Dynamics in two-dimensions

We have done:

kinematics in two dimensions

Newton's second law in one dimension

$$(F_{\text{net}})_x = \sum F_x = ma_x$$

$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\,\Delta t \qquad v_{fy} = v_{iy} + a_y\,\Delta t$$

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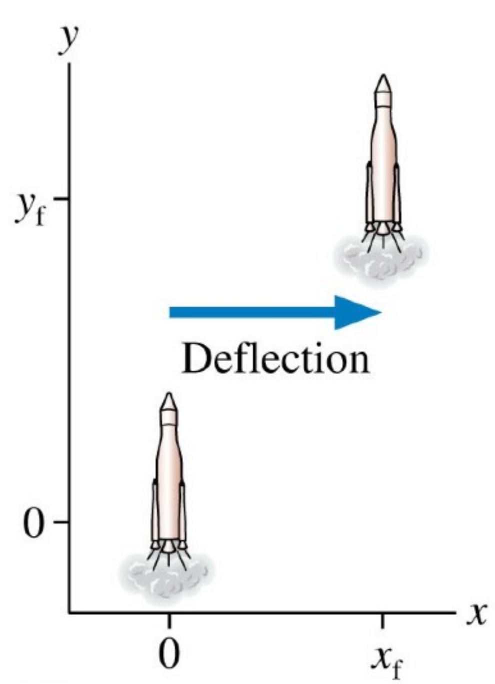
As an example

A small rocket for gathering weather data has a mass of 30 kg and generates 1500 N of thrust. On a windy day, the wind exerts a 20 N horizontal force on the rocket. If the rocket is launched straight up, by how much has it been deflected sideways when it reaches a height of 1.0 km?

- Draw a free body diagram for the rocket.
- Apply Newton's second law ($F_{net} = ma$) to the rocket in the y-direction.
- Apply Newton's second law to the rocket in the x direction.



Known

$x_i = y_i = 0$ m
$v_{ix} = v_{iy} = 0$ m/s
$y_f = 1000$ m
$m = 30$ kg
$F_{thrust} = 1500$ N
$D = 20$ N

Find

x_f

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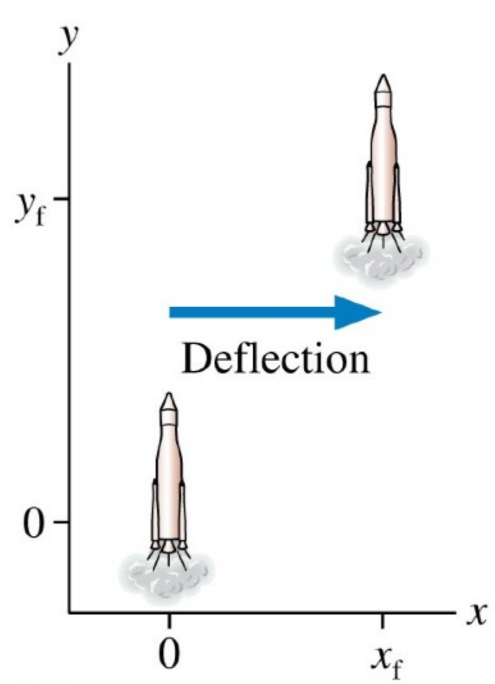

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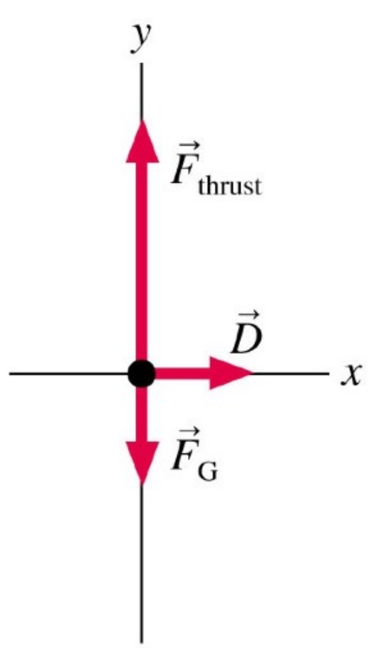
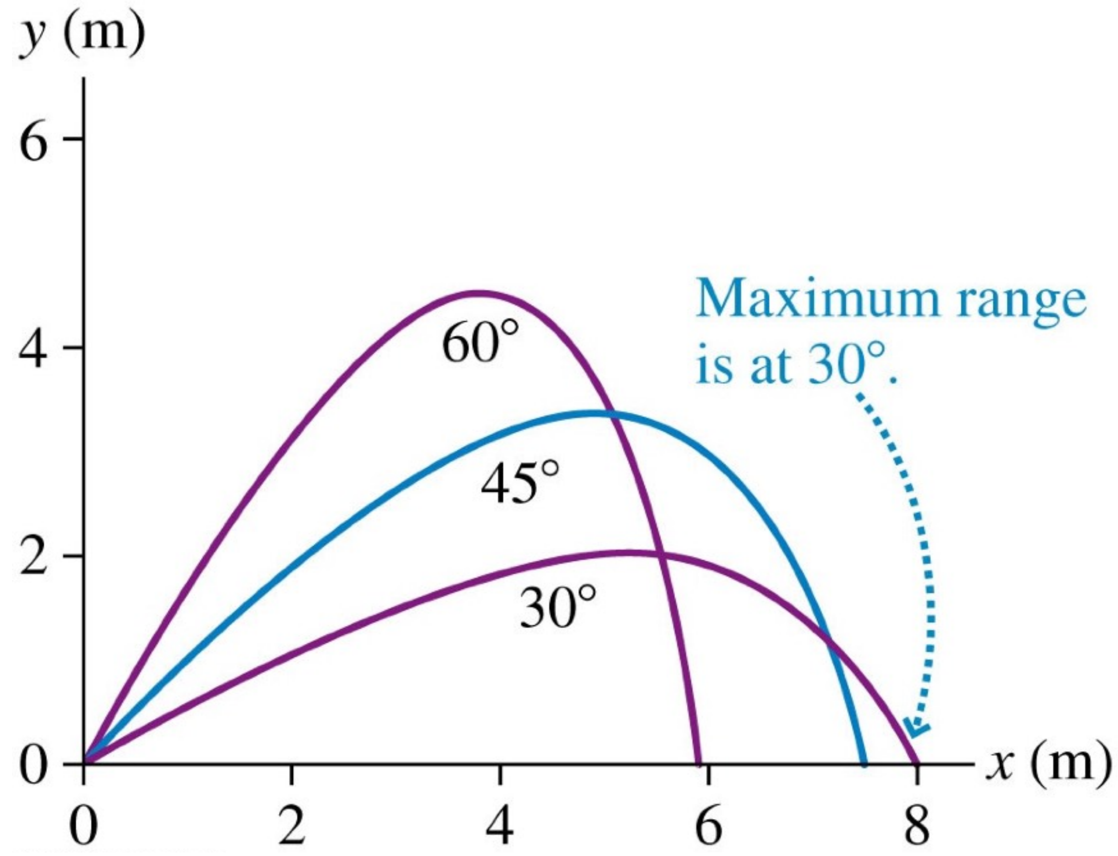


