Remaining Useful Life Prediction for Nonlinear Degrading Systems with Maintenance

Hanwen Zhang[1], Maoyin Chen[1], Donghua Zhou[2,1*]
[1]Department of Automation, TNList, Tsinghua University, Beijing, P. R. China
[2]College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, P.R. China
zhanghw14@ mails.tsinghua.edu.cn, mychen@mail.tsinghua.edu.cn, zdh@mail.tsinghua.edu.cn

Abstract—**Remaining useful life (RUL) prediction is one of the most critical procedures of the prognostics and health management (PHM). In the existing literature, most RUL prediction methods are under the assumption that there is no maintenance activity during the whole life time of the degrading system. However, most practical systems experience various kinds of maintenance activities when they are in operation. This article presents an approach to predict the RUL of a class of nonlinear degrading systems with stochastic maintenance. To predict the RUL for systems with stochastic maintenance, a wiener process based degradation model is proposed. The switches between states of normal operation and maintenance are described by a continuous time Markov chain (CTMC). In addition, the maximum likelihood estimation (MLE) is adopted to estimate both unknown parameters in the degradation model and the transition probability between normal operation and maintenance. The analytical form of first hitting time (FHT) of degradation process is difficult to derive with the presence of maintenance activities. To avoid complicated mathematical derivation of stochastic differential, Monte Carlo method is used to obtain a numerical result of the RUL distribution. A numerical study is presented to illustrate and validate the proposed method.**

Keywords—Remaining useful life; maintenance; continuous time Markov chain; first hitting time.

I. INTRODUCTION

Since remaining useful life (RUL) prediction has been widely considered as one of the most important procedures of prognostics and health management (PHM) [1-3], it has attracted comprehensive attention and experienced substantial development in the past decades [3]. However, most of the existing degradation models are developed based on the assumption that there is no maintenance activity taken into practice, or the degradation processes will not be influenced by the maintenance activities. This assumption is not reasonable for practical industrial equipment. In practice cases, there are varies maintenance activities will be carried out during the whole life time of the equipment, such as periodic, stochastic, on-condition and predictive maintenance. These maintenance activities will inevitably change the RUL and reliability of the equipment [4,5].

The difficulties of predicting the RUL of degrading systems with maintenance can be concluded in two aspects. One is that the maintenance is hard to be quantized in the degradation model. And the other one is that the first hitting time (FHT) of degradation process with maintenance is complex to deduce. Literature [6] proposed a degradation model of a kind of degrading systems with maintenance by the Wiener process with jumps. However, it is under the assumption that the maintenance activities are finished instantaneously or executed during downtime. For some industrial equipment which does not allow for downtime, this method loses the efficiency.

It is worth mentioning, a similar problem of RUL prediction for degrading systems with environment or operational state switches has been studied with similar models [7-9]. Literature [7] used a discrete-time Markov chain (DTMC) to describe the controlled environments, but this literature did not provide the PDF of RUL. Besides, literature [8] and [9] only consider the linear degradation.

In this paper, we develop a nonlinear degradation model which describes the switches between normal operation and maintenance by a continuous time Markov chain (CTMC). That is to say, every maintenance activity will last for some time, and the durations of maintenance activities obey a certain probability. The remaining contents of this paper are organized as follows. In Section II, the degradation model is proposed to describe the degradation of degrading system with maintenance. An estimation method is proposed in Section III to estimate the unknown parameters in the degradation model. In Section IV, an RUL prediction algorithm using Monte Carlo method is given. To demonstrate the effectiveness of the proposed method, a numerical study is given in Section V. The main conclusions of this paper are discussed in Section VI.

