

Vertical Kinematics

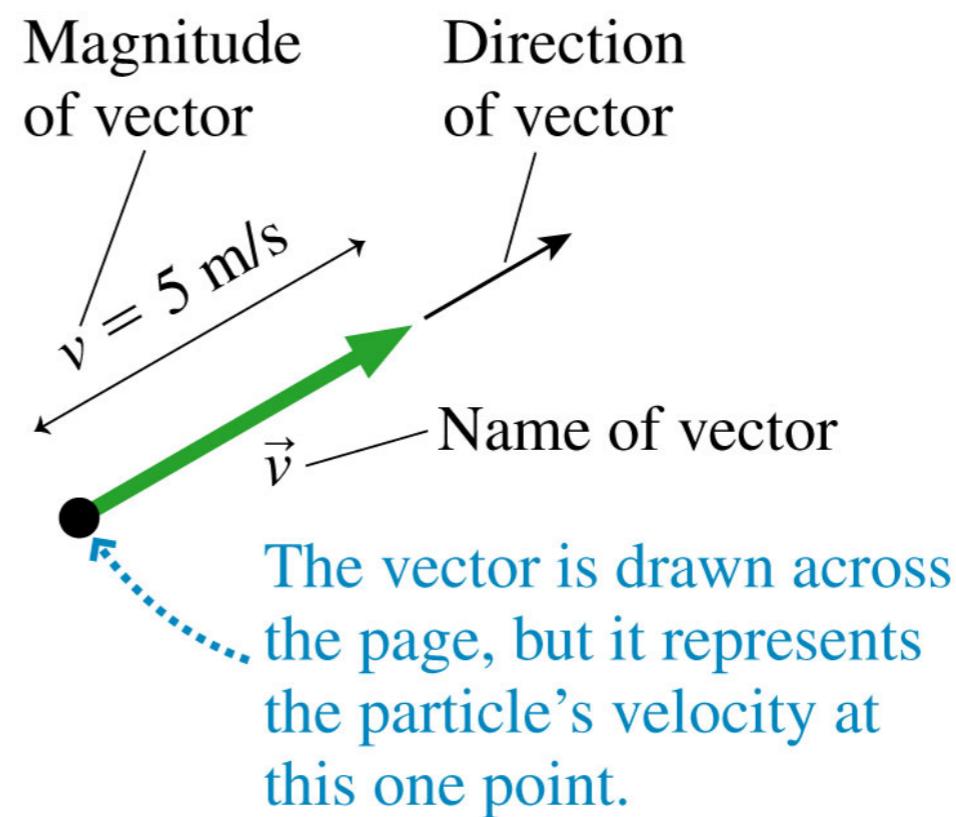
A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at 30 m/s^2 for 30 s, then runs out of fuel.

- a) What is the rocket's max altitude
- b) How long is the rocket in the air before hitting the ground.

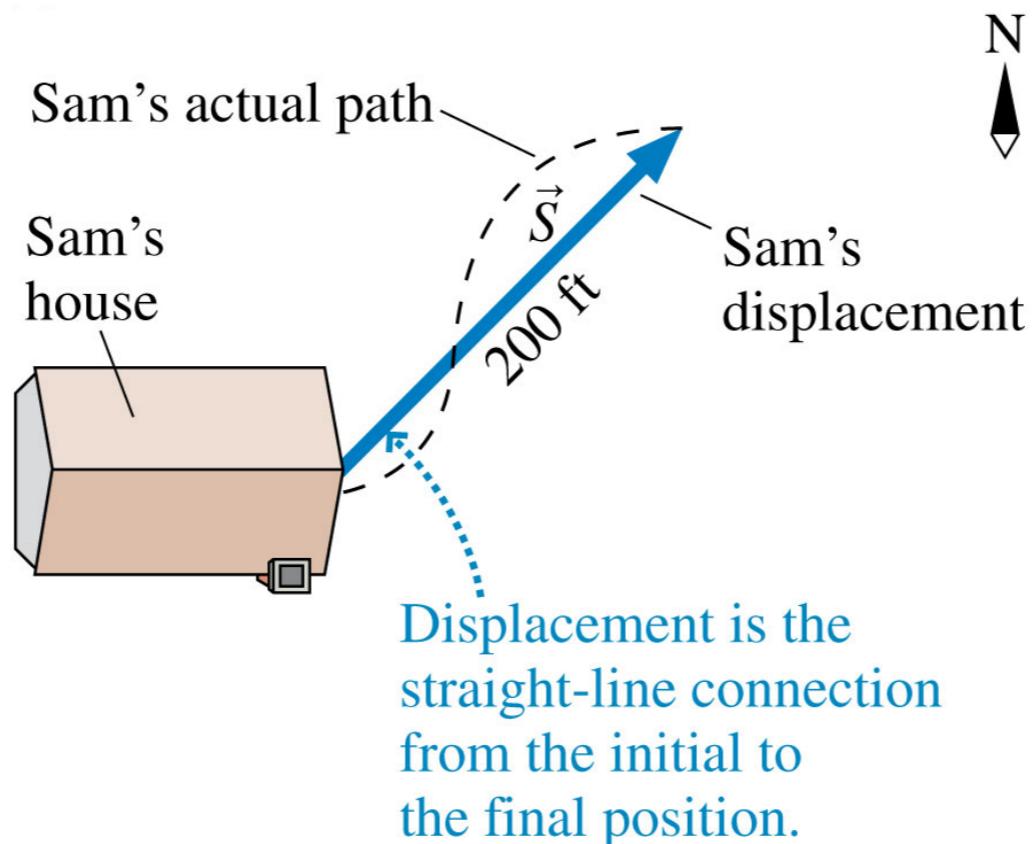
Vectors

Magnitude

Direction



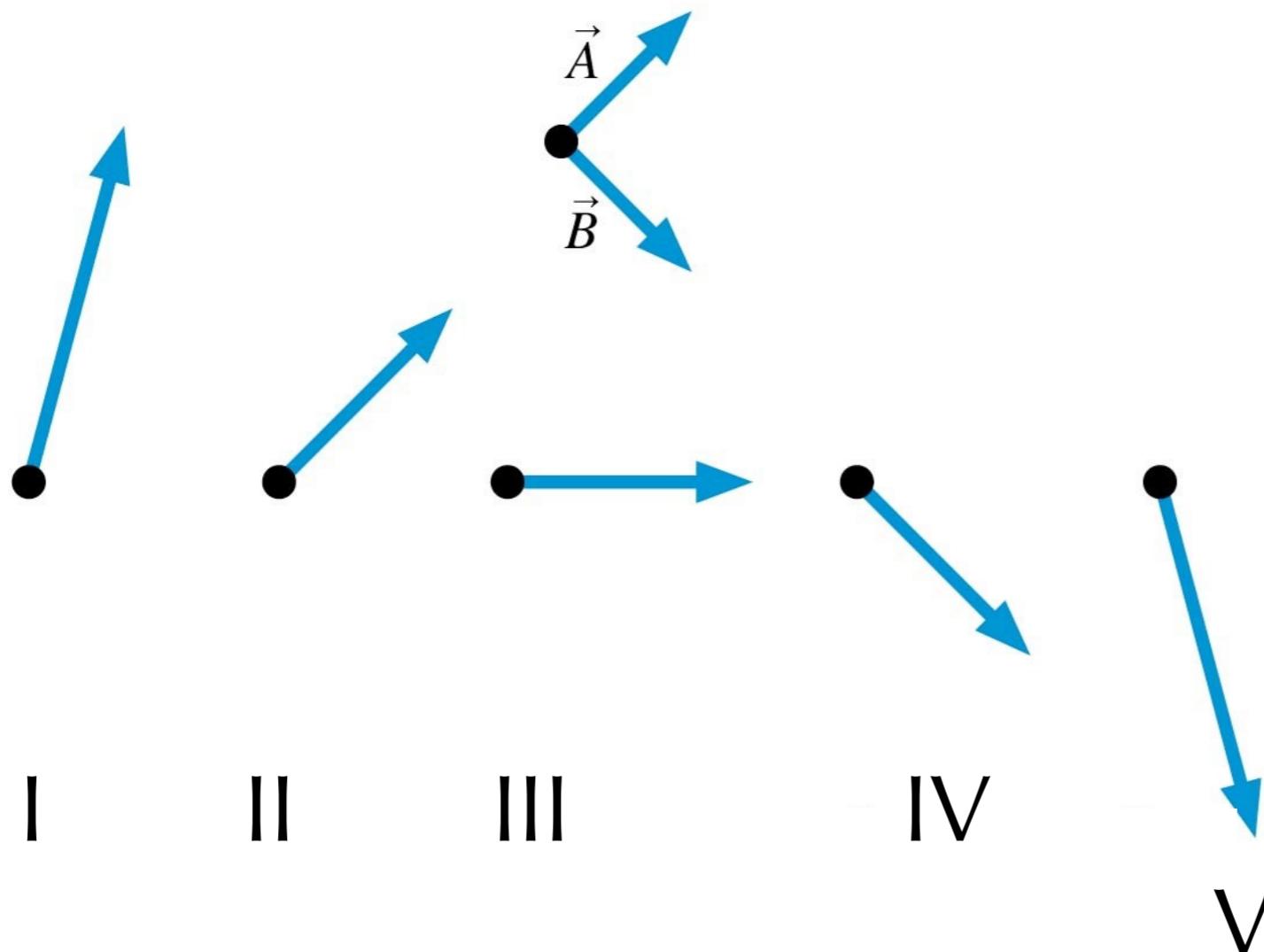
Displacement



$$\vec{S} = (200 \text{ ft, northeast})$$

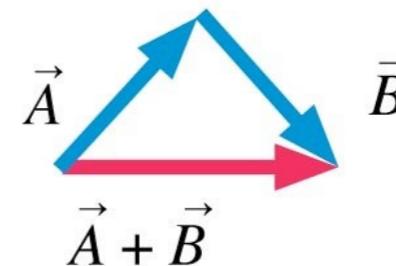
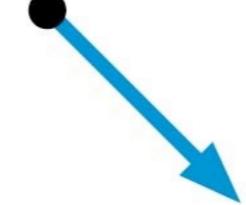
Quiz

Which of the vectors represents $\vec{A} + \vec{B}$



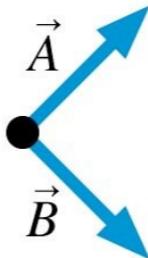
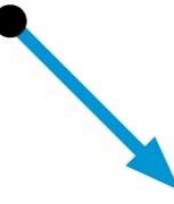
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Which of the vectors represents $\vec{A} + \vec{B}$

- 
- The diagram illustrates the parallelogram law of vector addition. Two vectors, \vec{A} and \vec{B} , originate from the same point. Their sum, $\vec{A} + \vec{B}$, is represented by the vector connecting the tail of \vec{A} to the head of \vec{B} . This vector is shown in red.
- A.  Vector \vec{A} is a blue arrow pointing upwards and to the right.
- B.  Vector \vec{B} is a blue arrow pointing upwards and to the left.
- C.  A blue arrow originating from the tip of \vec{A} and ending at the tip of \vec{B} . A red checkmark is placed near the tip of \vec{B} .
- D.  Vector $\vec{A} - \vec{B}$ is a blue arrow originating from the tip of \vec{B} and ending at the tip of \vec{A} .
- E.  Vector $\vec{B} - \vec{A}$ is a blue arrow originating from the tip of \vec{A} and ending at the tip of \vec{B} .

Quiz

Which vector represents $2\vec{A} - \vec{B}$

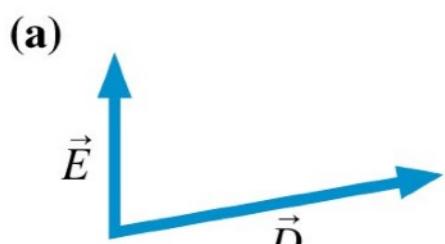
- 
- The diagram shows two vectors originating from the same point. Vector \vec{A} is a horizontal blue arrow pointing to the right. Vector \vec{B} is a blue arrow pointing diagonally upwards and to the right, forming an acute angle with \vec{A} .
- A. 
 - B. 
 - C. 
 - D. 
 - E. 

