

## Auxiliary Particle Filter-Based Remaining Useful Life Prediction of Rolling Bearing

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**Abstract**—Rolling bearing's running state has an important influence on the health condition of rotate machinery. This work focuses on the remaining useful life prediction of the rolling bearing. An auxiliary particle filter-based predictor for rolling bearing is presented. The energy spectrum feature of vibration signal is selected as the representation of system degraded states. The wavelet packet decomposition is employed to extract the energy spectrum feature of rolling bearing. The process of prediction includes two stages: the energy spectrum feature is extracted firstly; and then auxiliary particle filter (APF) is trained by using degraded energy spectrum states; APF-based predictor is constructed in the end. Based on an experiment platform of rolling bearing life, a whole life cycle data set of vibration signal is collected for evaluating the performance of the presented APF-based predictor. The classical particle filter (PF) is utilized for comparing with the proposed method. The investigation results indicate that the proposed method can efficiently forecast the remaining useful life of the rolling bearing and has higher accuracy than PF-based predictor.

**Keywords**—remaining useful life; rolling bearing; particle filter; auxiliary particle filter

### I. INTRODUCTION

Rolling bearing is one of the most used components in rotating machinery, whose running state has significantly influence on the performance of machinery [1]. Each kind of fault of bearings may give rise to the machine breakdown, which could bring about some severe consequences, such as time consuming, economic loss and even security issues occurrence. To predict the uptime of machine and to minimize its maintenance cost, condition-based research on rolling bearings has becoming a hotspot in the last few decades. Three stages are included in machinery monitoring: fault detection, fault diagnosis and remaining useful life (RUL) prediction [2]. Most researchers pay their attention to the first two stages and plenty of papers have been published, but the last stage, RUL prediction has been reported far fewer than the first stages [3].

In general, the RUL prognostic approaches can mainly be divided into model-based methods and data-driven methods [4]. Model-based methods usually adopt a comprehensive mathematical model which depends on physical law to describe the system physical states and update model parameters by using measured data. The popular used model-based techniques include the Markov process model [5], the Winner process model [6], exponential degradation model [7] and Paris' law model [8], etc. However, model-based methods cannot be used in many industrial cases because its parameter and failure modes may vary under different operational

conditions, especially in a complicated system whose internal states are difficult to direct measurement. Data-driven methods rely on data collected from working condition's machinery by sensors to forecast future system states according to the trend of feature degradation. Therefore, the quality and quantity of historical data have a significant impact on prognostic accuracy [4]. The commonly used data-driven methods include artificial neural network (ANN) [9], support vector machine (SVM) [10], Kalman filter [11], and particle filter (PF) [3, 4, 12].

PF is a technique for implementing a recursive Bayesian filter by Monte Carlo simulation and suitable to estimate model parameters and system states. Furthermore, it has been introduced to predict the RUL of rolling bearings and Li-ion battery [3-4]. However, the classical PF can be inefficient and sensitive to outliers in many complex applications. Main reason is that the prior distribution is independent of the current measurement, which is usually chosen as its importance sampling density to simplify computation. Thus, a large number of particles may be produced in the area of low likelihood.

In order to address this problem, an improved PF named auxiliary particle filter (APF) was proposed. In APF, the samples are drawn from a joint distribution which is related with the latest observation and more particles would be generated in the region of larger likelihood. In this paper, APF is applied to forecast the RUL of rolling bearings. Firstly, energy values feature is extracted from vibration signal of rolling bearing. Secondly, state space model is constructed by energy values feature. Finally, APF-based predictor is used to the RUL prediction of rolling bearing.

The remainder of this paper is organized as follows: Section II is the APF-based RUL prediction. Section III analysis the experimental results and together with some discussions. Conclusions are finally made in Section IV.

### II. APF-BASED RUL PREDICTION

#### A. Review of PF

A general state space model is defined as follows:

$$x_k = f(x_{k-1}) + w_{k-1} \quad (1)$$

$$y_k = h(x_k) + v_k \quad (2)$$

where  $x_{k-1}$ ,  $w_{k-1}$ ,  $f(\cdot)$ ,  $y_k$ ,  $v_k$  and  $h(\cdot)$  are described as follows:

$x_{k-1}$  : Internal system state at time  $t_{k-1}$ .

