An Adaptive State-Space Model for Predicting Remaining Useful Life of Planetary Gearbox

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***Abstract*-State-space model is often used to estimate system degradation. However, the state-space model is usually only one observation equation in existing research, and it only applies to one observation indicator. This paper puts forward an adaptive state-space model, which can be applied to multiple observation indicators. The modeling method and parameter estimation approach of adaptive state-space model are studied. In order to show that the proposed model is better than traditional state-space model, planetary gearbox remaining useful life (RUL) estimation based on adaptive state-space model is carried out.**

Keywords- state-space model; observation equation; remaining useful life; planetary gearbox

I. INTRODUCTION

System degradation process is the result of a series of physical or chemical reaction. State indicator that can directly describe system degradation state, but it is often unable or difficult to directly measure, such as abrasion, corrosion, crack, etc.. To be clear, condition monitoring is an indirect observation for system degradation process. Condition monitoring data indirectly reflects system health status. The evolution of the system degradation state is the main root cause for changing in condition monitoring data. Therefore, it can use the state-space model to describe system degradation process through observation indicator [1,2].

The state-space model is usually only one observation equation in existing research [1-8], and it only applies to one observation indicator. However, there is more than one observation parameter in system condition monitoring in some cases. For example, when some mechanical systems (such as engine, gearbox) are on condition monitoring, not only monitoring its vibration state, but also monitoring its oil state. For bearing condition monitoring, in addition to collecting vibration data will also collect pulse data by SPM. In these cases, the observation indicator on condition monitoring is more than one. And traditional state-space model cannot be applicable to these cases.

In order to solve this problem, this paper proposes an adaptive state-space model, which will multiple observation equations. And it can be applied to multiple observation indicators.

II. TRADITIONAL STATE-SPACE MODEL

A. Model Formulation

State-space model consists of two components. The first component is state equation, which used to represent the system degradation process. The second component is termed as the observation equation. The traditional state-space model is shown as formula (1), there is only one observation equation.

$$\begin{cases} x_{i+1} = F_i(x_i, \vartheta) + \xi \\ y_i = H_i\left(x_i, \varphi\right) + \varepsilon \end{cases} \tag{1}$$

where y_i is the observation quantity for degradation state x_i. $F_i(*)$ is the state evolution equation, $H_i(*)$ is the observation equation. ϑ, φ are function parameters and ξ, ε are noises for state evolution equation and observation equation, respectively.

As Gamma process has the properties such as stable and independent increment, it is suitable for describing monotonic degradation [3,4]. System degradation process is assumed to be Gamma distribution in this study. In other words, the degradation process of system is considered to be smooth Gamma process, the shape-parameter is $\eta(t) = a \cdot t^b$ and the scale-parameter is ξ, the observation equation is $y = c \cdot x^d + \varepsilon$. Therefore, formula (1) can be expressed as:

$$\begin{cases} x_i - x_{i-1} \sim Gamma(\eta(t_i) - \eta(t_{i-1}), \xi) \\ y_i = c \cdot \left(x_i\right)^d + \varepsilon \end{cases} \tag{2}$$

When the observation noises are assumed to be normal distribution and the expectation is 0, the observation indicator $y \sim N(c \cdot x^d, \sigma)$. Therefore, the system degradation state space model can be determined after estimating the parameters a, b, ξ, c, d, σ.

B. Parameter Estimation

To facilitate the description, let $\theta = (\theta_1, \theta_2)$ and $\theta_1 = (a, b, \xi)$, $\theta_2 = (c, d, \sigma)$. If the observation indicator sequence $y_{0:n}$ and the monitoring time sequence $t_{0:n}$ have been given,

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the E steps assesses the expectation of the complete likelihood function that can be decomposed into two component as

$$
\begin{aligned}
&Q\left(\theta|\theta^{(l)}\right)=E\left\{L(\theta|z)\,|\,y_{0:n},\theta^{(l)}\right\}= \\
&E\left\{\log f_{Y|X}(y_{0:n}\,|\,x_{0:n},\theta_2)\,|\,y_{0:n},\theta^{(l)}\right\}+E\left\{\log f_X(x_{0:n}\,|\,\theta_1)\,|\,y_{0:n},\theta^{(l)}\right\}
\end{aligned}
\tag{3}
$$

where, $\theta^{(l)}$ is the initial parameters of the l-th iteration cycle. The two components of (3) can be further calculated as (4) and (5), respectively.

