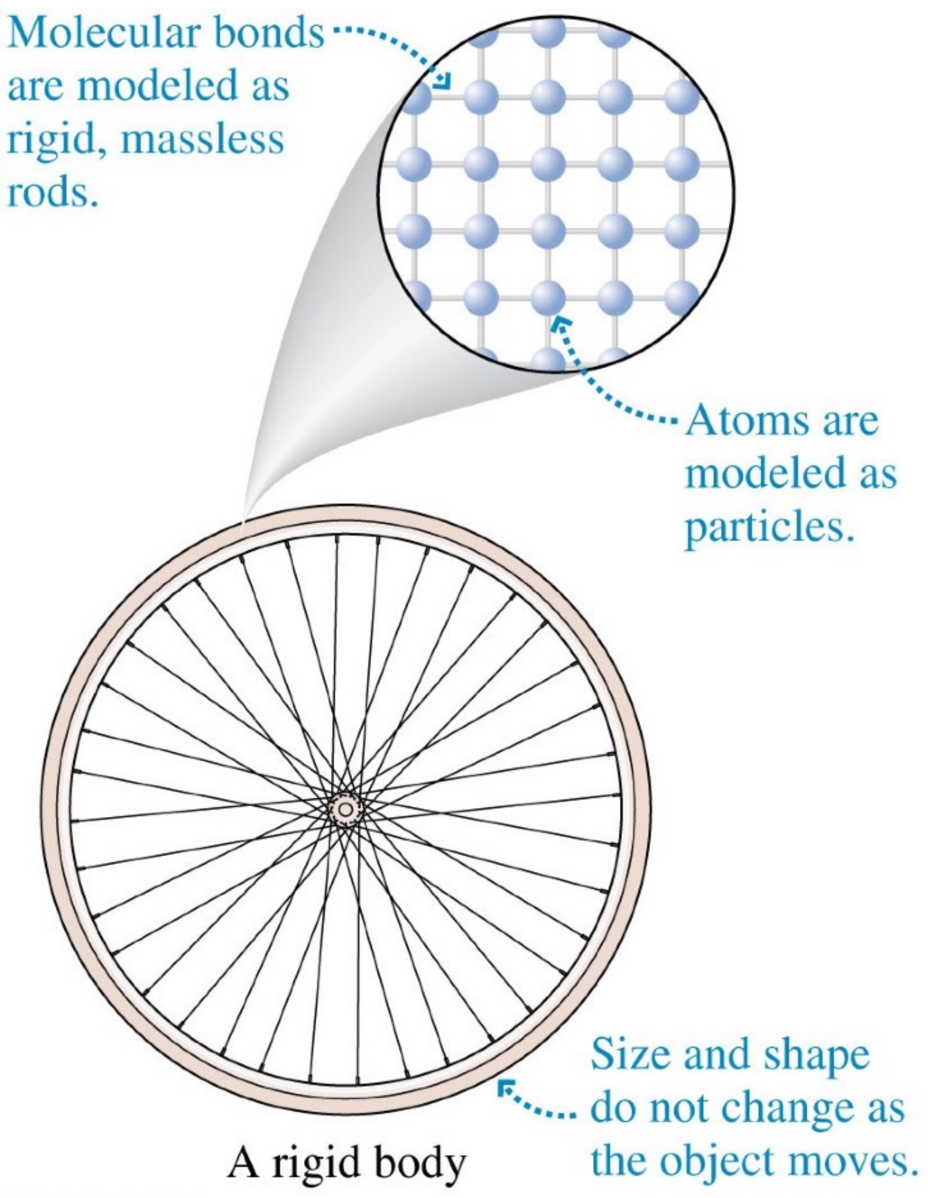
Rigid-Body Model

A **rigid body** is an extended object whose size and shape do not change as it moves.



Rotational motion review

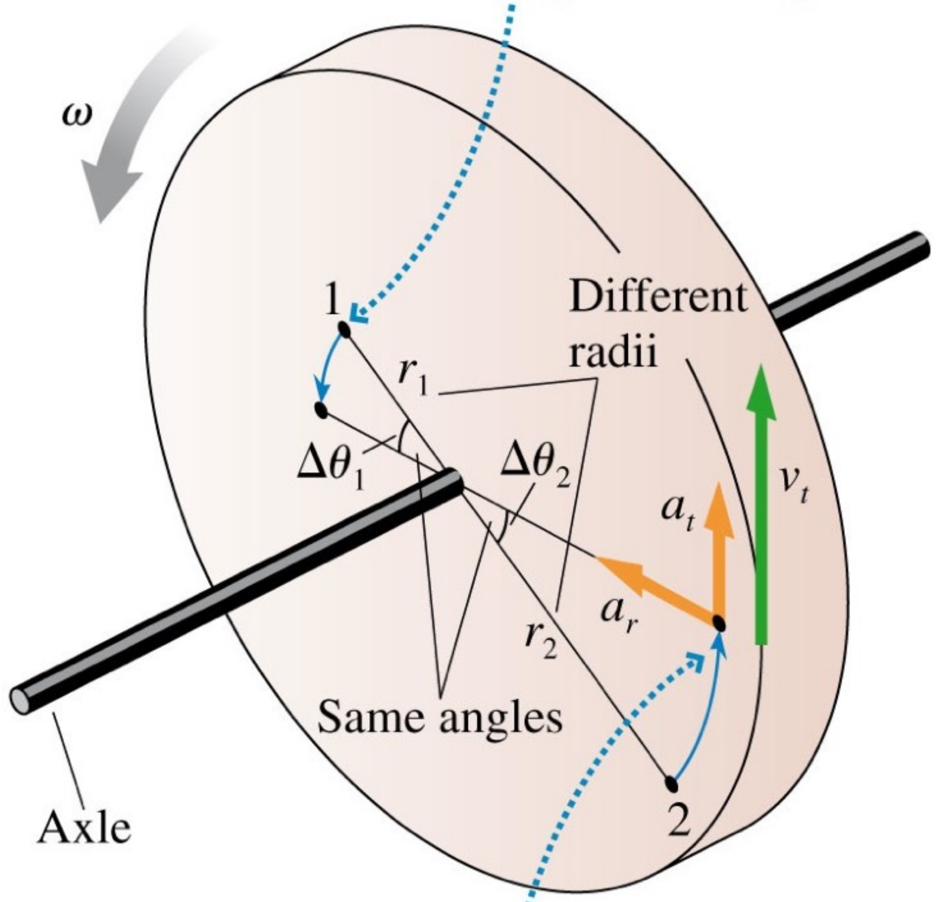
Recall that angular velocity is

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

All points on a rotating rigid body have the same ω and the same α.

Every point on the wheel turns through the same angle and thus undergoes circular motion with the same angular velocity ω.



All points on the wheel have a tangential velocity and a radial (centripetal) acceleration. They also have a tangential acceleration if the wheel has angular acceleration.

Rotational Motion Review

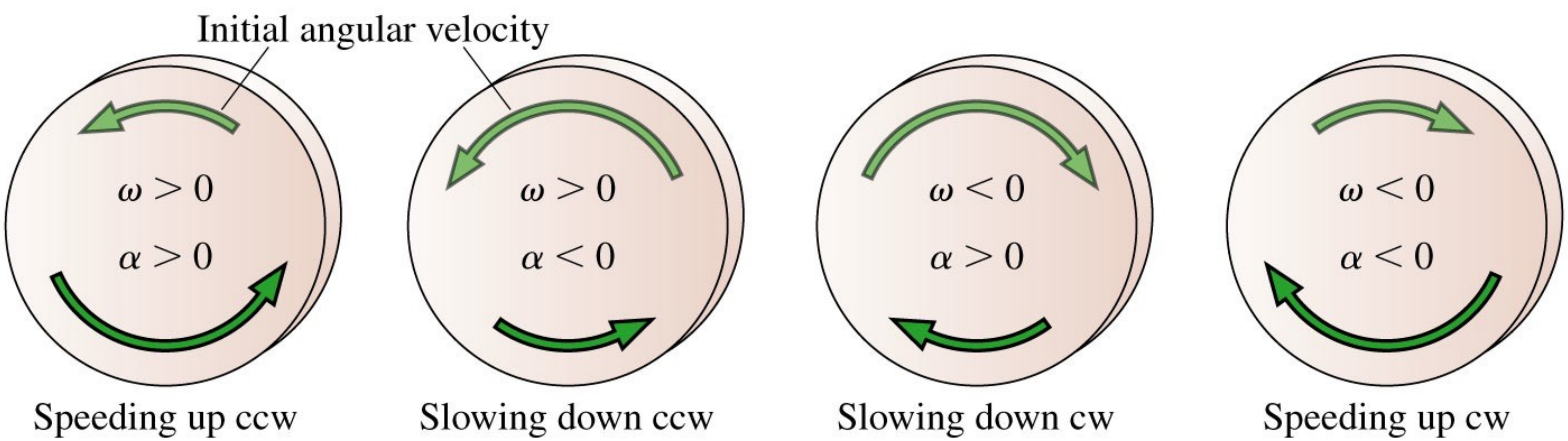
Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$