$w_{k-1}$  : Process noise which independence of time.

$f(\cdot)$  : System state transition function.

$y_k$  : Measurement value at time constants  $t_k$ , which has relationship with  $x_k$ .

$v_k$  : Measurement noise which also independence of time.

$h(\cdot)$  : Measurement function.

From a Bayesian perspective, supposed that the required transition probability density function (PDF)  $p(x_k|x_{k-1})$  and the state PDF  $p(x_{k-1}|Y_{k-1})$ ,  $Y_{k-1} = \{y_1, y_2, \dots, y_{k-1}\}$  is available at time  $t_{k-1}$ . Thus, the prior PDF of system state  $p(x_k|Y_{k-1})$ , in prediction stage at time  $t_k$  can be computed by the Chapman-Kolmogorov equation:

$$p(x_k|Y_{k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|Y_{k-1})dx_{k-1} \quad (3)$$

In update stage, the measurement value  $y_k$  can be obtained at time  $t_k$ , which is used to modify prior density to get the posterior PDF of the current system state at time  $t_k$  via Bayes' law:

$$p(x_k|Y_k) = \frac{p(y_k|x_k)p(x_k|Y_{k-1})}{p(y_k|Y_{k-1})} \quad (4)$$

where the normalizing constant is calculated by:

$$p(y_k|Y_{k-1}) = \int p(y_k|x_k)p(x_k|Y_{k-1})dx_k \quad (5)$$

The Bayesian recursive estimation is consisted of (3) to (5). Nevertheless, it's difficult to address integrals. Therefore, a numerical approximated method is employed to estimate the posterior PDF.

PF algorithm is introduced to solve the Bayesian recursive estimation problem, whose main idea is to approximate the posterior PDF of system state by a series of samples  $x_k^{(i)} \{i=1,2,\dots,N\}$  and their associated normalized weights  $\tilde{w}_k^{(i)}$ .

$$p(x_k|Y_{k-1}) = \sum_{i=1}^N \tilde{w}_k^{(i)} \delta(x_k - x_k^{(i)}), \sum_{i=1}^N \tilde{w}_k^{(i)} = 1 \quad (6)$$

where  $N$  is the number of particles,  $\delta$  is the Dirac delta function, the estimation will be more accuracy as  $N \rightarrow \infty$ , the associated weights  $w_k^{(i)}$  can be recursive update as follows:

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)})p(x_k^{(i)}|x_{k-1}^{(i)})}{q(x_k^{(i)}|x_{k-1}^{(i)}, Y_k)} \quad (7)$$

where  $q(\cdot)$  is an important density which is used to generate particles. The prior density is chosen as the important density.

$$p(x_k|x_{k-1}, Y_k) = p(x_k|x_{k-1}) \quad (8)$$

Substituting (8) into (7) yields

$$w_k^{(i)} \propto w_{k-1}^{(i)} p(y_k|x_k^{(i)}) \quad (9)$$

In PF algorithm, the prior density  $p(x_k|x_{k-1})$  is chosen as the important sampling density which is independent of measurement value  $y_k$  and without any knowledge of the latest observation, thus, the classical PF could be sensitive to outliers and exist a higher error [13]. To handle those issues, an APF algorithm is applied in this paper.

### B. Review of APF

In APF framework, the important sampling density is defined as a joint density of  $q(x_k, i|Y_k)$ , which is used to draw samples and also satisfy the proportionality

$$q(x_k, i|Y_k) \propto p(y_k|x_k^{(i)})p(x_k|x_{k-1}^{(i)})w_{k-1}^{(i)} \quad (10)$$

where  $u_k^{(i)}$  is a mean, mode or a sample from  $p(x_k|x_{k-1}^{(i)})$ , the index  $i$  is related with the initial particle  $x_{k-1}^{(i)}$ , it so-called an auxiliary variable. It can be clearly seen that in (10) the latest observation  $y_k$  has been considered in generating new samples. Therefore, more particles would be generated in the region of larger likelihood. Correspondingly, the weights are updated by

$$w_k^{(j)} \propto \frac{p(y_k|x_k^{(j)})}{p(y_k|u_k^{(i')})}, \{j=1,2,\dots,N\} \quad (11)$$

where  $j$  is the particle index of current step and  $i$  related with the index of particle drawn from the last step. Thus, the particles generated from a joint density can describe the true state of posterior PDF more accuracy.