Projectile Motion with drag

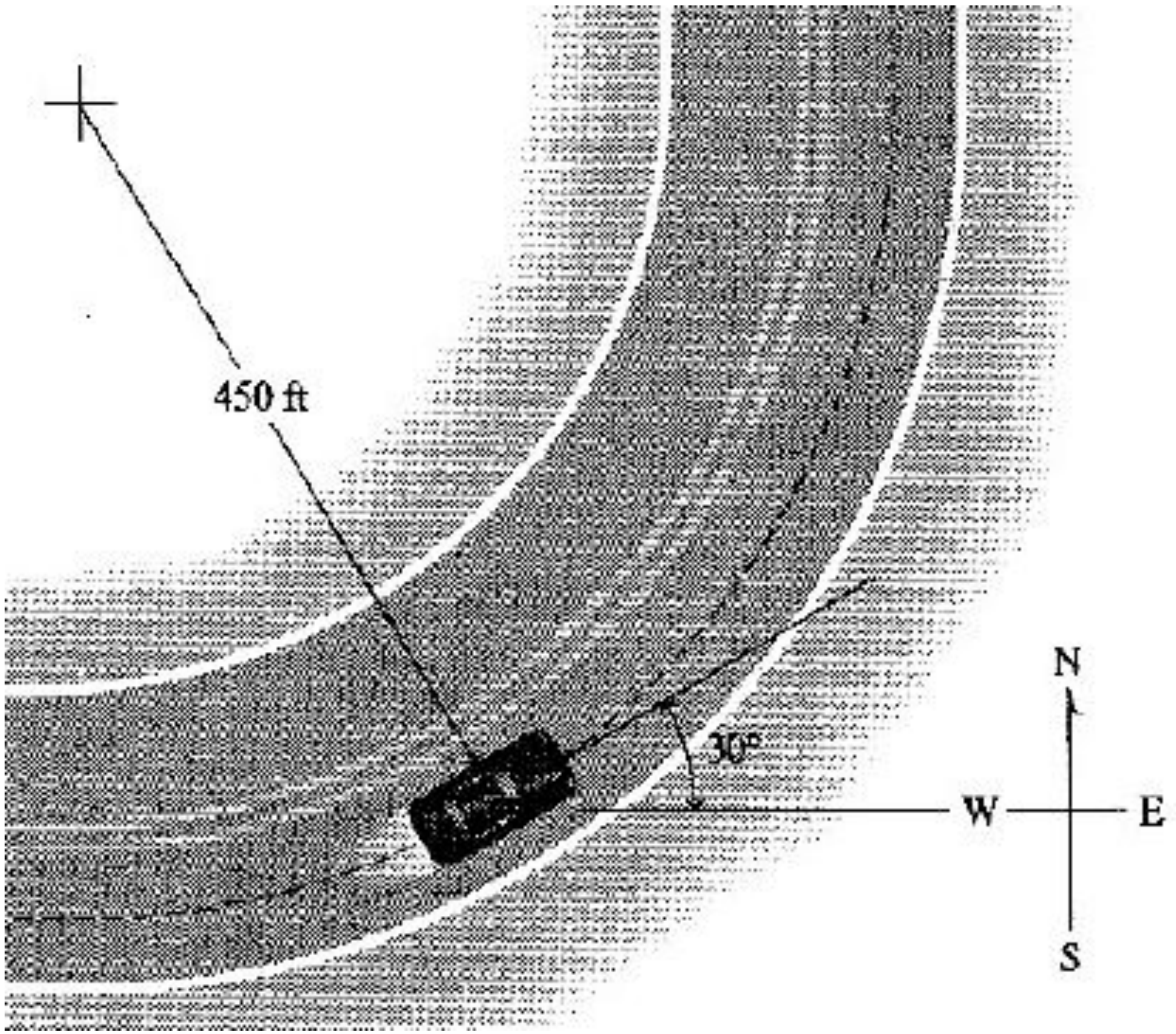


$$a_x = -\frac{\rho C A}{2m} v_x \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$a_y = -g - \frac{\rho C A}{2m} v_y \sqrt{v_x^{\,2} + v_y^{\,2}}$$

Question #16

A car travels around a curve at constant speed without sliding. What force is responsible for the acceleration of the car?



a) Static Friction
b) Normal Force
c) Tension
d) Kinetic Friction
e) Thrust

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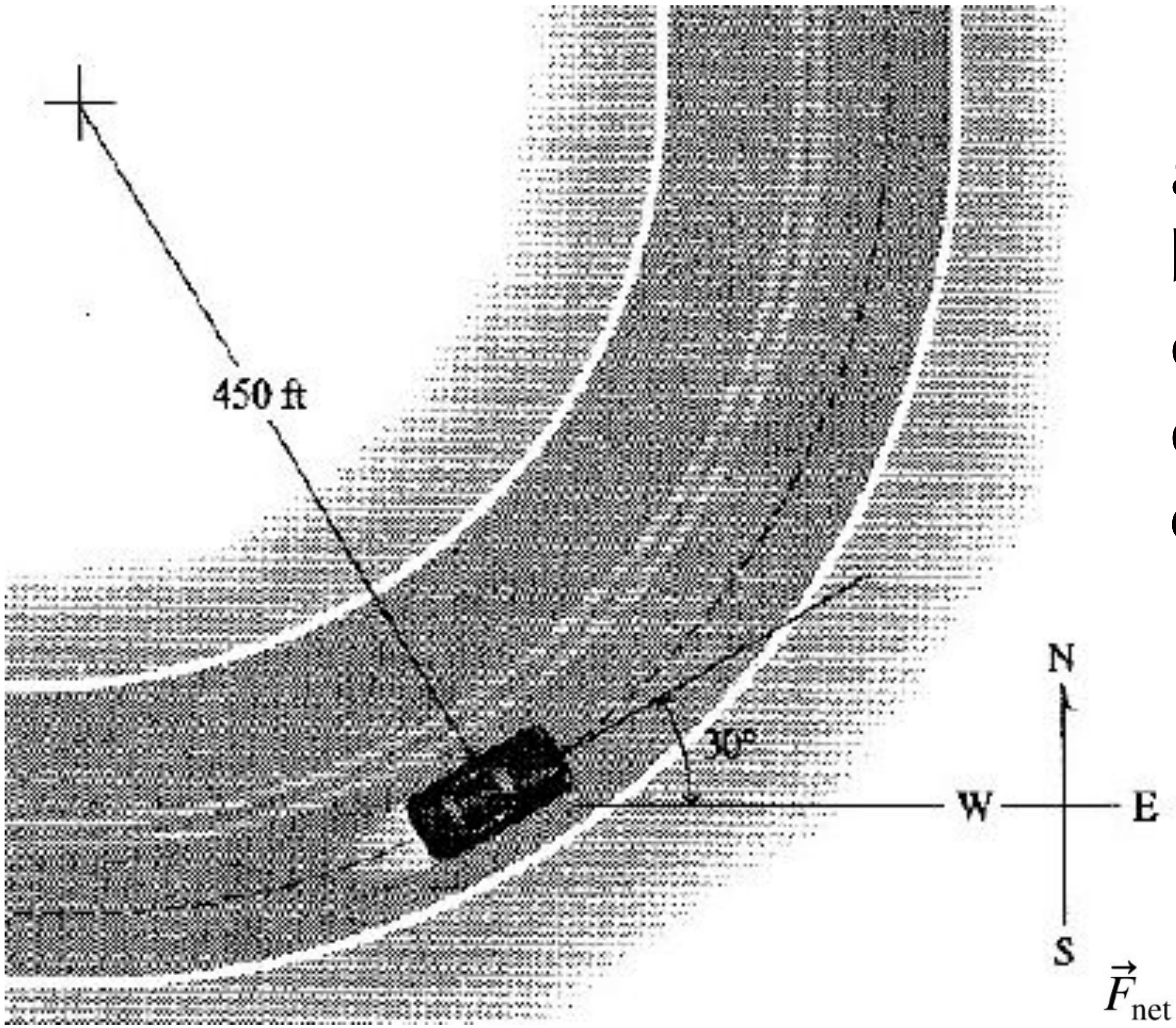

a) Static Friction
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e) Thrust

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{toward center of circle}\right)$$

Question #17

A satellite orbits the earth. What force is responsible for the acceleration of the satellite?

a) Static Friction
b) Gravity
c) Tension
d) Kinetic Friction
e) Thrust

Question #18

A little girl holds tight to the bars on a merry-go-round as it rotates steadily. What force is responsible for the acceleration of the girl?

a) Static Friction between shoes and floor
b) Gravity
c) Kinetic Friction
d) Tension in arms
e) Thrust

Quiz

Question #19

An ice hockey puck is tied by a string to a stake in the ice. The puck is then swung in a circle. What force is producing the centripetal acceleration of the puck?

