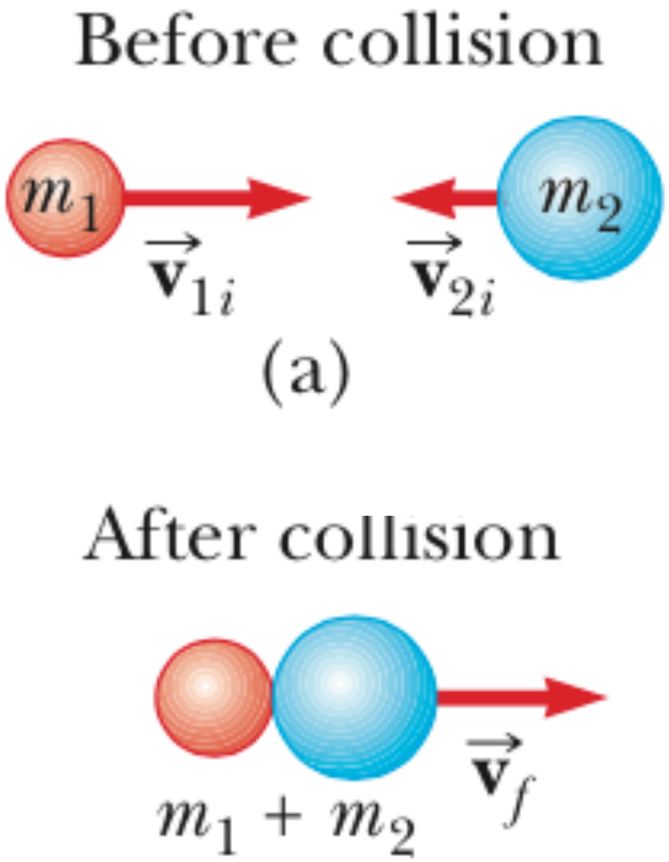
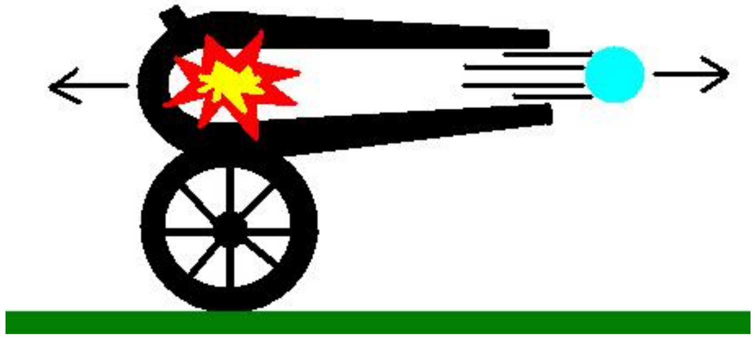
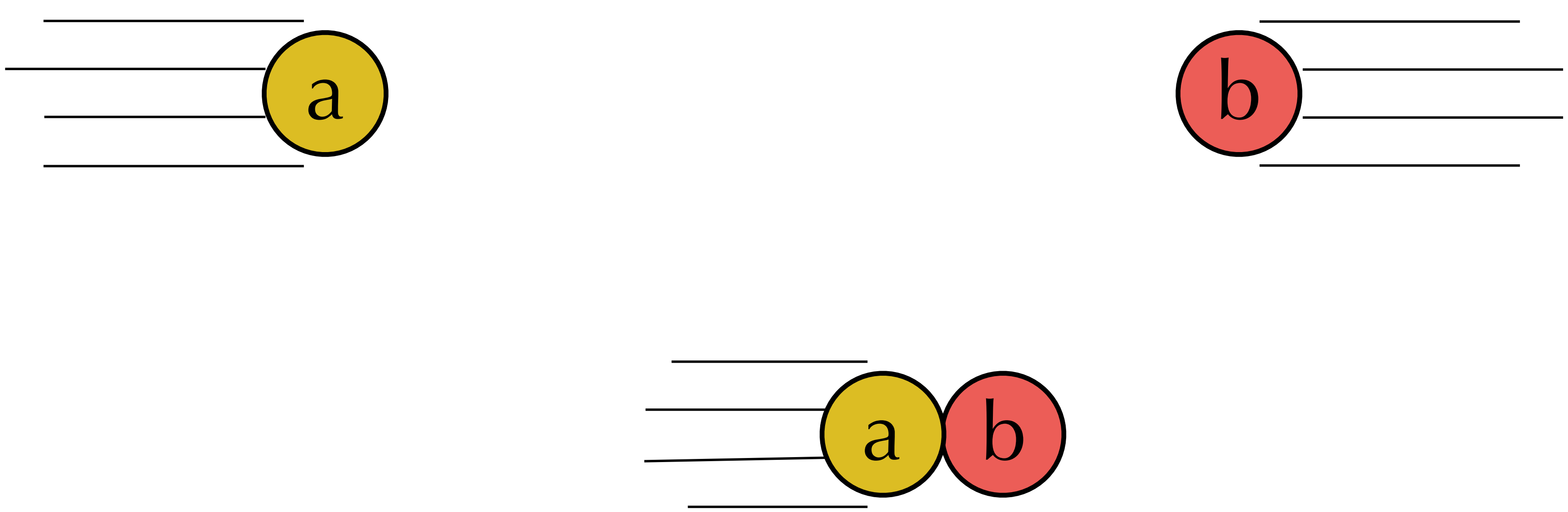
Law of conservation of momentum

Describe how conservation of momentum applies in the following situations.





Two balls are moving towards each other and collide. After the collision they stick together and move to the right. Which ball had the larger speed before the collision?



a. ball a
b. ball b
c. they had the same speed
d. No way of knowing without knowing the masses.

Collision applet

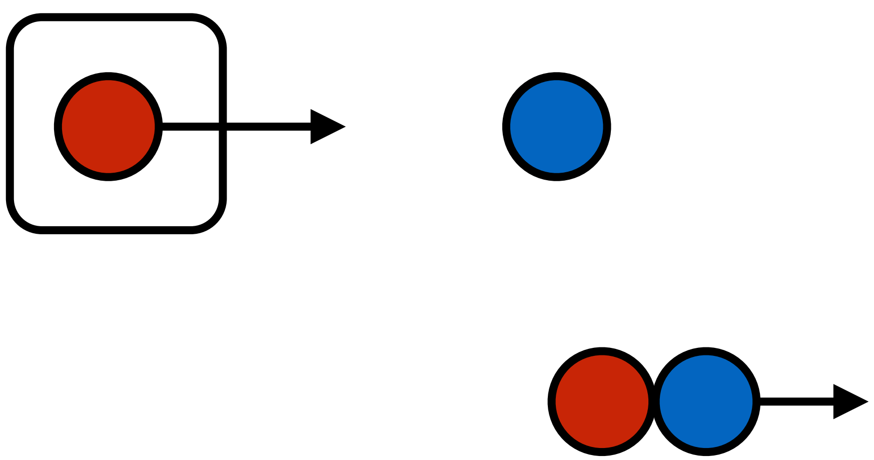
Choice of system is important

For which choice of system will momentum be conserved?

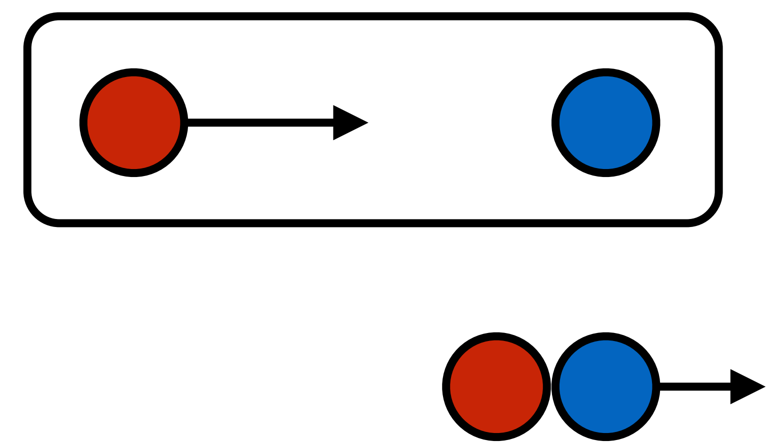
Choice of system is important

For which choice of system will momentum be conserved?