### C. Representation of System Degraded State

A suitable feature indicator not only can simplify modeling but also enhance the accuracy of prediction. In this subsection, energy values were used to describe the degraded states of rolling bearing.

Previous researches have shown that different features have various impacts on different stages of the degradation process. In this work, kurtosis, energy values and impulse factor have been calculated to describe the degradation trend of rolling bearing.

As shown in Fig.1, it can be clearly seen that kurtosis and impulse factors are difficult to describe the degradation process of system, however, energy value have an obvious monotonic trend. Consequently, energy value is selected as the representation of rolling bearing state.

### D. State Space Model Construction

In this subsection, the APF-based state space model is described to predict the RUL of rolling bearing, where a variant of Paris-Erdogan model are used [3]. The growth rate of crack size is defined in Paris-Erdogan model as (12)

$$\frac{dx}{dM} = C(\Delta F)^n, \Delta F = \beta\sqrt{x} \quad (12)$$

where  $x$  is the crack size,  $M$  denotes load cycles,  $\Delta F$  is the amplitude of stress intensity factor,  $C$ ,  $n$  and  $\beta$  are material

constants which are estimated by test. For convenient application, (12) can be rewritten to

$$\frac{dx}{dM} = e^a \cdot x^b \quad (13)$$

Assumed that  $y_k$  have the linear relationship with  $x_k$ , thus, the state space model (1) and (2) can be transformed into (14)

$$\begin{cases} x_k = x_{k-1} + e^a \cdot x_{k-1}^b \cdot \Delta t + w_{k-1} \\ y_k = x_k + v_k \end{cases} \quad (14)$$

where  $a$  and  $b$  are estimated parameters whose initial value is derived from training data (energy values) using least-square estimator,  $\Delta t = t_k - t_{k-1}$ ,  $w$  and  $v$  are process noise and measurement noise, respectively, which caused by operating condition or equipment.

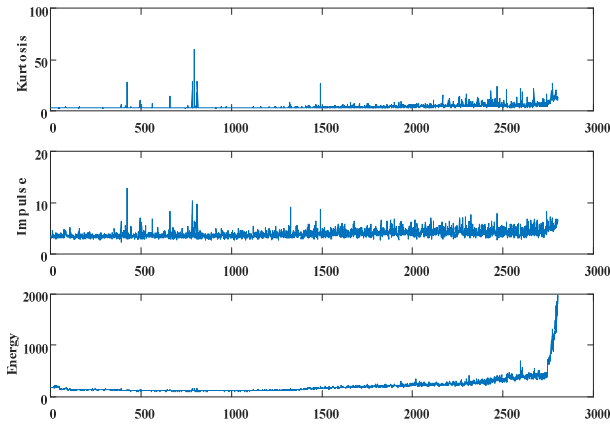


Figure 1. Degradation trend with different features.

#### E. Framework of APF-based Predictor

The feature come from subsection C was used to train the state space model to obtain the initial values of parameters  $a$  and  $b$ , and then, APF technique was utilized to predict RUL. Equation (15) is defined to calculate the RUL of rolling bearing.

$$RUL = P_{end} - P_{start} \quad (15)$$

where  $P_{start}$  means the start of prediction,  $P_{end}$  represents the end of prediction. The flowchart of APF-based predictor shown in Fig.2 and detail explain as follows:

- 1) Energy feature extraction. The feature that can track the rolling bearing degradation is extracted using wavelet packet decomposition (WPD) with 3-layer
- 2) Feature preprocessing. Smooth the energy feature to reduce the influence of noise and resample the feature according to equal time intervals.
- 3) Acquiring the degradation of rolling bearing.
- 4) Defining the threshold  $X_{th}$ .
- 5) Using part of preprocessed feature to train state space model and obtain the initial values of parameters and setting the beginning time  $k$  of prediction.