Quiz

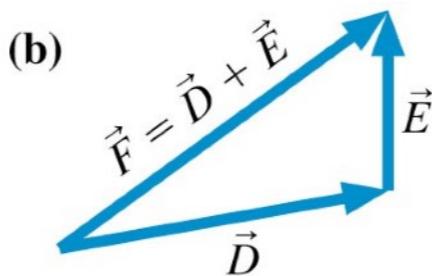
Which vector represents $2\vec{A} - \vec{B}$

- A. A blue vector originating from a black dot and pointing upwards and to the right.
- B. A blue vector originating from a black dot and pointing upwards and to the left.
- C. A horizontal blue vector originating from a black dot and pointing to the right.
- D. A blue vector originating from a black dot and pointing downwards and to the right.
- E. A blue vector originating from a black dot and pointing downwards and to the left.
- ✓** A diagram showing the geometric addition of vectors. It consists of two vectors originating from the same point: a blue vector labeled \vec{A} pointing upwards and to the right, and a blue vector labeled \vec{B} pointing downwards and to the right. A pink vector labeled $2\vec{A}$ is shown as twice the length of \vec{A} , also pointing upwards and to the right. A blue vector labeled $-\vec{B}$ is shown as the negative of \vec{B} , pointing upwards and to the left. The resulting vector, which is the sum $2\vec{A} + (-\vec{B})$, is shown in blue and points upwards and to the right, matching option A.

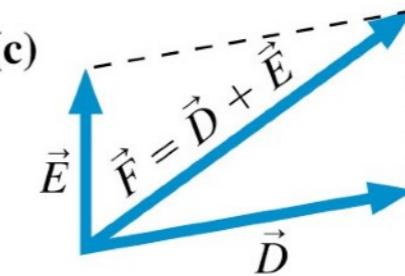
Parallelogram Rule



What is $\vec{D} + \vec{E}$?

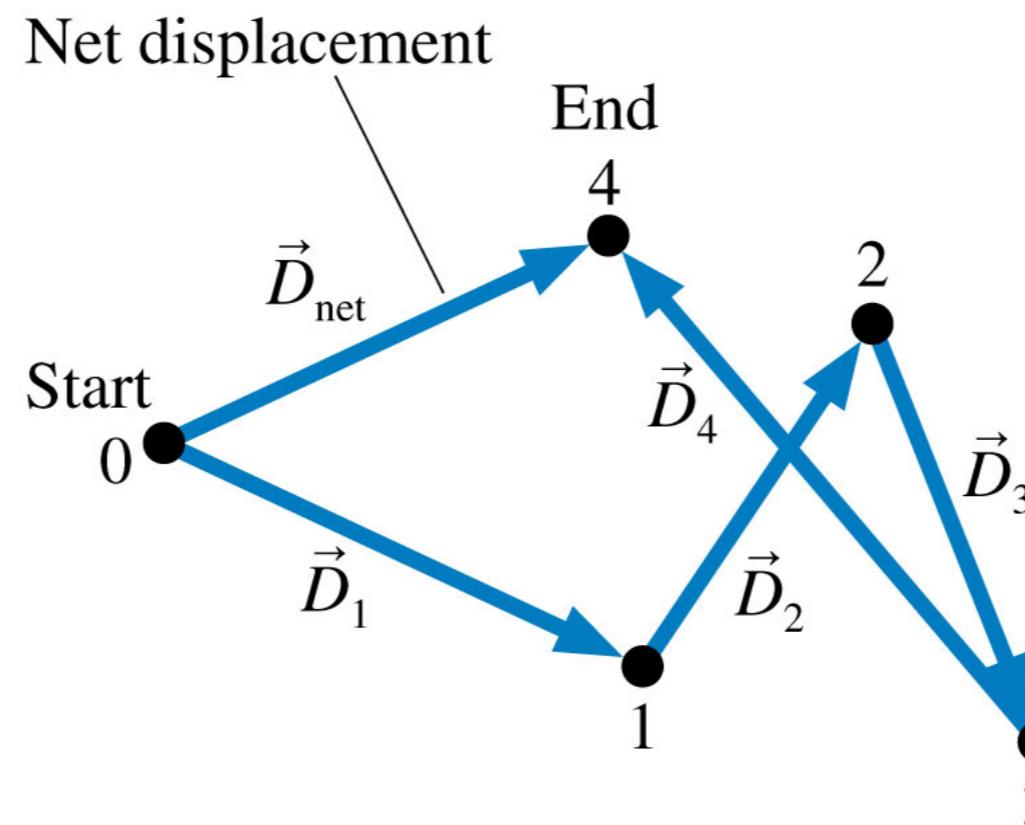


Tip-to-tail rule:
Slide the tail of \vec{E}
to the tip of \vec{D} .



Parallelogram rule:
Find the diagonal of
the parallelogram
formed by \vec{D} and \vec{E} .

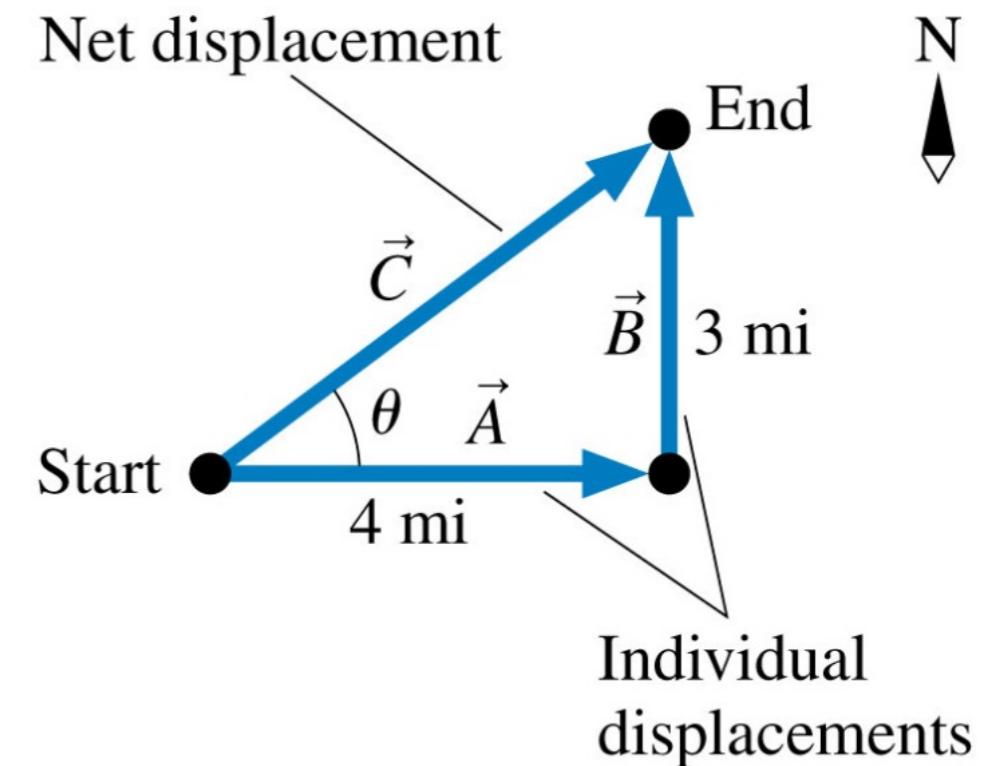
Addition of multiple vectors



$$\vec{D}_{\text{net}} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$$

Vector Addition

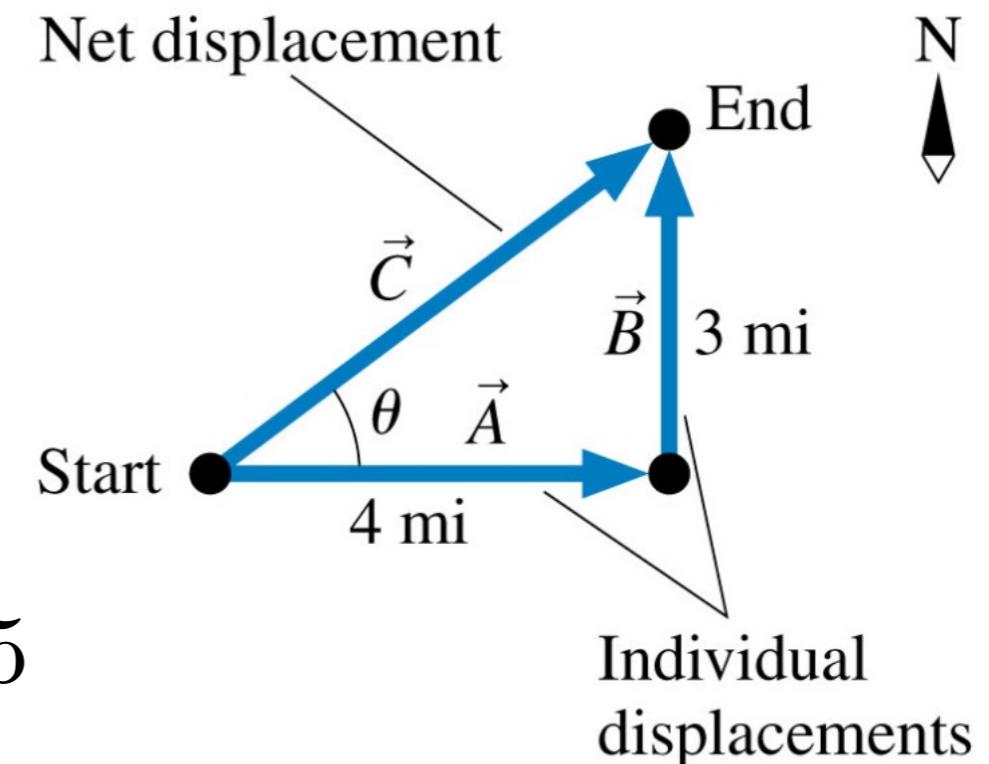
$$\vec{C} = \vec{A} + \vec{B}$$



Vector Addition

$$\vec{C} = \vec{A} + \vec{B}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = 5$$

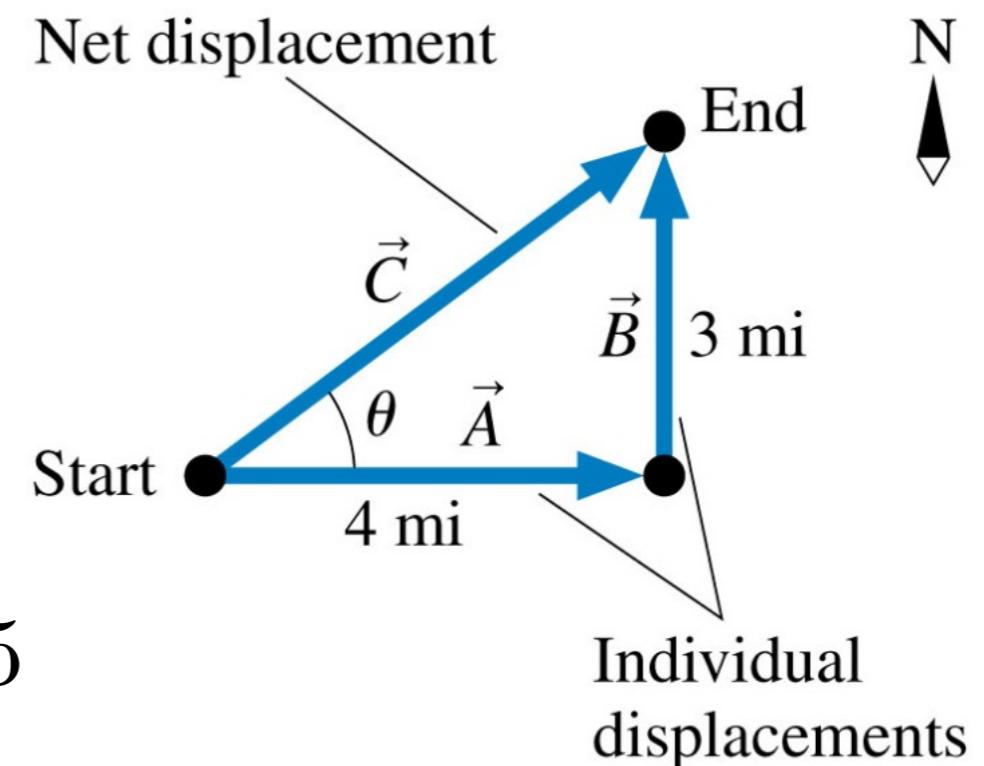


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$$\theta = \tan^{-1} \left(\frac{3}{4} \right) = 37^\circ$$

