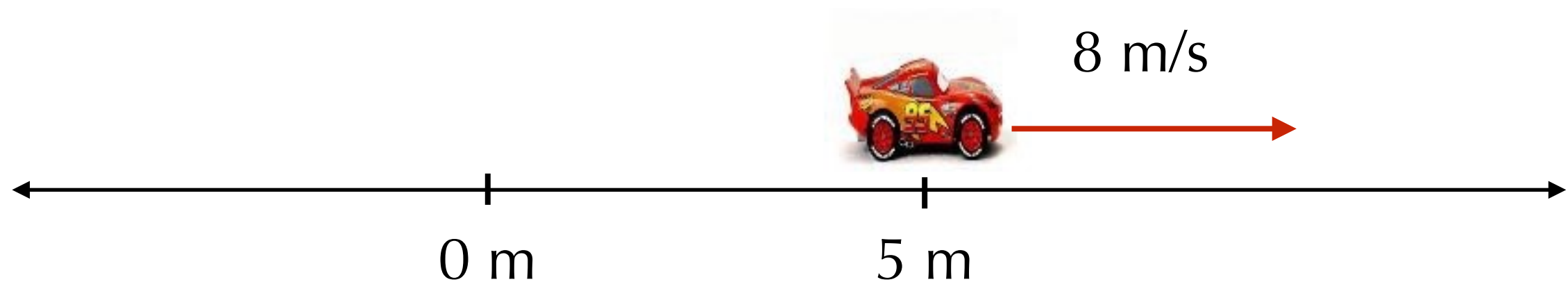
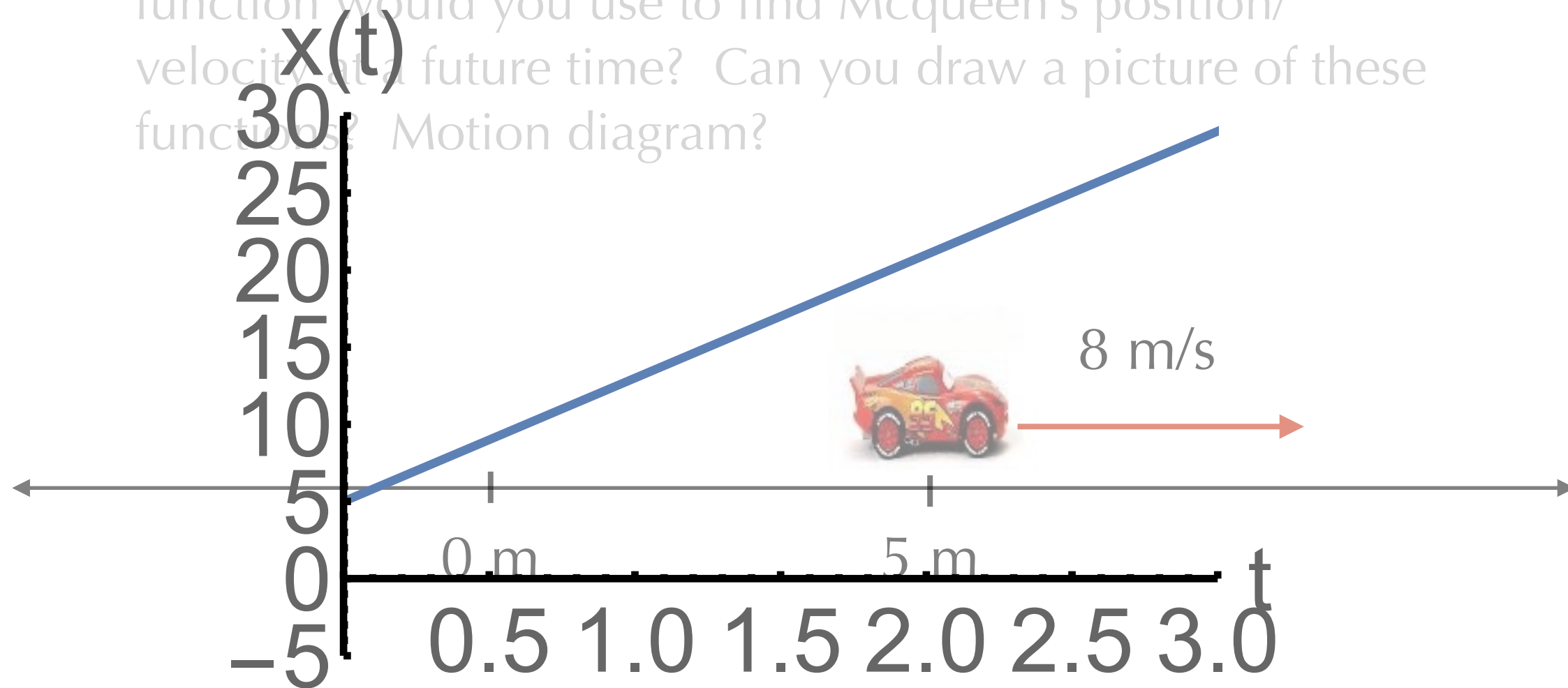


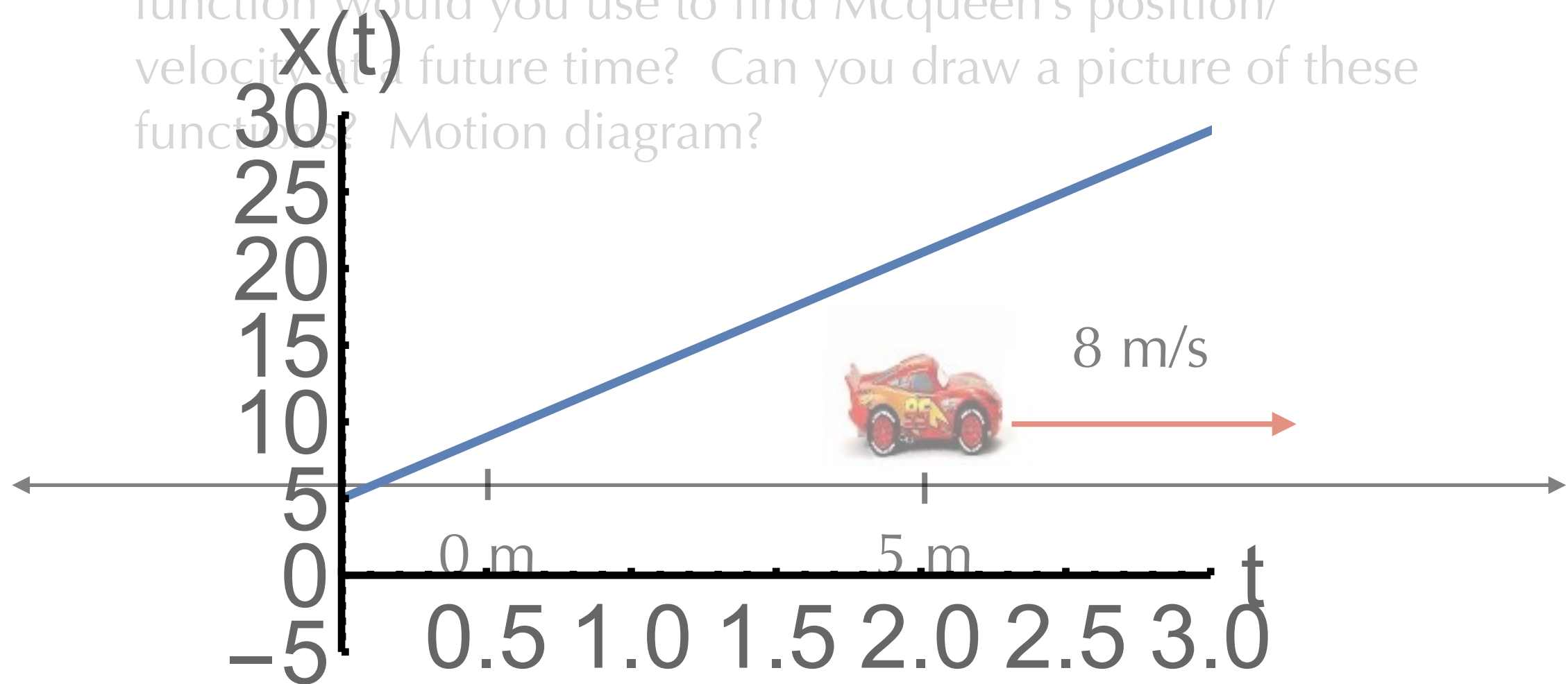
You notice Lightning McQueen traveling to the right at a constant speed of 8 m/s. When you first notice him, he is located 5 m to the right of where you are standing. What function would you use to find McQueen's position/velocity at a future time? Can you draw a picture of these functions? Motion diagram?



You notice Lightning McQueen traveling to the right at a constant speed of 8 m/s. When you first notice him, he is located 5 m to the right of where you are standing. What function would you use to find McQueen's position/velocity at a future time? Can you draw a picture of these functions? Motion diagram?



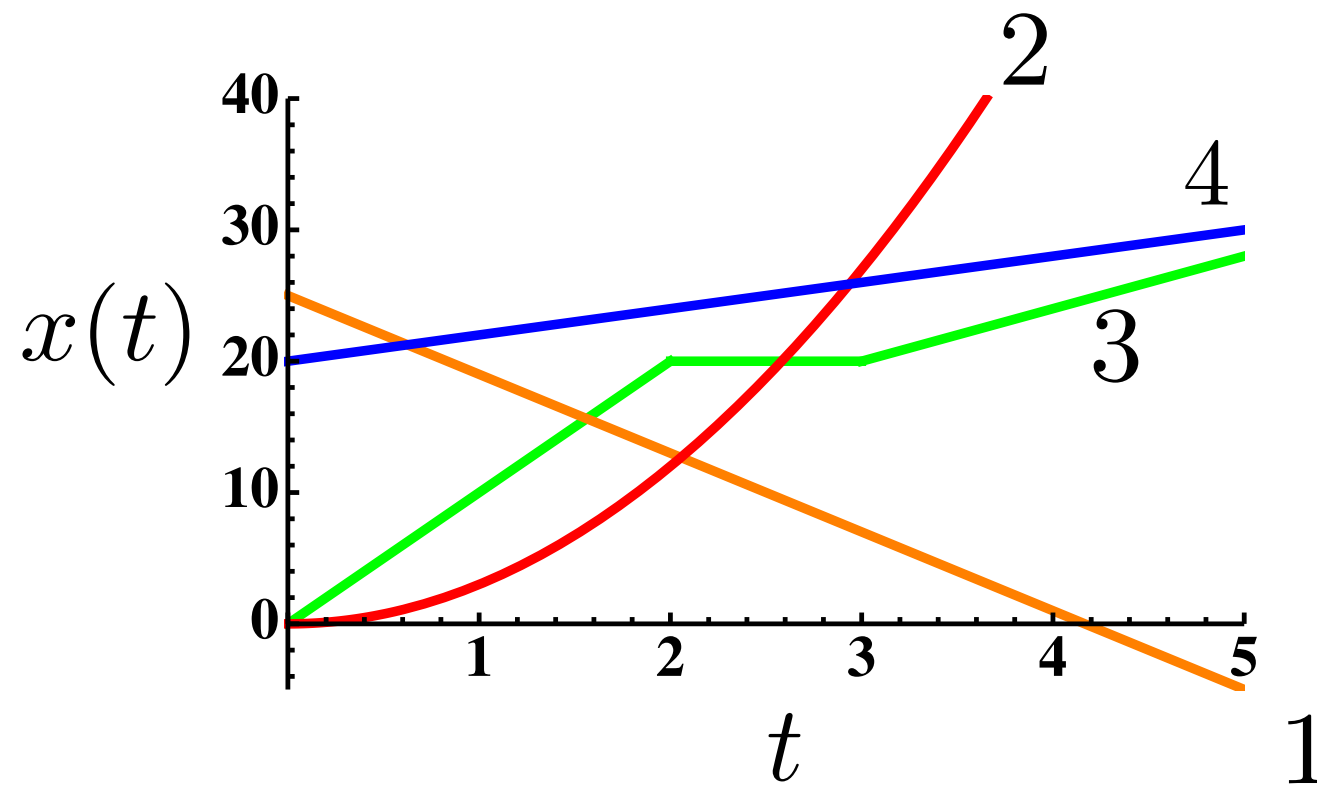
You notice Lightning McQueen traveling to the right at a constant speed of 8 m/s. When you first notice him, he is located 5 m to the right of where you are standing. What function would you use to find McQueen's position/velocity at a future time? Can you draw a picture of these functions? Motion diagram?



$$x(t) = x_i + v\Delta t$$

Quiz

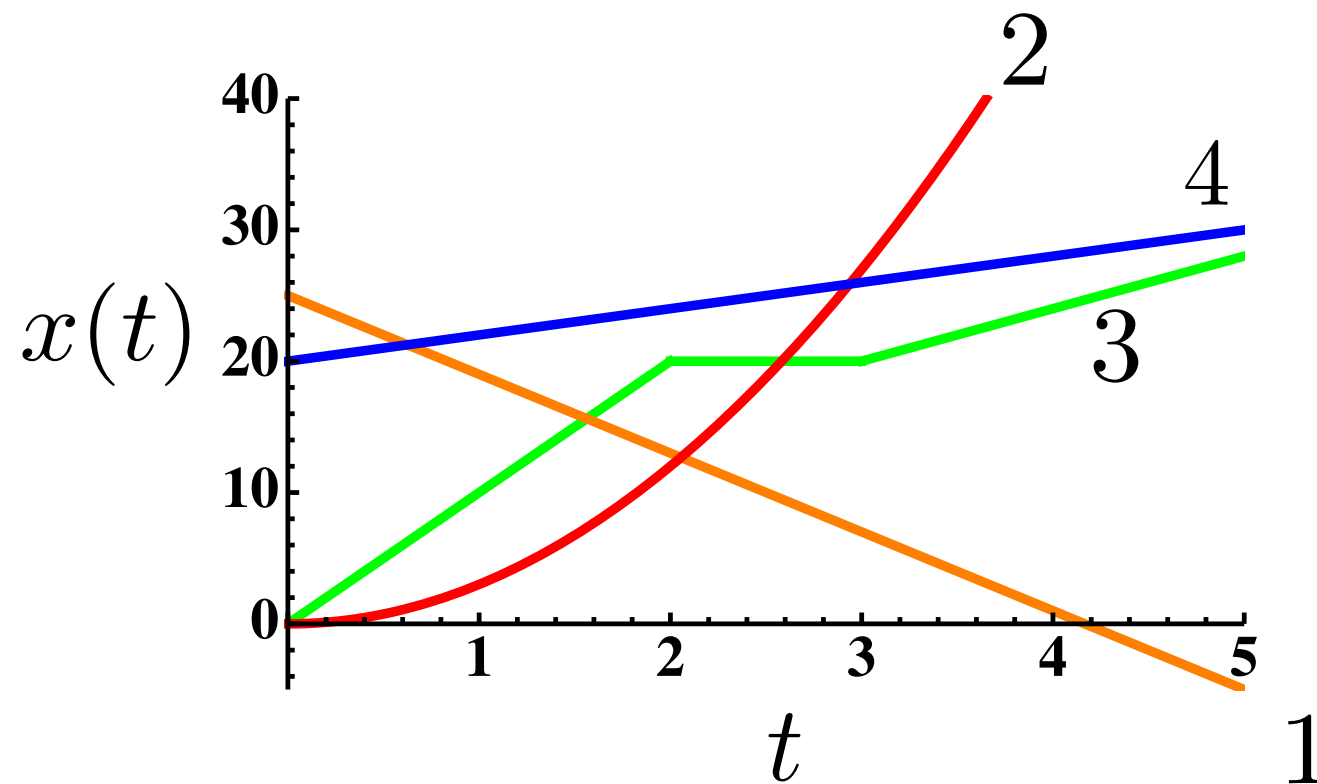
Which curve(s) do(es) **not** represent uniform motion?



- a) 1 and 3
- b) 1 and 4
- c) 2
- d) 2 and 3
- e) none of the above

Quiz

Which curve(s) do(es) **not** represent uniform motion?



a) 1 and 3

b) 1 and 4

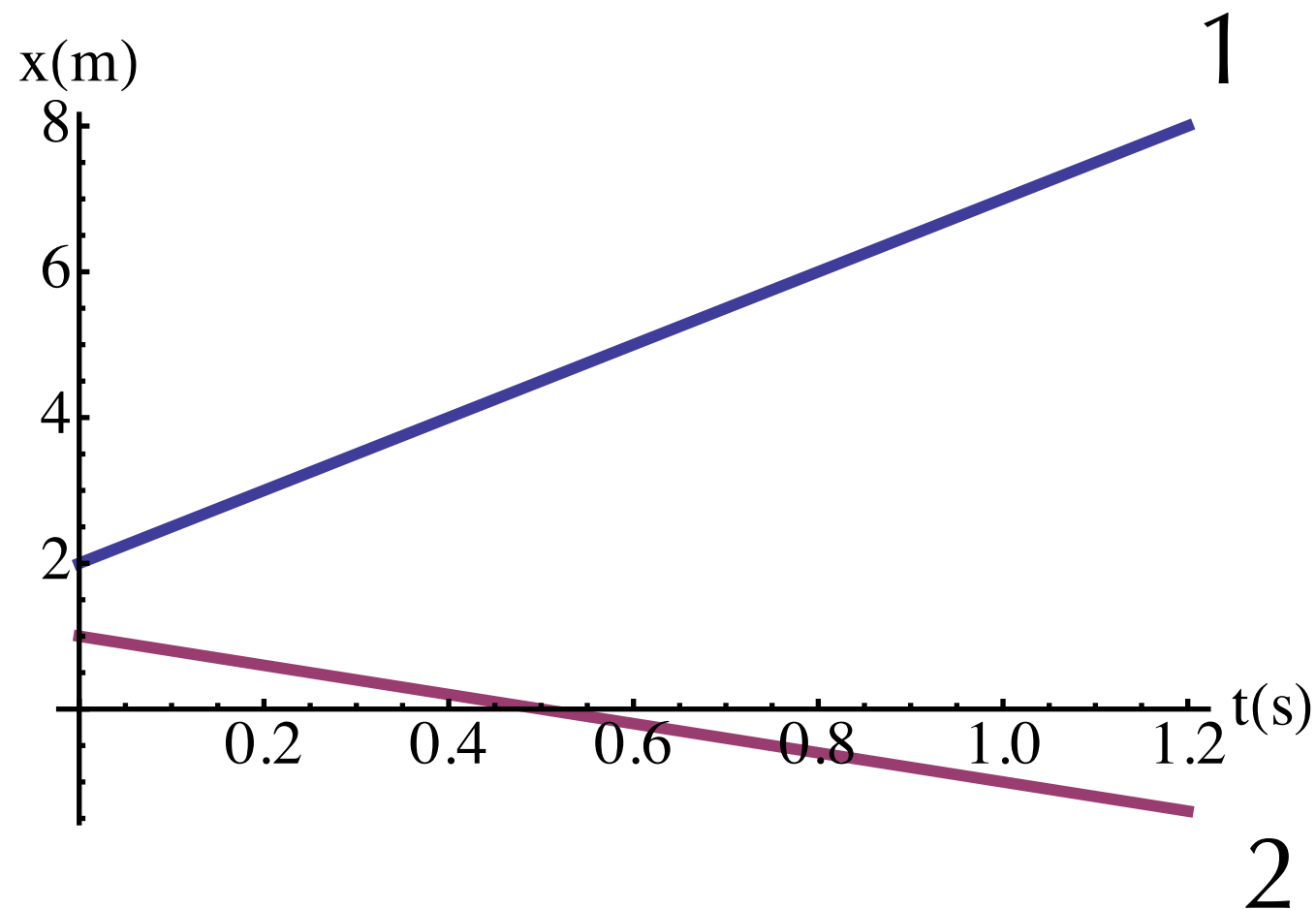
c) 2

d) 2 and 3

e) none of the above

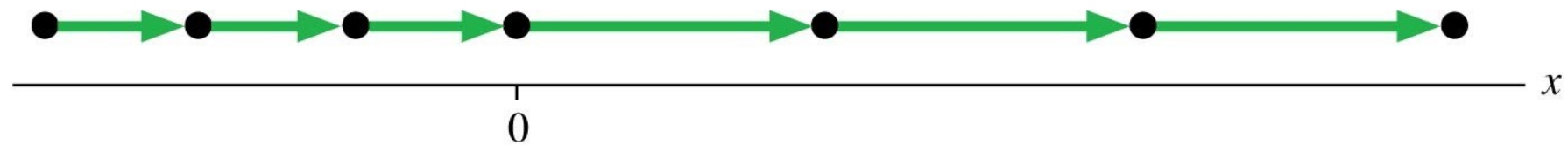
Example

The position-versus-time graph of the figure represents the motion of two students on roller blades. Determine their velocities and describe their motion.

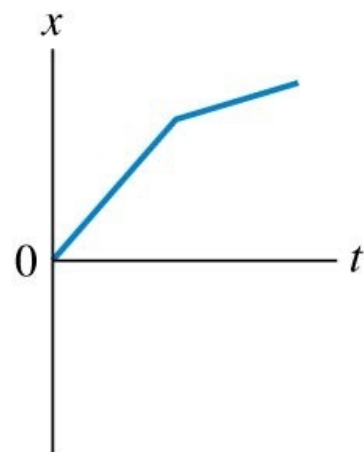


Quiz

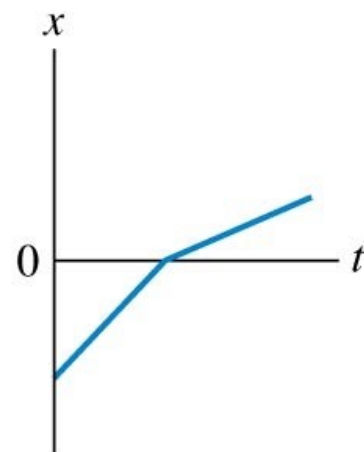
Here is a motion diagram of a car moving along a straight road:



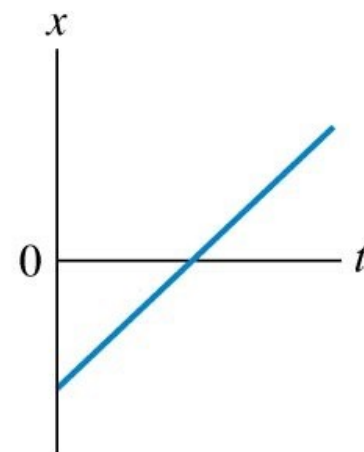
Which position-versus-time graph matches this motion diagram?



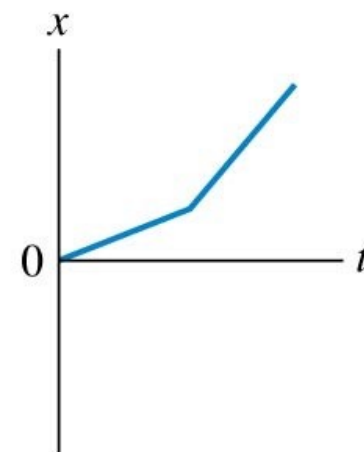
A.



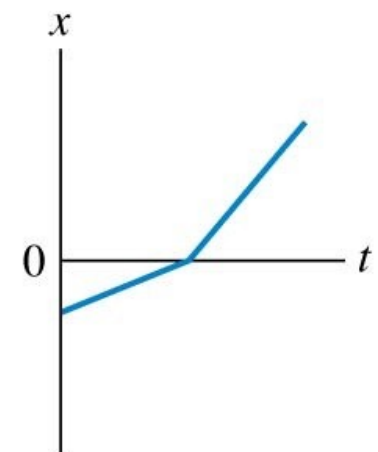
B.



C.



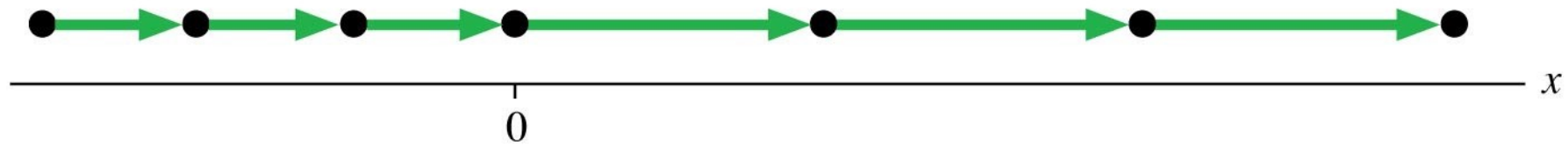
D.



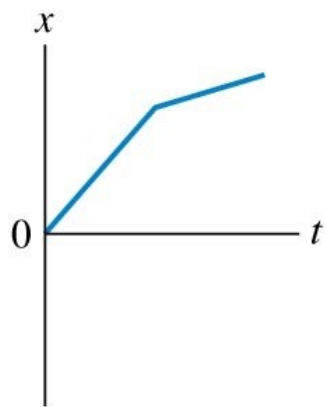
E.

Quiz

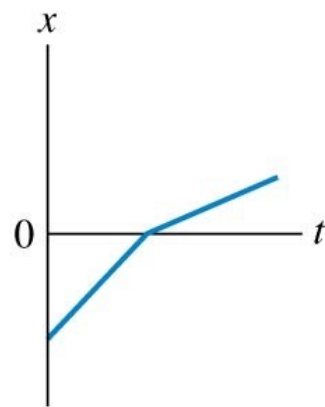
Here is a motion diagram of a car moving along a straight road:



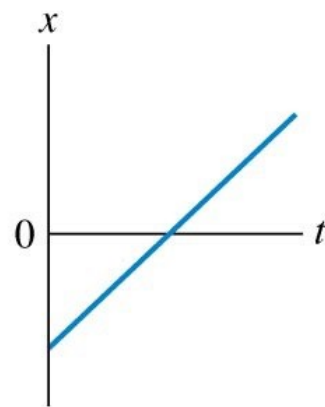
Which position-versus-time graph matches this motion diagram?



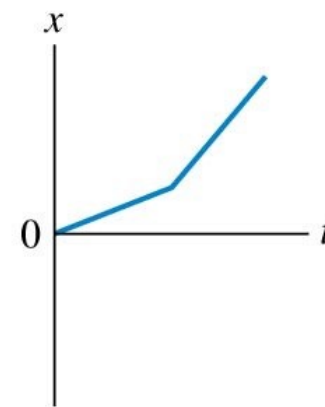
A.



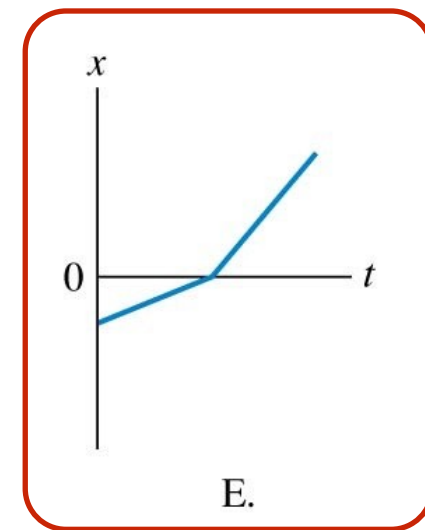
B.



C.



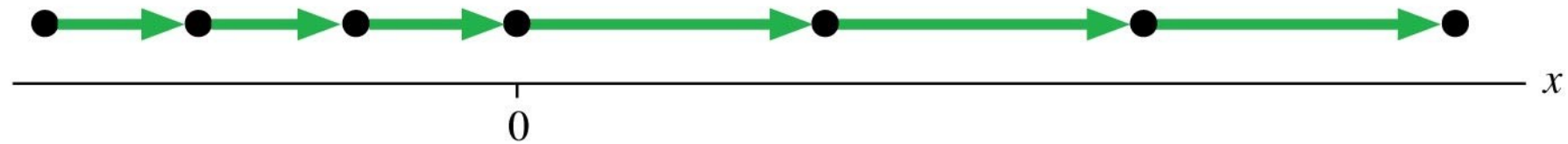
D.



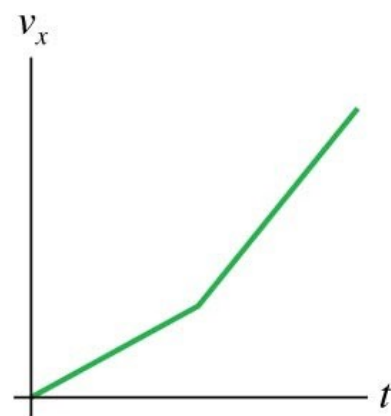
E.

Quiz

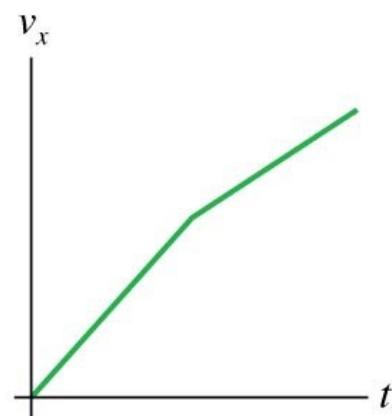
Here is a motion diagram of a car moving along a straight road:



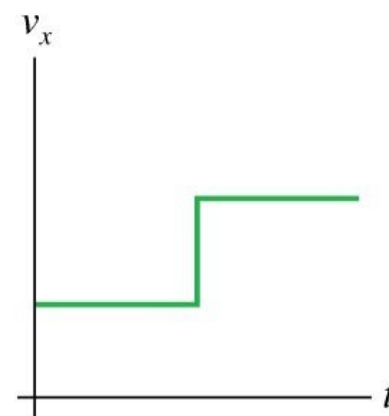
Which velocity-versus-time graph matches this motion diagram?



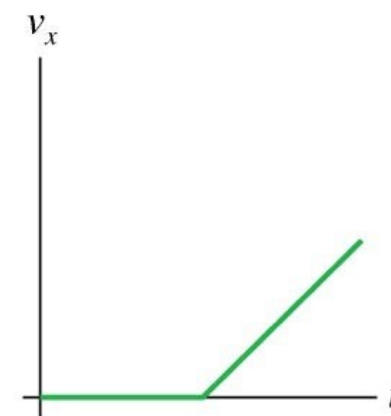
I



II



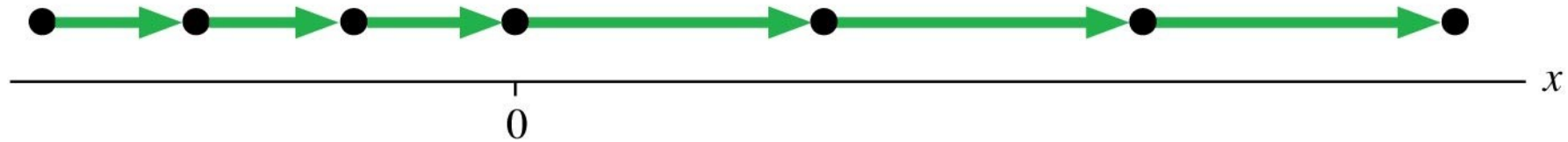
III



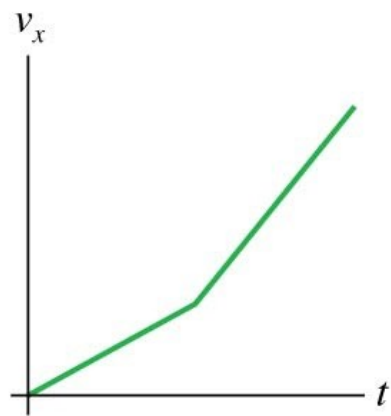
IV

Quiz

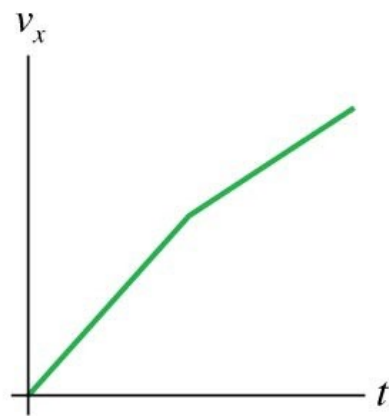
Here is a motion diagram of a car moving along a straight road:



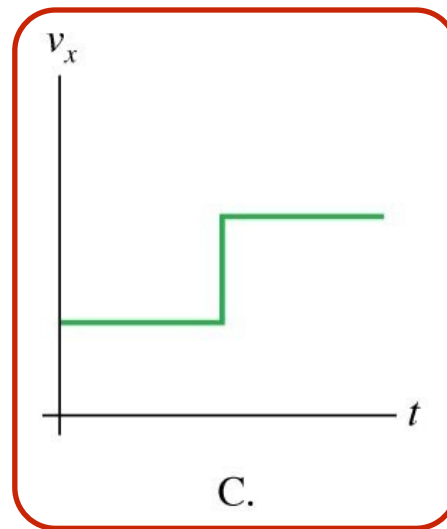
Which velocity-versus-time graph matches this motion diagram?



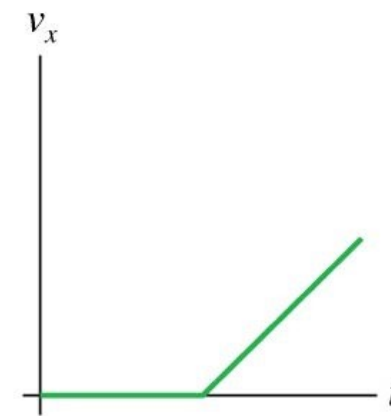
A.



B.



C.

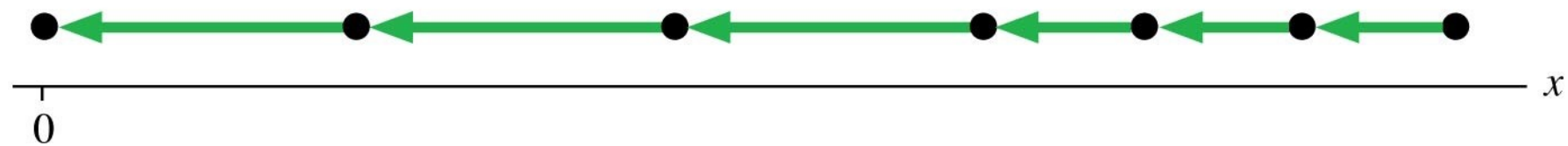


D.