$$
\begin{aligned}
&E\left\{\log\prod_{i=1}^{n} f_{Y|X}(y_{0:n}\,|\,x_{0:n},\theta_2)\,|\,y_{0:n},\theta^{(l)}\right\}= \\
&-n\ln\sigma-\frac{n}{2}\ln 2\pi-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(\left(y_i\right)^2-2y_i cE\left(\left(x_i\right)^d\right)+c^2E\left(\left(x_i\right)^{2d}\right)\right)
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&E\left\{\log f_X(x_{0:n}\,|\,\theta_1)\,|\,y_{0:n},\theta^{(l)}\right\}= \\
&\sum_{i=2}^{n}\left(\begin{aligned}&a\cdot\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)\ln\xi-\ln\Gamma\left(a\cdot\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)\right)+ \\ &\left(a\cdot\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)-1\right)E\left(\ln(x_i-x_{i-1})\right)-\xi\left(E(x_i)-E(x_{i-1})\right)\end{aligned}\right)
\end{aligned}
\tag{5}
$$

Set $u_i=a\cdot\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)$, $v_i=x_i-x_{i-1}$ and $i=2,3,\ldots,n$. (5) can be written as

$$
\begin{aligned}
&E\left\{\log f_X(x_{0:n}\,|\,\theta_2)\,|\,y_{0:n},\theta^{(l)}\right\}= \\
&\sum_{i=2}^{n}\left(-u_i\ln\xi-\ln\Gamma\left(u_i\right)+\left(u_i-1\right)E\left(\ln v_i\right)\right)-\frac{1}{\xi}\left(E\left(x_n\right)-E\left(x_1\right)\right)
\end{aligned}
\tag{6}
$$

By the extremum solution conditions, it can be known that $Q\left(\theta\,|\,\theta^{(l)}\right)$ is maximum when $\frac{\partial Q(\theta\,|\,\theta^{(l)})}{\partial\theta}=0$. Therefore, the initial parameters of the (l+1)-th iteration cycle $\theta^{(l+1)}$ can be obtained as follow.

By calculating the partial derivative of variables c,σ,d in (4), respectively. The parameters c,d,σ can be described as

$$
c=\frac{\sum_{i=1}^{n}\left(y_iE\left(\left(x_i\right)^d\right)\right)}{\sum_{i=1}^{n}E\left(\left(x_i\right)^{2d}\right)}
\tag{7}
$$

$$
\sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(y_i\right)^2-2y_icE\left(\left(x_i\right)^d\right)+\left(c\right)^2E\left(\left(x_i\right)^{2d}\right)\right)}
\tag{8}
$$

$$
\begin{aligned}
&\sum_{i=1}^{n}E\left(\left(x_i\right)^{2d}\right)\cdot\sum_{i=1}^{n}\left(y_iE\left(\left(x_i\right)^d\ln x_i\right)\right)- \\
&\sum_{i=1}^{n}\left(y_iE\left(\left(x_i\right)^d\right)\right)\cdot\sum_{i=1}^{n}E\left(\left(x_i\right)^{2d}\ln x_i\right)=0
\end{aligned}
\tag{9}
$$

The parameter estimate result $\hat{d}$ can be obtained according to (9). Thereafter, the parameter estimate results $\hat{c}$ and $\hat{\sigma}$ can be calculated as (7, 8).

Similarly, by calculating the partial derivative of variables ξ,a,b in (6), respectively. The relationships between parameters ξ,a,b can be represented as ($a\neq 0$)

$$
\xi=\frac{\sum_{i=1}^{n}\left(E(x_i)-E(x_{i-1})\right)}{a\cdot\sum_{i=1}^{n}\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)}=\frac{E(x_n)-E(x_1)}{a\cdot\left(\left(t_n\right)^b-\left(t_1\right)^b\right)}
\tag{10}
$$

$$
\left(\left(t_n\right)^b-\left(t_1\right)^b\right)\cdot\ln\xi+\sum_{i=2}^{n}\left(\left(\left(t_i\right)^b-\left(t_{i-1}\right)^b\right)\left(\frac{\Gamma'(u_i)}{\Gamma(u_i)}-E\left(\ln v_i\right)\right)\right)=0
\tag{11}
$$

$$
\sum_{i=2}^{n}\left(\left(\ln\xi+\frac{\Gamma'(u_i)}{\Gamma(u_i)}-E\left(\ln v_i\right)\right)\left(\left(t_i\right)^b\cdot\ln t_i-\left(t_{i-1}\right)^b\cdot\ln t_{i-1}\right)\right)=0
\tag{12}
$$