A. Gravity

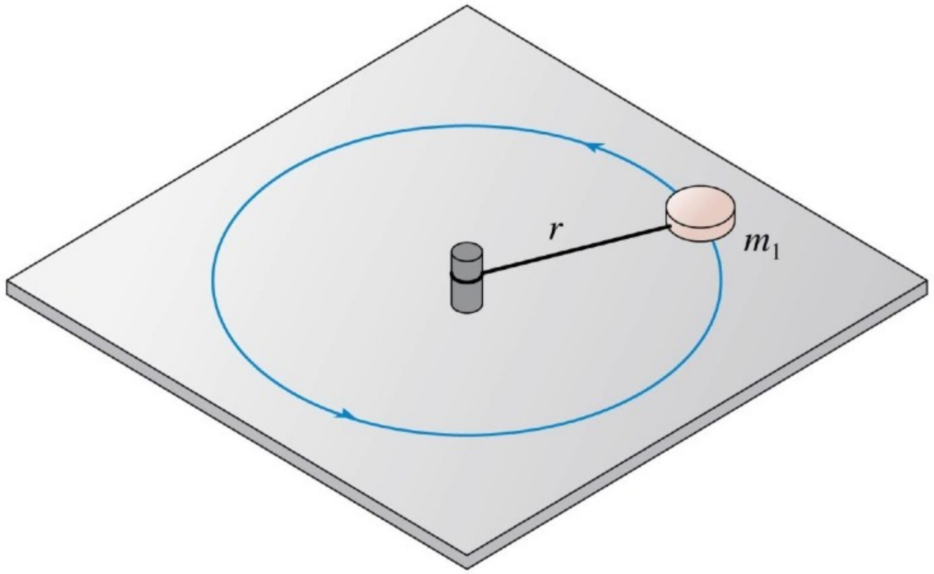
B. Air resistance

C. Friction

D. Normal force

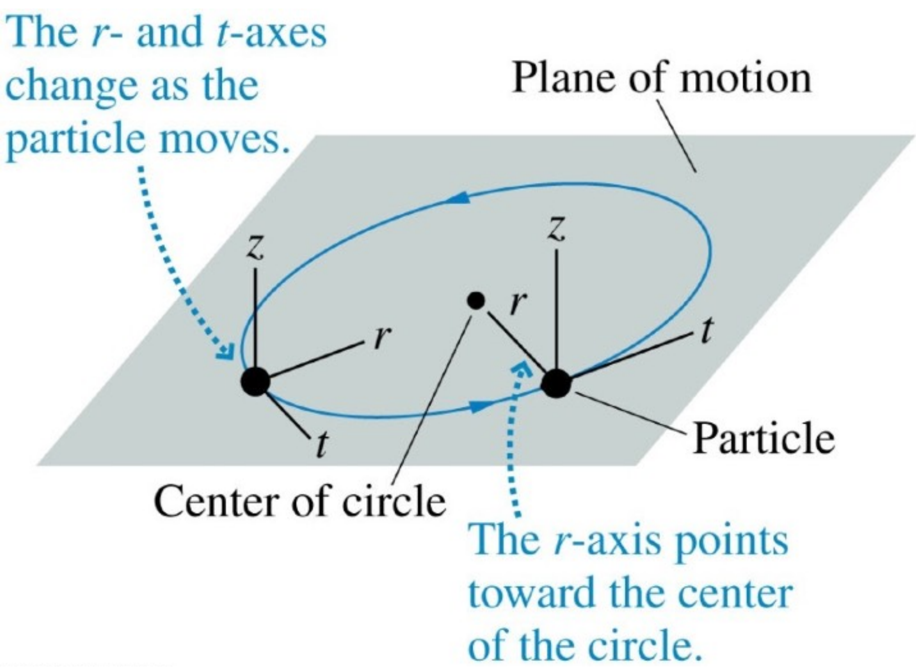
E. Tension in the string

F. A new force: the centrifugal force.



Uniform circular motion

rtz-coordinate system



- The *r*-axis (radial) points *from* the particle *toward* the center of the circle.
- The *t*-axis (tangential) is tangent to the circle, pointing in the ccw direction.
- The *z*-axis is perpendicular to the plane of motion.

Dynamics of Uniform Circular Motion

An object in uniform circular motion is not traveling at constant velocity. It is accelerating!

There must be a force that causes this acceleration

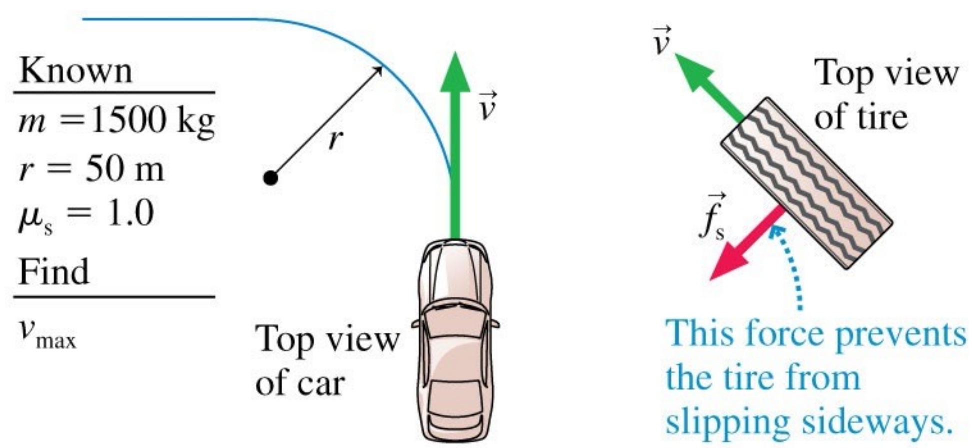
$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

Highway and racetrack curves are banked to allow the normal force of the road to provide the centripetal acceleration of the turn.

Example Problem I

What is the maximum speed with which a 1500 kg car can make a left turn around a curve of radius 50 m on a level (unbanked) road without sliding?

- Draw a free-body diagram for the car (rear view) as it travels around the corner.
- Identify your r-t-z coordinate system.
- Assemble Newton's second law ($F_{net} = ma$) in the "r" and "z" dimensions.