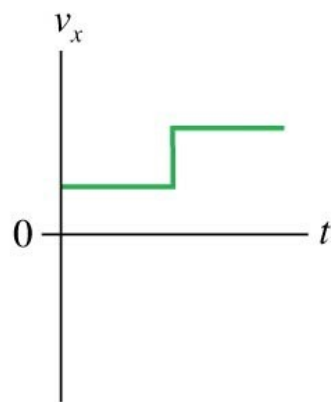
E. None of the above.

Quiz

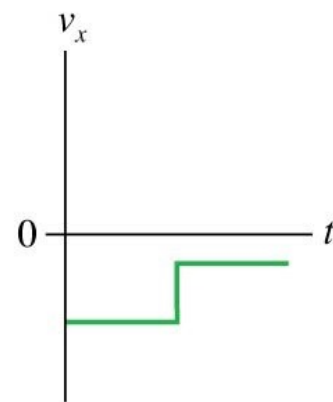
Here is a motion diagram of a car moving along a straight road:



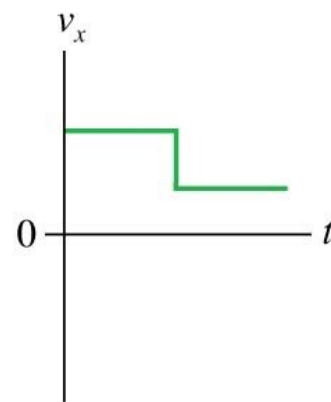
Which velocity-versus-time graph matches this motion diagram?



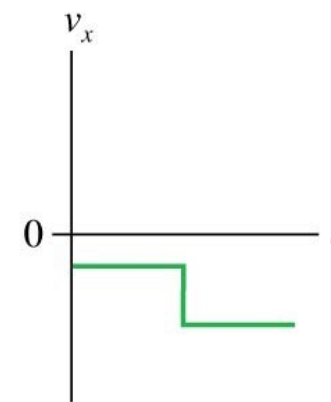
I



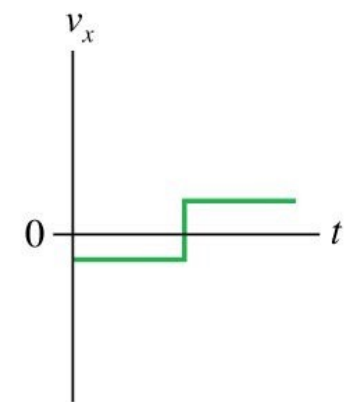
II



III



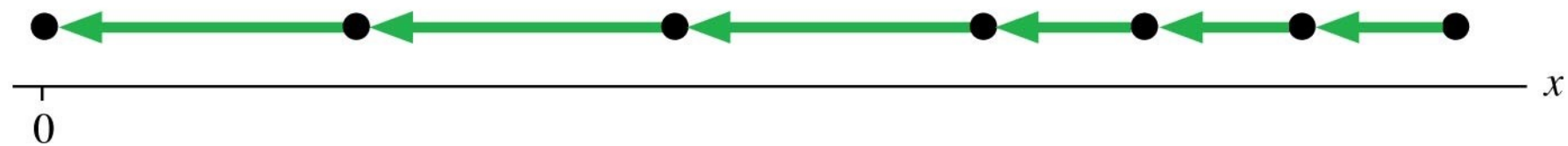
IV



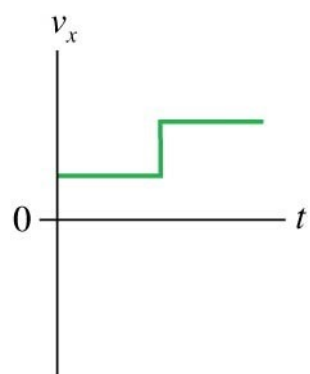
V

Quiz

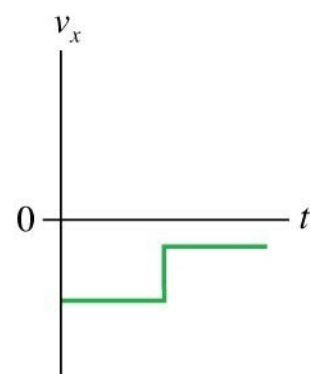
Here is a motion diagram of a car moving along a straight road:



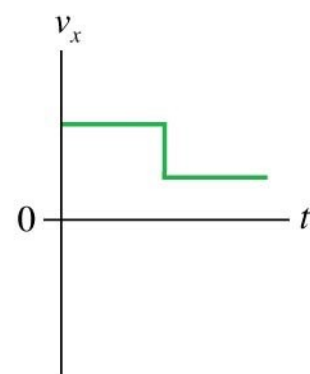
Which velocity-versus-time graph matches this motion diagram?



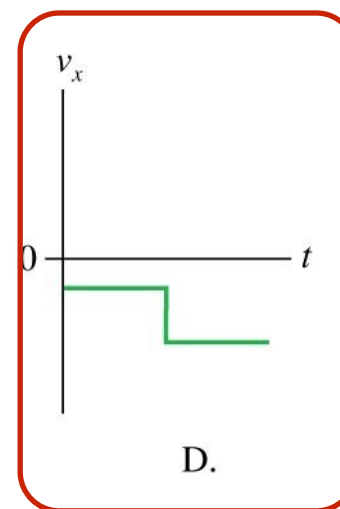
A.



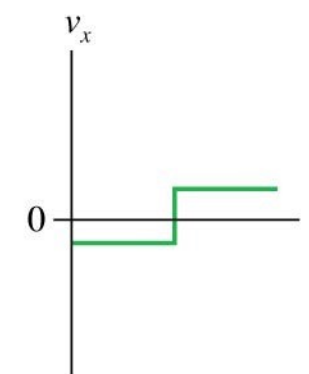
B.



C.



D.

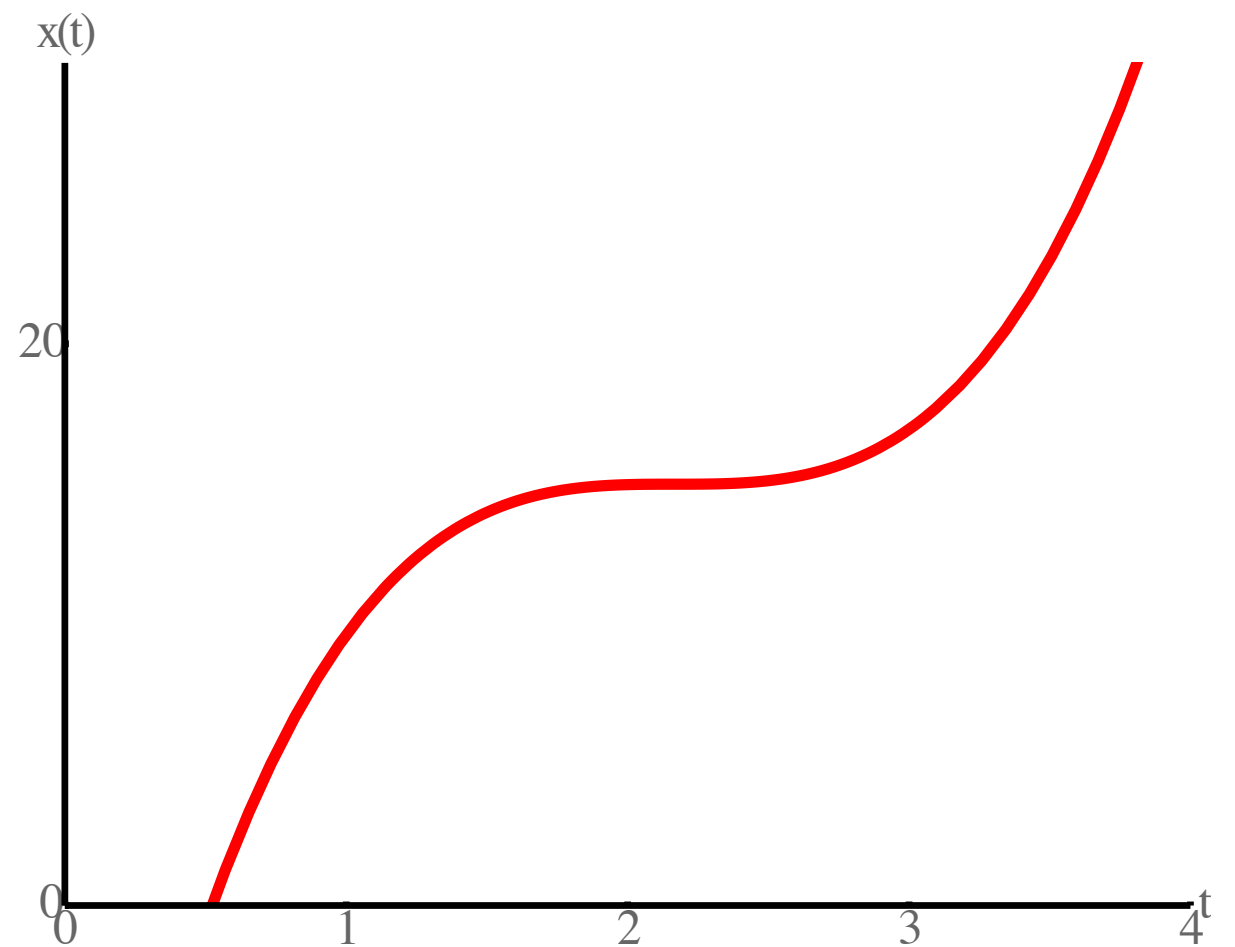
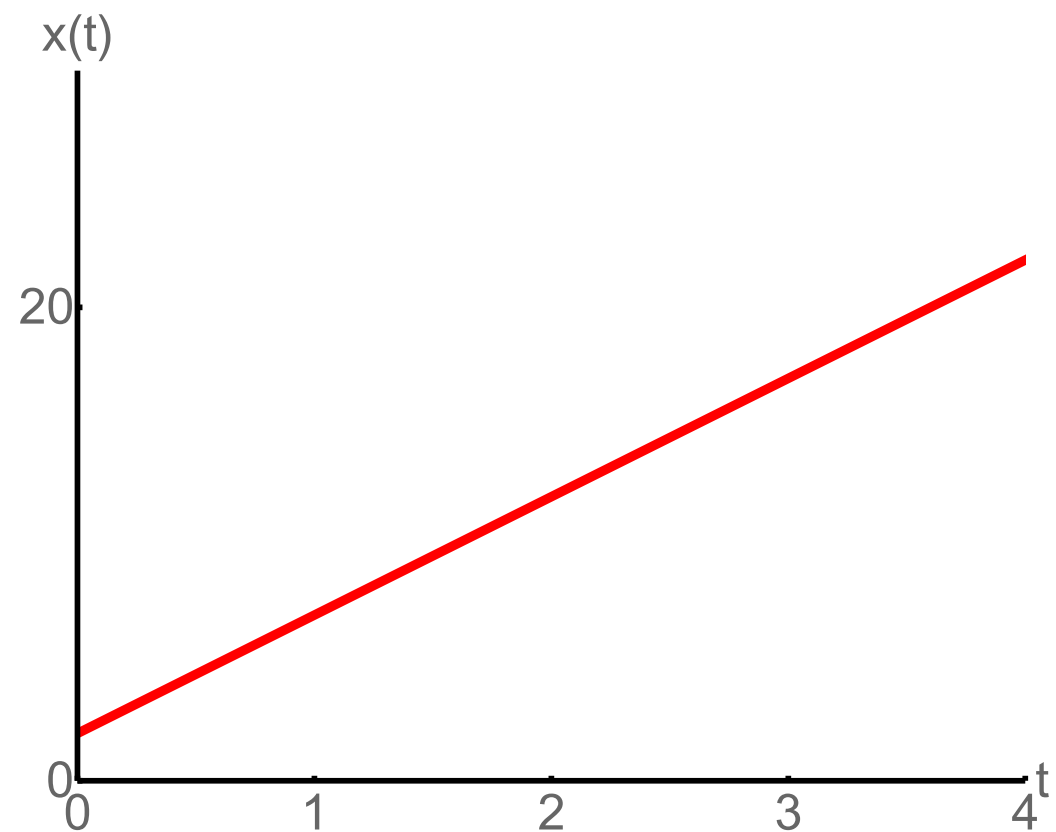


E.

Instantaneous Velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

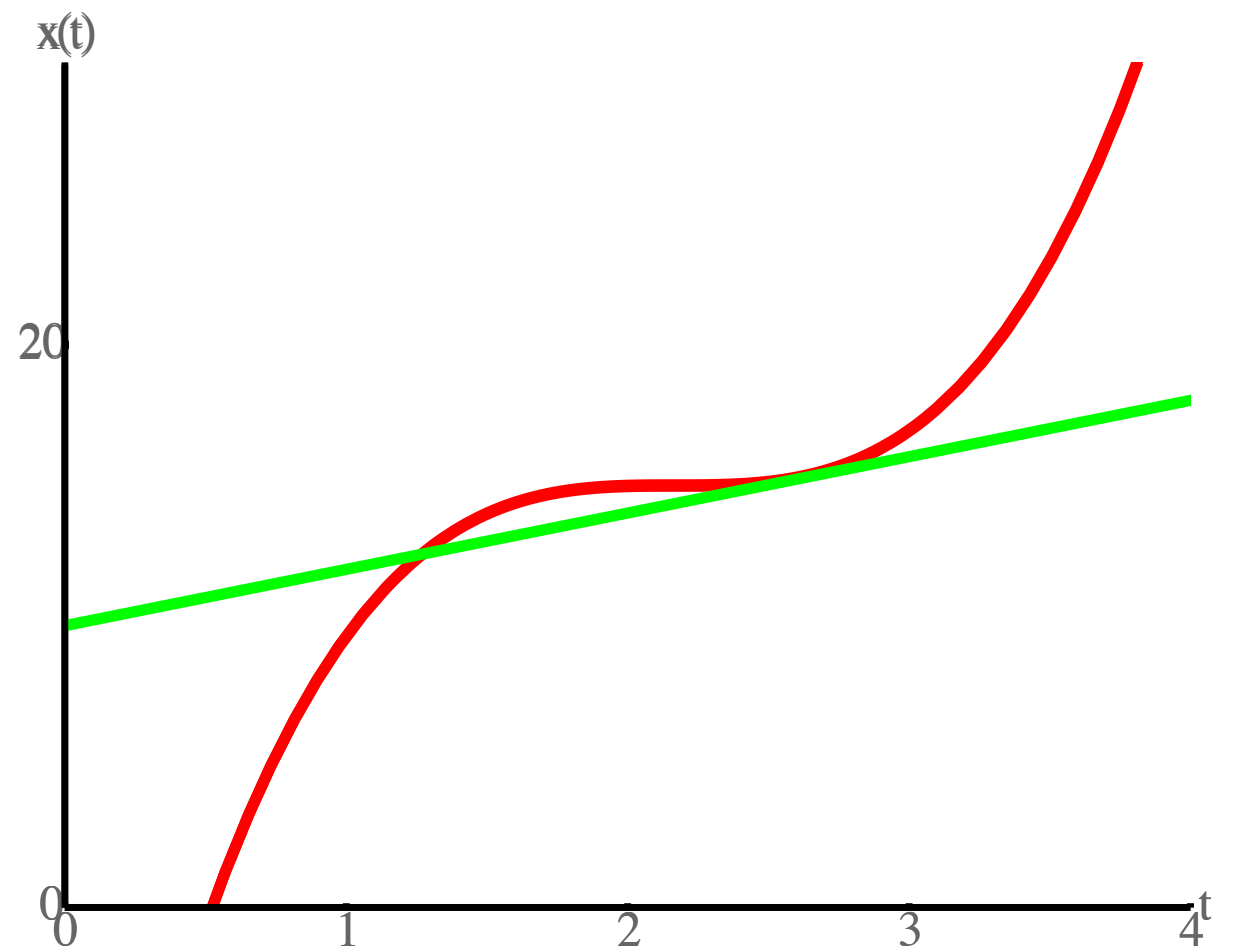
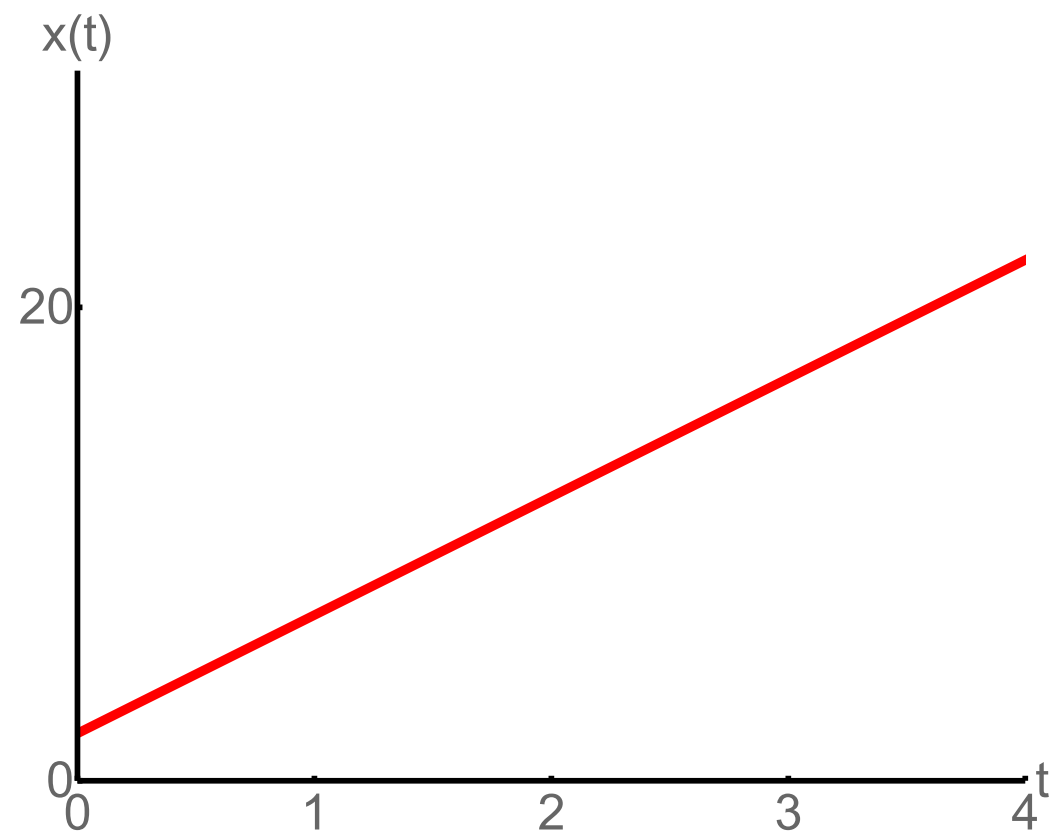
$$x(t) = 5t + 2$$



Instantaneous Velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

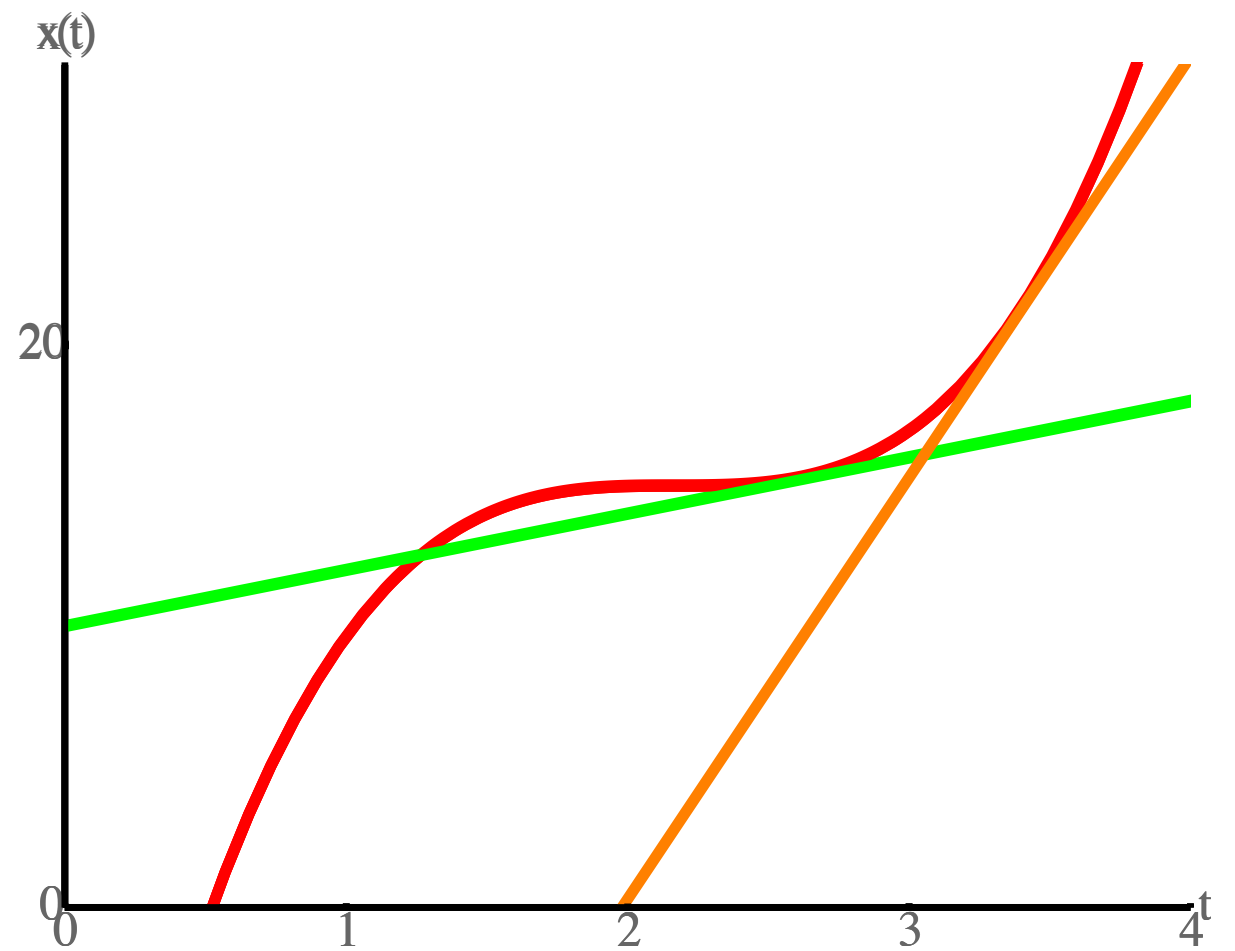
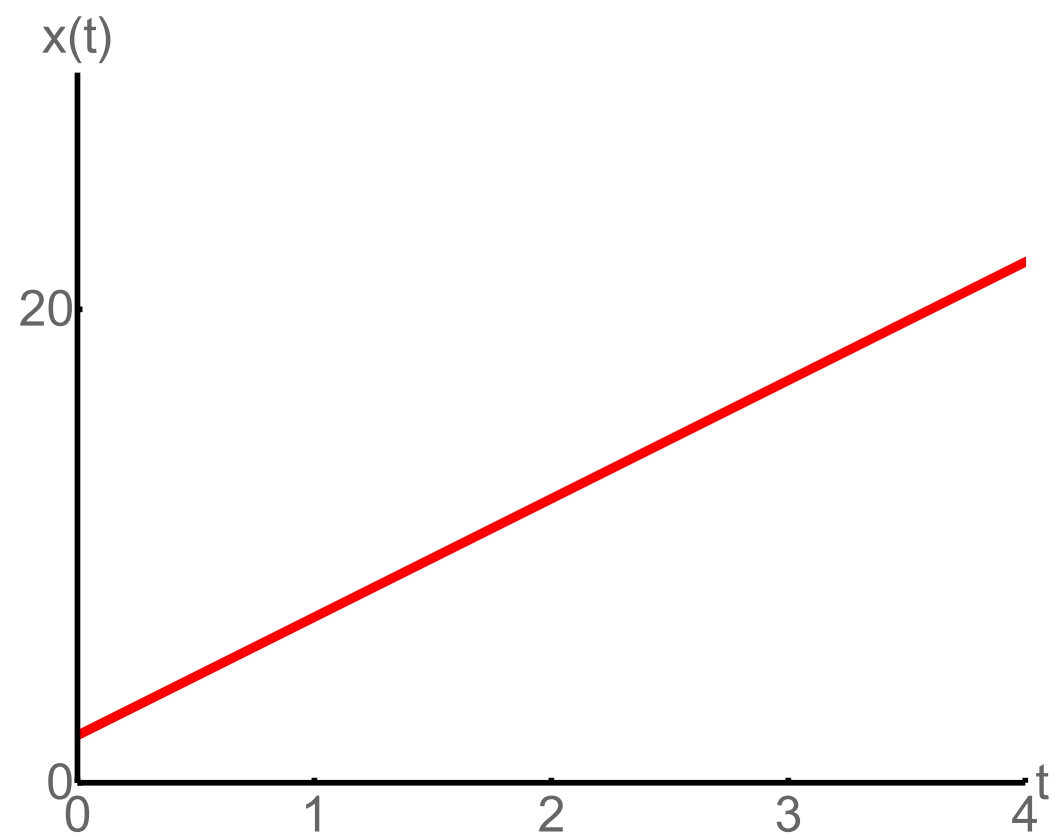
$$x(t) = 5t + 2$$



Instantaneous Velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

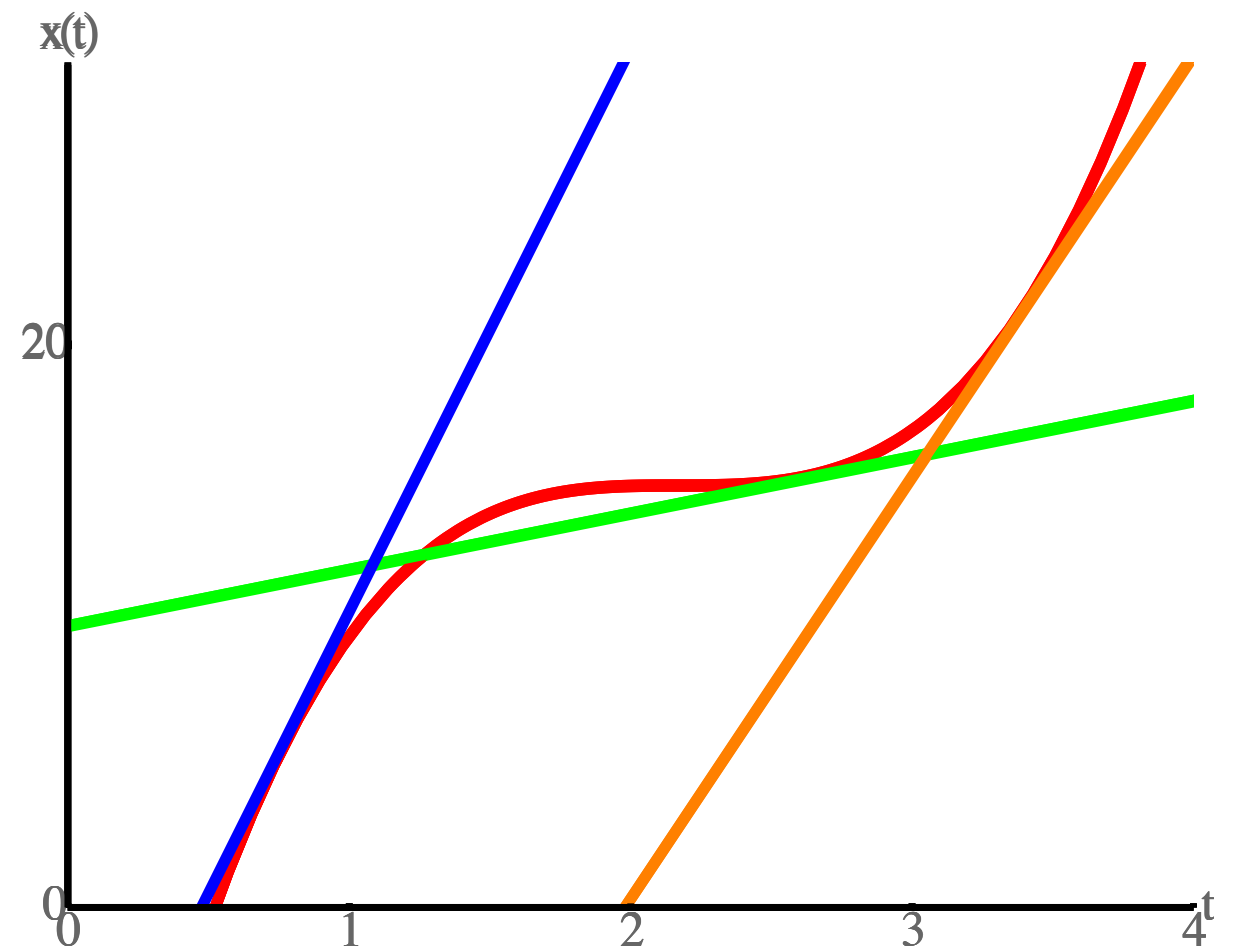
$$x(t) = 5t + 2$$



Instantaneous Velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

$$x(t) = 5t + 2$$

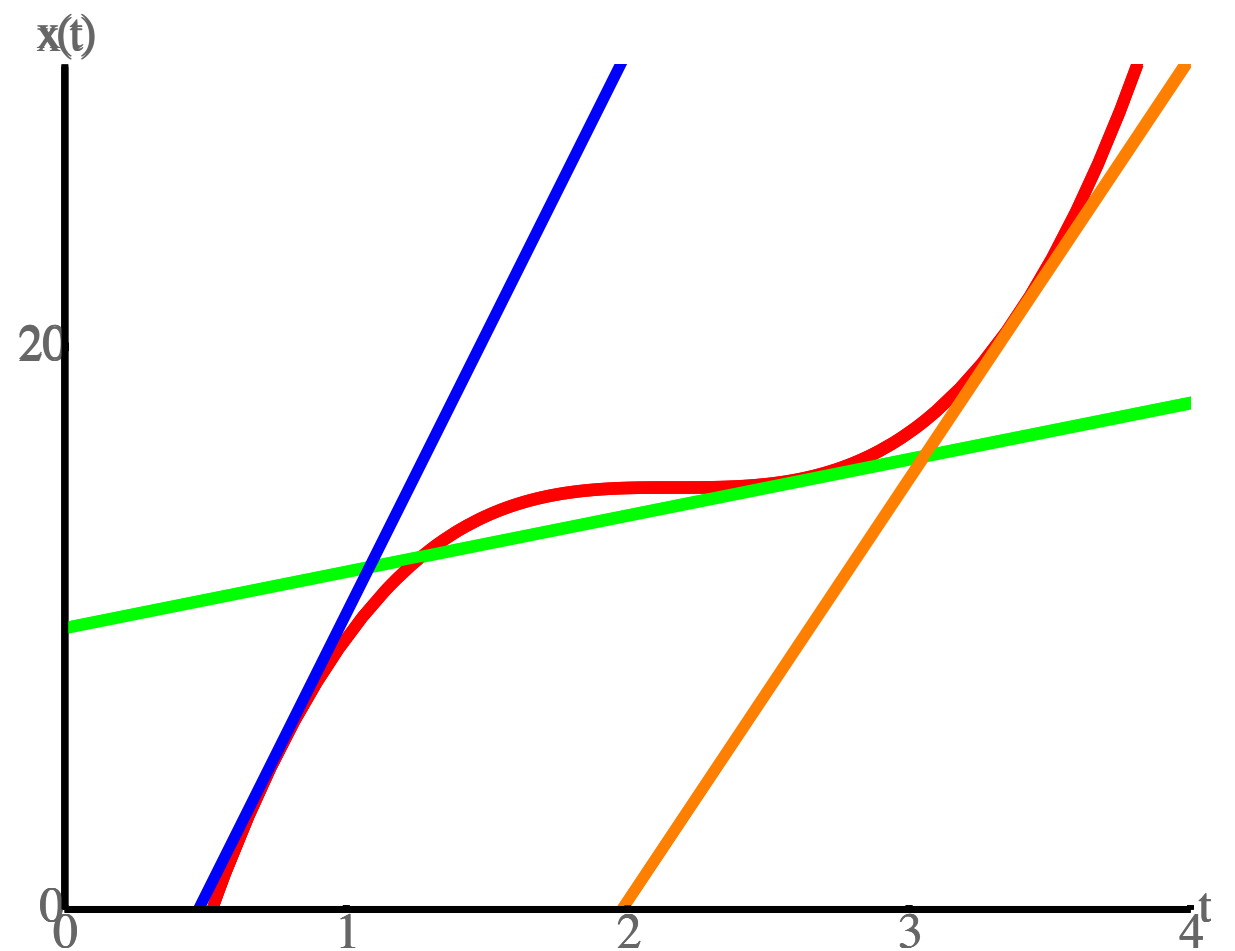
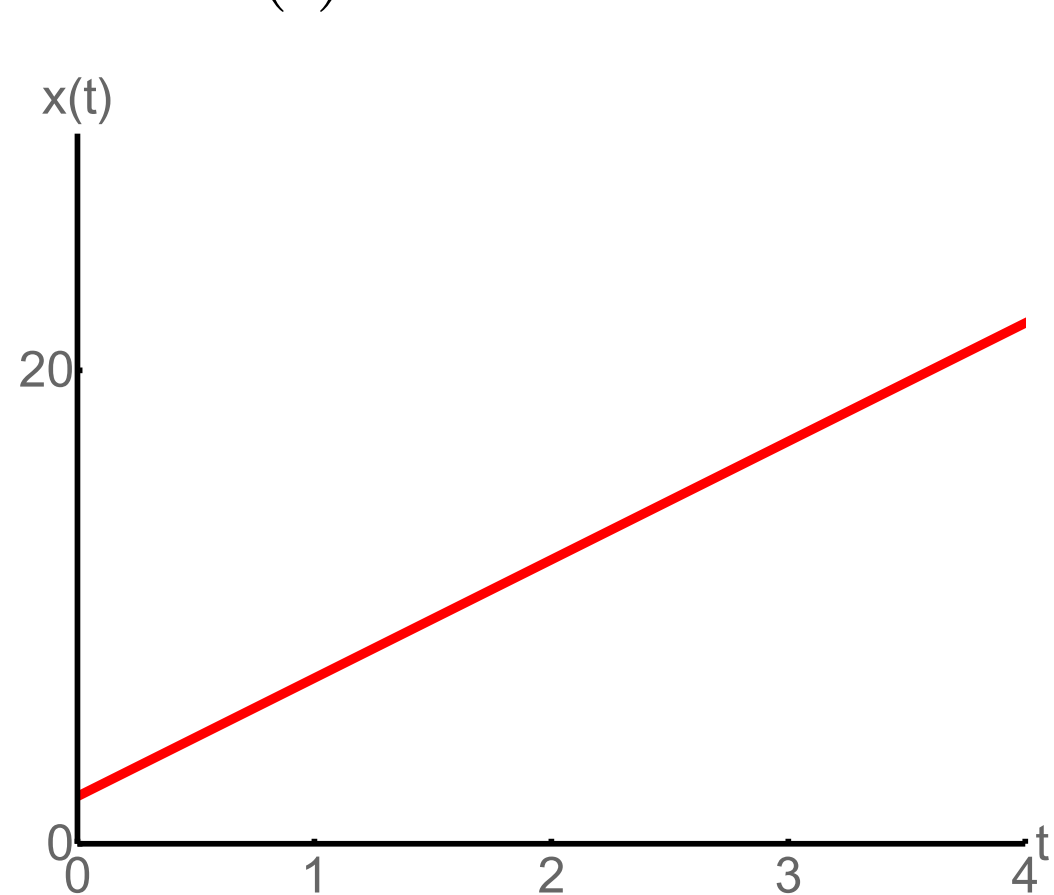