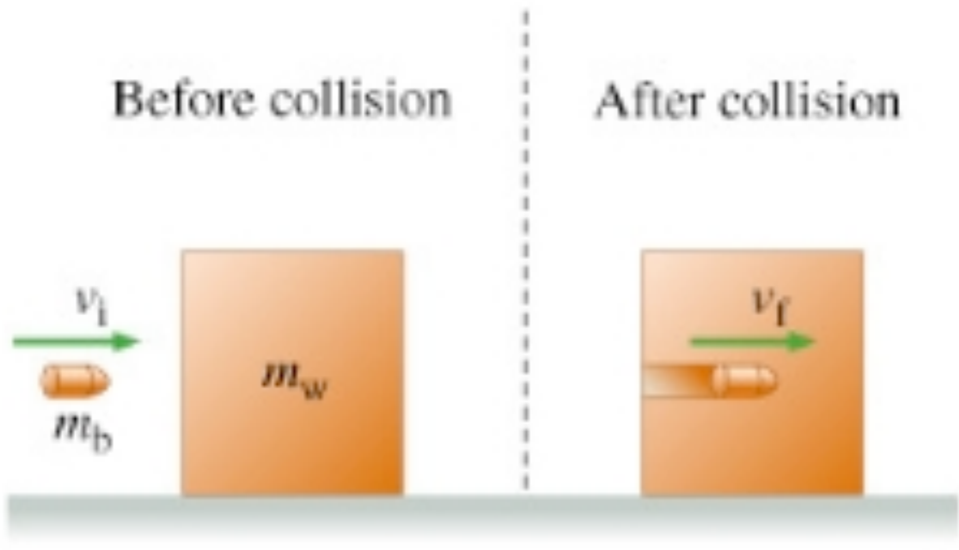
Momentum is not conserved

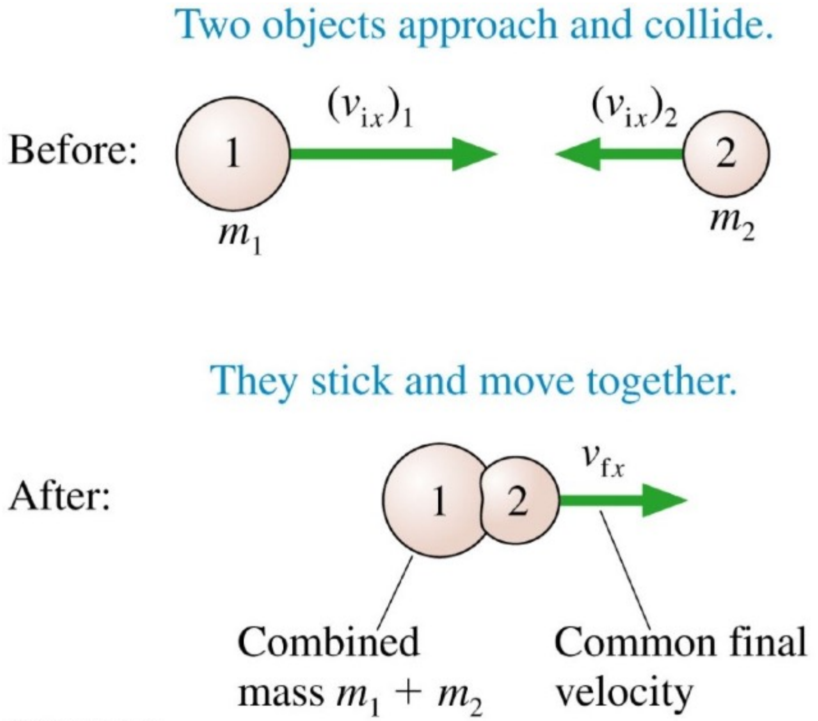


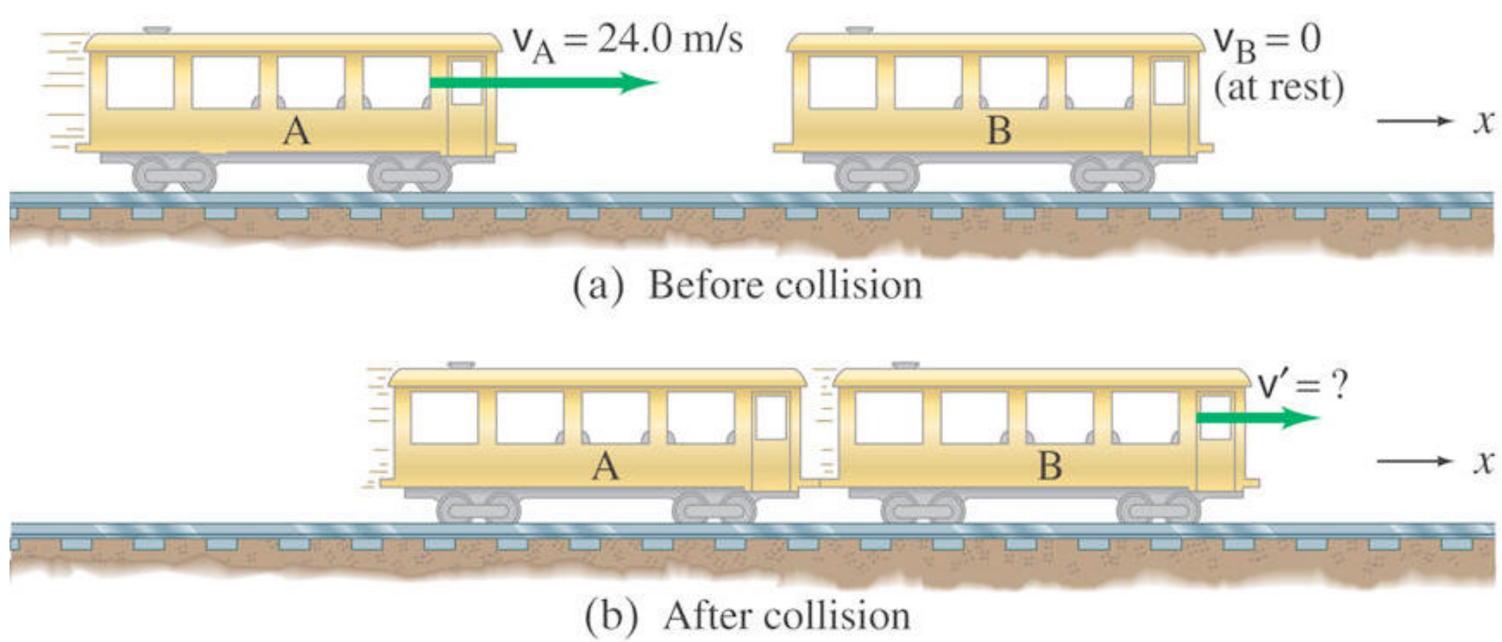
Momentum is conserved



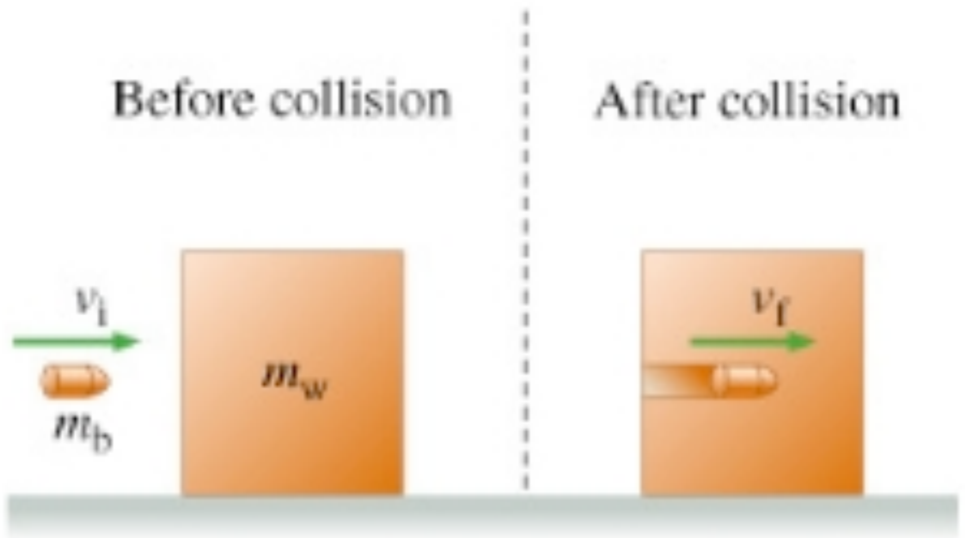
What do these three situations have in common?

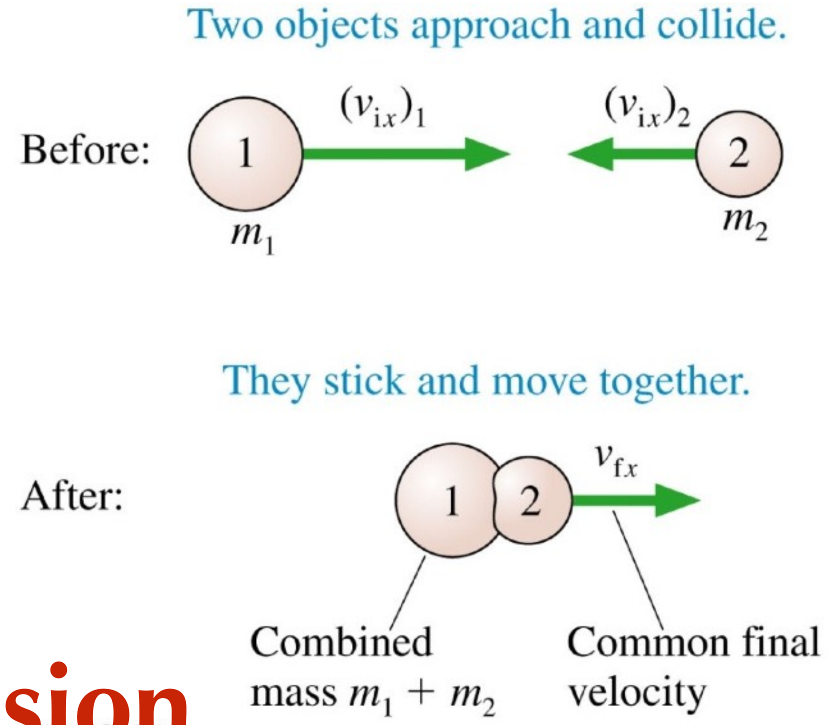




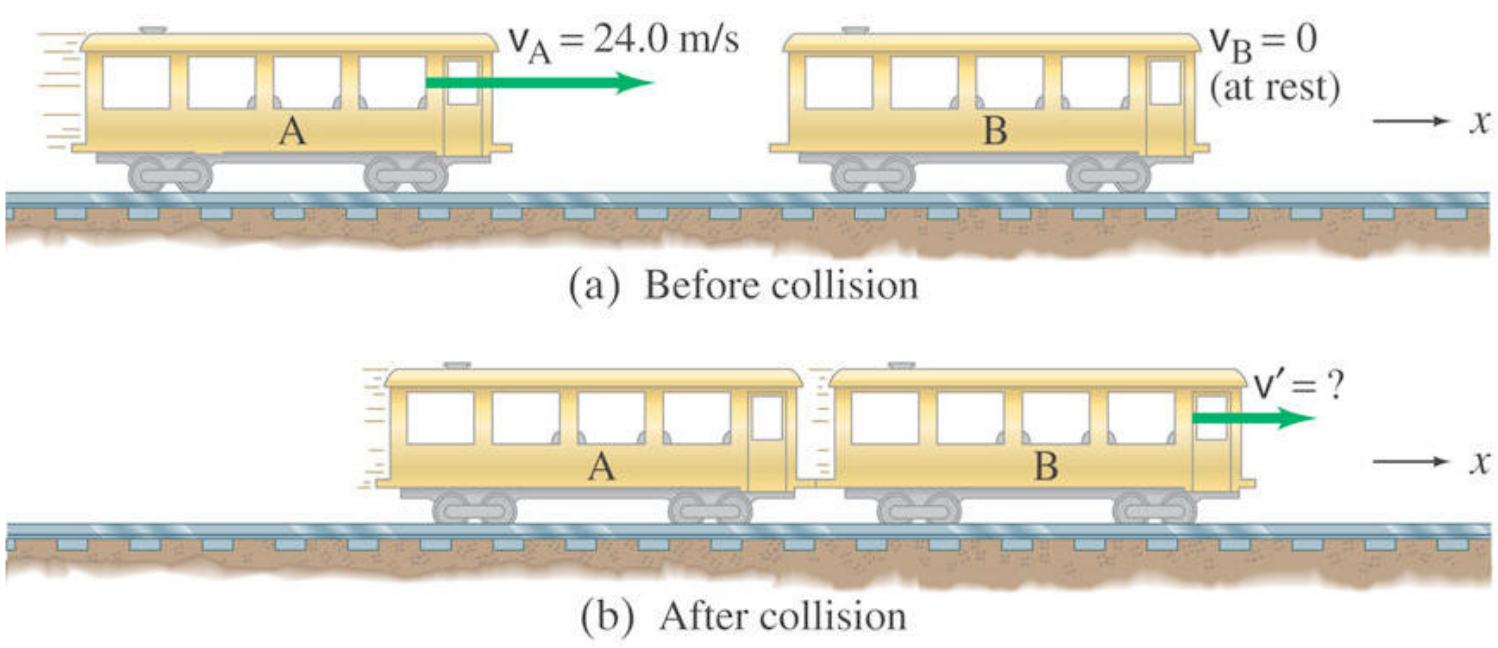


What do these three situations have in common?