The parameter estimate results $\hat{\xi},\hat{a},\hat{b}$ can be calculated according to (10, 11, 12).

Substituting parameter estimate results $\theta^{(l)}$ and $\theta^{(l+1)}$ into (3), if it satisfies the condition of convergence, the parameter estimate result is considered to be $\hat{\theta}=\theta^{(l+1)}$; if it does not satisfy the condition of convergence, let $\theta^{(l+1)}$ as the initial parameter of (l+1)-th step cycle for the E steps until the condition of convergence is satisfied. The condition of convergence has been studied in Zhou's literature [5,6]. And it also can be facilitated as

$$
\left|\,Q\left(\theta\,|\,\theta^{(l+1)}\right)-Q\left(\theta\,|\,\theta^{(l)}\right)\,\right|<Q_{thre}
\tag{13}
$$

Q_{thre} is the threshold value of the condition of convergence, it can be given as a rule of thumb.

It is not difficult to find from (7) to (12) that parameter estimate result $\theta^{(l+1)}$ can be calculated as long as $E(x_i)$, $E((x_i)^d)$, $E((x_i)^{2d})$ and $E(\ln(x_i-x_{(i-1)}))$ are known. However, there are only a small number of degradation state quantity x can be achieved in actual condition monitoring process. And, cannot be ignored, the degradation state quantity x has a certain degree of uncertainty.

As the advantages of particle filter (PF) [7,8], $E(x_i)$, $E((x_i)^d)$, $E((x_i)^{2d})$ and $E(\ln(x_i-x_{(i-1)}))$ can be estimated by PF algorithm. The results are shown as follows.

$$\begin{cases} E\left(x_i\right) \approx \sum_{s=1}^{N_s} w_i^s \cdot x_i^s \\ E\left(\left(x_i\right)^d\right) \approx \sum_{s=1}^{N_s} w_i^s \cdot \left(x_i^s\right)^d \\ E\left(\left(x_i\right)^{2d}\right) \approx \sum_{s=1}^{N_s} w_i^s \cdot \left(x_i^s\right)^{2d} \\ E\left(\ln\left(v_i\right)\right) \approx \sum_{s=1}^{N_s} w_i^s \cdot \ln\left(x_i^s - x_{i-1}^s\right) \end{cases} \tag{14}$$

where, N_s is the particle number of x_i, x_i^s is the s-th particle of x_i, and w_i^s is the weight of x_i^s.

The process of state space model parameter estimation is shown in Fig. 1.

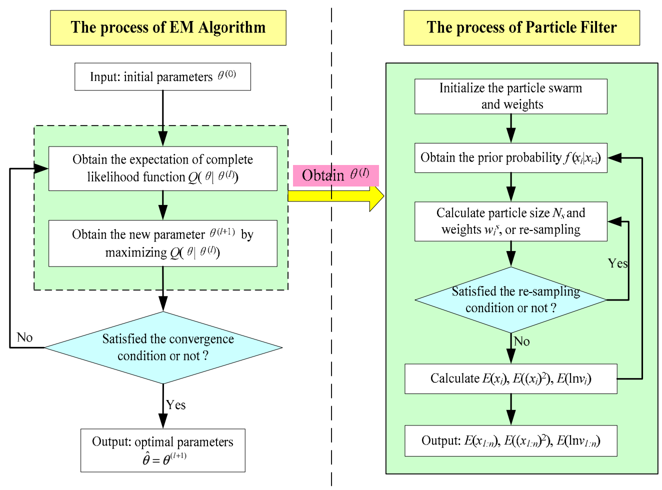

Figure 1. The parameter estimation procedure of traditional state-space model

III. ADAPTIVE STATE-SPACE MODEL

A. Model Formulation

Adaptive state-space model is put forward according the situation that there may be a variety of observation indicators. As a result, in adaptive state-space model, the number of observation equations should be modeled as the number of observation indicators collected in condition monitoring. That is to say, the established state-space model should be have n observation equations if there are n observation indicators collected in condition monitoring. The common format of adaptive state-space model, which is proposed in this study, can be represented as:

$$\begin{cases} x_{i+1} = F_i(x_i, \vartheta) + \xi \\ y_i^1 = H_i^1\left(x_i, \varphi^1\right) + \varepsilon^1 \\ y_i^2 = H_i^2\left(x_i, \varphi^2\right) + \varepsilon^2 \\ \qquad \vdots \\ y_i^n = H_i^n\left(x_i, \varphi^n\right) + \varepsilon^n \end{cases} \tag{15}$$

where, x_i (i=1,2,$\cdots$,m) is the system degradation state at working time t_i, x_0 is initial degradation state (as shown in Fig. 2), y_i^j is the jth (j=1,2,$\cdots$,n) observation indicator of x_i, $F_i(\bullet)$ is the degradation state evolution equation, $H_i^j(\bullet)$ is the observation equation of y_i^j, ϑ is the function parameter of $F_i(\bullet)$, φ^j is the function parameter of $H_i^j(\bullet)$, ξ and ε^j are noises of $F_i(\bullet)$ and $H_i^j(\bullet)$, respectively.

B. Parameter Estimation Framework

The sampling time and sampling frequency of different observation indicator might be inconformity due to the different data collection method. As a result, it is improperly select observation indicator according to MSE and contribution ratio as Zhou proposed in literature [5,6]. Meanwhile, the number of function parameters in adaptive state-space model is too many to evaluate, parameter evaluation cannot be completed just one-time. In order to solve this problem, function parameters can be separately evaluated.

Refer to parameter estimation approach of state-space model with single observation equation [5,6], the parameter estimation process of proposed state-space model is respectively carried out by the state indicator x and the j-th observation indicator y^j. In parameter estimation process, as shown in Fig. 1, ϑ_j, φ_j are the assessment results by x and y^j. ϑ can be obtained from comprehensive analysis all ϑ_j. For example, ϑ_j can be given corresponding weight α_j, $\sum_{j=1}^{n}\alpha_j = 1$, and ϑ can be obtained as

$$\vartheta = \sum_{j=1}^{n} \alpha_j \bullet \vartheta_j \tag{16}$$

The l-th final assessment result is $\theta^{(l)} = \left(\vartheta^{(l)}, \xi^{(l)}, \varphi_j^{(l)}, \varepsilon_j^{(l)}\right)$. Put the l-th final assessment result $\theta^{(l)}$ into the (l+1)-th parameter estimation process, and the (l+1)-th final assessment result $\theta^{(l+1)}$ can be obtained. The parameter estimation procedure of adaptive state-space model is shown in Fig. 3.

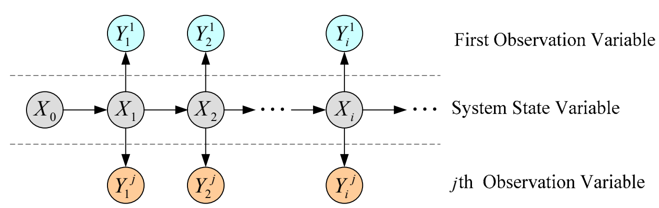

Figure 2. The relationship between state indicator and observation indicators

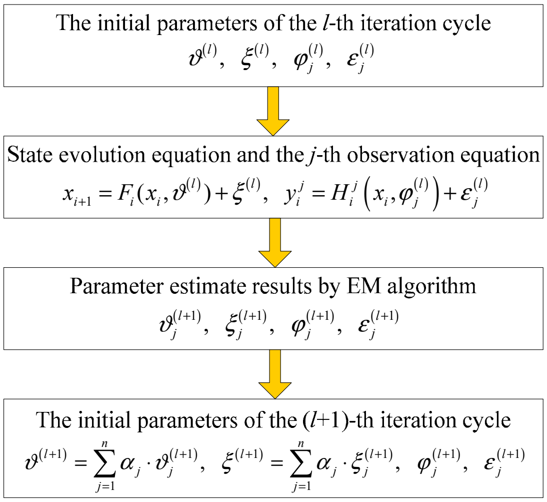

Figure 3. The parameter estimation procedure of adaptive state-space model

C. RUL Prediction

The system RUL cumulative distribution function (CDF) can be obtained as (17).