6) One-step prediction using APF-based to get the predictive values  $X_p$ .

7) if  $X_p$  smaller than threshold  $X_{th}$ , back to step 6, otherwise, stop to prediction and calculate the RUL of rolling bearing.

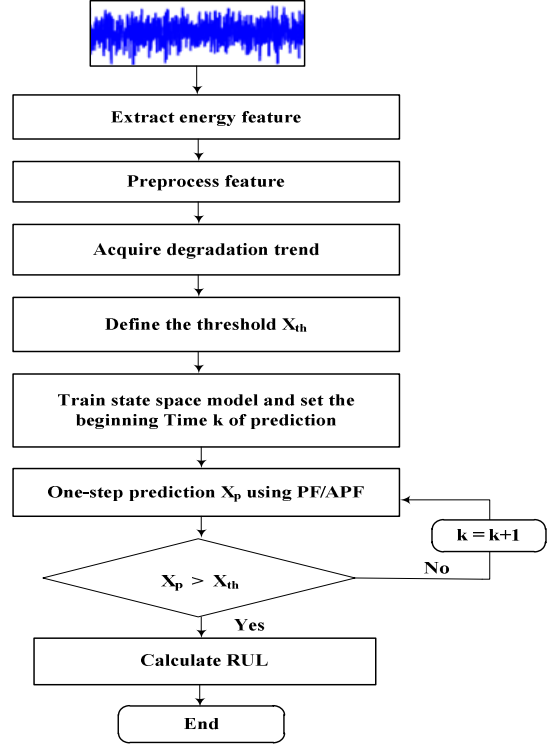


Figure 2. Flowchart of APF-based predictor

### III. EXPERIMENTAL RESULTS AND ANALYSIS

#### A. Experimental Introduction

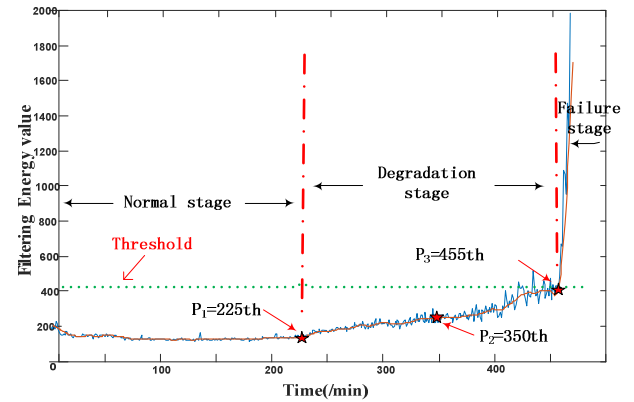


Figure 3. Energy degradation trend of test rolling bearing

Experimental data comes from IEEE PHM 2012 prognostic challenge [14]. The test rolling bearing's speed is 1800 rpm, load is 4000 N. The sampling frequency is 25.6 kHz, each sampling time is 0.1s, thus, 2560 points are collected in each

sample and sampling is repeated every 10s. Finally, 2802 samples are collected. All samples are decomposed by WPD with 3-layer. Accordingly, the degradation trend of rolling bearing can be obtained, and then, smoothing out the degradation trend and evenly resampling every 6 points. Therefore, 467 points are acquired and each point represents 1 minute. The degradation trend after preprocessing of rolling bearing is shown in Fig. 3.