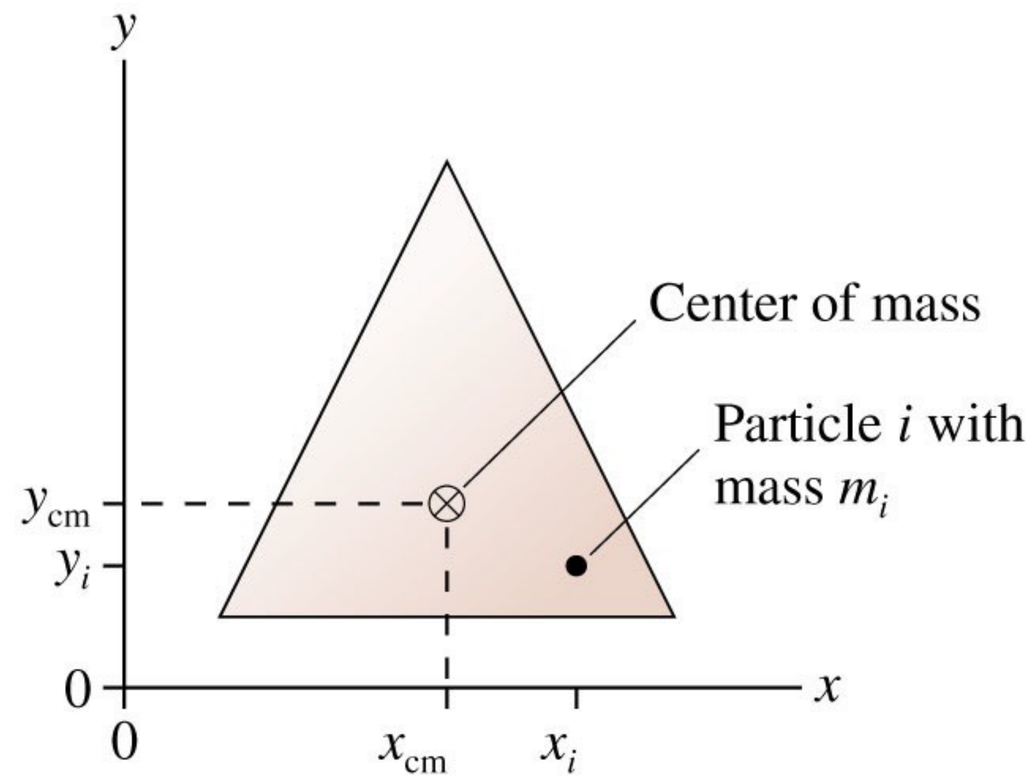
$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

The signs of angular velocity and angular acceleration.



Finding the Center of Mass

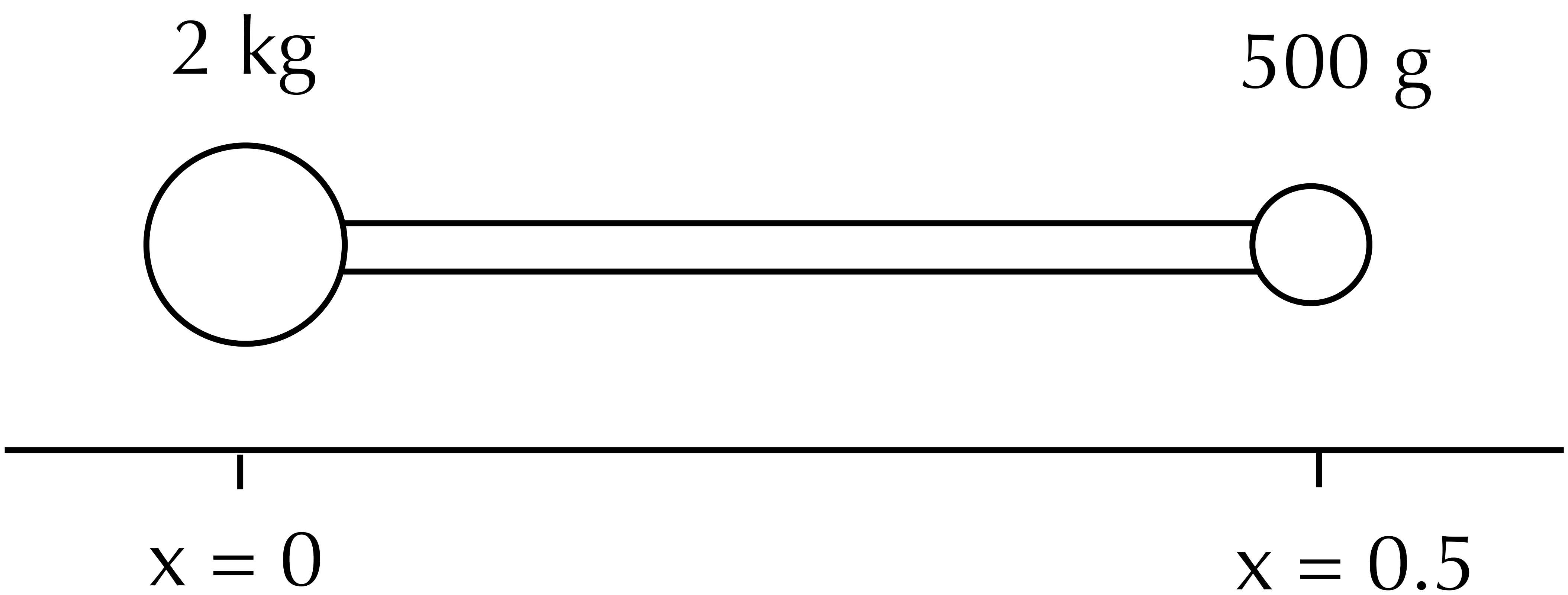


$$x_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$y_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

What is the center of mass for this system?

