Little g and big G

According to observer on earth

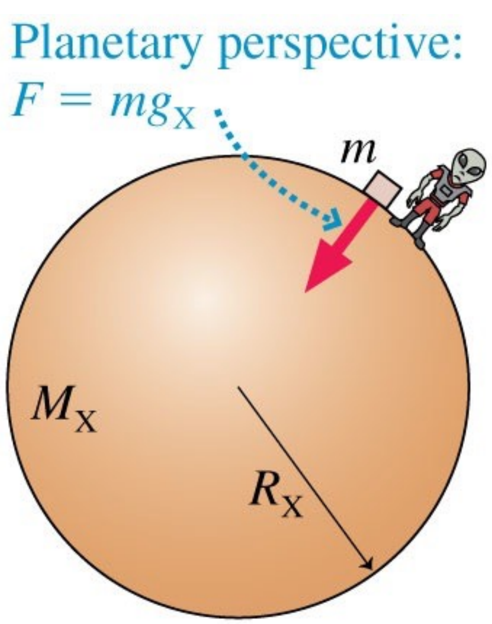
$$F_G = mg_{\text{surface}}$$

According to Newton's Law of gravity

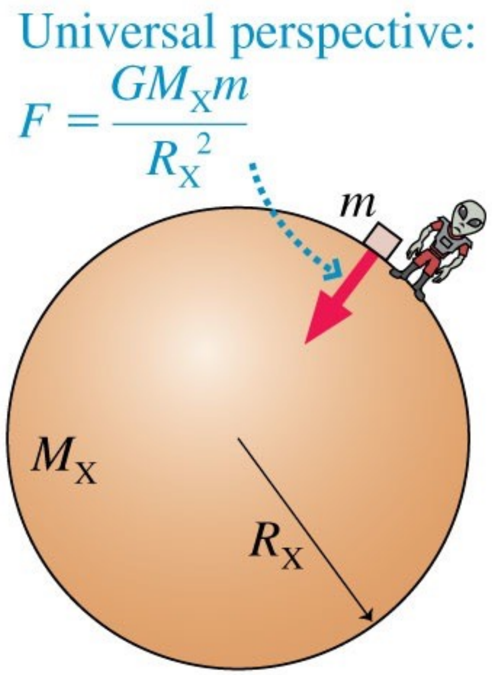
$$F_G = \frac{GMm}{r^2}$$

These are equal if:

$$g_{\text{surface}} = \frac{GM}{R_{\text{earth}}^2}$$



Planet X



Planet X

Question #17

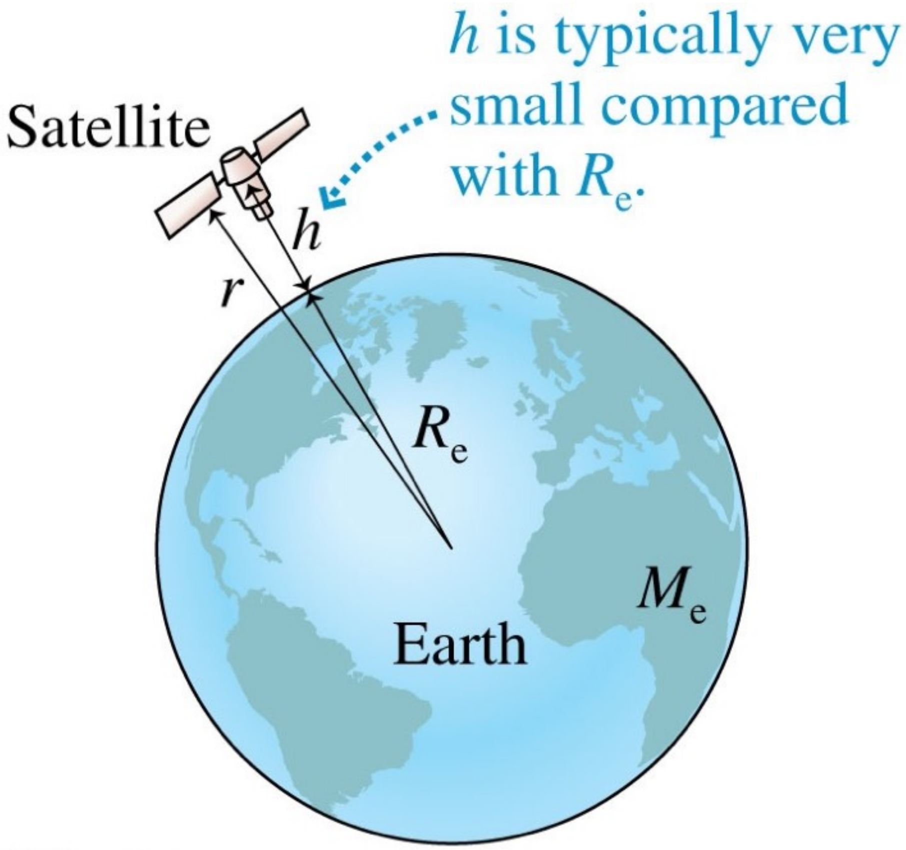
Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

a. $g = 2$ m/s^2.
b. $g = 8$ m/s^2.
c. $g = 4$ m/s^2.
d. $g = 16$ m/s^2.
e. $g = 32$ m/s^2.

Decrease of g with distance

$$g = \frac{GM_e}{(R_e + h)^2} = \frac{GM_e}{R_e^2(1 + h/R_e)^2} = \frac{g_{earth}}{(1 + h/R_e)^2}$$

where $g_{earth} = 9.83$ m/s^2 and $R_e = 6.37 \times 10^6$ m.



