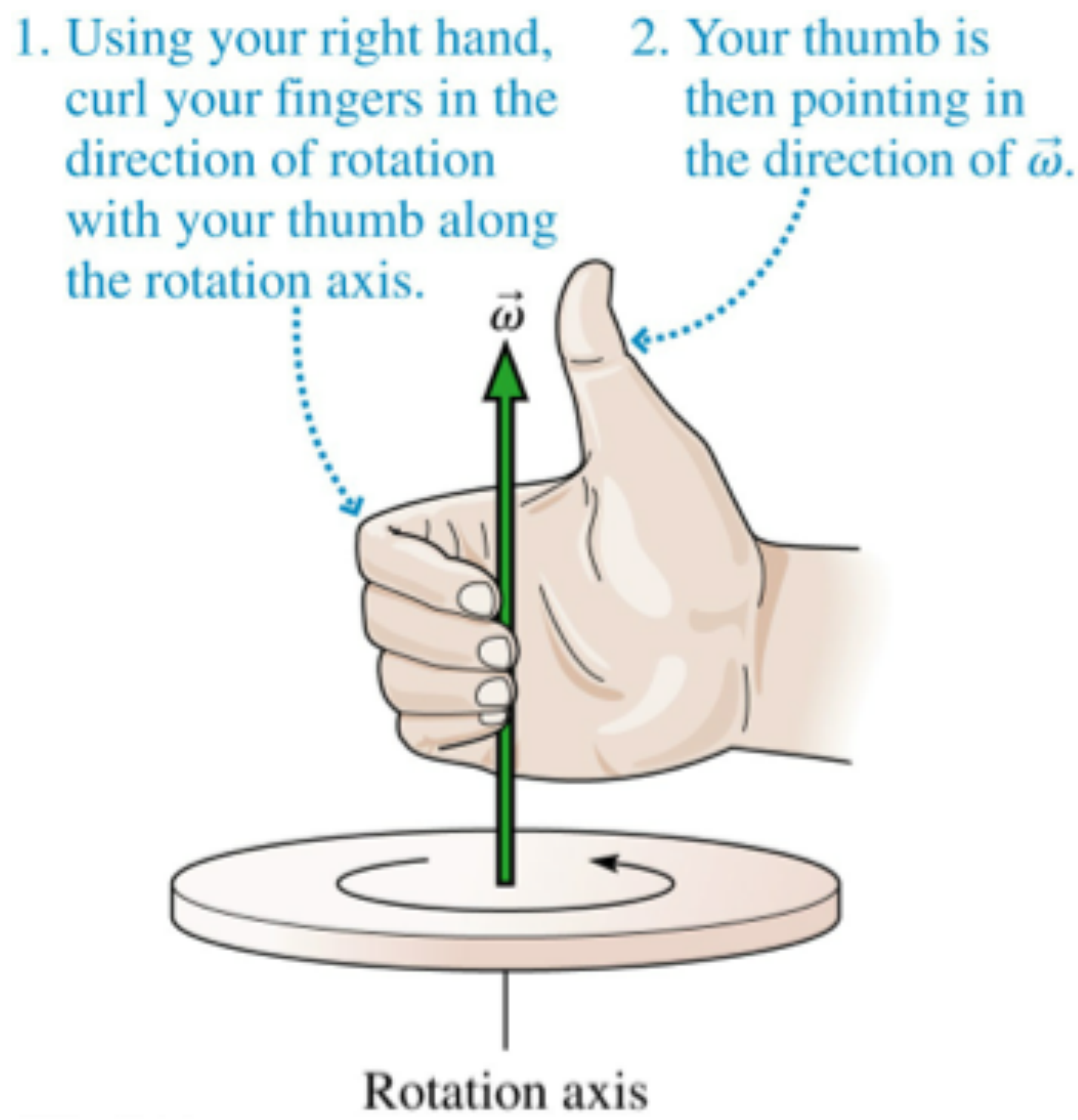
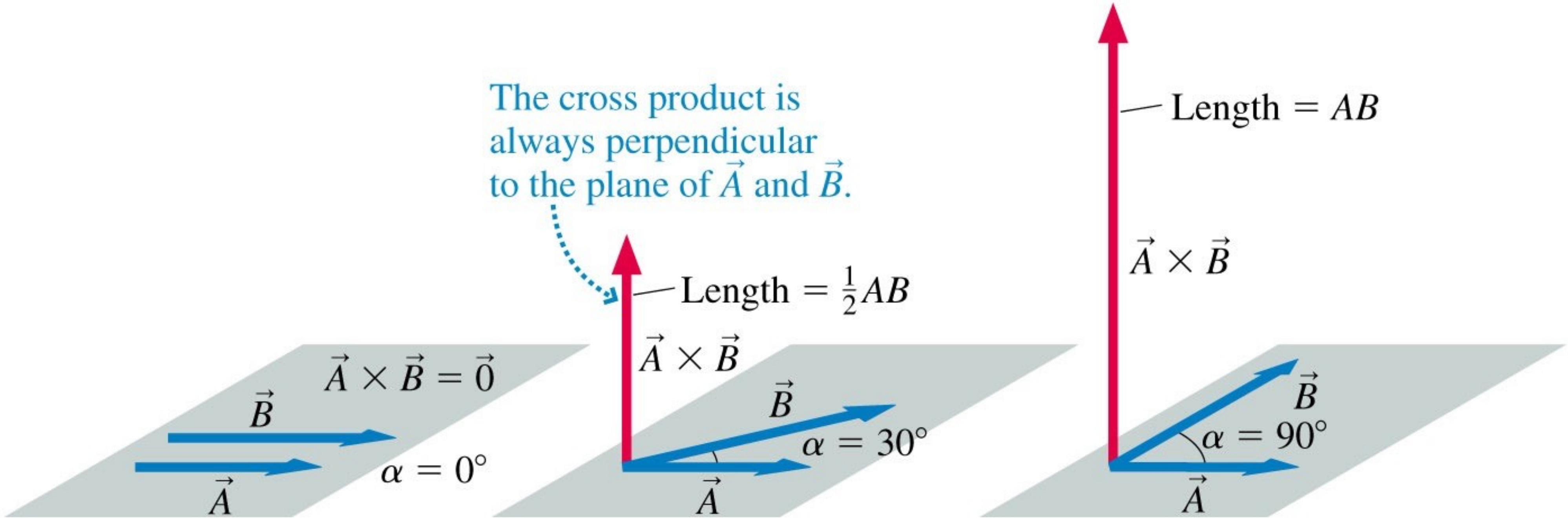
Angular Velocity Vector