Question #1



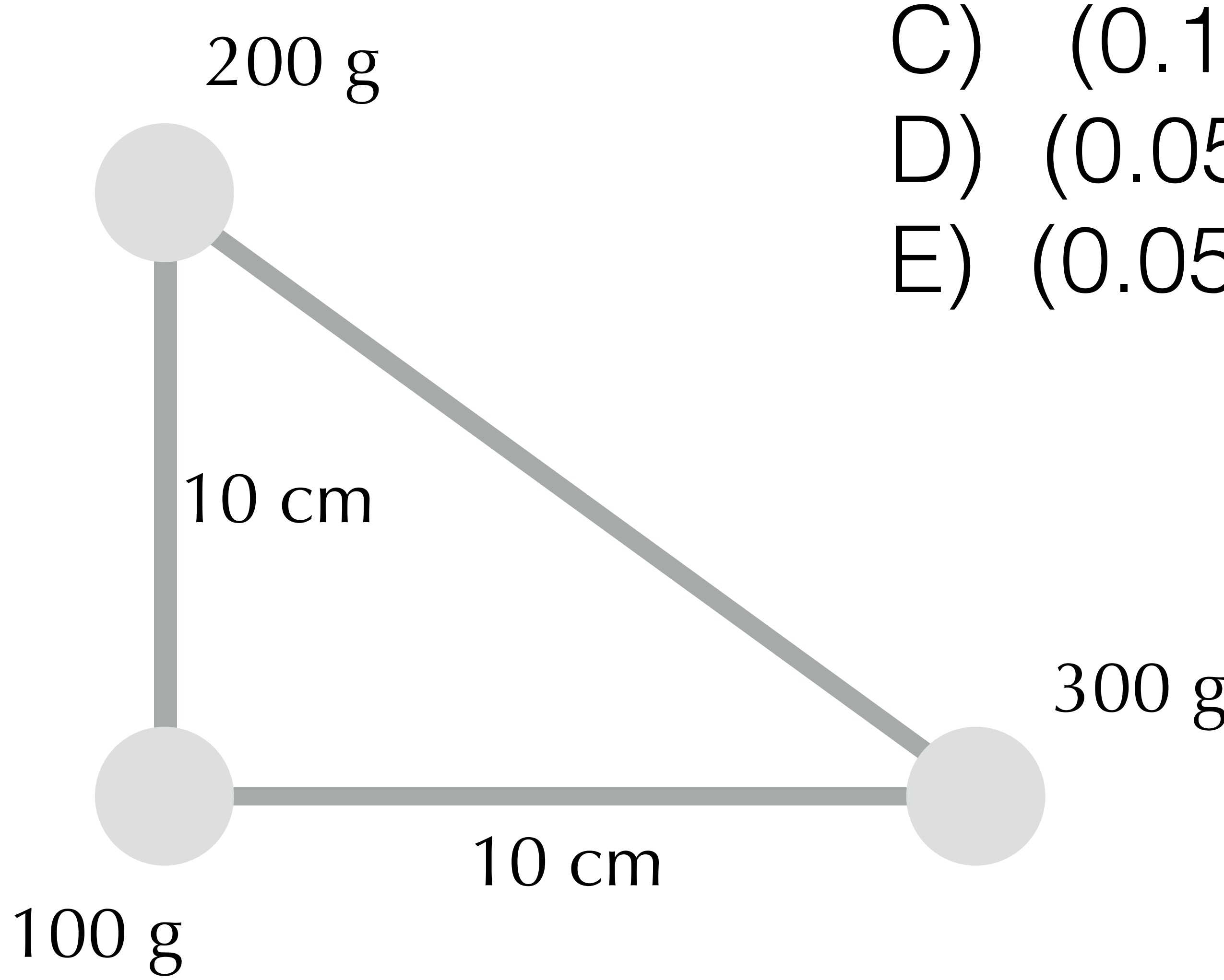
A) 0.05 m
B) 0.25 m
C) 0.1 m
D) 0.5 m
E) 0.75 m

Two-Dimensional Problem

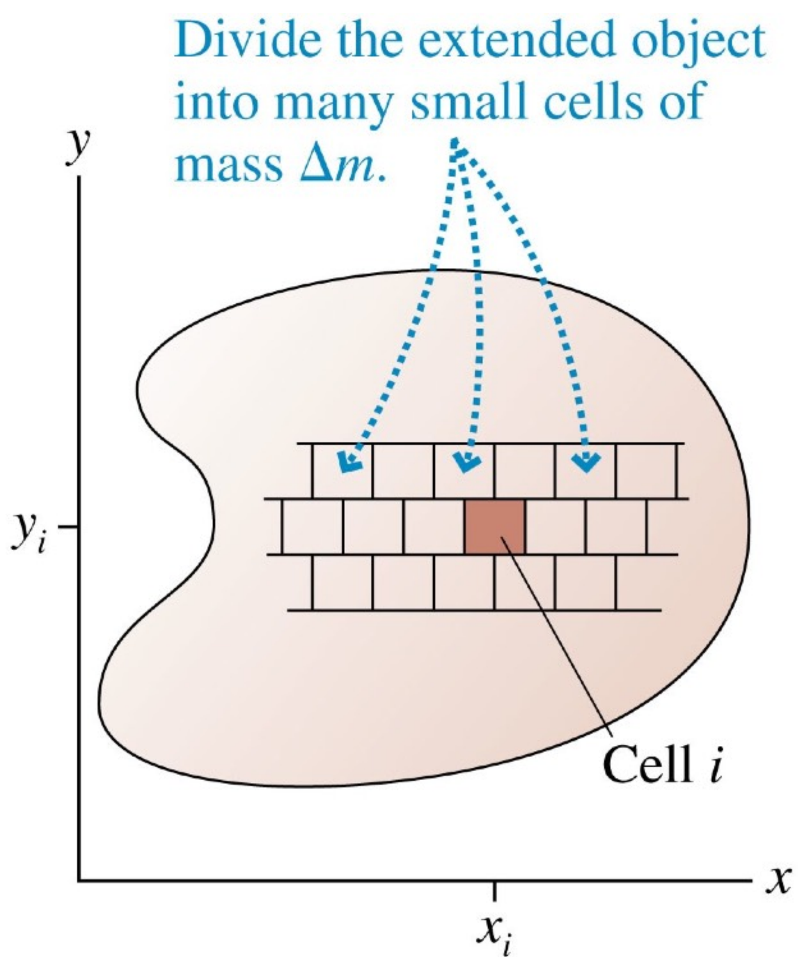
Question #2

The three balls shown are connected by massless, rigid rods. What are the coordinates of the center of mass?

A) (0.05, 0.05) m
B) (0.1, 0.03) m
C) (0.1, 0.05) m
D) (0.05,0.075)m
E) (0.05 , 0.03) m



Center of Mass of a solid object



$$x_{\rm cm} = \frac{1}{M} \int x \, dm$$

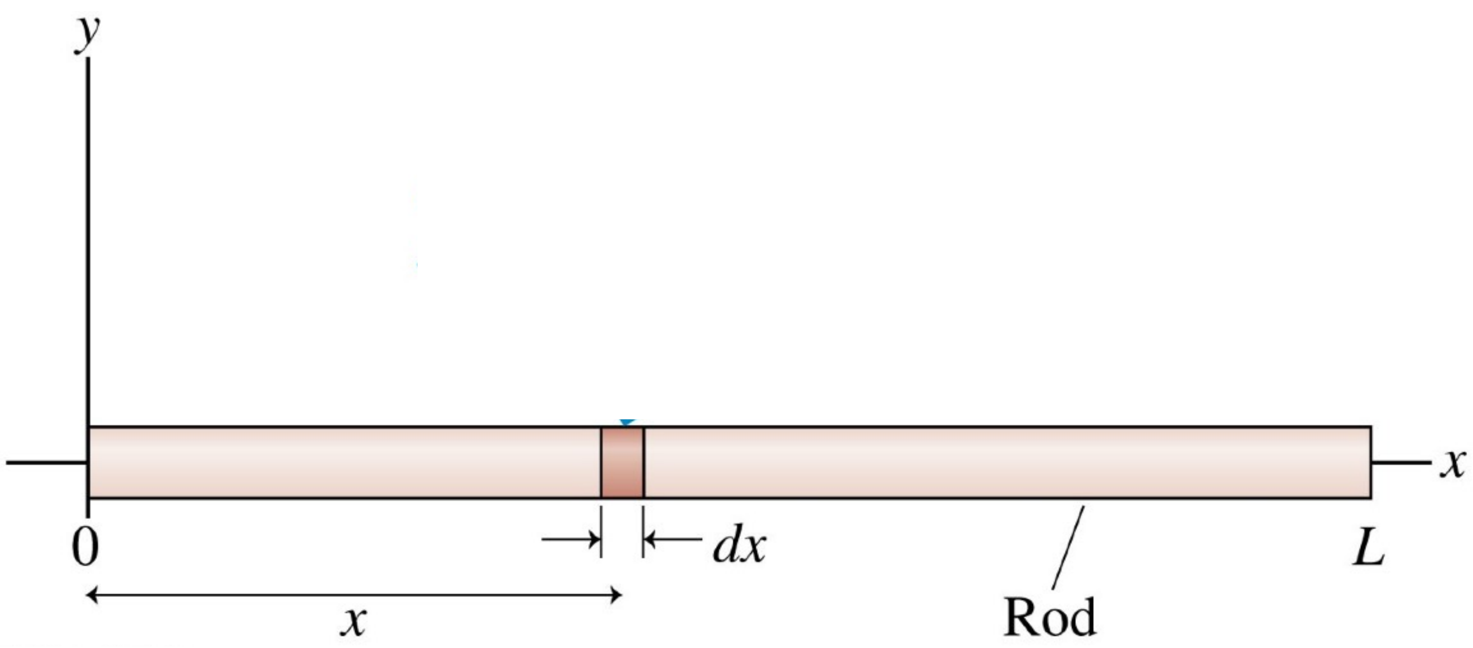
$$y_{\rm cm} = \frac{1}{M} \int y \, dm$$

To do these integrals:

- *dm* must be replaced with expressions involving *dx* and *dy*
- *I*ntegration limits must be set

Example

Find the center of mass of a thin, uniform rod of length L and mass M. Use your result to find the tangential acceleration of one tip of a 1.60-m-long rod that rotates about its center of mass with an angular acceleration of 6.0 rad/s^2.