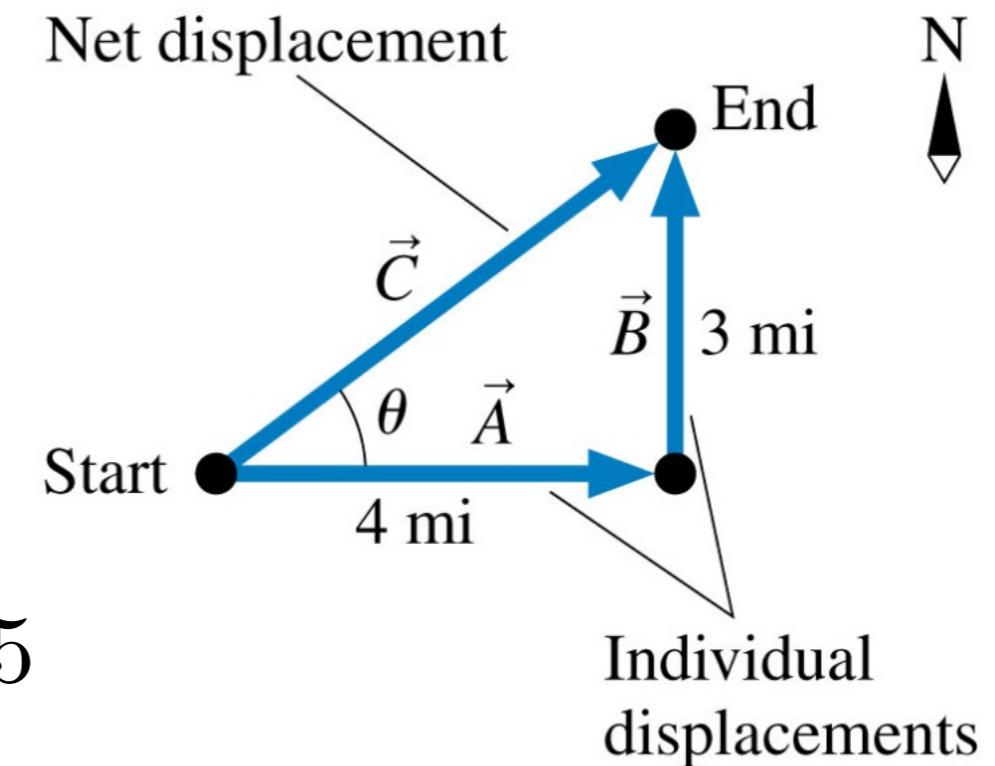


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$$\vec{C} = \vec{A} + \vec{B} = (5 \text{ mi}, 37^\circ \text{ north of east})$$

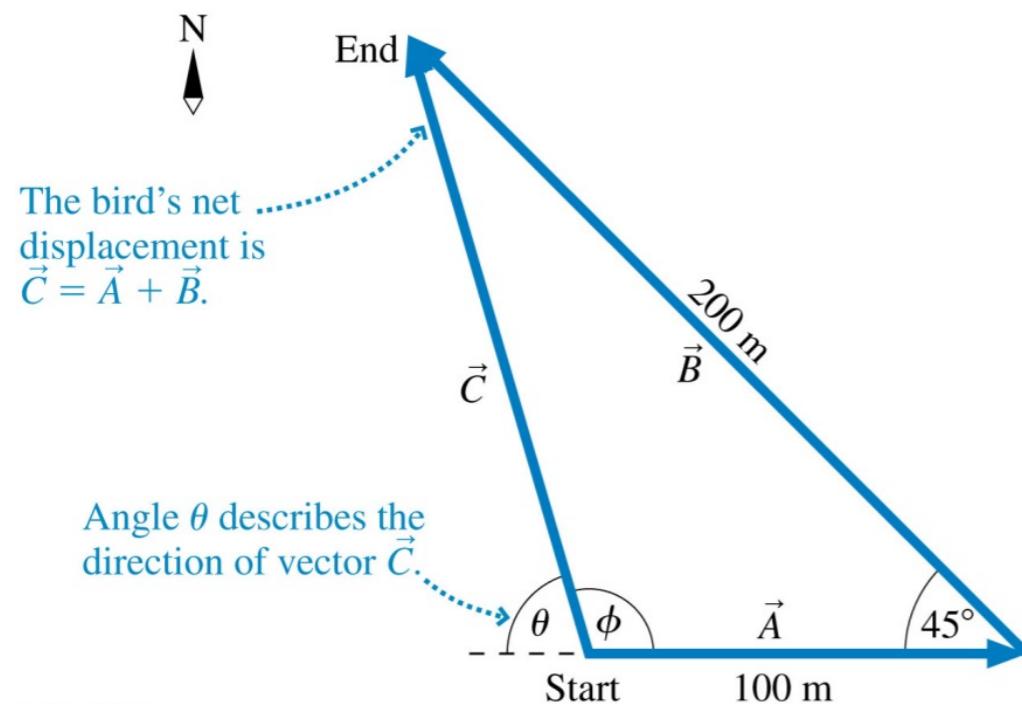
Addition of vectors(more challenging)

A bird flies 100 m due east from a tree, then 200 m northwest (45 degrees north of west). What is the bird's displacement vector?

Describe to your neighbor how you would attack this problem! Draw a picture, describe the math, but don't actually do the math.

Addition of vectors(more challenging)

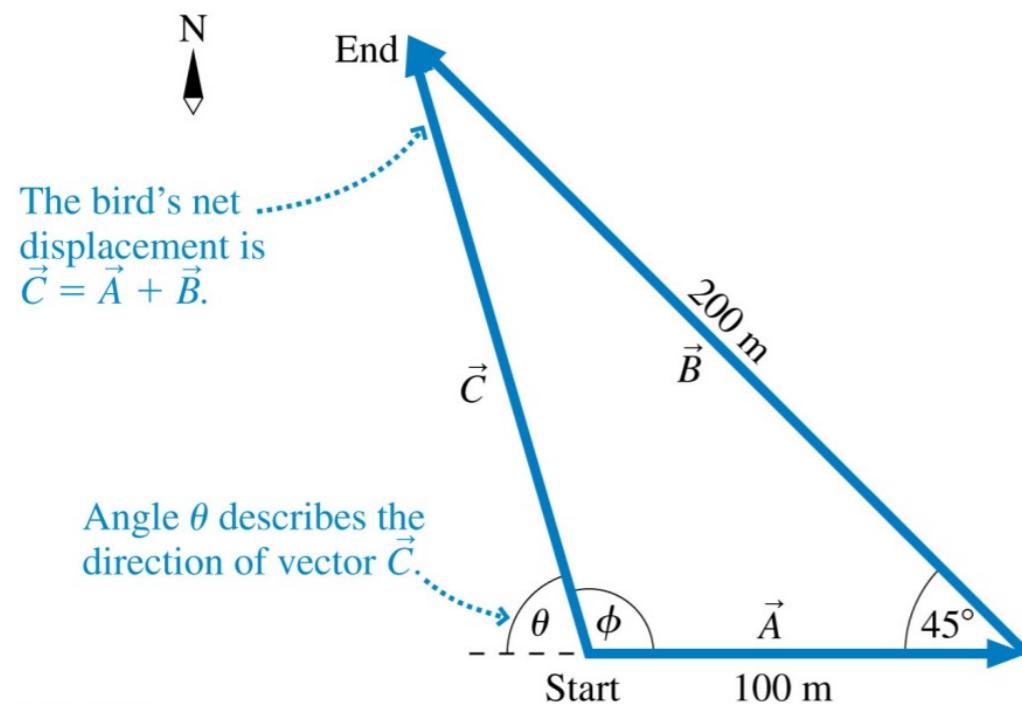
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$$C^2 = A^2 + B^2 - 2AB \cos 45^\circ$$

$$= 21,720$$

$$C = \sqrt{21,720} = 147 \text{ m}$$

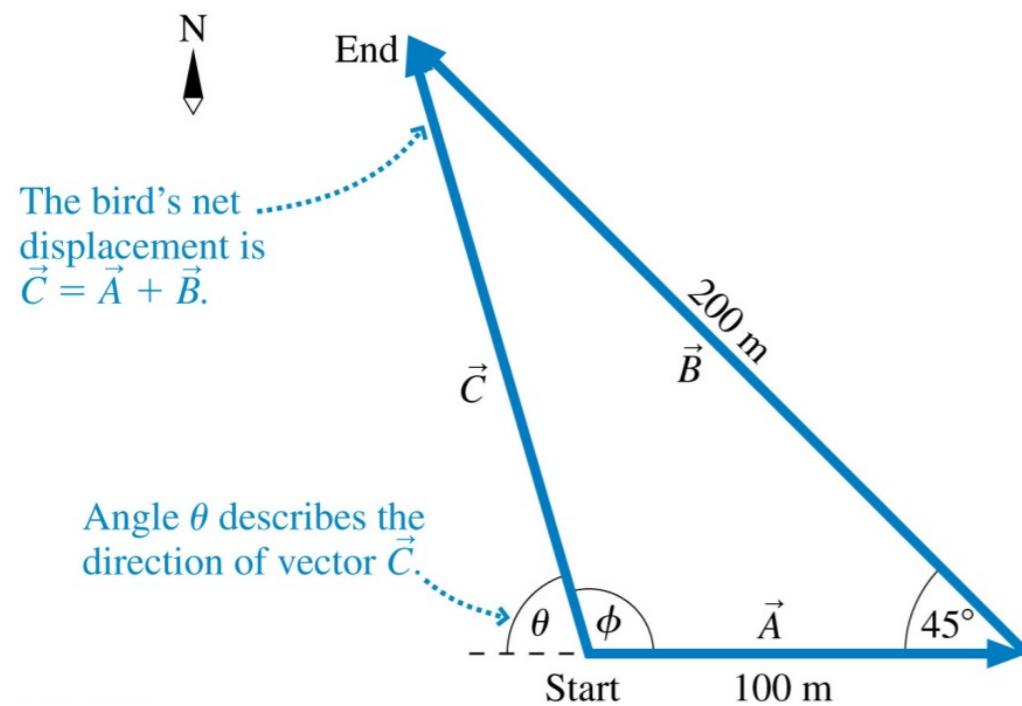
$$B^2 = A^2 + C^2 - 2AC \cos \phi$$

$$\phi = \cos^{-1} \left[\frac{A^2 + C^2 - B^2}{2AC} \right] = 106^\circ$$

Addition of vectors(more challenging)

A bird flies 100 m due east from a tree, then 200 m northwest (45 degrees north of west). What is the bird's displacement vector?

Yuck!!



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$$= 21,720$$

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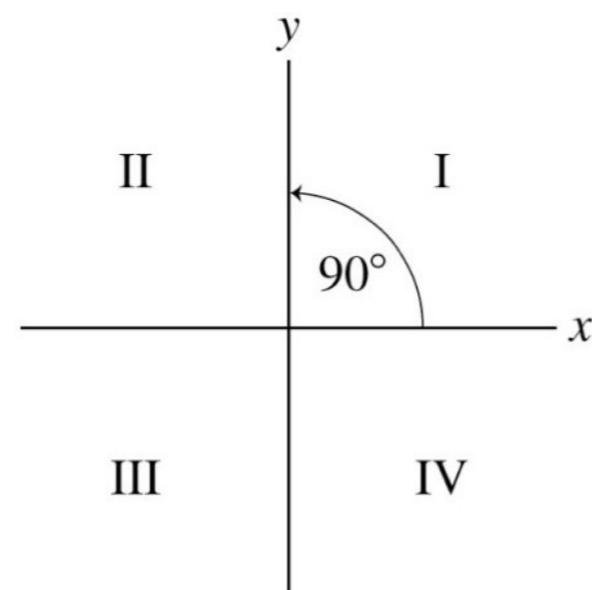
$$\phi = \cos^{-1} \left[\frac{A^2 + C^2 - B^2}{2AC} \right] = 106^\circ$$

Coordinate Systems and Vector Components

- A *coordinate system* is an artificially imposed grid that you place on a problem.
- You are free to choose:
 - Where to place the origin, and
 - How to orient the axes.