The cross product of two vectors

$$\vec{A} \times \vec{B} \equiv (AB \sin\alpha, \text{in the direction given by the right-hand rule})$$



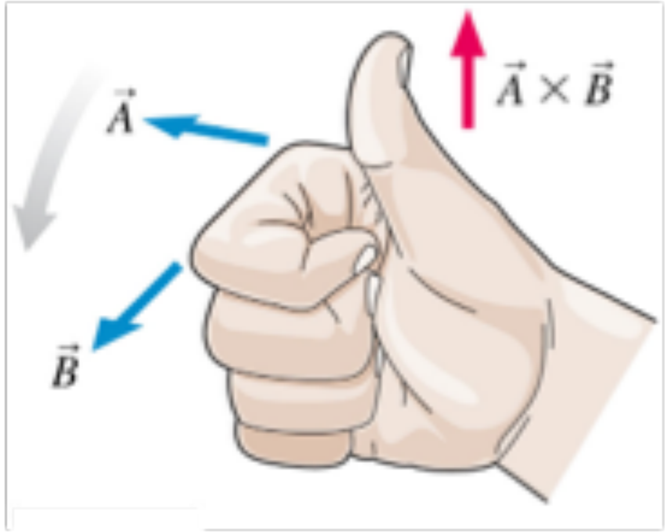
The cross product is zero when $\vec{A}$ and $\vec{B}$ are parallel.

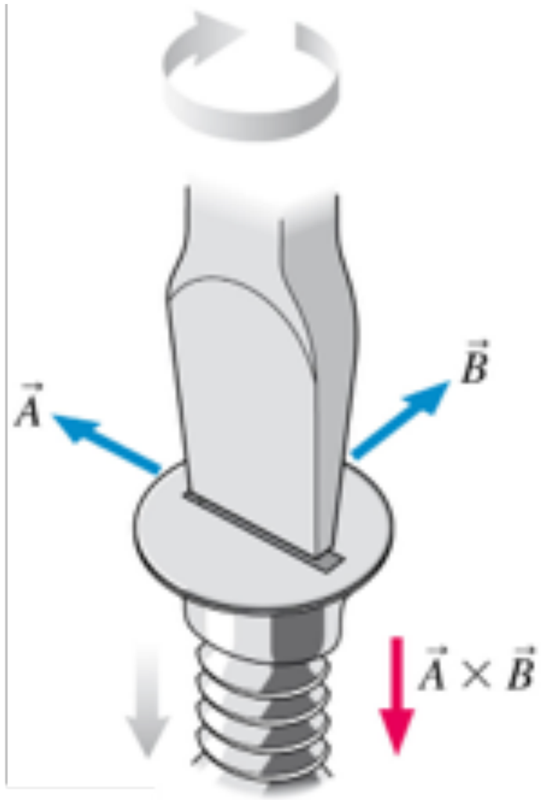
As α increases from 0° to 90°, the length of $\vec{A} \times \vec{B}$ increases.

The cross product is maximum when $\vec{A}$ and $\vec{B}$ are perpendicular.

Right Hand Rule

Several ways to determine the direction of a cross product





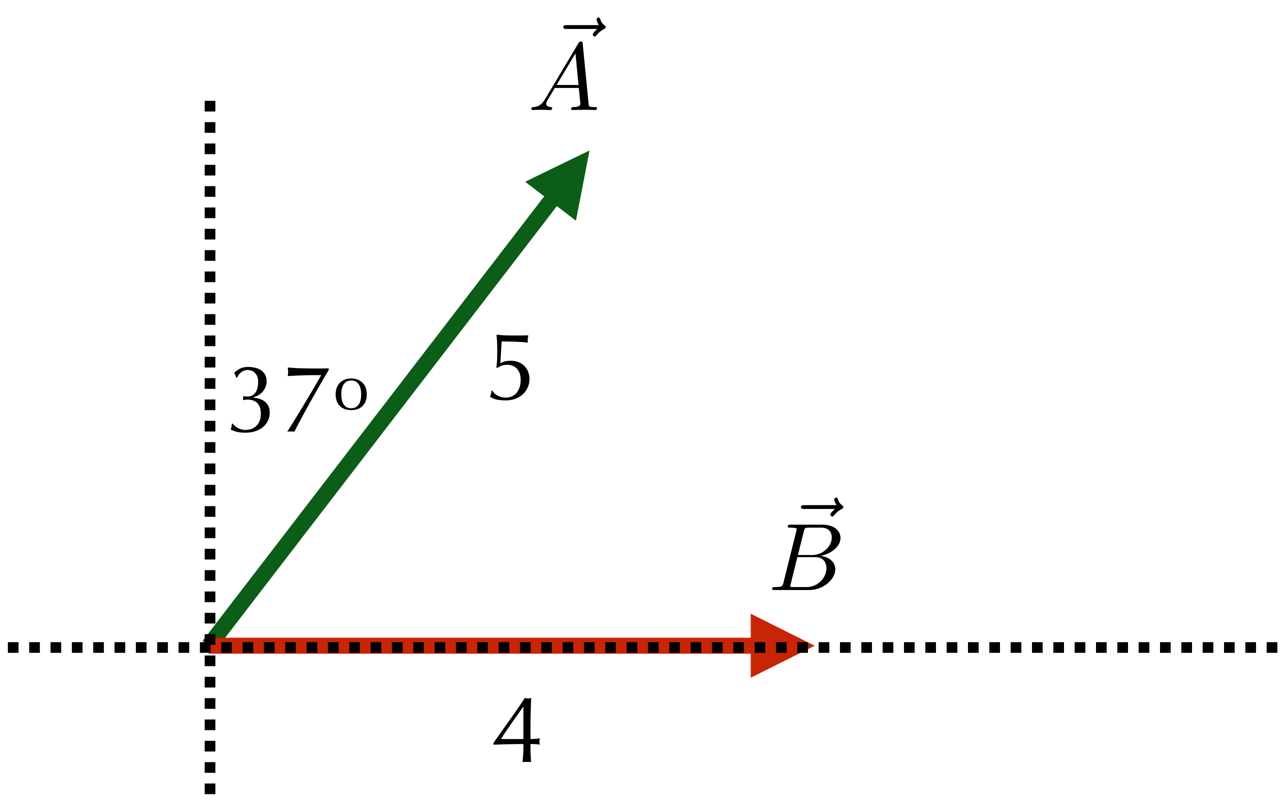
$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Calculating the cross product