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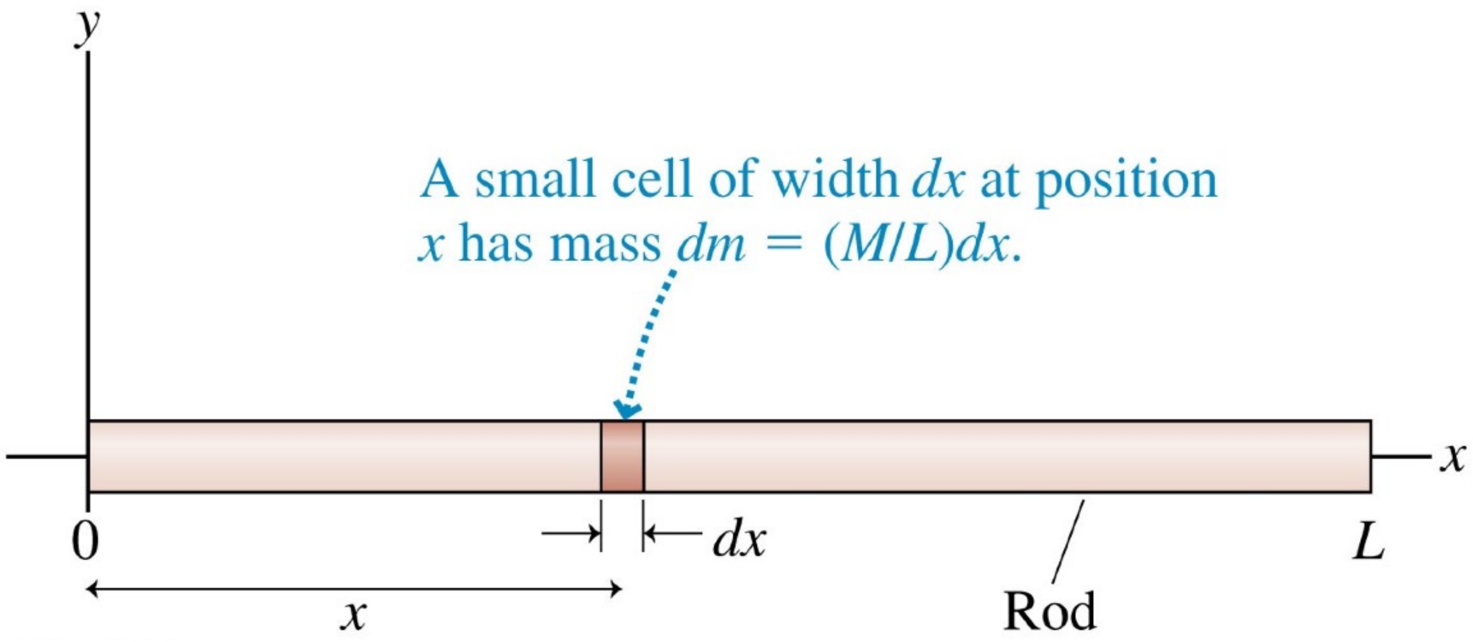
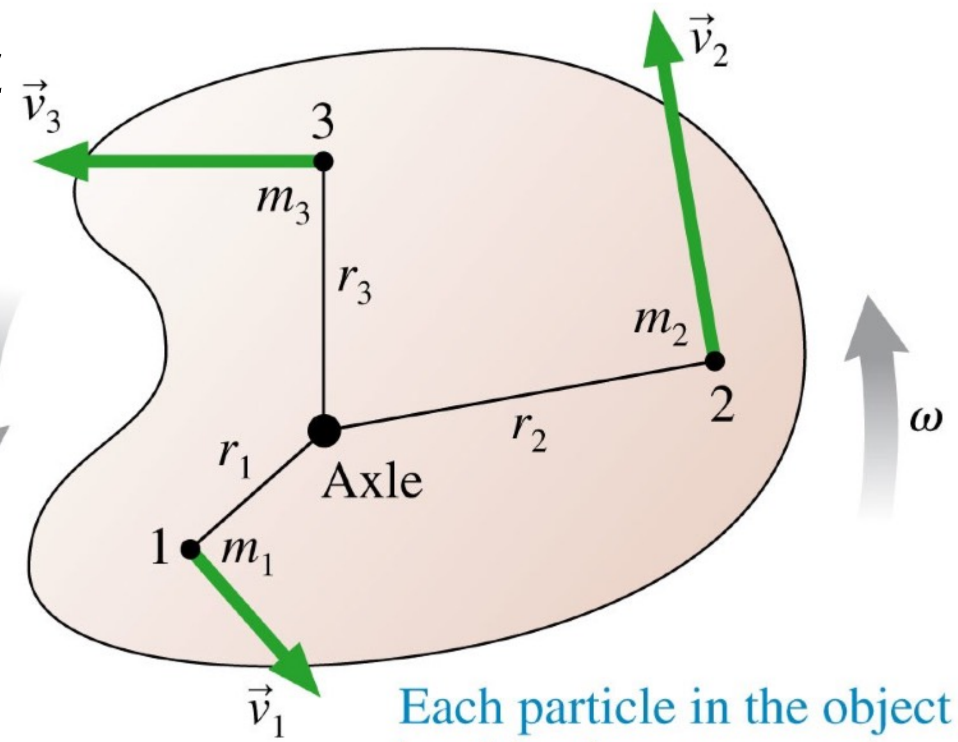


Rotational Energy

- When an object rotates, each individual piece of mass has kinetic energy.
- This is called rotational kinetic energy



$$K_{\text{rot}} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \cdots$$

recall that $v = r\omega$

$$= \frac{1}{2}m_1 r_1^2 \omega^2 + \frac{1}{2}m_2 r_2^2 \omega^2 + \cdots = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

Rotational Energy

Define the object's **moment of inertia**:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