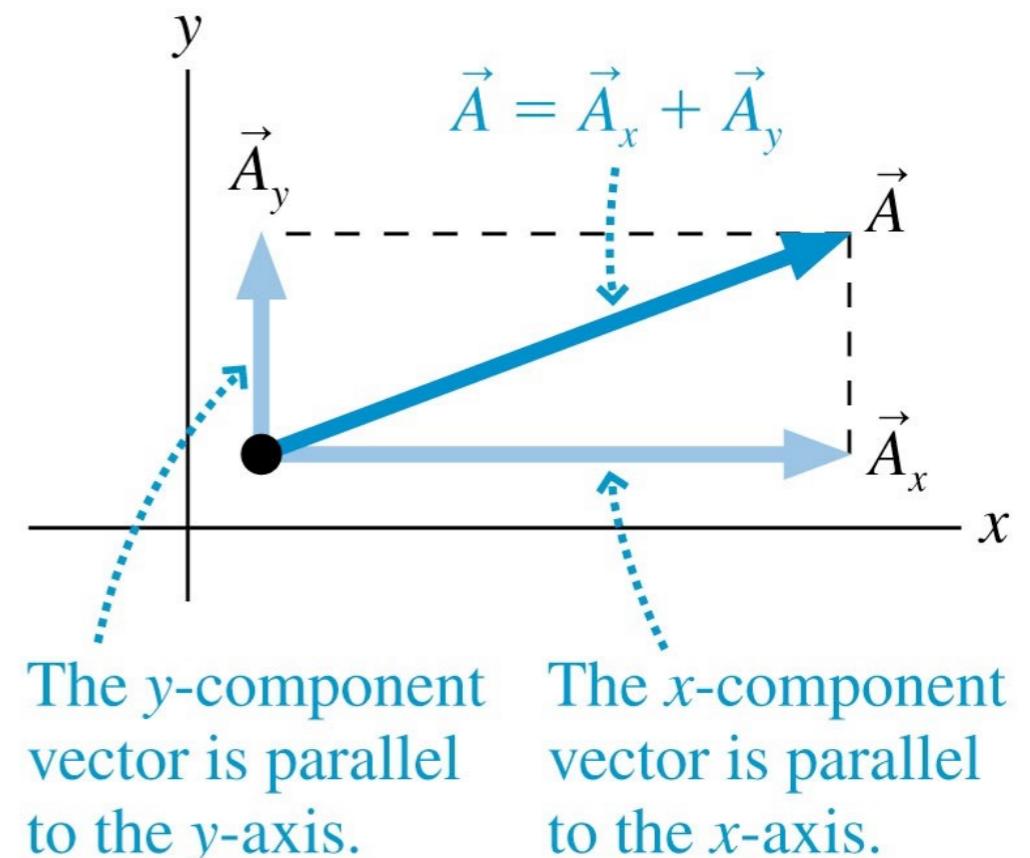
The navigator had better know which way to go, and how far, if she and the crew are to make landfall at the expected location.



Component Vectors

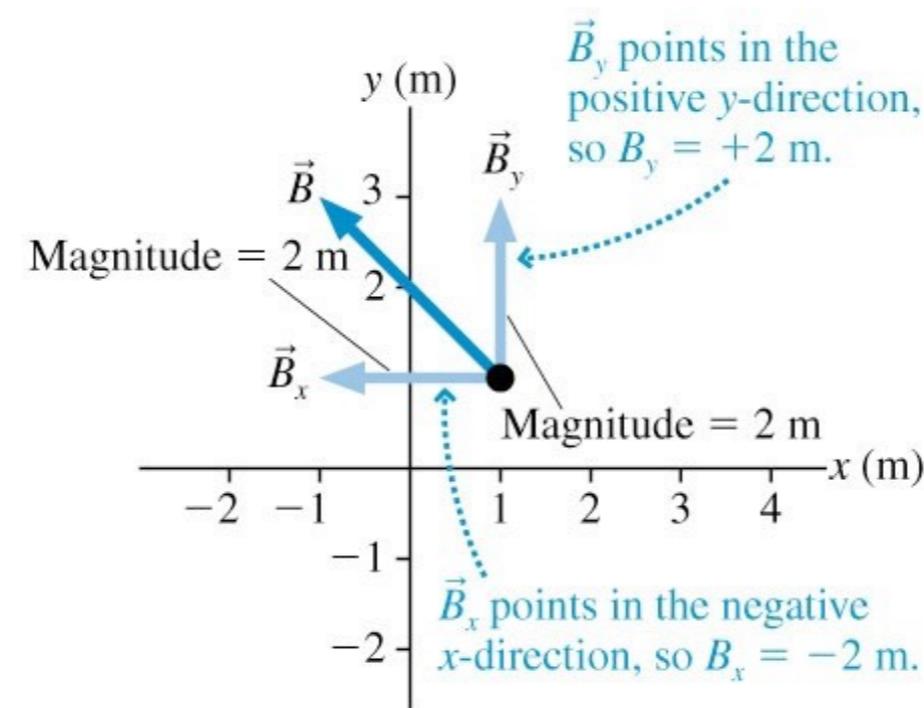
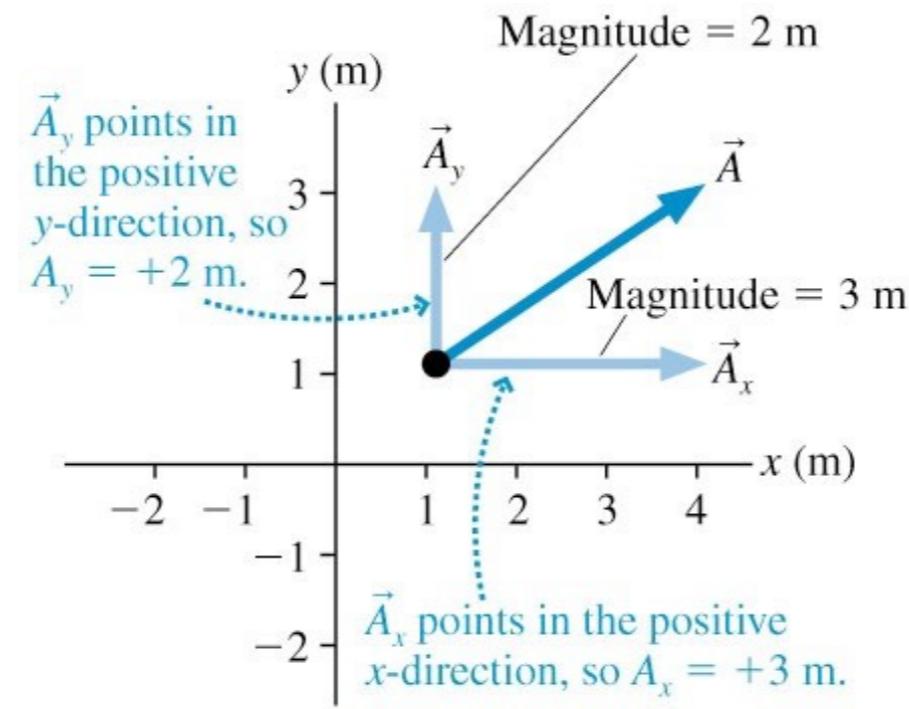
- The figure shows a vector A and an xy -coordinate system that we've chosen.
- We can define two new vectors *parallel to the axes* that we call the **component vectors** of A , such that:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



- We have broken A into two perpendicular vectors that are parallel to the coordinate axes.
- This is called the **decomposition** of A into its component vectors.

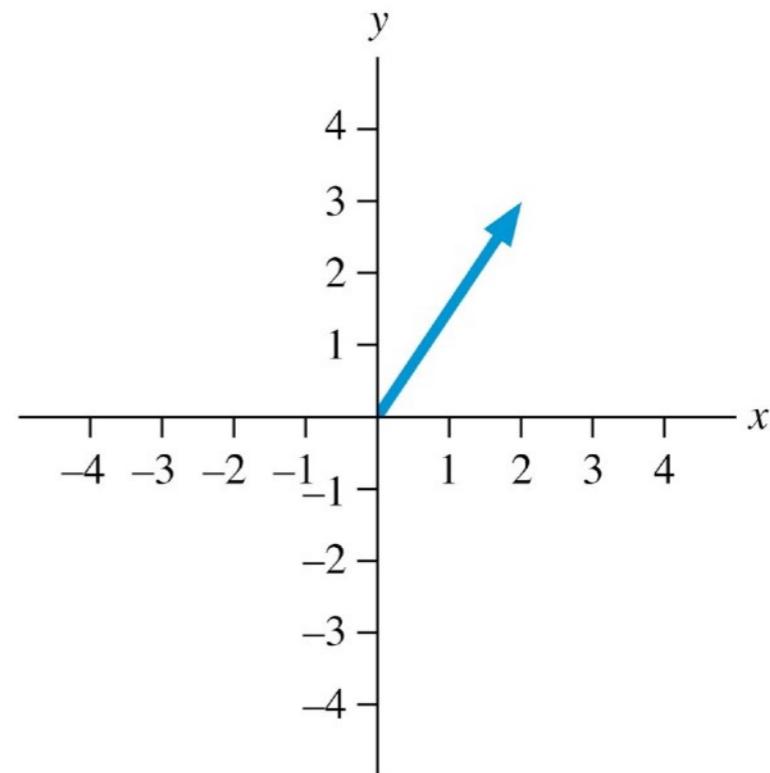
Components



Quiz

What are the x- and y- components of this vector?

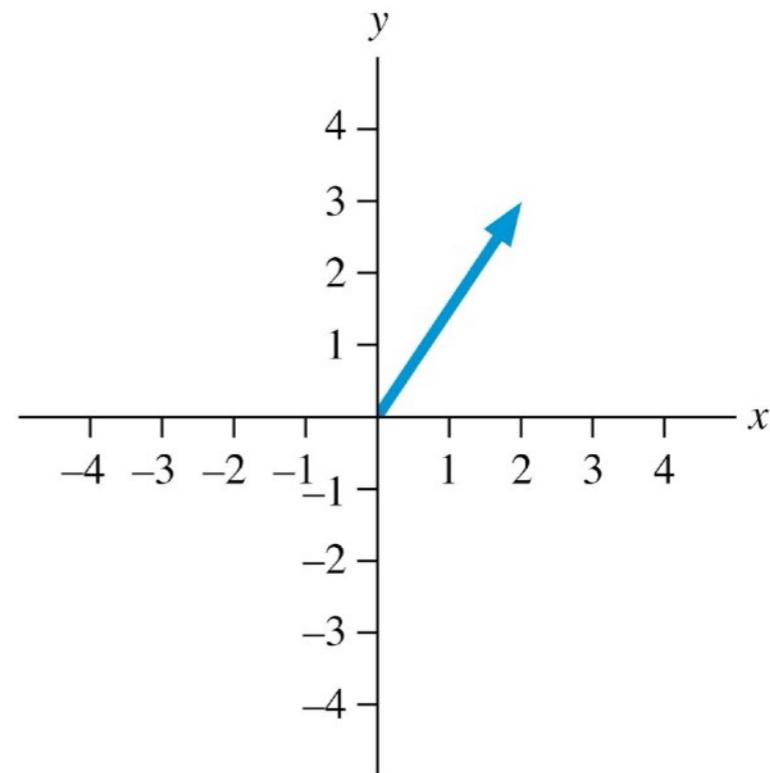
- a) (3,2)
- b) (2,3)
- c) (-3,2)
- d) (2,-3)
- e) (-3,-2)



Quiz

What are the x- and y- components of this vector?

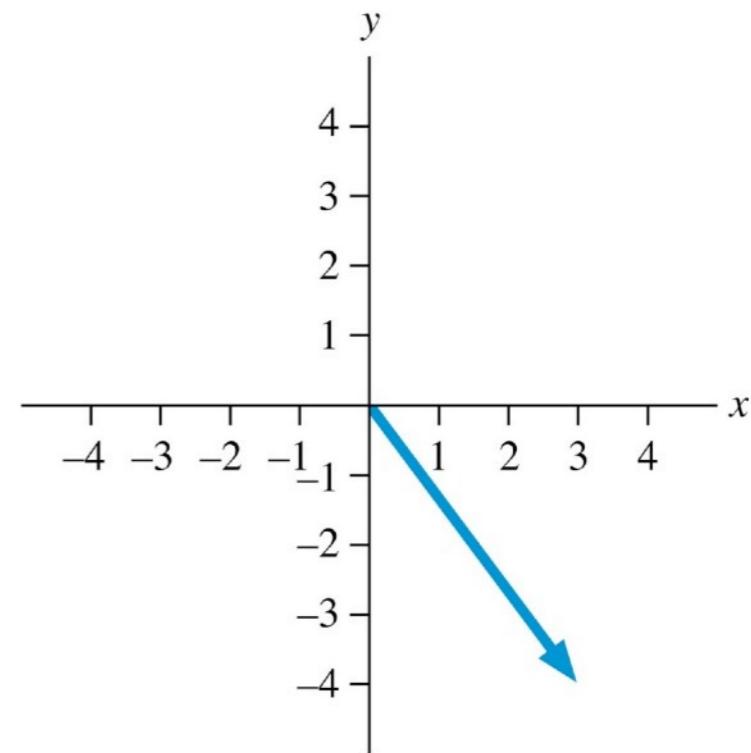
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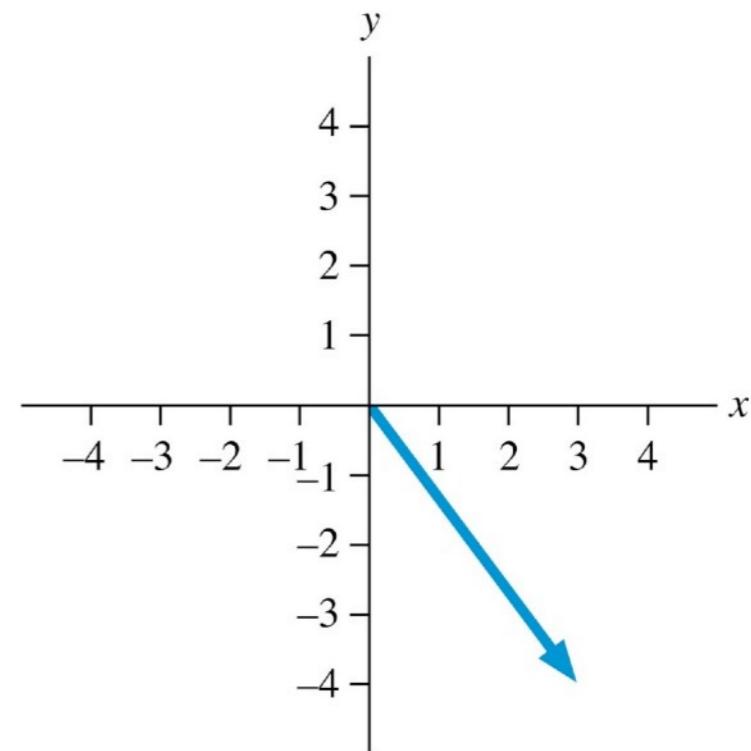
- a) (3,4)
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Quiz

What are the x- and y- components of this vector?