Question #21

What is the magnitude of the cross product?

a) 16
b) 12
c) 20
d) 15



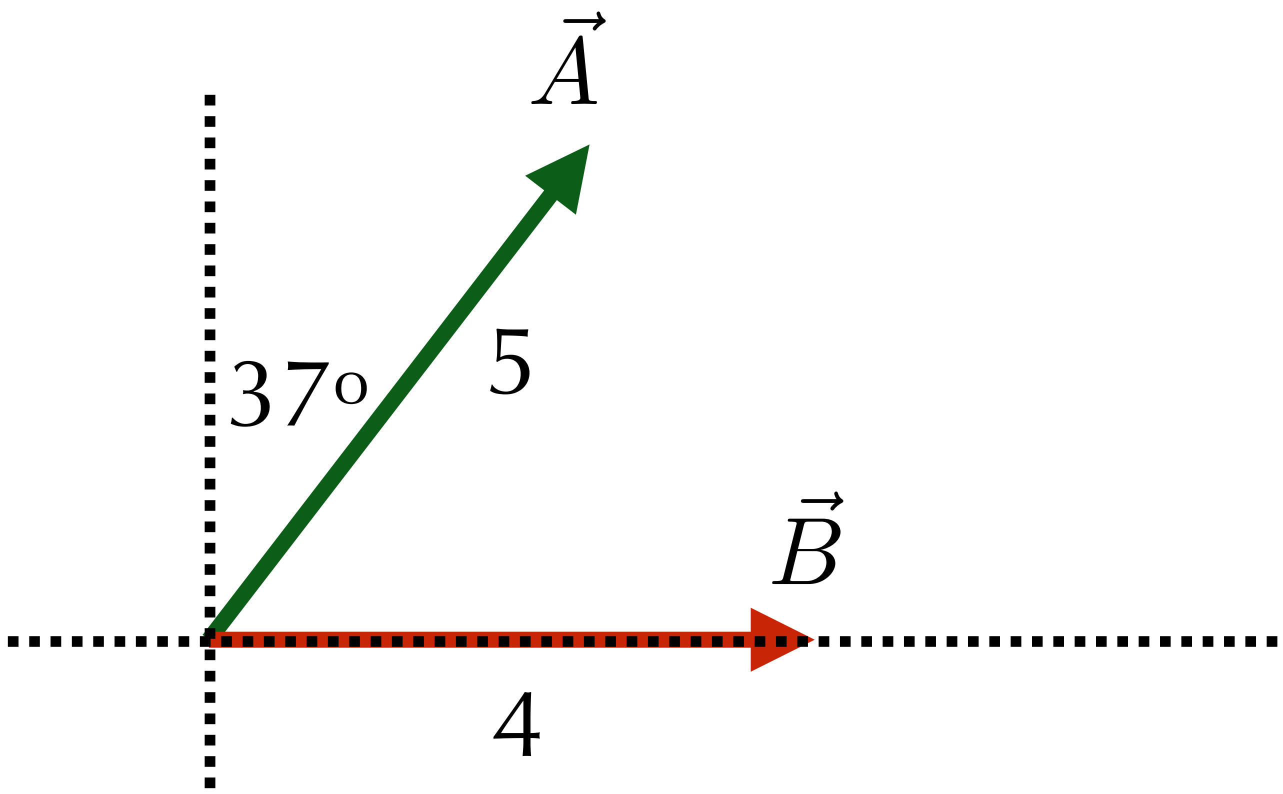
Calculating the cross product

Question #22

What is the direction of the cross product?

$$\vec{A} \times \vec{B}$$

a) Out of the screen
b) To the right
c) Into the screen
d) To the left



What if the vectors were given to you in component form?

$$\vec{A} \times \vec{B}$$

$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

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$$(A_y B_z - A_z B_y)\hat{i}$$

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$$\vec{A} \times \vec{B}$$

$$\begin{vmatrix} A_x & A_y & \text{[redacted]} \\ B_x & B_y & \text{[redacted]} \end{vmatrix}$$

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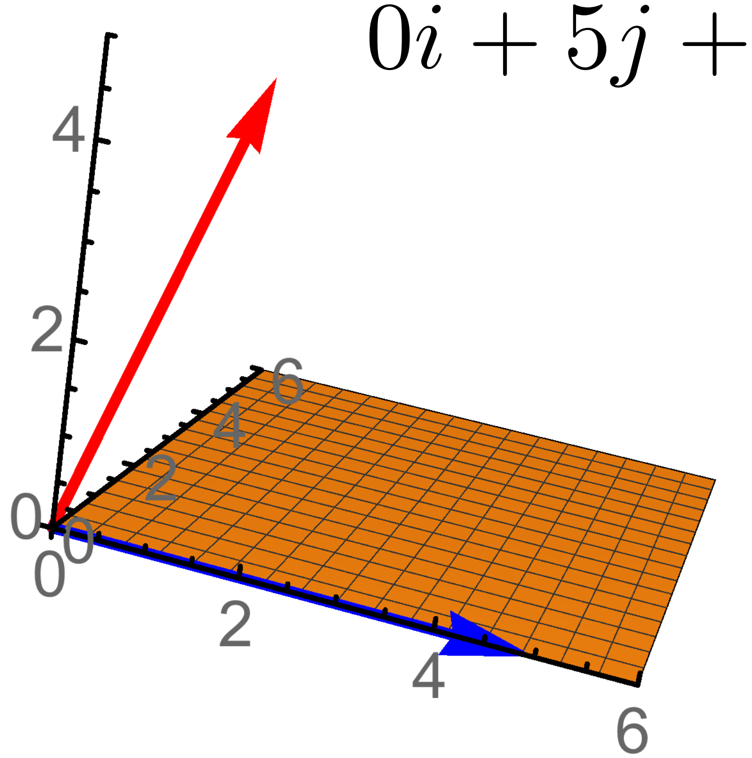
$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$(A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

Question #23

What is the cross product of the blue vector with the red vector?