### B. Contrastive Analysis

In Fig. 3, the blue line is original energy values and the red line is the filtering energy values, what's more, the trend of system where data points from 1<sup>st</sup> to  $P_1$  (the 225<sup>th</sup> energy point) is almost steady, so we think the rolling bearing run under normal operating conditions. The energy values from  $P_1$  to  $P_3$  (the 455<sup>th</sup> energy point) increase gradually, which mean the operating conditions of rolling bearing is under degradation stage. After data point  $P_3$ , the energy values change abruptly. Thus, we can think the rolling bearing under failure stage. Based on above analysis, data points from  $P_1$  to  $P_2$  (the 350<sup>th</sup> energy point) are used to train state space model to get model initial parameters, and remaining data is used to test the performance of the proposed APF-based predictor. The value of  $P_3$  was defined as threshold, that is, the RUL predictions stop it until the predictive energy value larger than the value of  $P_3$ . Equation (15) was adopt to compute the RUL of rolling bearing and  $P_2$  and  $P_3$  were chosen as  $P_{start}$  and  $P_{end}$ , respectively. Correspondingly, the real RUL of rolling bearing is 105 minutes (455 - 350 = 105). To evaluate the validity of proposed model, the mean absolute error (MAE) is computed as follows:

$$MAE = \frac{1}{T} \sum_{i=1}^T |f_p - f_r| \quad (16)$$

where  $T$  is time length,  $f_p$  is the energy value of prediction,  $f_r$  is real energy value.

Firstly, energy values from  $P_1$  to  $P_2$  are selected to train state space model and obtained model initial parameters are follows:  $a = 9.8671$ ,  $b = -1.4432$ ;  $w$  and  $v$  are zero-mean Gaussian noise, and the initial value is the energy value of  $P_2$ , which equal to 245.128. The threshold is the energy value of  $P_3$  which is equal to 412.8898. Then, the classical PF and APF framework were used to predict the RUL of rolling bearing. The data fitting of PF and APF prediction as showed in Fig. 4 and Fig. 5, respectively.

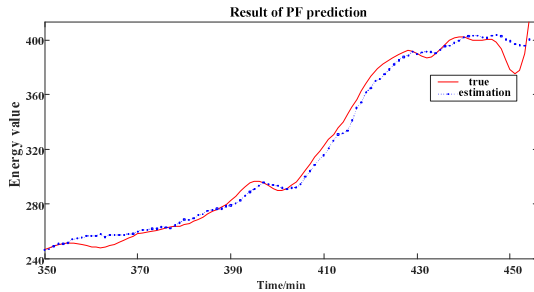


Figure 4. Prediction trajectories of PF-based predictor

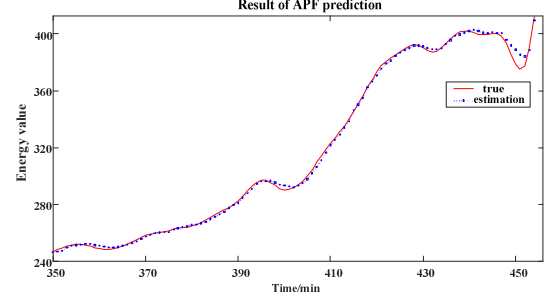


Figure 5. Prediction trajectories of APF-based predictor

Fig. 5 shows that the prediction results of APF-based method is closer to real sequences than PF-based one, in addition, (15) and (16) are used to calculate the RUL and MAE based on PF and APF predictor, respectively. Finally, the detail prediction results are shown in Table I.

TABLE I. DETAIL PREDICTION RESULTS BY USING PF/APF

Description	Real sequence	PF prediction sequence	APF prediction sequence
Beginning point	350 <sup>th</sup>	350 <sup>th</sup>	350 <sup>th</sup>
Energy value of beginning point	245.128	245.128	245.128
Ending point	455 <sup>th</sup>	456 <sup>th</sup>	455 <sup>th</sup>
Energy value of ending point	412.8898	421.2208	421.3535
RUL(/min)	105	106	105
MAE	—	4.4731	1.5617

From Table I, it can be known that the predictive energy value equal to 421.2208 at 456<sup>th</sup> point which trigger the stopping criterion based on classical PF method. Accordingly, the RUL of rolling bearing based on classical PF method is 106 minutes (456-350=106). APF-based algorithm stopped to predict energy value when the predictive energy value equal to 421.3535 at 455<sup>th</sup> point. Thus, the RUL of rolling bearing based on proposed method is 105 minutes (455-350=105). Additionally, the mean square error (MAE) between the real sequence and the prediction sequence are calculated. It also can be seen that the APF-based predictor can provide a lower MAE than PF-based predictor. In other words, APF-based predictor can get better experimental results than PF-based predictor. This is because the APF technique makes full use of the latest observation to generate particles which can more effective to approximate the real PDF of system.