Decrease of g with distance

TABLE 13.1 Variation of g with height above the ground

Height h	Example	g (m/s^2)
0 m	ground	9.83
4500 m	Mt. Whitney	9.82
10,000 m	jet airplane	9.80
300,000 m	space shuttle	8.90
35,900,000 m	communications satellite	0.22

Question #18

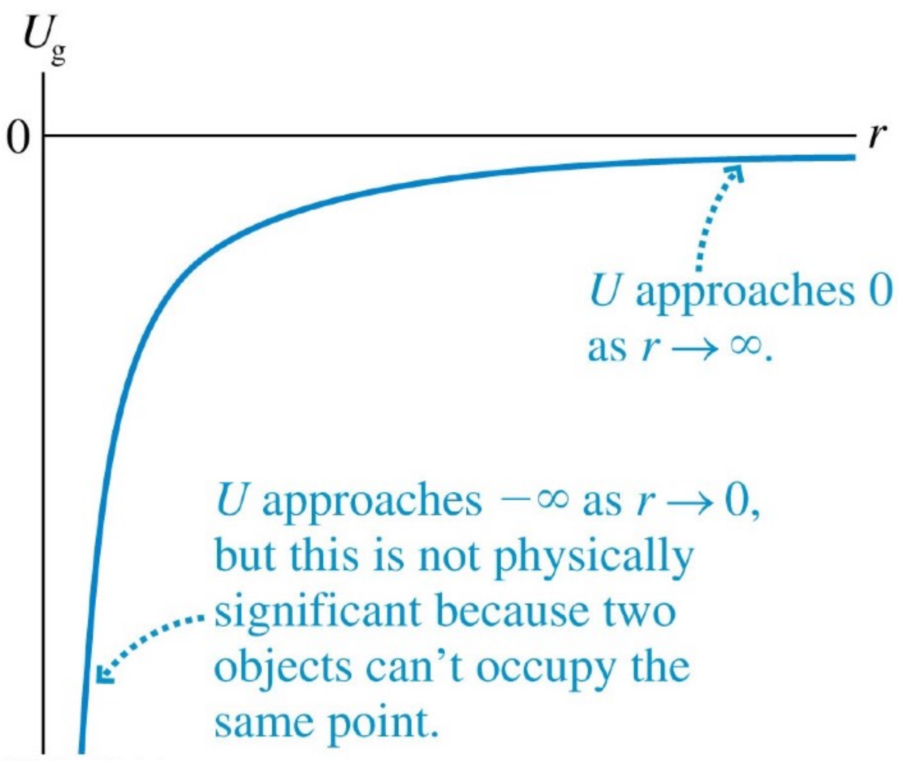
Astronauts on the International Space Station are weightless because

a. There's no gravity in outer space.
b. The net force on them is zero.
c. They are in free fall.
d. g is very small, although not zero.
e. The centrifugal force balances the gravitational force.

Gravitational Potential Energy

When two isolated masses m_1 and m_2 interact over large distances, they have a gravitational potential energy of:

$$U_g = -\frac{Gm_1 m_2}{r}$$

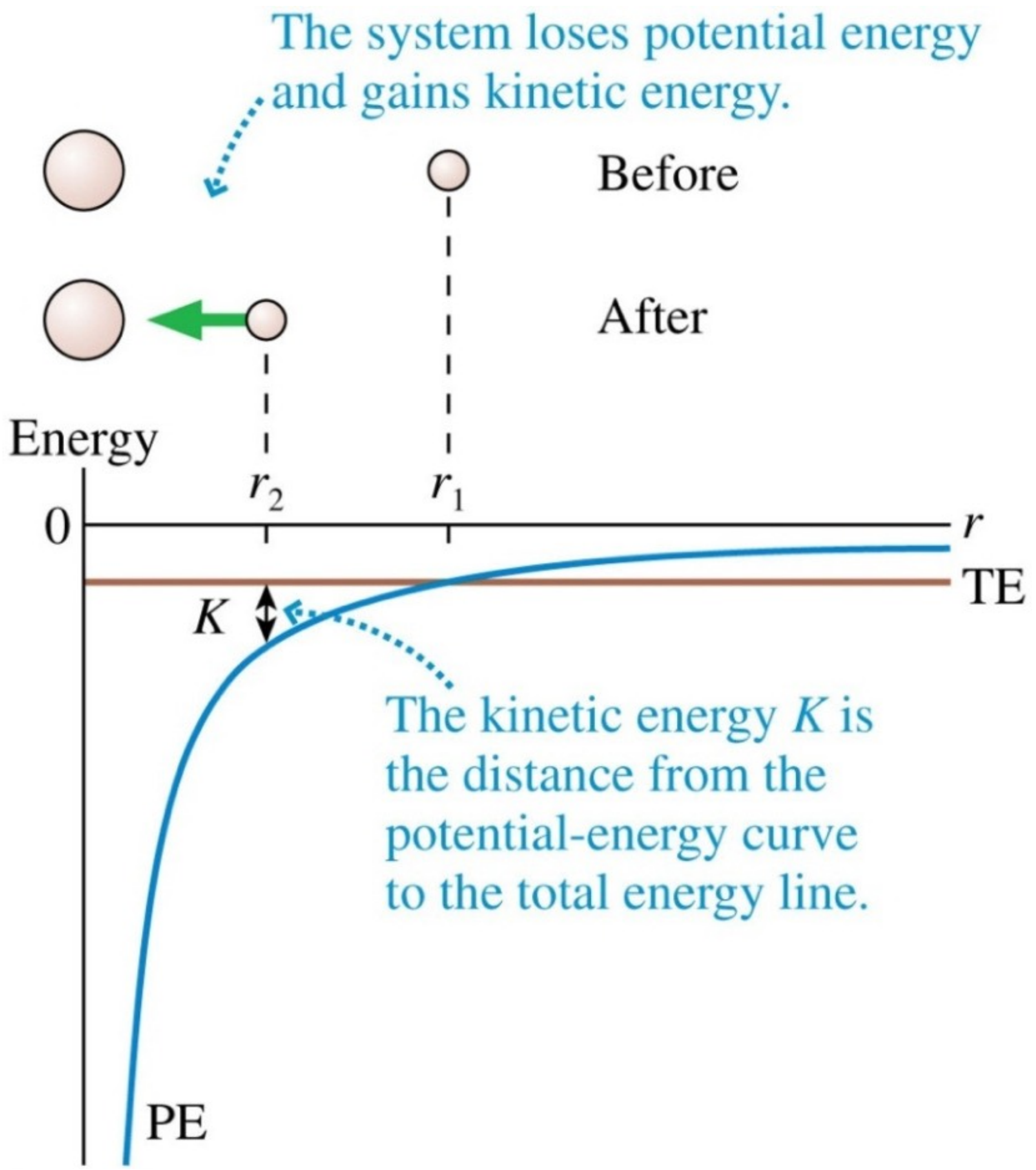


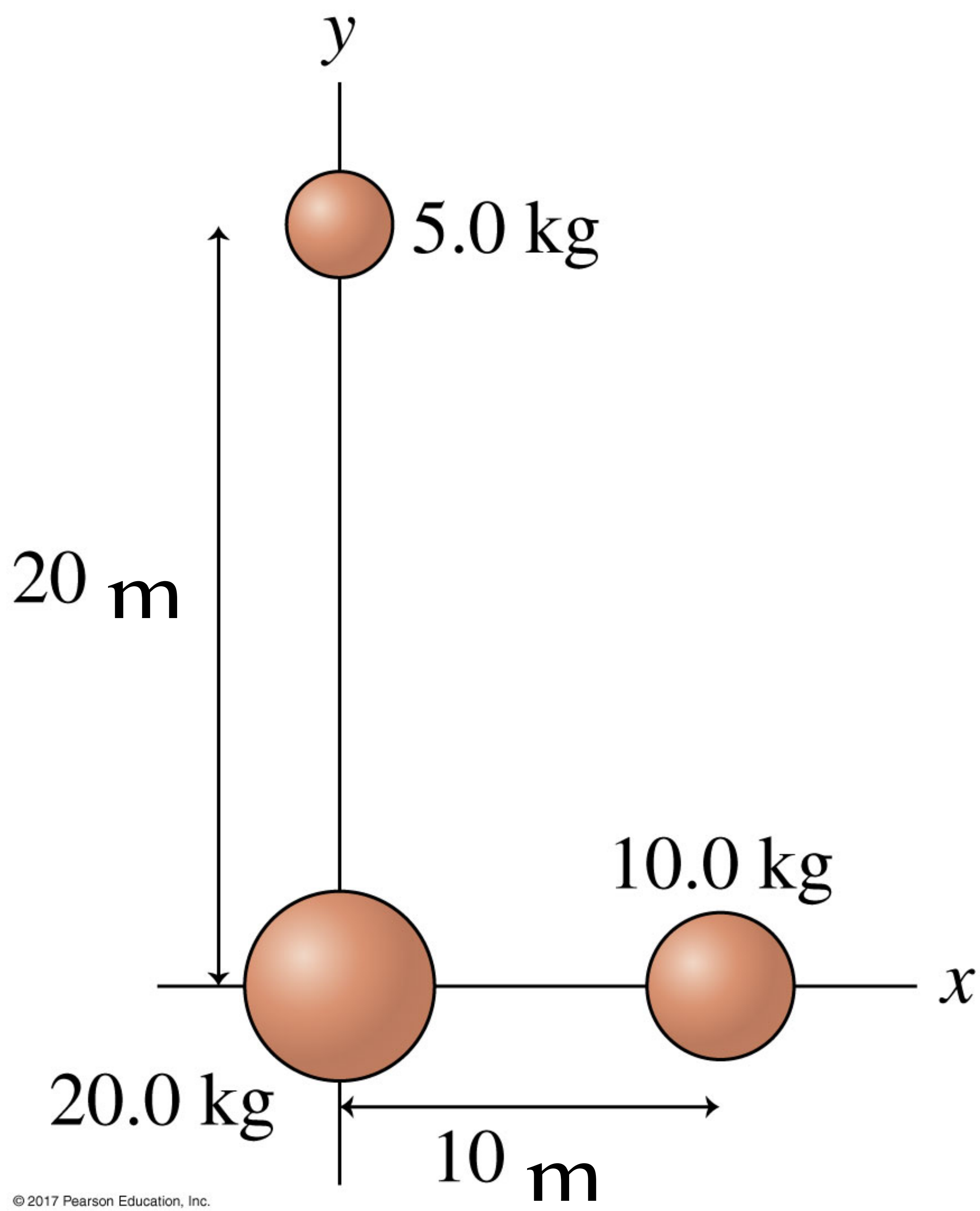
where we have chosen the zero point of potential energy at $r = \infty$, where the masses will have no tendency, or potential, to move together.

Note that this equation gives the potential energy of masses m_1 and m_2 when their *centers* are separated by a distance r.

Gravitational Potential Energy

a. At r_1 U is negative.
b. At r_2 |U| is larger and U is still negative, meaning that U has decreased.
c. As the system loses potential energy, it gains kinetic energy while conserving mechanical energy.
d. The smaller mass speeds up as it falls.




y
5.0 kg
20 m
10.0 kg
x
20.0 kg
10 m

Example: Escape Speed

Question # 19

A 1000 kg rocket is fired straight away from the surface of the earth. What speed does the rocket need to "escape" from the gravitational pull of the earth and never return?