$0\hat{i} + 5\hat{j} + 4\hat{k}$



$5\hat{i} + 0\hat{j} + 0\hat{k}$

b) $20\hat{i} + 0\hat{j} + 25\hat{k}$

c) $0\hat{i} + 20\hat{j} - 25\hat{k}$

d) $0\hat{i} + 20\hat{j} + 25\hat{k}$

e) $0\hat{i} - 20\hat{j} + 25\hat{k}$

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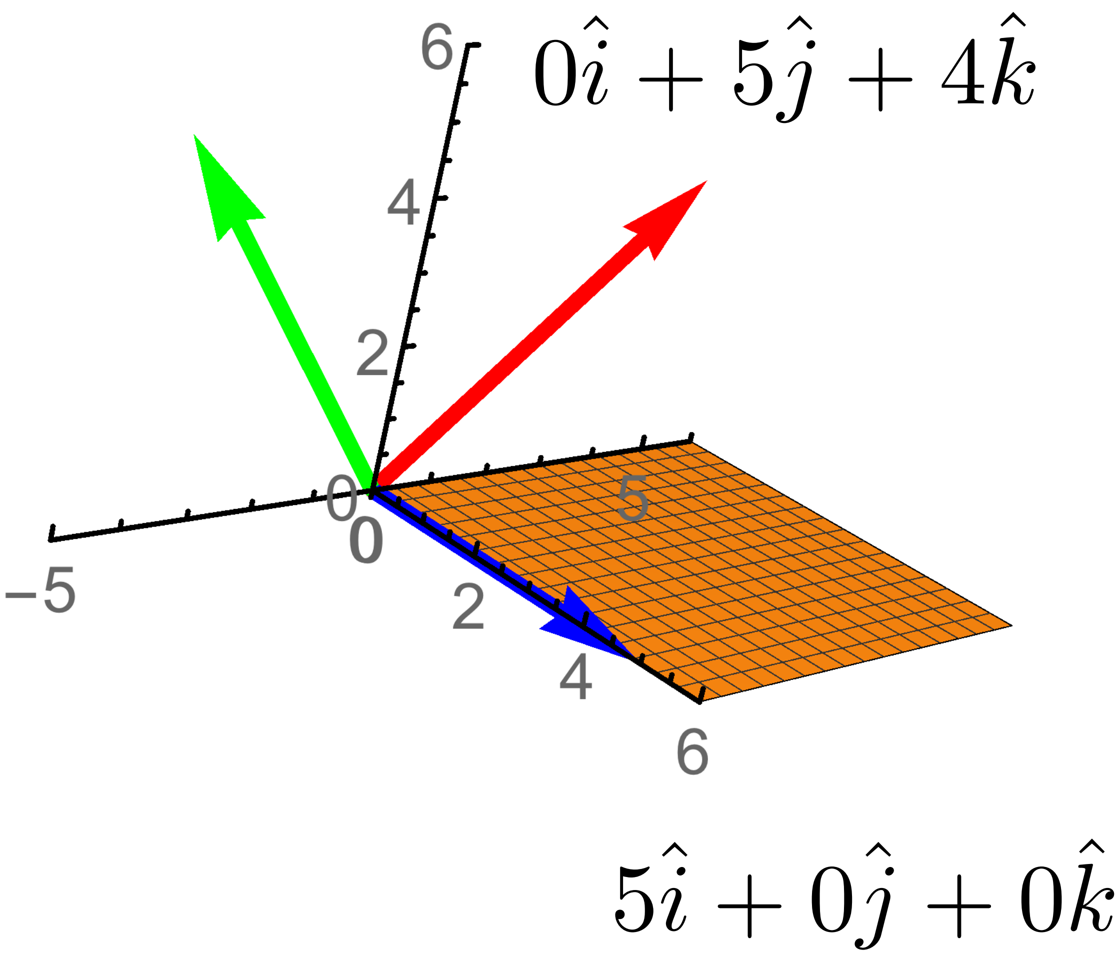