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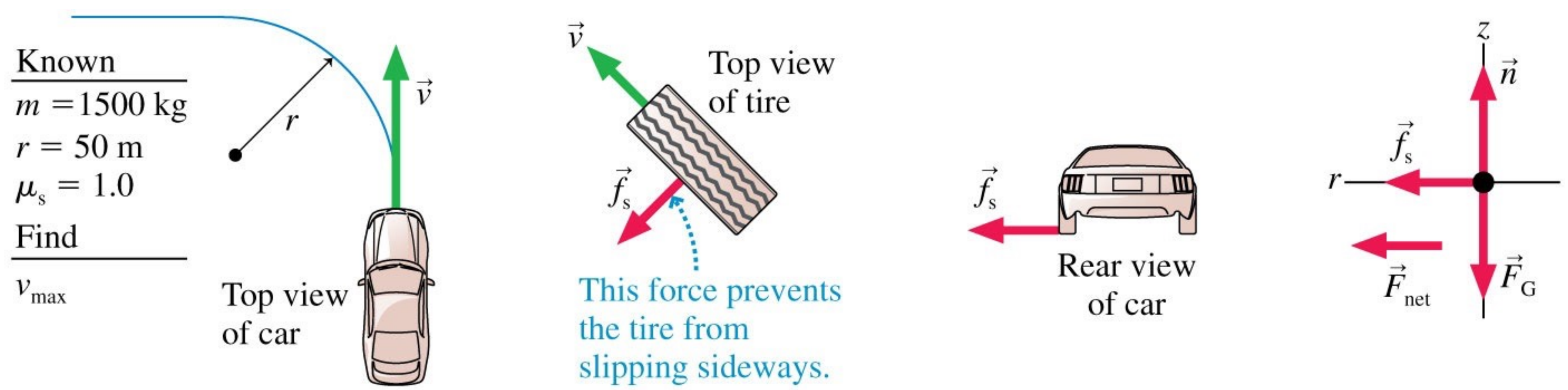
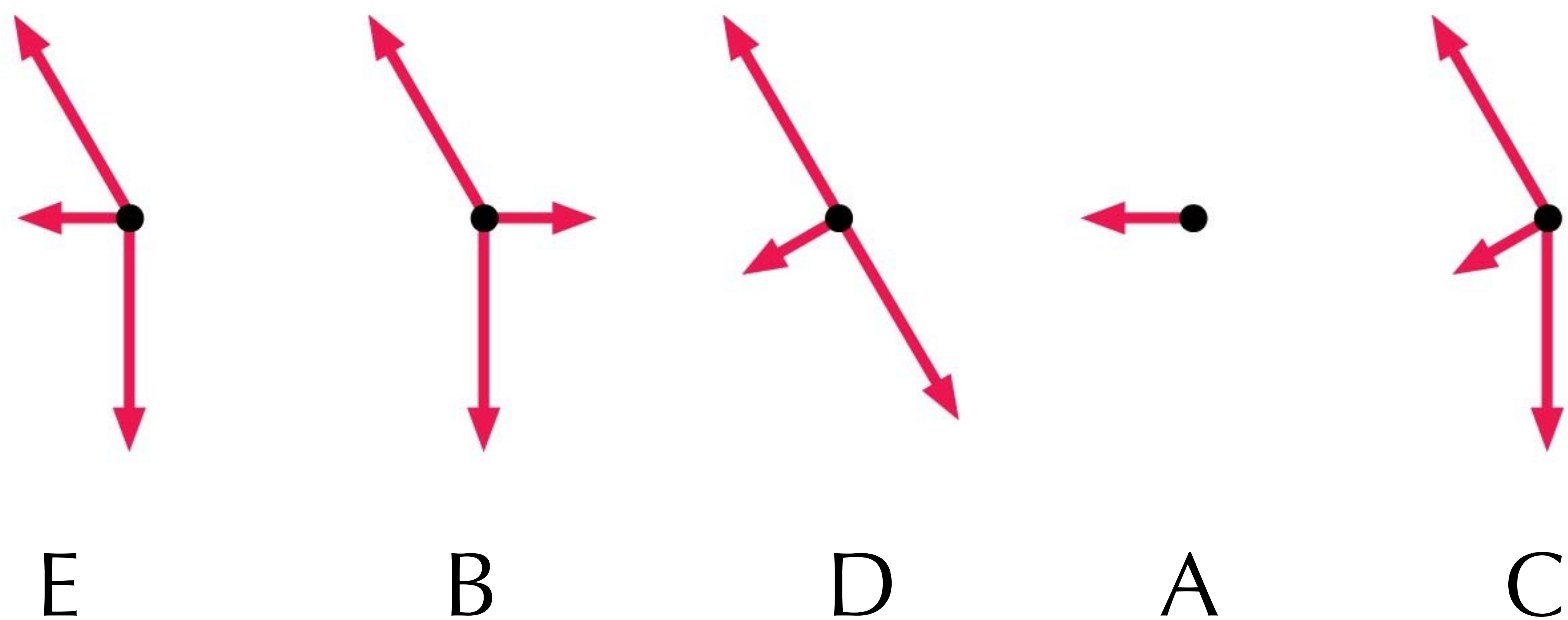


Quiz

Question #20

A car turns a corner on a banked road. Which of the diagrams <u>could</u> be the car's free-body diagram?



Quiz

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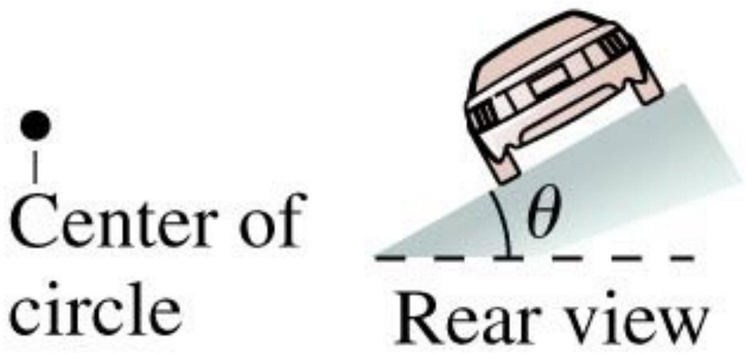
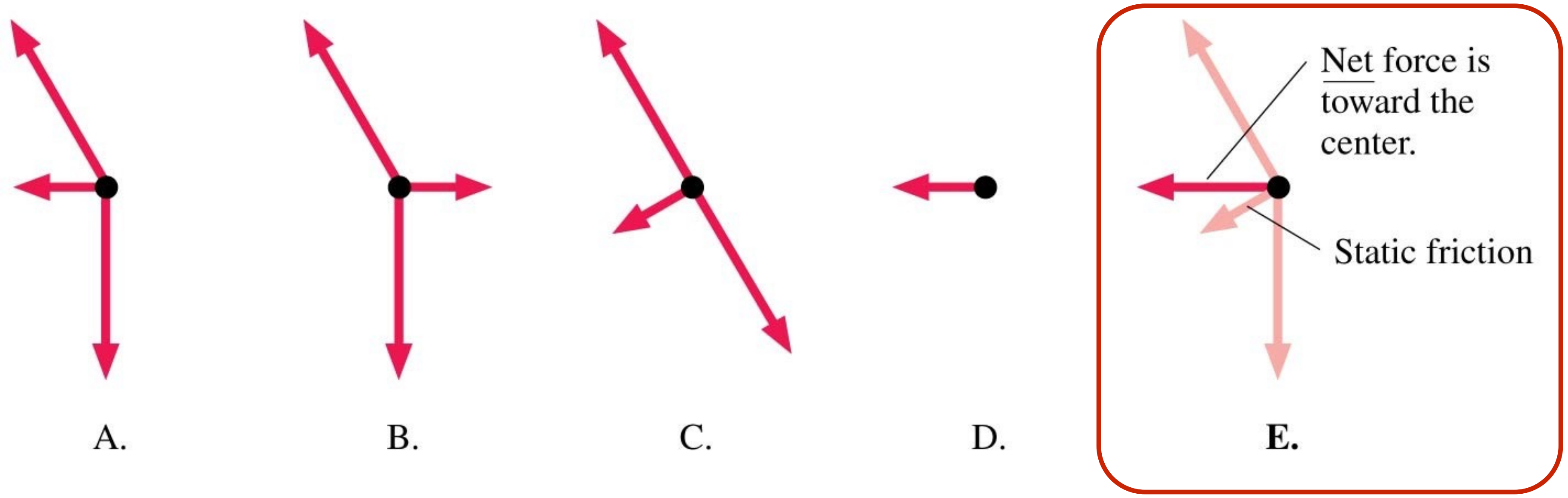
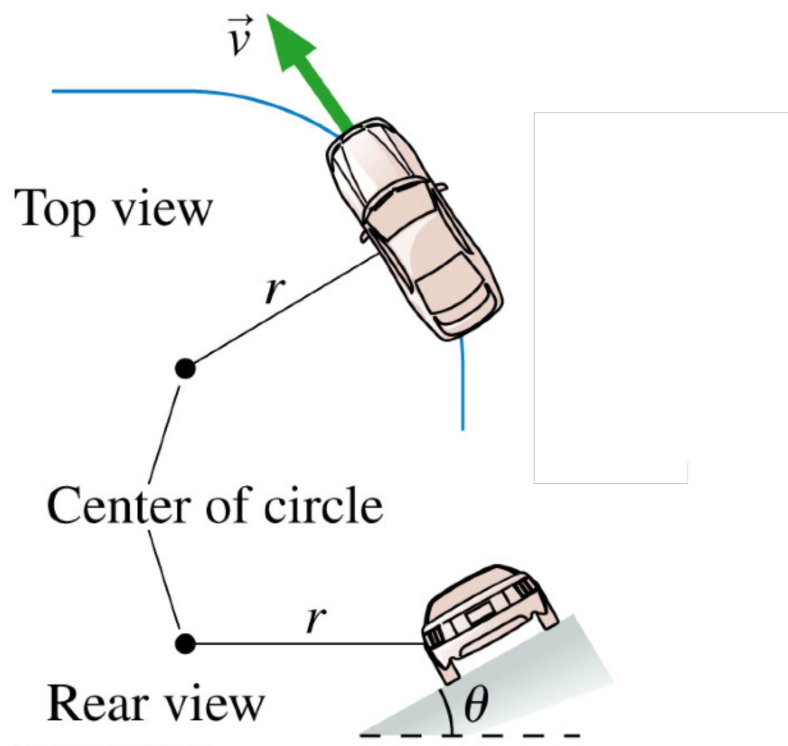




Banked Curves

If this were an icy road (no friction), what banking angle must the road have if you are going to be able to make it through the corner?

a) Draw the free-body diagram
b) Identify your coordinate system
c) Put together Newton's second law