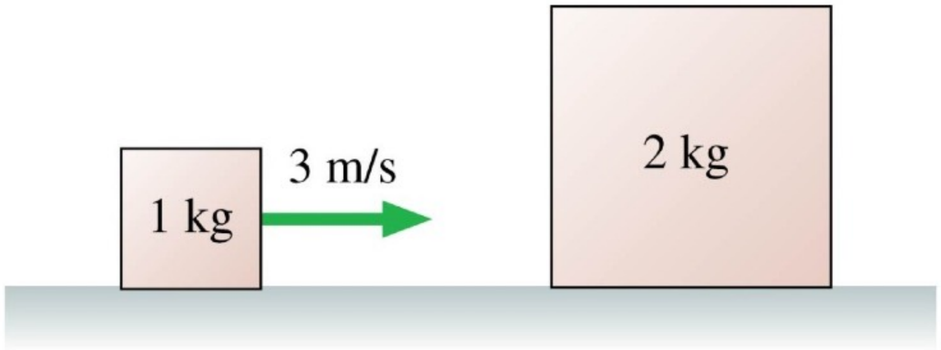


Inelastic collision



Quiz

The 1 kg box is sliding along a frictionless surface. It collides with and sticks to the 2 kg box. Afterward, the speed of the two boxes is



a. 0 m/s.

b. 1 m/s.

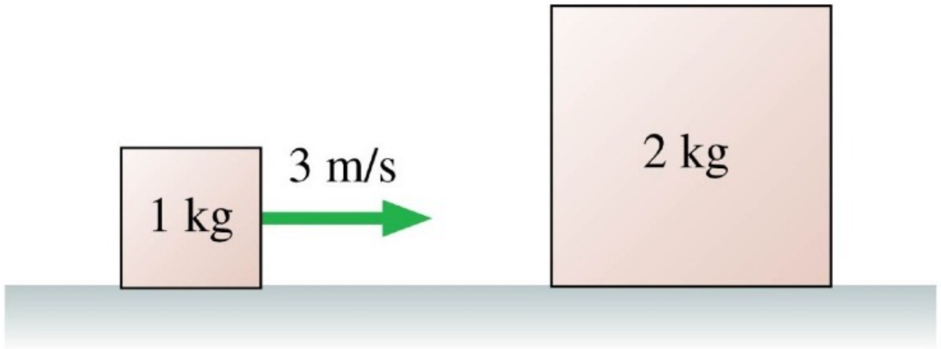
c. 2 m/s.

d. 3 m/s.

e. There's not enough information to tell.

Quiz

The 1 kg box is sliding along a frictionless surface. It collides with and sticks to the 2 kg box. Afterward, the speed of the two boxes is



a. 0 m/s.

b. 1 m/s.

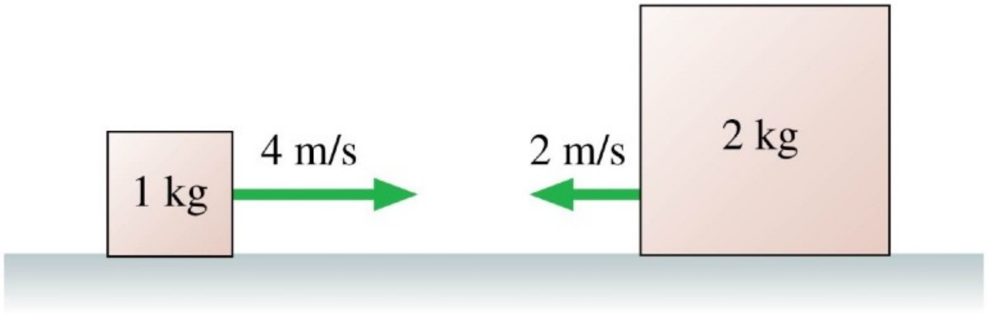
c. 2 m/s.

d. 3 m/s.

e. There's not enough information to tell.

Quiz

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the
two boxes is



a. 2 m/s to the left.

b. 1 m/s to the left.

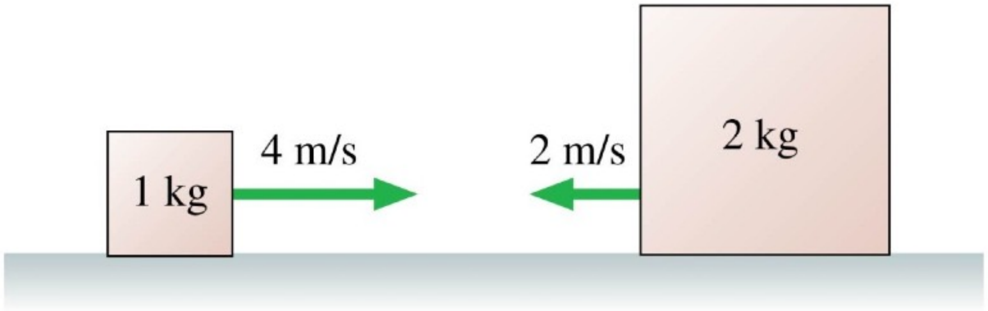
c. 0 m/s, at rest.

d. 1 m/s to the right.

e. 2 m/s to the right.

Quiz

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the
two boxes is



a. 2 m/s to the left.

b. 1 m/s to the left.

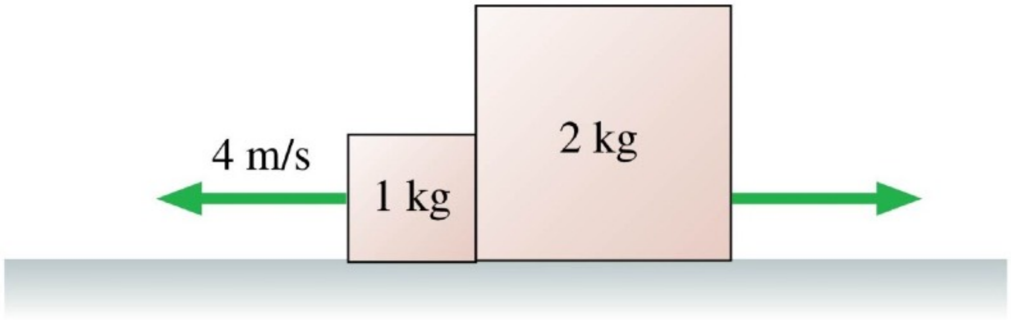
c. 0 m/s, at rest.

d. 1 m/s to the right.

e. 2 m/s to the right.

Quiz

The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2 kg box going?



A. 1 m/s.

B. 2 m/s.

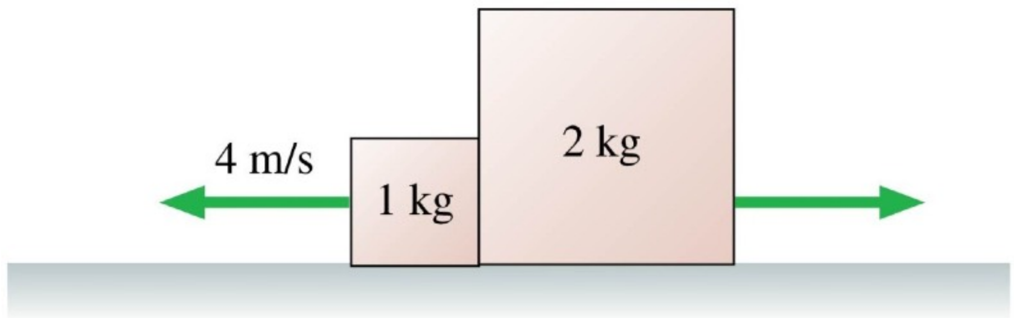
C. 4 m/s.

D. 8 m/s.

E. There's not enough information to tell.

Quiz

The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2 kg box going?



A. 1 m/s.

B. 2 m/s.

C. 4 m/s.

D. 8 m/s.

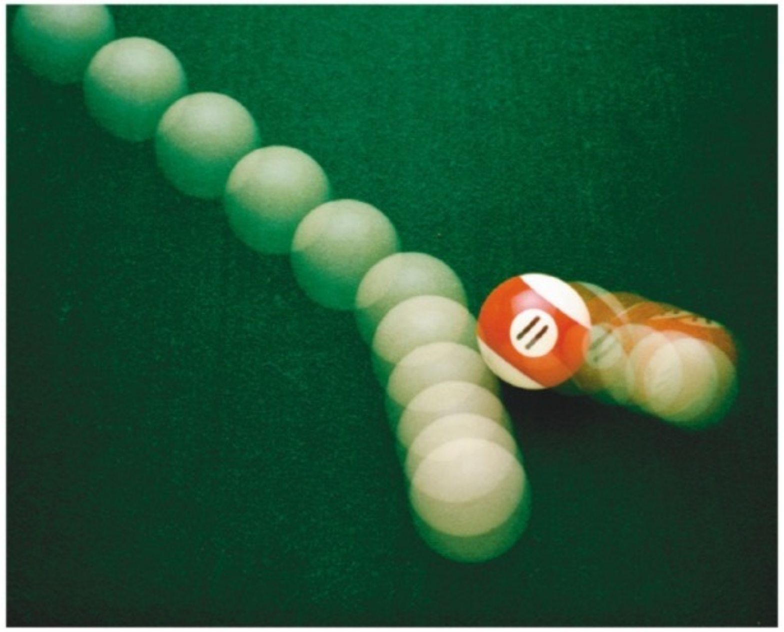
E. There's not enough information to tell.