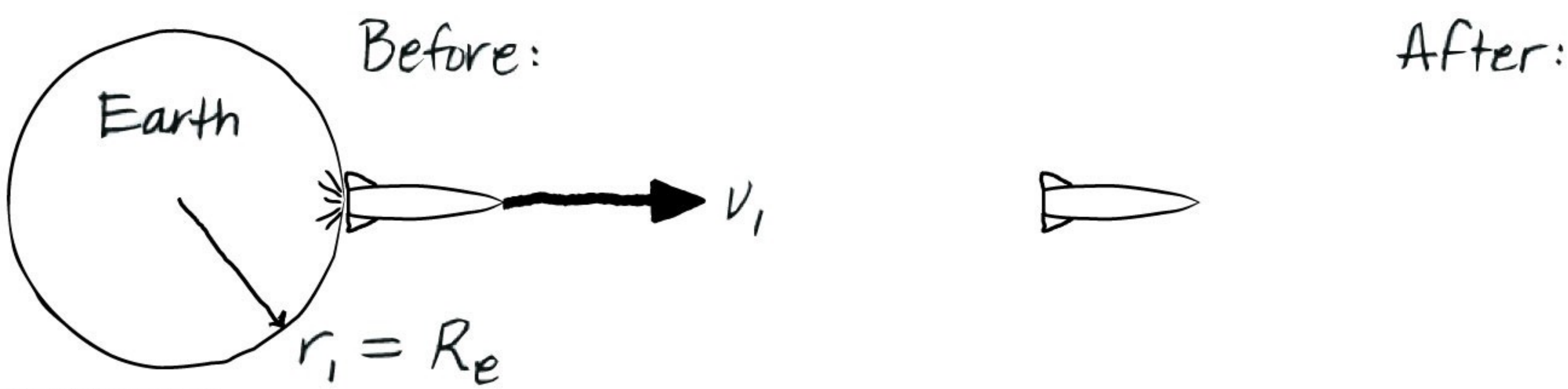
Which equation is a correct statement of conservation of energy?

D $$\frac{1}{2}mv_i^2 - \frac{GM_e m_R}{R_e} = 0$$

C $$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{GM_e m_R}{R_e}$$

B $$\frac{1}{2}mv_i^2 + \frac{GM_e m_R}{R_e} = 0$$

A $$\frac{1}{2}mv_i^2 - \frac{GM_e m_R}{R_e} = \frac{1}{2}mv_f^2$$



Example Problem **Question # 20**

A less-than-successful inventor wants to launch small satellites into orbit by launching them straight up from the surface of the earth at very high speed. a)With what speed should he launch the satellite if it is to have a speed of 500 m/s at a height of 400 km?

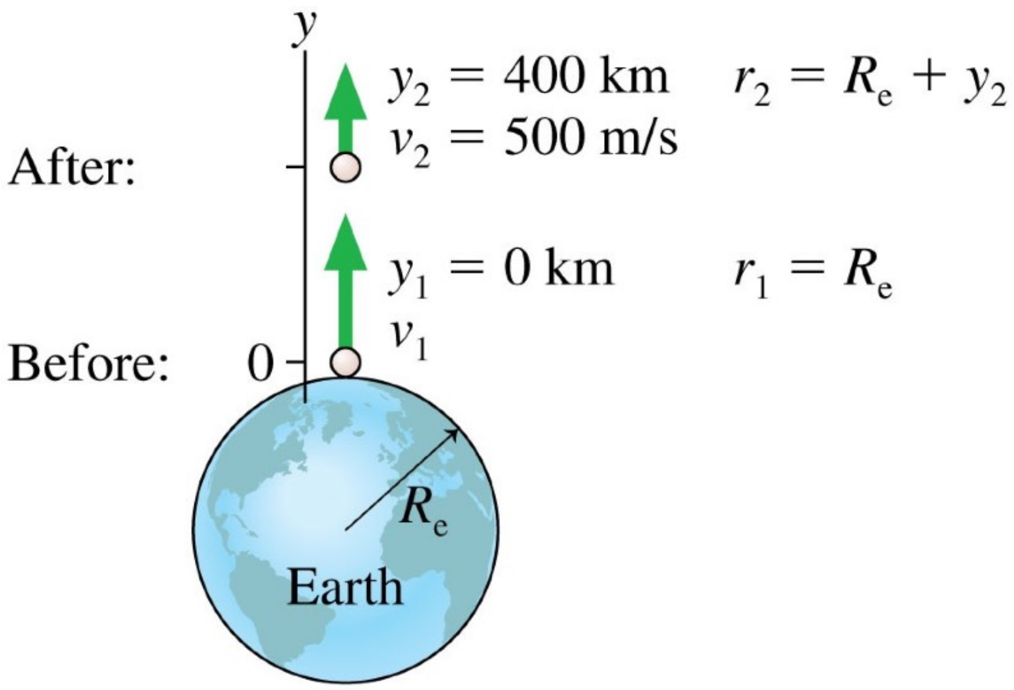
Which equation is a correct statement of conservation of energy?