$$K_{\mathrm{rot}} = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

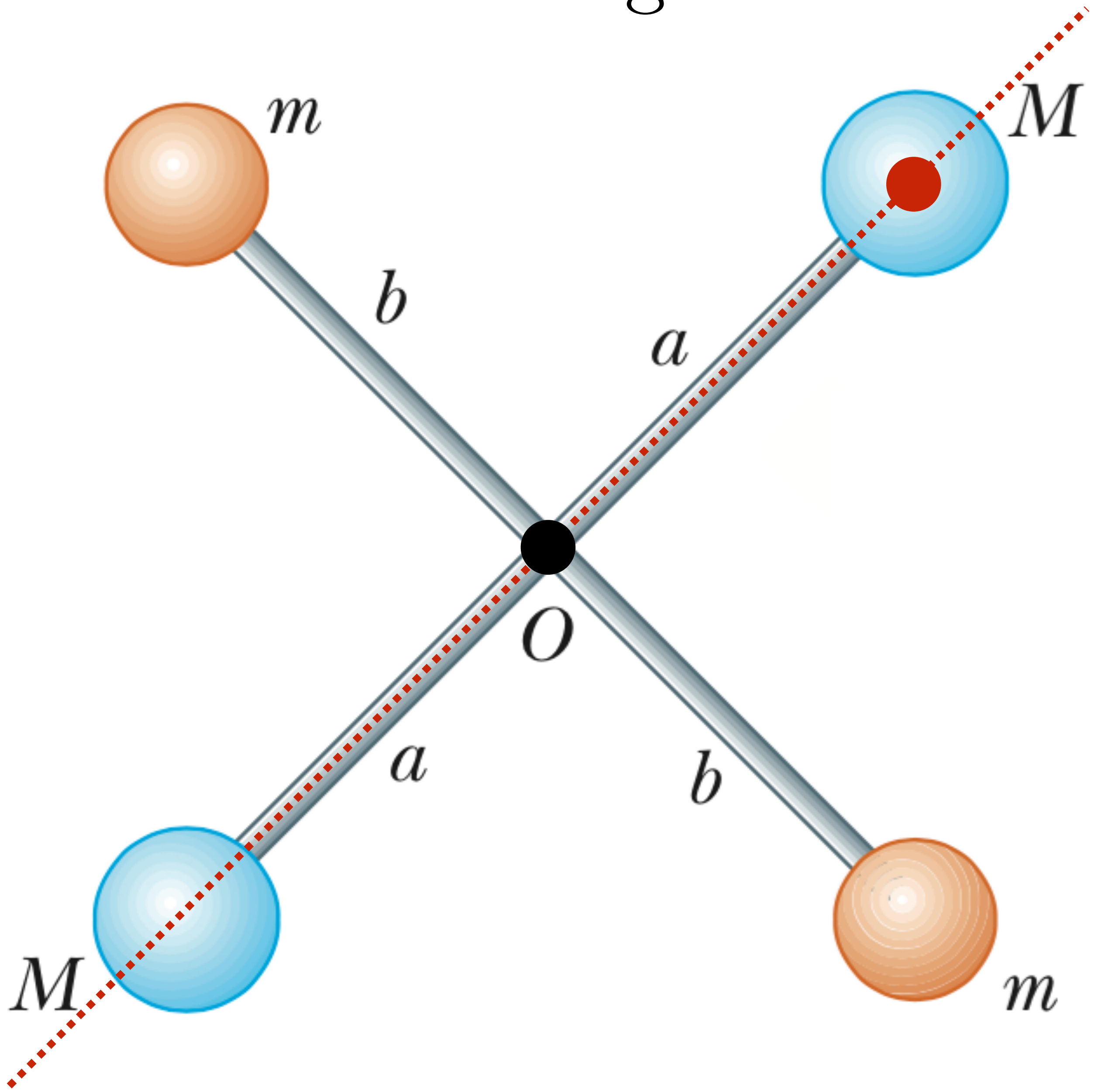
- The units of moment of inertia are kg m^2.
- **Moment of inertia depends on the axis of rotation.**
- Mass farther from the rotation axis contributes more to the moment of inertia than mass nearer the axis.
- This is *not* a new form of energy, merely the familiar kinetic energy of motion written in a new way.

Question #3

$a = 50 \text{ cm} \quad b = 75 \text{ cm} \quad m = 100 \text{ g}$

$M = 200 \text{ g}$

$$I = \sum mr^2$$

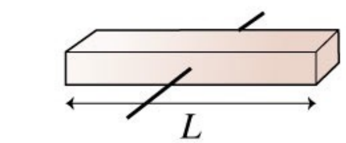
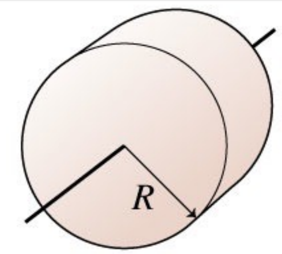
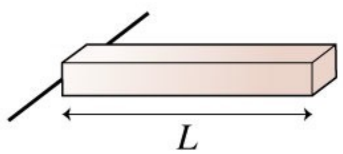
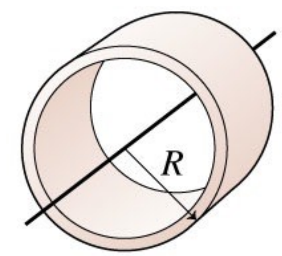
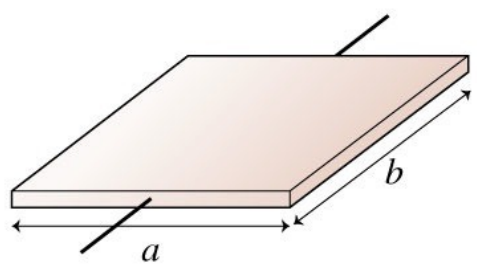
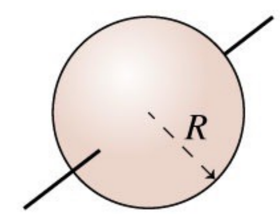
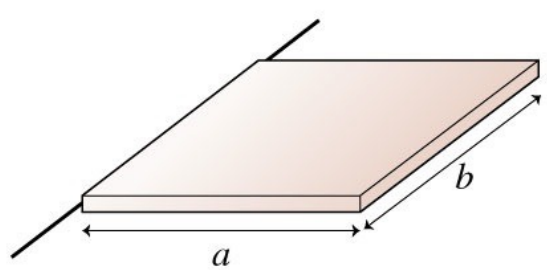
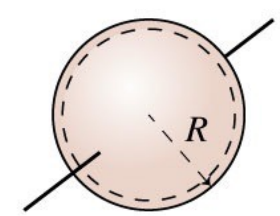


The moment of inertia about which axis will be greatest?

a) red dot
b) black dot
c) red dotted line.