b) $20\hat{i} + 0\hat{j} + 25\hat{k}$

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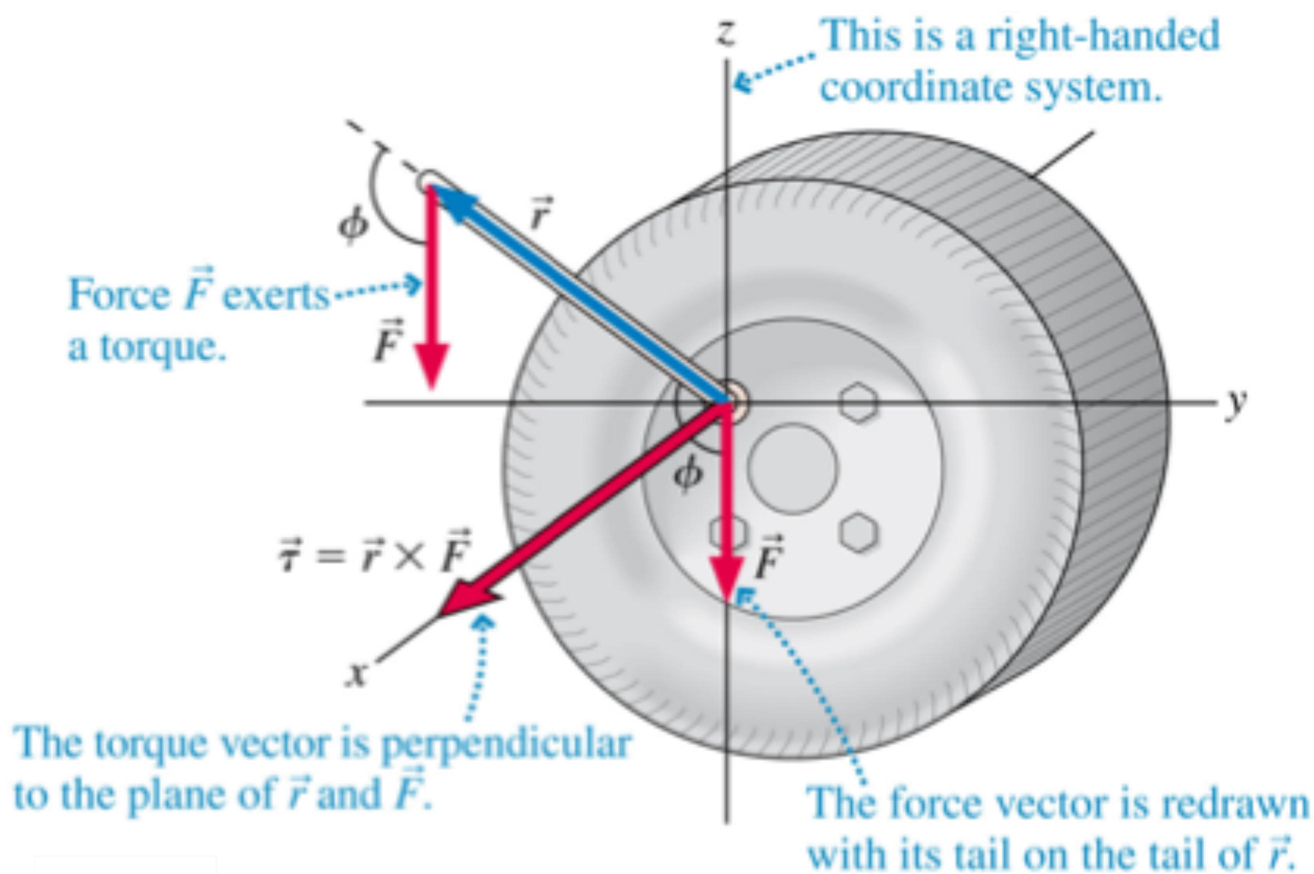
The Torque Vector

Previously we learned… ⟶ $\tau = Fr\sin\phi$

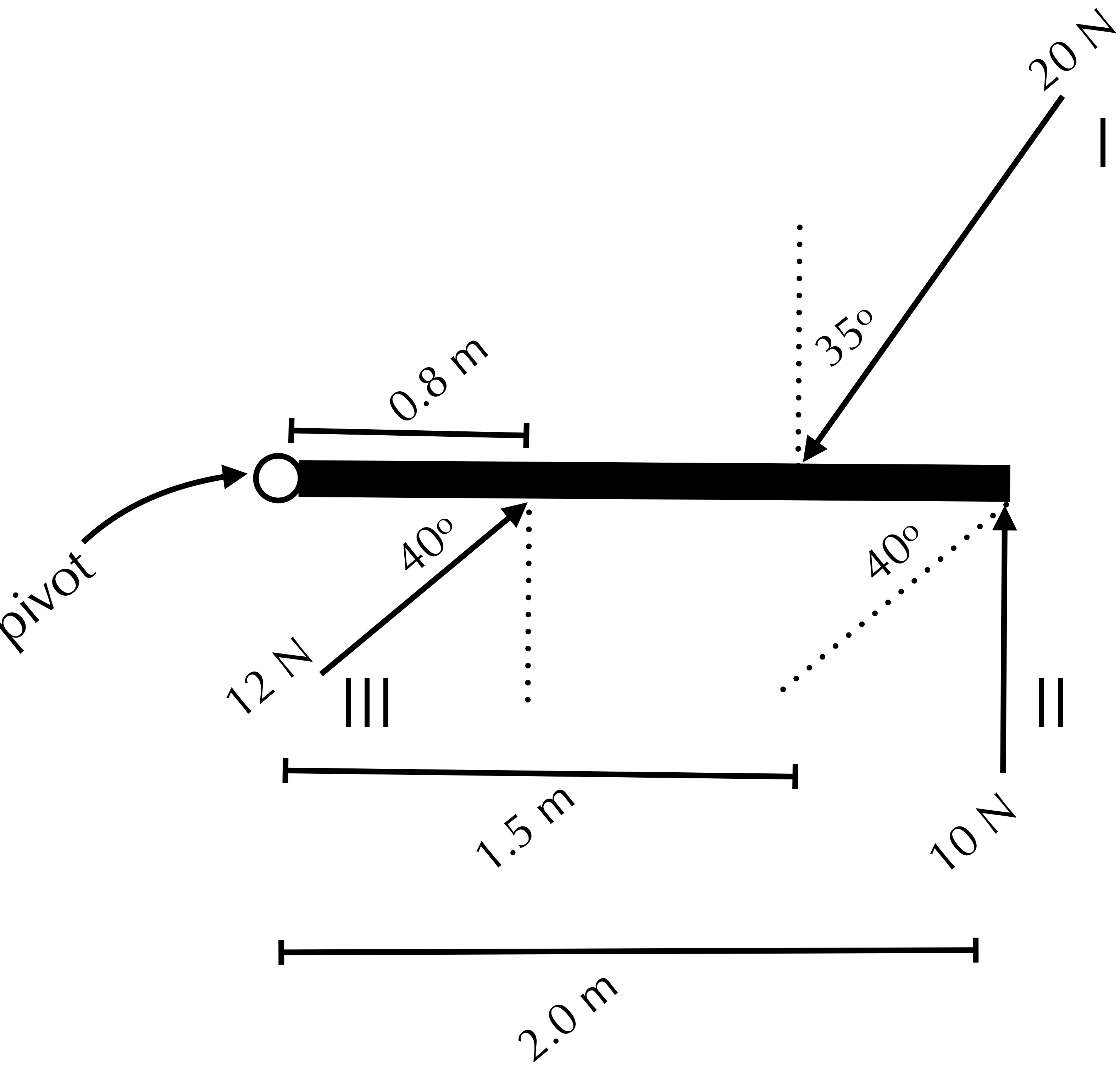
But actually…. ⟶ $\vec{\tau} = \vec{r} \times \vec{F}$



Torque

Which of the torque vectors point out of the screen?