II. PROBLEM FORMULATION

Owing to the clear physical significance and excellent mathematical properties, Wiener process has been widely used in the modeling for degradation [3,10]. In this section, to formulate the degradation of degrading systems with stochastic maintenance, we adopt a Wiener process based model as follows:

$$X(t) = x(0) + \int_0^t \mu\left[u;\theta,\Phi(u)\right]du + \sigma B(t) \tag{1}$$

where $x(0)$ is an initial value of the degradation, $B(t)$ is a standard Brown motion, σ is the diffusion coefficient, and $\mu\left[t;\theta,\Phi(t)\right]$ is the drift coefficient. For the degrading systems with stochastic maintenance, there are two states need

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to be considered in the modeling. One is the state of normal operation, and the other is the state of maintenance. To describe the degradation of these two states, assume that the drift coefficient $\mu\left[t;\theta,\Phi(t)\right]$ is a time-varying function depending on the unknown parameter set θ and the state at time t, which is as $\Phi(t)$.

$$\Phi(t)=\begin{cases}0, \text{ normal opertation}\\1, \text{ maintenance}\end{cases} \tag{2}$$

As shown in Fig.1, for the degrading systems with stochastic maintenance, the states of normal operation and maintenance will switch back and forth.

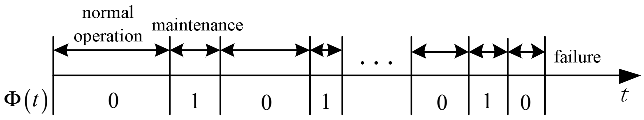

Fig.1. Sketch of the stochastic maintenance

Here assume that the switches between the two states follow a certain probability. Then we describe the switches as a CTMC with a transition rate matrix $\boldsymbol{Q}$:

$$\boldsymbol{Q}=\begin{bmatrix}-q_0 & q_0\\ q_1 & -q_1\end{bmatrix} \tag{3}$$

As a consequence, $\mu\left[t;\theta,\Phi(t)\right]$ is a piecewise function. To enlarge the scope of application, $\mu\left[t;\theta,\Phi(t)\right]$ can be a nonlinear function. In this paper, we focus on the following functional form:

$$\mu\left[t;\theta,\Phi(t)\right]=\begin{cases}abt^{b-1},\Phi(t)=0\\ c,\Phi(t)=1\end{cases} \tag{4}$$

where a, b and c are both unknown constants for a degrading system. According to the rule of degradation, the degradation process $X(t)$ will in a rising trend during the state of normal operation while in a declining trend during the state of maintenance, so we have $a>0$ and $c<0$. It is worth to mention that the RUL predictions can also be obtained for other functional forms of $\mu\left[t;\theta,\Phi(t)\right]$. Without loss of generality, the initial value of the degradation is assumed as zero, i.e. $x(0)=0$.

III. Estimation of Parameters

For the degradation model (1), there are two parts of parameters need to be estimated. One part includes the parameters in the transition rate matrix of the CTMC, and the other part includes the parameters in the drift coefficient and the diffusion coefficient. Here we present a two-stage estimation method to estimate the two parts of unknown parameters.

A. Estimation of transition rate matrix

In this stage we estimate q_0 and q_1 by the maximum likelihood estimation (MLE). Because most of practical systems have maintenance records, the history of maintenance is assumed to be known. According to the characteristics of CTMC, we can obtain the probability of residence time as follows:

$$f_{\delta_{i,j}}\left(\delta_{i,j}\right)=q_i e^{-q_i\delta_{i,j}} \tag{5}$$

where $\delta_{i,j}$ denotes the residence time of the *j*th time reach the state *i*. Then we deduce the following log-likelihood function of the transition rates q_0 and q_1:

$$\ell\left(q_0,q_1\right)=n_{01|\tilde{t}_k}\ln q_0+n_{10|\tilde{t}_k}\ln q_1-q_0\delta_{0|\tilde{t}_k}-q_1\delta_{1|\tilde{t}_k} \tag{6}$$

where $\tilde{t}_k$ is the last transition time before t_k, $n_{\bar{i}i|\tilde{t}_k}$ is the amount of transition from state $\bar{i}$ to state i before time $\tilde{t}_k$, and $\delta_{i|\tilde{t}_k}=\sum_{j=1}^{n_{\bar{i}i|\tilde{t}_k}}\delta_{i,j}$ denotes the total residence time of the state i before time $\tilde{t}_k$.