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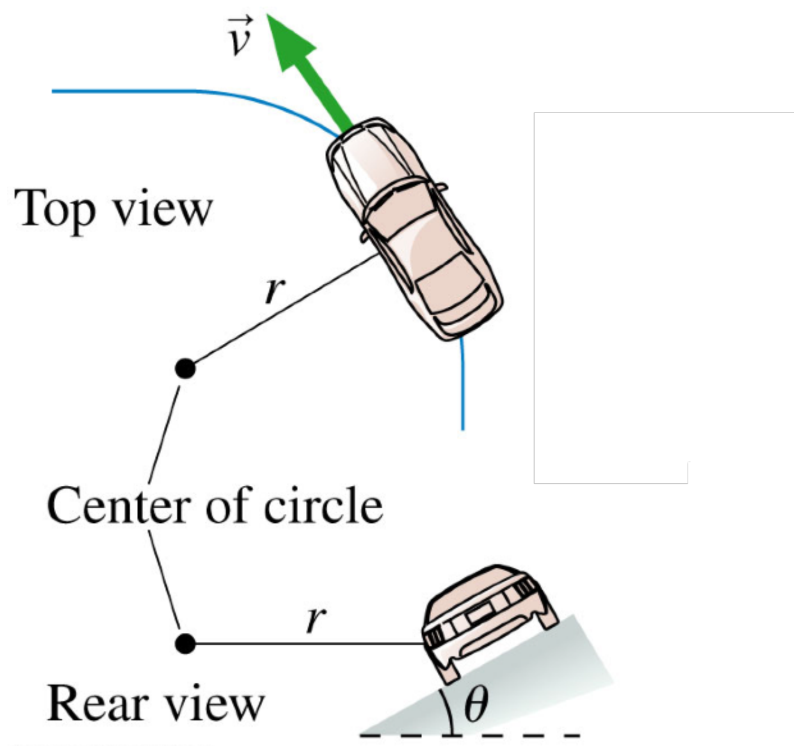

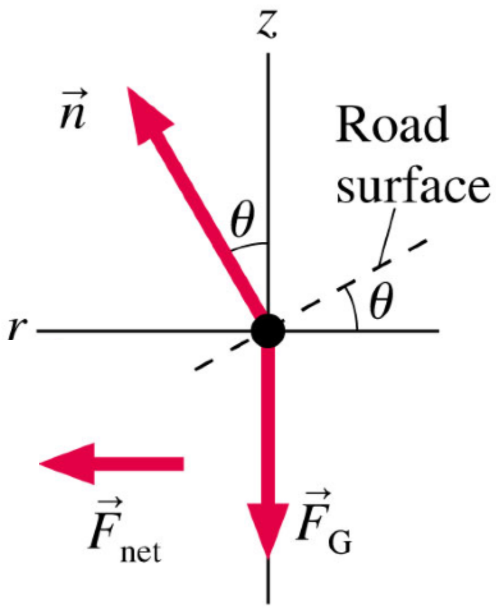

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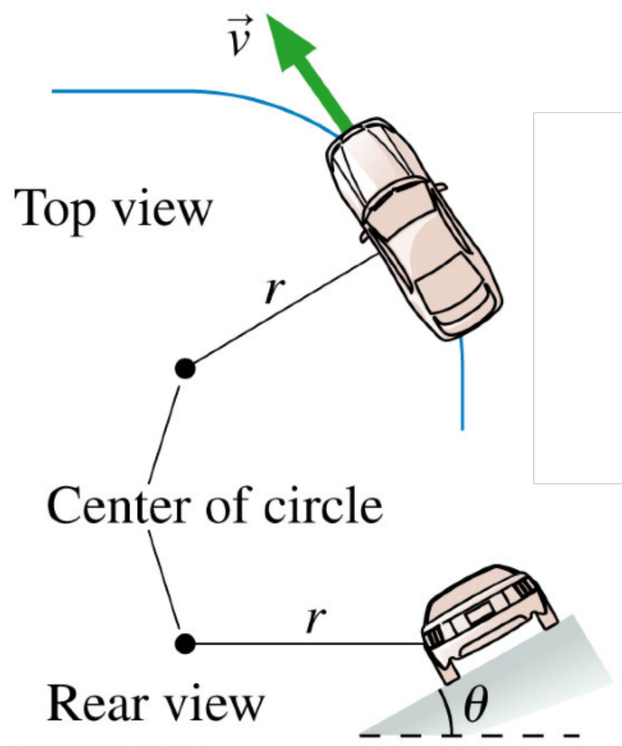
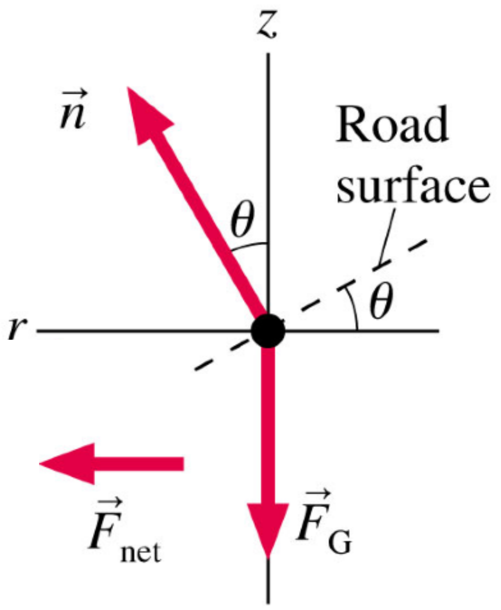