D $$\frac{1}{2}mv_i^2 - \frac{GM_e m_R}{R_e} = \frac{1}{2}mv_f^2 - \frac{GM_e m_R}{R_e + h}$$

C $$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - \frac{GM_e m_R}{R_e + h}$$

B $$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - \frac{GM_e m_R}{R_e}$$

A $$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{GM_e m_R}{R_e}$$



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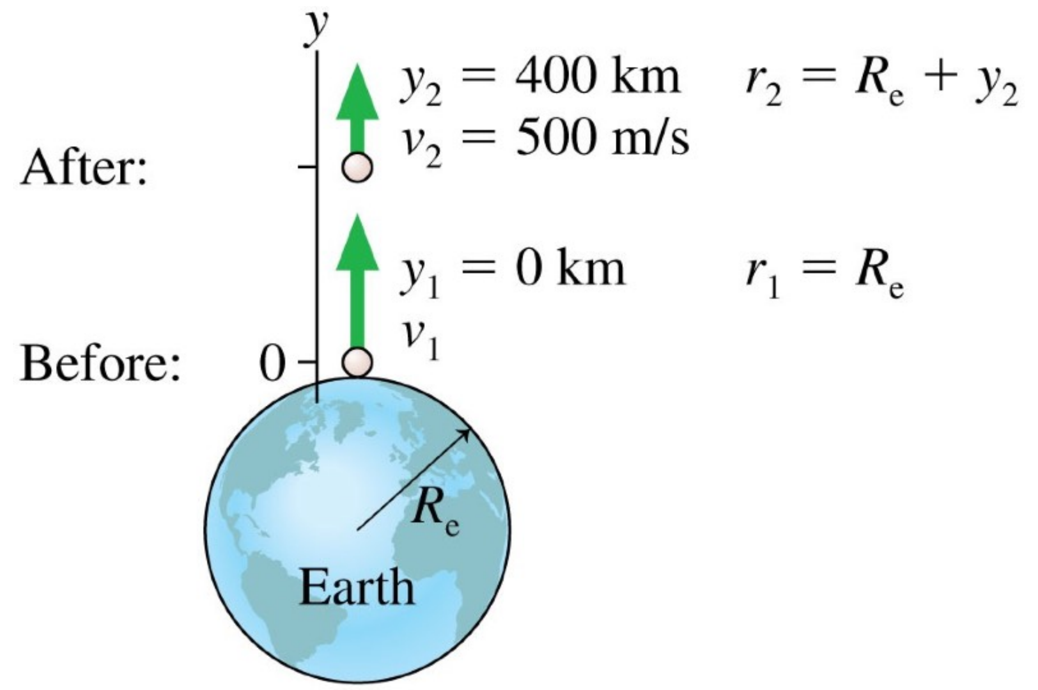


old approach

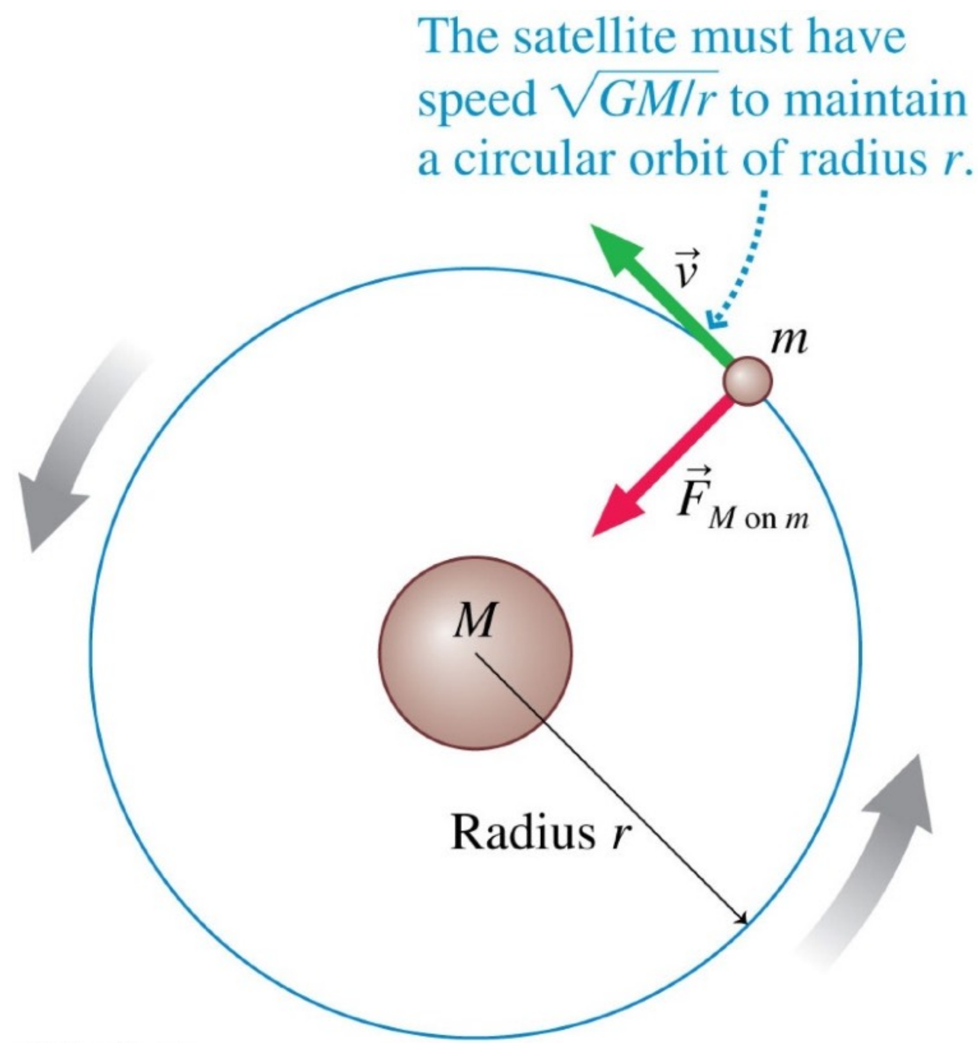
$v_i = 2844 \text{ m/s}$

correct way

$v_i = 2763 \text{ m/s}$

Satellite Orbits

For a given altitude, what must the speed of an orbiting craft be?