$$F(t_r \mid y_{0:c})=\sum_{i=1}^{N_s}\frac{\Gamma(a\cdot t_r,(X_f-x_c^s)/\xi)}{\Gamma(a\cdot t_r)}\cdot w_c^s \tag{17}$$

where t_c is the current time, t_r is the RUL, X_f is the system failure threshold, w_c^s is particle weight.

The system mean RUL is

$$T_r=\int_0^{\infty}\left(1-F\left(t_r \mid t_c\right)\right)dt \tag{18}$$

IV. Planetary Gearbox RUL Estimation

In order to validate the validity of the proposed method, a case study of predicting planetary gearbox RUL is carried out. And a planetary gearbox life-cycle experiment has been done to collect degradation process data.

The planetary gearbox experiment rig is shown in Fig. 4. In the life-cycle experiment, the sampling frequency is 20kHz, the input speed of gearbox is 1000rpm, the load provided by magnetic powder brake is about 340Nm. The total experimental time is 1003 hours. Schematic map of planetary gearbox structure is shown in Fig. 5.

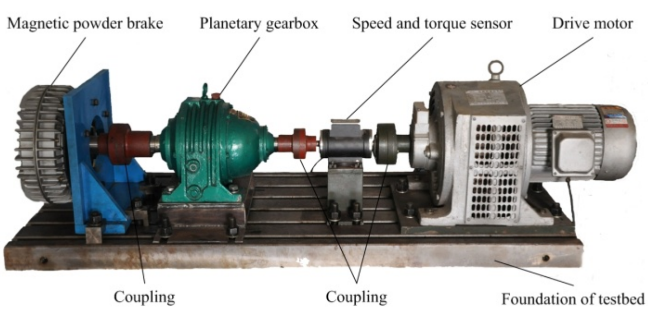

Figure 4. Planetary gearbox experiment rig

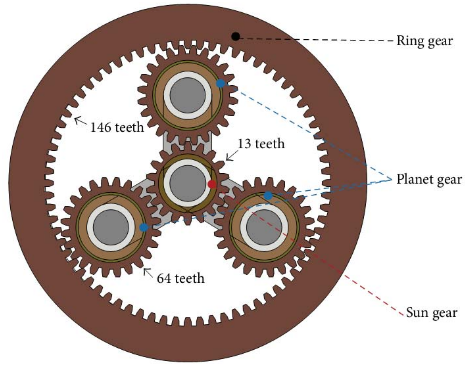

Figure 5. Schematic map of planetary gearbox structure

Three degradation indicators were collected in experiment process, wear data, backlash data and vibration data. Wear data of planetary gear is expressed in Table I, a total of 6 sets of data. The energy of vibration signals is presented in Fig. 6.

TABLE I. WEAR DATA OF PLANETARY GEAR

Time (h)	0	350	720	865	1003
Wear (mm)	0	0.2	0.35	0.45	0.55

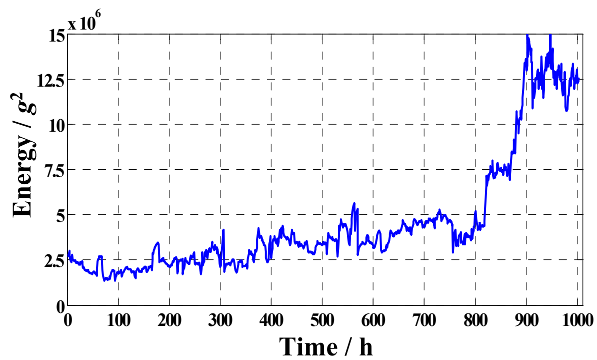

Figure 6. Energy of vibration signals

As shown in Fig. 7, if the gears are intact (namely standard gears), the gap between gears is very small (even without gap) when the gears are meshing; if the gears are wearing, there will be a gap between two gears when gears are meshing, and the more serious the wear is, the bigger the gap is. By gear meshing theory, if there is a gap between two meshing gears, keeping one of gear still, there will be a no-load rotation distance when another gear rotates back and forth. The no-load rotation distance is gear backlash. The gear backlash, to a certain extent, reflects the wear degree of gear.

