Operation Optimization for Centrifugal Chiller Plants Using Continuous Piecewise Linear Programming

Xiaolin Huang, Jun Xu, Shuning Wang
Department of Automation, Tsinghua University
Tsinghua National Laboratory for Information Science and Technology (TNList)
Beijing, 100084, P.R. China
huangxl06@mails.tsinghua.edu.cn, yun-xu05@mails.tsinghua.edu.cn, swang@mail.tsinghua.edu.cn

***Abstract*—Centrifugal chiller plants (CCP) are widely used in air conditioning systems, its operation optimization can save lots of energy and has great significance in environmental protection. The optimization is a large-scale nonlinear problem and there is no practical algorithm until now. This paper proposes a new method to do this operation optimization using continuous piecewise linear programming (CPWLP). The main idea is transforming the nonlinear problem into a series of linear programmings by approximating the original system using piecewise linear representation. For CPWLP, some properties are discussed and an algorithm is given. Using CPWLP, CCP system is optimized and its energy performance is improved significantly.**

***Index Terms*—operation optimization, continuous piecewise linear modeling, nonlinear programming, centrifugal chiller plants**

I. Introduction

Centrifugal chiller plants (CCP) are commonly used to provide chilled water for air-conditioning in buildings. The system includes a series of equipments, which drive refrigerant, cooling water and chilled water cycling in CCP system, see Fig. 1. The optimization objective is to find proper operation meeting the demand of terminal unit with minimal power consuming.

Compared with the environmental variance, the CCP system can achieve steady state very quickly. Hence for the operation optimization of CCP system, we can focus on its steady state which can be described by a physical model. In this physical model, there are hundreds of variables and equations. Many of these equations and the objective function are nonlinear. Due to the complexity, the existing researches only optimize some control variables independently. For example, [1] [2] consider the energy performance at part load operation and do optimization on chiller speed when other parameters are fixed. Some researchers try to save pumping energy by optimizing variable speed pumps ([3][4]). Besides, the performance under different chilled water set points are considered and some optimization strategies are given ([5]). Because these optimization method do not consider all the control variables together, they are not effective when the condition changes. Therefore, in practice, the operation of CCP is obtained by experience or some strategies. The strategies come from experiments on simulation platforms like EOP, HVACSIM$^+$ and TRANSYS, which are built based on physical model ([6][7]).

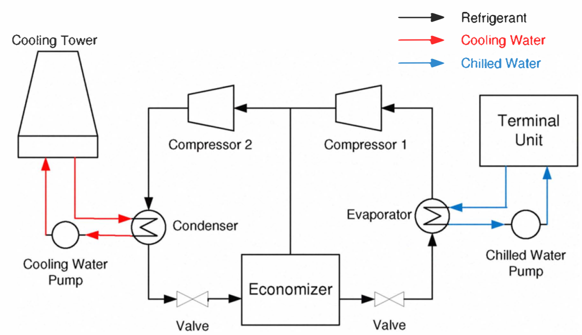

Fig. 1. Physical Structure of Centrifugal Chiller Plant

In this paper, we propose a new optimization method and apply it to CCP system. The method consists of two procedures. First, continuous piecewise linear model is used to approximate the nonlinear relationship. Then in the approximated system, piecewise linear optimization is applied. Using this method, the energy performance of CCP is improved.

Intuitively, piecewise linear approximation is useful for solving nonlinear optimization. On one hand, any continuous nonlinear function can be approximated by a continuous piecewise linear (CPWL) function to arbitrary precision and some practical identification algorithms have been developed. On the other hand, once in a system, the functions are replaced by CPWL functions, the problem may be transformed into a series of linear programming (LP) problems which can be solved by very efficient algorithms. Based on this idea, there are some researches on solving nonlinear optimization through continuous piecewise linear programming (CPWLP). [16] [17] [18] [19] discuss a special problem with a separable nonlinear objective function when all the constrains are linear. The function $f(x)$ is separable if it can be written as a sum of univariate functions, say $f(x) = \sum_{j=1}^{n} f_j(x_j)$. The main idea for solving this problem is to approximate each f_j with a univariate CPWL function, and formulate an equivalent mixed-integer programming by introducing some 0-1 variables and some auxiliary inequalities. Besides this problem, [20] considers a general nonlinear objective function under linear constrains.