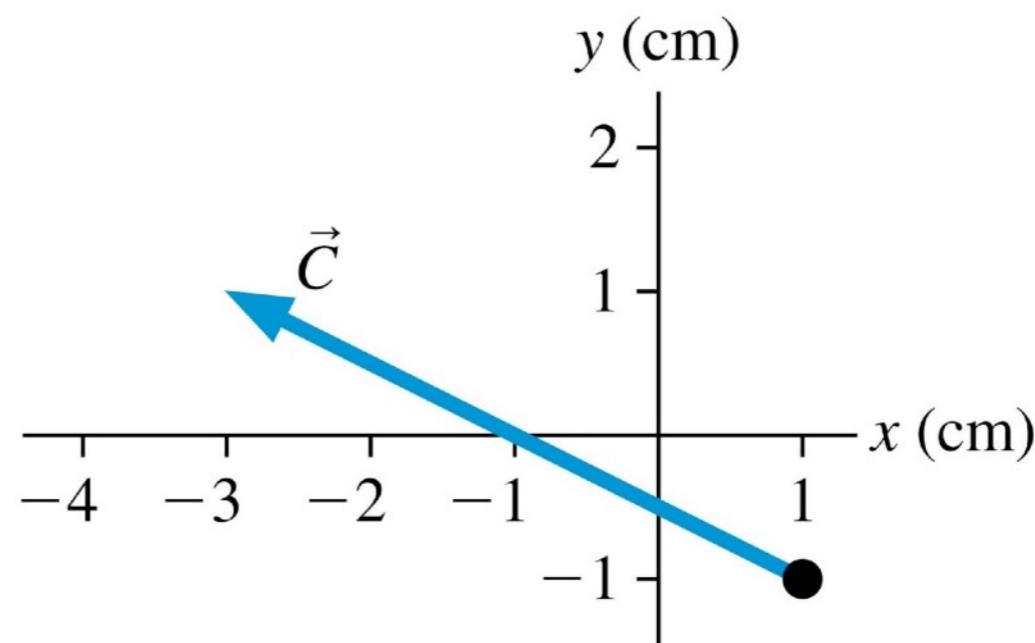
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Quiz

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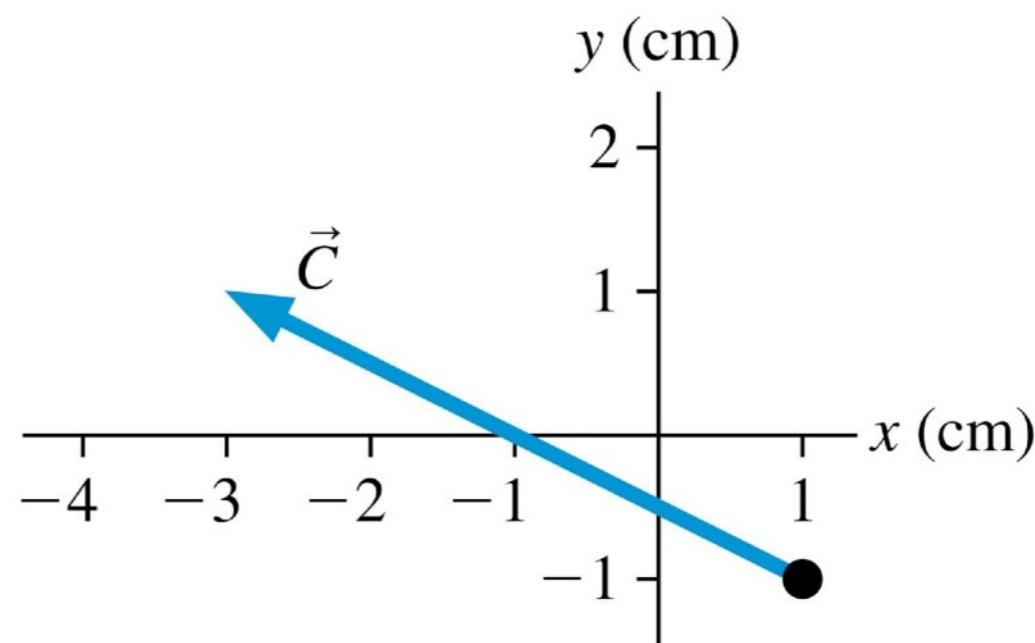
- a) (1,-3)
- b) (-3,1)
- c) (1,-1)
- d) (-4,2)
- e) (2,-4)



Quiz

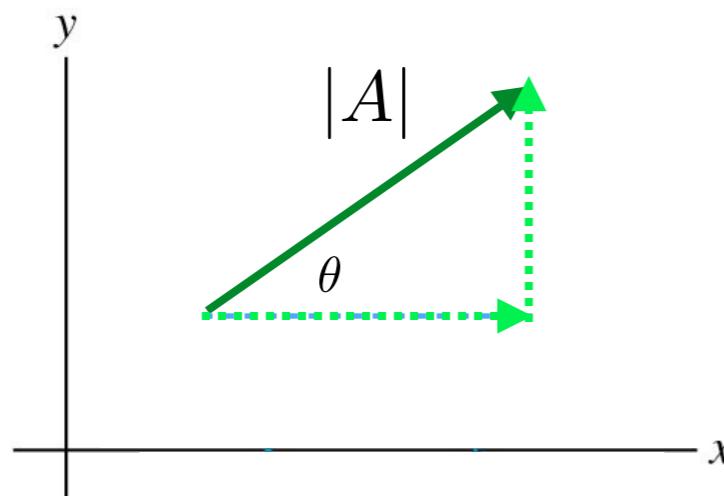
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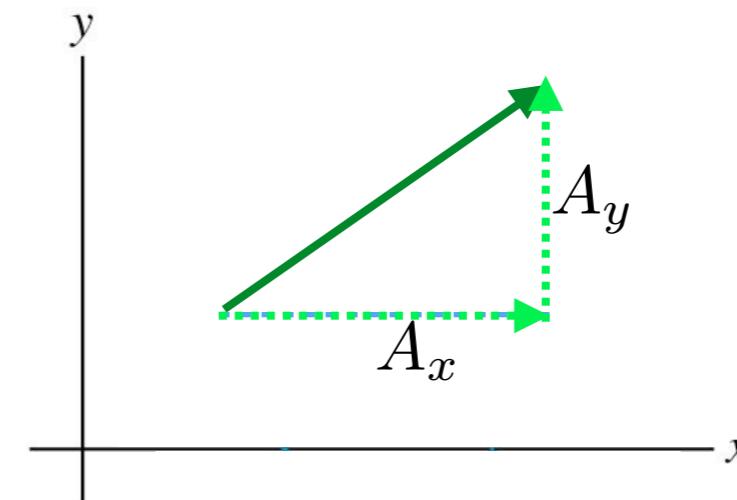


Moving between representations

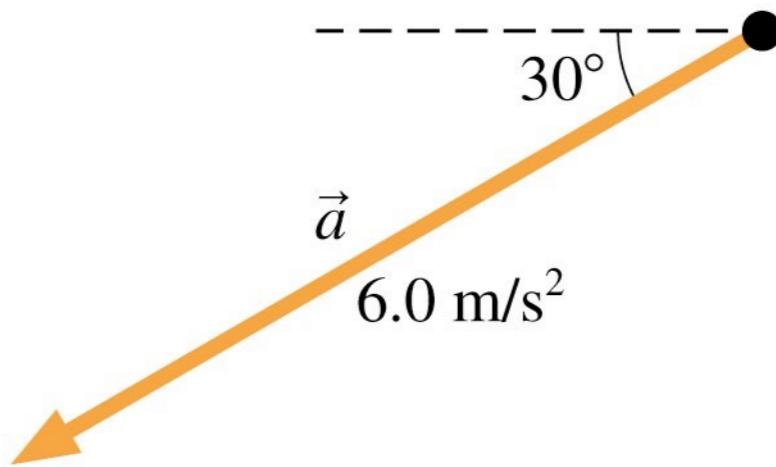
$$\vec{A} = (|A|, \theta)$$



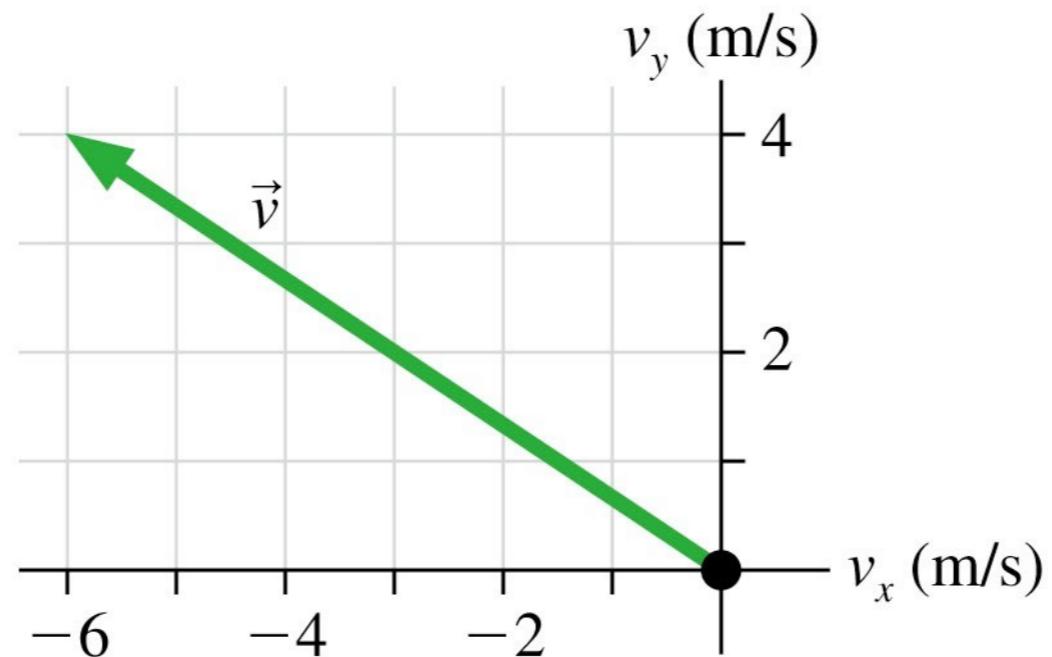
$$\vec{A} = (A_x, A_y)$$



Find the x- and y- components of the acceleration vector shown below.



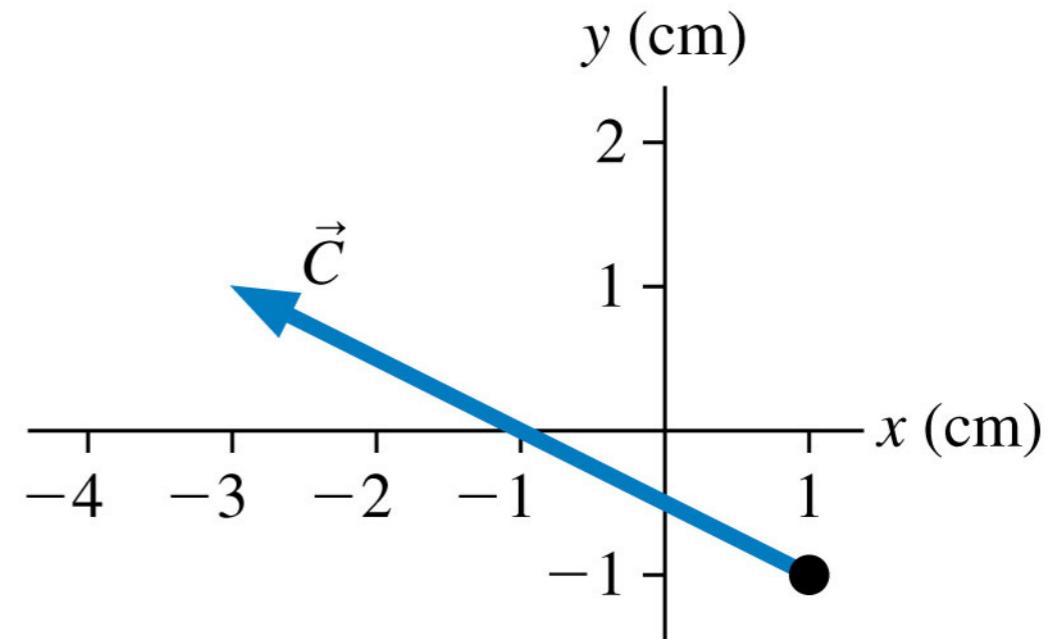
The figure below shows a car's velocity vector.
Determine the car's speed and direction of motion.