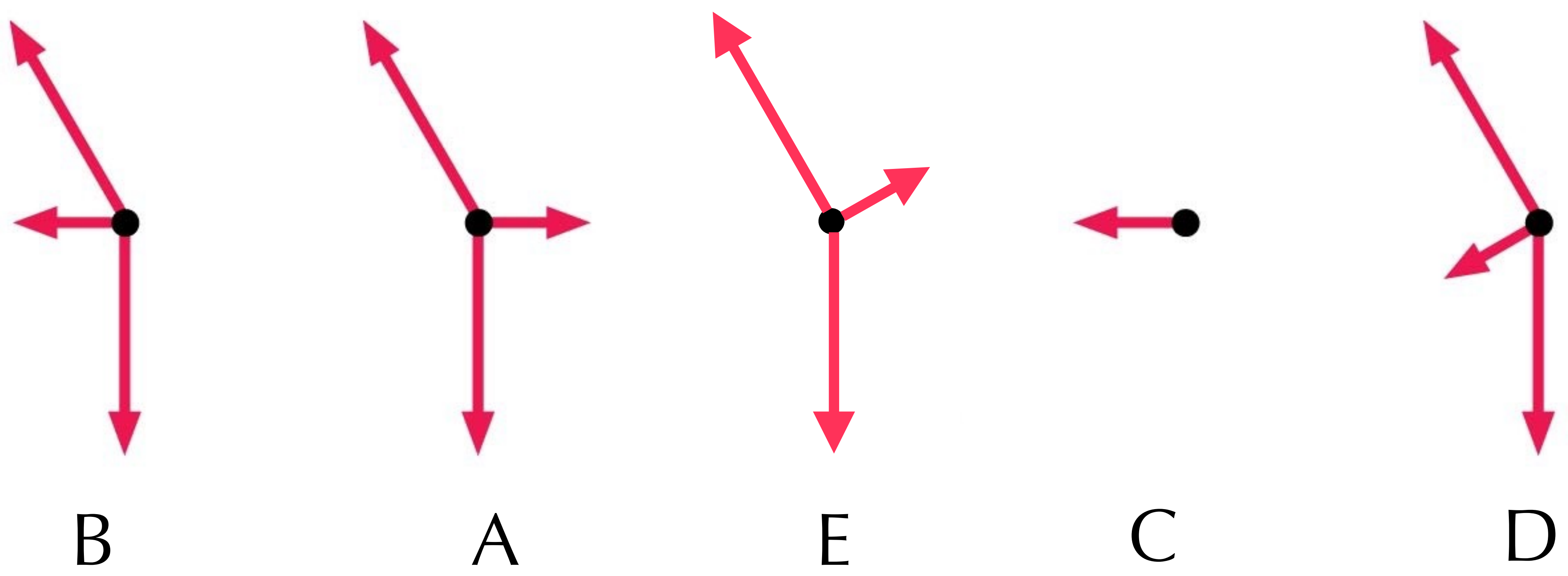


Banked Curves

Question #21

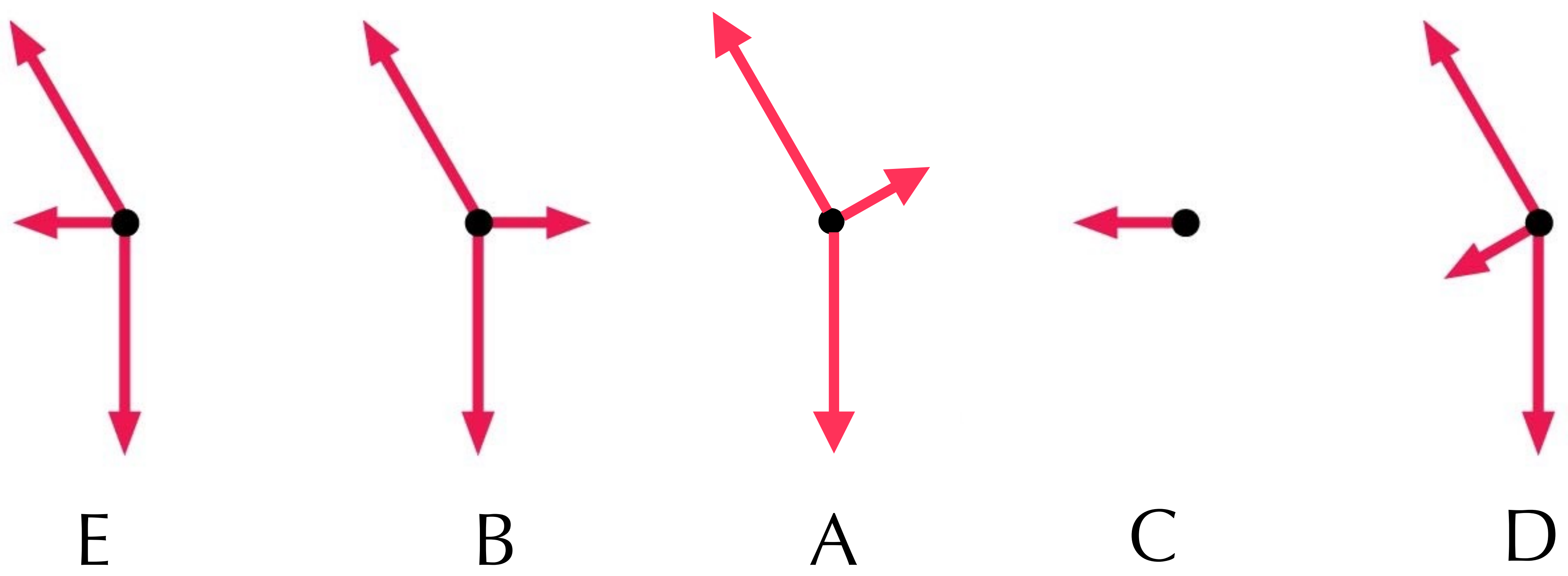
If you travel faster than this speed, what must the free-body diagram look like?

$$v_0 = \sqrt{rg \tan\theta}$$



If you travel slower than this speed, what must the free-body diagram look like?

$$v_0 = \sqrt{rg\tan\theta}$$



Banked Curves with friction

What is the maximum speed that this car can travel through the banked curve without slipping off the road. (The road is not frictionless)

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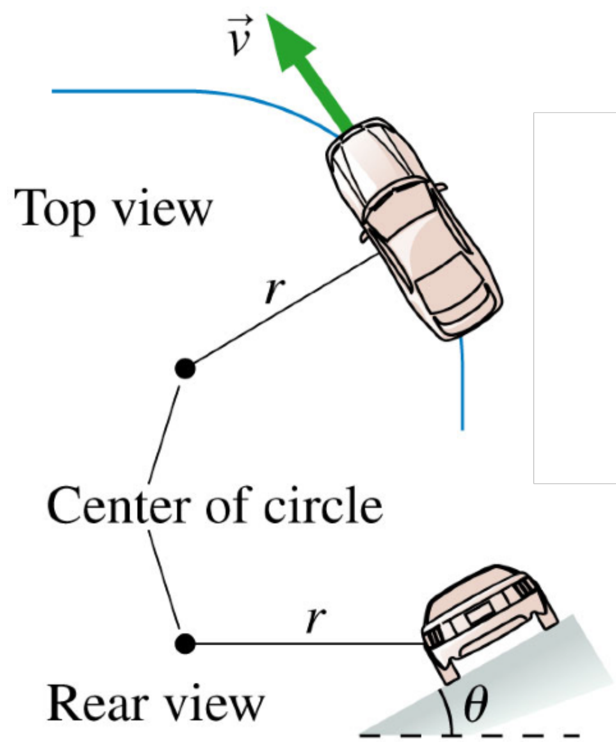

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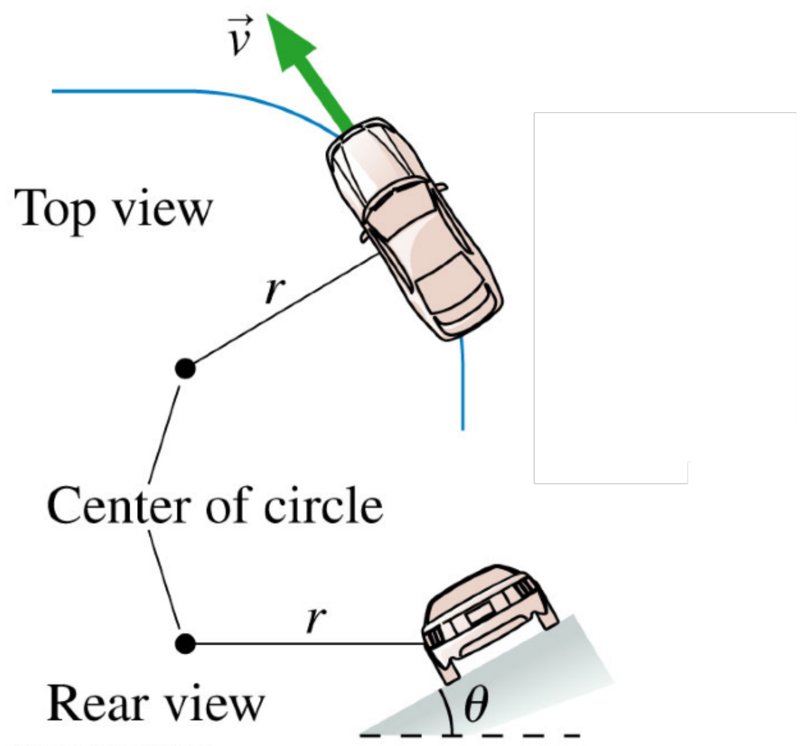
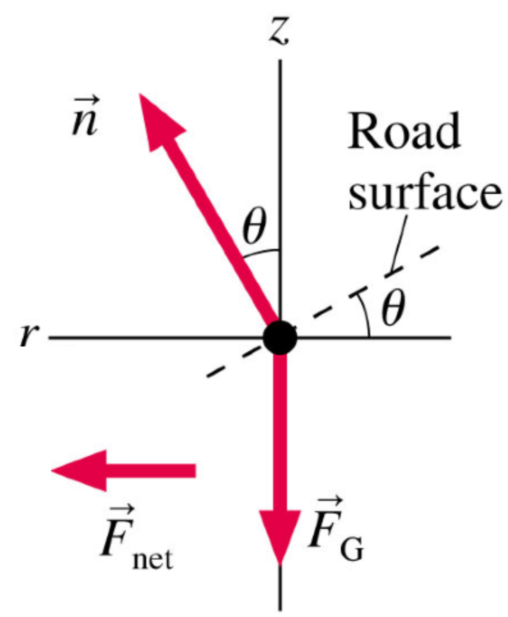
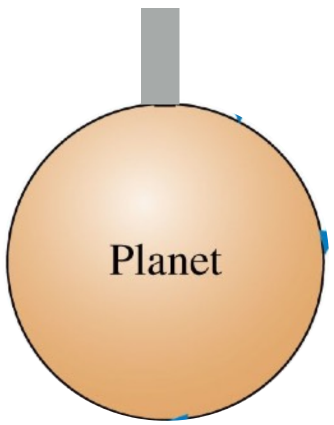




Circular Orbits

An object is launched from the top of a tall tower.