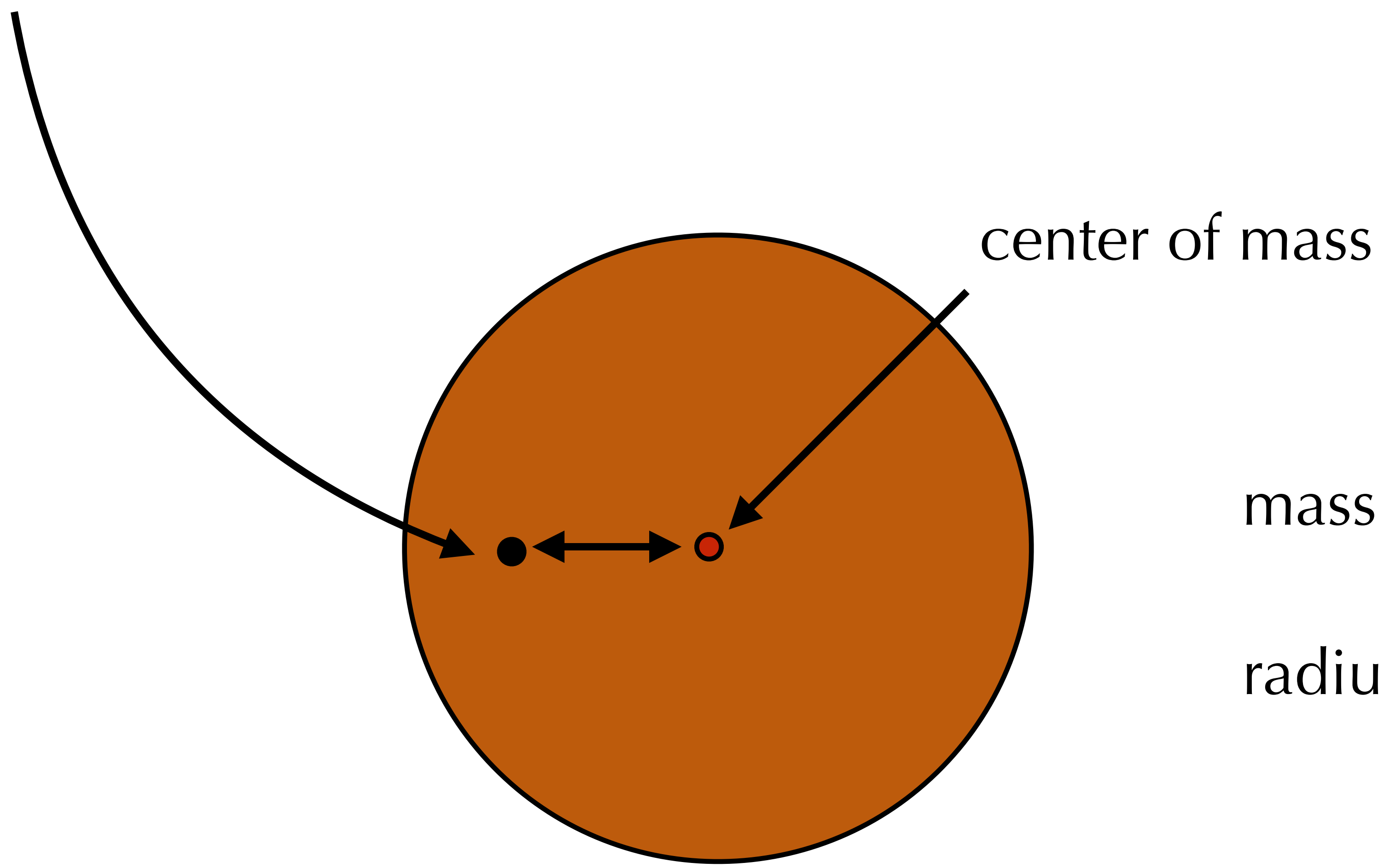
Calculating the Moment of Inertia

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Example (Parallel Axis Theorem)

What is the moment of inertia if this disk is rotated about this axis?

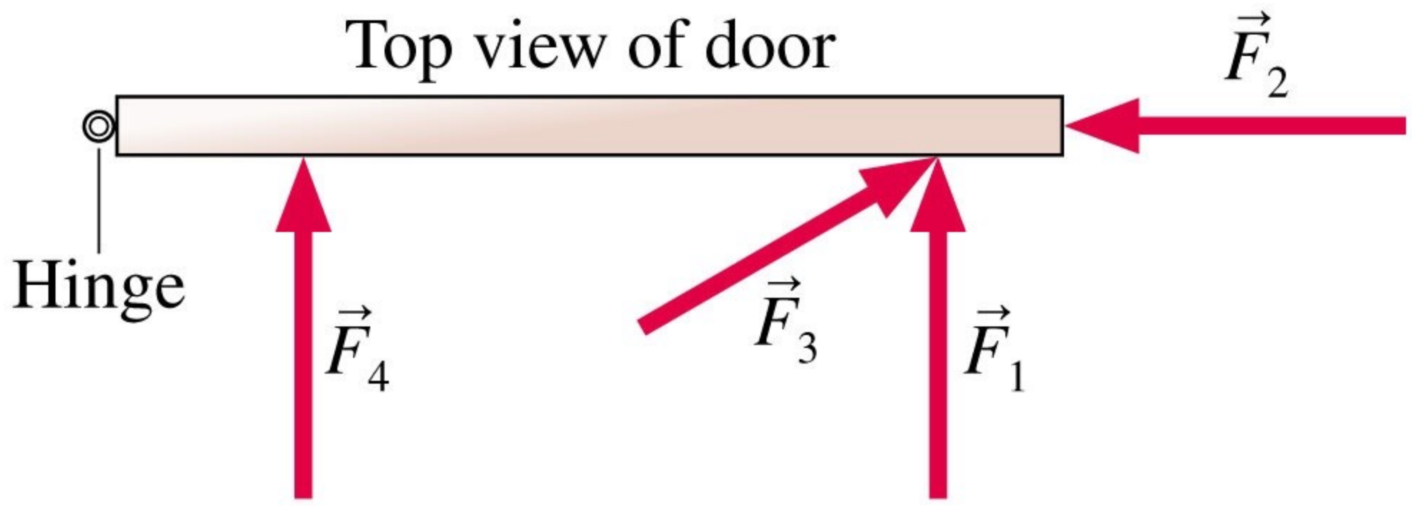


mass = 10 kg

radius = 50 cm

$$I = I_{\mathrm{cm}} + Md^2$$

The four forces shown have the same strength. Which force would be most effective in opening the door?

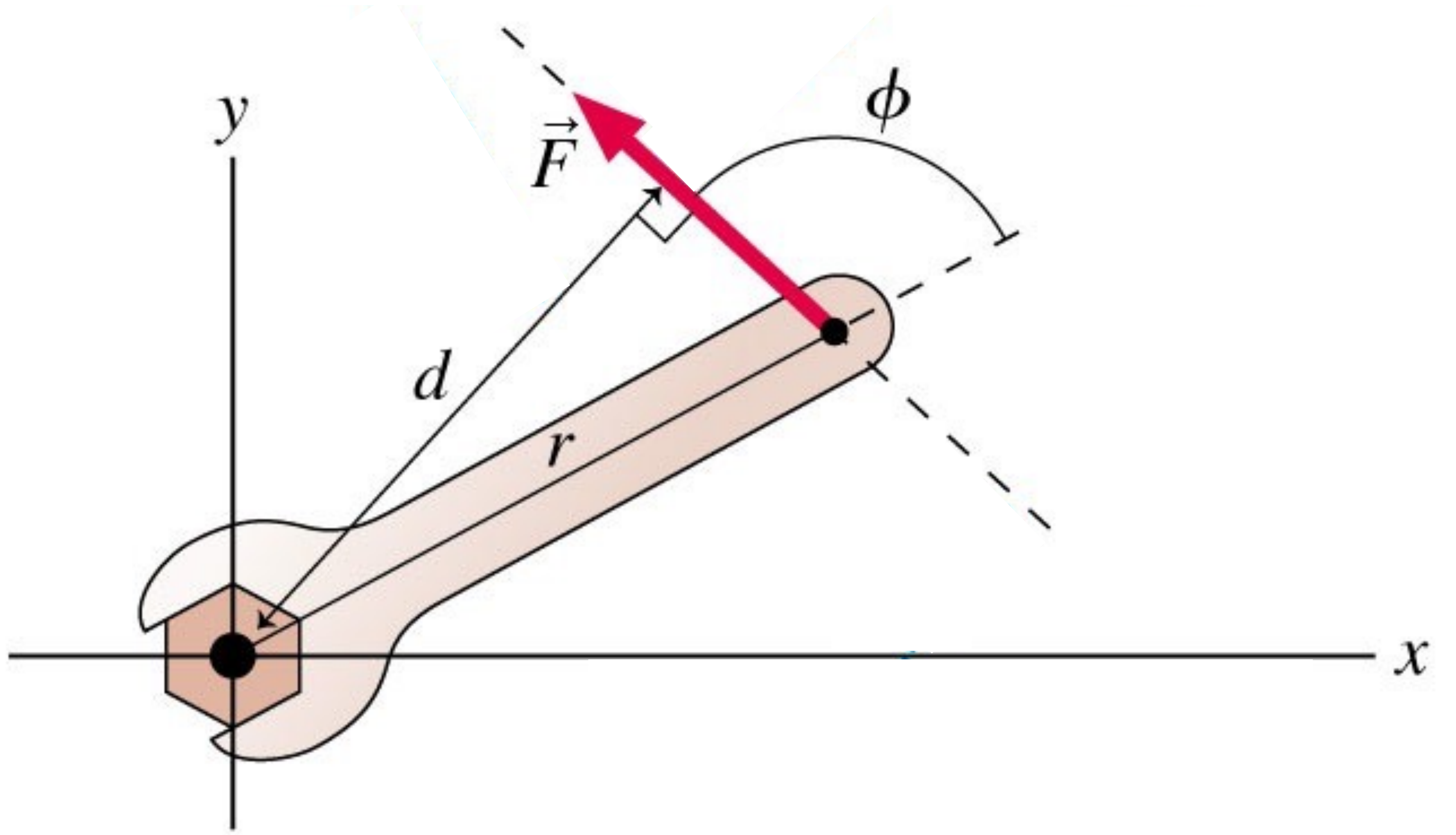


a. Force F_2.
b. Force F_1.
c. Force F_3.
d. Force F_4.
e. Either F_1 or F_3.