Try another one

Two ice skaters, with masses 50 kg and 75 kg, are at the center of a 60-m-diameter circular rink. The skaters push off against each other and glide to opposite edges of the rink. If the heavier skater reaches the edge in 20 s, how long does the other skater take to reach the edge?

Momentum in two dimensions

- The total momentum is a vector sum of the momenta of the individual particles.



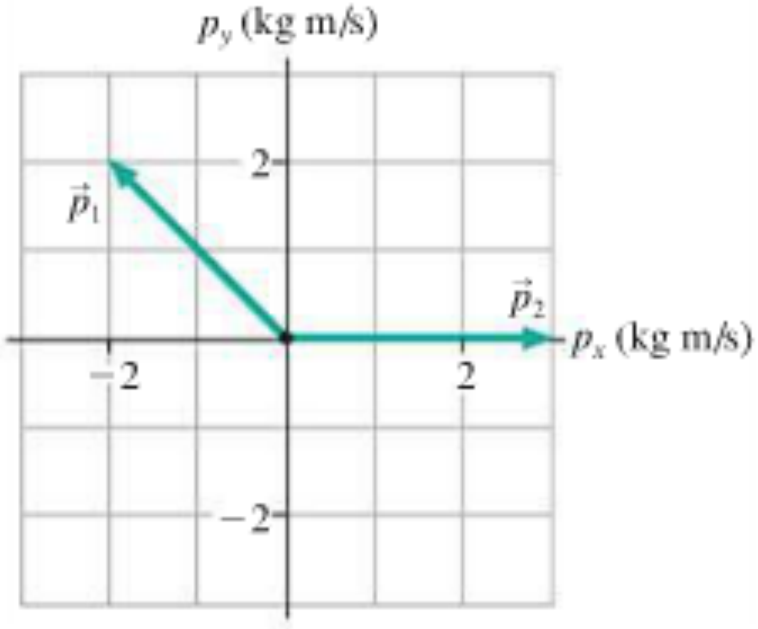
- Momentum is conserved only if each component of is conserved:

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

Quiz

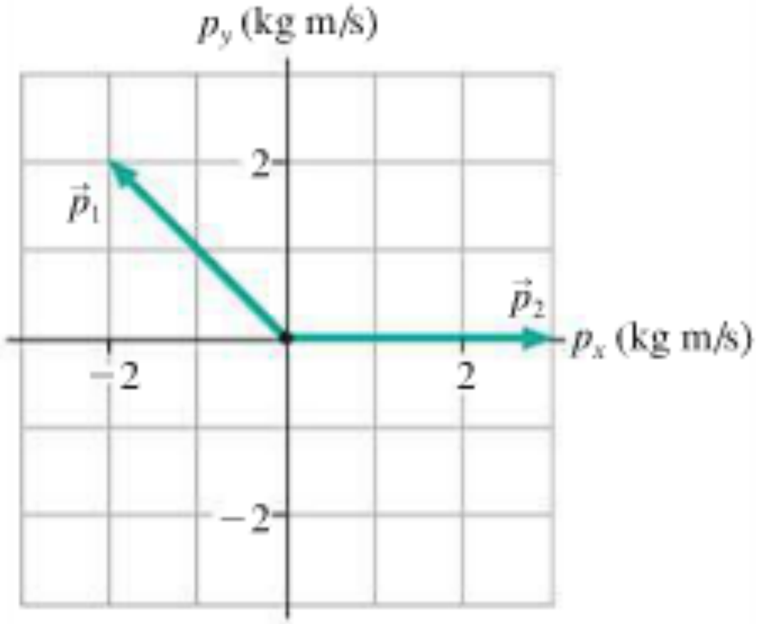
An object at rest explodes into three fragments. The figure shows the momentum vectors of two of the fragments. What are p_x and p_y of the third fragment?



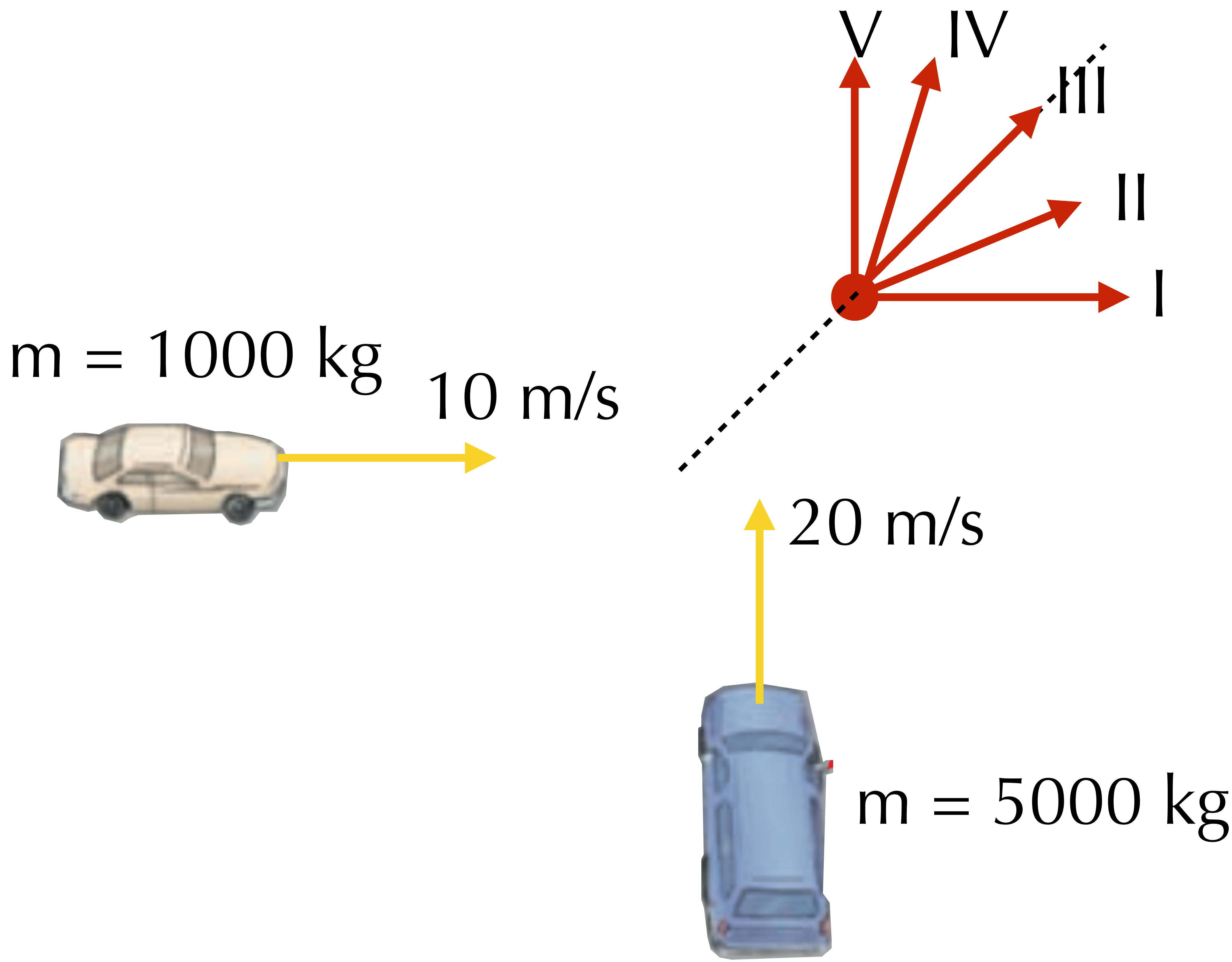
a. $p_x = 2$, $p_y = 3$
b. $p_x = 1$, $p_y = 2$
c. $p_x = -1$, $p_y = -2$
d. $p_x = -2$, $p_y = -1$
e. $p_x = 2$, $p_y = 1$

Quiz

An object at rest explodes into three fragments. The figure shows the momentum vectors of two of the fragments. What are p_x and p_y of the third fragment?

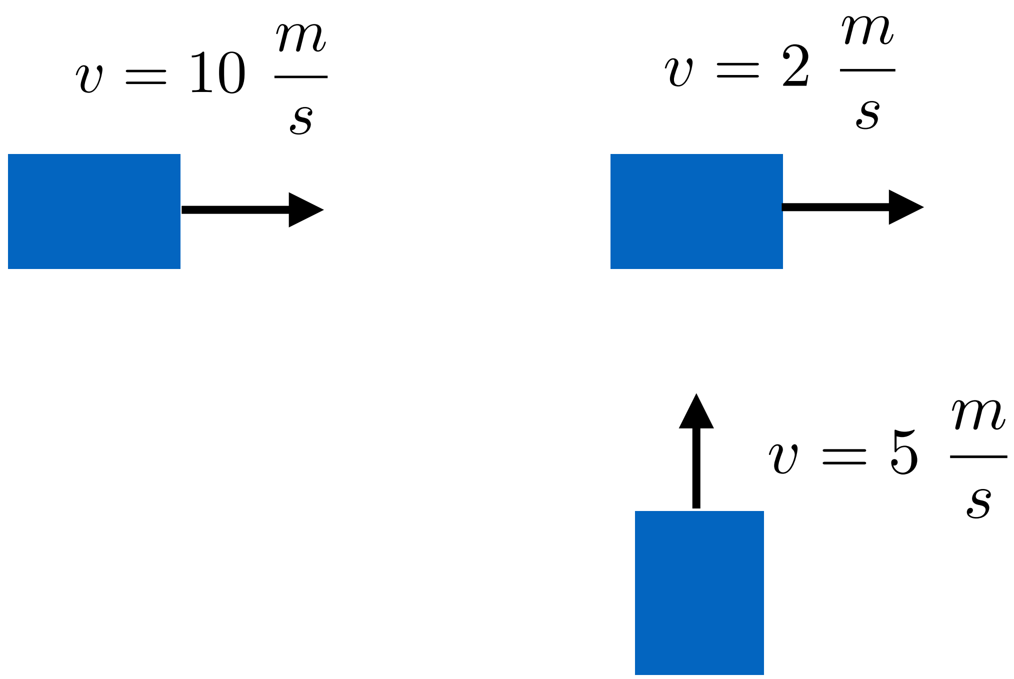


a. $p_x = 2$, $p_y = 3$
b. $p_x = 1$, $p_y = 2$
c. $p_x = -1$, $p_y = -2$
d. $p_x = -2$, $p_y = -1$
e. $p_x = 2$, $p_y = 1$