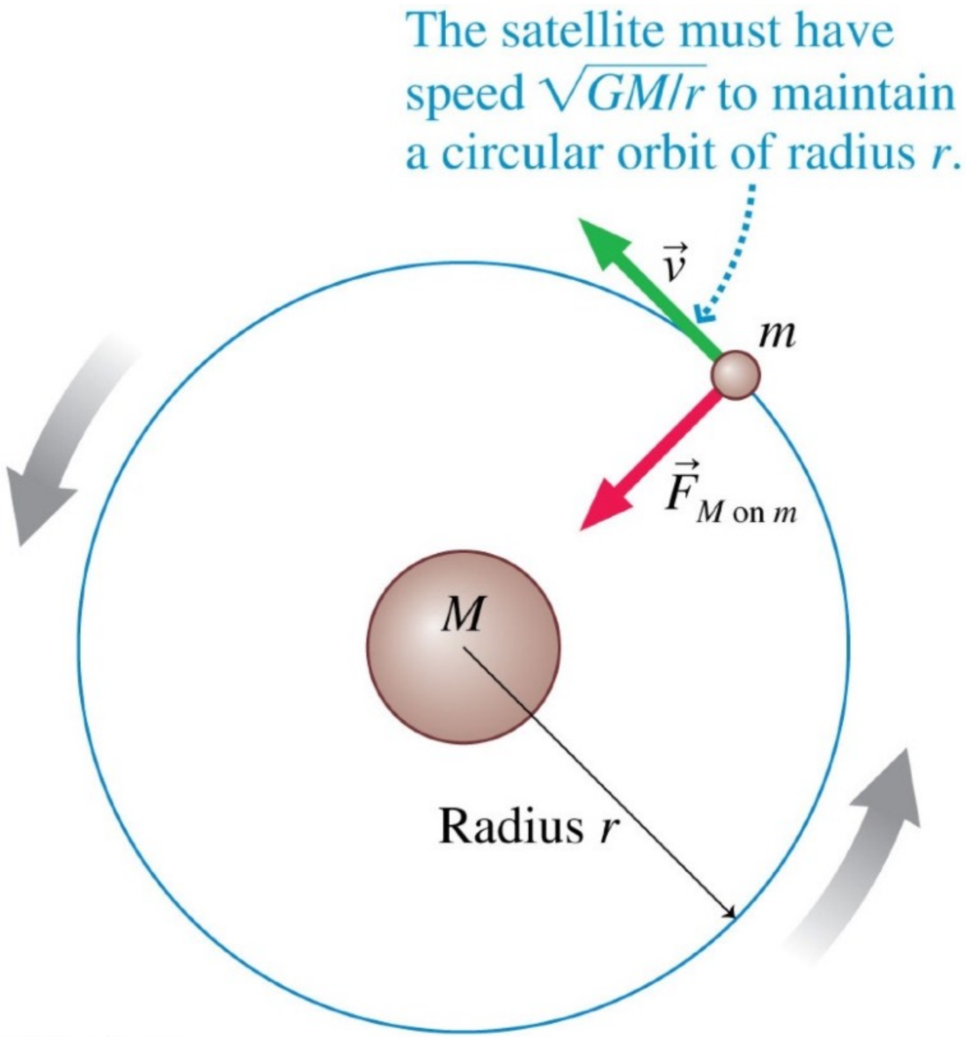


movie

Satellite Orbits

For a given altitude, what must the speed of an orbiting craft be?

$$v = \sqrt{\frac{GM}{r}}$$



movie

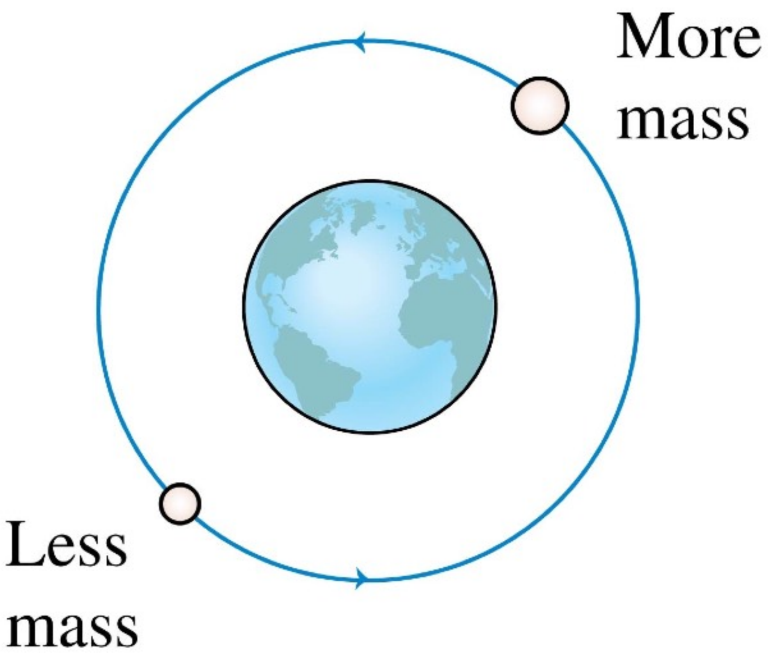
Question #21

Two satellites have circular orbits with the same radius. Which has a higher speed?`

B- They have the same speed.