Instantaneous Velocity

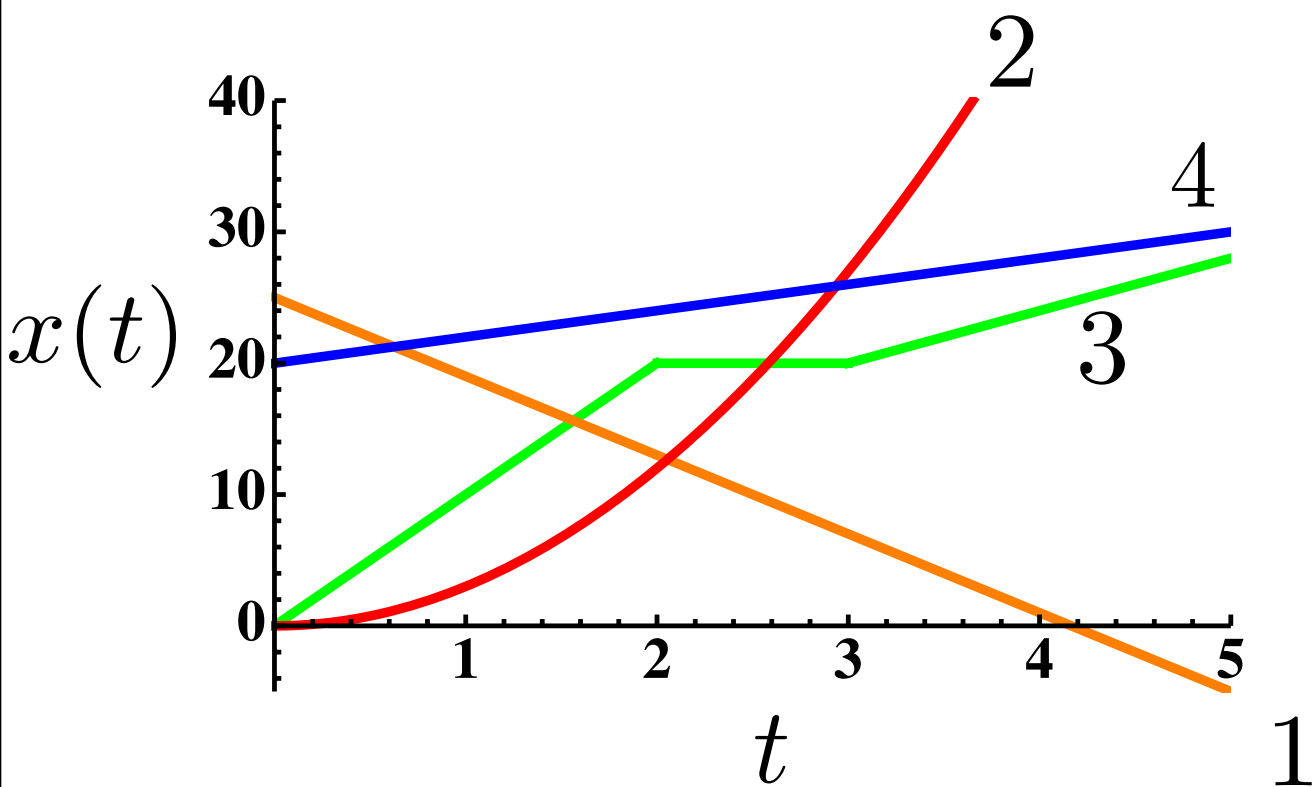
$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$

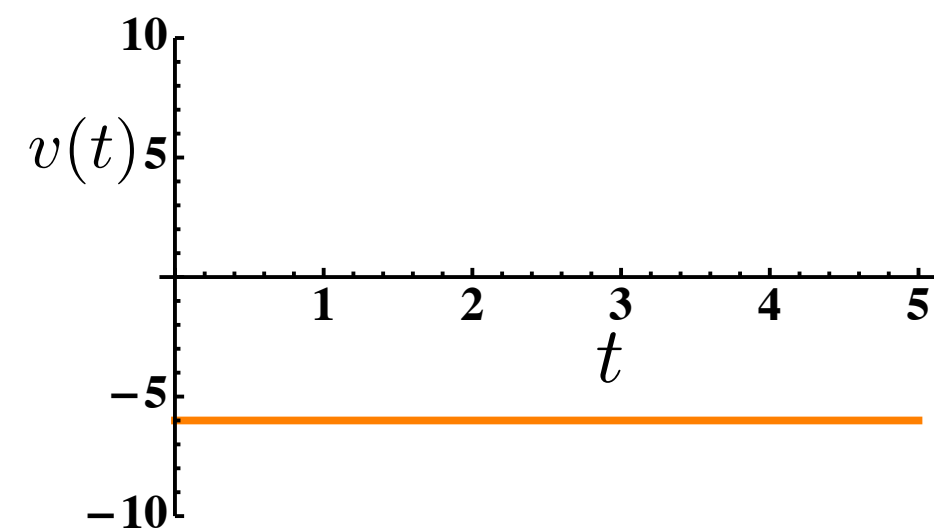
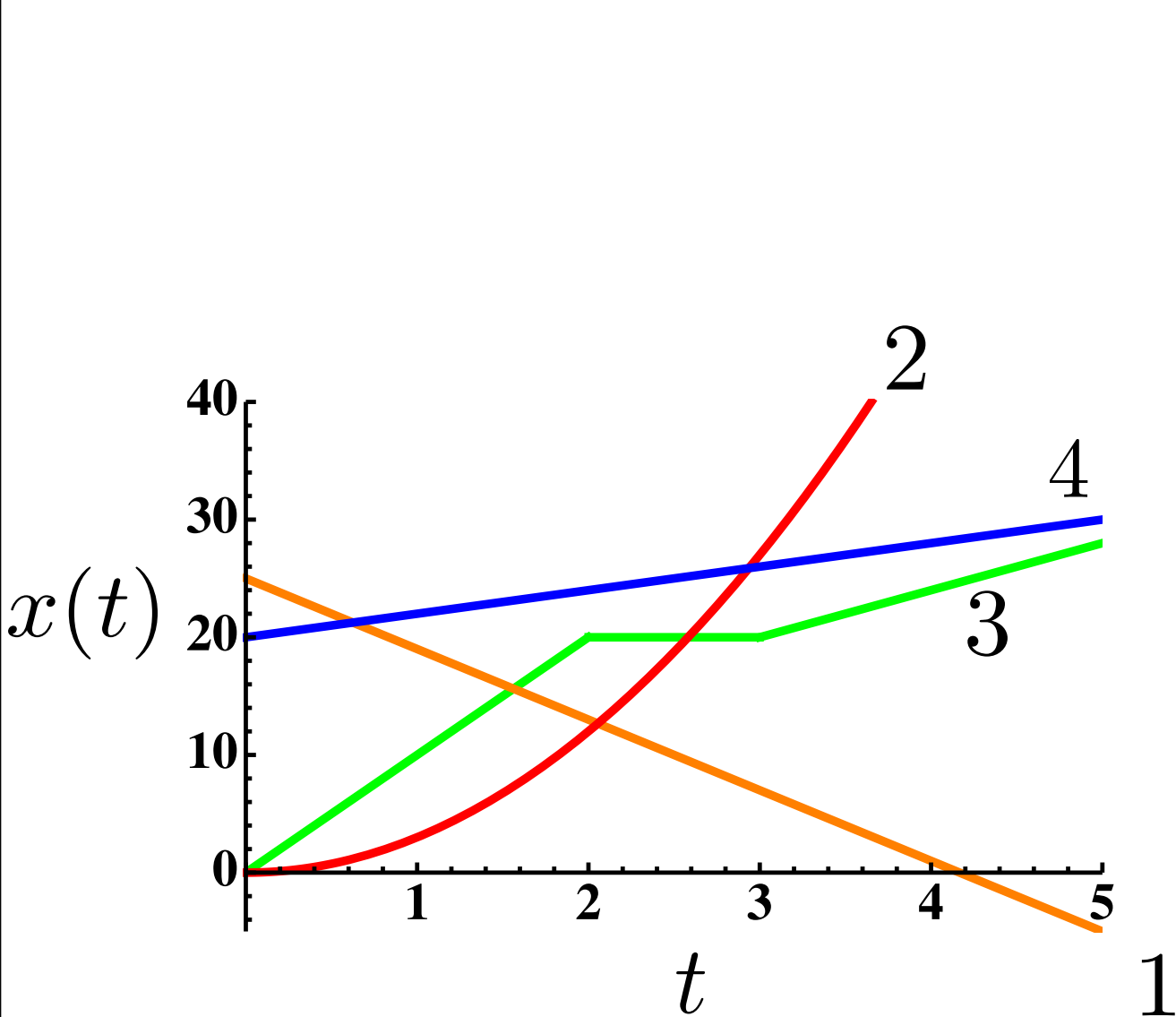
$$x(t) = 5t + 2$$



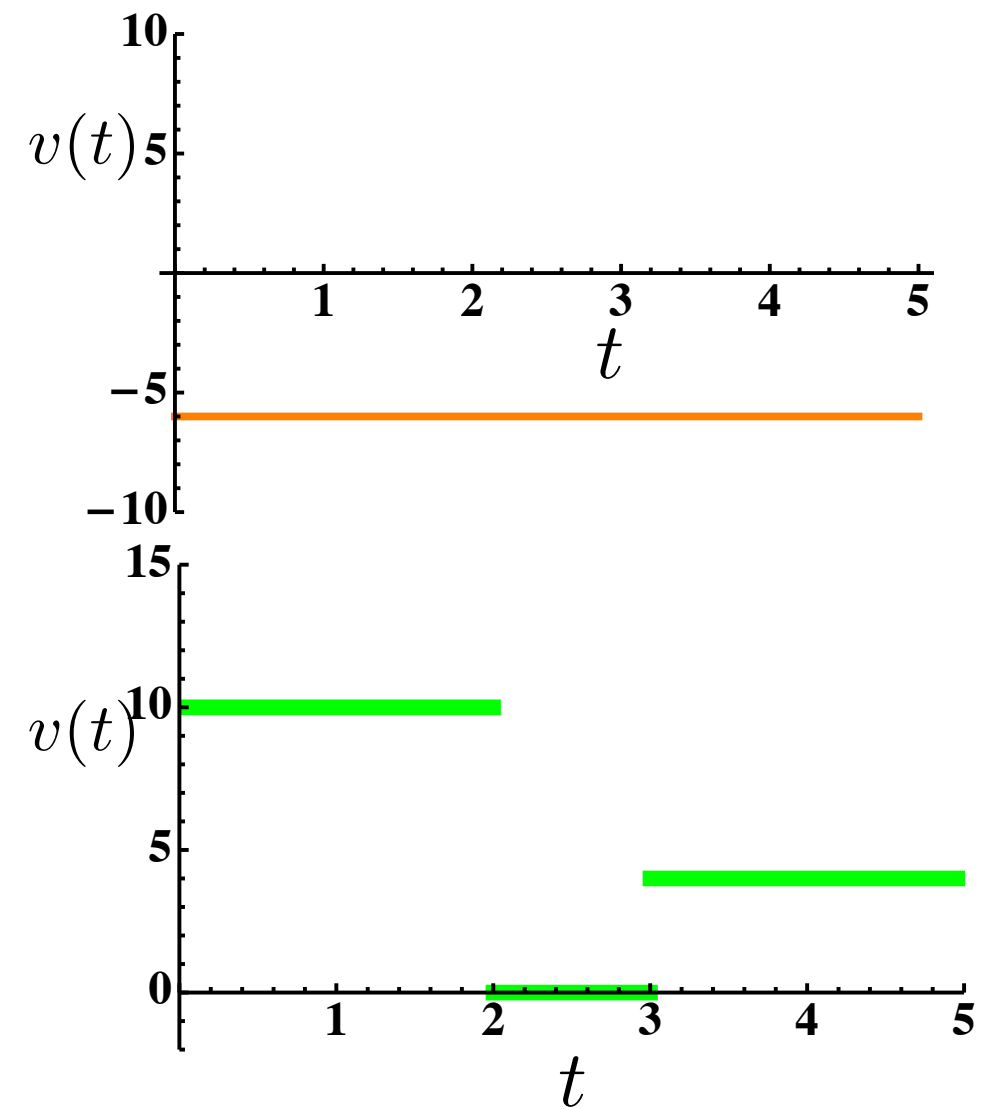
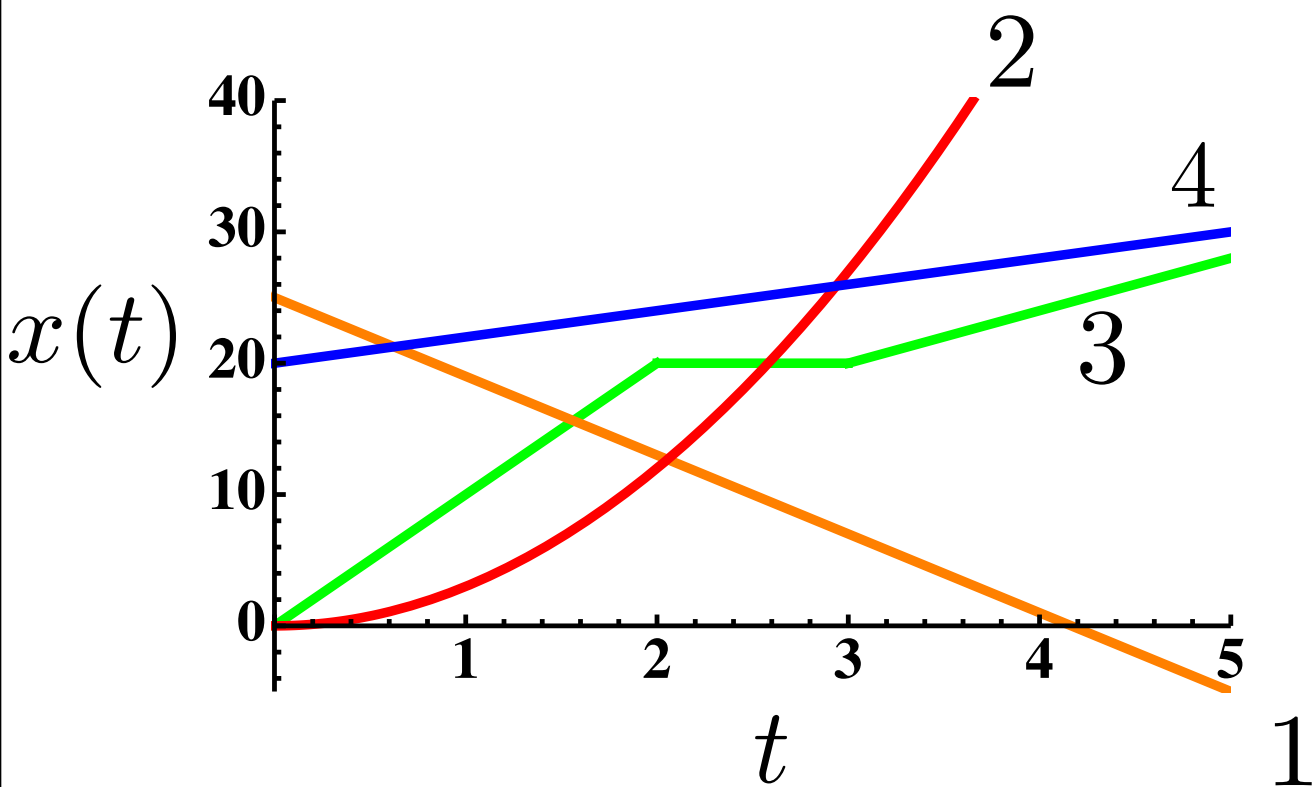
Velocity from Position



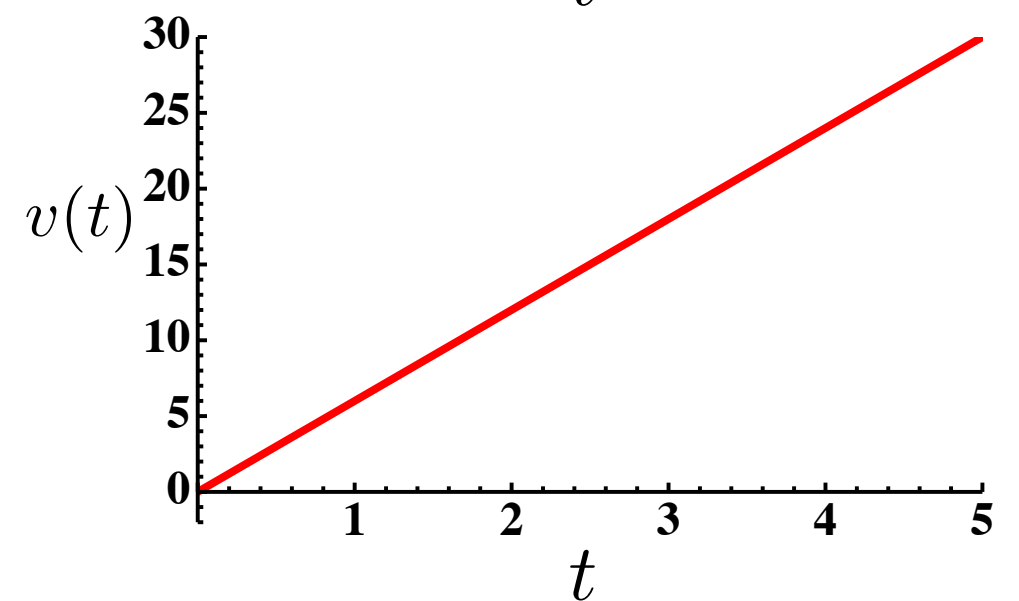
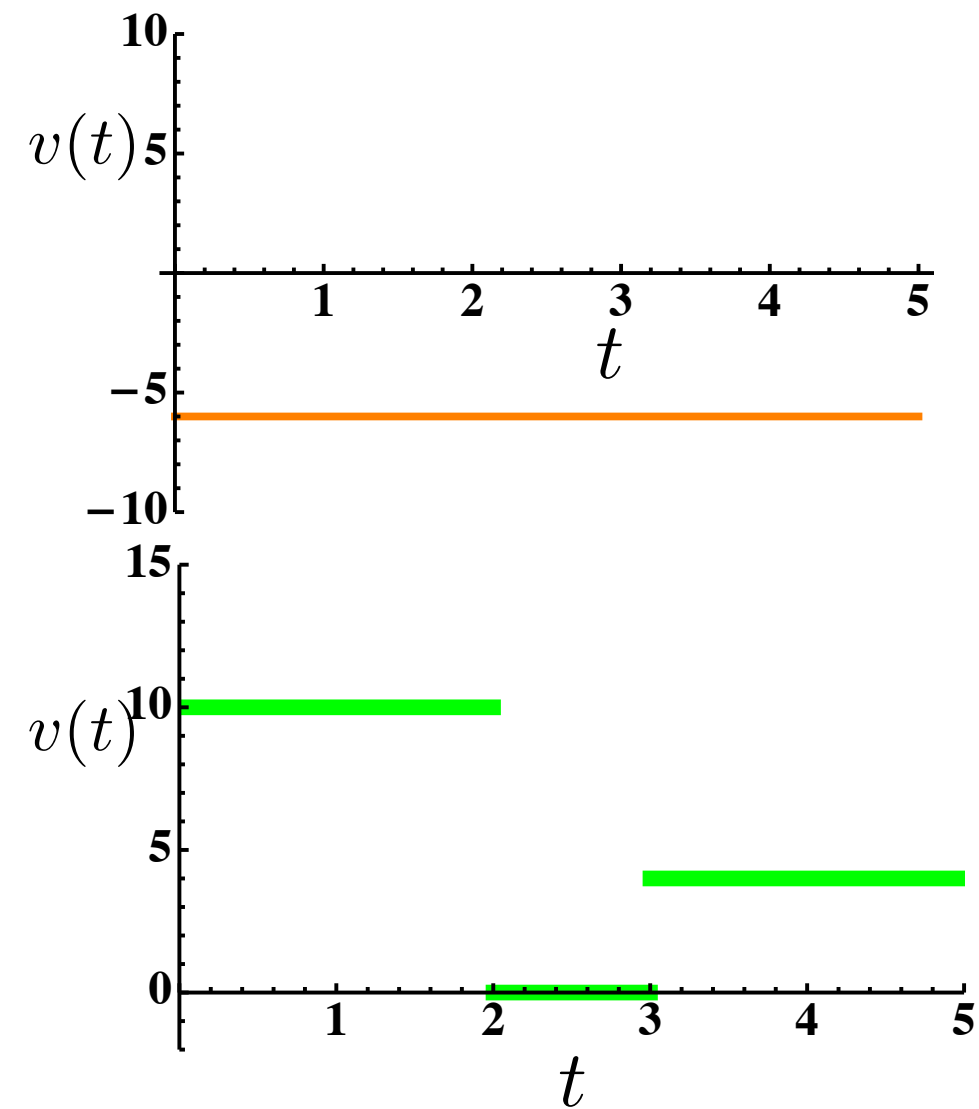
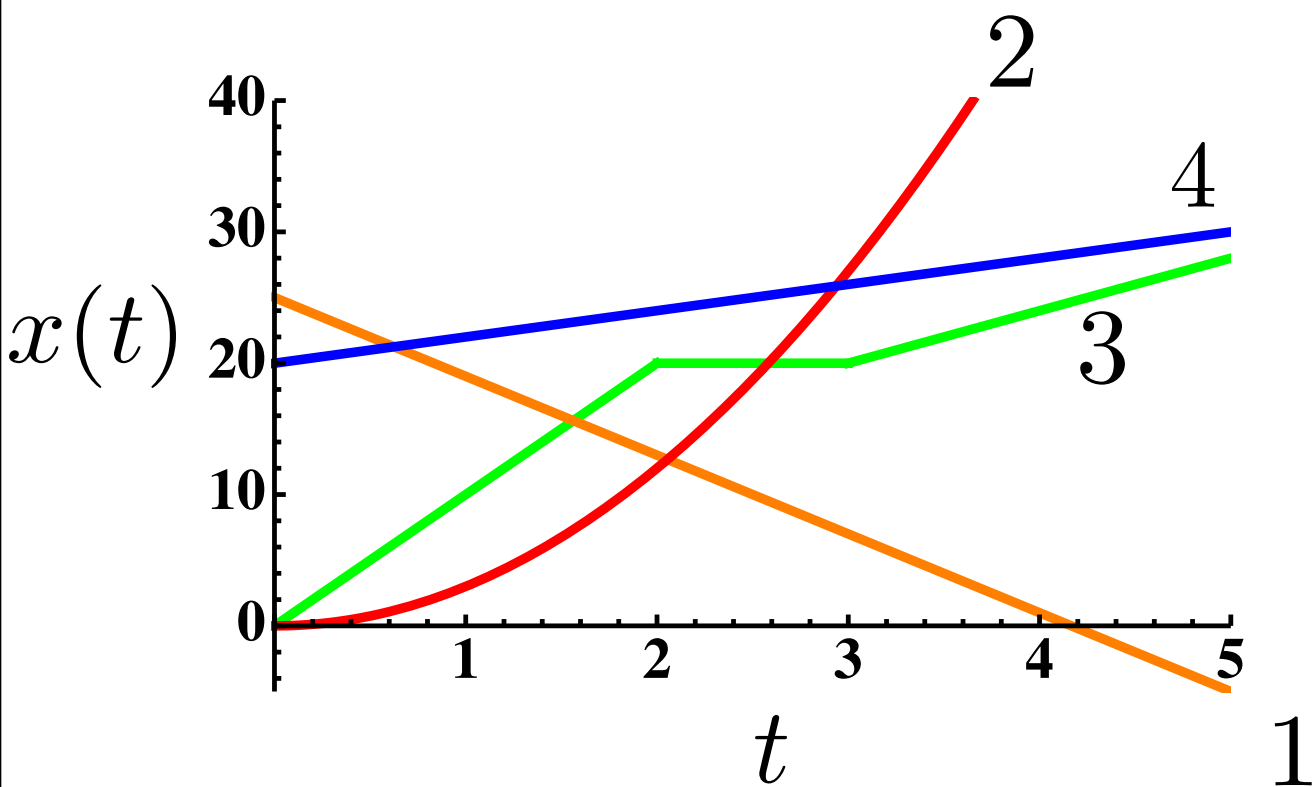
Velocity from Position



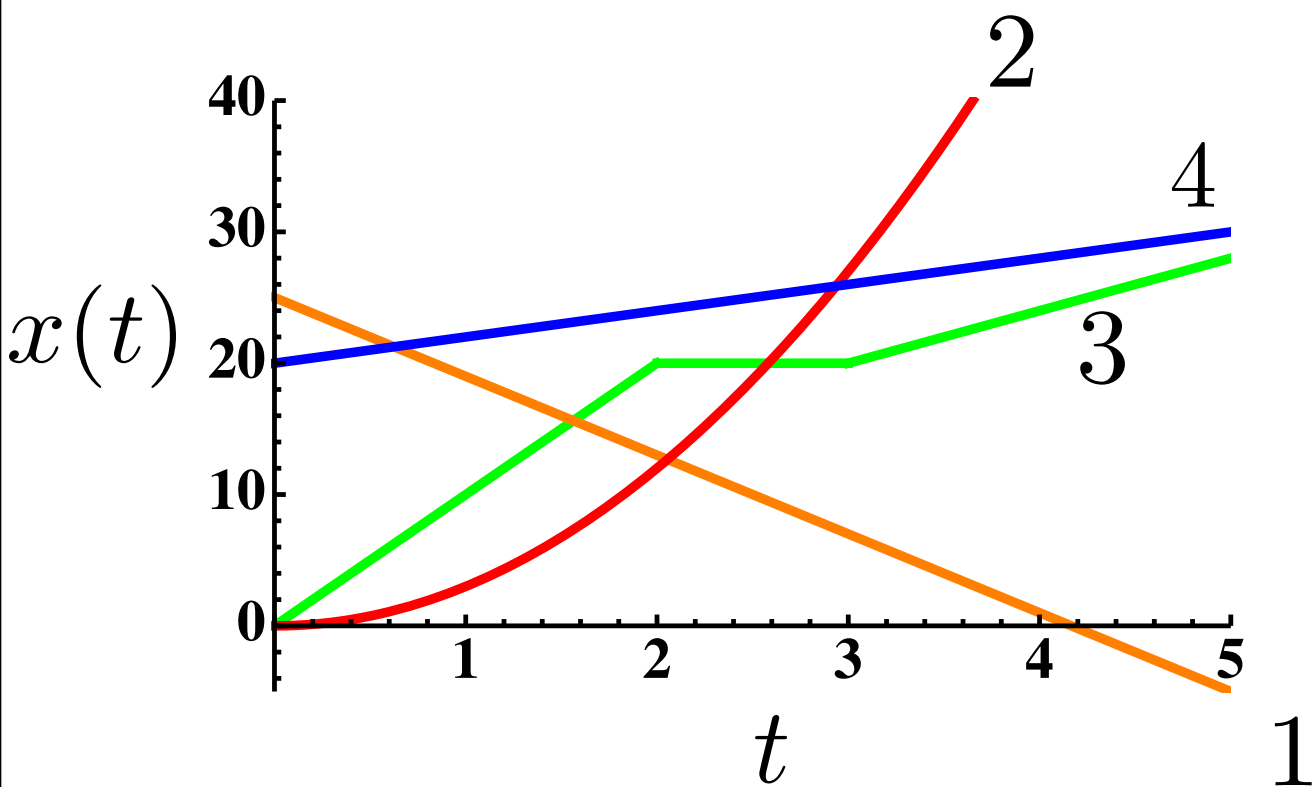
Velocity from Position



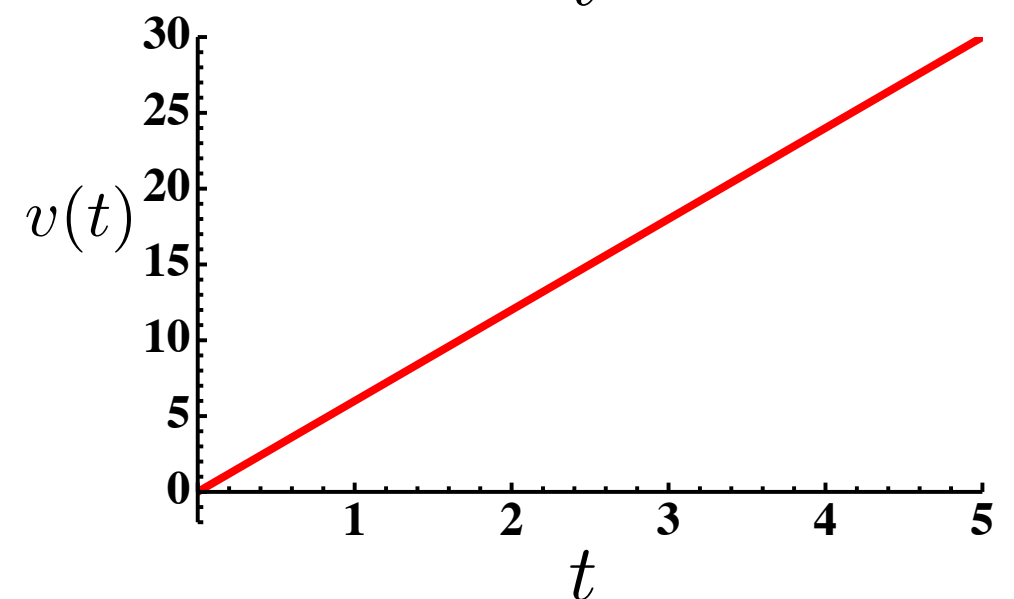
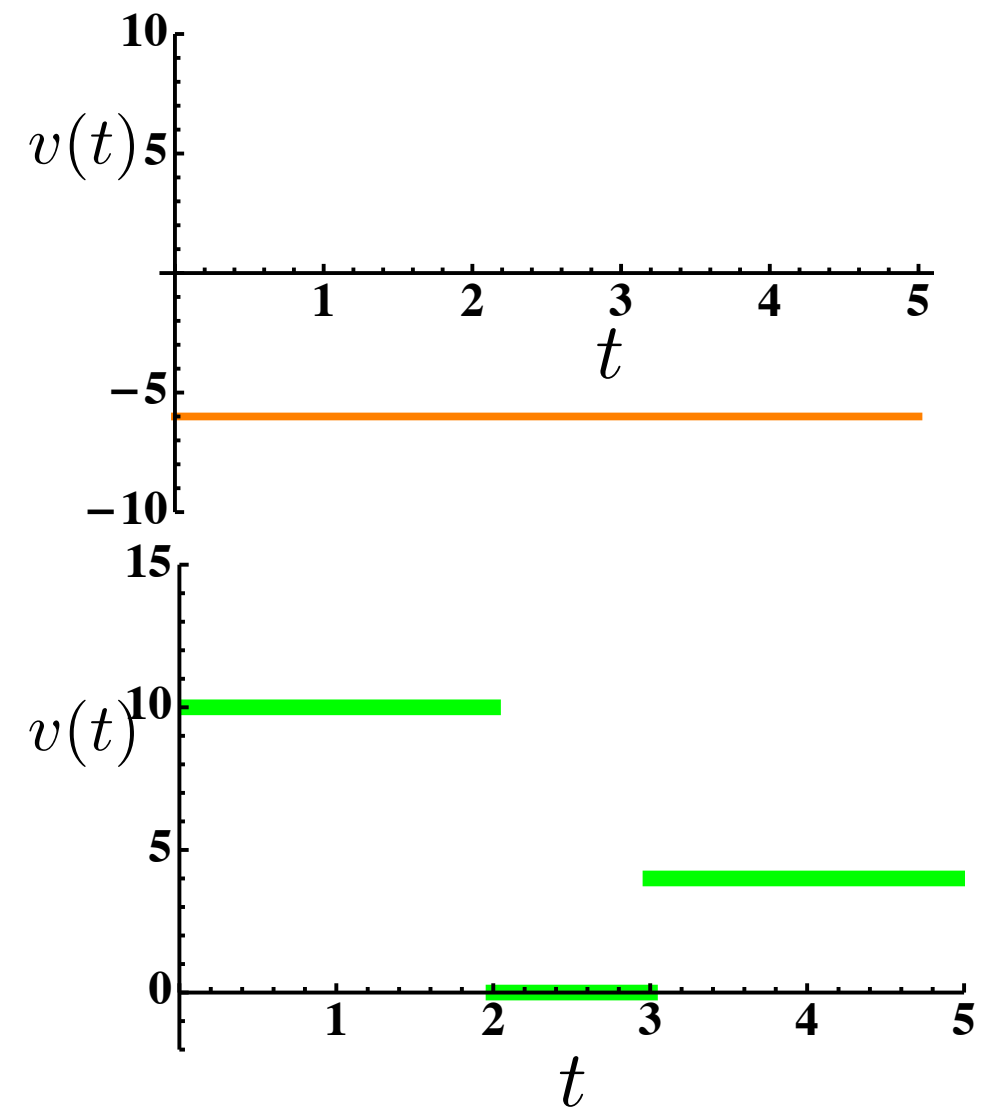
Velocity from Position