V
IV
III
II
I
m = 1000 kg
10 m/s
20 m/s
m = 5000 kg

Problem

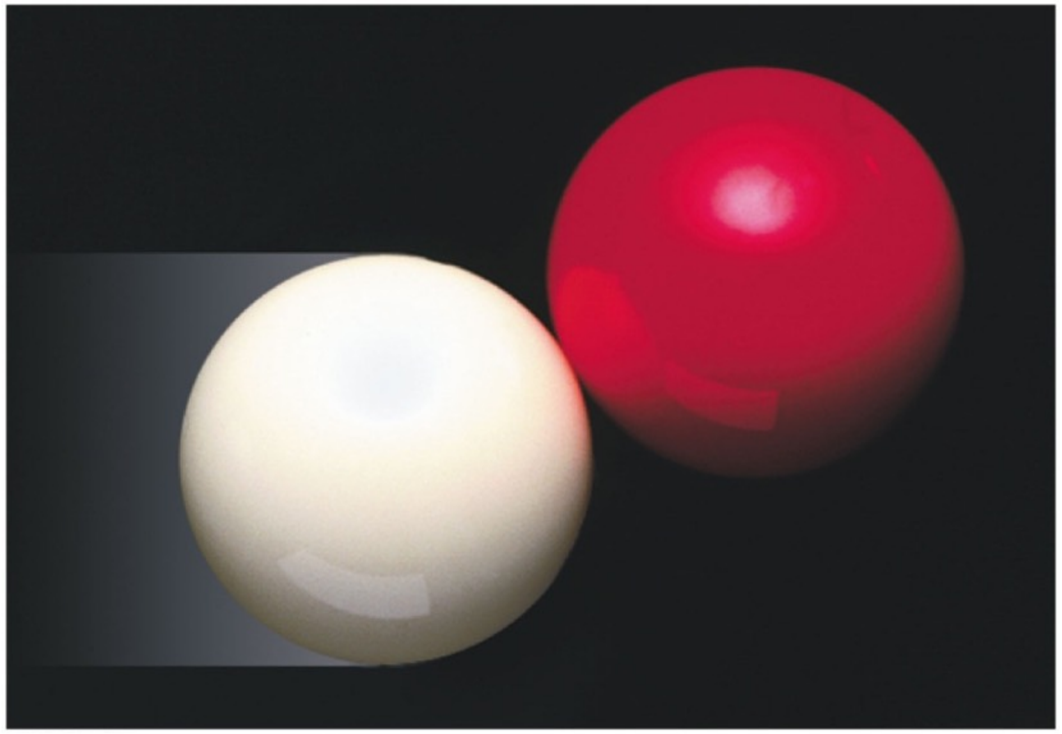
A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously by two cars. One car is a 1200 kg compact traveling north at 5.0 m/s. The other is a 1500 kg midsize traveling east at 10 m/s. The three vehicles become entangled and slide as one body. What are their speed and direction just after the collision



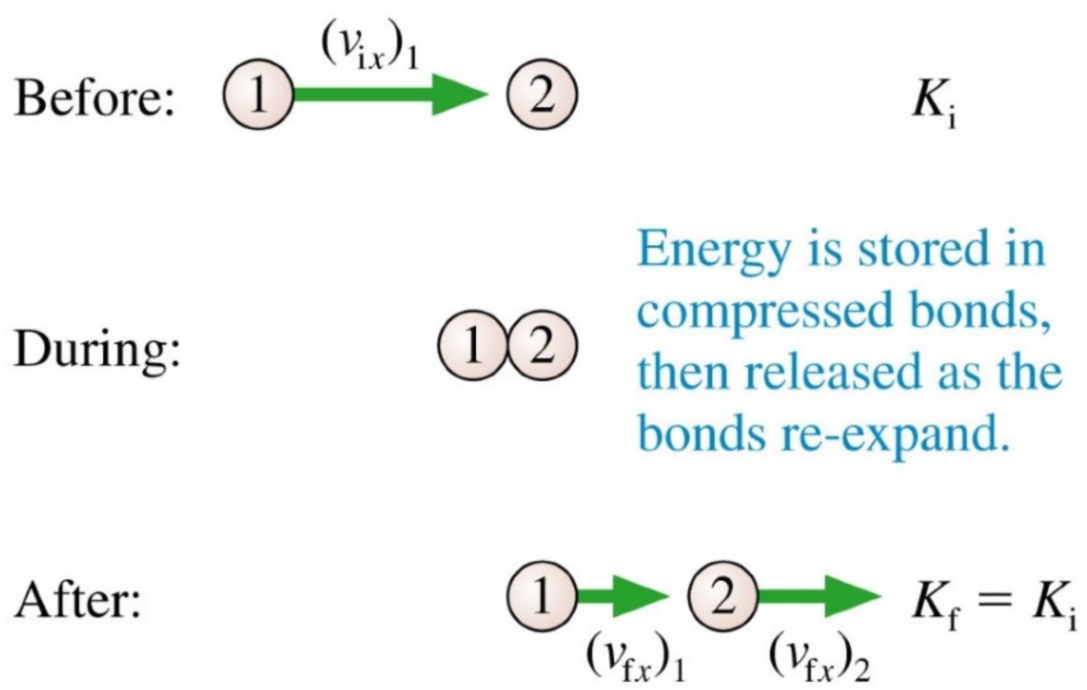
Elastic Collisions

For most collisions, some of the mechanical energy is dissipated inside the objects as thermal energy

A collision in which mechanical energy is conserved is a **perfectly elastic collision**



A perfectly elastic collision

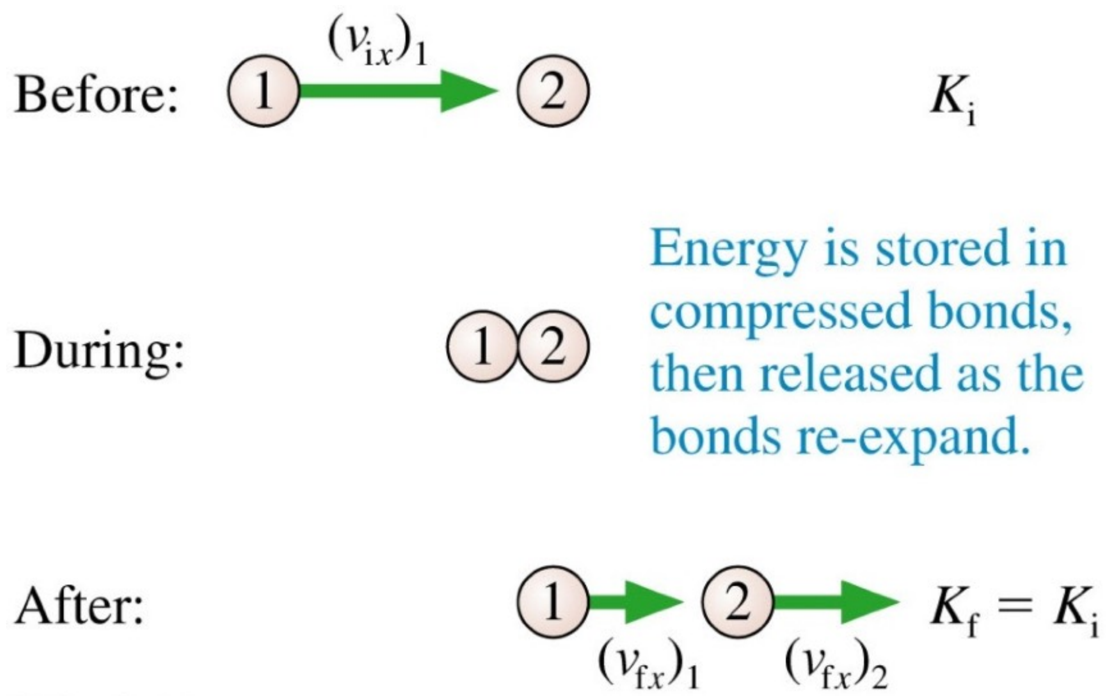


- Momentum is conserved in all isolated collisions.
- In a perfectly elastic collision, the kinetic energy must also be conserved.

momentum conservation: $m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$

energy conservation: $\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

A perfectly elastic collision



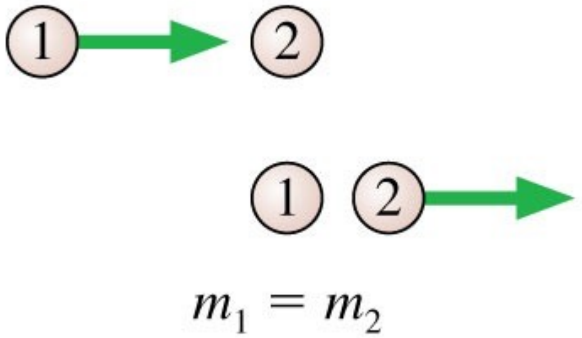
$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

A perfectly elastic collision: special case 1

$$\boxed{\begin{aligned}(v_{fx})_1 &= \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \\ (v_{fx})_2 &= \frac{2m_1}{m_1 + m_2}(v_{ix})_1\end{aligned}}$$

$$\underline{m_1 = m_2}$$



Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

The first balls stops and transfers all of its momentum to the second particle.