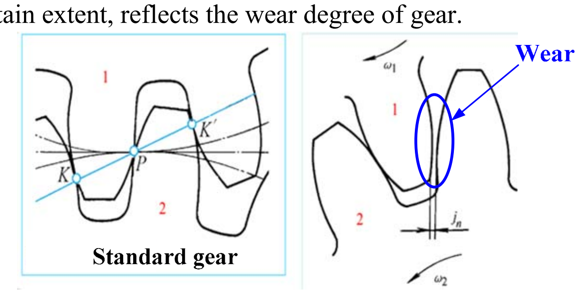

Figure 7. The gap after gear wear

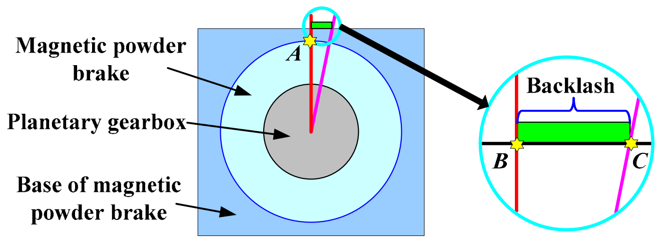

Figure 8. The measuring method of backlash

In the experiment, several sets of gear backlash data of planetary gearbox in different working time are collected. But the time interval of working time is not fixed. The backlash measuring steps can be expressed as follows: (a) Clamping the input shaft and output shaft of gearbox and make sure the teethes is fully engaged; (b) Hold the gearbox input shaft and ensure the input shaft is no longer rotating, then selected a label point A in the magnetic powder brake and mark the label point location B on magnetic powder brake base; (c) Slowly and gently rotate the output shaft until can't turn to make the teeth in output shaft turn from original mesh teeth to another teeth in input shaft. (d) Mark the new label point location C on magnetic powder brake base for the selected label point A, then measuring the distance L between B and C, and L is the collecting backlash in this study, as shown in Fig. 8.

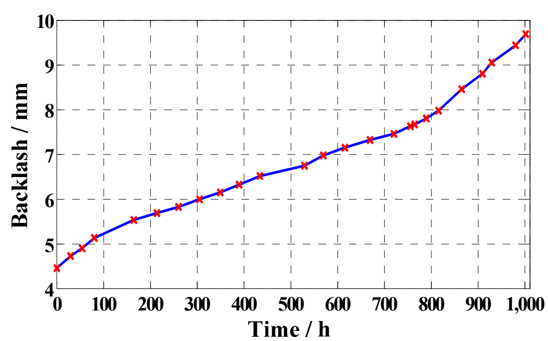

Figure 9. Backlash in experiment

When collect backlash data, after measuring a backlash data, magnetic powder brake turned a certain angle, then measuring another backlash data. A set of backlash data measured at least eight data in a moment, remove the maximum and the minimum, then averaging the remaining data as the final backlash in the working time. For example, the backlash data measured in initial time 0 are 4.40,4.60,4.20, 4.30,4.35,4.55,4.65,4.50 (units are mm), remove the maximum 4.65mm and the minimum 4.20mm, the mean value of the remaining 6 sets data is 4.4500mm and it is the final backlash data. The all backlash data in planetary gearbox life-cycle experiment are shown in Fig. 9.

Wear values is the system state indicator x, backlash is the first observation indicator y, and vibration is the second observation indicator w. The adaptive state-space model of planetary gearbox degradation process is established as:

$$\begin{cases} x_i - x_{i-1} \sim Gamma(a \cdot (t_i - t_{i-1}), \xi) & (21a) \\ y_i = b \cdot x_i + \varepsilon & (21b) \\ w_i = c \cdot x_i + \delta & (21c) \end{cases}$$

where observation noises ε, δ are assumed to be normal distribution, $\varepsilon \sim N\left(0, \sigma_y^2\right)$, $\delta \sim N\left(0, \sigma_w^2\right)$.