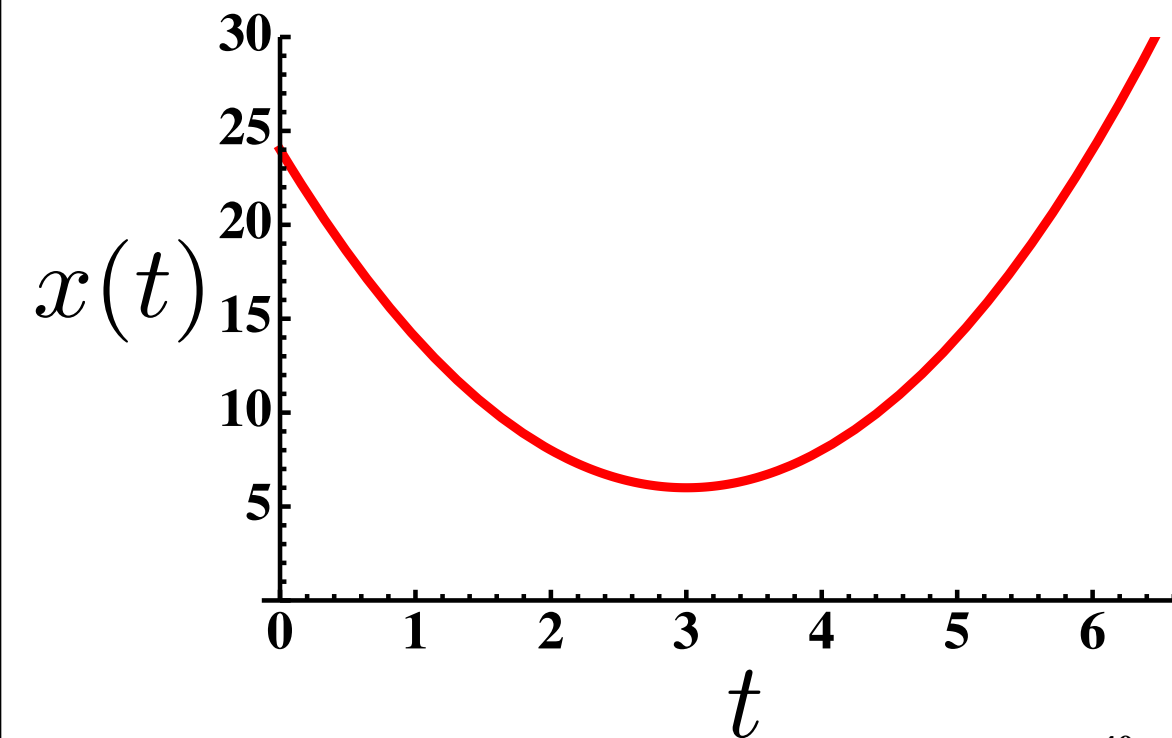
Velocity from Position



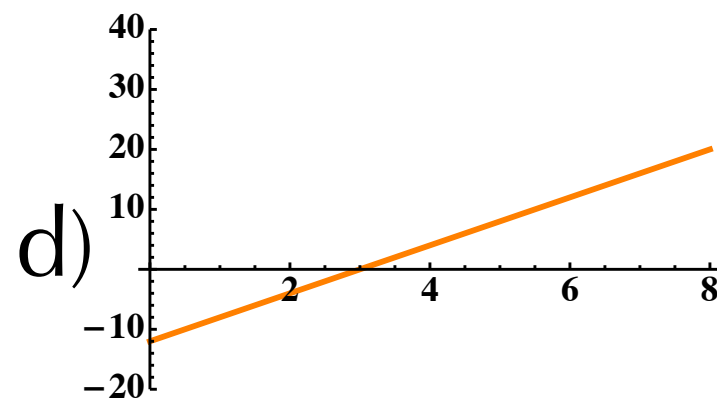
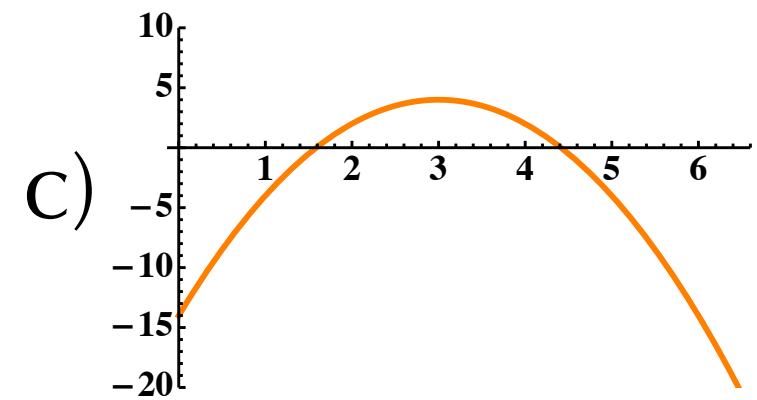
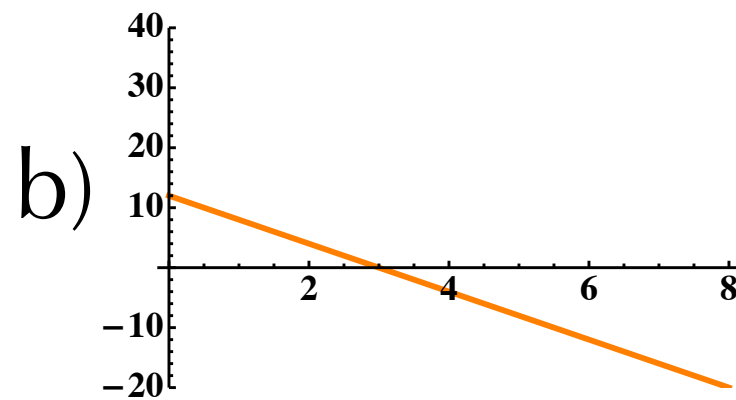
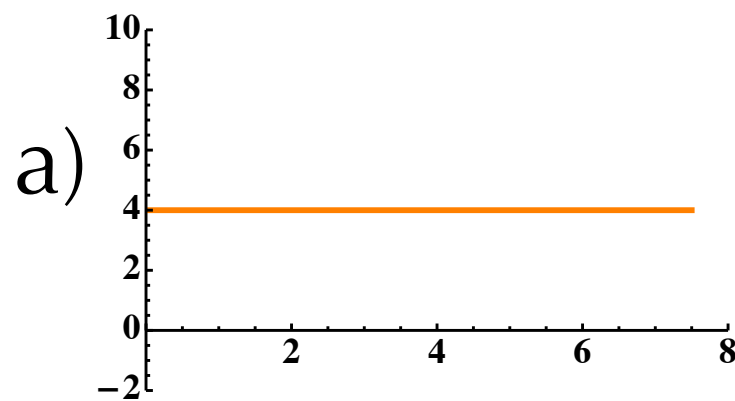
$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$



iClicker quiz

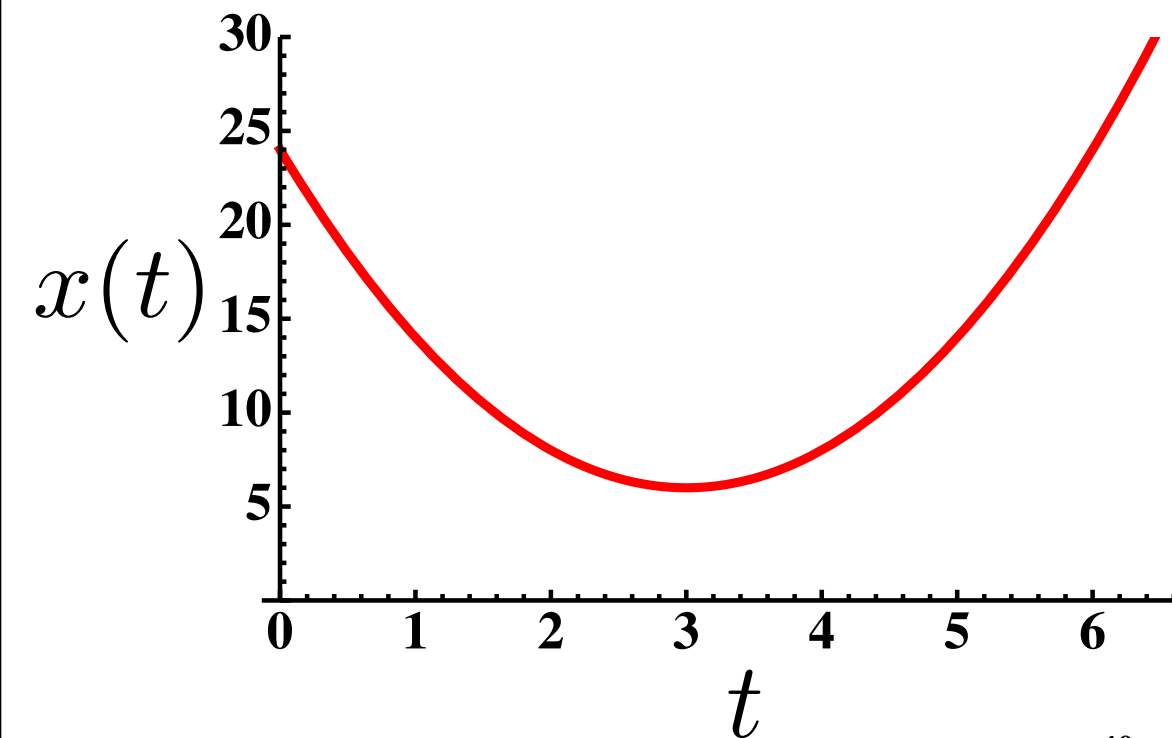


Find the velocity vs. time graph that corresponds to the position vs. time graph seen at left.

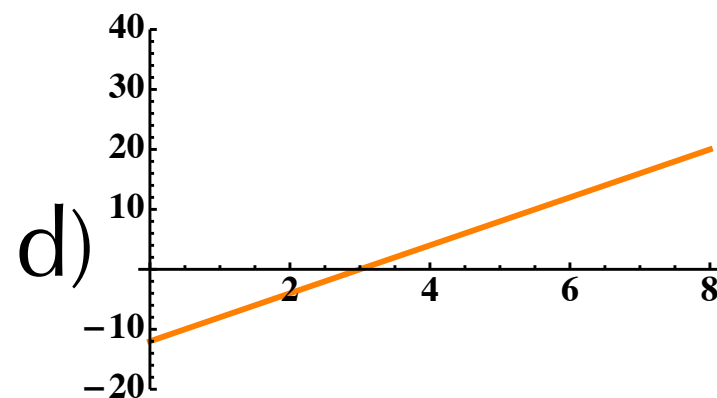
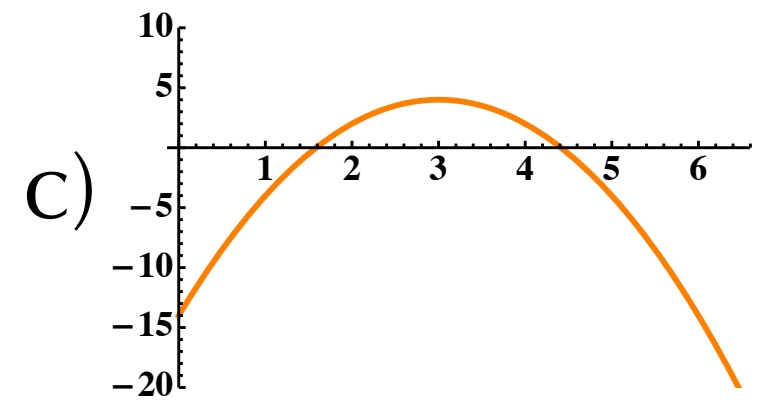
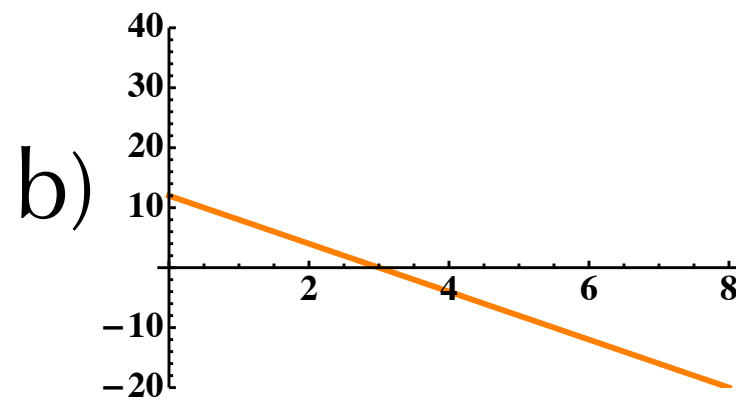
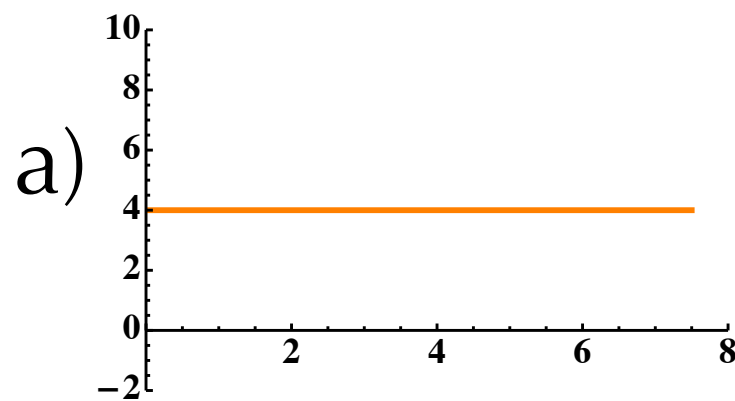


e) none of the above

iClicker quiz

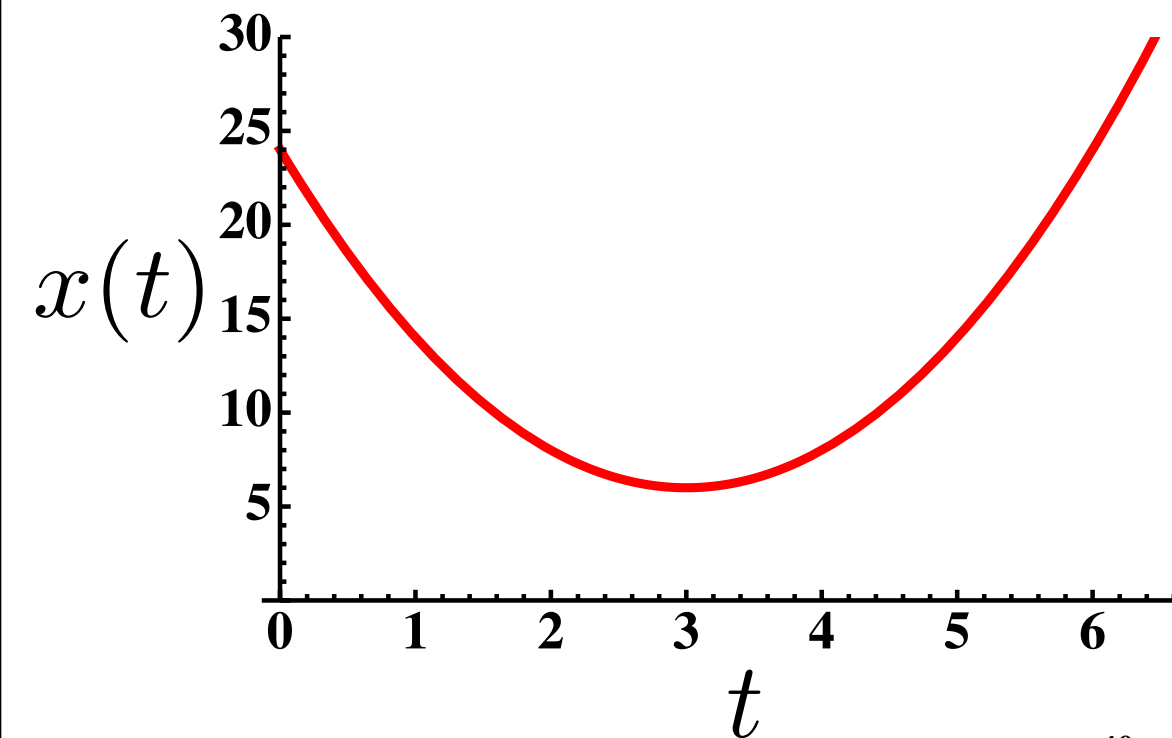


Find the velocity vs. time graph that corresponds to the position vs. time graph seen at left.

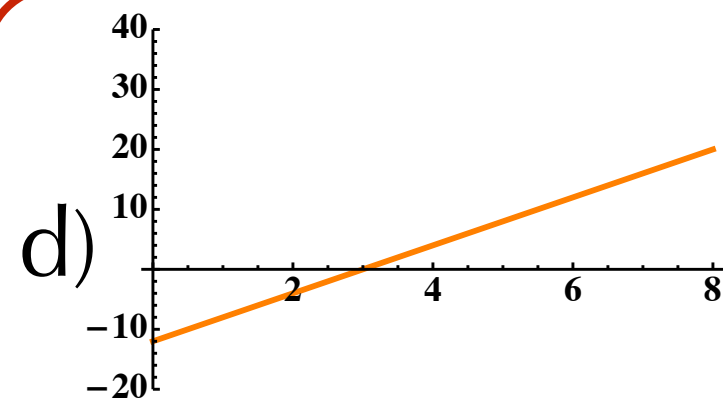
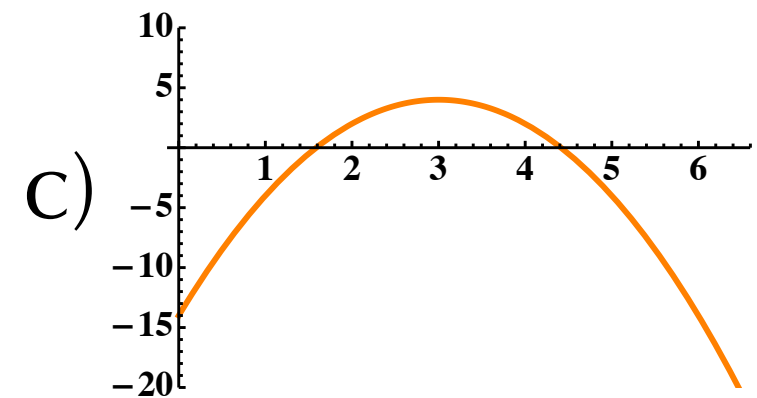
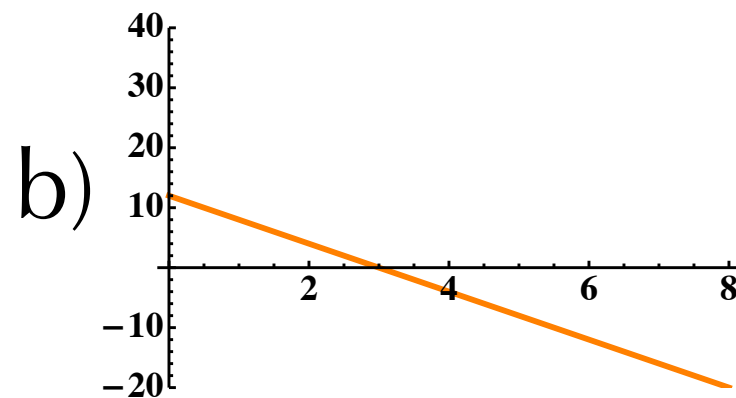
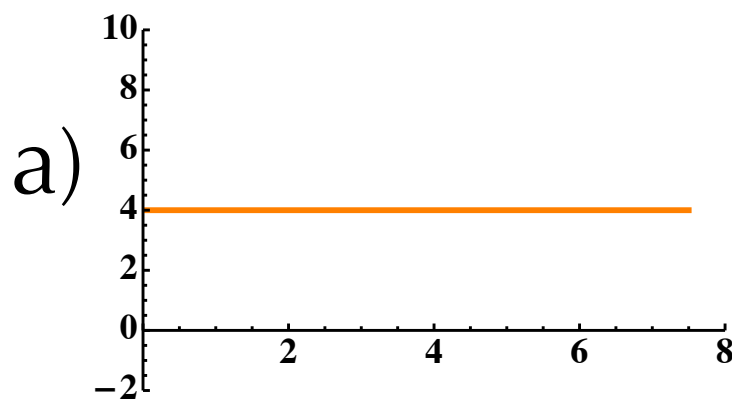


e) none of the above

iClicker quiz

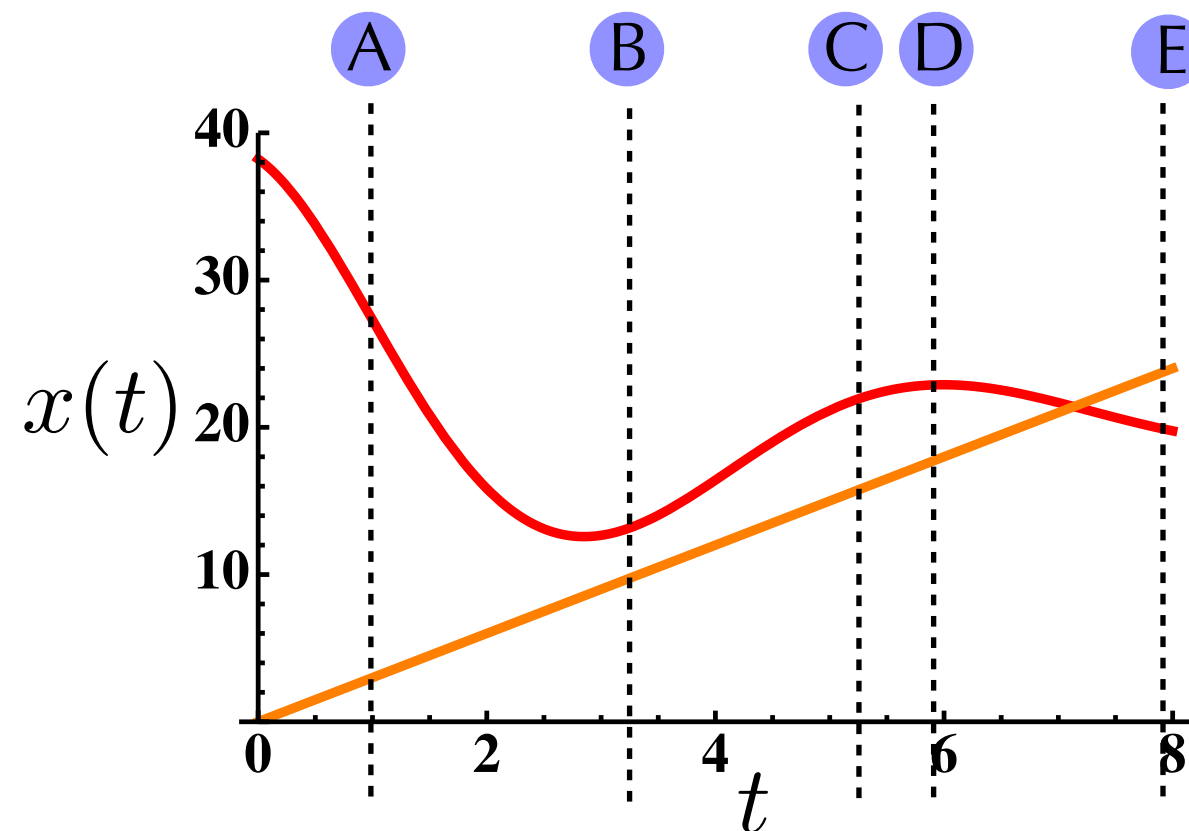


Find the velocity vs. time graph that corresponds to the position vs. time graph seen at left.



e) none of the above

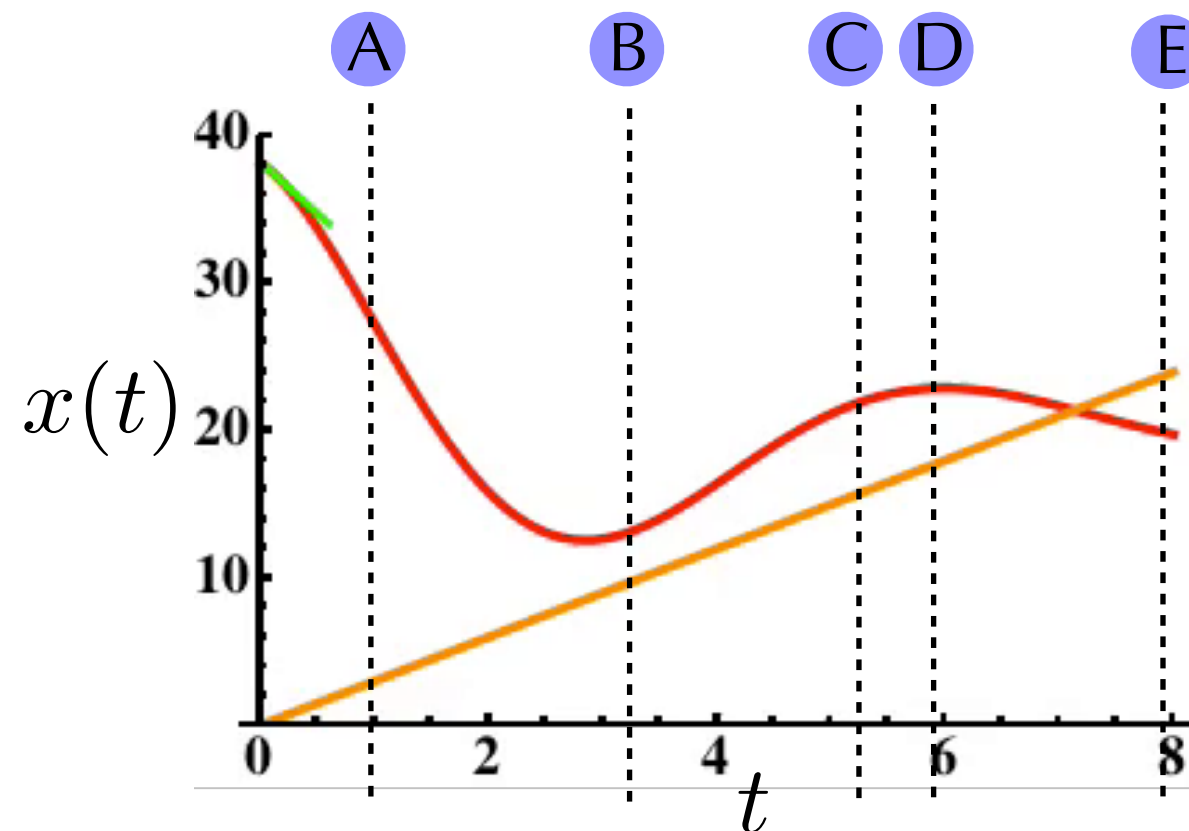
iClicker quiz



At what time(s) do these two objects have the same speed?

- a) A & E
- b) B only
- c) B, C, & E
- d) B & C
- e) C only
- f) The two objects never have the same speed

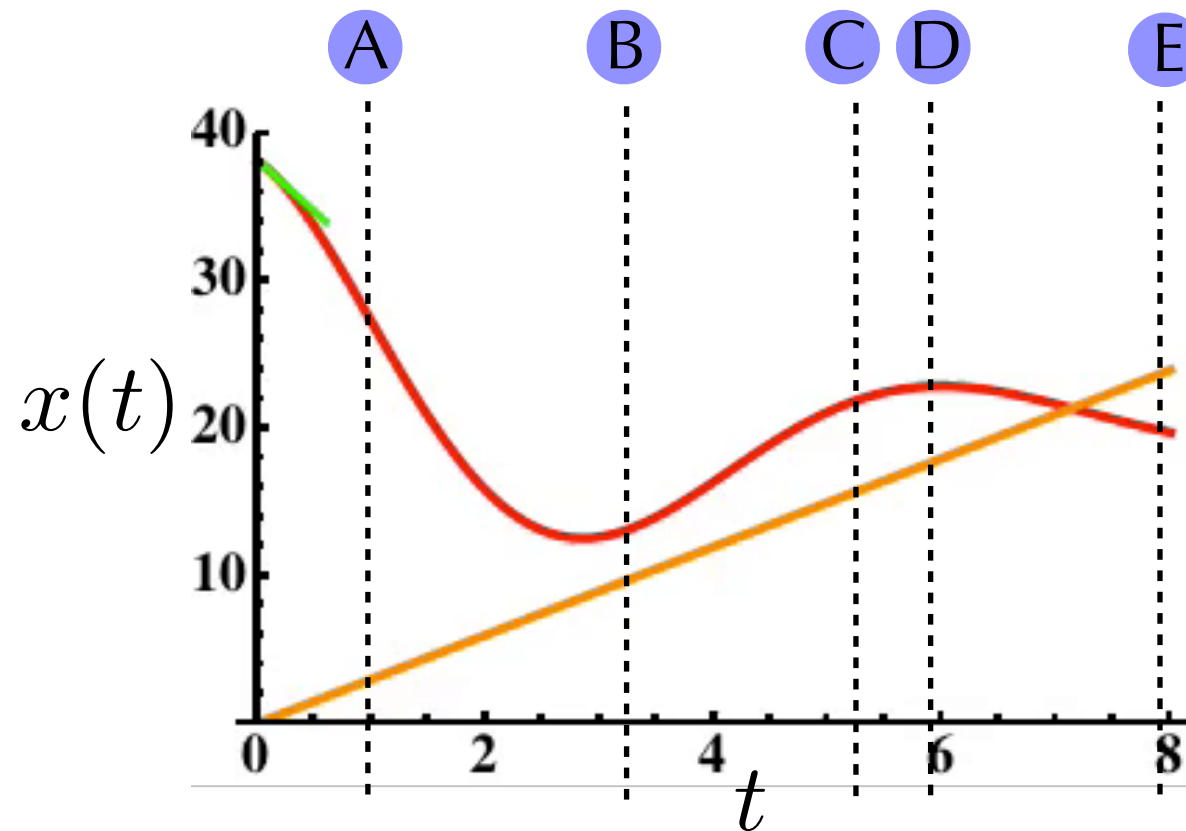
iClicker quiz



At what time(s) do these two objects have the same speed?