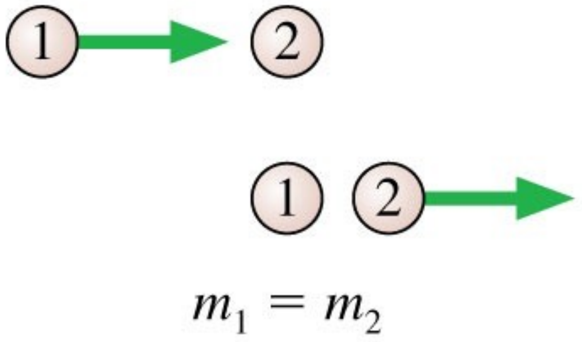
A perfectly elastic collision: special case 1

$$\boxed{\begin{aligned}(v_{fx})_1 &= \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \\[2ex] (v_{fx})_2 &= \frac{2m_1}{m_1 + m_2}(v_{ix})_1\end{aligned}}$$

$$\underline{\;m_1 = m_2\;}$$

$$(v_{fx})_1 = 0$$

$$(v_{fx})_2 = (v_{ix})_1$$



Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

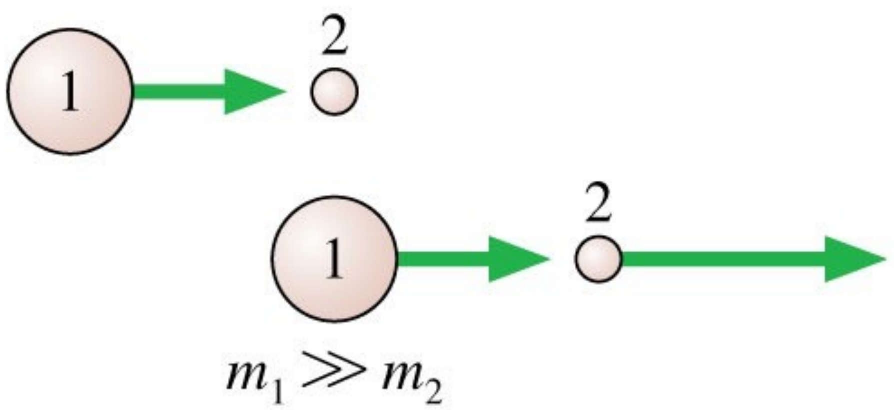
The first balls stops and transfers all of its momentum to the second particle.

A perfectly elastic collision: special case 2

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

$$m_1 \gg m_2$$



Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

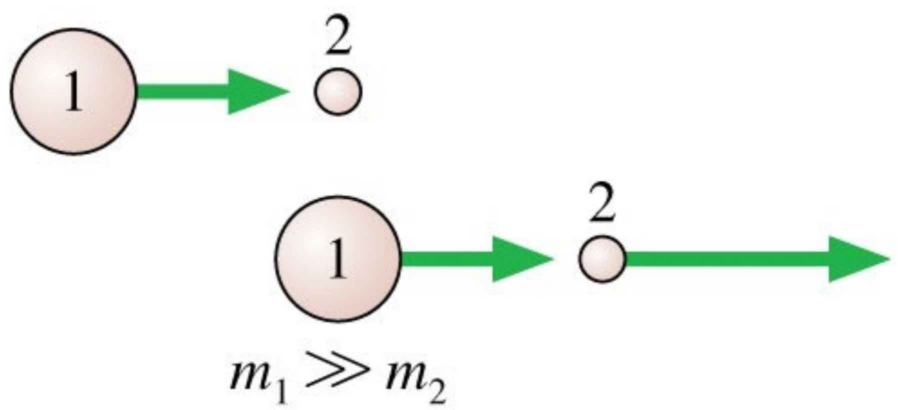
A perfectly elastic collision: special case 2

$$\boxed{\begin{aligned}(v_{fx})_1 &= \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \\ (v_{fx})_2 &= \frac{2m_1}{m_1 + m_2}(v_{ix})_1\end{aligned}}$$

$$m_1 \gg m_2$$

$$(v_{fx})_1 \approx (v_{ix})_1$$

$$(v_{fx})_2 \approx 2(v_{ix})_1$$

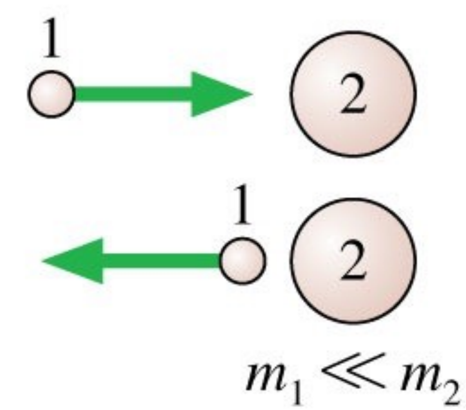


Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

A perfectly elastic collision: special case 3

$$\boxed{\begin{aligned}(v_{fx})_1 &= \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \\ (v_{fx})_2 &= \frac{2m_1}{m_1 + m_2}(v_{ix})_1\end{aligned}}$$

$$m_1 \ll m_2$$



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

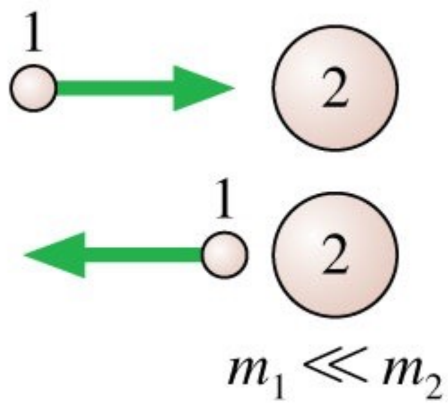
A perfectly elastic collision: special case 3

$$\boxed{\begin{aligned}(v_{fx})_1 &= \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \\ (v_{fx})_2 &= \frac{2m_1}{m_1 + m_2}(v_{ix})_1\end{aligned}}$$

$$m_1 \ll m_2$$

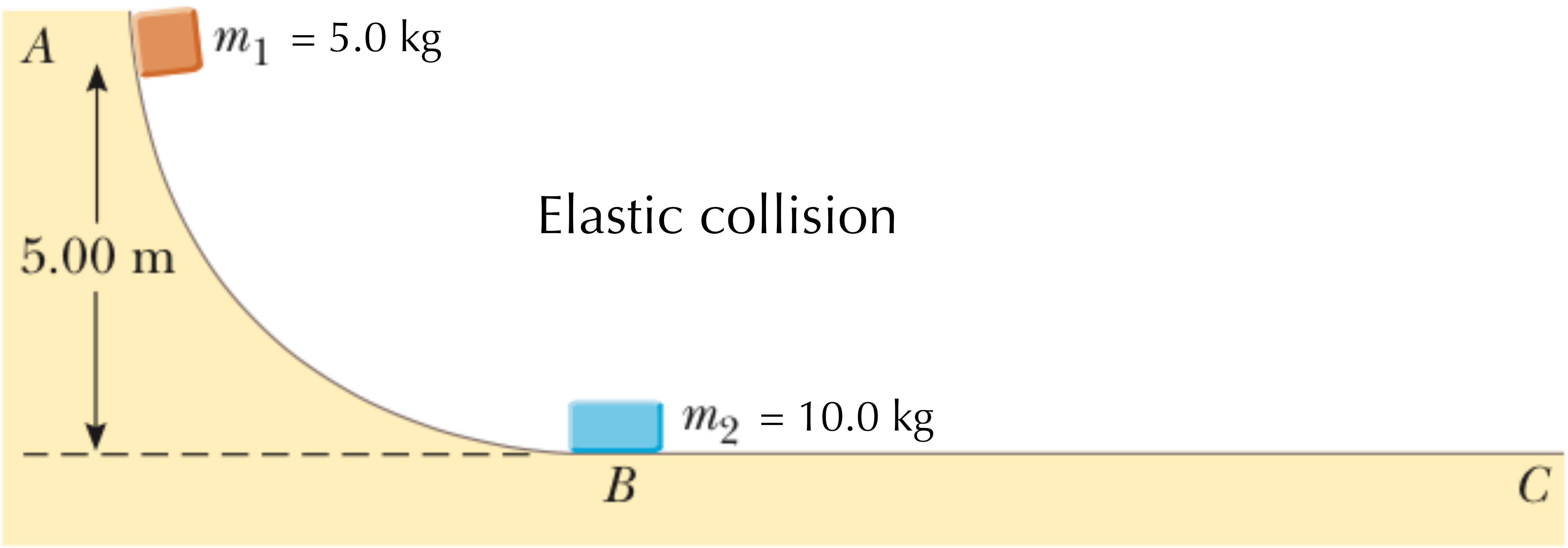
$$(v_{fx})_1 \approx -(v_{ix})_1$$

$$(v_{fx})_2 \approx 0$$



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

What is the max height of m_1 after the collision?

