+1} = f(x_t) + w_t \tag{3}$$

$$y_{t+1} = h(x_t) + e_t \tag{4}$$

where, f is the dynamic model function and h is the measurement model function. $w_{t-1} \sim N(0, Q_{t-1})$ is process noise and $e_{t-1} \sim N(0, R_{t-1})$ is the measurement noise. The extended kalman filter is separated to two steps. The steps for the first order EKF are as follows: for t =1,2…,n-1.

Prediction: $F_t = \frac{\partial f}{\partial x}|_{x_{t-1|t-1}}$, $H_t = \frac{\partial h}{\partial x}|_{x_{t|t-1}}$.

Update: $\tilde{y}_t = y_t - h(x_{t|t-1}, t-1)$, $S_t = H_t P_{t|t-1} H_t^T + R_t$ $K_t = P_{t|t-1} H_t^T S_t^{-1}$.

$$x_{t|t} = x_{t|t-1} + K_t \tilde{y}_t \tag{5}$$

$$P_{t|t} = (I - K_t H_t) P_{t|t-1} \tag{6}$$

where the matrices F_t and H_t are the Jacobians of *f* and *h*.

B. Maximum Likelihood Estimation Using the EM Algorithm

Generally the EM algorithm solves the problem by alternating between two steps: firstly, maximizing the likelihood function with respect to the system states (E-step) and, secondly, with respect to the parameters (M-step) [11].

(1) E-step

We assume the initial state is $x_0 \sim (x_{0|0}, p_{0|0})$. Then, The parameters of the sate space model given by Eq(3-4) for estimating is $\theta = \{F, H, Q, R, x_{0|0}, P_{0|0}\}$, The state vector x_t of a state space model is estimated by Extend kalman filter based on all of the observed data. A set of recursions for calculating x_t and P_t from the extend kalman filter results are given Eq. (5) and Eq. (6).

Given measurement data $(y_1, y_2, ..., y_n)$, then we wish to find the parameters θ, making the $p(Y | X, \theta)$ maximum. Particularly:

$$p(x_{0:T}, y_{1:T} | \theta) = p(x_0 | \theta) \prod_{t=1}^{T} p(x_t | x_{t-1}, \theta) \prod_{t=1}^{T} p(y_t | x_t, \theta).$$

(2) M-step

The follow equation can be obtained with basic theory of EM algorithm [12]:

$$\begin{aligned}
&E[\ln p(x_{0:T}, y_{1:t} | \theta)] = \\
&-\frac{1}{2}\ln|p_{0|0}| - \frac{N}{2}\ln|Q| - \frac{N}{2}\ln|R| \\
&-\frac{(n+m)N+m}{2}\ln(2\pi) \\
&-\frac{1}{2}tr(p_{0|0}^{-1}E[(x_0 - x_{0|0})(x_0 - x_{0|0})']) \\
&-\frac{1}{2}\sum_{t=1}^{T} tr(Q^{-1}E[(x_t - F_t x_{t-1})(x_t - F_t x_{t-1})']) \\
&-\frac{1}{2}\sum_{t=1}^{T} tr(R^{-1}E[(y_t - H_t x_t)(y_t - H_t x_t)]
\end{aligned} \tag{7}$$

$$\begin{cases}
A = \sum_{t=1}^{T}(x_{t|T}x'_{t|T} + P_{t|T}) \\
B = \sum_{t=1}^{T}(x_{t-1|T}x'_{t-1|T} + P_{t-1|T}) \\
C = \sum_{t=1}^{T}(x_{t|T}x'_{t-1|T} + P_{t,t-1|T}) \\
D = \sum_{t=1}^{T}(y_t y_t') \\
E = \sum_{t=1}^{T} y_t x_{t|n}
\end{cases} \tag{8}$$

Complete data log likelihood function can be written as:

$$\begin{aligned}
&E[\ln p(x_{0:T}, y_{1:t} | \theta)] = -\frac{1}{2}\ln|p_{0|0}| - \frac{T}{2}\ln|Q| - \frac{T}{2}\ln|R| - \\
&\frac{(n+m)T+m}{2}\ln(2\pi) - \frac{1}{2}tr(p_{0|0}^{-1}E[(x_0 - x_{0|0})(x_0 - x_{0|0})' + P_{0|T}]) \\
&-\frac{1}{2}\sum_{t=1}^{T} tr(Q^{-1}[A - 2F_t C' + F_t B F_t']) \\
&-\frac{1}{2}\sum_{t=1}^{T} tr(R^{-1}[D - EH_t' - H_t E' + H_t A H_t']
\end{aligned} \tag{9}$$

Calculating derivatives of the Eq. (9) we can obtain the optimum parameters [13]. $x_0 = x_{0|0}$, $p_0 = p_{0|0}$.