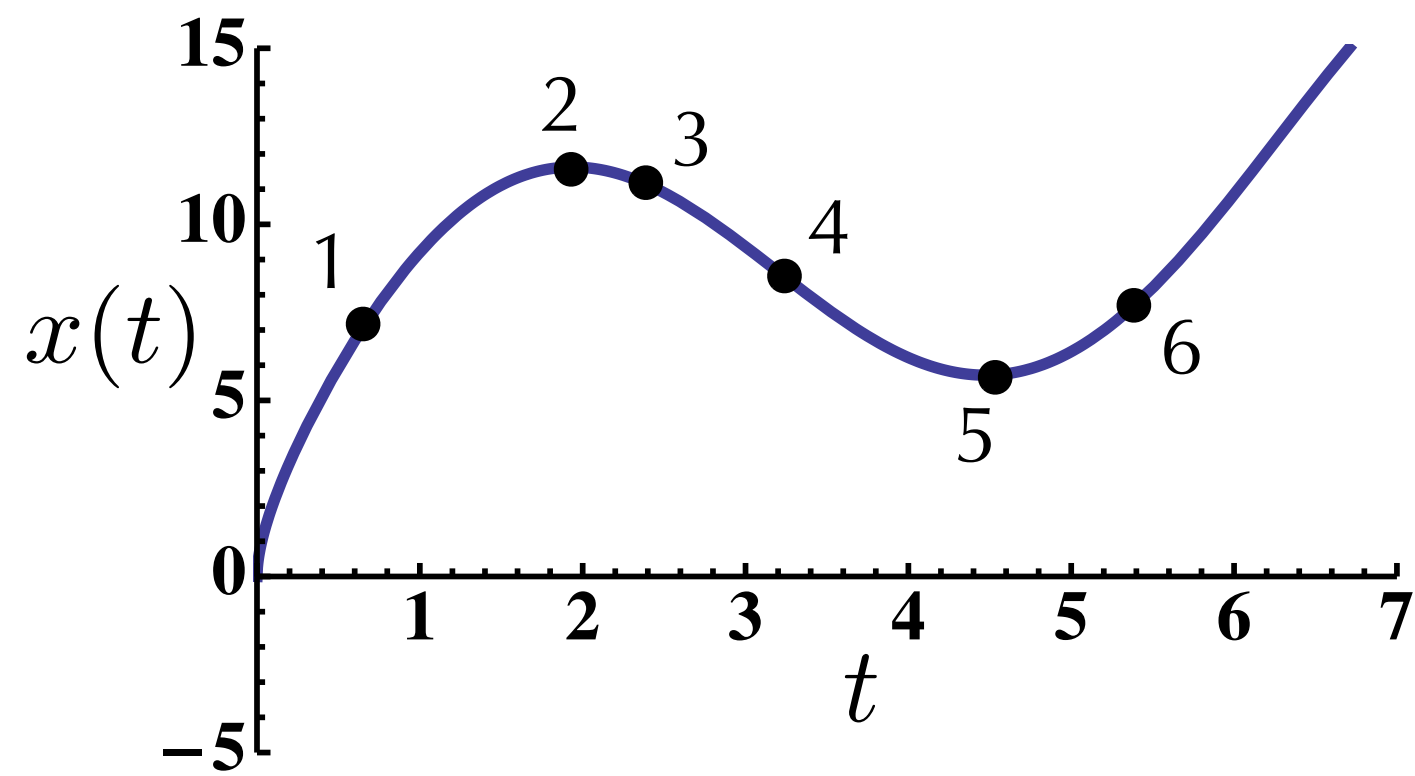
- a) A & E
- b) B only
- c) B, C, & E
- d) B & C
- e) C only
- f) The two objects never have the same speed

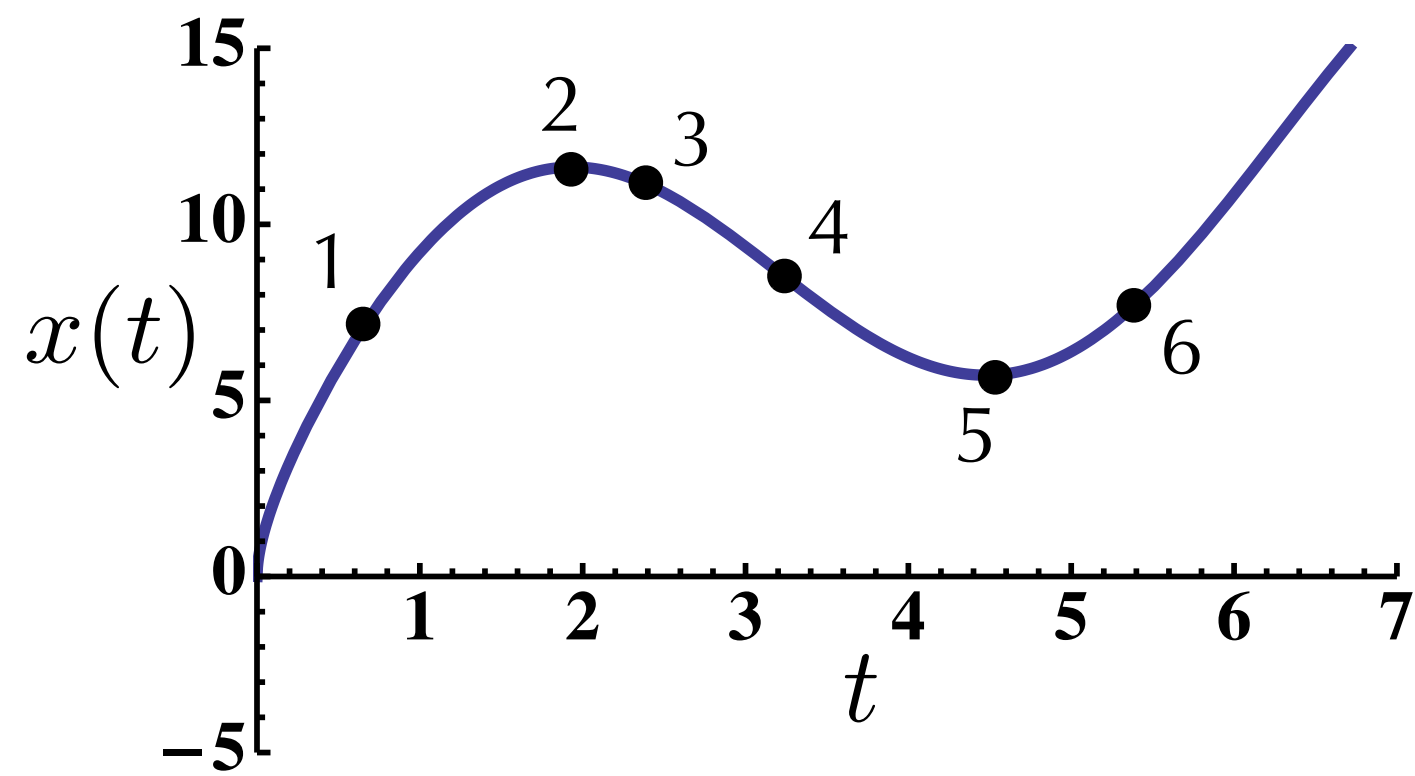
iClicker quiz

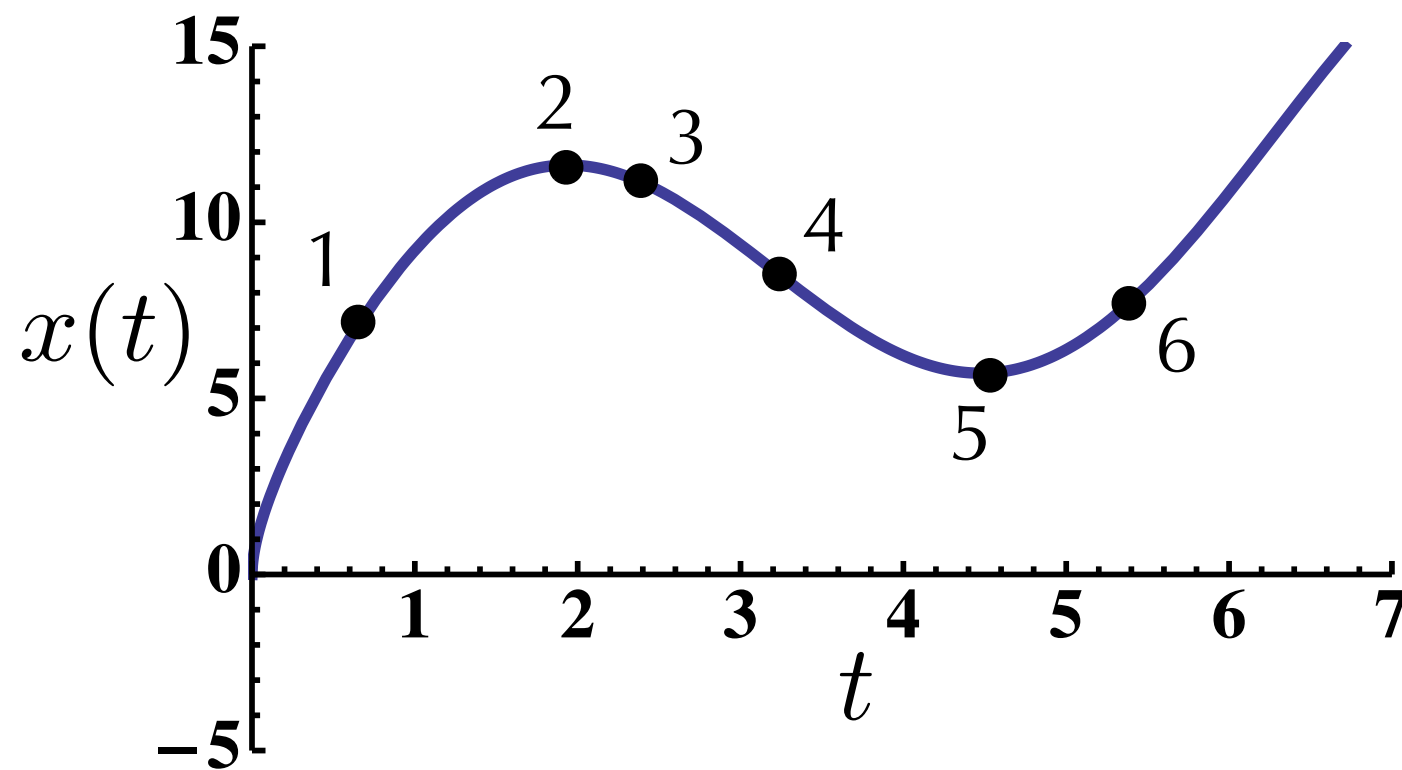


At what time(s) do these two objects have the same speed?

- a) A & E
- b) B only
- c) B, C, & E
- d) B & C
- e) C only
- f) The two objects never have the same speed

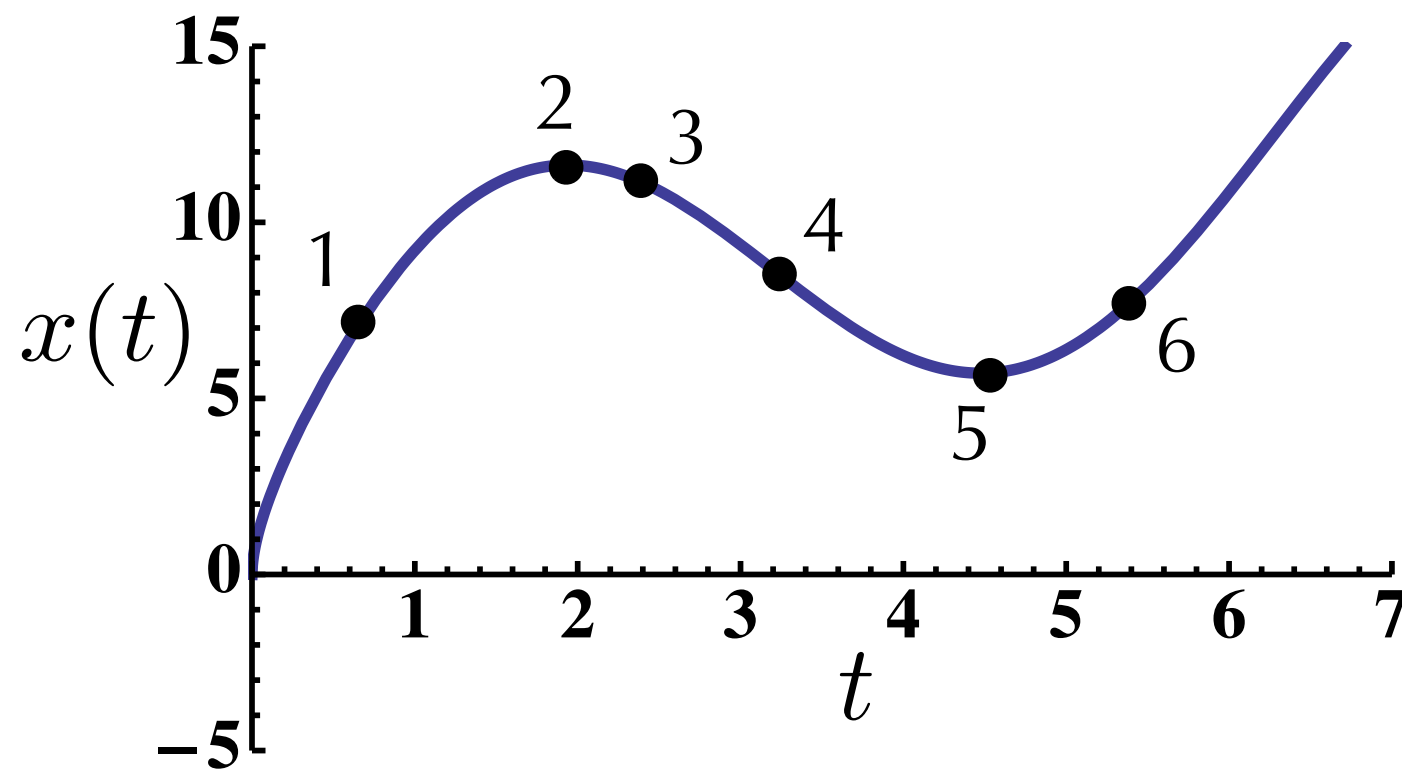






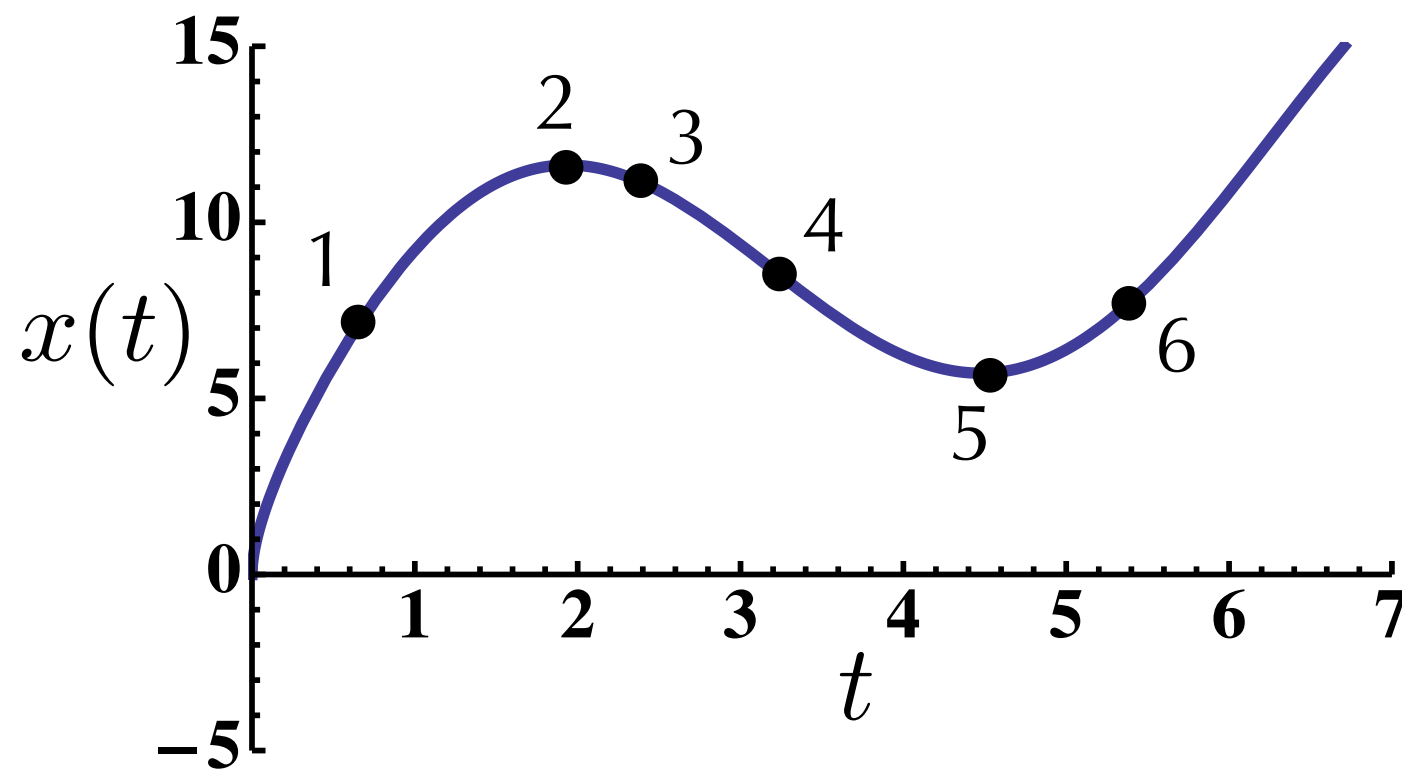
- a) 1 and 6
- b) 3 and 5
- c) 3
- d) 3 and 4
- e) never

At which point(s) is the object moving to the left?



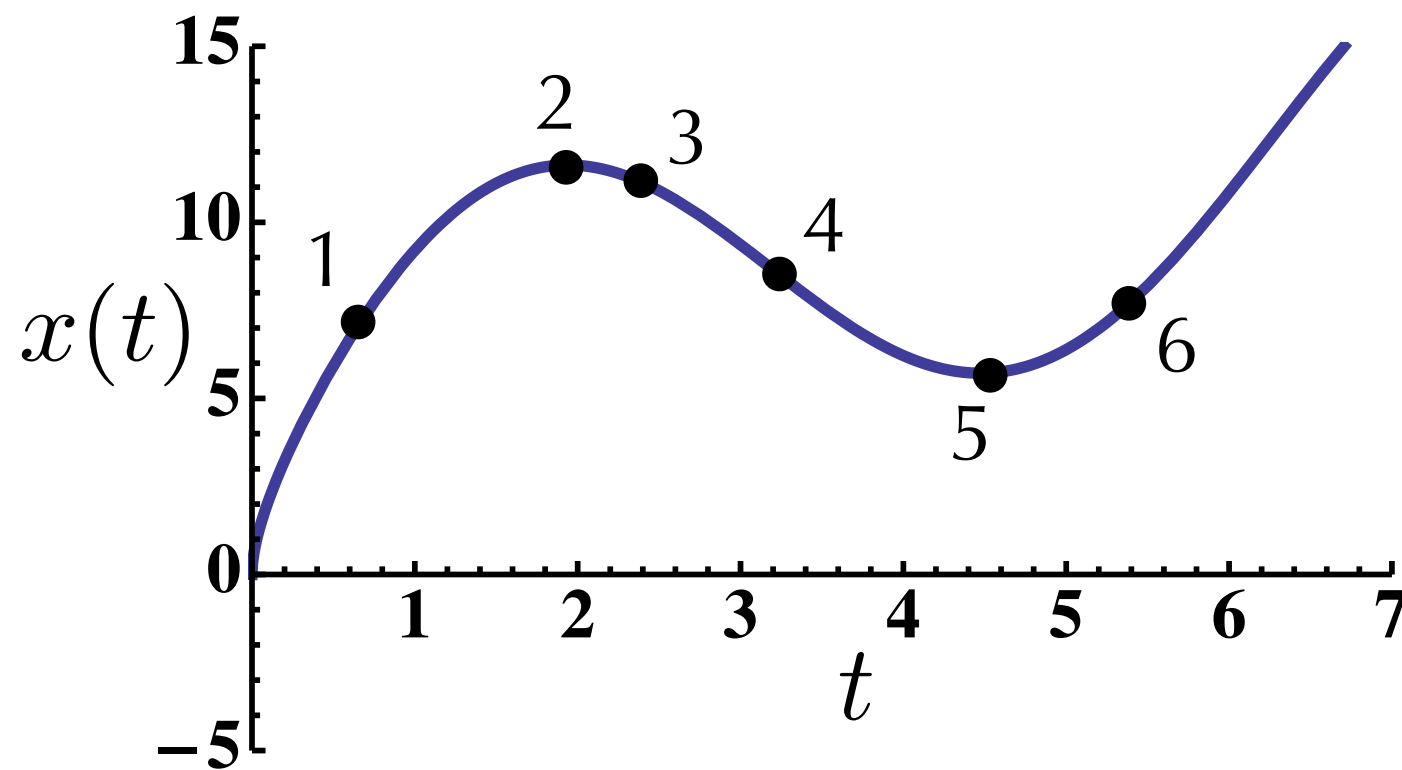
- a) 1 and 6
- b) 3 and 5
- c) 3
- d) 3 and 4
- e) never

At which point(s) is the object moving to the left? d) 3 and 4



- a) 2 and 5
- b) 4 only
- c) 3 and 4
- d) 5 only
- e) never

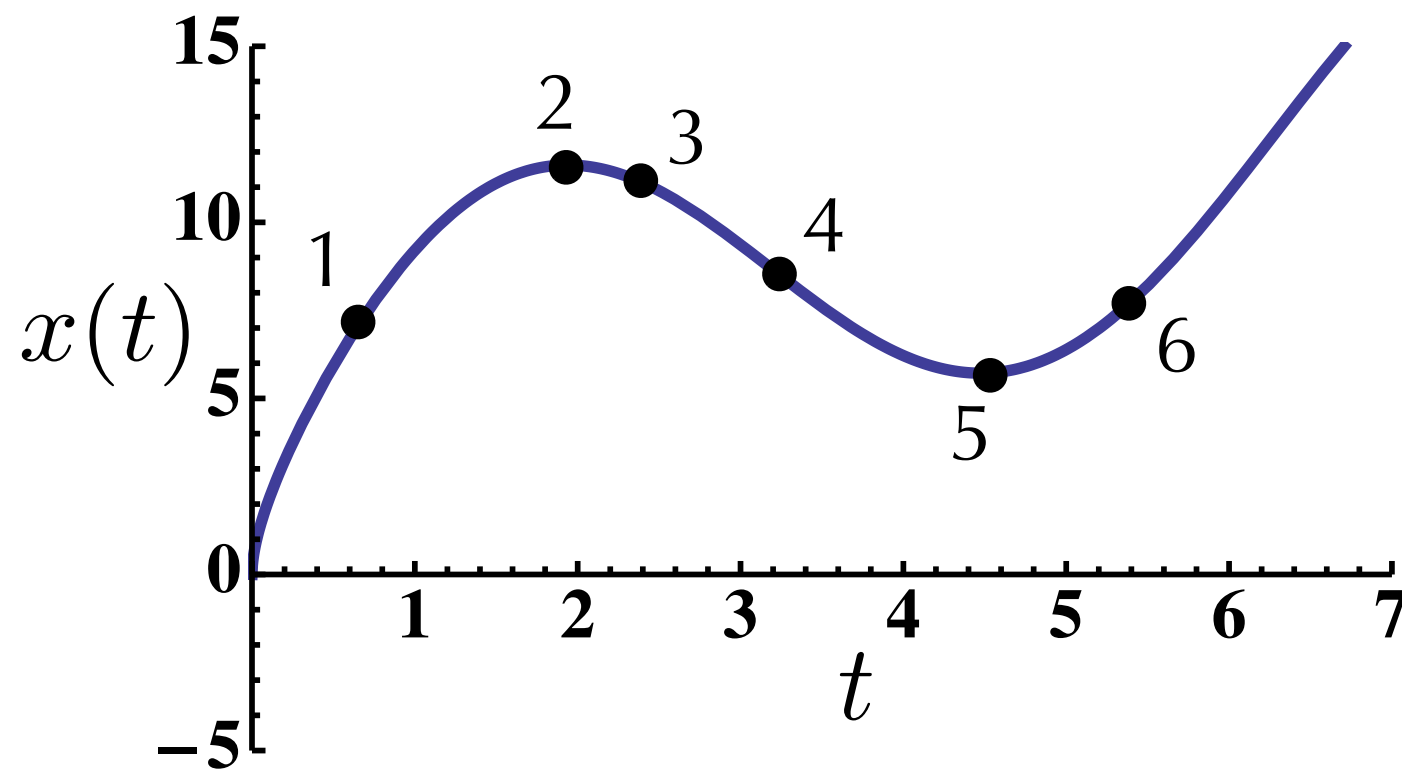
At which point(s) is the object moving to the left? **d) 3 and 4**
At which point(s) is the object at rest?



- a) 2 and 5
- b) 4 only
- c) 3 and 4
- d) 5 only
- e) never

At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

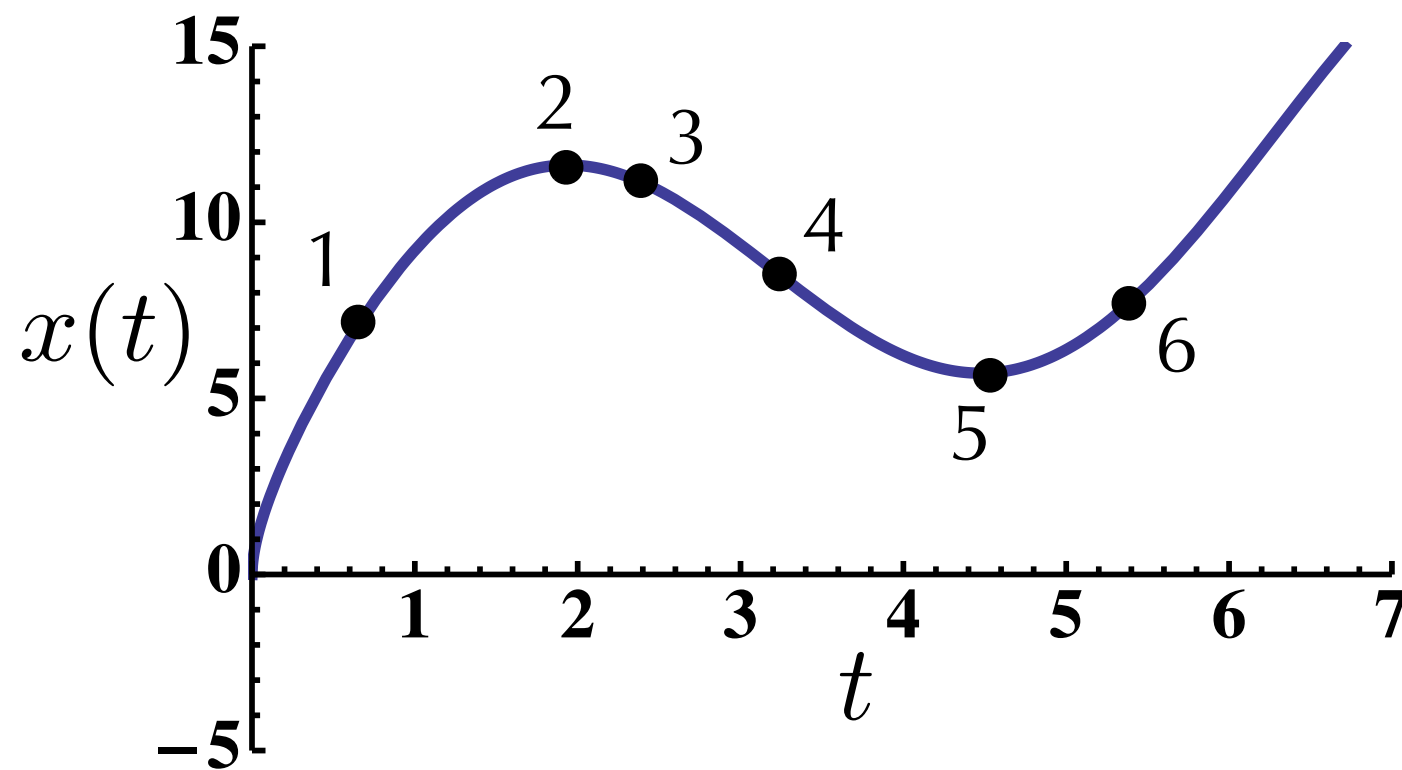


- a) 1 and 3
- b) 3 and 6
- c) 1 only
- d) 1, 3 and 6
- e) none of the above

At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

At which point(s) is the object **slow**ing down?

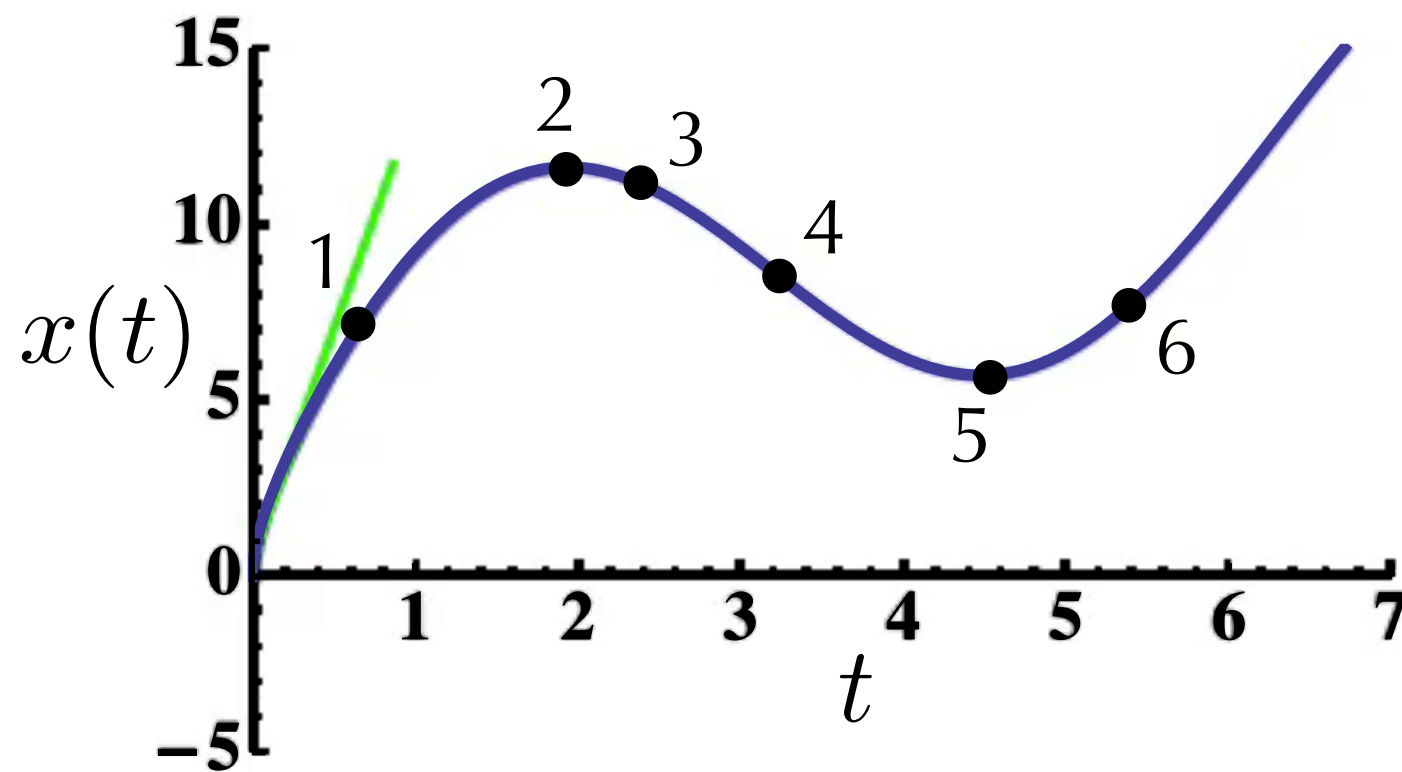


- a) 1 and 3
- b) 3 and 6
- c) 1 only
- d) 1, 3 and 6
- e) none of the above

At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

At which point(s) is the object **slow**ing down? c) 1 only

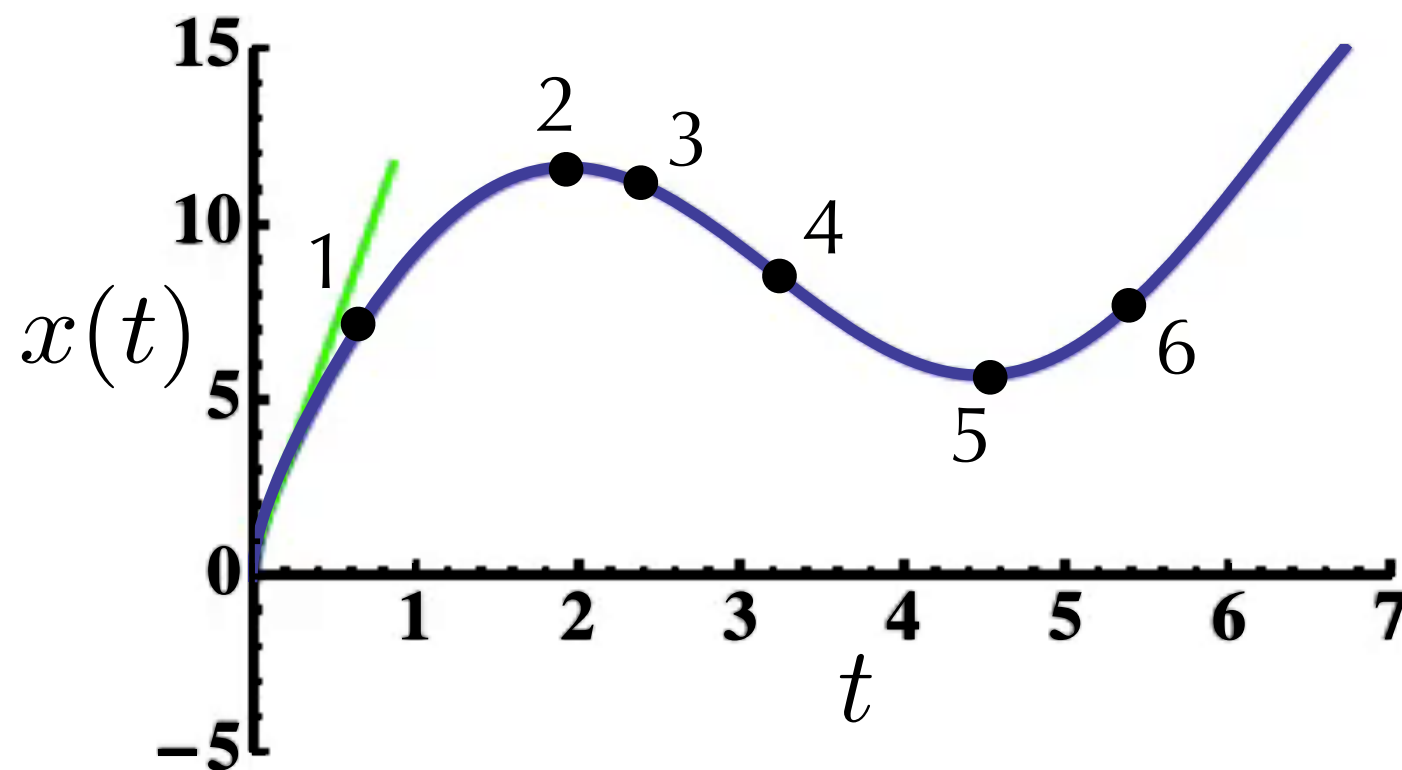


- a) 1 and 3
- b) 3 and 6
- c) 1 only
- d) 1, 3 and 6
- e) none of the above

At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

At which point(s) is the object **slow**ing down? c) 1 only



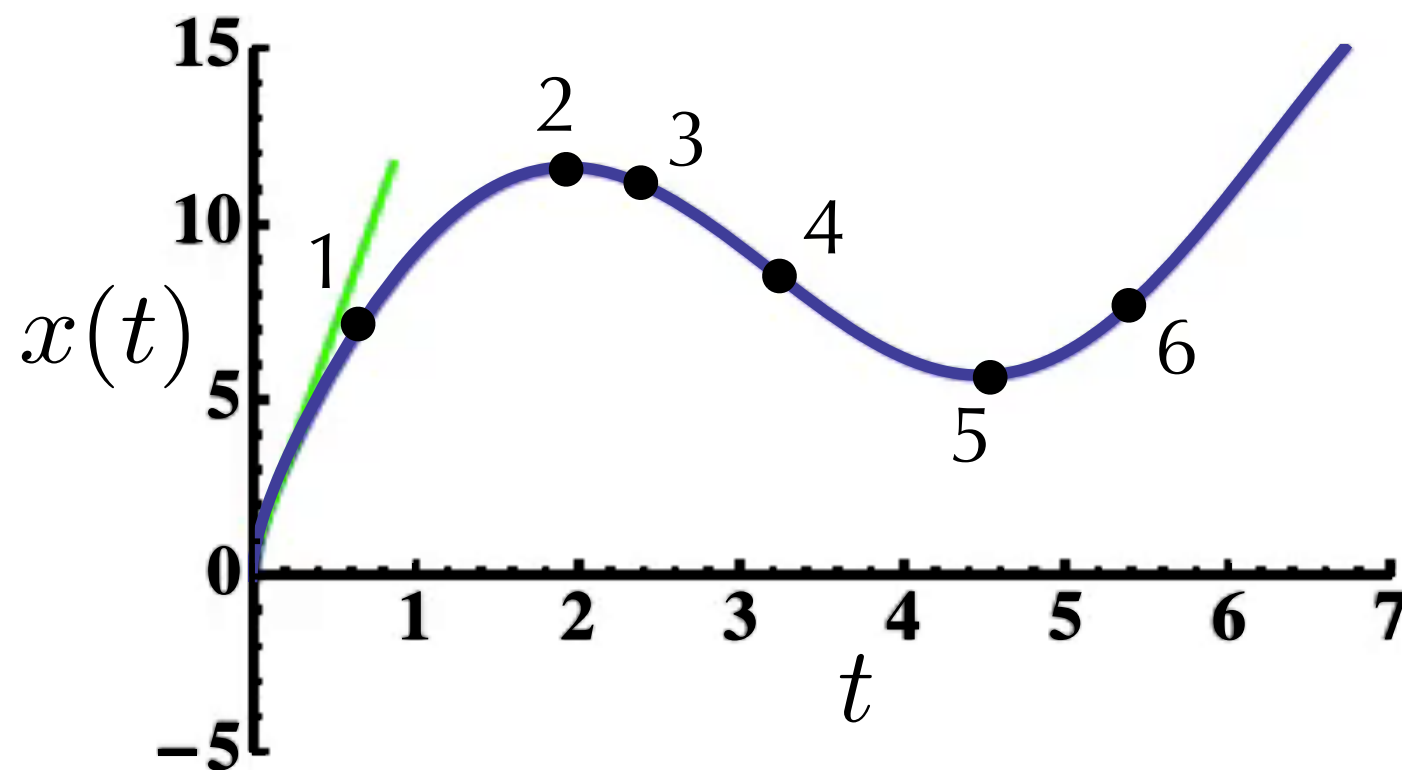
- a) 1 and 3
- b) 3 and 6
- c) 6 only
- d) 4 only
- e) none of the above

At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

At which point(s) is the object **slow**ing down? c) 1 only

At which point(s) is the object **speed**ing up?