Question #24

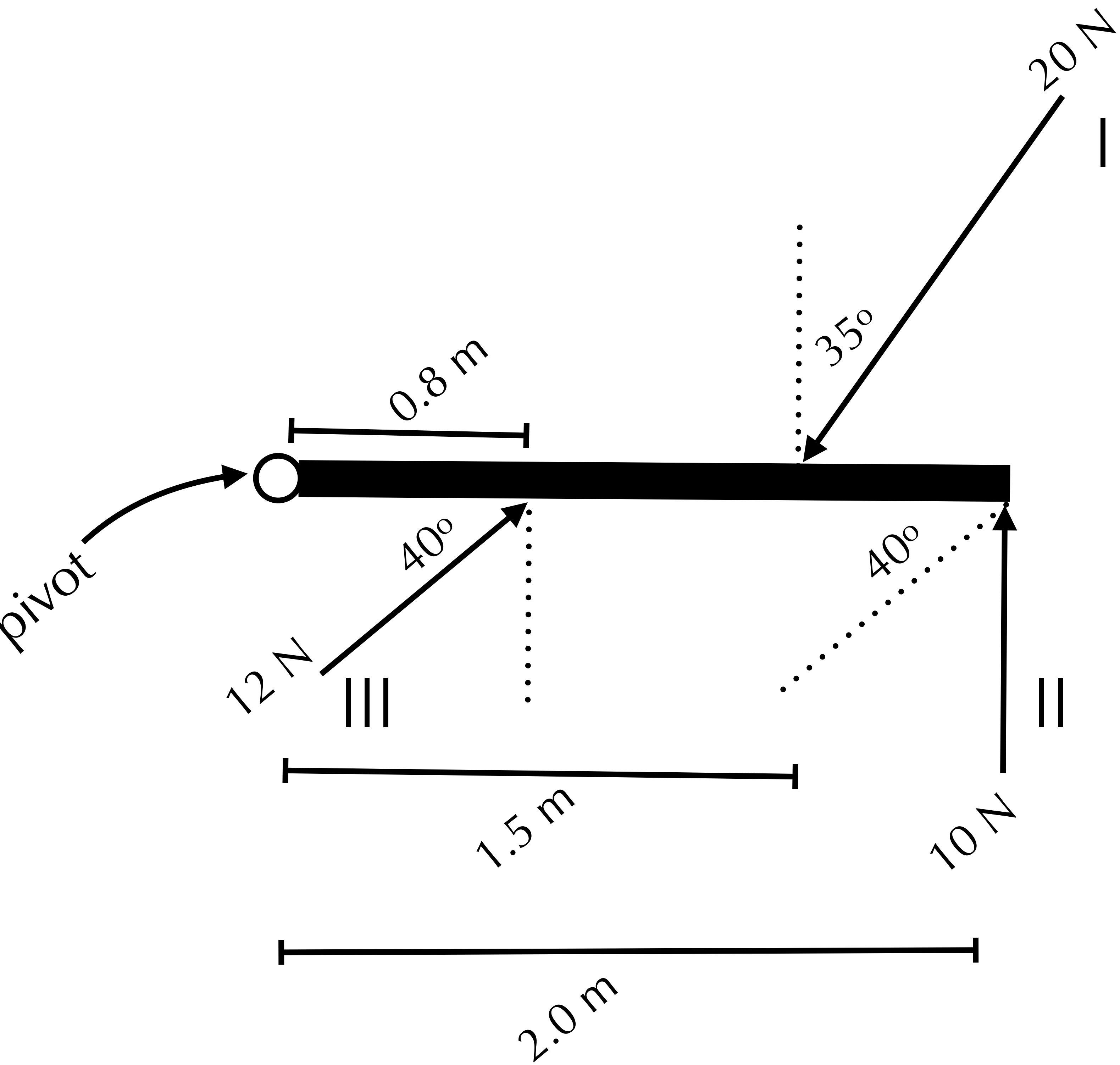


A) I and II
B) II and III
C) I and III
D) All three

Torque

Which of the torque vectors point out of the screen?

Question #24



A) I and II
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Linear Equation	Angular Equation
$\vec{F} = m\vec{a}$	$\tau = I\alpha$
$\vec{p} = m\vec{v}$	
$\vec{F} = \dfrac{d\vec{p}}{dt}$	

Linear Equation		Angular Equation
$\vec{F} = m\vec{a}$	$\longrightarrow$	$\tau = I\alpha$
$\vec{p} = m\vec{v}$	$\xrightarrow{?}$	
$\vec{F} = \frac{d\vec{p}}{dt}$	$\xrightarrow{?}$	

Linear Equation		Angular Equation
$\vec{F} = m\vec{a}$	$\longrightarrow$	$\tau = I\alpha$
$\vec{p} = m\vec{v}$	$\xrightarrow{?}$	$\vec{L} = I\vec{\omega}$ stay tuned!
$\vec{F} = \dfrac{d\vec{p}}{dt}$	$\xrightarrow{?}$	

Linear Equation		Angular Equation	
$\vec{F} = m\vec{a}$	$\longrightarrow$	$\tau = I\alpha$	
$\vec{p} = m\vec{v}$	$\xrightarrow{?}$	$\vec{L} = I\vec{\omega}$	stay tuned!
$\vec{F} = \frac{d\vec{p}}{dt}$	$\xrightarrow{?}$	$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$	

Anti-grav wheel

Precessing wheel demo

Conservation of Momentum

An isolated system that experiences no net torque has

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\mathrm{net}} = 0$$

and thus the angular momentum vector $\vec{L}$ is a constant.