The crucial issue for applying CPWLP to nonlinear optimization is that how to approximate a nonlinear system by CPWL functions. Recent progress on CPWL representation and identification provides useful tools to do CPWL approximation. There are some compact representations and practical identification algorithms, like canonical piecewise linear representation ([8]), hinging hyperplanes ([9]), lattice representation ([10]) and generalized hinging hyperplanes (GHH) ([11]). Recently, [13] proposes a new CPWL representation called adaptive hinging hyperplanes (AHH) and gives an adaptive identification algorithm. AHH is based on GHH and multivariate adaptive regression splines ([14]) and shares the advantages of the two. Its approximation capability to any continuous functions with arbitrary precision has been proved. Some experiments for identification in static and dynamic systems show its power and flexibility in black-box modeling ([15]).

The progress on CPWL approximation algorithm make it possible to apply CPWLP in nonlinear optimization. In this paper, the algorithm for CPWLP is proposed. Combined with AHH, we propose the method for solving nonlinear optimization via CPWLP. This method is successfully applied to the operation optimization of CCP system. The rest of this paper is organized as follows. Section II will discuss some properties of CPWLP and propose the algorithm. Then CPWLP and AHH are combined together for optimizing CCP system, reported in Section III. Section IV ends the paper with some conclusions.

II. CONTINUOUS PIECEWISE LINEAR PROGRAMMING

A. Continuous Piecewise Linear Programming Model

A continuous piecewise linear programming (CPWLP) model can be represented as follows,

$$\begin{aligned} &\min\ g(x) \\ \text{s.t.}\quad & H(x)=0 \\ & A(x)\le 0 \\ & x\in\Omega \end{aligned} \tag{1}$$

where $x \in R^n$, Ω is a convex polyhedron, $H(x) = [h_1(x), h_2(x), \dots, h_m(x)]^T$ and $A(x) = [a_1(x), a_2(x), \dots, a_l(x)]^T$. Here $H(x)$ and $A(x)$ are CPWL vector functions, i.e., their elements are all CPWL functions.

Basically, a CPWL function is a continuous function which equals to one of the finite linear functions at any point in its definitional domain. Let $f(x)$ be a CPWL function defined on a convex compact polyhedron $\Upsilon \subseteq R^n$. There will be a set of distinct linear functions $l_i(x), 1 \le i \le N$ as the following,

$$f(x) \in \{l_1(x), l_2(x), \dots, l_N(x)\}, \forall x \in \Upsilon. \tag{2}$$

All the functions in CPWLP are piecewise linear. Using the above definition of a single CPWL function, the functions in CPWLP equal to one of the following functions at any point in Ω, that is,

$$\begin{bmatrix} g(x) \\ H(x) \\ A(x) \end{bmatrix} \in \left\{ \begin{bmatrix} p_1(x) \\ Q_1(x) \\ B_1(x) \end{bmatrix}, \dots, \begin{bmatrix} p_T(x) \\ Q_T(x) \\ B_T(x) \end{bmatrix} \right\}, \forall x \in \Omega,$$

where $\forall i, p_i(x), Q_i(x), B_i(x)$ are linear (vector) functions.

With this description, Ω is identical to the union of the following T sets,

$$\Omega_i = \left\{ x : \begin{bmatrix} g(x) \\ H(x) \\ A(x) \end{bmatrix} = \begin{bmatrix} p_i(x) \\ Q_i(x) \\ B_i(x) \end{bmatrix} \right\}, 1 \le i \le T, \tag{3}$$

and

$$\overset{\circ}{\Omega}_i \bigcap \overset{\circ}{\Omega}_j = \emptyset, \forall i \ne j,$$

where $\overset{\circ}{\Omega}_i$ denotes the inner part of Ω_i. Since $g(x)$, $H(x)$, $A(x)$ are continuous, there are

$$\begin{bmatrix} p_i(x) \\ Q_i(x) \\ B_i(x) \end{bmatrix} = \begin{bmatrix} p_j(x) \\ Q_j(x) \\ B_j(x) \end{bmatrix}, \forall x \in \Omega_i \bigcap \Omega_j, \forall i, j.$$

It is stated in [21] that every subregions like Ω_i is either a convex polyhedron or a union of several convex polyhedra if Ω is a polyhedron. So without loss of generality, we assume Ω and all the sets in (3) are convex polyhedra. Therefore, CPWLP can be transformed into a series of LPs in subregions. Take the one in Ω_i for example, the related LP is

$$\begin{aligned} &\min\ p_i(x) \\ \text{s.t.}\quad & Q_i(x)=0 \\ & B_i(x)\le 0 \\ & x\in\Omega_i \end{aligned} \tag{4}$$

Since (4) is an LP problem, it can be solved by very efficient algorithms. There are three situations: 1) the optimal value is unbounded; 2) there is no feasible solution; 3) get an optimal solution x_i^*.