The wear quantity of planetary gear in 1003h is 0.55mm, set it as failure threshold, X_f=0.55mm, and $\alpha_1 = \alpha_2 = 0.5$. The parameters of different state-space models are evaluated when working time are 720h and 865h, respectively. The RUL of planetary gearbox are also calculated. The parameter estimation results and predicting RUL are presented in Table III.

It can be known from Table III that the prediction effect of Mod 3 (adaptive state-space model) is better than Mod 1 and Mod 2 (traditional state-space model) no matter the working time is 720h or 865h. The Mod 3 makes full use of backlash data and vibration data to improve the accuracy of state equation, but Mod 1 and Mod 2 just use one of observation data.

TABLE II. THE PARAMETER ESTIMATION RESULTS AND PREDICTING RUL

Model	Time	a	ξ	b	σ_y	c	σ_w	Actual RUL	Predicting RUL	Relative error
Mod 1 $\begin{cases}(21a)\\(21b)\end{cases}$	720	0.0293	47.7714	28.0523	1.7564	--	--	283	342.6443	21.08%
	865	0.0313	46.0445	23.0350	1.9972	--	--	138	162.5775	17.81%
Mod 2 $\begin{cases}(21a)\\(21c)\end{cases}$	720	0.0567	91.1423	--	--	1.255×10^7	5.294×10^5	283	329.8080	16.54%
	865	0.0575	84.3856	--	--	1.322×10^7	6.382×10^5	138	154.9532	12.28%
Mod 3 $\begin{cases}(21a)\\(21b)\\(21c)\end{cases}$	720	0.0442	67.0023	26.5412	1.9071	1.202×10^7	5.135×10^5	283	313.9900	10.95%
	865	0.0465	64.6856	23.5325	1.9080	1.294×10^7	6.563×10^5	138	149.3612	8.23%

V. CONCLUSIONS

This paper proposes a state-space model that can be applied to multiple observation indicators. The modeling method and parameter estimation approach of adaptive state-space model were studied. Adaptive state-space model was used to predict planetary gearbox RUL, and the predicting result shows that adaptive state-space model is better than traditional state-space model.

REFERENCES

[1] J. Z. Sun, H. F. Zuo, W. B. Wang, and M. G. Pecht, "Application of a state space modeling technique to system prognostics based on a health index for condition-based maintenance," Mechanical Systems and Signal Processing, vol. 28, pp. 585-596, 2012.

[2] J. Z. Sun, and H. F. Zuo, "Bayesian method for system real-time performance reliability assessment," Systems Engineering and Electronics, vol. 3, no. 8, pp. 1790-1796, 2013. (in Chinese)

[3] X. L. Ni, J. M. Zhao, J. C. Chen, and H. P. Li. "Planetary gearbox remaining useful life estimation based on state space model," Vibroengineering Procedia, vol. 5, pp. 253-258, 2015.

[4] H. Meier, F. Sourget, and M. Roussignol, "Maintenance optimization for a system with a gamma deterioration process and intervention delay: application to track maintenance," Journal of Risk and Reliability, vol. 223, no. 3, pp. 189-198, 2009.

[5] Y. F. Zhou, L. Ma, J. Mathew, Y. Sun, and R. Wolff. "Asset Life Prediction Using Multiple Degradation Indicators and Failure Events: a Continuous State Space Model Approach," EKSPLOATACJA I NIEZAWODNOSC-Maintenance and Reliability, vol. 4, pp. 72-81, 2009.

[6] Y. F. Zhou, Y. Sun, J. Mathew, R. Wolff, and L. Ma. "Latent Degradation Indicators Estimation and Prediction: A Monte Carlo Approach," Mechanical system and Signal Processing, vol. 25, pp. 222-236, 2011.

[7] F. Cadini, D. Avram, E. Zio. "System state estimation by particle filtering for fault diagnosis and prognosis," Proc. IMechE, Part O: J. Risk and Reliability, vol. 224, pp. 149-158, 2010.

[8] E. Marcos, J. George. "A particle-filtering approach for on-line fault diagnosis and failure prognosis," Transactions of the Institute of Measurement and Control, vol. 3, pp. 221-246, 2009.