- a) 1 and 3
- b) 3 and 6
- c) 6 only
- d) 4 only
- e) none of the above

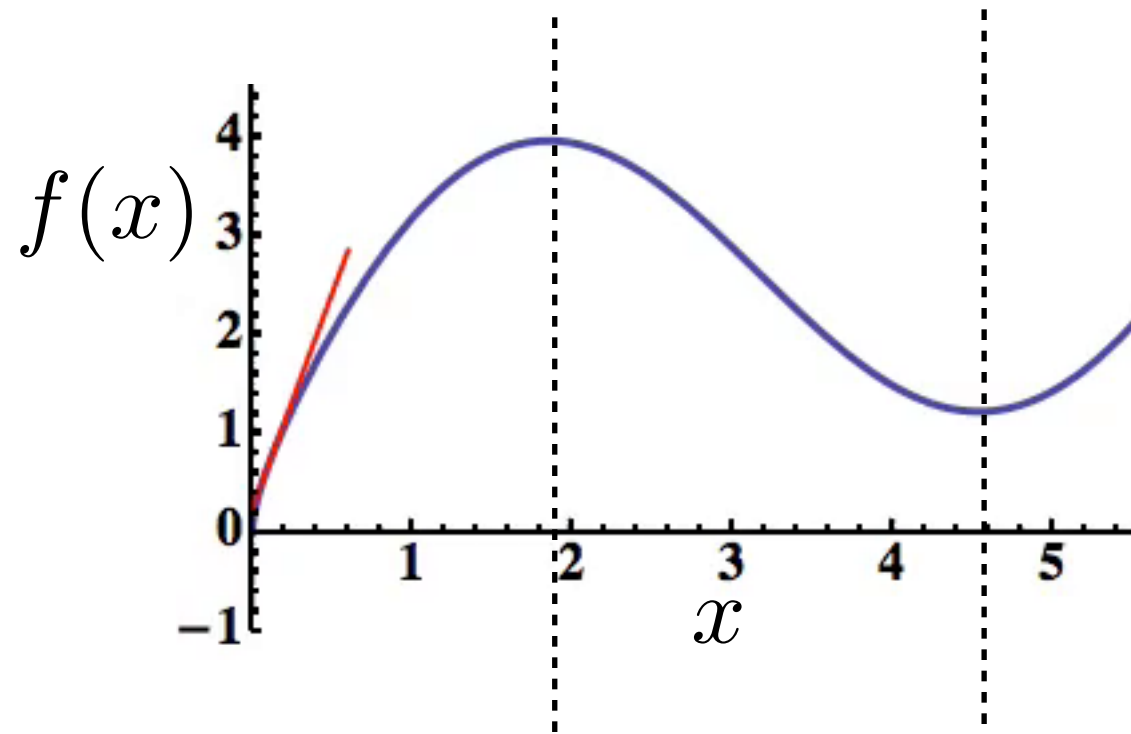
At which point(s) is the object moving to the left? d) 3 and 4

At which point(s) is the object at rest? a) 2 and 5

At which point(s) is the object **slow**ing down? c) 1 only

At which point(s) is the object **speed**ing up? b) 3 and 6

Calculus review: The derivative



$$u(t) = ct^n$$

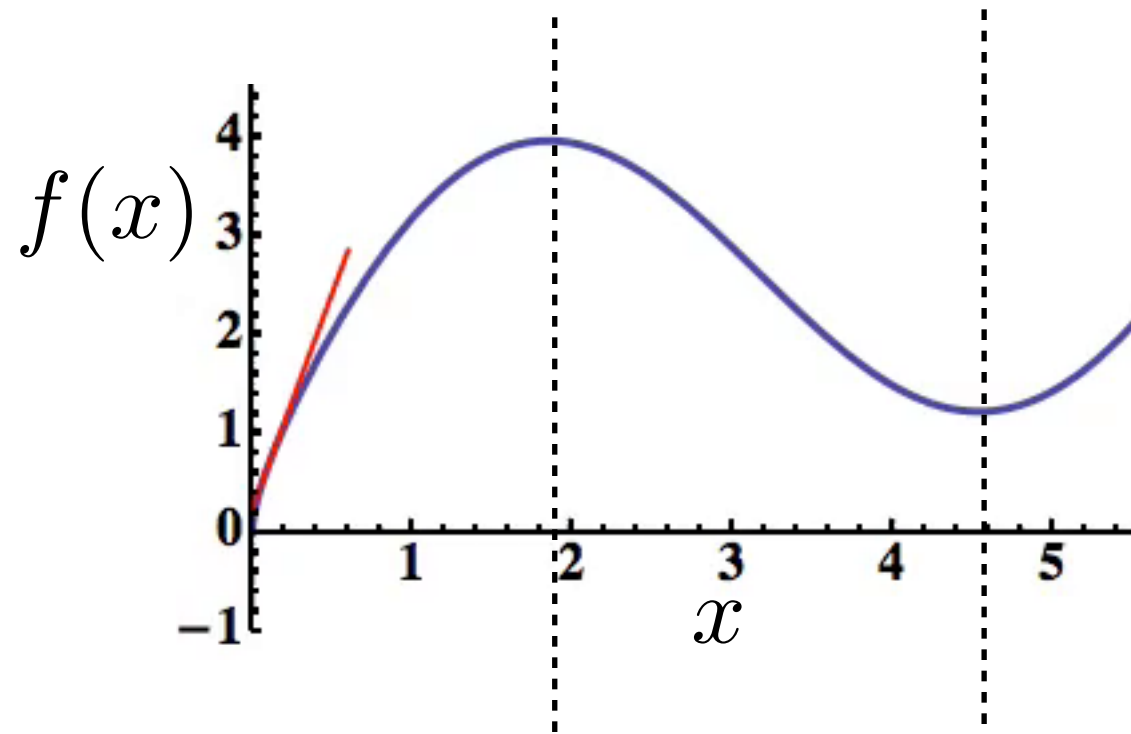
$$\frac{du(t)}{dt} = nct^{n-1}$$

$$\frac{d}{dt} (u(t) + v(t)) = \frac{du(t)}{dt} + \frac{dv(t)}{dt}$$

$$\frac{d}{dt} \sin(t) = \cos(t)$$

$$\frac{d}{dt} \cos(t) = -\sin(t)$$

Calculus review: The derivative



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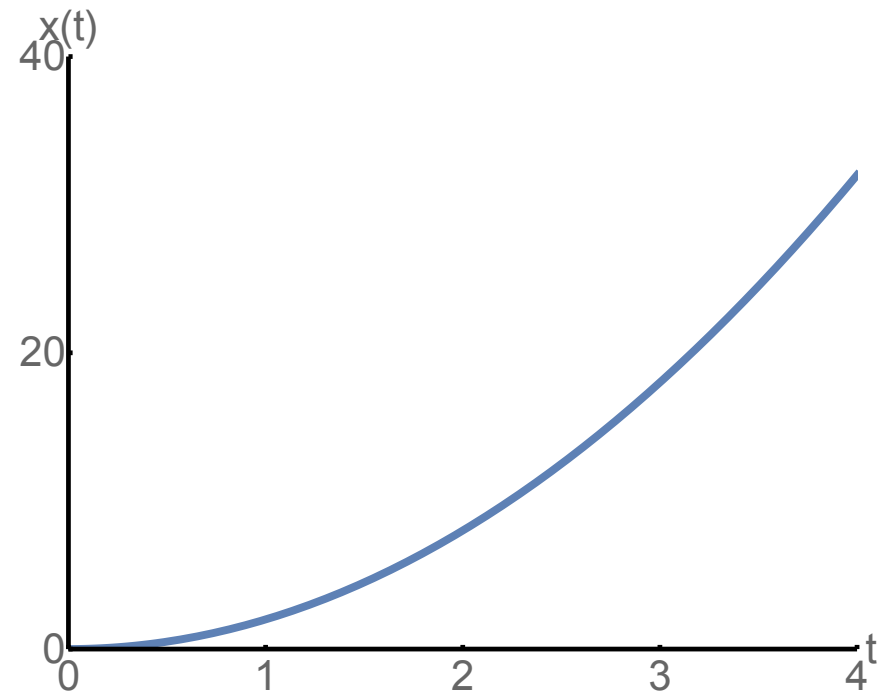
$$\frac{d}{dt} \sin(t) = \cos(t)$$

$$\frac{d}{dt} \cos(t) = -\sin(t)$$

Derivative Example

Find the velocity for this position function.

$$x(t) = 2t^2$$

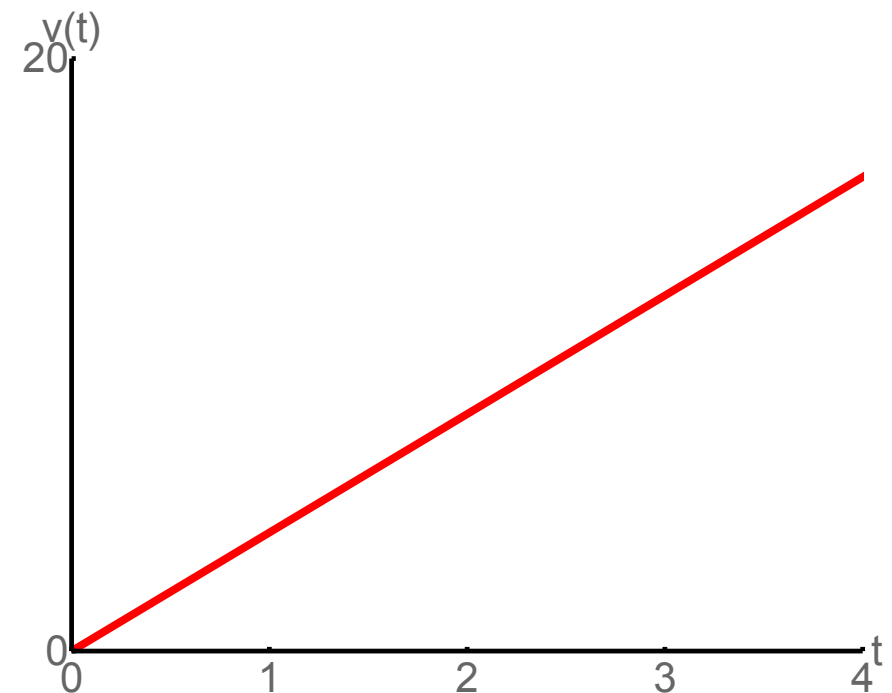
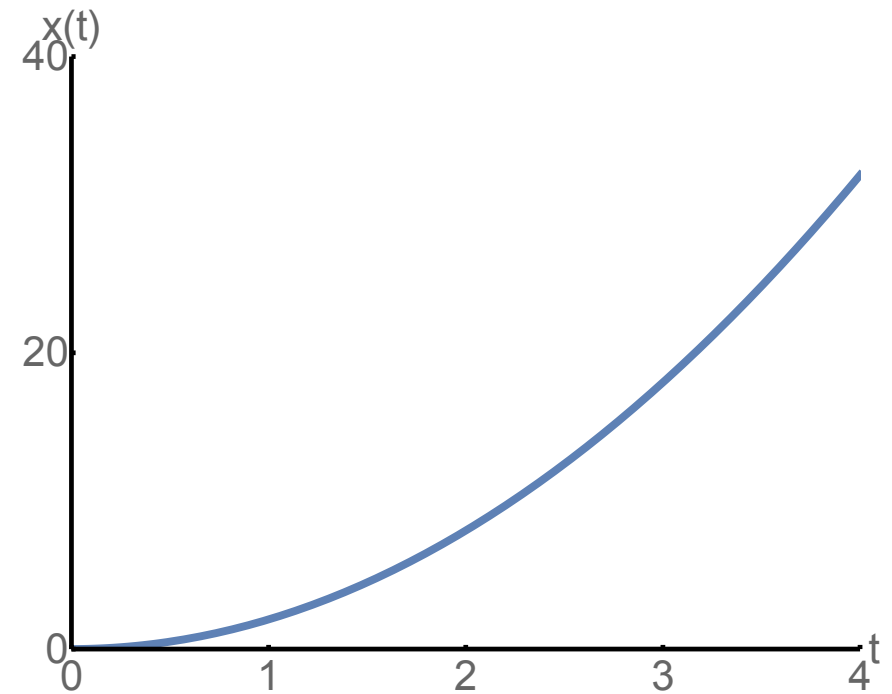


Derivative Example

Find the velocity for this position function.

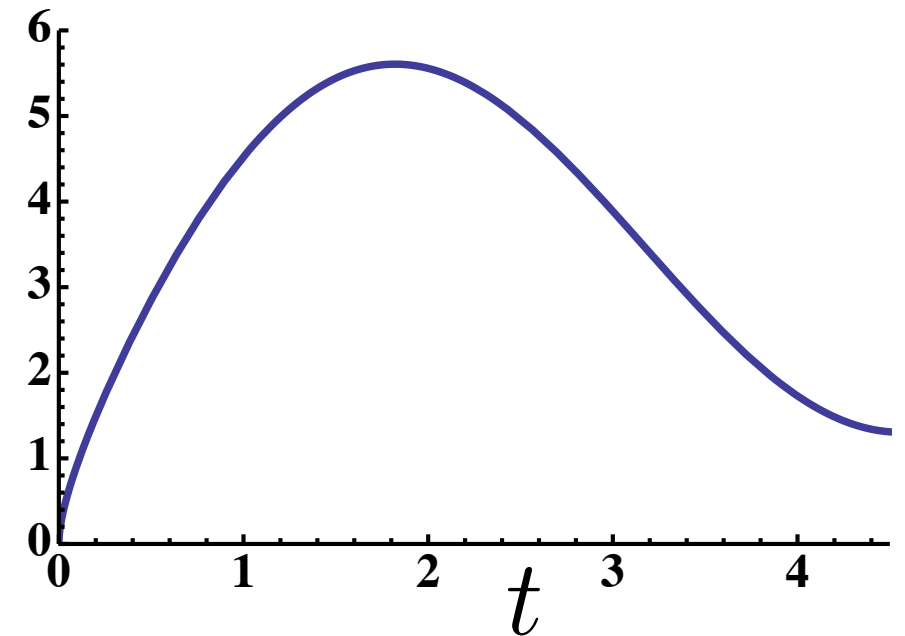
$$x(t) = 2t^2$$

$$v = \frac{dx}{dt} = 4t$$

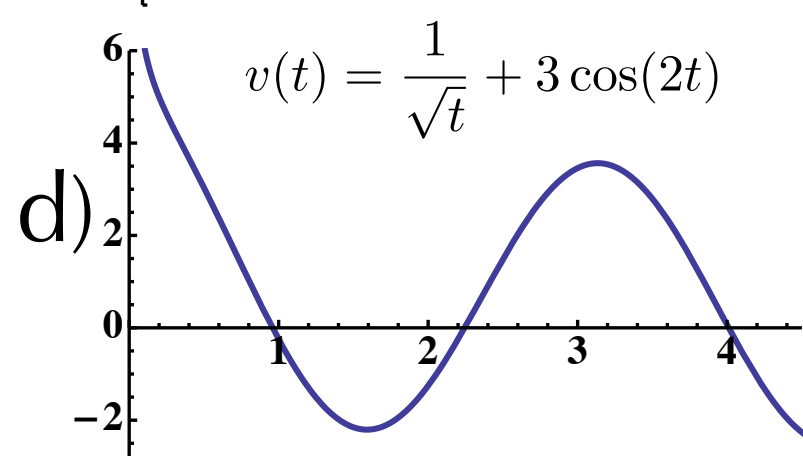
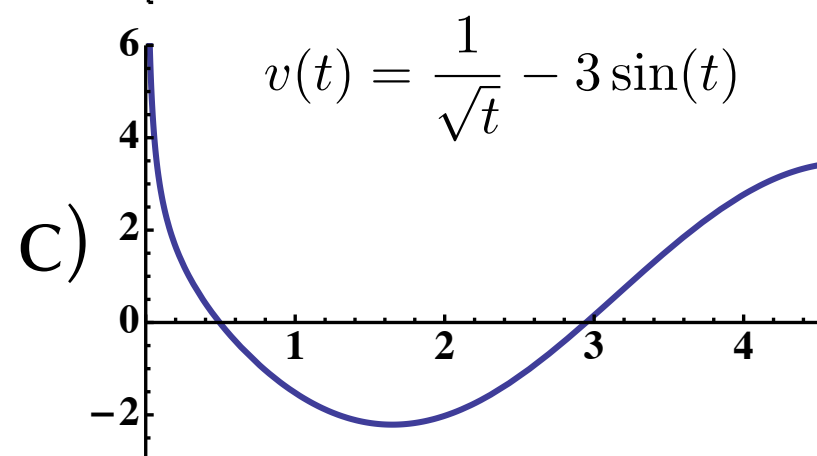
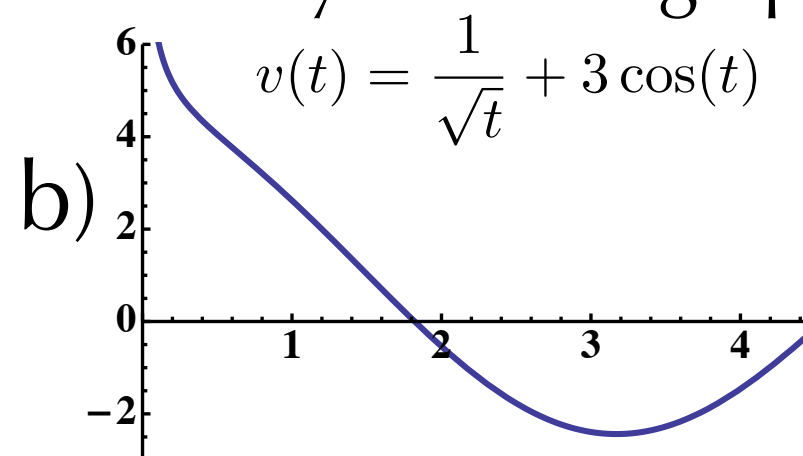
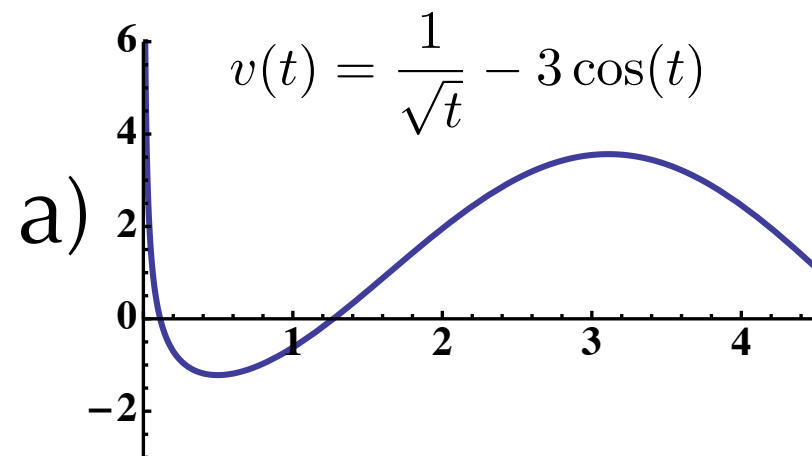


The motion of a bus is given by the following position vs. time graph

$$x(t) = 2\sqrt{t} + 3\sin(t)$$



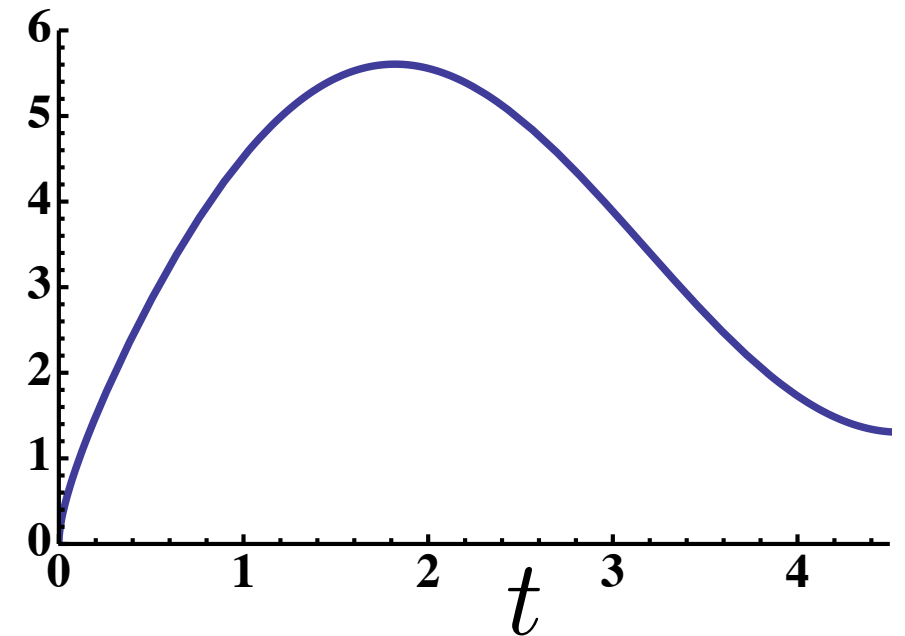
Which of the following is the velocity vs. time graph for the bus?



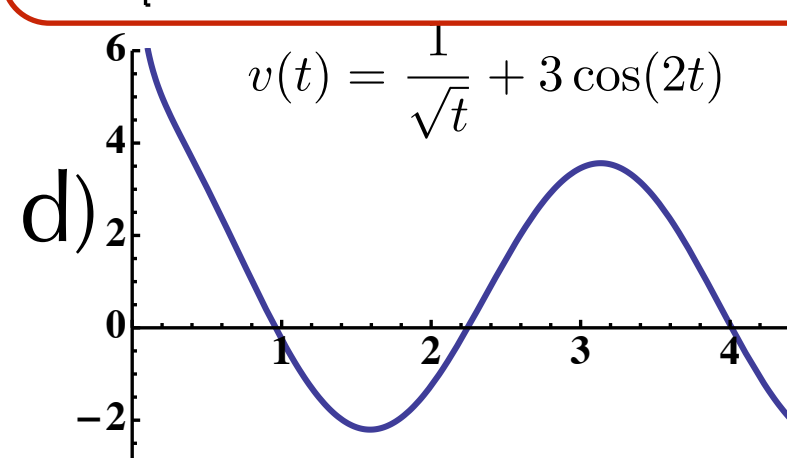
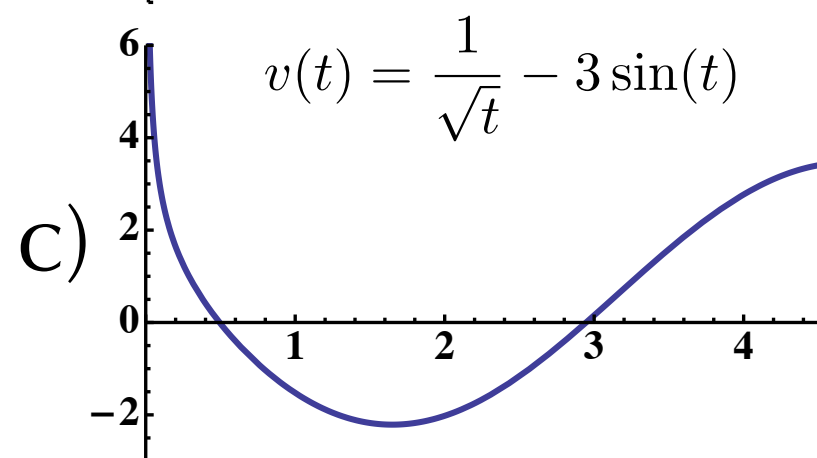
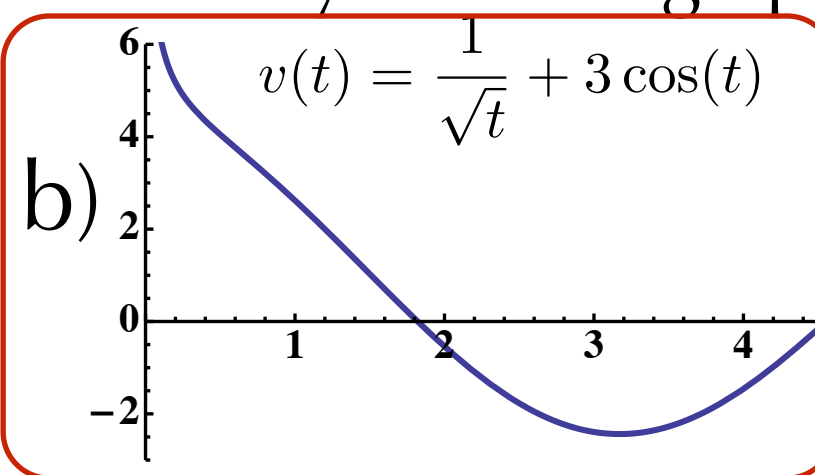
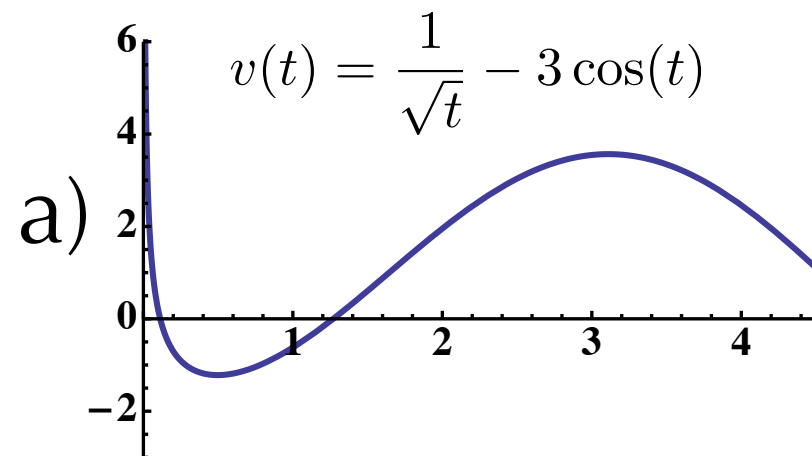
e) b & c are both correct

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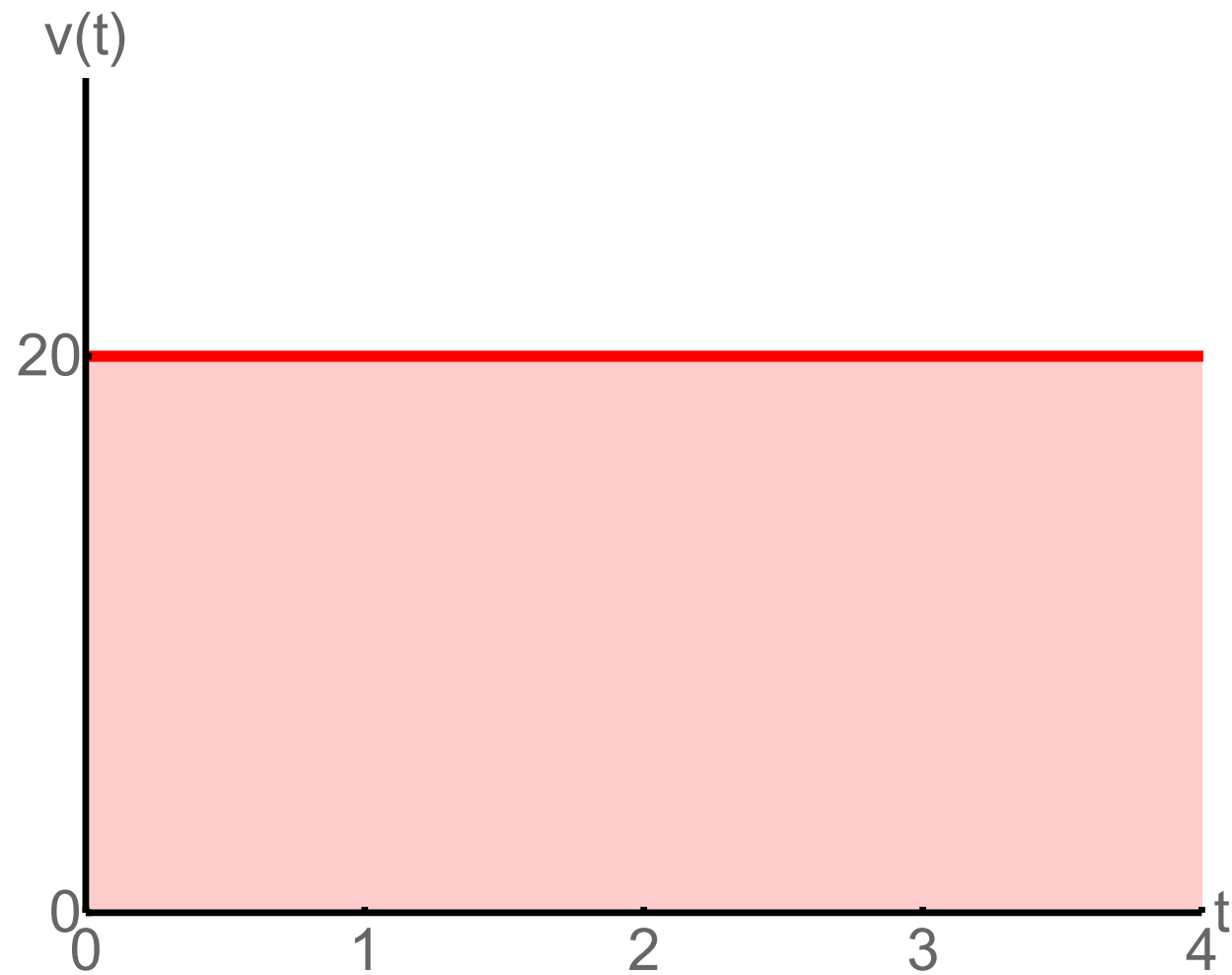


Which of the following is the velocity vs. time graph for the bus?