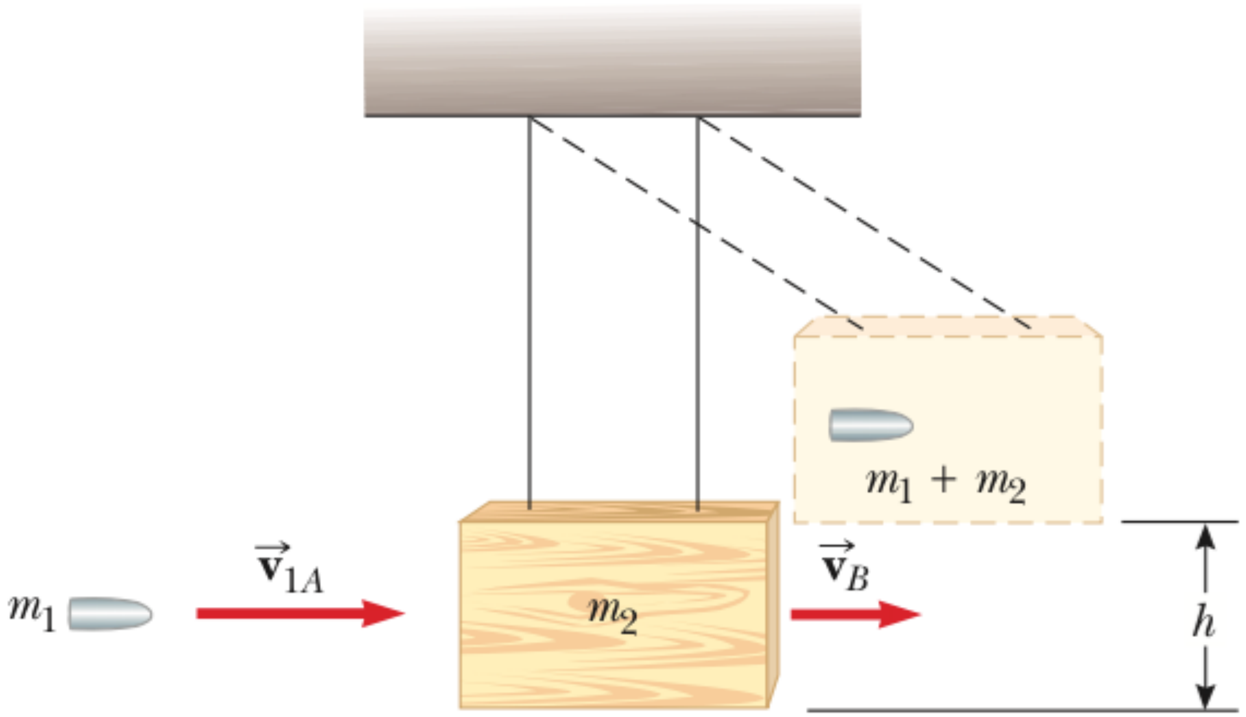


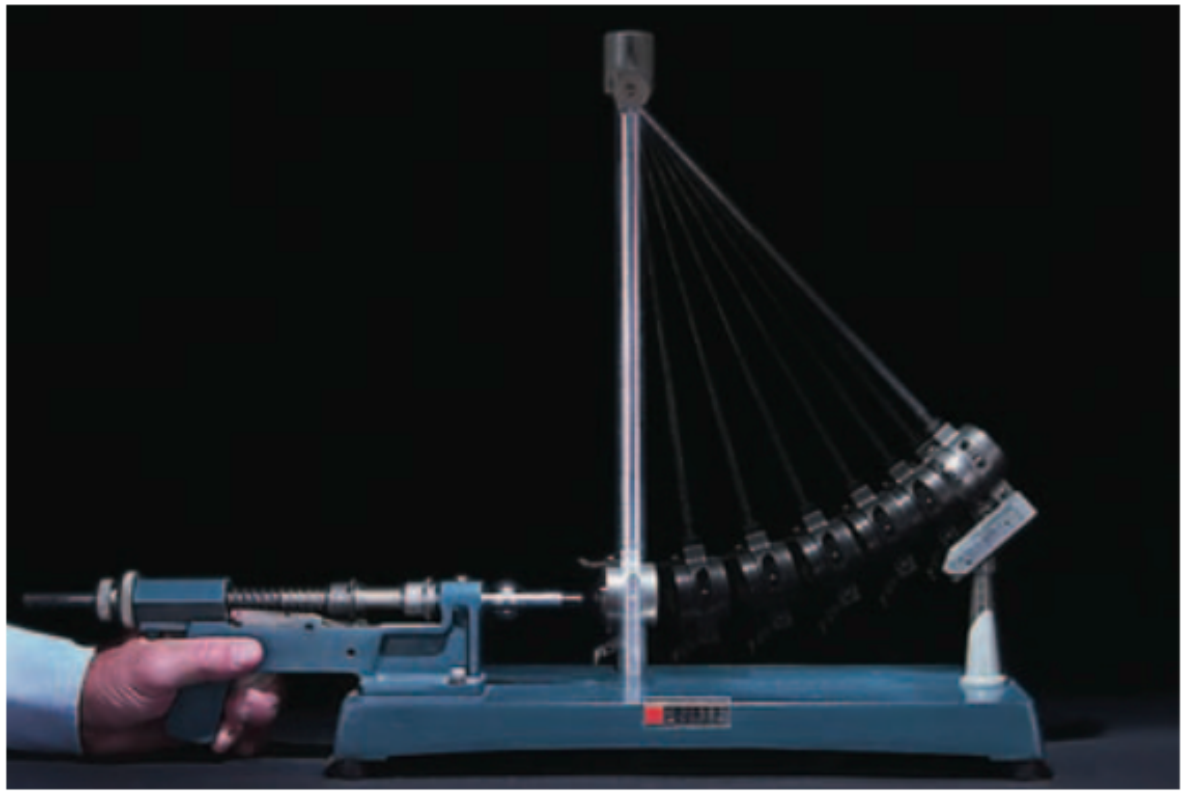
Ballistic Pendulum

Symbolic problem alert

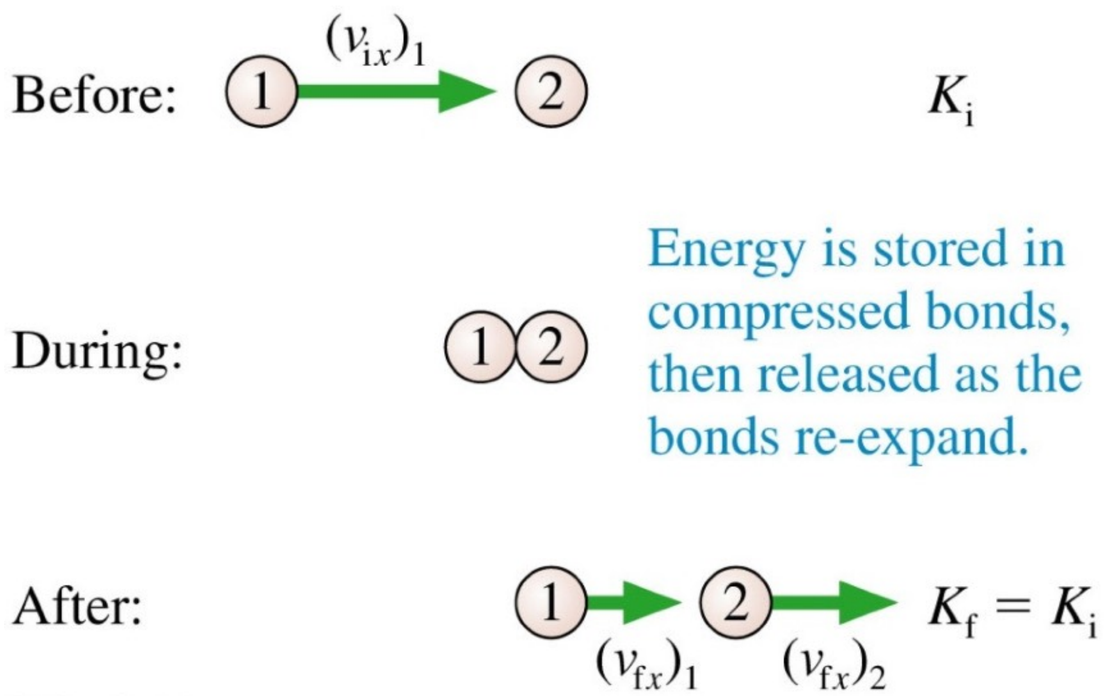
If you knew: m_1, m_2, and h, find an expression for v_{1A}, the initial speed of the bullet.

Think about: Which principle applies to which part of the problem?





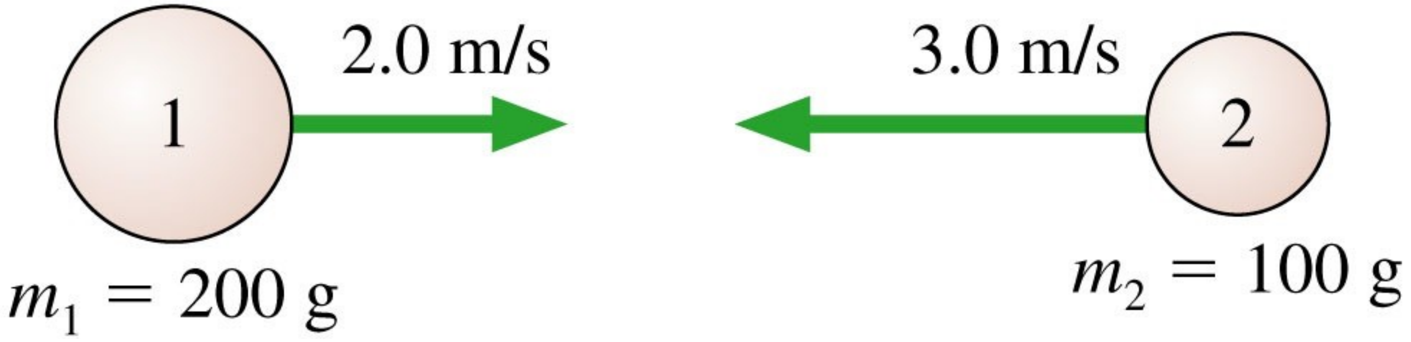
Perfectly elastic collisions: Using reference frames



$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

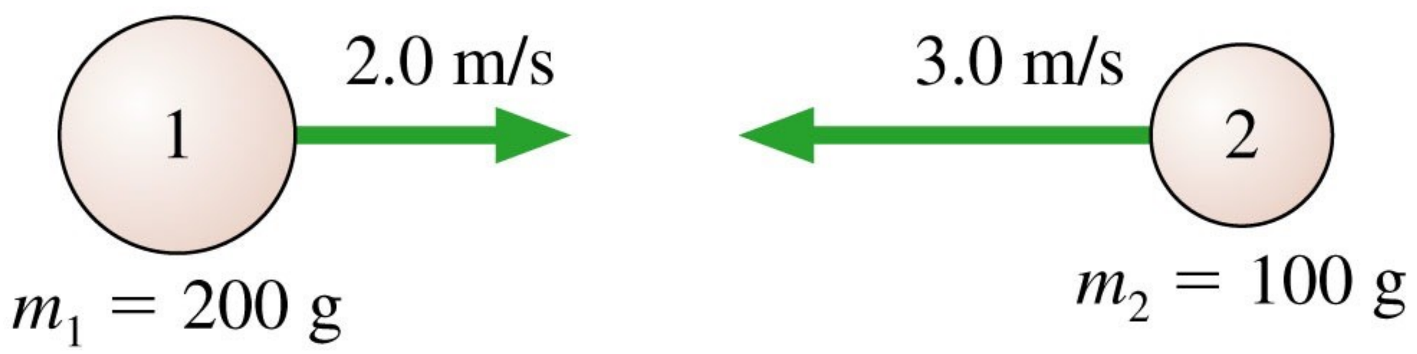
$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

What if.....