If there is one subregion, in which the corresponding LP (4) is unbounded, the CPWLP (1) is also unbounded. If in all the subregions, the feasible region of (4) are empty, (1) is infeasible. Otherwise, we can gain the optimal solutions $x_1^*, x_2^*, \dots, x_{T'}^*$ in some subregions, here $1 \le T' \le T$. Comparing $x_1^*, x_2^*, \dots, x_{T'}^*$, the globally optimal solution of (1) can be got. Summing up the above three situations, we know that CPWLP can be solved by finite LPs, since the number of subregions T is finite.

B. Algorithm for Piecewise Linear Programming

Practical problems should not be unbounded or infeasible, the existence of the global optimum of CPWLP thus can be guaranteed. However, if the number of subregions is huge, the searching for the global optimum is very time-consuming. Therefore, we will develop a descent algorithm for local optimum in application.

The definitional domain of CPWLP can be divided into T non-overlapping subregions as (3). For any feasible point x_0, there exists at least one subregion Ω_i satisfying $x_0 \in \Omega_i$. Solving LP problem (4) will get an optimal solution x_i^* in Ω_i. Since $g(x_i^*) \le g(x_0)$, the objective value of CPWLP is nonincreasing. Regarding x_i^* as x_0, another LP can be constructed and solved, then the objective value will be decreased until x_0 is locally optimal. The following proposition will give the necessary and sufficient condition for a feasible point x_0 to be locally optimal.

Proposition 1: Assume x_0 is a feasible point of CPWLP problem (1), there exists an index set $\Psi = \{i : x_0 \in \Omega_i\}$, where Ω_i is the subregion as (3). Then x_0 is locally optimal for (1) if and only if x_0 is the optimum of corresponding LP (4) in $\Omega_i, \forall i \in \Psi$.

Proof: x_0 being locally optimal means $\exists \varepsilon > 0, \forall x \in \Omega$, if $\|x - x_0\| < \varepsilon$ and $H(x) = 0, A(x) \leq 0$, then $g(x_0) \leq g(x)$. If $\exists l \in \Psi$, x_0 is not the optimum in Ω_l, then $\exists x_1 \in \Omega_l, H(x_1) = 0, A(x_1) \leq 0$ and $g(x_1) < g(x_0)$. All the functions of (1) are linear in Ω_l, therefore, $\forall 0 < \xi < 1$, $x_2 = x_0 + \xi \times (x_1 - x_0)$ is a feasible point and $g(x_2) = g(x_0) + \xi \times (g(x_1) - g(x_0)) < g(x_0)$. As ξ is an arbitrary small positive number, given any $\varepsilon > 0$, we can find $\xi > 0$ to get a feasible point $x_2 = x_0 + \xi \times (x_1 - x_0)$, let $\|x_2 - x_0\| < \varepsilon$ and $g(x_2) < g(x_0)$. Then x_0 is not locally optimal.

Now we will consider the sufficiency. As long as $\varepsilon > 0$ is small enough, for any $x \in \Omega, \|x - x_0\| < \varepsilon$, we can find an $l \in \Psi$ making $x \in \Omega_l$. Because x_0 is the optimal solution in Ω_l, there is $g(x_0) \leq g(x)$. Therefore, x_0 is a local optimum. ■

When given a feasible solution x_0, we need a method to get Ω_i satisfying $x_0 \in \Omega_i$. This method differs for different CPWL models. In the latter of this paper, we will use AHH to construct CPWL model. The method for constructing subregion for AHH will be explained in the next section and here we firstly present a descent algorithm for (1). The descent algorithm terminates at a local optimum and Proposition 1 provides the method to judge if a solution x_0 is locally optimal. According to the procedure of proving Proposition 1, we can see that if x_0 is not locally optimal, there will be a subregion in which the corresponding LP (4) can strictly decrease the objective value. Summing up the above discussions, the descent algorithm for CPWLP is proposed below,

Step 1. Let M be a positive integer and generate an initial feasible point x_0;

Step 2. Let $\Psi = \{i : x_0 \in \Omega_i\}$ and $k = 1$;

Step 3. Randomly select an $l \in \Psi$, set $\Psi = \Psi \backslash \{l\}$ and construct an LP in Ω_l;

Step 4. Solve the LP. If it is unbounded, end the algorithm and the CPWLP is unbounded. If the LP is infeasible, go to Step 6; else, denote the optimum by x^*;

Step 5. If $g(x^*) < g(x_0)$, then set $x_0 = x^*$ and turn to Step 2;

Step 6. If $k < M$ and $\Psi \neq \emptyset$, let $k = k + 1$ and turn to Step 3, else end the algorithm and x_0 is the optimized point.