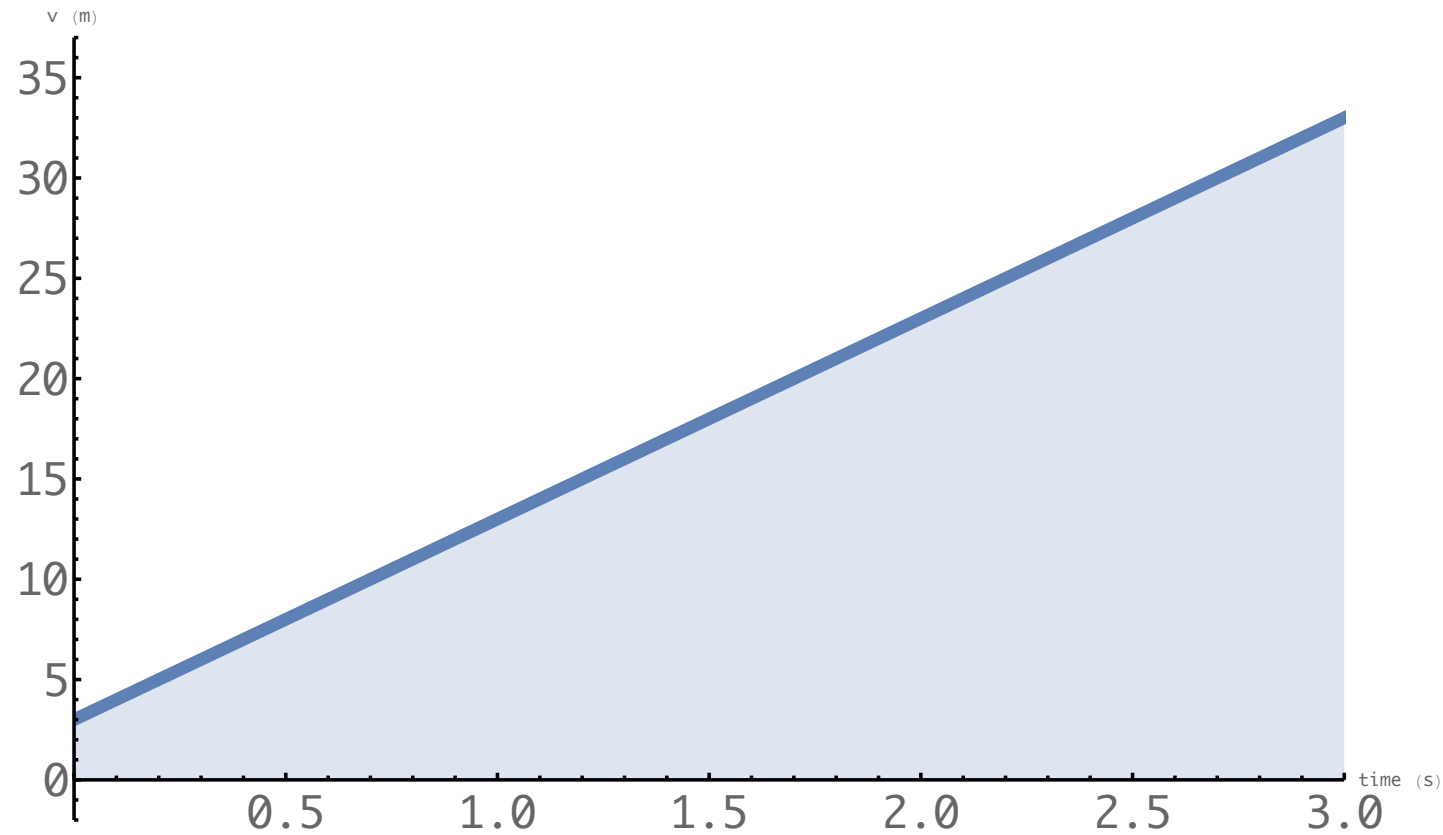
e) b & c are both correct

Finding Position from Velocity



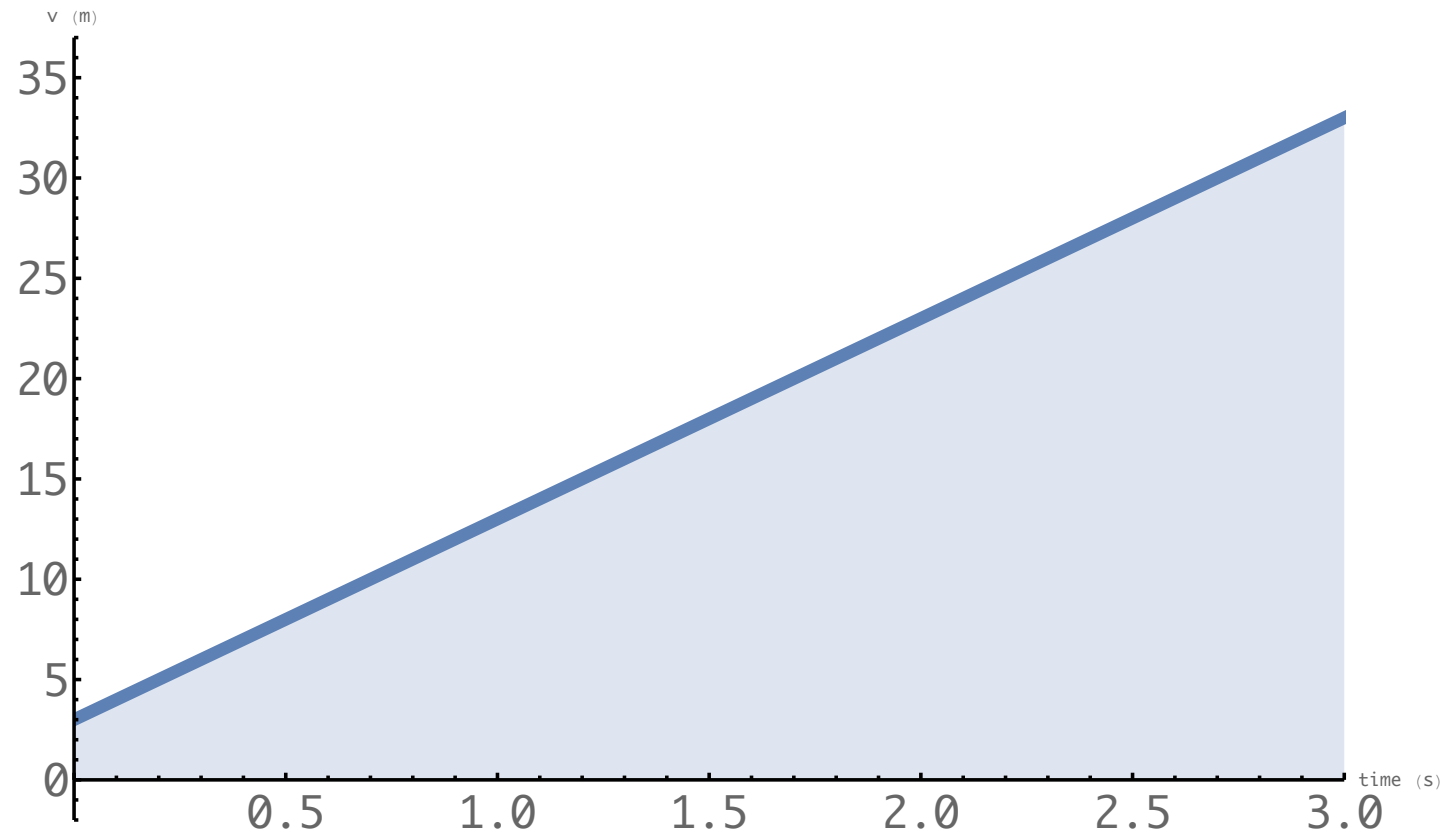
$$s_f = s_i + v_s \Delta t$$

Finding Position from Velocity



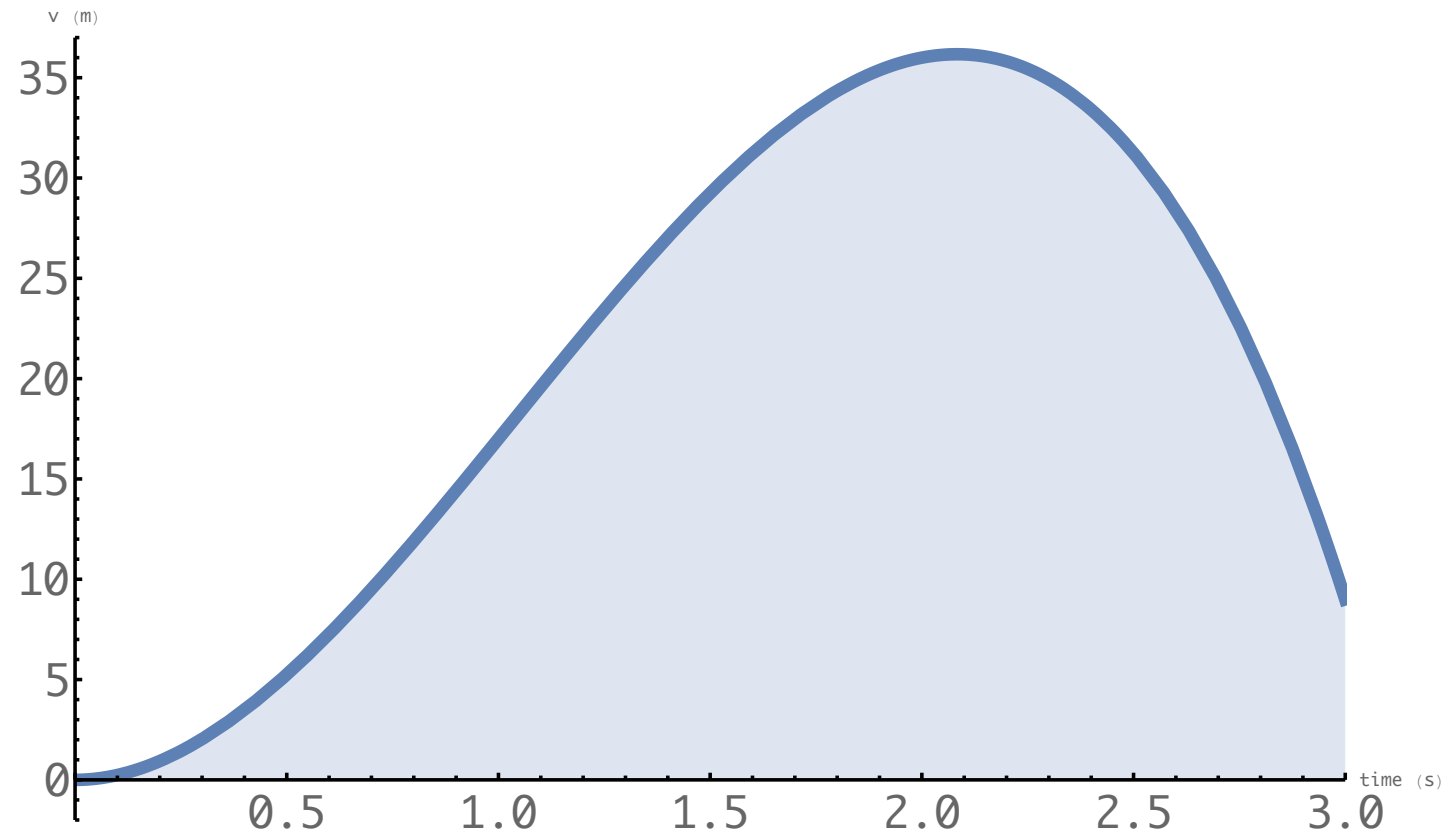
$$s_f = s_i + v_s \Delta t$$

Finding Position from Velocity



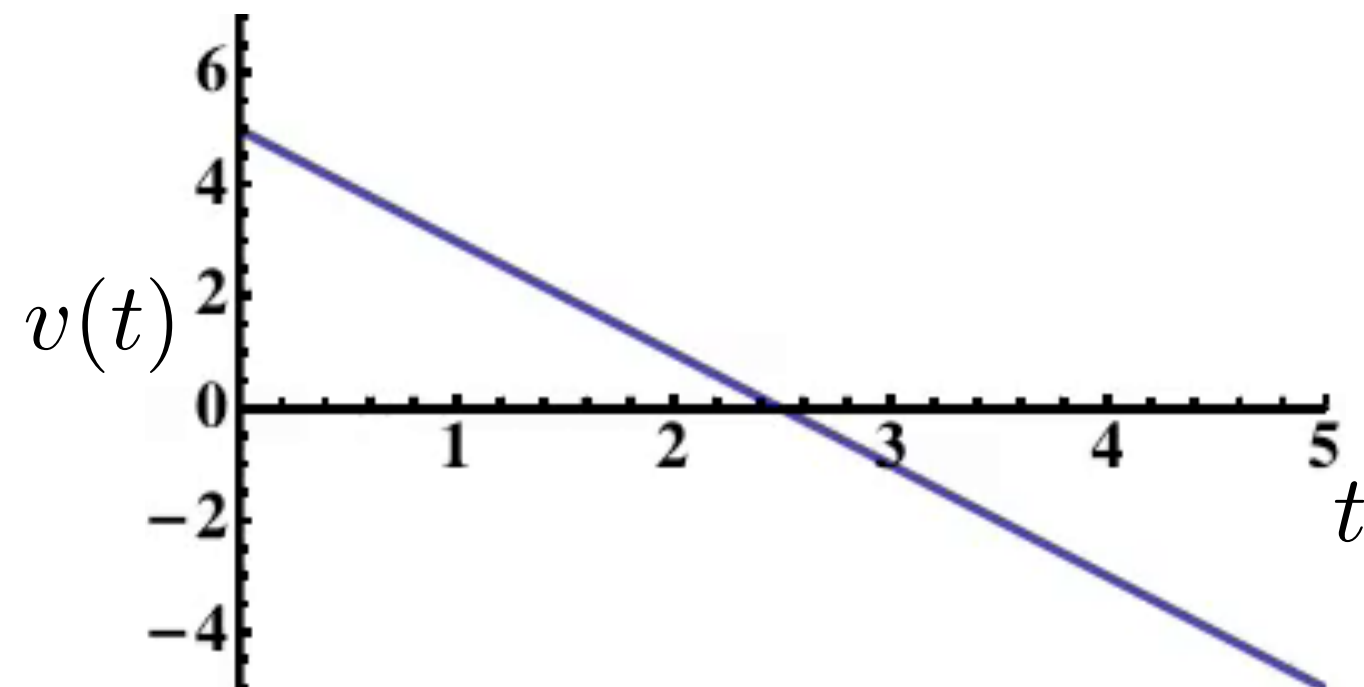
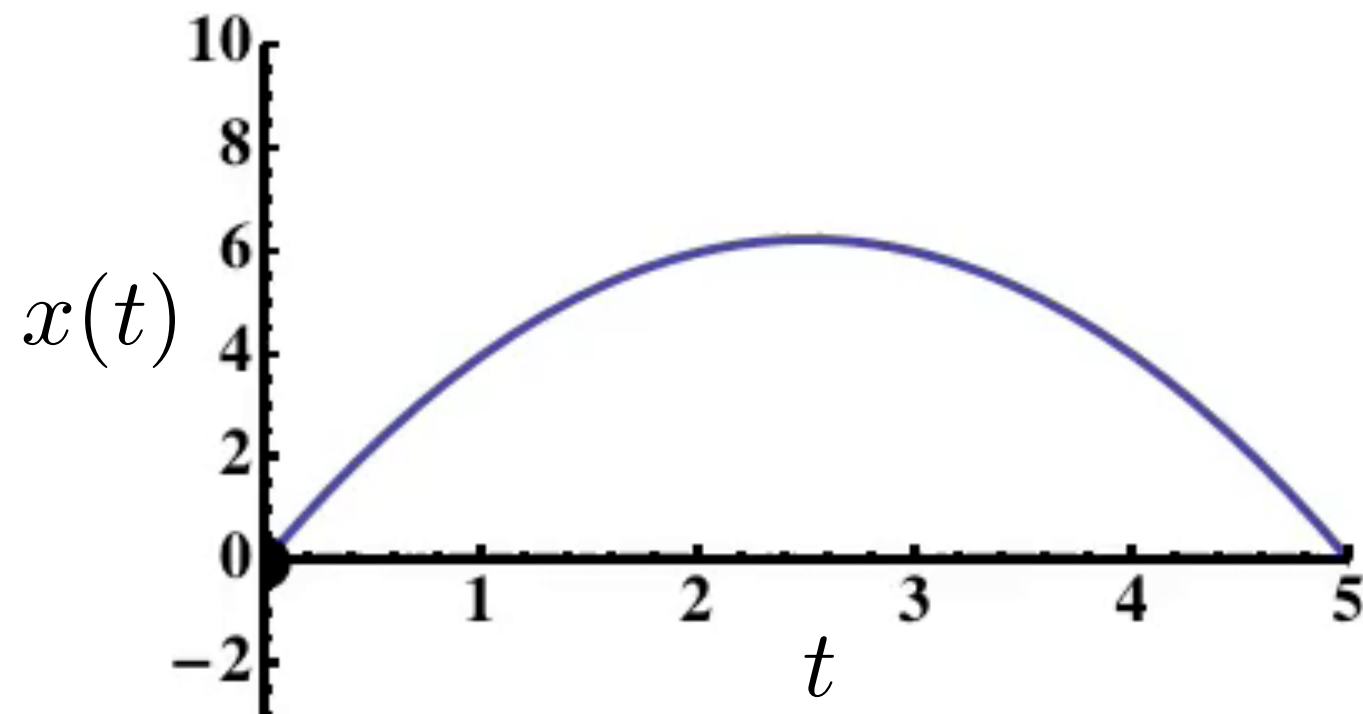
$$s_f = s_i + \text{area under curve}$$

Finding Position from Velocity

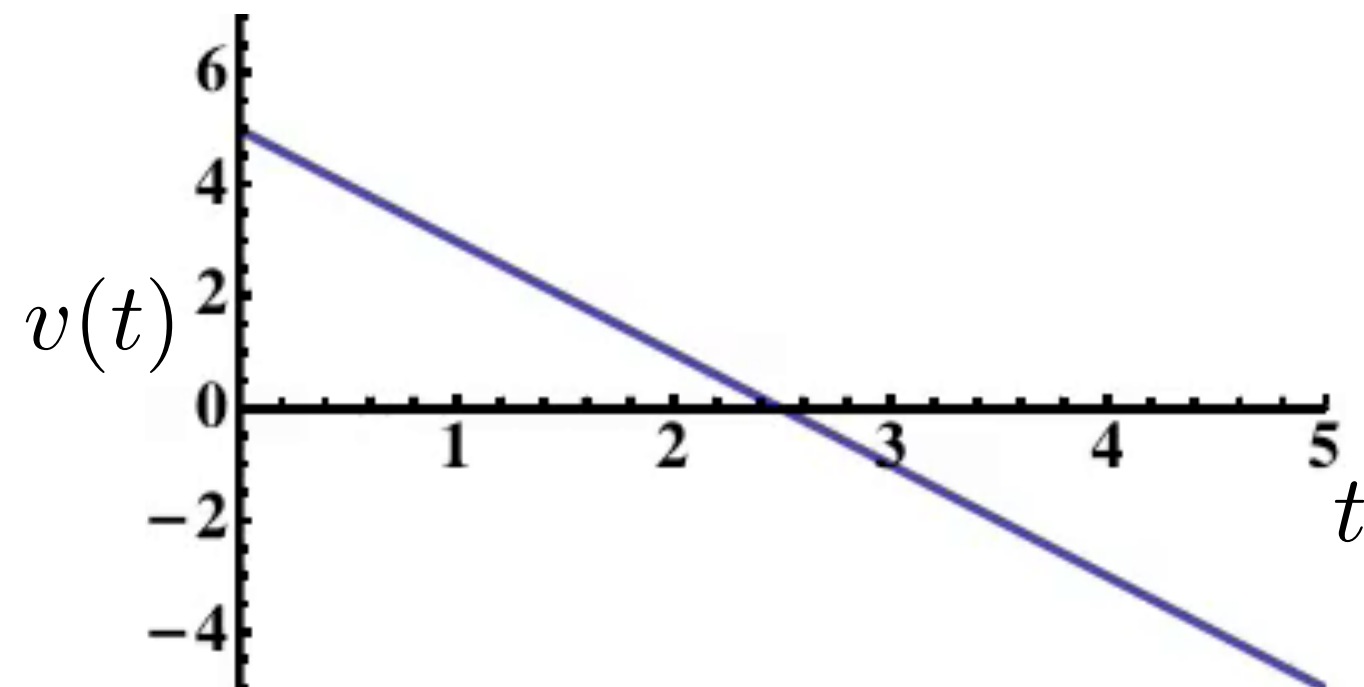
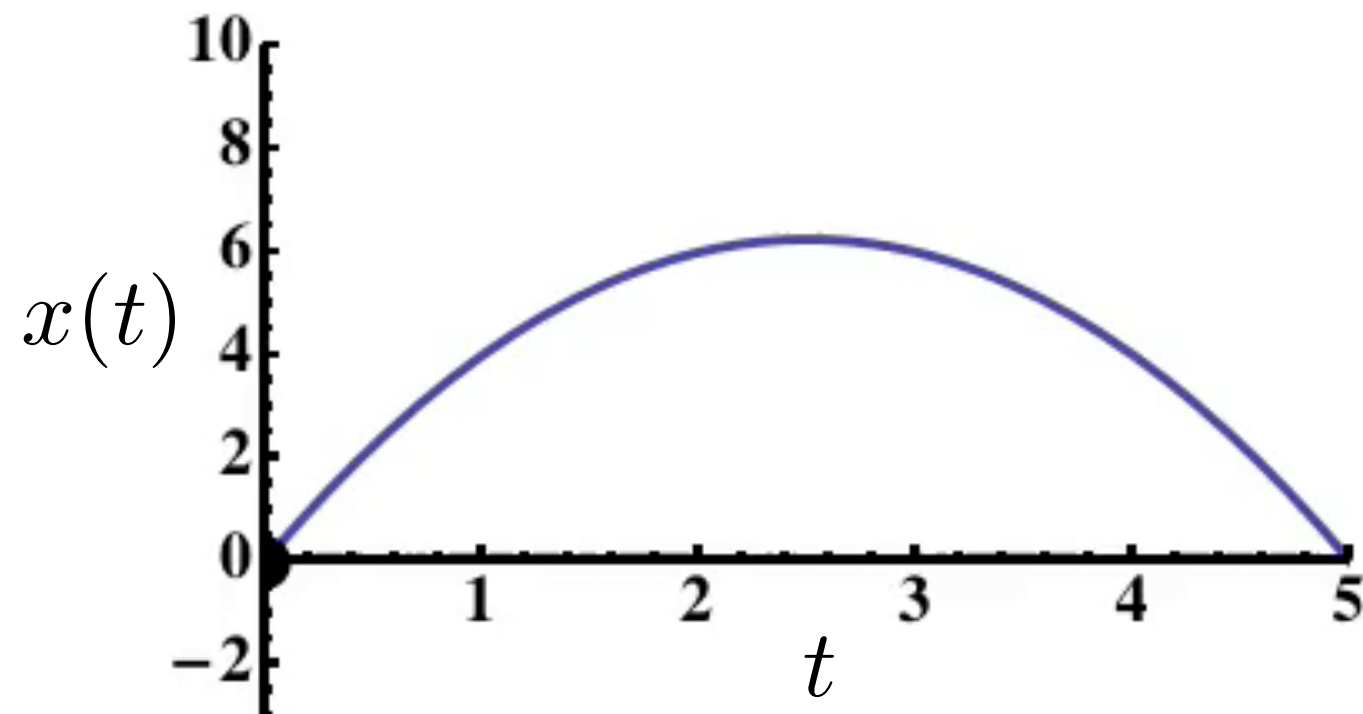


$$s_f = s_i + \text{area under curve}$$

Integral illustration



Integral illustration



Quiz

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

At $t = 4.0$ s, the object's position is

- A. 20 m.
- B. 16 m.
- C. 12 m.
- D. 8 m.
- E. 4 m.

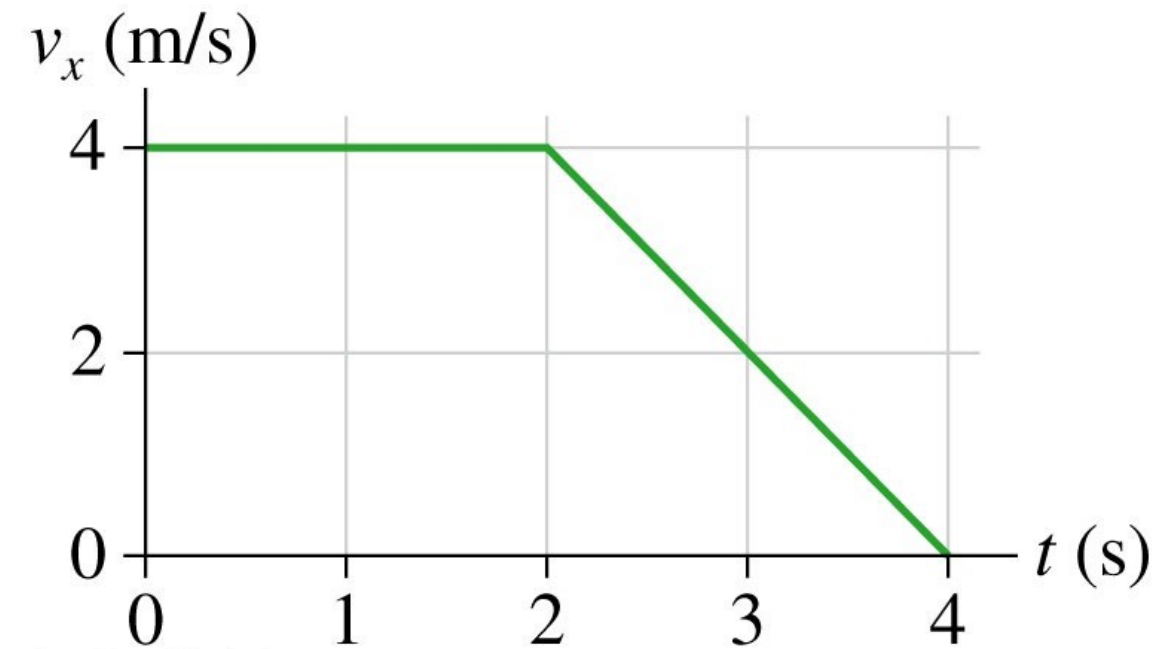


Quiz

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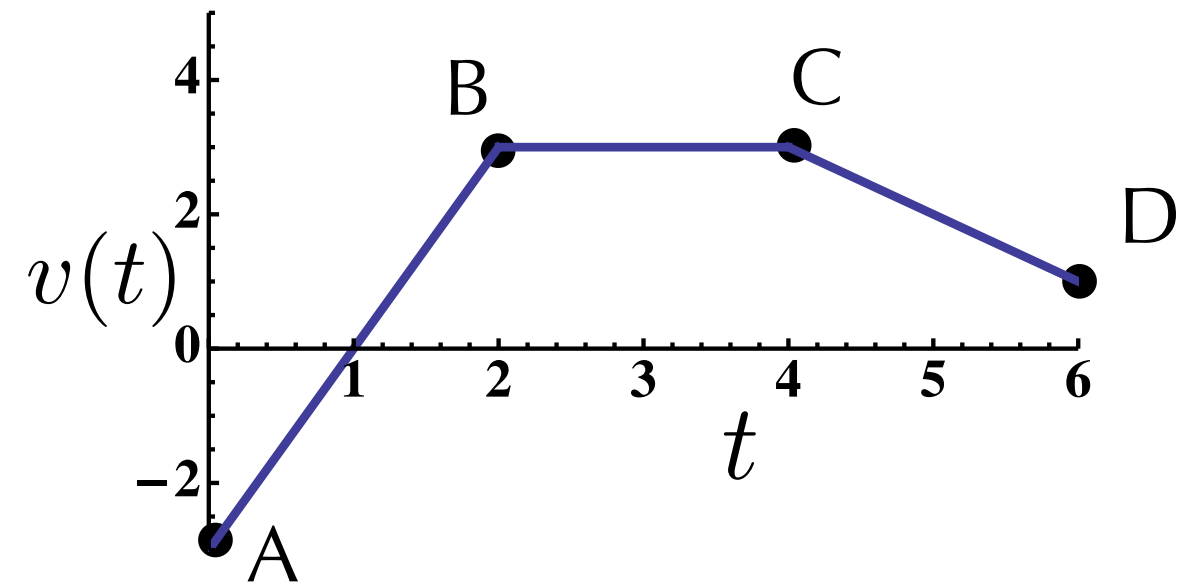
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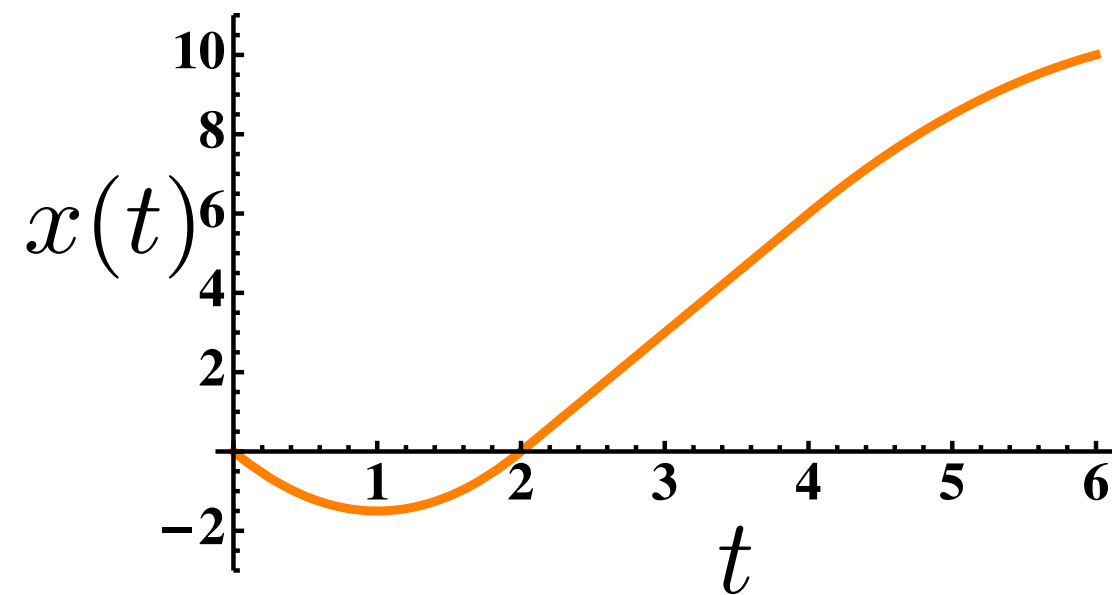
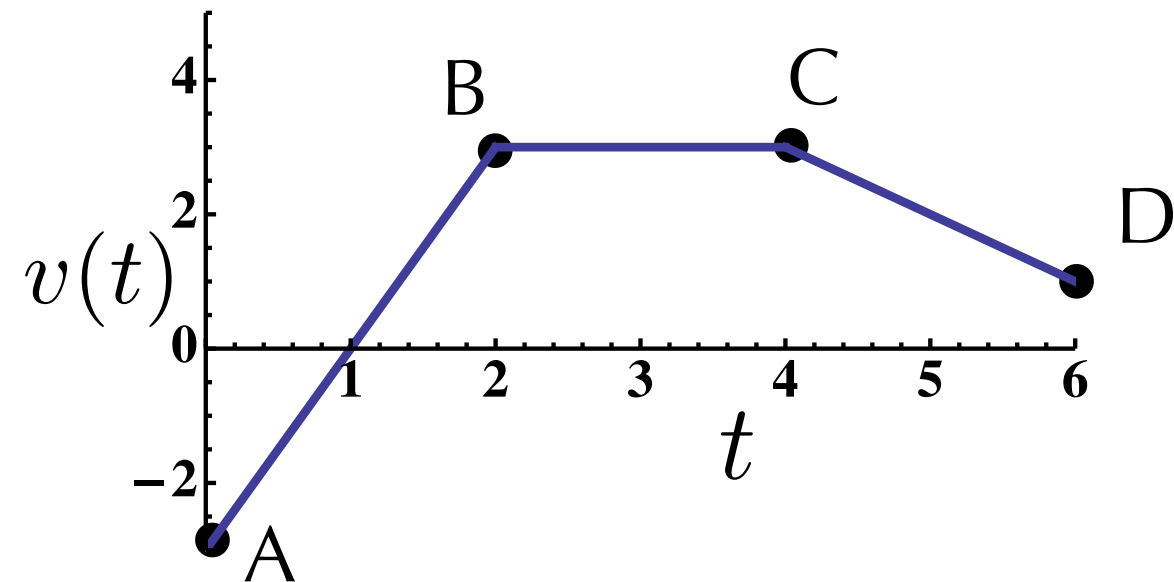