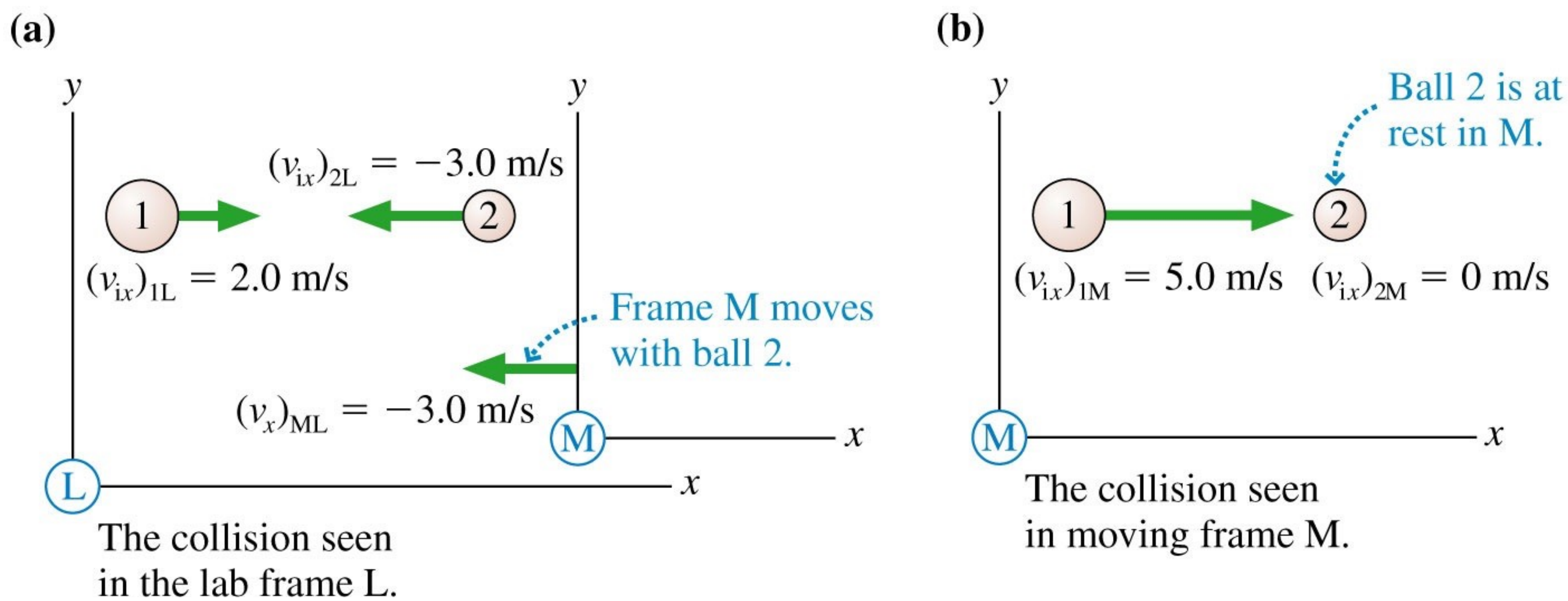
Using Reference Frames: Quick Example

A 200 g ball moves to the right at 2.0 m/s. It has a head on, perfectly elastic collision with a 100 g ball that is moving toward it at 3.0 m/s. What are the final velocities of both balls?



Using Reference Frames: Quick Example

$$(v_{ix})_{1M} = (v_{ix})_{1L} + (v_x)_{LM} = 2.0\text{ m/s} + 3.0\text{ m/s} = 5.0\text{ m/s}$$
$$(v_{ix})_{2M} = (v_{ix})_{2L} + (v_x)_{LM} = -3.0\text{ m/s} + 3.0\text{ m/s} = 0\text{ m/s}$$



Using Reference Frames: Quick Example

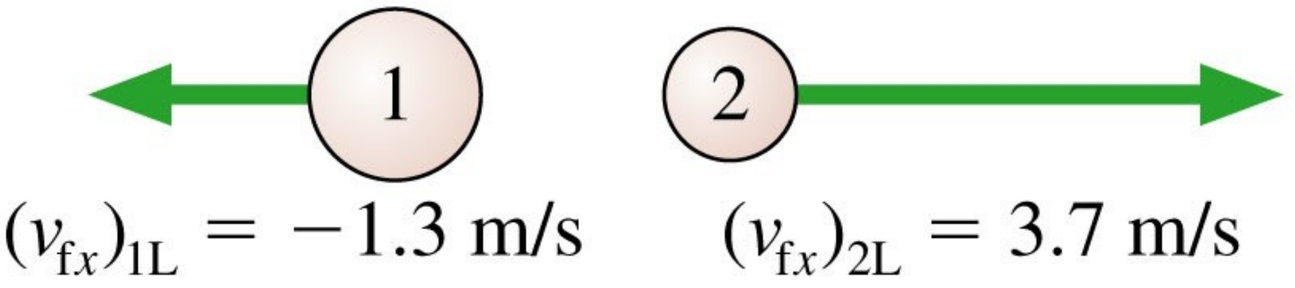
- We can use Equations 10.42 to find the post-collision velocities in the moving frame M:

$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_{1M} = 1.7 \text{ m/s}$$

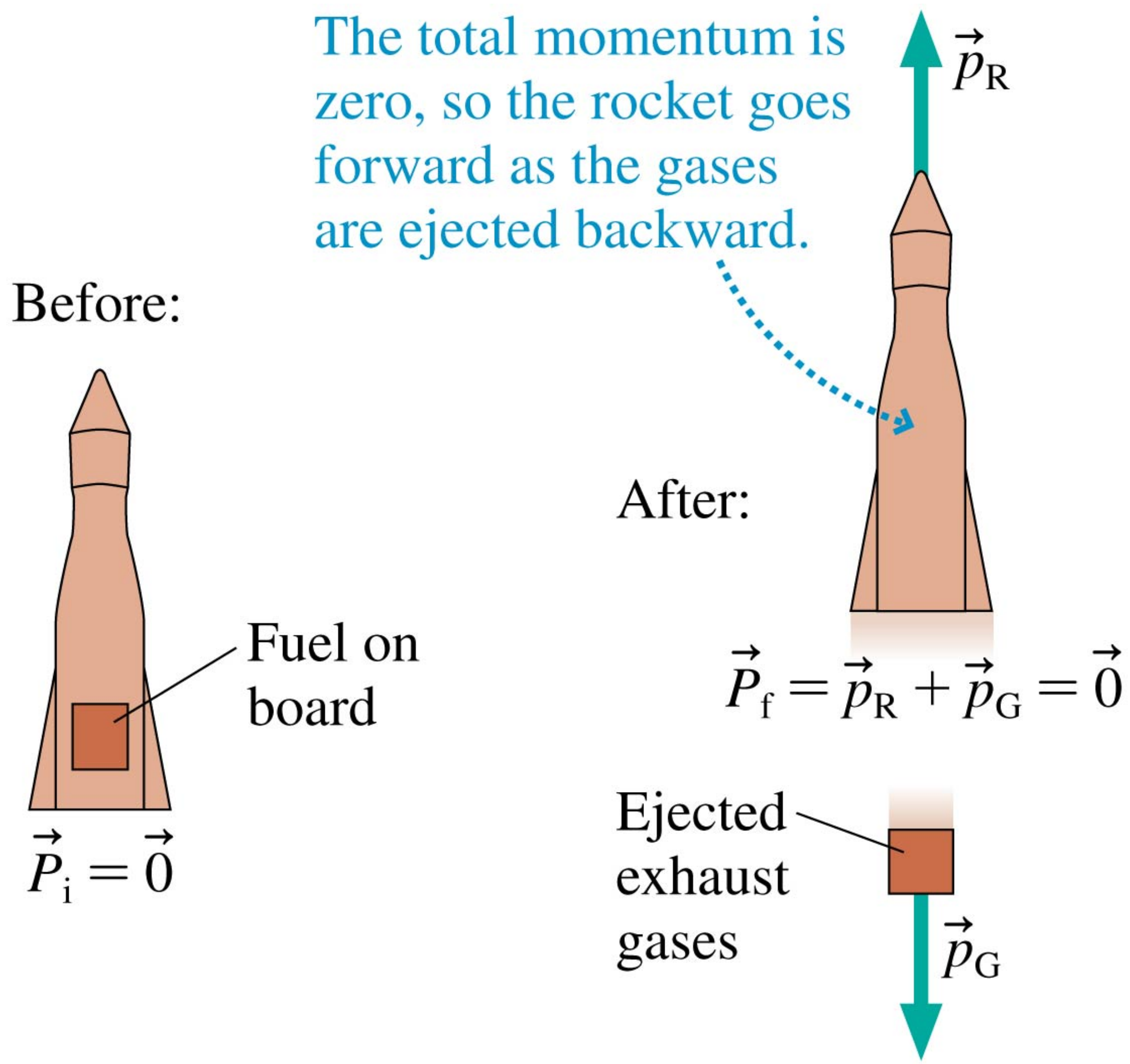
$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2}(v_{ix})_{1M} = 6.7 \text{ m/s}$$

- Transforming back to the lab frame L:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = 1.7 \text{ m/s} + (-3.0 \text{ m/s}) = -1.3 \text{ m/s}$$
$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 6.7 \text{ m/s} + (-3.0 \text{ m/s}) = 3.7 \text{ m/s}$$



Rocket propulsion



New Equations (Chapters 9 & 10)

$$\vec{p} = m\vec{v}$$

$$K = \frac{1}{2}mv^2$$

$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{i1}$$

$$J = \int_{t_i}^{t_f} F dt$$

$$U_g = mgy$$

$$J = \Delta p$$

$$F = -k\Delta x$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{i1}$$

$$\vec{P}_i = \vec{P}_f$$

$$U_s = \frac{1}{2}k\Delta x^2$$

Conservation of Energy

New Equations (Chapters 9 & 10)

$\vec{p} = m\vec{v}$ 5

$K = \frac{1}{2}mv^2$ 9

$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2}v_{i1}$ 7

8 $J = \int_{t_i}^{t_f} F dt$

$U_g = mgy$ 4

$J = \Delta p$ 2

$F = -k\Delta x$ 10

$v_{f2} = \frac{2m_1}{m_1 + m_2}v_{i1}$ 3

$\vec{P}_i = \vec{P}_f$ 1

$U_s = \frac{1}{2}k\Delta x^2$ 11

Conservation of Energy 6