Quiz

Vector \vec{C} can be written as

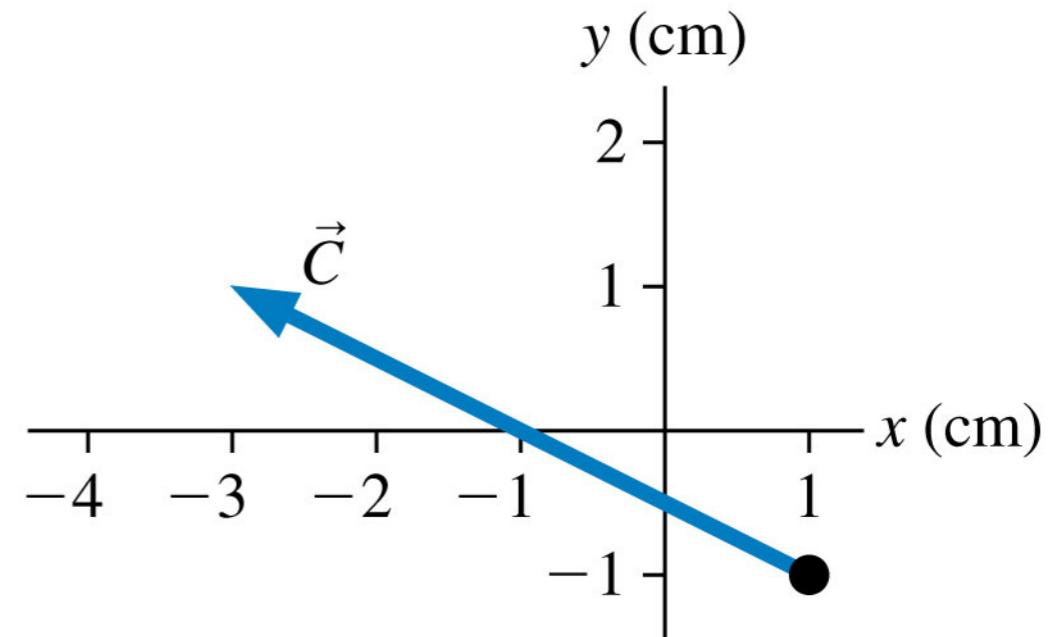
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- c) $\hat{i} - 3\hat{j}$
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- e) $\hat{i} - \hat{j}$



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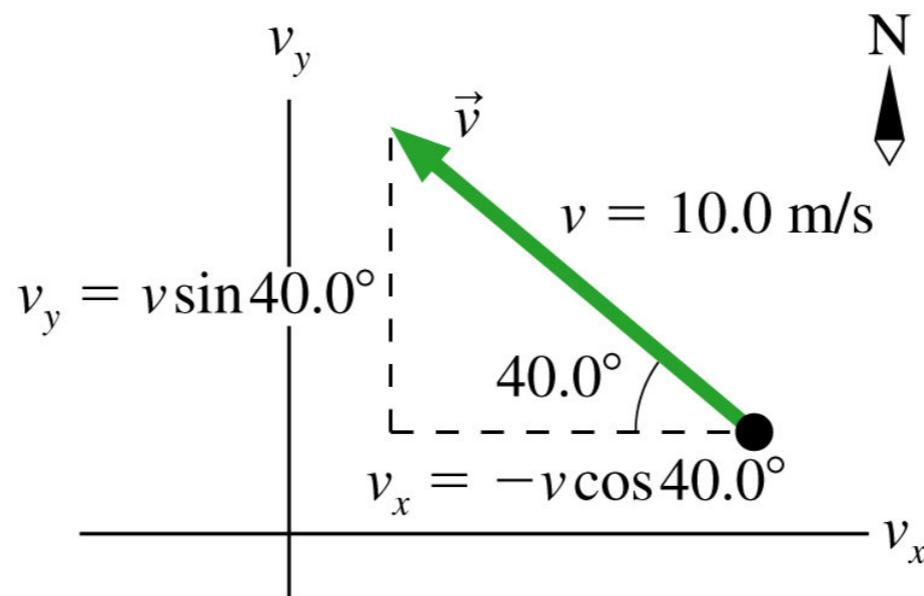


Quiz

A rabbit, escaping a fox, runs 40.0 degrees north of west at 10.0 m/s. A coordinate system is established with the positive x-axis to the east and the positive y-axis to the north. Write the rabbit's velocity using unit vectors.

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Adding vectors (the easy way)