It should be noted that in every iteration except the initial one, x_0 is always a basic feasible solution for all LP constructed in Step 3. This property can save lots of computing time for solving LP. In the algorithm, the objective value keeps descending and the number of subregions is finite. Therefore, the algorithm will naturally converge. In Step 2, if there are too many subregions Ω_i satisfying $x_0 \in \Omega_i$, we use M to control the computing time. In CCP optimization, we set $M = 100$. If x_0 is the optimum in 100 randomly selected subregions, it should be possible that x_0 is a locally optimal point.

The above algorithm will gain an optimized point from an initial one. Therefore, an initial feasible point of (1) is needed. That means there should be an $x_0 \in \Omega$, satisfying $H(x_0) = 0$ and $A(x_0) \leq 0$. To get x_0, we will solve equations $H(x) = 0$ and then check whether the solution satisfies $A(x) \leq 0$ and $x \in \Omega$.

$H(x) = 0$ are nonlinear equations with m equations in n-dimension. If $H(x)$ are separable piecewise linear functions, $H(x) = 0$ can be solved by the sign tests algorithm ([22]) or the linear programming algorithm ([23]). However, the general piecewise linear equations are still hard to solve. We will use the piecewise linear property to get one solution.

In subregion Ω_i, there are $H(x) = Q_i(x), A(x) = B_i(x)$, $\forall x \in \Omega_i$. As $Q_i(x) = 0, B_i(x) \leq 0$ are linear, they can be solved and a solution space Γ_i is gained. If $\Gamma_i \bigcap \Omega_i \neq \emptyset$, we will get a feasible point $x_0 \in \Gamma_i \bigcap \Omega_i$. If $\Gamma_i \bigcap \Omega_i = \emptyset$, another subregion will be tried. Since the number of subregions is finite, a feasible point will be found if the feasible region of (1) is nonempty.

When the number of subregions is small (for example, less than ten thousands), the above procedure will find a feasible point quickly. But when the number grows up, this procedure may cost lots of time. In this case, we use experience or some approximation algorithms to generate an approximate solution x'. Then the subregion $\Omega_{x'}$ satisfying $x' \in \Omega_{x'}$ should be tested. For x' is the approximate solution, the probability that $\Omega_{x'}$ has feasible points is higher than other subregions. In theory, this method for solving piecewise linear equations can not guarantee to get a feasible point, but it works well in CCP optimization.

III. OPERATION OPTIMIZATION FOR CENTRIFUGAL CHILLER PLANT

A. Adaptive Hinging Hyperplane and Subregion Construction

The proposed CPWLP algorithm is applied to the optimization for CCP system. First, we use AHH as the piecewise linear model to approximate the CCP system. AHH is proposed as a regression model in [13]. It can be seen as one kind of basis function expansion which is a combination of basis functions as follows,

$$f(x) = \beta_0 + \sum_{m=1}^{M} \beta_m B_m(x), \tag{5}$$

where $B_m(x)$ is the basis function, β_m is the coefficient of the corresponding basis function and β_0 is for the constant basis function. The basis function $B_m(x)$ takes the following form, here $x = [x(1), x(2), \ldots, x(n)]^T \in R^n$,

$$B_m(x) = \min_{k \in \{1,2,\ldots,K_m\}} \{\max\{0, s_{km}(x(\nu_{km}) - \lambda_{km})\}\}, \tag{6}$$

in which K_m is the number of factors $\max\{0, s_{km}(x(\nu_{km}) - \lambda_{km})\}$ of the basis function $B_m(x)$. $s_{km} = \pm 1$ and the item

$\max\{0, s_{km}(x(\nu_{km}) - \lambda_{km})\}$ is a univariate function. It is noted that k must be different for the same m, so the maximal number of K_m is n.

It is easy to see that AHH is piecewise linear, and the approximation capability of AHH to any continuous function with arbitrary precision has been proved in [13]. The parameters s_{km}, ν_{km} and λ_{km} are nonlinear. If we choose λ_{km} from a discrete set, AHH model can be interpreted as a tree where each node of the tree represents a basis function (for details, please see [15]). An adaptive tree-based algorithm is used for model constructing and parameters identification. Given a set of training data, we can construct an AHH model, where the number of basis functions can be controlled.