What will the trajectory look like if you give the object a **small** initial velocity?



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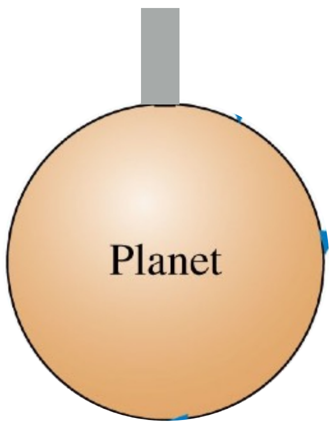

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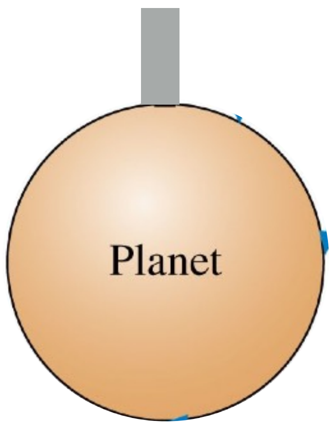

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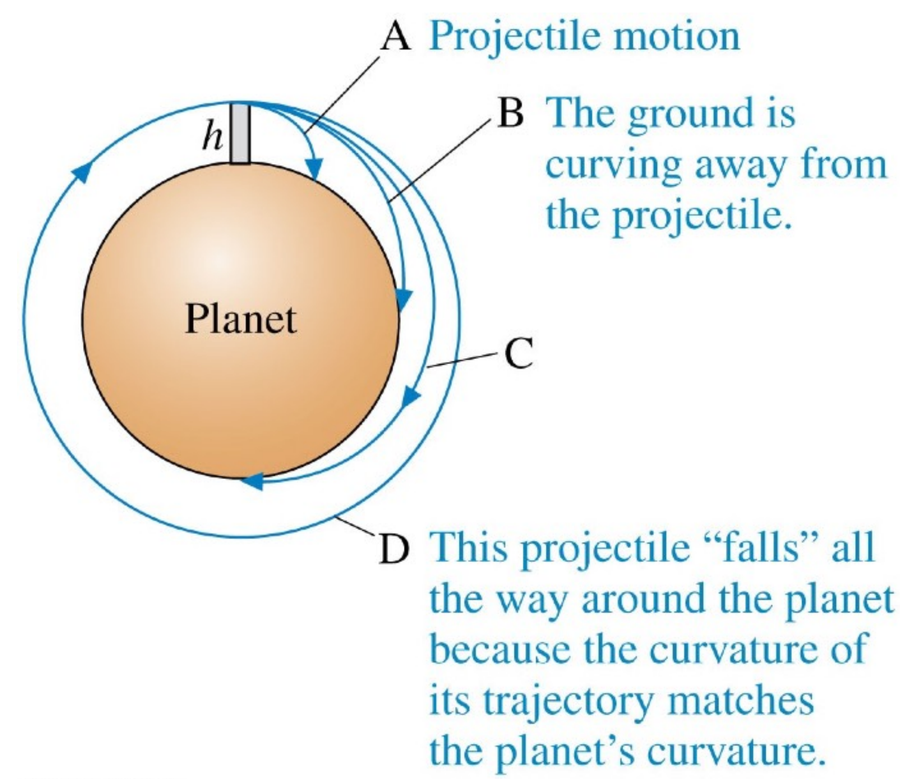


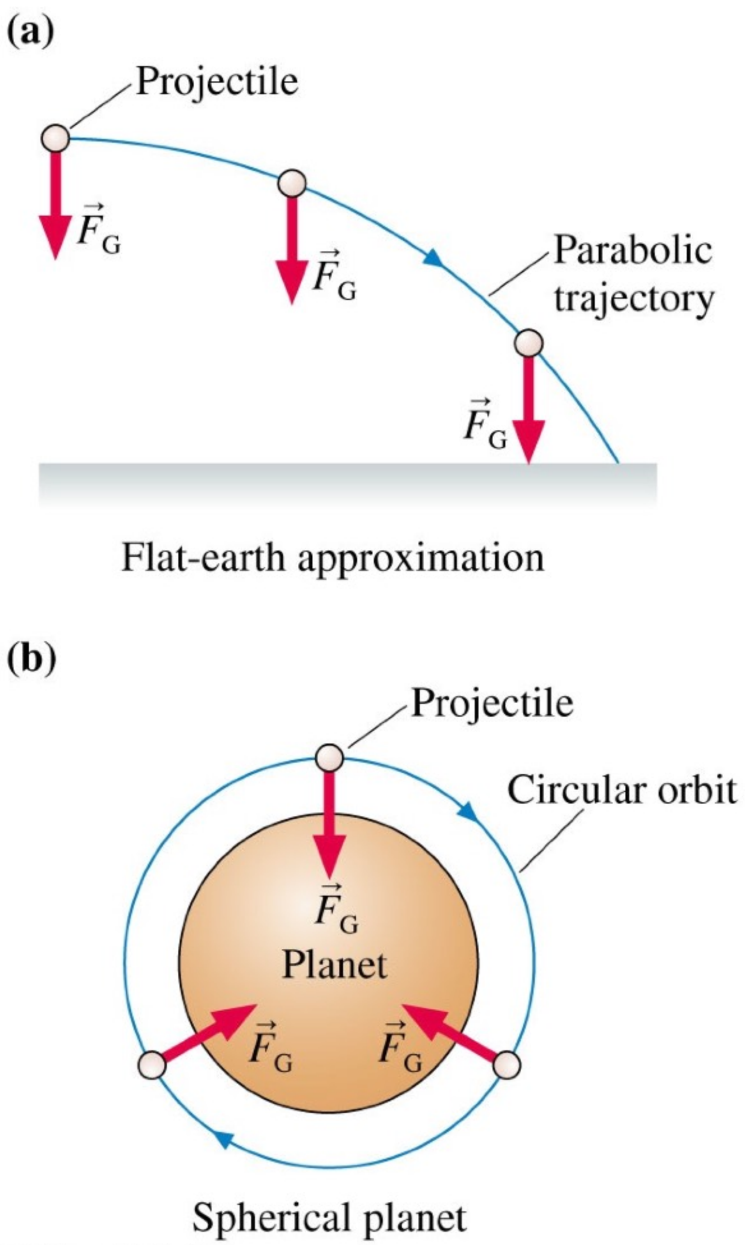
Circular Orbits

Flat-earth approximation

$$\vec{F}_G = (mg, \text{ vertically downward})$$

Actual Planet

$$\vec{F}_G = (mg, \text{ toward center})$$



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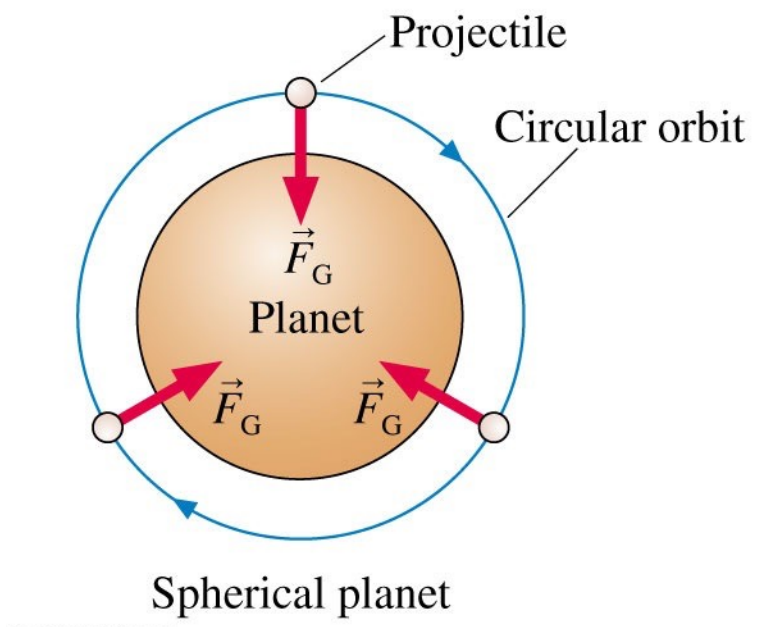
Question: An object is orbiting a planet a distance r from the center of the planet. What speed does the orbiting object have?