Next, we maximize the log-likelihood function (6) by setting the partial derivatives with respect to q_0 and q_1 equal to zero. The estimates can be deduced as follows:

$$\hat{q}_i=\frac{n_{\bar{i}i|\tilde{t}_k}}{\delta_{i|\tilde{t}_k}} \tag{7}$$

B. Estimation of parameters in degradation model

To predict the RUL of degradation process (1), we also need to estimate the unknown parameters in the drift coefficient and the diffusion coefficient. According to (1) and (2), we construct the unknown parameter set $\Theta=\{a,b,c,\sigma\}$. The degradation process (1) can be discretized as:

$$x_{k+1}=x_k+\int_{t_k}^{t_{k+1}}\mu\left[u;\theta,\Phi(u)\right]du+\varepsilon_k \tag{8}$$

where t_k is the *k*th sampling time, $x_k=X\left(t_k\right)$ is the degradation state at the *k*th sampling time. Since $\varepsilon_k=\sigma\left[B\left(t_{k+1}\right)-B\left(t_k\right)\right]$, then we have $\varepsilon_k\sim N\left(0,\sigma^2\tau_k\right)$, where $\tau_k=t_{k+1}-t_k$. Thus the distributions of increments of the degradation can be obtained as follows:

$$x_{k+1}-x_k\sim\begin{cases}N\left(at_{k+1}^b-at_k^b,\sigma^2\tau_k\right),\phi_k=0\\ N\left(c\tau_k,\sigma^2\tau_k\right),\phi_k=1\end{cases} \tag{9}$$

where $\phi_k=\Phi\left(t_k\right)$ denotes the state at time t_k. Then the log-likelihood function of the unknown parameter set Θ can be written as:

$$\ell(\Theta) = -\frac{k-1}{2}\ln 2\pi - (k-1)\ln\sigma - \frac{1}{2\sigma^2}\times \left\{ \sum_{j=1,\phi_j=0}^{k-1}\left[x_{j+1}-x_j-\left(at_{j+1}^b-at_j^b\right)\right]^2 + \sum_{j=1,\phi_j=1}^{k-1}\left[x_{j+1}-x_j-c\tau_k\right]^2 \right\} \tag{10}$$

To maximize the log-likelihood function (10), we obtain the estimates of a and c by setting the partial derivatives of the log-likelihood function (10) with respect to a and c equal to zero, where

$$\hat{a} = \frac{\sum_{j=1,\phi_j=0}^{k-1}\left(x_{j+1}-x_j\right)\left(t_{j+1}^b-t_j^b\right)}{\sum_{j=1,\phi_j=0}^{k-1}\left(t_{j+1}^b-t_j^b\right)^2} \tag{11}$$

$$\hat{c} = \frac{\sum_{j=1,\phi_j=1}^{k-1}\left(x_{j+1}-x_j\right)\tau_j}{\sum_{j=1,\phi_j=1}^{k-1}\tau_j^2} \tag{12}$$

However, owing to the existence of the nonlinear term, the analytical form of estimates of b and σ are difficult to obtain. Here optimizing approach can be adopted. To simplify the likelihood function, we feed $\hat{a}$ into the likelihood function (10) and only reserve the terms related to b and σ. Then a new log-likelihood function of b and σ can be obtained that

$$\ell'(b,\sigma) = -(k-1)\ln\sigma - \frac{1}{2\sigma^2}\times \sum_{j=1,\phi_j=0}^{k-1}\left[x_{j+1}-x_j-\frac{\sum_{i=1,\phi_j=0}^{k-1}\left(x_{i+1}-x_i\right)\left(t_{i+1}^b-t_i^b\right)}{\sum_{i=1,\phi_j=0}^{k-1}\left(t_{i+1}^b-t_i^b\right)^2}\left(t_{j+1}^b-t_j^b\right)\right]^2 \tag{13}$$

The log-likelihood function (13) can be maximized by the joint optimizing approach offered by the function "fminsearch" in Matlab [11]. Then $\hat{a}$ can be deduced by (11) with the estimate of b.