To apply the descent algorithm for CPWLP based on AHH, we need a method to construct the expression of subregion. Suppose the point x_0 and parameters of (5) and (6) are given, we want to get an expression of the subregion Ω_i satisfying $x_0 \in \Omega_i$. To do this, we first consider some constraints related to one basis function $B_m(x)$. Then we combine all constraints for all basis functions together and get the expression of Ω_i. The procedure for dealing with $B_m(x)$ starts from computing the value of (6).

If $\max\{0, s_{km}(x_0(\nu_{km}) - \lambda_{km})\} \geq 0$, we can gain a constraint for Ω_i, i.e., $s_{km}(x_0(\nu_{km}) - \lambda_{km}) \geq 0$. If $\max\{0, s_{km}(x_0(\nu_{km}) - \lambda_{km})\} = 0$, this constraint should be $s_{km}(x_0(\nu_{km}) - \lambda_{km}) \leq 0$. For the same m and different k, there are totally K_m constraints. Next, we should check the value of $B_m(x_0)$, if $B_m(x_0) \leq 0$, the above constraints are enough, otherwise, we set $k^* = \arg\max_k\{s_{km}(x_0(\nu_{km}) - \lambda_{km})\}$ and gain the following $K_m - 1$ inequalities: $s_{km}(x_0(\nu_{km}) - \lambda_{km}) \leq s_{k^*m}(x_0(\nu_{k^*m}) - \lambda_{k^*m}), \forall k \neq k^*$.

Therefore, there are at most $2K_m - 1$ inequalities for basis function $B_m(x)$. And combine the constraints for all basis functions together, we can get the inequalities for determining Ω_i. Though there may be a lot of inequalities and some of them are redundancy, all of the inequalities are linear. Hence, Ω_i is a polyhedron and the corresponding problem (4) on Ω_i is an LP problem. It should be noted that using this method, one can gain all subregions containing x_0. If there are more than one Ω_i satisfying $x_0 \in \Omega_i$, we will meet the situations that $\max\{0, s_{km}(x_0(\nu_{km}) - \lambda_{km})\} = 0$ or there are more than one k^*. Take the former case for example, we can get two subregions with constraints $s_{km}(x_0(\nu_{km}) - \lambda_{km}) \geq 0$ and $s_{km}(x_0(\nu_{km}) - \lambda_{km}) \leq 0$, respectively. Hence the expressions for all the subregions with x_0 can be constructed.

B. AHH Approximation

Using AHH identification algorithm proposed in [15], we build three black-box models (Chiller, Cooling Tower & Cooling Water Pump, Terminal Unit & Chilled Water Pump). The connections of these models are illustrated in Fig. 2.

The variables in Fig. 2 are listed as follows,

α_1: Temperature of chilled water entering the chiller;
α_2: Temperature of chilled water out of the chiller;
α_3: Temperature of cooling water entering the chiller;

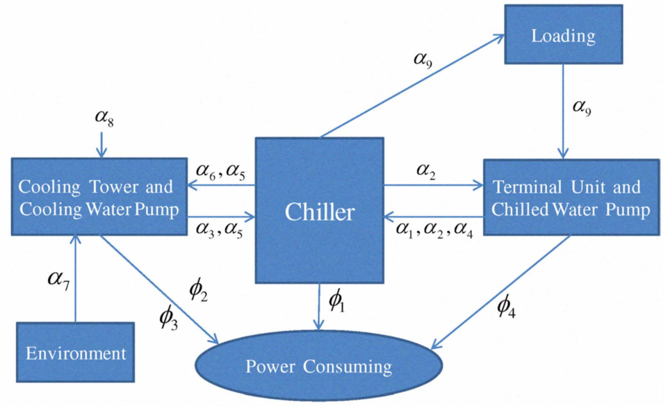

Fig. 2. The Structure of Black-box Model

α_4: Flux of the chilled water;
α_5: Flux of the cooling water;
α_6: Temperature of cooling water out of the chiller;
α_7: Wet-bulb temperature;
α_8: Air volume of the cooling tower;
α_9: Overall Loading;
ϕ_1: Power consuming of the chiller;
ϕ_2: Power consuming of the cooling pump;
ϕ_3: Power consuming of the cooling tower fans;
ϕ_4: Power consuming of the chilling pump.