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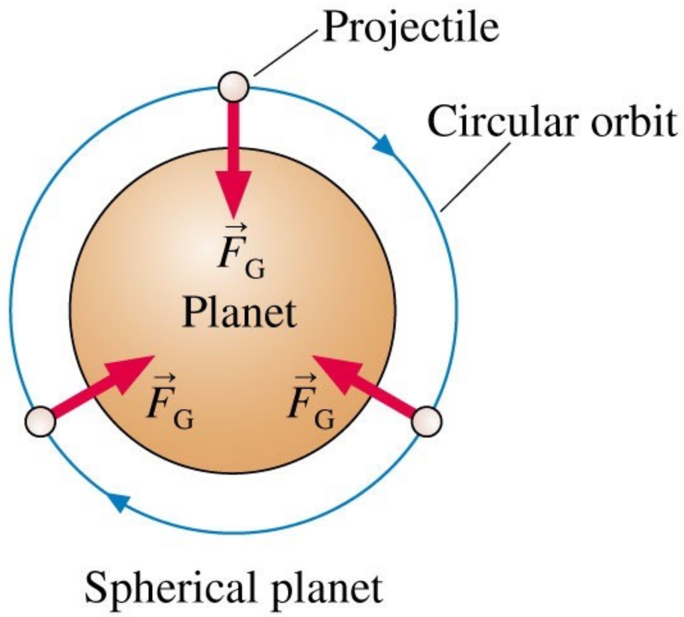
$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = (g, \text{toward center})$$



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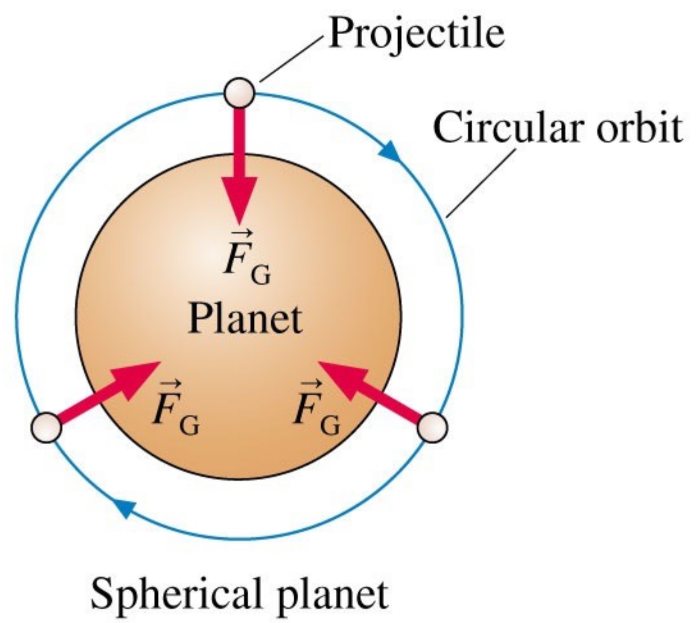

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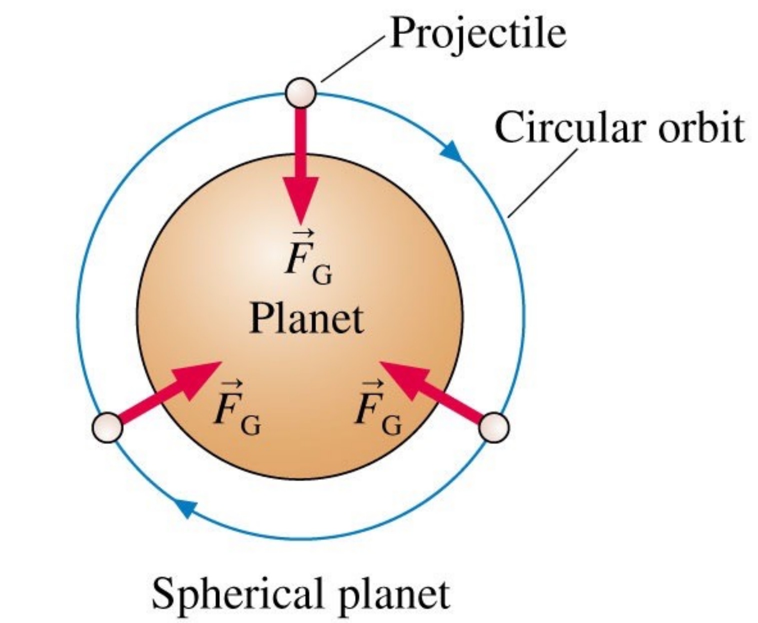

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The required speed for a circular orbit near the planet's surface

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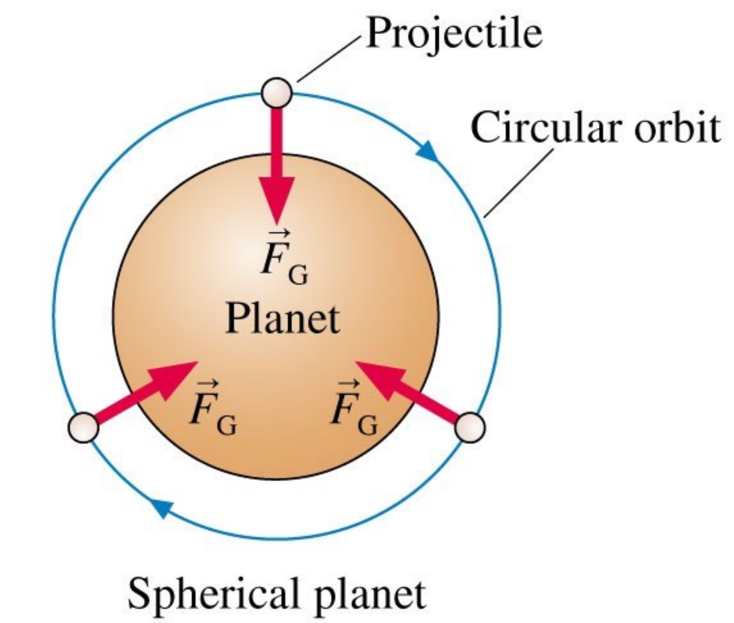

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The required speed for a circular orbit near the planet's surface

$$v_{\text{orbit}} = \sqrt{rg}$$

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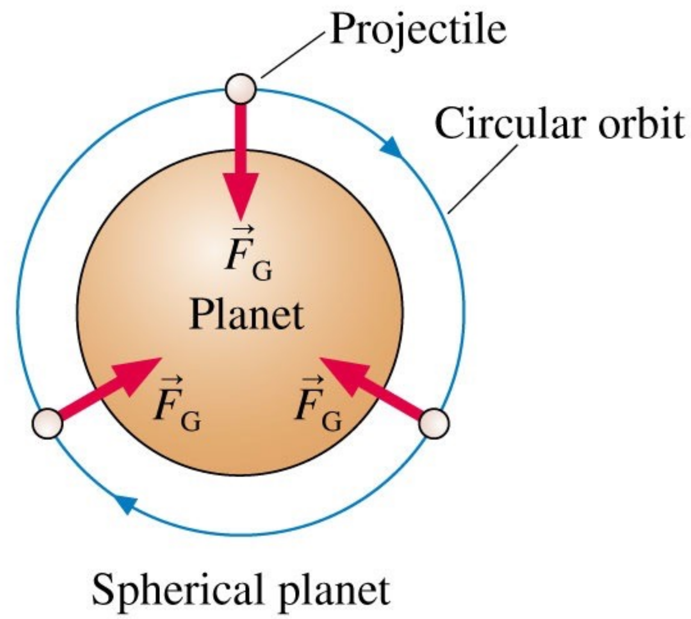

The required speed for a circular orbit near the planet's surface

$$v_{\text{orbit}} = \sqrt{rg}$$

$$v_{\text{orbit}} = \frac{2\pi r}{T}$$