$$\tau = F_{\perp} r$$

or

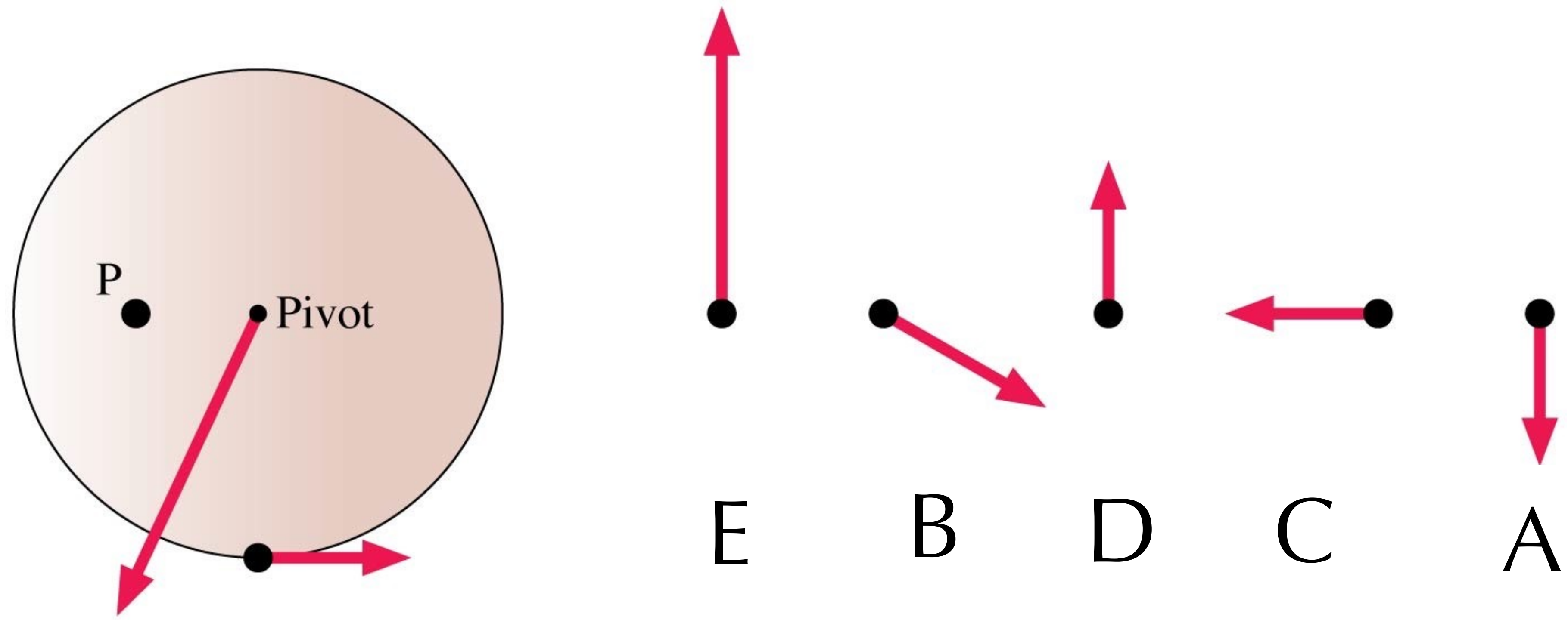
$$\tau = F r_{\perp}$$



Quiz

Question #5

Which third force on the wheel, applied at point P, will make the net torque zero?



Gravitational Torque

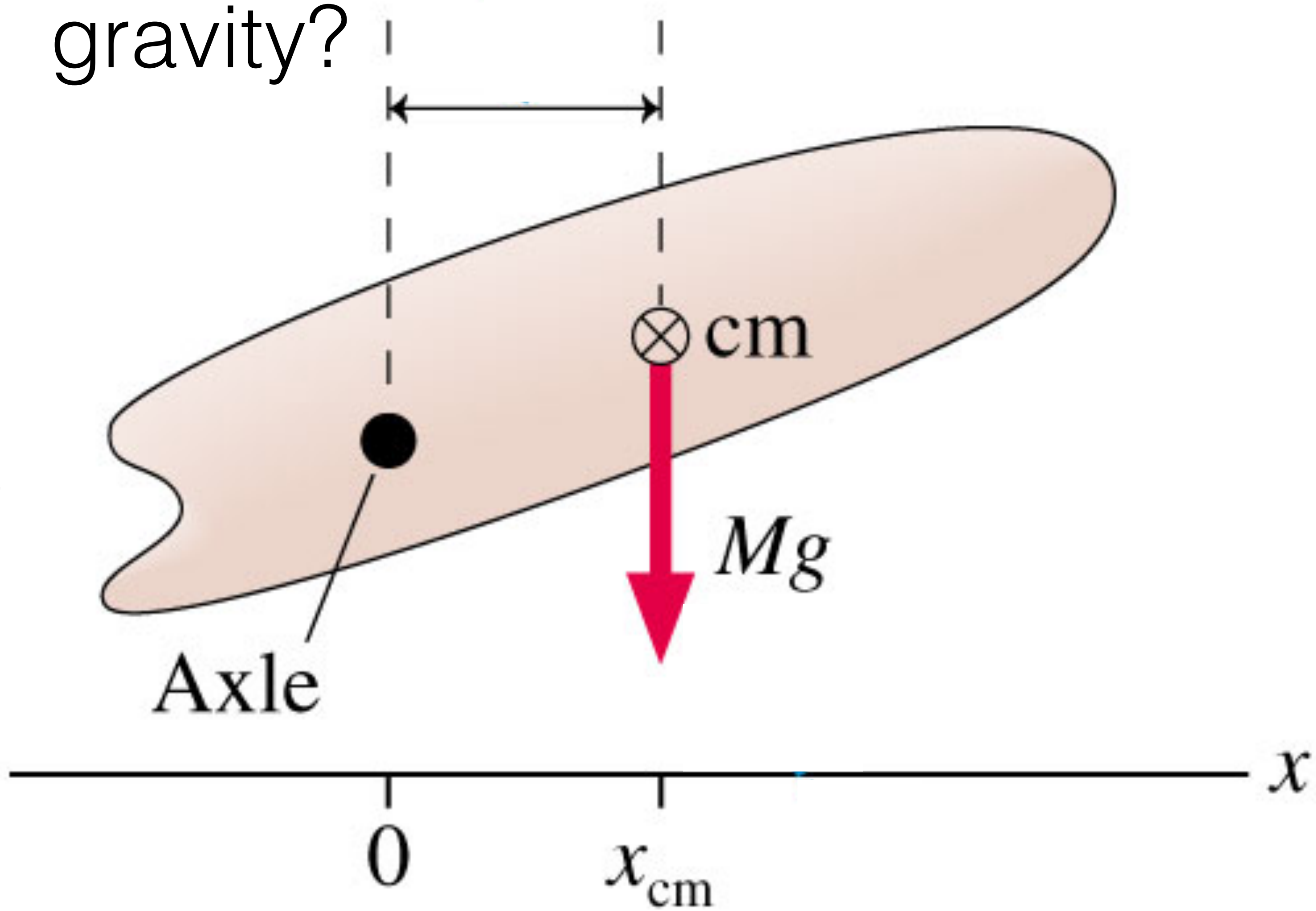
Question #6

Which is the correct expression for the torque produced by gravity?