The arrows show the inputs and outputs of each black-box model. For example, the input of the chiller is a 5-dimensional vector $[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5]^T$, and there are three outputs ϕ_1, α_6, and α_9. They can be approximated by three 5-dimensional functions denoted by $f_{\text{cm1}}, f_{\text{cm2}}, f_{\text{cm3}}$ respectively. The relationships from the inputs to the outputs can be described by a physical model which will generate enough data for AHH approximation.

Similarly, for the cooling tower and cooling water pump, the input is a 4-dimensional vector $[\alpha_5, \alpha_6, \alpha_7, \alpha_8]^T$. There are three outputs ϕ_2, ϕ_3 and α_3 approximated by $f_{\text{cl1}}, f_{\text{cl2}}$ and f_{cl3}. Noted that the input α_6 is also an output of the chiller and some other variables are also used in different black-box functions. Finally, for the terminal unit and chilled water pump, the input is a 2-dimensional vector $[\alpha_2, \alpha_9]^T$ and there are 3 outputs ϕ_4, α_1 and α_4, which can be approximated by functions $f_{\text{cp1}}, f_{\text{cp2}}$ and f_{cp3}.

The procedure of AHH approximation can be explained by constructing $f_{\text{cm1}}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$. The physical model will generate a series of data $\{[\alpha_1(t), \alpha_2(t), \alpha_3(t), \alpha_4(t), \alpha_5(t)]^T, \phi_1(t)\}, t = 1, 2, \ldots, N$ off-line. To construct f_{cm1}, 30509 samples are provided through the calculation of the physical model, where 6000 samples are used as the estimation data and the left are the validation data. The accuracy of approximation can be evaluated by the relative squared sum of error (RSSE),

$$\text{RSSE} = \frac{\sum_{t=1}^{N}\left(\phi_1(t) - f_{\text{cm1}}(\alpha_1(t), \alpha_2(t), \ldots, \alpha_5(t))\right)^2}{\sum_{t=1}^{N}\left(\phi_1(t) - E(\phi_1)\right)^2},$$

where $E(\phi_1)$ is the average value of ϕ_1.

In this sense, the validation error for f_{cm1} is 0.0069, where 40 basis functions are used. It can be seen that RSSE for f_{cm1} is small enough and hence AHH approximates the data accurately. Meanwhile, RSSEs of the other functions are all small, which means AHH can describe CCP system well.

C. CPWLP and Optimization Result

After AHH approximation, the CPWLP model can be constructed as follows,

$$\begin{aligned} \min \quad & f_{\text{cm1}}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) + f_{\text{cl1}}(\alpha_5, \alpha_6, \alpha_7, \alpha_8) + \\ & \qquad f_{\text{cl2}}(\alpha_5, \alpha_6, \alpha_7, \alpha_8) + f_{\text{cp1}}(\alpha_2, \alpha_9) \end{aligned}$$

s.t.

$$\begin{aligned} f_{\text{cm2}}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) &= \alpha_6 \\ f_{cm3}(\alpha_1, \alpha_2, \alpha_4) &= \alpha_9 \\ f_{\text{cl3}}(\alpha_5, \alpha_6, \alpha_7, \alpha_8) &= \alpha_3 \\ f_{\text{cp2}}(\alpha_2, \alpha_9) &= \alpha_4 \\ f_{\text{cp3}}(\alpha_2, \alpha_9) &= \alpha_1 \\ [\alpha_1, \alpha_2, \ldots, \alpha_9]^T &\in \Omega \end{aligned}$$

where Ω is a convex polyhedron representing the boundaries of variables and some physical constraints which are linear. All the functions are piecewise linear, hence the above is a CPWLP problem and can be optimized by the algorithm proposed in Section II.B.

In a typical work condition, the wet-bulb temperature is 15 °C and the overall loading is 1500 kW. After optimization, the total power consuming is decreased from 170.1 kW to 134.0 kW, where the initial operation point is got by experience. The proposed method is a descent algorithm, and the descending of each LP is shown in Fig. 3. Notice that in the descent algorithm, the objective value may keep the same after solving one LP. Therefore, the iteration steps shown in Fig. 3 are the ones decreasing the power consuming.