$$\begin{aligned}
&Q = \frac{1}{n}(A - CB^{-1}C') \\
&R = \frac{1}{n}(D - EA^{-1}E') \\
&H = EA^{-1} \\
&F = CB^{-1}
\end{aligned} \tag{10}$$

The result of the EM algorithm is parameters θ and the corresponding smoothed state distributions. with the filter estimate of the state vector distribution $(x_{n|n}, p_{n|n})$ and EM algorithm estimating state space model parameter, At every future time step *t>n*, the mean and variance of the output vector y_t can be calculated by propagating the initial distribution through the state space model.. Additionally, based on the estimated model, a Monte-Carlo simulation of future feature trend is repeated n times, propagating and predicting the time of every feature achieves critical value .The RUL can be obtained based the feature (y_t) achieve the critical value.

IV. Full Life Test Of Gearbox

A. *Experiment Set*

The experimental system includes a gearbox, a speed and torque sensor, electromotor, magnetic powder brake and so on. The function is showed as Fig. 1.

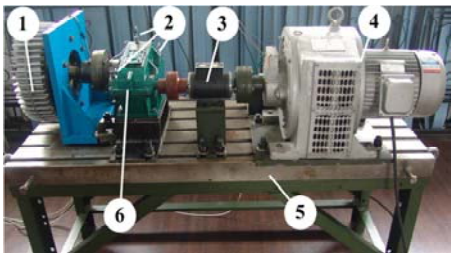


Figure 1. The image of test bed.
(1-Load, 2-Accelerometers, 3-Sensor of speed and torque, 4-Electromotor, 5-Test bed, 6-Gearbox system)

Test gearbox is the core component of this experiment .as shown in the Fig. 2. It is a second level Oblique two gearbox, Rated transmission power is 0.75kw. We have four transducers to acquire data. The Experiment process load is 2~2.5 double to gearbox nominal load and belong to high stress accelerated life experiment. The experiment has acquired data every 5 minute .for each sample, the sample frequency is 20kHz lasting 40k.

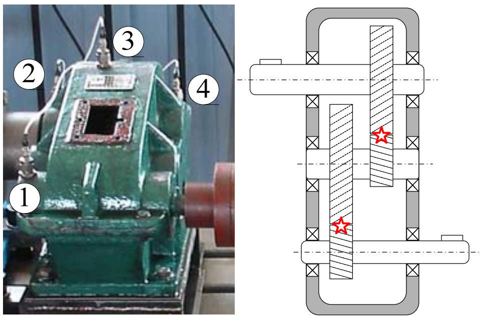


Figure 2. Gear case of the text and structure.

The highest sensitivity in our experiment has been observed in sensors labeled vibration 4, which measure the vibrations on gearbox output and input shafts respectively. Signals acquired at each acquisition session, were analyzed using envelope analysis This work does not consider anomaly detection or diagnostics and instead focuses on the Prognostic aspects. As the proportional energy feature major is our measurement feature of model for sensibility to the detection of the failure. This feature are expected to be good indicators of the damage level. From the vibration 4, a feature time series vector $\{y_1, y_2, ..., y_n\}$ is derived, where n is the total life number of extracted features.

Each element of feature represents the gearbox wear degradation condition, as shown in Fig. 3.

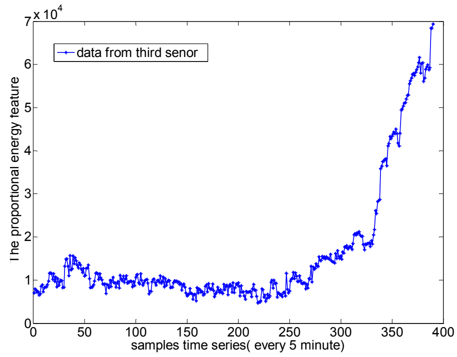


Figure 3. Total life cycle vibration feature signal of gearbox.

For the crack growth data, the true end-of-life (EOL) time series is 390 cycles.