IV. RUL Prediction

Now we return to predict the RUL of the degradation process (1). Identical to most existing literature, here we define the RUL as the first time when the degradation process reaches a known failure threshold, namely the first hitting time (FHT) [12], which can be denoted as

$$RUL_k : \Delta_k = \inf\{l_k : X(t_k + l_k) \geq \omega | x_{0:k}, \hat{\Theta}_k, l_k \geq 0\} \tag{14}$$

On account of the effect of the maintenance, analytical form of the PDF of the RUL is very difficult to infer. So, in this section we attempt to obtain a simulation solution by Monte Carlo method, as it can get an accurate result by a large quantity of random sampling, and thus avoid the complicated mathematical derivation sequentially. According to the transition rate matrix $\boldsymbol{Q}$, the corresponding state transition probability with the time interval τ_k can be calculated by

$$p_{01}(\tau_k) = \frac{q_0}{q_0+q_1} - \frac{q_0}{q_0+q_1}e^{-(q_0+q_1)\tau_k} \tag{15}$$

$$p_{10}(\tau_k) = \frac{q_1}{q_0+q_1} - \frac{q_1}{q_0+q_1}e^{-(q_0+q_1)\tau_k} \tag{16}$$

where $p_{ij}(\tau_k) = P\{\Phi(t+\tau_k) = j \mid \Phi(t) = i\}, t \geq 0, i, j = 0,1$. With the time interval τ_k, the degradation state x_k can be recurred by x_{k-1},

$$\hat{x}_k = \begin{cases} x_{k-1} + a\left(t_{k+1}^b - t_k^b\right) + \gamma_k, \phi_k = 0 \\ x_{k-1} + c\tau_k + \gamma_k, \phi_k = 1 \end{cases} \tag{17}$$

where γ_k is a random number obeys the normal distribution $N\left(0,\sigma^2\tau_k\right)$. The specific RUL prediction algorithm at the kth sampling time t_k based on Monte Carlo method is presented in Algorithm 1.

Algorithm 1

1) Initialize the related parameters, choose the Monte Carlo sampling times n, and set $i = k+1$;
2) Estimate q_0 and q_1 by (7);
3) Obtain the estimates of b and σ by the "fminsearch" function in Matlab;
4) Calculate the estimates of a and c by (11) and (12);
5) Generate n dimensional vector $\hat{\boldsymbol{X}}_i$ of $\hat{x}_i$ by (17);
6) If $\phi_{i-1} = 0$, generate a random number ξ_i from 0 to $\frac{1}{p_{01}}$, else generate a random number ξ_i from 0 to $\frac{1}{p_{10}}$;
7) If $\xi_i \leq 1$, set $\phi_i = \overline{\phi}_{i-1}$, else set $\phi_i = \phi_{i-1}$;
8) If all the elements in $\hat{\boldsymbol{X}}_i$ reach the threshold ω, go to the next step. Otherwise, set $i = i+1$ and go to step 5;
9) The probability of RUL can be obtained approximatively by recording the frequency of the FHT of the samples.

V. Numerical Example

To illustrate the effectiveness of the developed method, we present a numerical example in this section. The simulation data is generated by Matlab according to the degradation process (1). Set $q_0 = 0.01$, $q_1 = 0.125$, $a = 0.6$, $b = 1.2$,

$c = -20$, $\sigma = 8$ and $\tau = 1$. The data length is $N = 5000$, and the generated degradation path is shown in Fig.2.