### IV. CONCLUSIONS

In this paper, an APF-based prediction technique was developed for remaining useful life prediction of rolling bearing. Experimental data from IEEE PHM 2012 prognostic challenge were used to evaluate the presented method. Some conclusions can be drawn as follows: i) The energy feature of vibration signal can efficiently describe the degradation trend of rolling bearing; ii) The APF-based predictor outperforms the PF-based one for the RUL prediction of rolling bearing; iii)

the APF-based predictor efficient utilizes the knowledge of the latest observation of system to generate samples.

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#### REFERENCES

- [1] K. Teotrakool, M.J. Devaney, and L. Eren, "Adjustable-speed drive bearing-fault detection via wavelet packet decomposition," *IEEE Trans. Instrum. Meas.*, vol. 58, pp. 2747–2754, Aug. 2009.
- [2] J. Lee, F.J. Wu, W.Y. Zhao, M. Ghaffari, L.X. Liao, and D. Siegel, "Prognostics and health management design for rotary machinery systems-Reviews, methodology and applications," *Mech. Syst. Signal Process.*, vol. 42, pp. 314–334, Jan. 2014.
- [3] Y.G. Lei, N. Li, S. Gontarz, and J. Lin, "A Model-Based Method for Remaining Useful Life Prediction of Machinery," *IEEE Trans. Rel.*, vol. 65, pp. 1314–1326, Jun. 2016.
- [4] J. Liu, W. Wang, F. Ma, Y.B. Yang, and C.S. Yang, "A data-model-fusion prognostic framework for dynamic system state forecasting," *Eng. Appl. Artif. Intell.*, vol. 25, pp. 814–823, Jun. 2012.
- [5] H.Y. Dui, S.B. Si, M.J. Zuo, and S.D. Sun, "Semi-Markov process-based integrated importance measure for multi-state systems," *IEEE Trans. Rel.*, vol. 64, pp. 754–765, Jun. 2015.
- [6] X.S. Si, W.B. Wang, C.H. Hu, D.H. Zhou, and M.G. Pecht, "Remaining useful life estimation based on a nonlinear diffusion degradation process," *IEEE Trans. Rel.*, vol. 61, pp. 50–67, Mar. 2012.
- [7] Gebräel N, "Sensory-updated residual life distributions for components with exponential degradation patterns," *IEEE Trans. Autom. Sci. Eng.*, vol. 3, pp. 382–293, Oct. 2006.
- [8] Liu T. An integrated bearing prognostics method for remaining useful life prediction. Concordia University, 2013.
- [9] R. Huang, L. Xi, X. Li, et al., "Residual life predictions for ball bearings based on self-organizing map and back propagation neural network methods," *Mech. Syst. Signal Process.*, vol. 21, pp. 193–207, Jan. 2007.
- [10] A. Widodo, and B.S. Yang, "Machine health prognostics using survival probability and support vector machine," *Expert Syst. Appl.*, vol. 38, pp. 8430–8437, Jul. 2011.
- [11] Y. Qian, R. Yan, and S. Hu, "Bearing degradation evaluation using recurrence quantification analysis and Kalman filter," *IEEE Trans. Instrum. Meas.*, vol. 63, pp. 2599–2610, Nov. 2014.
- [12] M. E. Orchard and G. J. Vachtsevanos, "A particle-filtering approach for on-line fault diagnosis and failure prognosis," *Trans. Inst. Meas. Control*, vol. 31, pp. 221–246, 2009.
- [13] M. Sanjeev Arulampalam, Simon Maskell, Neil Gordon, and Tim Clapp. "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Trans. Signal Process.*, vol. 50, pp. 174–188, Feb. 2002.
- [14] FEMTO-ST, "IEEE PHM 2012 Data Challenge", online website, <http://www.femto-st.fr/en/Researchdepartments/AS2M/Researchgroups/PHM/IEEE-PHM-2012-Datachallenge.php>