B. *Model Structrue*

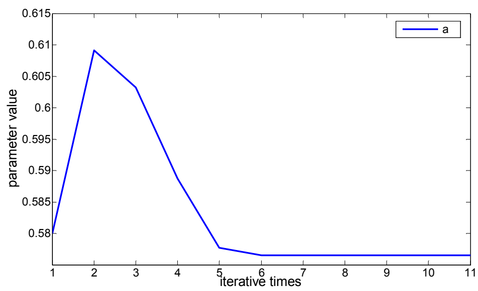
State-space representation is a very general model, that can describe a whole set of different models. In our case we assume that condition of the gearbox is a dynamic process influenced by random nonlinear process. It is relatively easy to establish the SSM if there is a physics-based degradation model, e.g., Paris' law for a crack growth model subject to fatigue [7]. Then the interest lies in predicting when the crack size reaches a critical level based on the monitoring information. Can be in many cases simplified by employing a non-linearization of the model as used in our work , which can be describe by a state space model [14]:

$$
\begin{aligned}
x_t &= x_{t-1} + x_{t-1}^{\alpha} + w_{t-1} \\
y_t &= c x_{t-1} + e_{t-1}
\end{aligned}
\tag{11}
$$

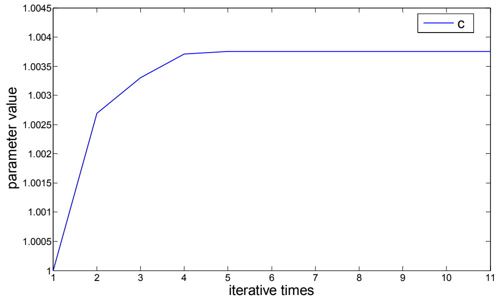
C. *Model Parameters Estimation*

The algorithm estimates model parameters using all samples before estimation point. Initial parameters of the model are set to the following values: $\partial = 0.58$, C=1, Q=0, R=0, $x_0 = 7410$, $P_0 = 10$. The Fig. 4 to Fig. 6 show that estimation result converge to the true optimum.

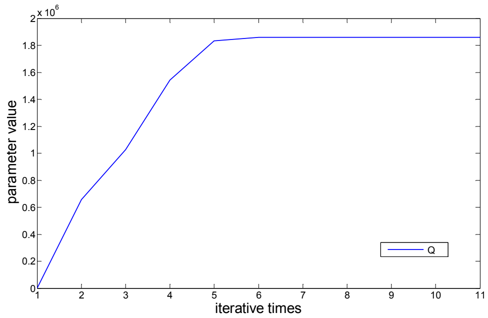
The model parameters is estimated in the 340*th* sample (32.5 hours), using the $\{y_1, y_2, ... y_{340}\}$ feature sample. The dynamic model that construct as the estimated parameters correspond to the change of the feature before 340th sample and predict the feature in the future.



Firgure 4. Parameter ∂ convergent process.



Firgure 5. Parameter C convergent process.



Firgure 6. Parameter Q convergent process.

D. *Remaining Useful Life Prediction*

RUL prediction starts at potential failure point (PF is based failure identification or history knowledge),using all sample data before the prediction estimation point, with the filter estimate of the state vector distribution $(x_{n|n}, p_{n|n})$ and EM algorithm estimating state space model parameter. Then at every future time step *t*>*n*, the mean and variance of the output vector y_t can be calculated by propagating the initial distribution through the state space model. Based on the estimated model, a Monte-Carlo simulation of future feature trend is repeated n times, propagating and predicting the time of every feature achieves critical value.

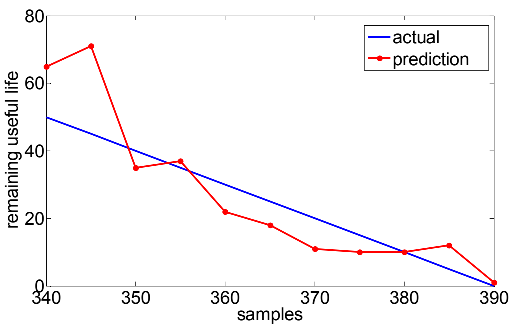


Figure 7. The estimated mean RUL and the actual RUL at monitoring points.

Fig. 7 illustrates the performance of our predictive model against the full life cycle data. We can see that the filtering process adjusts the prediction and the predicted remaining useful life (RUL) is close to the actual life. The prediction starting at the 340th cycle can still produce accurate results. This can be attributed to the flexible SSM technique, which can effectively describe the degradation process of gearbox.

V. Conclusion

In this paper a data-driven prognostic framework for gearbox has been demonstrated that uses measurements, a nonlinear dynamic model, extend kalman filter and EM algorithm estimator to predict remaining useful life. Advantage of our approach is used to online predict remaining useful life. When the measurement is obtained, we can quickly make a prediction and provide manager to take action in time. The EKF is probably the most widely used estimation algorithm for nonlinear systems. However, experience in the estimation community has shown that is difficult to implement, difficult to tune, and only reliable for systems that are almost linear on the time scale of the updates. Many of these difficulties arise from its use of linearization, .a nonlinear Kalman filter which shows promise as an improvement over the EKF is the Unscented Kalman filter (UKF). The UKF tends to be more robust and more accurate than the EKF in its estimation of error. Therefore, the next step in the development of the procedure is to improve the approach using UKF in the future.

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