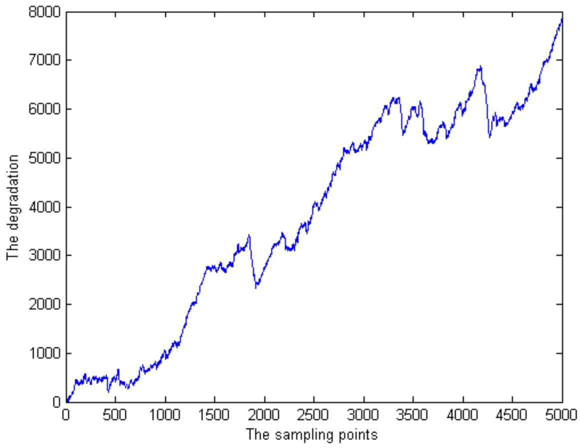

Fig.2. The degradation of a system with stochastic maintenance

Let $k = 4000$, and we use the data before t_k to estimate the unknown parameters. According to (7), we have $\hat{q}_0 = 0.0094$ and $\hat{q}_1 = 0.1068$. Then b and σ can be estimated by the "fminsearch" function in Matlab. To verify the profile likelihood function (13) is convex, we plot a 3-D surface of the profile likelihood function in Fig.3. The results are $\hat{b} = 1.1961$ and $\hat{\sigma} = 8.0745$. Using (11) and (12), we also obtain the estimates of a and c, where $\hat{a} = 0.6211$ and $\hat{c} = -20.1605$.

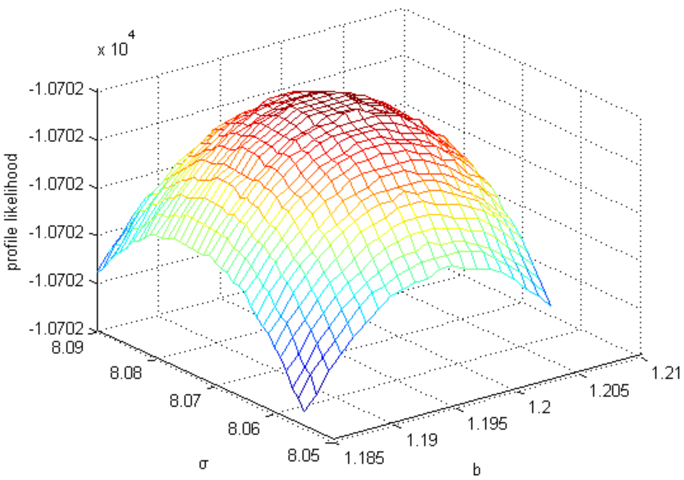

Fig.3. The profile likelihood of b and σ

According to (15) and (16), we have $\hat{p}_{01}(\tau) = 0.0089$ and $\hat{p}_{10}(\tau) = 0.1009$. Then we predict the PDF of the RUL of the degradation. In order to illustrate the superiority of our model compared with the traditional model, which neglects maintenance activities, we provide a comparison here. Considering a degradation model as

$$X(t) = x(0) + a_s t^{b_s} + \sigma_s B(t) \tag{18}$$

Here we denote the degradation process (1) as M_0, and denote the degradation process (18) as M_1 for convenience. For fair comparison, we also utilize MLE to estimate the unknown parameters in M_1. The log-likelihood function is

$$\ell(a_s, b_s, \sigma_s) = -\frac{k}{2}\ln 2\pi - k\ln\sigma_s - \frac{1}{2\sigma_s^2}\sum_{j=1}^{k}\left[x_j - x_{j-1} - a_s\left(t_j^{b_s} - t_{j-1}^{b_s}\right)\right]^2 \tag{19}$$

Then the estimate of a_s can be obtained directly, while the estimates of b_s and σ_s can be obtained by the function "fminsearch" in Matlab, where

$$\hat{a}_s = \frac{\sum_{j=1}^{k}\left(x_j - x_{j-1}\right)\left(t_j^{b_s} - t_{j-1}^{b_s}\right)}{\sum_{j=1}^{k}\left(t_j^{b_s} - t_{j-1}^{b_s}\right)^2} \tag{20}$$

The estimate results are $\hat{a}_s = 0.0424$, $\hat{b}_s = 1.4543$ and $\hat{\sigma}_s = 4.2340$, and the RUL of M_1 can also be obtained by Monte Carlo method. In order to contrast the RUL prediction results, we draw the PDFs of the RUL at different time obtained by M_0 and M_1 respectively in Fig.4 and Fig.5.