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{R} = \vec{P} - \vec{Q}$$

$$D_x = A_x + B_x + C_x$$

$$R_x = P_x - Q_x$$

$$D_y = A_y + B_y + C_y$$

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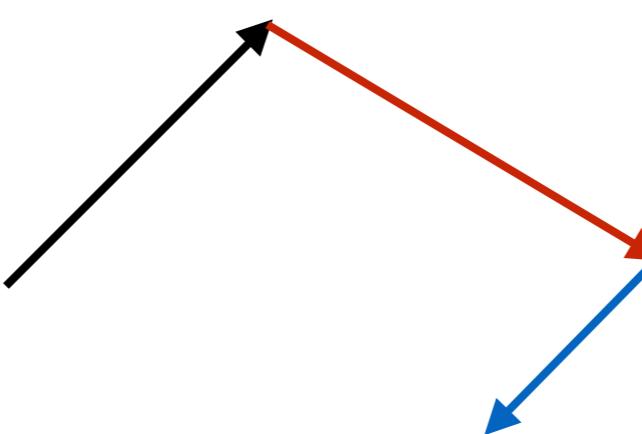
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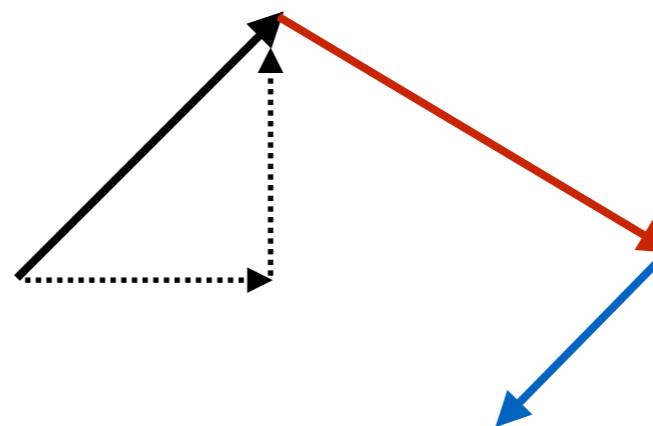
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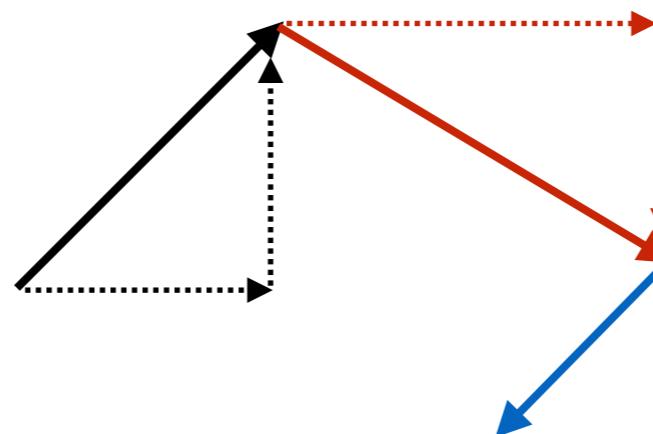
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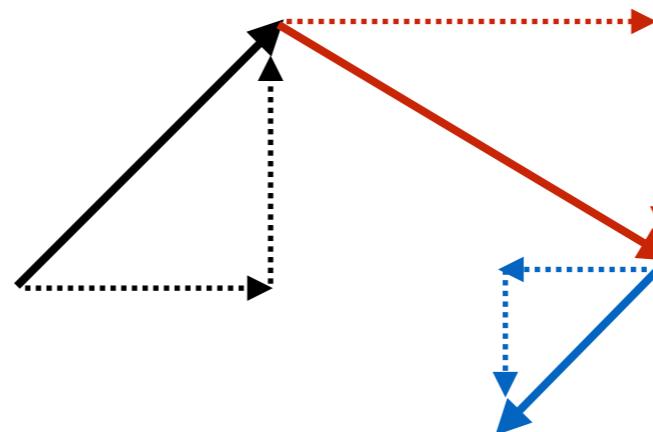
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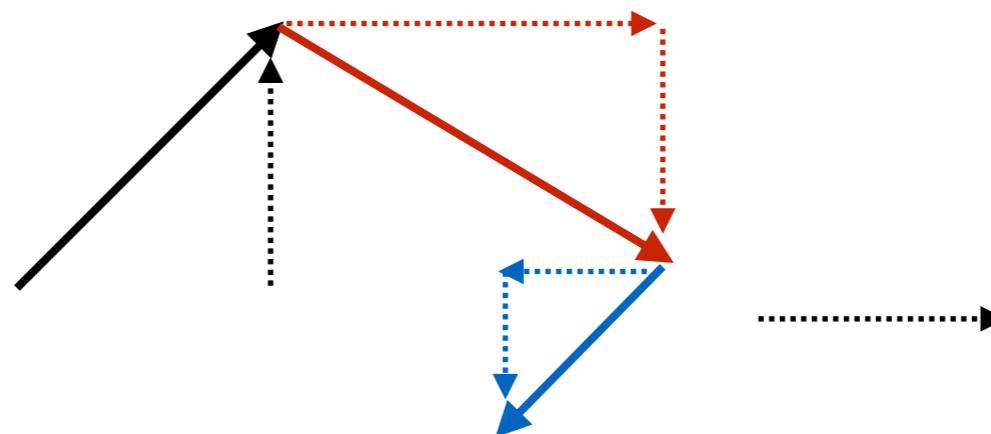
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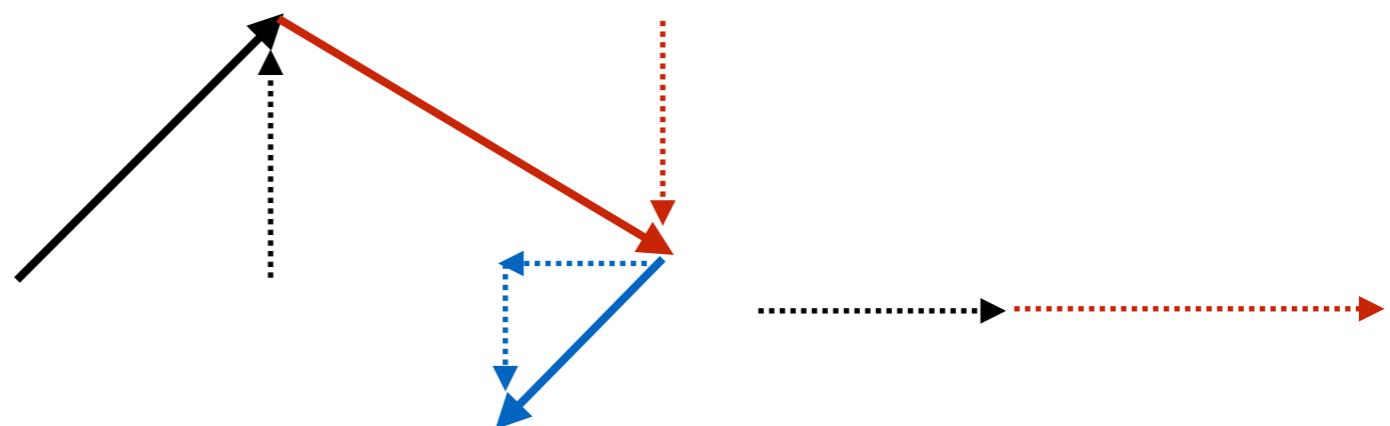
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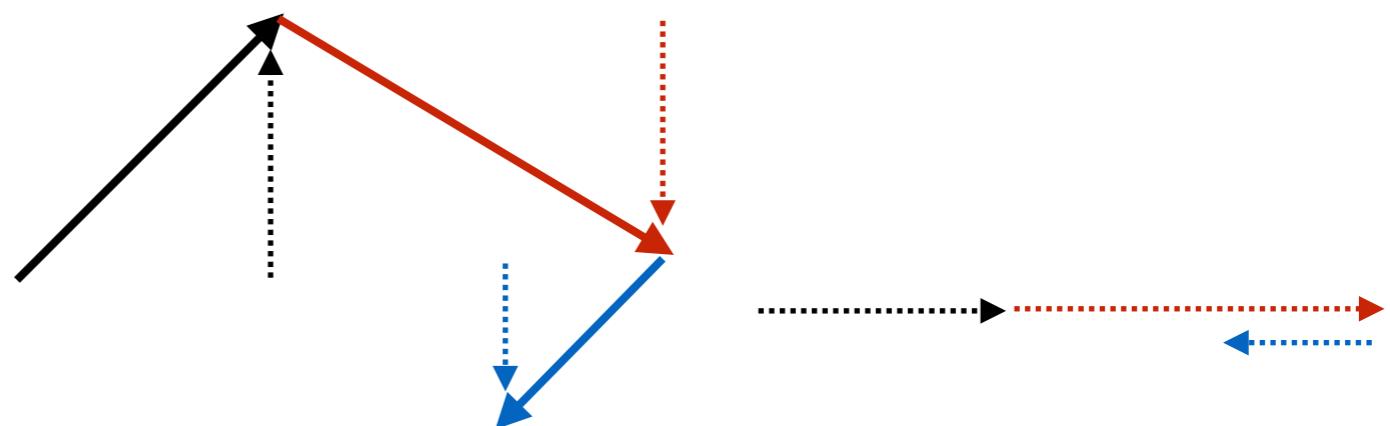
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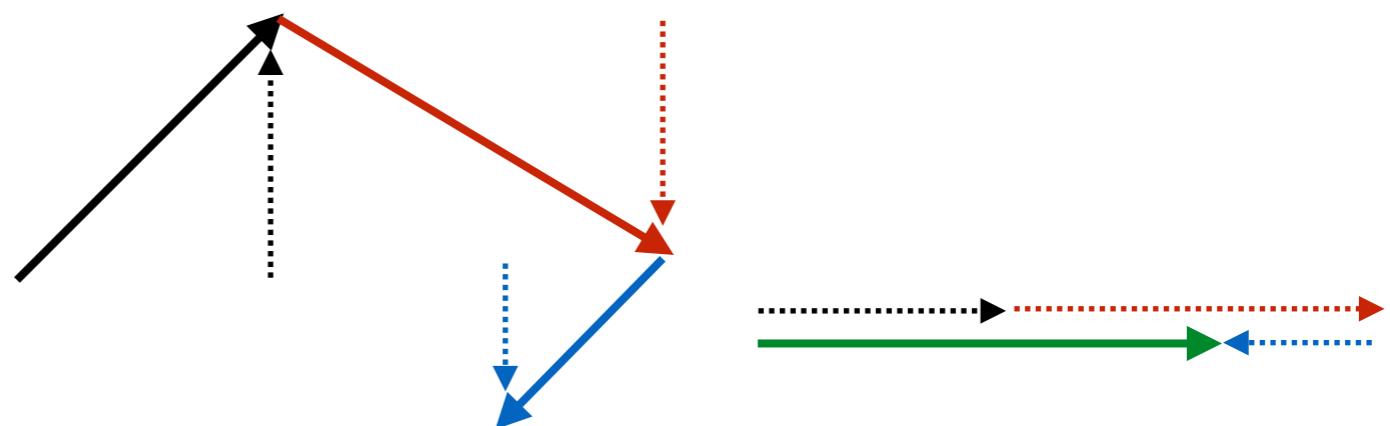
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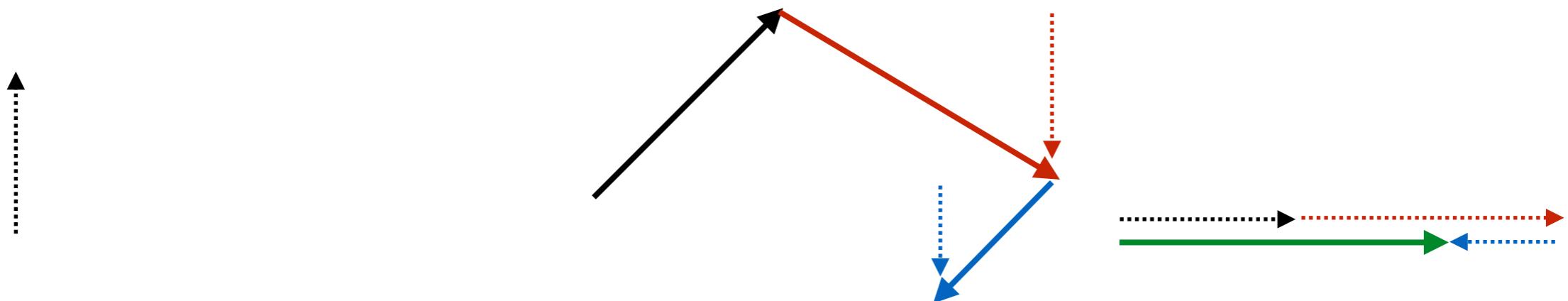
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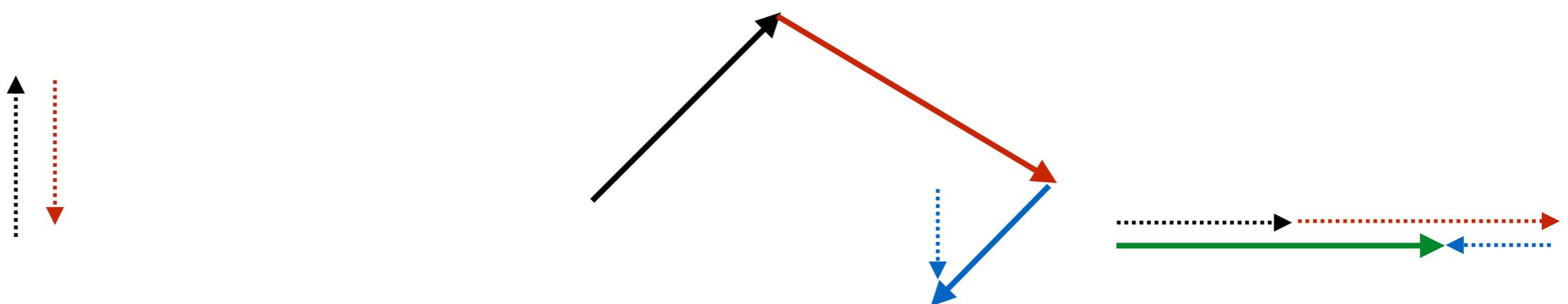
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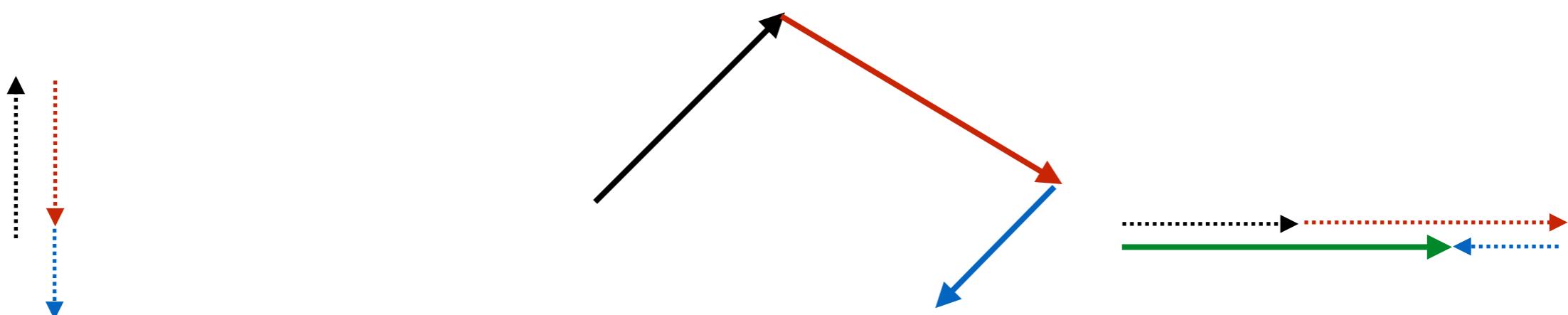
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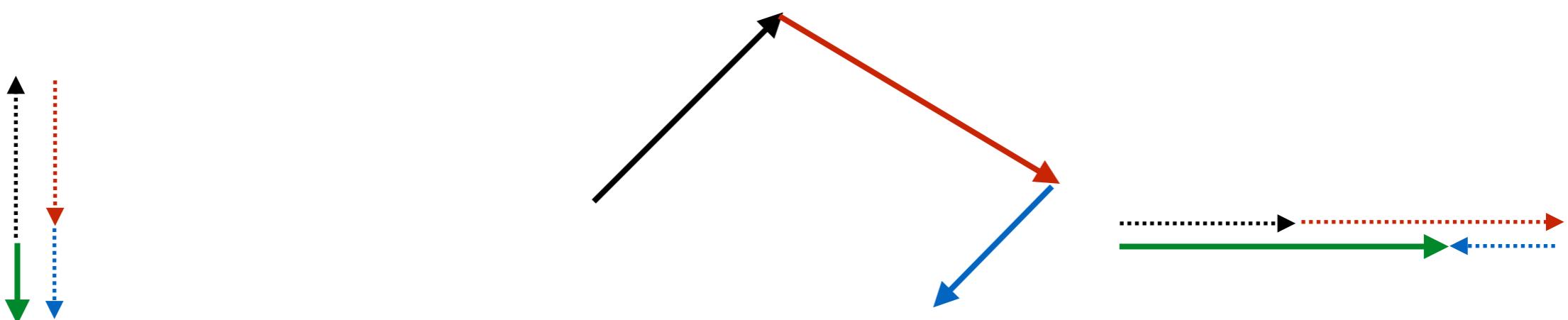
$$\vec{R} = \vec{P} - \vec{Q}$$

$$D_x = A_x + B_x + C_x$$

$$R_x = P_x - Q_x$$

$$D_y = A_y + B_y + C_y$$

$$R_y = P_y - Q_y$$



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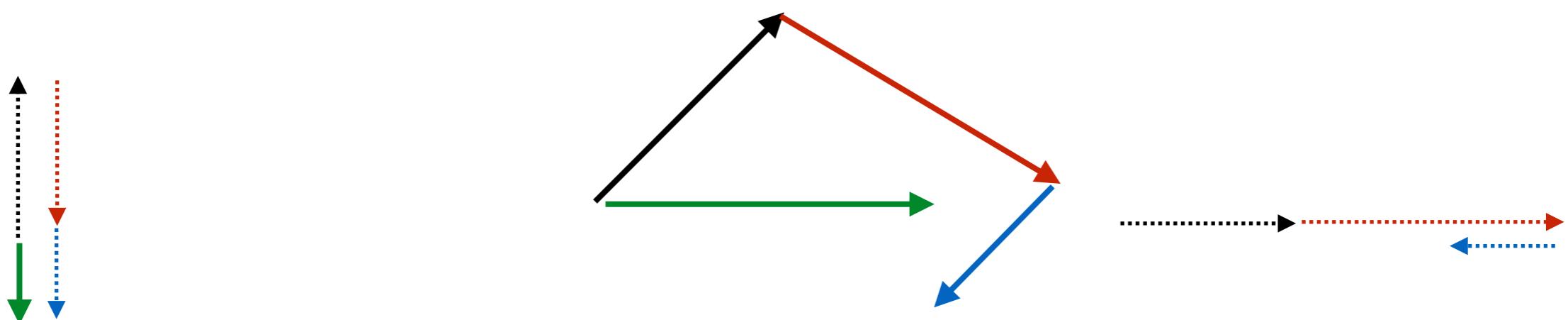
$$\vec{R} = \vec{P} - \vec{Q}$$

$$D_x = A_x + B_x + C_x$$

$$R_x = P_x - Q_x$$

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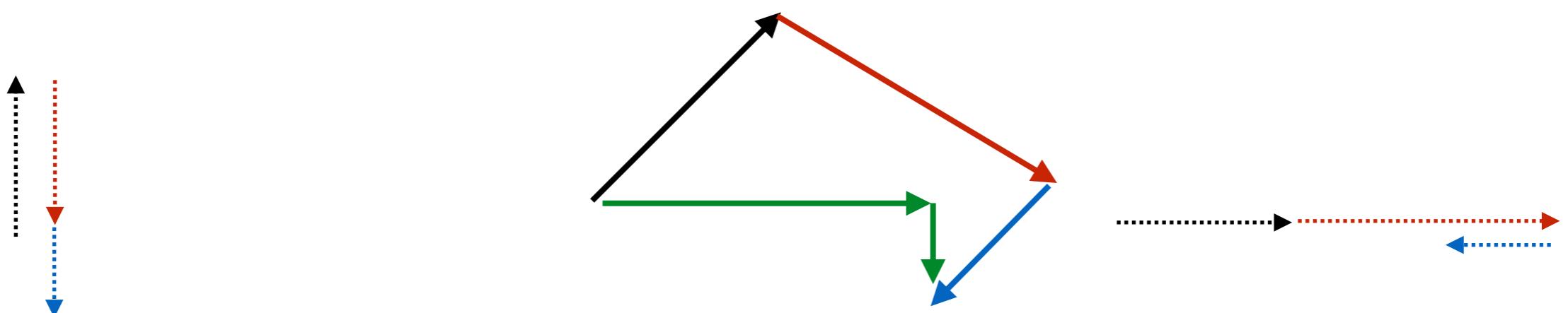
$$\vec{R} = \vec{P} - \vec{Q}$$