D $\tau_{\text{grav}} = -Mgx_{\text{cm}}$

C $\tau_{\text{grav}} = -Mgx_{\text{cm}} \sin\theta$

B $\tau_{\text{grav}} = -Mgx_{\text{cm}} \cos\theta$

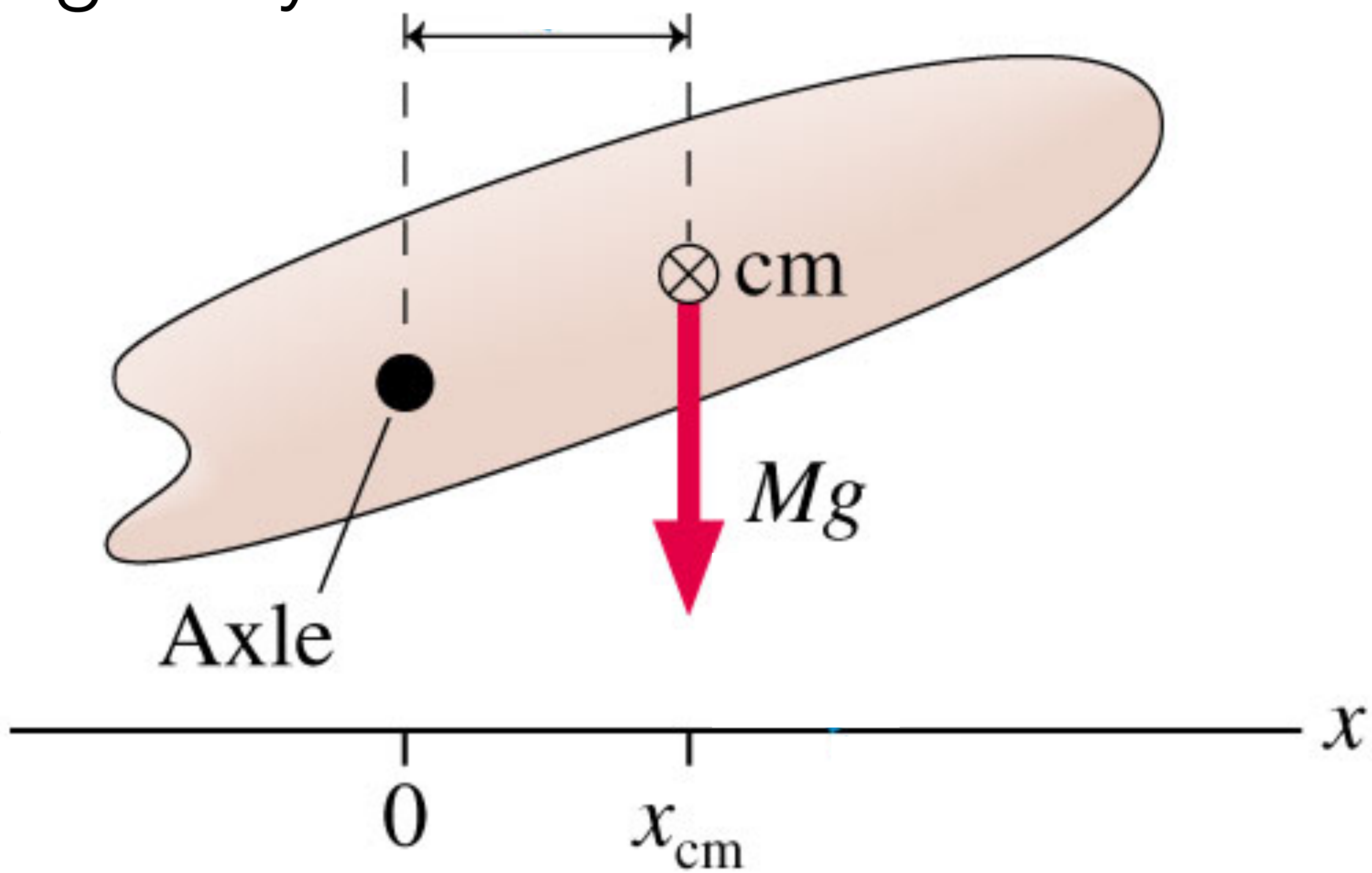


When I release the object what will happen?

A) It will oscillate back and forth. B) Nothing. It will stay where you put it. C) It will first rotate and the quickly come to rest.

Question #6

Which is the correct expression for the torque produced by gravity?



D $\tau_{\mathrm{grav}} = -Mgx_{\mathrm{cm}}$

C $\tau_{\mathrm{grav}} = -Mgx_{\mathrm{cm}}\sin\theta$

B $\tau_{\mathrm{grav}} = -Mgx_{\mathrm{cm}}\cos\theta$

Question #7

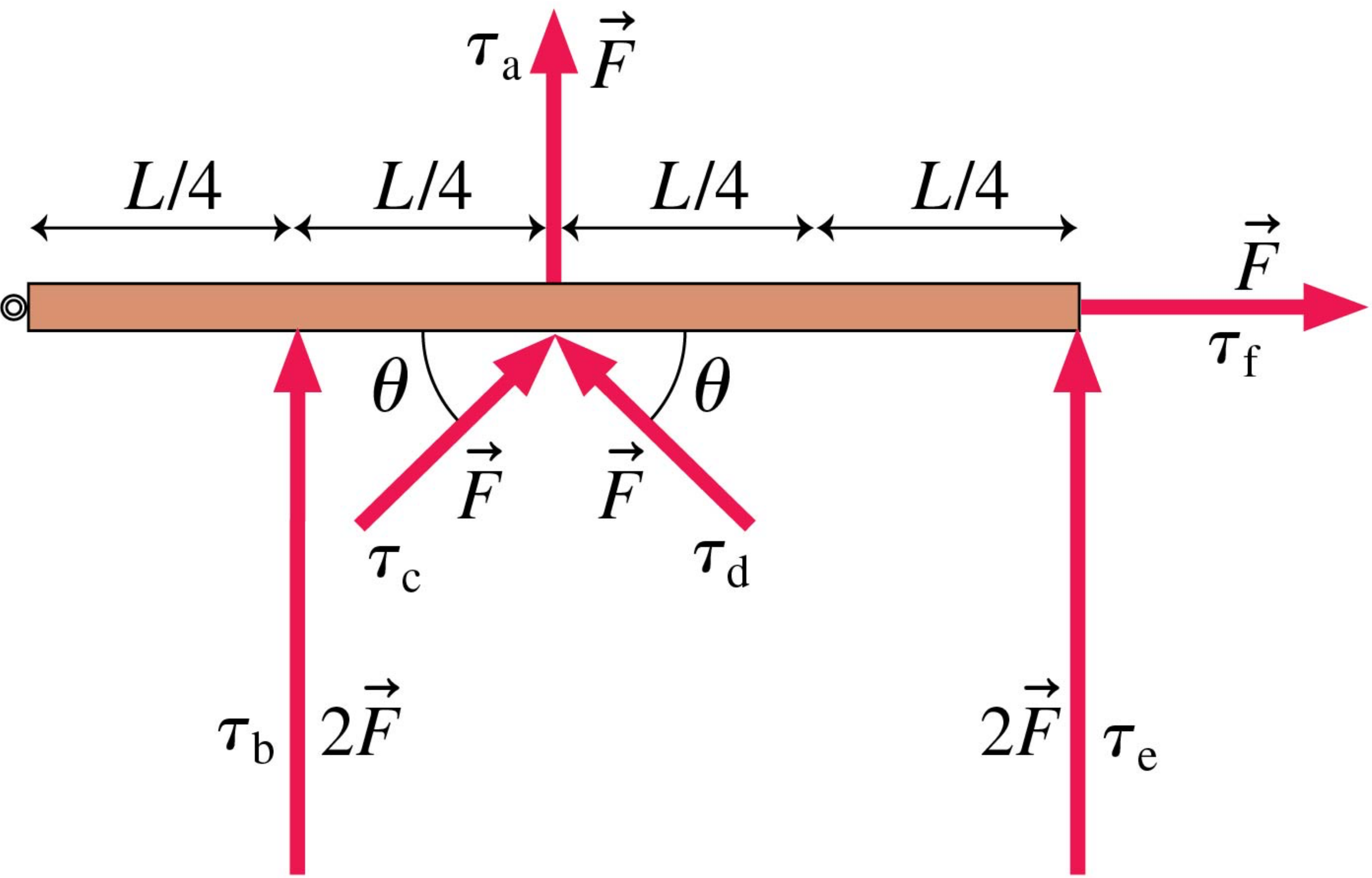
When I release the object what will happen?

A) It will oscillate back and forth. B) Nothing. It will stay where you put it.
C) It will first rotate and the quickly come to rest.

Question #8

Rank the torques

A) e > a = b > c = d > f
B) e = a > b > c > d > f
C) e > b > a > d = c > f



Conservation of Energy with Rotational Kinetic Energy

A 1.0-m-long, 200 g rod is hinged at one end and connected to a wall. It is held horizontal, then released. What is the speed of a the tip of the rod as it hits the wall?