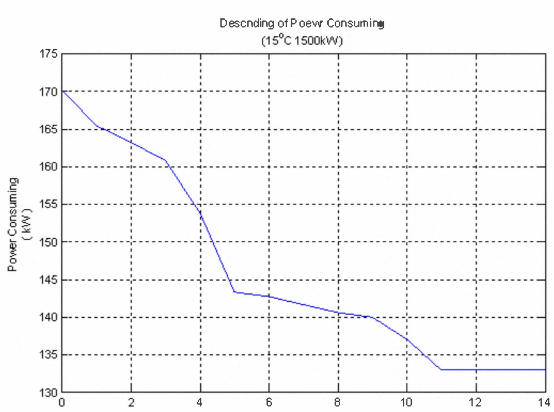

Fig. 3. Descending of Power Consuming in Every Efficient LP

Using CPWLP, the operation of CCP system is optimized for different overall loadings and wet-bulb temperatures. Since the proposed descent algorithm will get different optimized operations from different initial ones, we repeat the algorithm in 10 minutes and choose the best result. For there is no practical method for CCP optimization, we use a traversal method to compare with CPWLP. In the traversal method, each control variable is chosen from L possible values evenly spaced along its definitional domain. Given α_7 and α_9, there are totally L^7 operation candidates. The traversal method chooses one feasible operation with the minimal power consuming. Since it costs too much time even when L is small (in the following example, using the same computer the traversal method will take about 3 hours for each pair of α_7 and α_9 when $L = 10$), the traversal method is not practicable. However, it is applied off-line for the wet-bulb being 20 °C and $L = 10$. The result can be roughly regarded as the global optimum and used to evaluate the performance of CPWLP. This comparison show the effectiveness of CPWLP, see Fig. 4, where the power consuming is computed by the physical model.

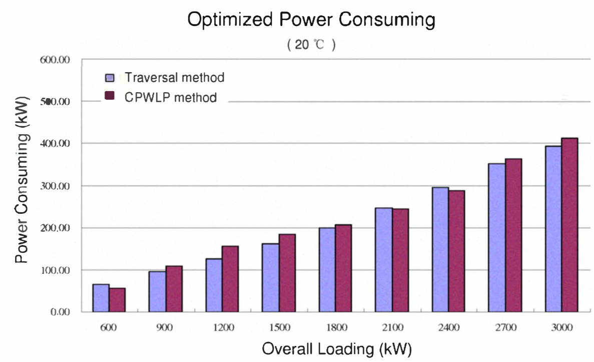

Fig. 4. Evaluation of the Optimized Results using CPWLP (20 °C)

For a real CCP system, the power consuming are saved significantly via CPWLP. In this system, the current operations are set by some strategies. After applying CPWLP, the power consuming are reported in Table I, where the first line represents different terminal request and the first row represents different wet-bulb temperature. "—" means there does not exist such situation in real world. For example, the terminal request keeps low when the temperature is not too high.

TABLE I
POWER SAVING USING CPWLP

	600 kW	1200 kW	1800 kW	2400 kW	3000 kW
10 °C	53.07 kW	110.4 kW	204.6 kW	—	—
	45.17 kW	92.36 kW	152.0 kW	—	—
15 °C	64.35 kW	144.9 kW	199.2 kW	285.2 kW	—
	47.74 kW	100.8 kW	162.9 kW	246.1 kW	—
20 °C	86.00 kW	181.1 kW	276.3 kW	324.3 kW	459.3 kW
	56.46 kW	156.6 kW	207.4 kW	290.9 kW	411.2 kW
25 °C	117.8 kW	205.1 kW	338.6 kW	374.4 kW	484.4 kW
	90.44 kW	185.7 kW	291.6 kW	336.1 kW	430.5 kW

For each situation, the first line is the current power consuming and the second line is the optimized result.

IV. Conclusion

In this paper, an optimization method for continuous nonlinear optimization is proposed. This method is based on CPWL approximation and CPWLP. CPWL approximation is used to simplify the nonlinear system and CPWLP can optimize the objective function based on the approximated piecewise linear system. After discussing some properties of CPWLP and the necessary and sufficient condition for the local optimum, we propose the descent algorithm to solve CPWLP. The proposed method is applied into CCP optimization and significantly saves power consuming from the currently used operations. The comparison with the traversal method also shows the effectiveness of CPWLP. The successful application on CCP optimization suggests that doing nonlinear optimization via piecewise linear approximation will be an attractive and efficient method in theory and practice, especially when the analytical relationships are unclear or complex.

Acknowledgment

The authors greatly appreciate Prof. Wenxing Shi, Prof. Da Yan and their group in the Department of Building Science, Tsinghua University, for providing the physical model, operation data and insightful discussion.

Project was jointly supported by the National Natural Science Foundation of China (60674025, 60974008), the 863 Program (2007AA04Z193), the Research Fund for Doctoral Program of Higher Education (200800030029) and Mitsubishi Heavy Industries, Ltd.