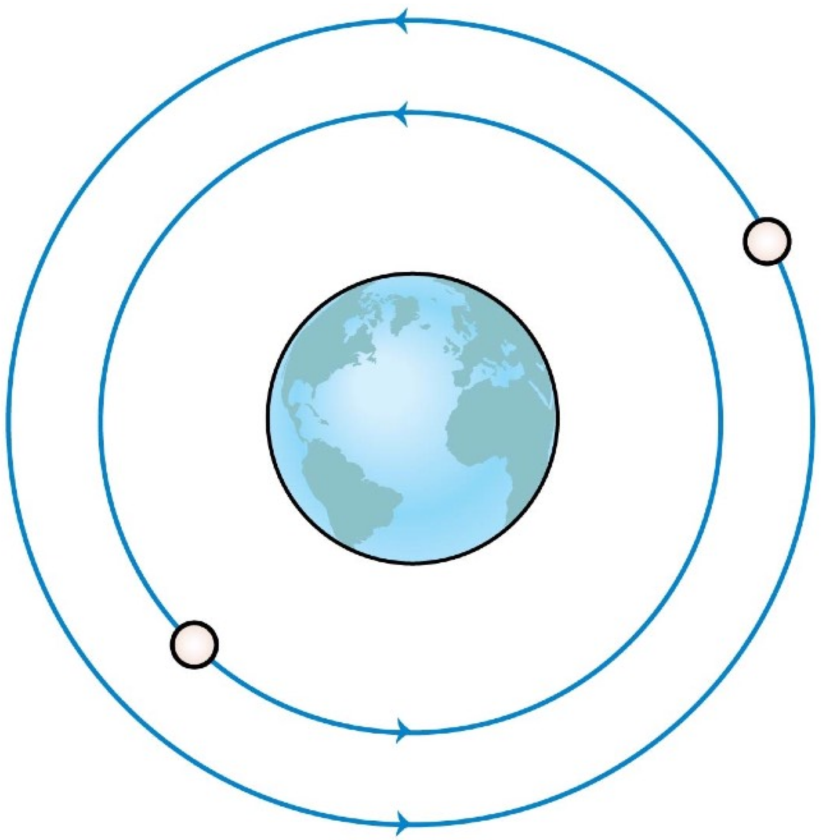
C- The one with less mass.

D- The one with more mass.



Question #22

Two identical satellites have different circular orbits. Which has a higher speed?



a. The one in the larger orbit.
b. They have the same speed.
c. The one in the smaller orbit.

Recall Kepler's Third Law of Planetary Motion

3. The square of a planet's orbital period is proportional to the cube of the semimajor-axis length.

The speed of a satellite in a circular orbit is:

$$v = \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring both sides and solving for T^2 gives:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Planets are satellites of the sun, in orbits that are almost circular. The orbital radius = the semimajor-axis length for a circle.

Use Kepler's third law to find the altitude of a satellite in a geosynchronous orbit.

$$T^2 = (\frac{4\pi^2}{GM})r^3$$

$$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$$

$$R_e = 6.37 \times 10^6 \text{ m}$$

$$M_e = 5.97 \times 10^{24} \text{ kg}$$

Use Kepler's third law to find the altitude of a satellite in a geosynchronous orbit.

$$T^2 = (\frac{4\pi^2}{GM})r^3$$

$$r = 3.6 \times 10^7 \text{ m}$$

$$\approx 22,000 \text{ miles}$$

$$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$$

$$R_e = 6.37 \times 10^6 \text{ m}$$

$$M_e = 5.97 \times 10^{24} \text{ kg}$$

Orbital Energetics

We know that for a satellite in a circular orbit, its speed is related to the size of its orbit by $v^2 = GM/r$. The satellite's kinetic energy is thus:

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

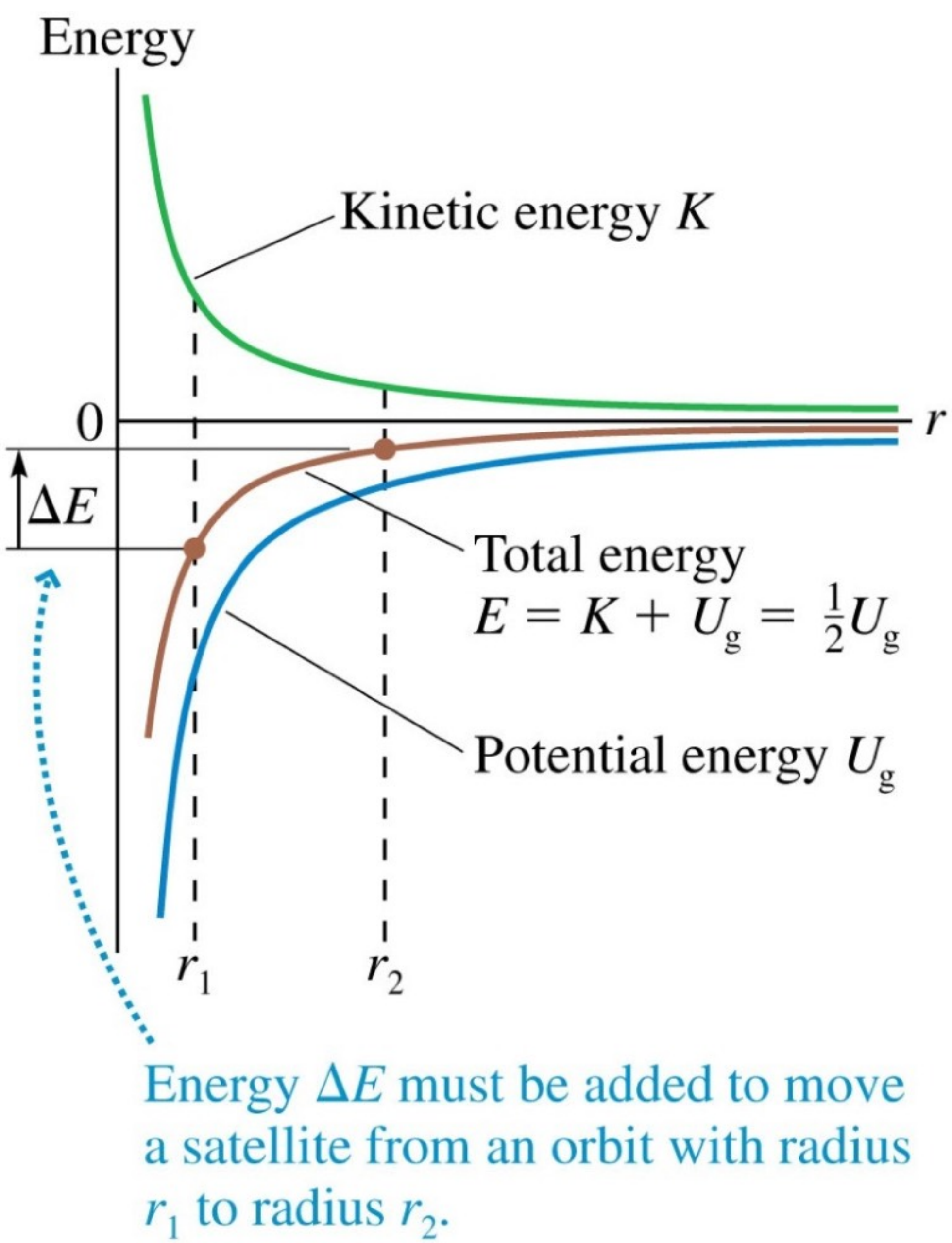
But $-GMm/r$ is the potential energy, U_g, so:

$$K = -\frac{1}{2}U_g$$

If K and U do not have this relationship, then the trajectory will be elliptical rather than circular. So, the mechanical energy of a satellite in a circular orbit is always:

$$E_{\text{mech}} = K + U_g = \frac{1}{2}U_g$$

Orbital Energetics

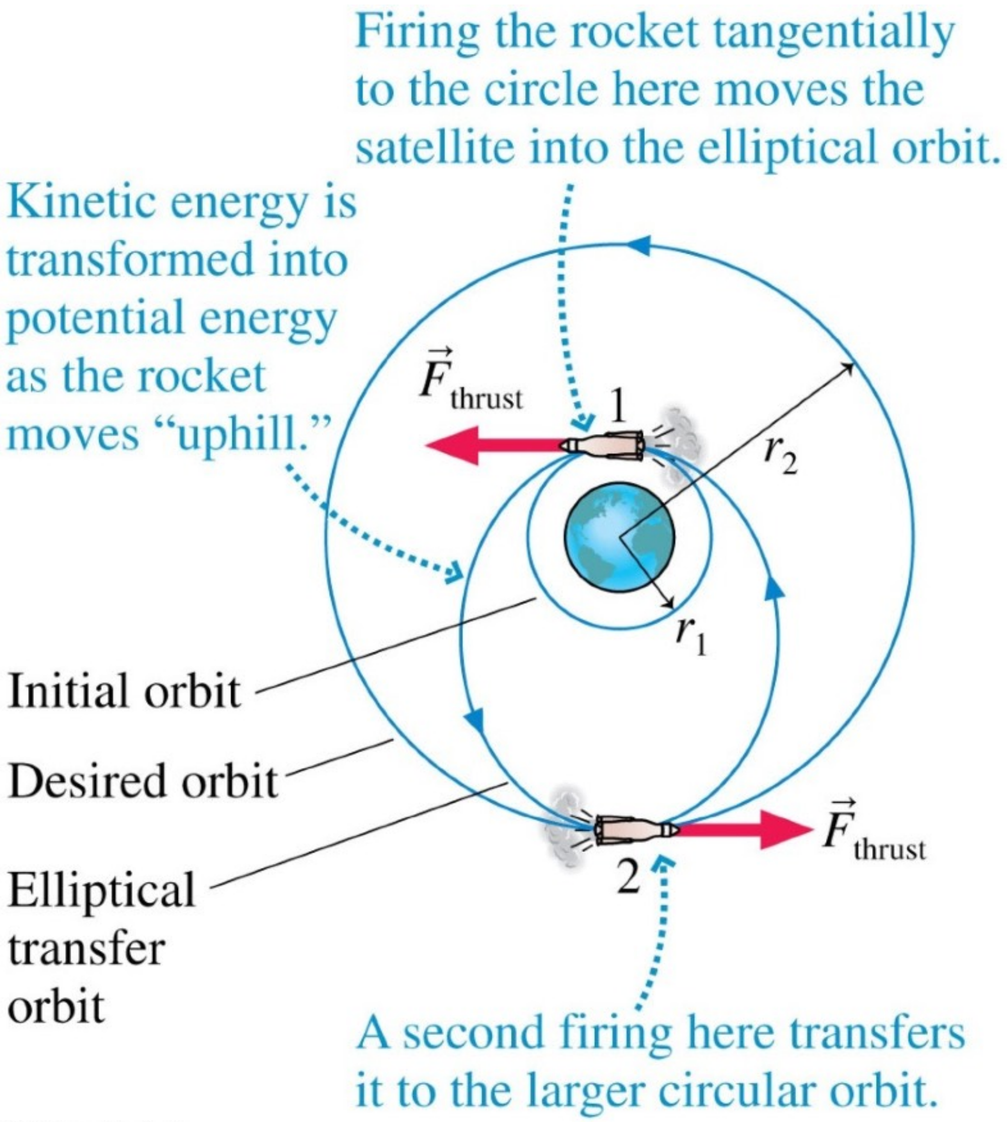


- The figure shows the kinetic, potential, and total energy of a satellite in a *circular orbit*.
- Notice how, for a circular orbit, $E_{mech} = U_g/2$.
- It requires positive energy in order to lift a satellite into a higher orbit.

Example: Raising a satellite

How much work must be done to boost a 1000 kg communications satellite from a low earth orbit with h = 300 km, where it is released by the space shuttle, to a geosynchronous orbit?

1. How much total energy (kinetic + potential) does the satellite have in the lower orbit?
2. How much total energy (kinetic + potential) does the satellite have in the higher orbit?
3. $K_i + U_i + W = K_f + U_f + \Delta E_{\text{th}}$



$$W = 9.84 \times 10^9 \text{ J}$$