Displacement = area under the curve

$$v(t) \rightarrow x(t)$$

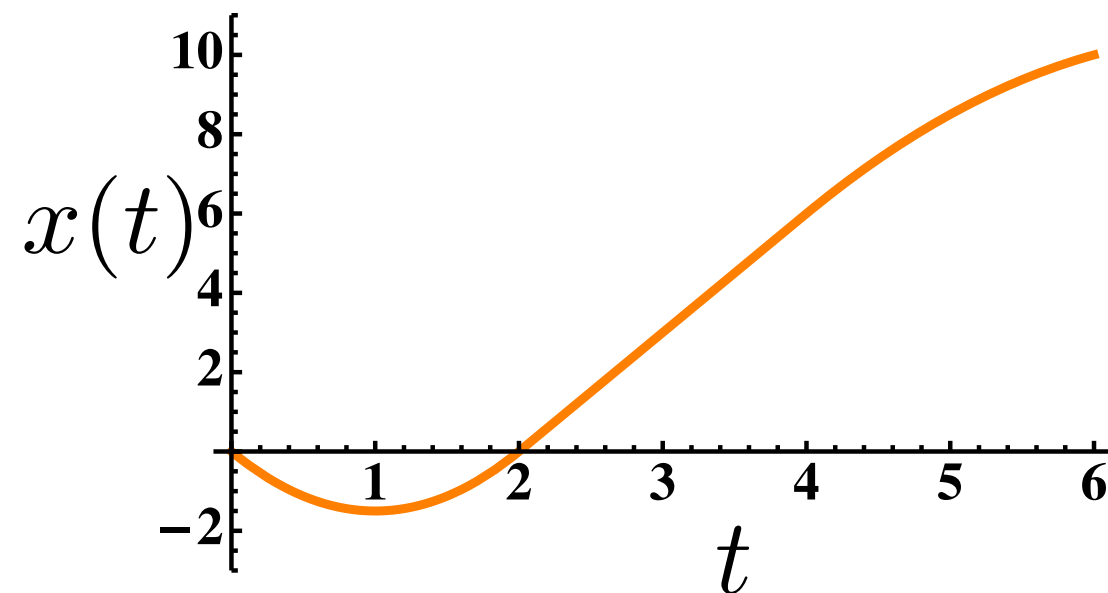
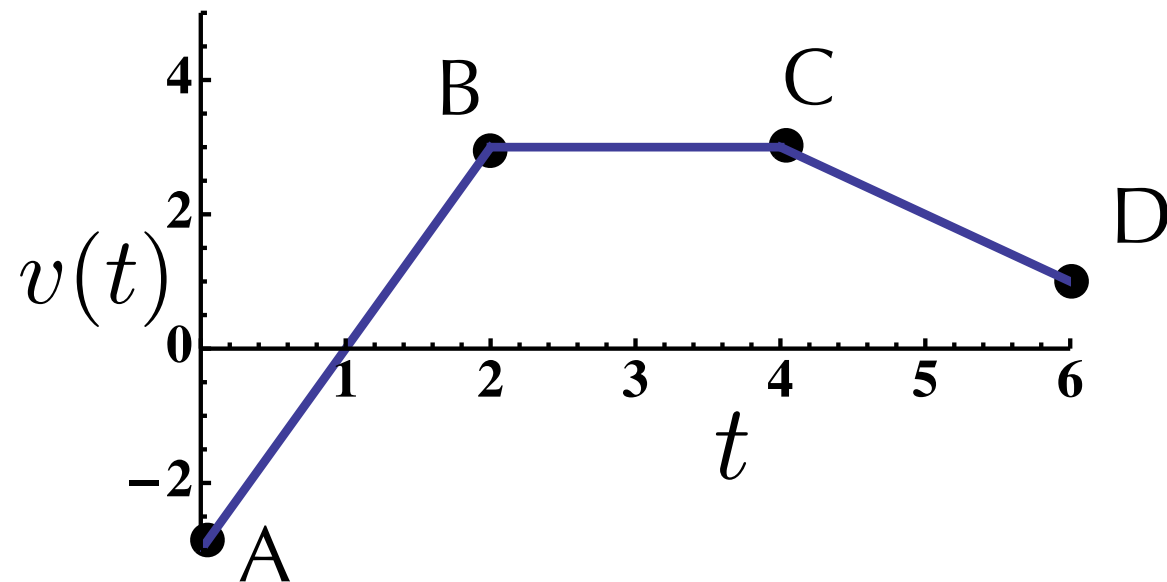


$$v(t) \rightarrow x(t)$$



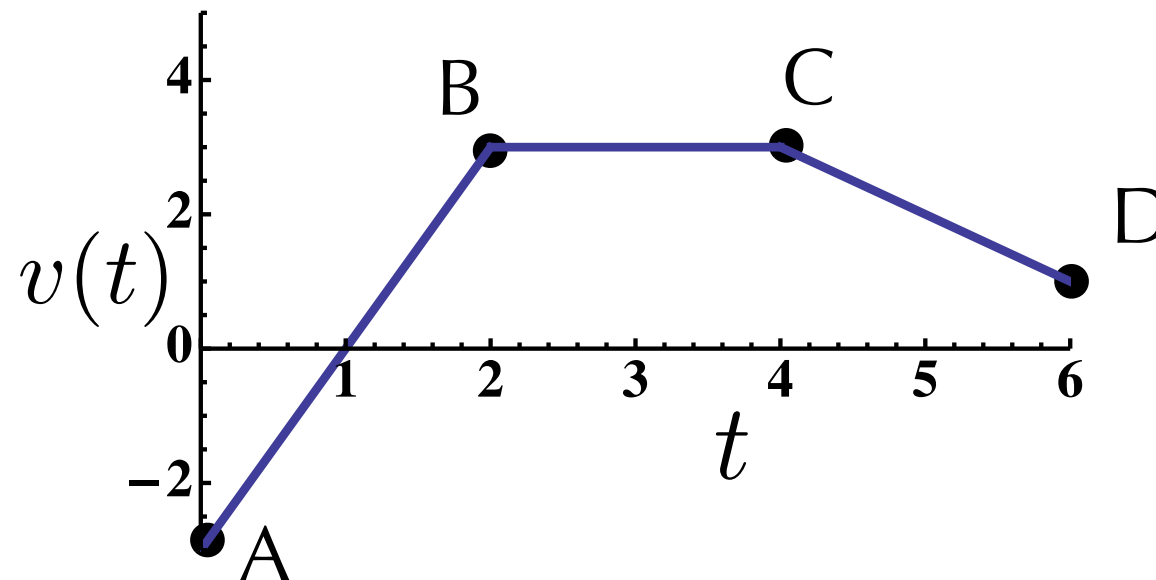
$$v(t) \rightarrow x(t)$$

The object is located at $x = 0$ at $t = 0$ s. What is the object's location at $t = 1$ s

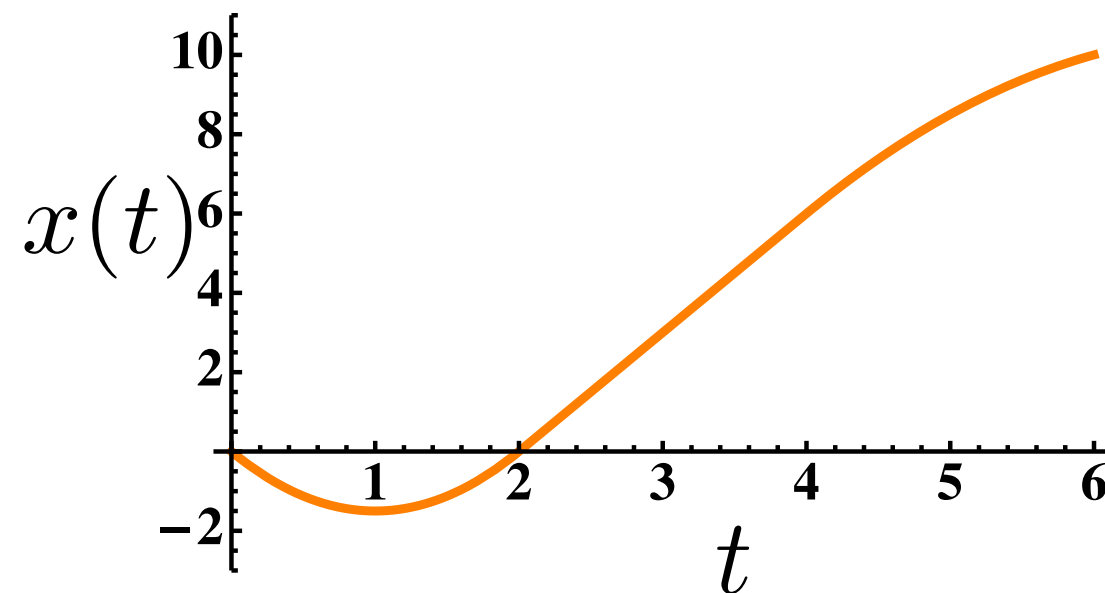


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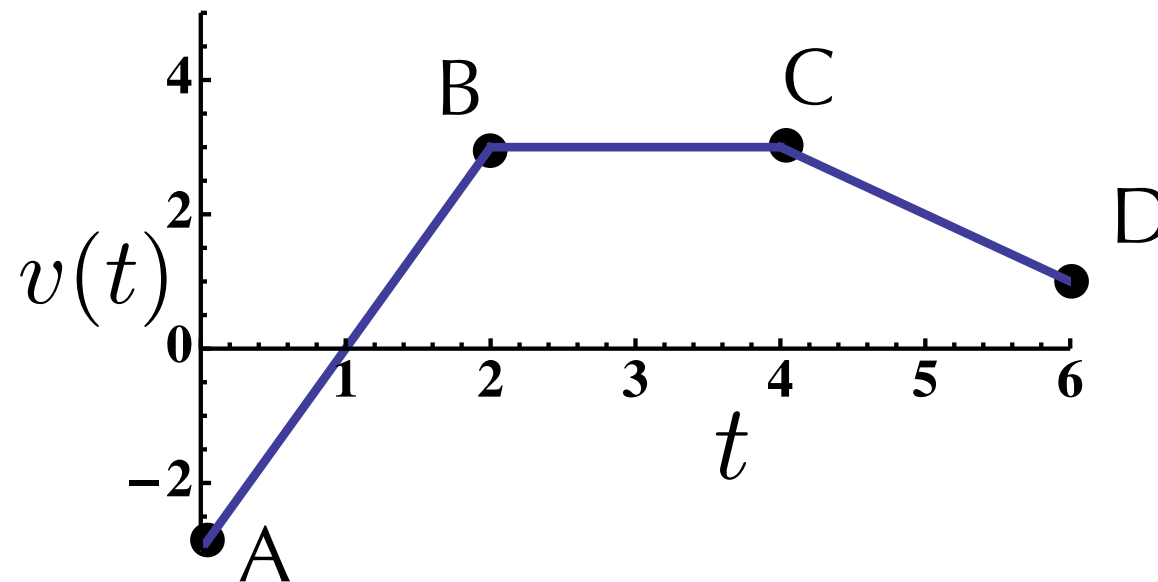


at $t = 2$ s?

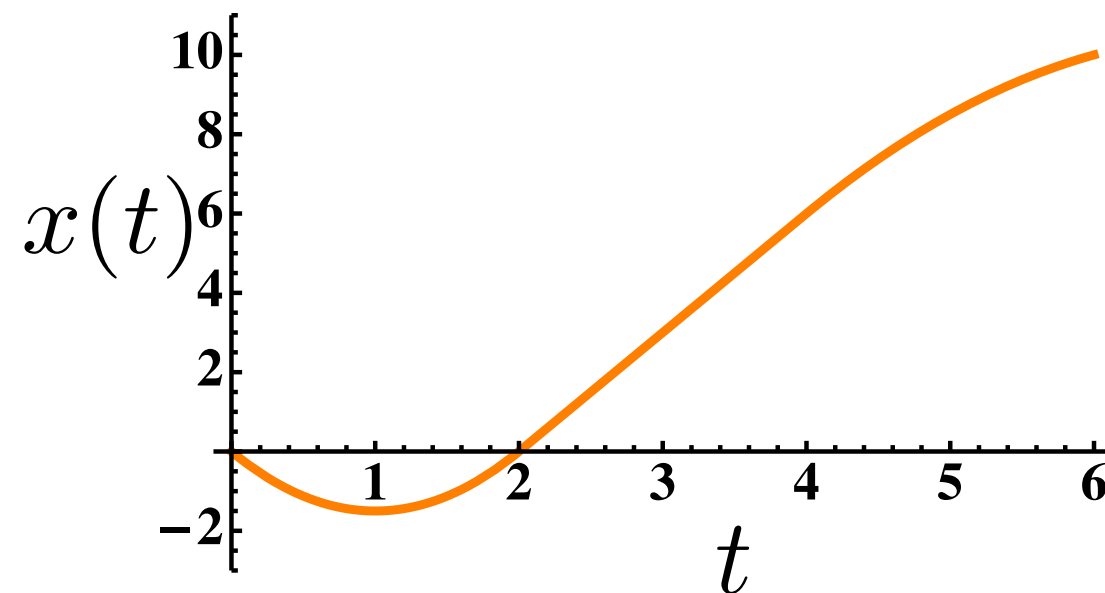


$$v(t) \rightarrow x(t)$$

The object is located at $x = 0$ at $t = 0$ s. What is the object's location at $t = 1$ s



at $t = 2$ s?



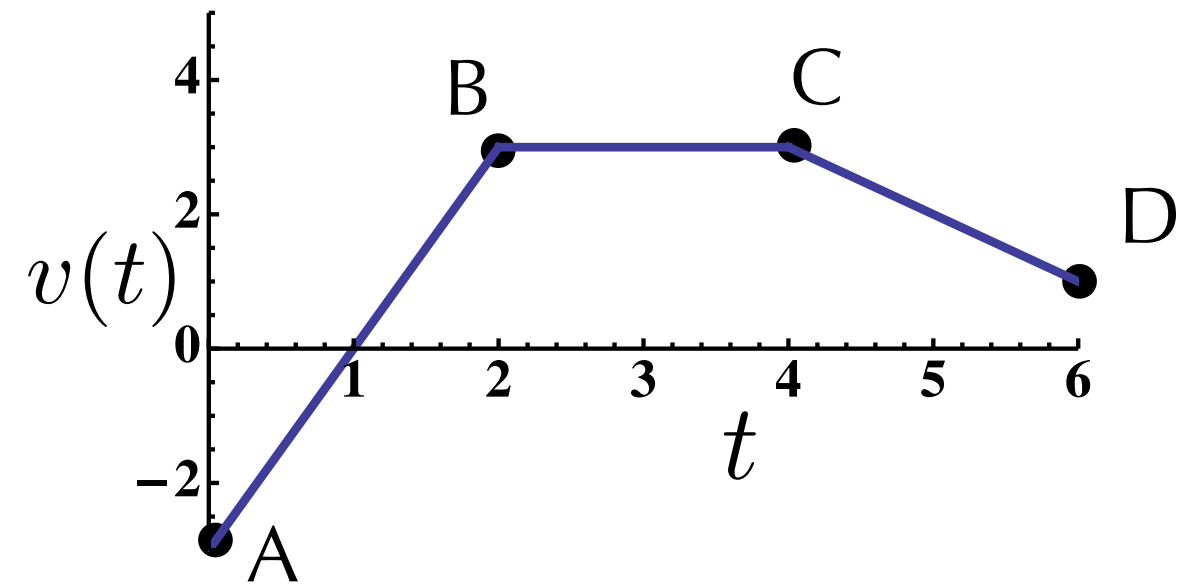
at $t = 4$ s?

Techniques for integration

$$\int_{t_i}^{t_f} ct^n dt = \left. \frac{ct^{n+1}}{n+1} \right|_{t_i}^{t_f} = \frac{c}{n+1} (t_f^{n+1} - t_i^{n+1})$$

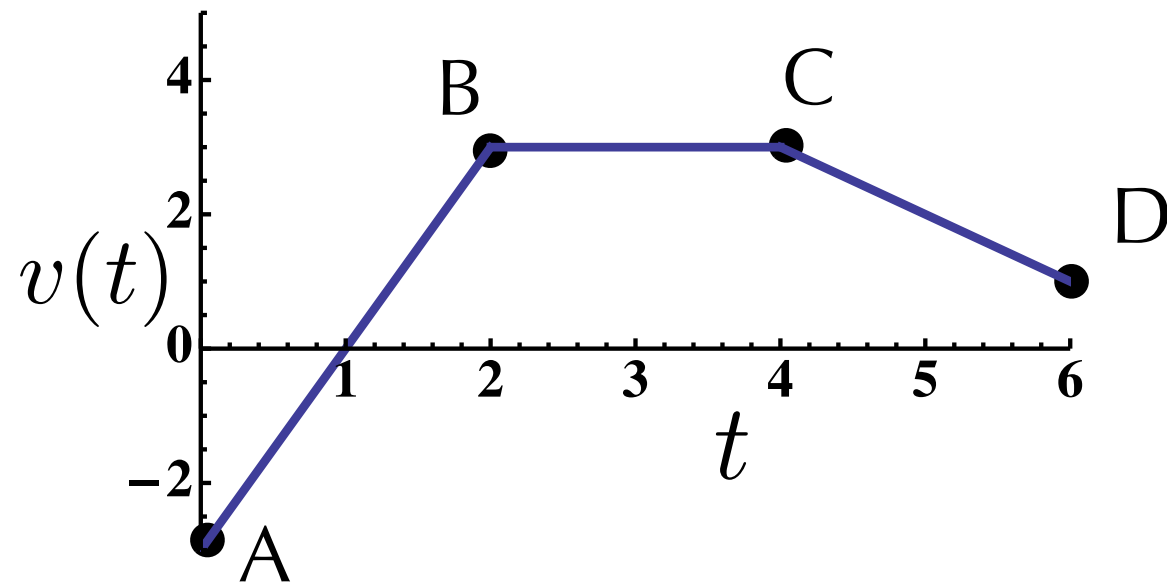
$$\int_{t_i}^{t_f} (u + w) dt = \int_{t_i}^{t_f} u dt + \int_{t_i}^{t_f} w dt$$

$$v(t) \rightarrow x(t)$$



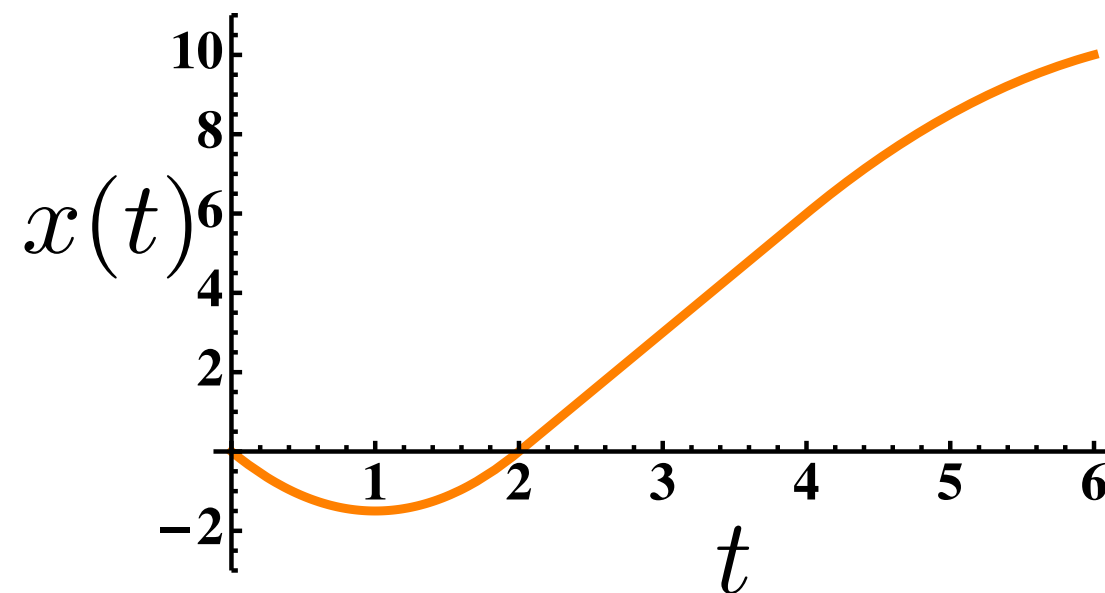
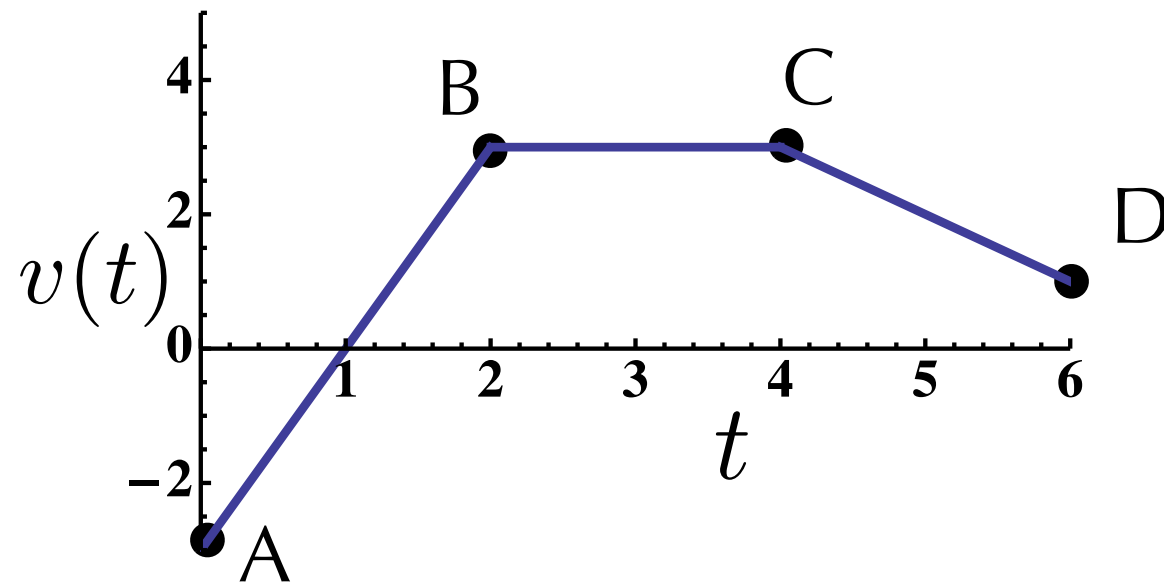
$$v(t) \rightarrow x(t)$$

Draw the
position-versus-
time plot
corresponding to
the velocity-time
graph shown (at
 $t=0$ the $x=0$).

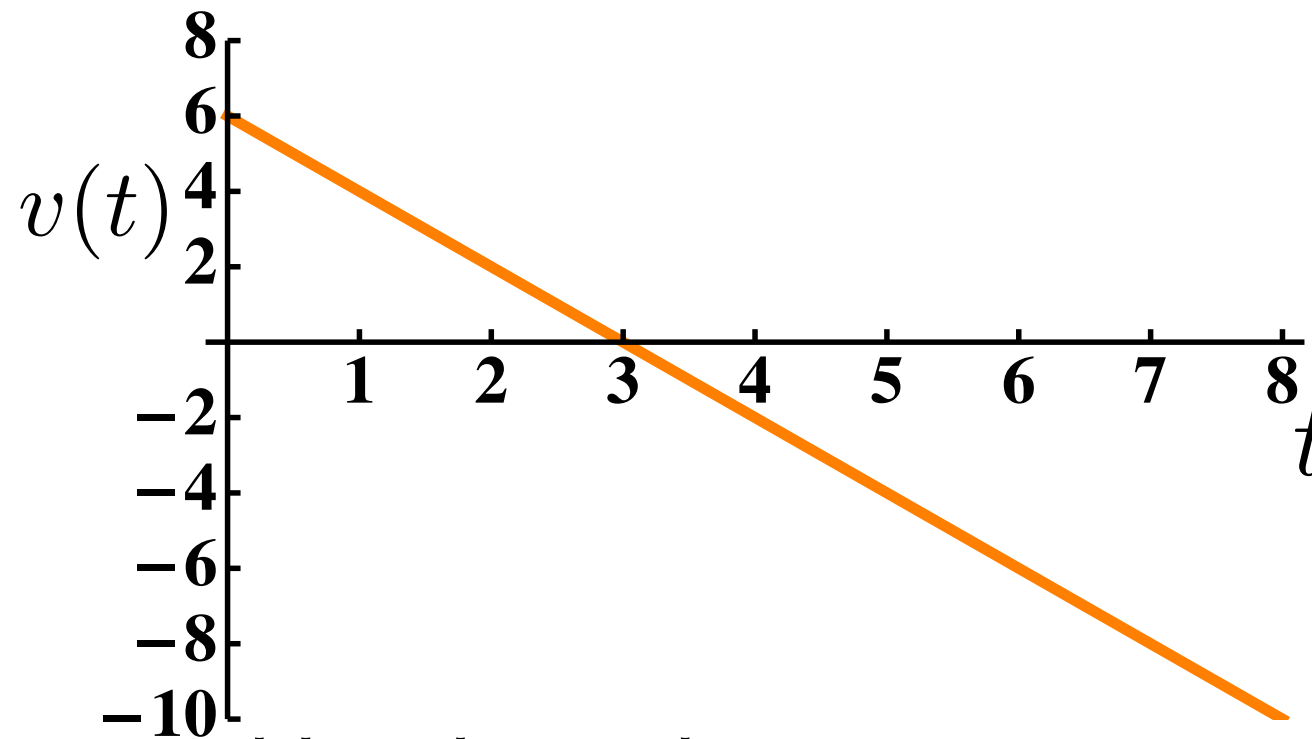


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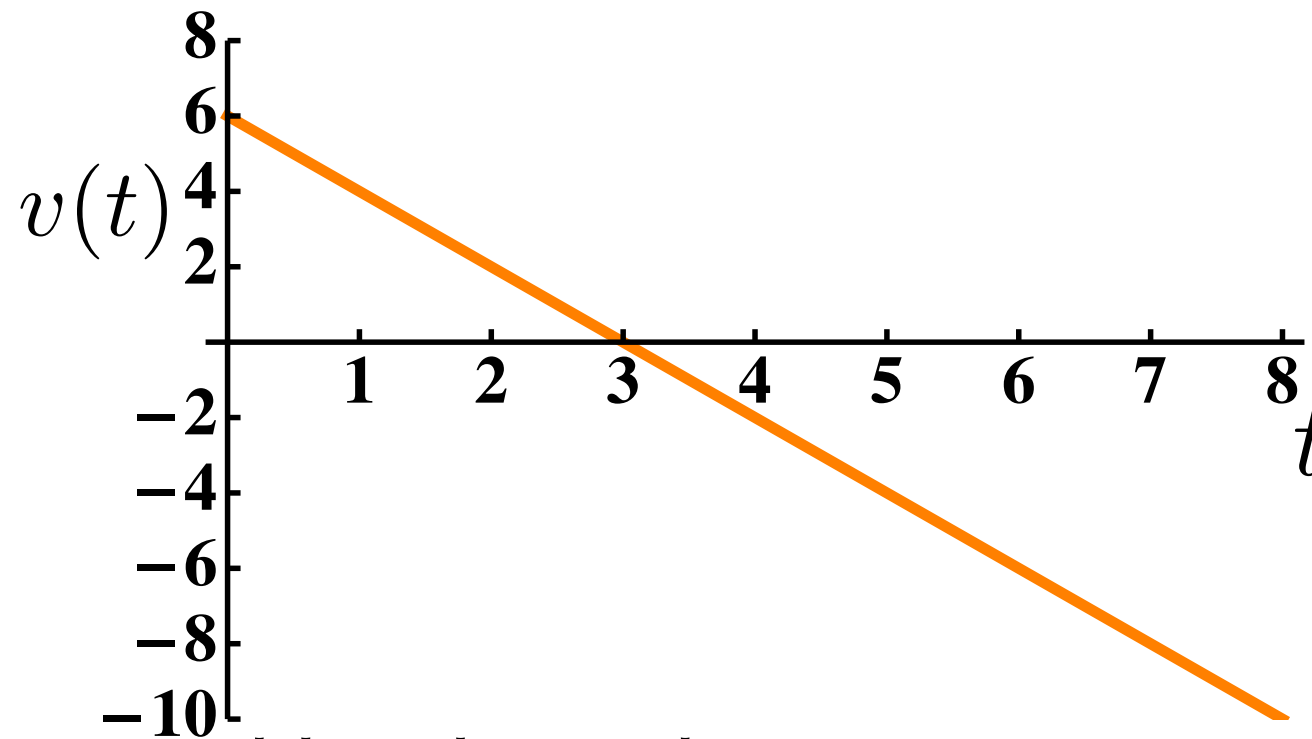
velocity \rightarrow position exercise



The object represented by the velocity-time graph above is at $x=7$ at $t=0$. Does the object ever reach $x=0$? If so, at what time does this happen?

- a) The object never reaches the origin.
- b) The object reaches the origin at $t = 3$.
- c) The object reaches the origin at $t = 7$.
- d) The object reaches the origin at $t = 6$.
- e) I have no idea.

velocity \rightarrow position exercise



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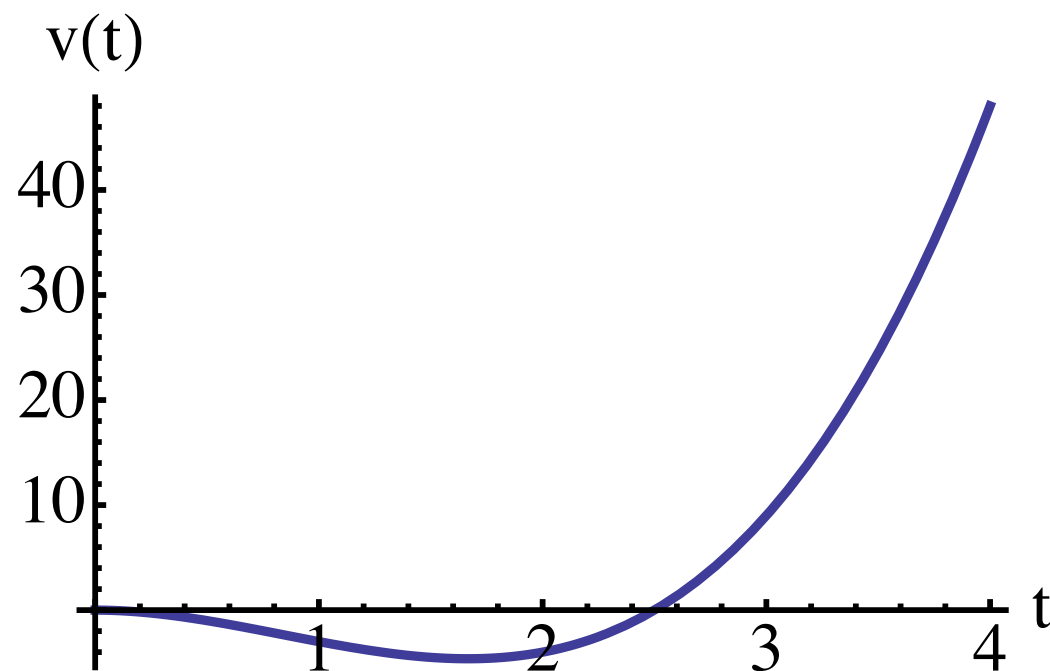
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Example Problem

The velocity of a particle is given by the expression

$$v(t) = 2t^3 - 5t^2$$

If the object's position at $t = 1$ is $x = 5$,
what is its position when $t = 3$?

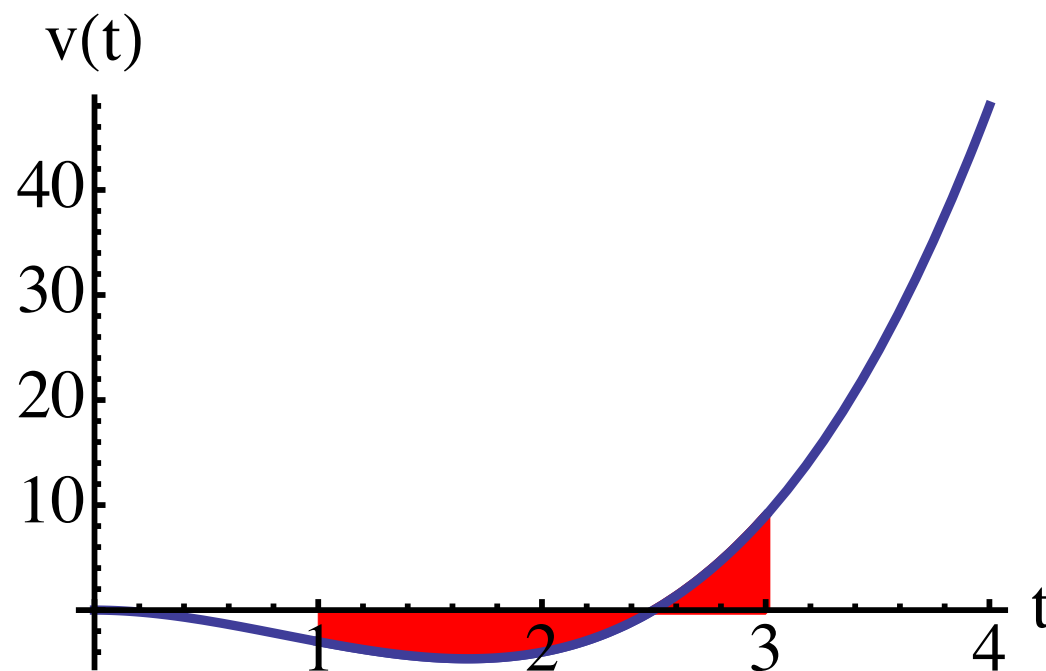


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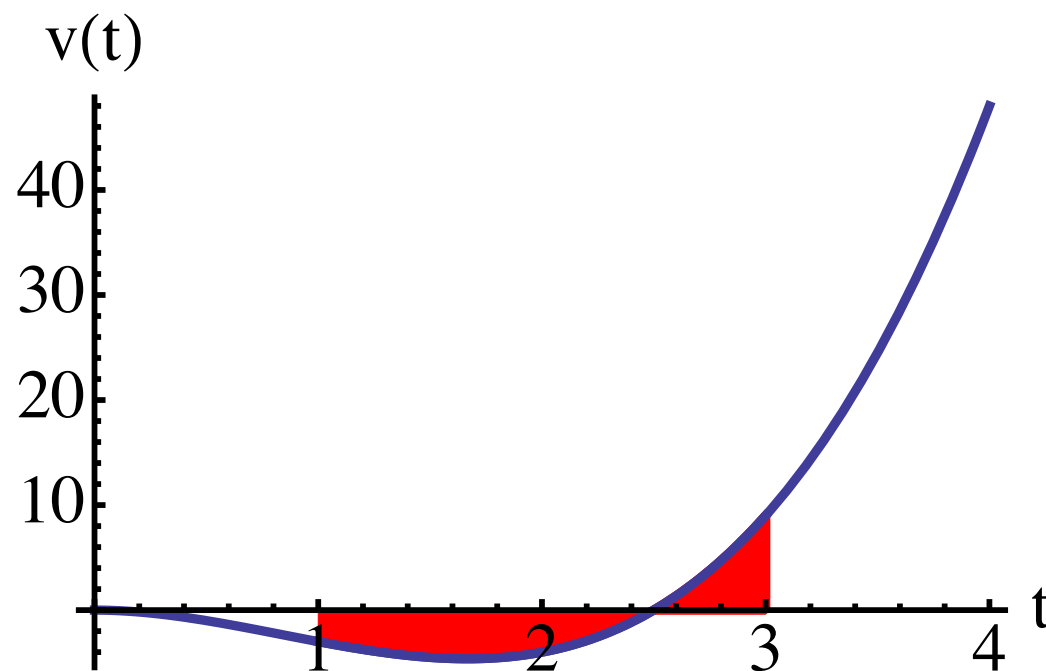


Example Problem

The velocity of a particle is given by the expression

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If the object's position at $t = 1$ is $x = 5$,
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$$\Delta x = \int_1^3 (2t^3 - 5t^2) dt = \left. \frac{1}{2}t^4 - \frac{5}{3}t^3 \right|_1^3 = -\frac{10}{3}$$