Law of conservation of angular momentum: The angular momentum of an isolated system is conserved. The final angular momentum is equal to the initial angular momentum. Both the magnitude and direction are unchanged.

ang mom game

Demo 1

Demo 2

Quiz

Question #25

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,

a. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
b. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
c. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
d. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
e. None of the above.

Quiz

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,

a. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
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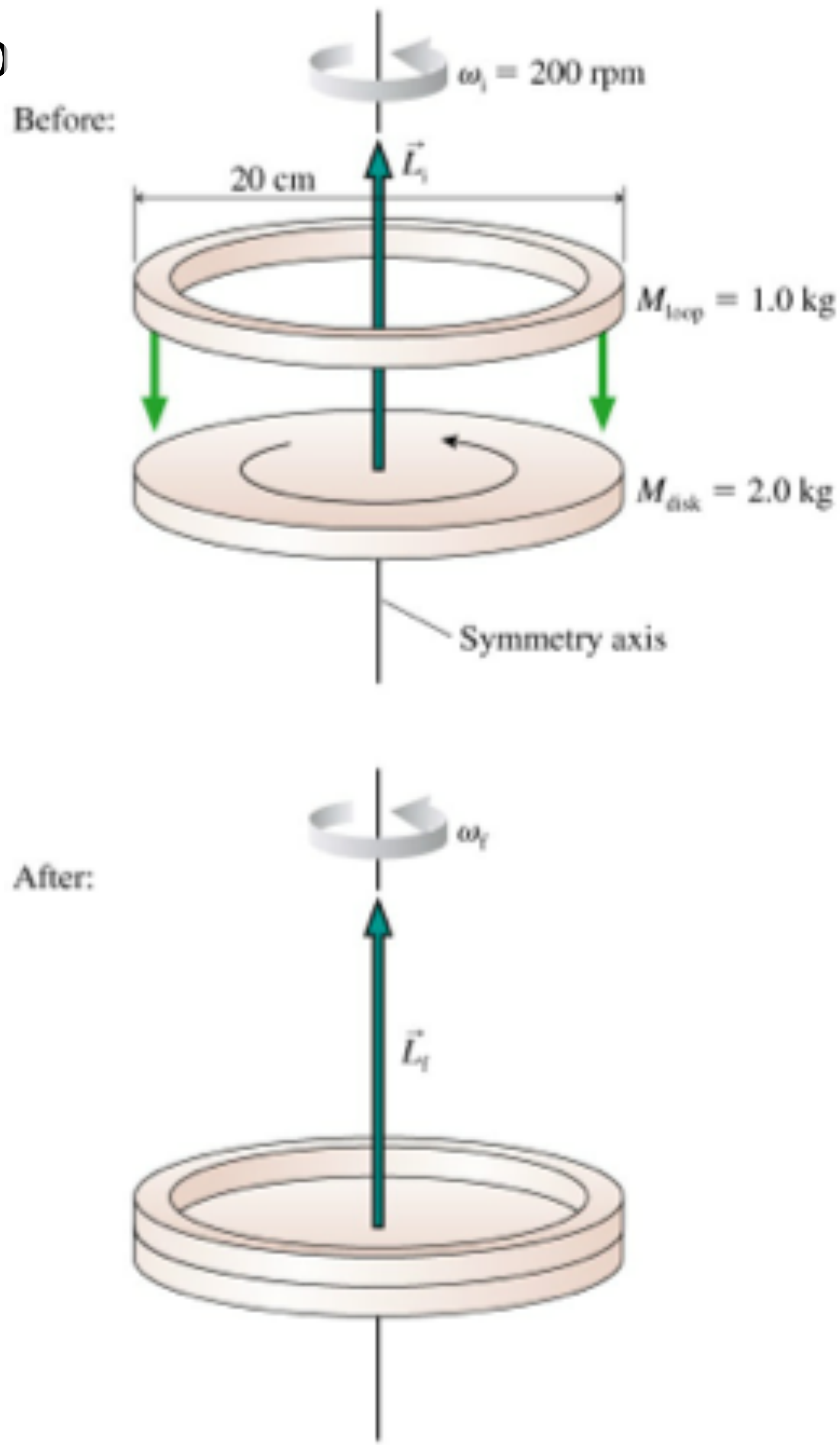
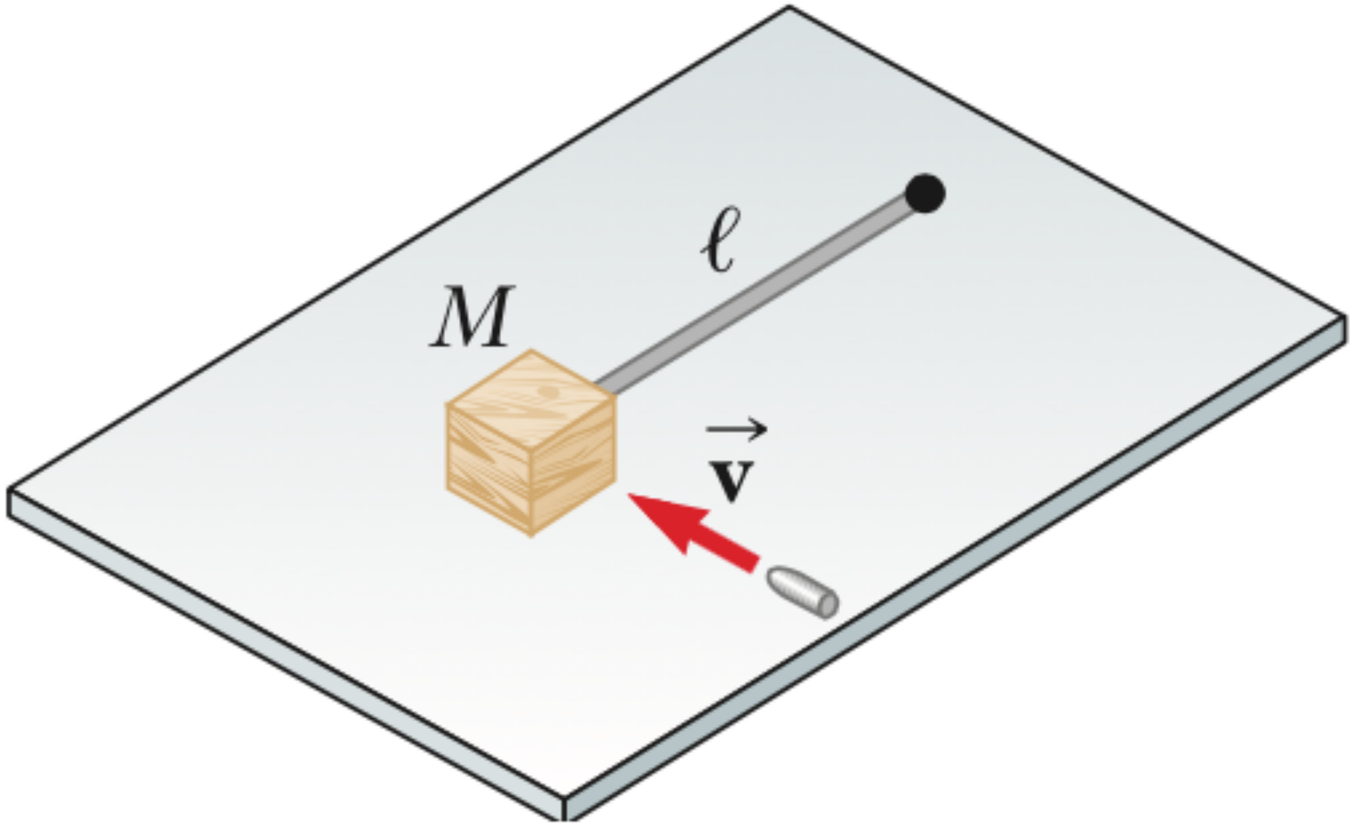
e. None of the above.