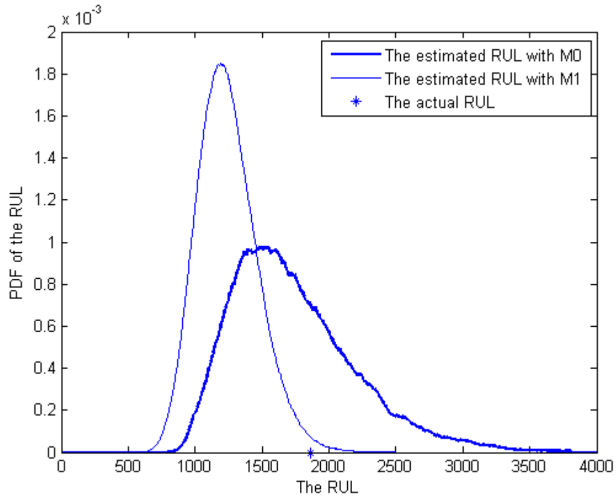

Fig.4. The estimated RUL at $t = 3000$ with M_0 and M_1, respectively

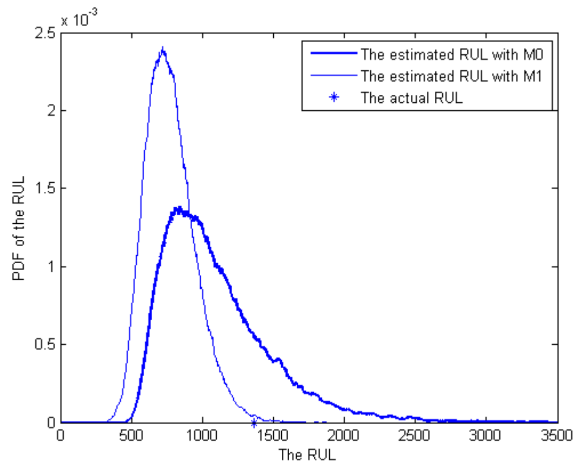

Fig.5. The estimated RUL at $t = 3500$ with M_0 and M_1, respectively

In Fig.4 and Fig.5, the thick lines are the predicted PDFs of the RUL obtained by the proposed degradation model (1), while the fine lines are the predicted PDFs of the RUL obtained by the traditional model neglecting maintenance activities. The true RUL of the generated data is marked by an asterisk. It is obvious that the RUL obtained by the proposed degradation model is more accurate. The PDFs of RUL at different sampling instants are given in Fig.6, from which we see the RUL prediction results are basically correct. In addition, the PDFs of RUL are more accurate and possess a smaller variance, if it is closer to the failure time.

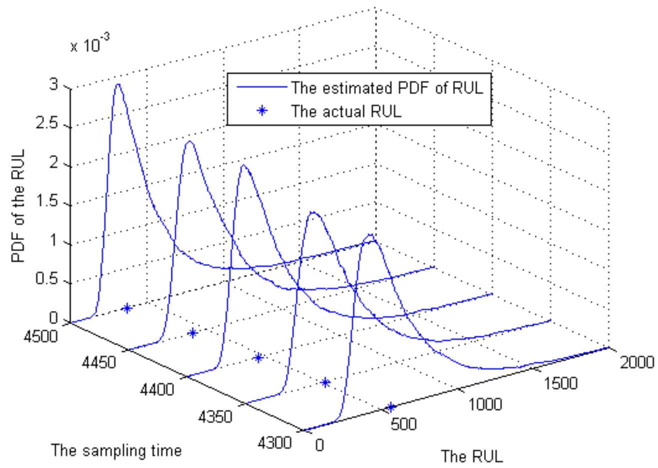

Fig.6. The estimated RUL

VI. Conclusion

It is noted that most of industrial equipment will experiences various maintenance activities during their whole life time. To predict the RUL of the degrading systems with maintenance, we utilize a Wiener process based model with a piecewise drift coefficient. The state switches between normal operation and maintenance is described by a CTMC. A parameter estimation method is proposed to estimate the transition probability and the unknown parameters in the degradation model. The PDF of RUL is obtained by Monte Carlo method. A numerical study demonstrates that the proposed model is more suitable than the traditional model for the degradation influenced by maintenance. However, in this paper we only considered one type of stochastic maintenance, while, in practical cases, the maintenance activities are different for different condition operation. The RUL prediction for degrading systems with maintenance still needs further research.

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