References

[1] T. Hartman, "All-variable speed centrifugal chiller plants", *ASHRAE Journal*, Vol. 43, 2001, pp. 43-53.

[2] G. Mazurkiewicz, "Better cooling via improve chillers", *Air Conditioning, Heating & Refrigeration News*, Vol.215, 2002, pp.37-38.

[3] S. Taylor, "Primary-only vs. primary secondary variable flow systems", *ASHRAE Journal*, Vol.44, 2002, 25-29.

[4] T. Moses, "Variable-primary flow: important lessons learned", *Heating/Piping/Air Conditioning Engineering*. Vol.76, 2004, pp.40-43.

[5] I. Dubov, "Chilled water plant efficiency", *ASHRAE Journal*, Vol.45, 2003, pp.37-40.

[6] J. Zhou, S. Deng, D. Turner and M. Liu, "Simulation and optimization on power plant operation using SEGA's EOP program", *Proceedings of the 12th Symposium on Improving Building System in Hot and Humid Climates*, San Antonio USA, 2000.

[7] M. Meckler, "Do the math: Chiller plant optimization strategies", *Engineered System*, Vol. 19, 2006, pp. 55-70.

[8] L. O. Chua and S. M. Kang, "Section-wise piecewise-linear functions: canonical representation, properties and applications", *IEEE Trans. Circuits Systems*, Vol.30, 1977, pp.185-040.

[9] L. Breiman, "Hinging hyperplanes for regression, classification, and function approximation", *IEEE Trans. Information Theory*, Vol.39, 1993, pp.999-1013.

[10] J. M. Tarela, E. Alonso and M. V. Martinez."A representation method for PWL functions oriented to parallel processing", *Mathematical & Computer Modeling*, Vol. 13, 1990, pp.75-83.

[11] S. Wang and X. Sun, "Generalization of hinging hyperplanes", *IEEE Trans. Information Theory*, Vol.51, 2005, pp.4425-4431.

[12] P. Julián, A. Desages and O. Agamennoni, "High-level canonical piecewise-linear representation using a simplicial partition", *IEEE Trans. Circuits Systems - I: Fundamental Theory and Applications*, Vol.46, 1999, pp.463-480.

[13] J. Xu, X. Huang and S. Wang, "Adaptive hinging hyperplanes", *Proceeding of the 17th World Congress, The International Federation of Automatic Control*, Seoul Korea, 2008, pp.4036-4041.

[14] J. H. Friedman, "Multivariate adaptive regression splines", *The Annals of Statistics*, Vol. 19, 1991, pp.1-67.

[15] J. Xu, X. Huang and S. Wang, "Adaptive hinging hyperplanes and its applications in dynamic system identification", *Automatica*, Vol. 45, 2009, pp. 2325-2332.

[16] H. Li and C. Yu, "A global optimization method for nonconvex separable programming problems", *European Journal of Operations Research* , Vol. 117, 1999, pp.275-292.

[17] M. W. Padbeg, "Approximating separable nonlinear functions via mixed zero-one programs", *Operation Research Letters*, Vol. 27, 2000, pp. 1-5.

[18] H. D. Sherali, "On mixed-integer zero-one representations for separable lower-semicontinunous piecewise-linear functions", *Operation Research Letters*, Vol. 28, 2001, pp. 155-160.

[19] H. Zhang and S. Wang, "Global optimization of separable objective functions on convex polyhedra via piecewise-linear approximation", *Journal of Computation and Applied Mathematics*, Vol.197, 2006, pp.212-217.

[20] H. Zhang and S. Wang, "Linearly constrained global optimization via piecewise-linear approximation", *Journal of Computation and Applied Mathematics*, Vol. 214, 2008, pp. 111-120.

[21] J. M. Tarela and M. V. Martínez, "Region configurations for realizability of lattice piecewise-linear models", *Mathematical and Computer Modelling*, Vol.30, 1999, pp.17-27.

[22] K. Yamamura, "An algorithm for finding all solution of piecewise-linear resistive circuits using sign tests", *IEEE Trans. Circuits and Systems - I: Fundamental Theory and Applications*, Vol. 40, 1993, pp. 546-551.

[23] K. Yamamura and K. Yomogita, "Finding all solution of piecewise-linear resistive circuits using an LP test", *IEEE Trans. Circuits and Systems - I: Fundamental Theory and Applications*, Vol. 47, 2000, pp. 1115-1120.