Conservation of Angular Momentum Problem

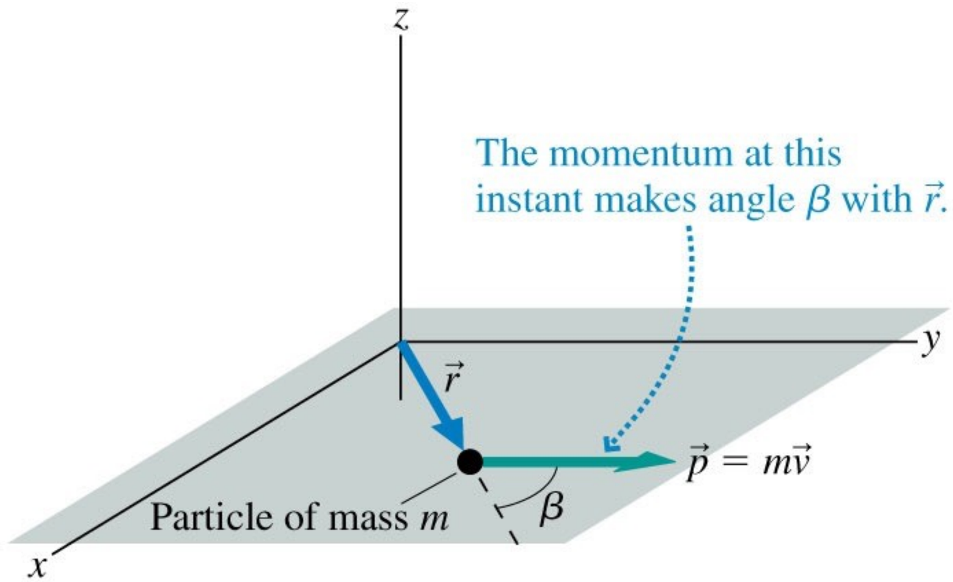
A 20-cm-diameter, 2.0 kg solid disk is rotating at 200 rpm. A 20-cm-diameter, 1.0 kg circular loop is dropped straight down onto the rotating disk. Friction causes the loop to accelerate until it is riding on the disk. What is the final angular velocity of the combined system?



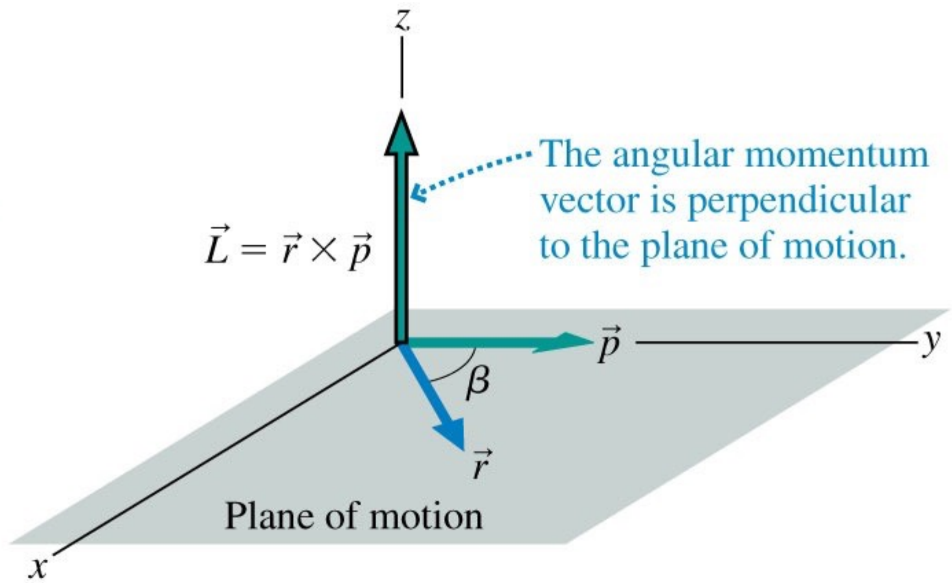
Find an expression for the angular velocity of the bullet block system after the collision



Angular Momentum of a particle



Vectors $\vec{r}$ and $\vec{p}$ define the plane of motion.



The vector tails are placed together to determine the cross product.

We define the particle's angular momentum vector relative to the origin to be

$$\vec{L} = \vec{r} \times \vec{p} = (mrv \sin \beta, \text{ direction of right} - \text{hand rule})$$

What is the angular speed of the combination after the ***<u>collision</u>***?