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Adding vectors (the easy way)

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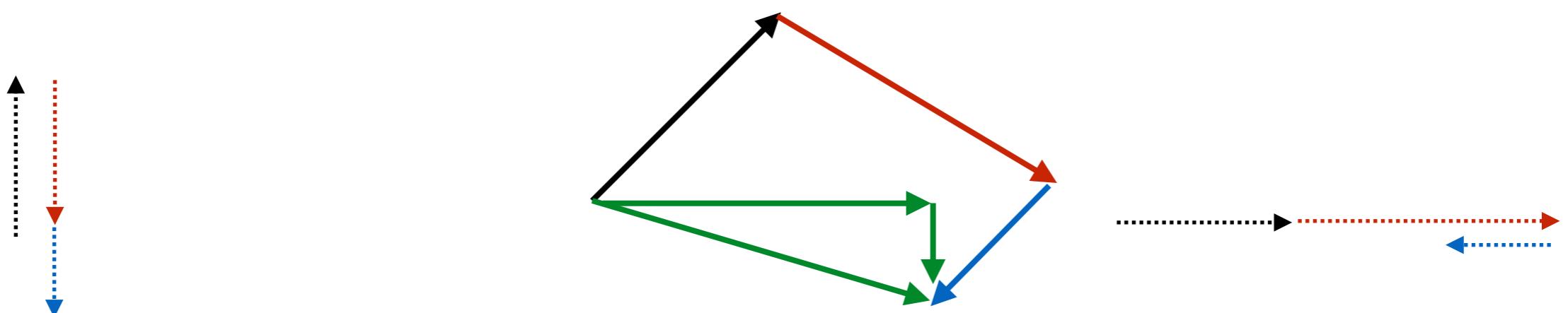
$$\vec{R} = \vec{P} - \vec{Q}$$

$$D_x = A_x + B_x + C_x$$

$$R_x = P_x - Q_x$$

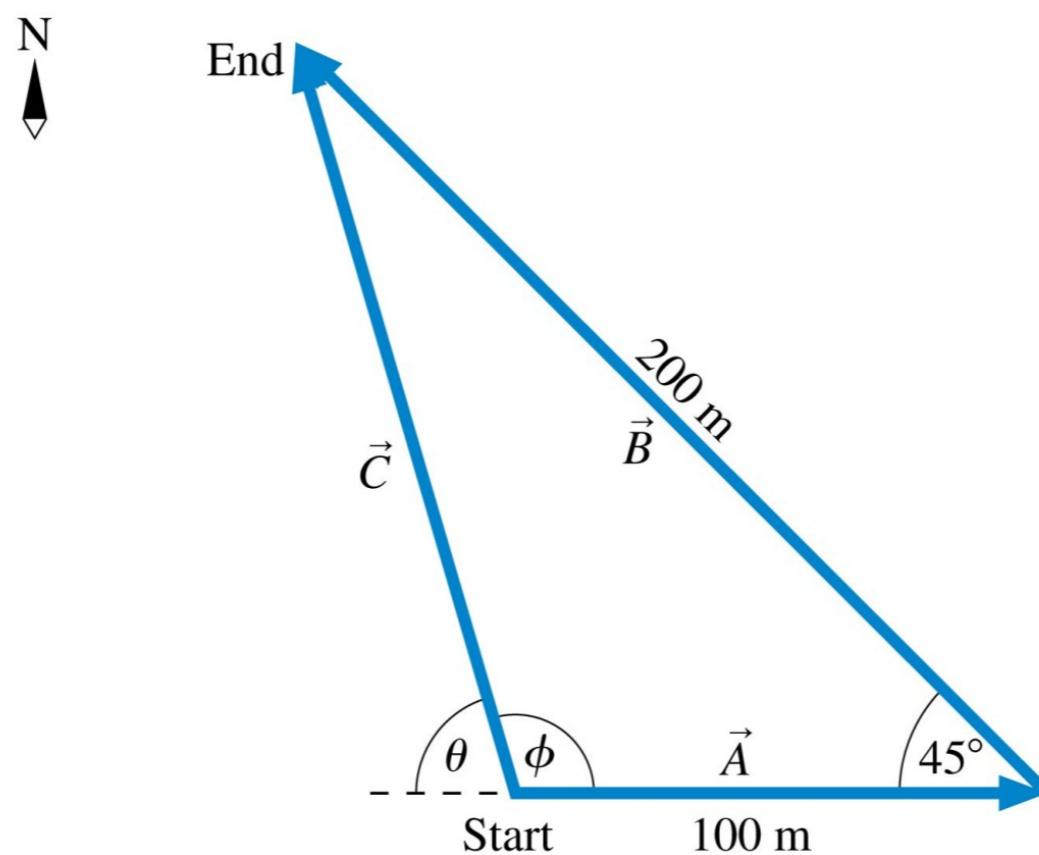
$$D_y = A_y + B_y + C_y$$

$$R_y = P_y - Q_y$$

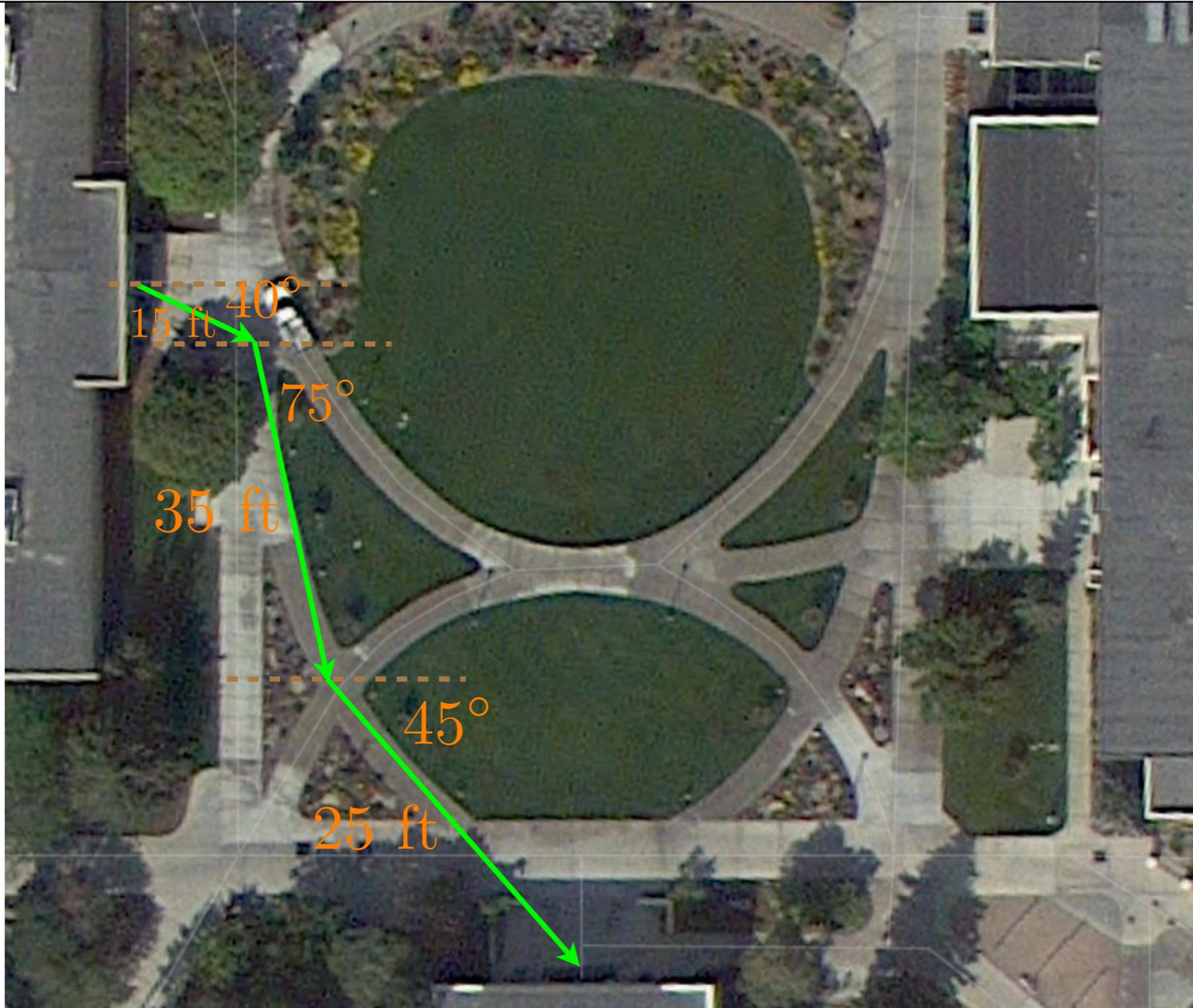


Try this one again using unit vectors

A bird flies 100 m due east from a tree, then 200 m northwest (45 degrees north of west). What is the bird's net displacement (use unit vectors this time).

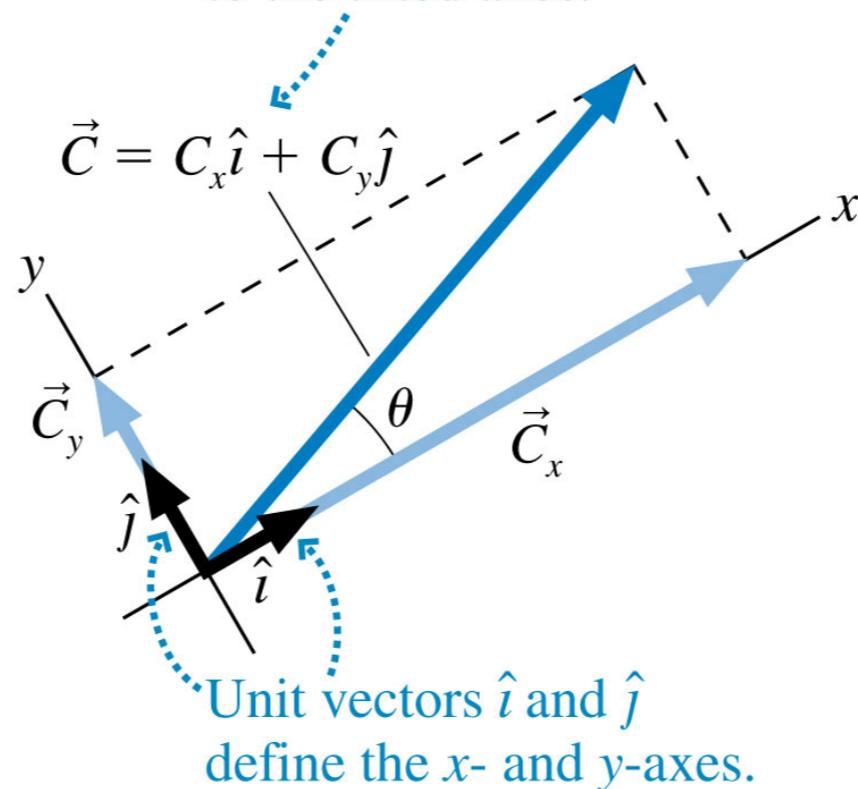


You want to measure the distance (as the crow flies) from the Romney to the Library, but can't walk in a straight line from one building to the next. You decide to walk the path shown. Can you find the distance?



Tilted Axes

The components of \vec{C} are found with respect to the tilted axes.



Problem

You apply a horizontal force of 10 N to a box on an incline. What are the components of the force perpendicular and parallel to the incline?

