

# Recommended Problems

## Elastic Collisions

11.63

11.57      11.65

11.25

## Center of Mass

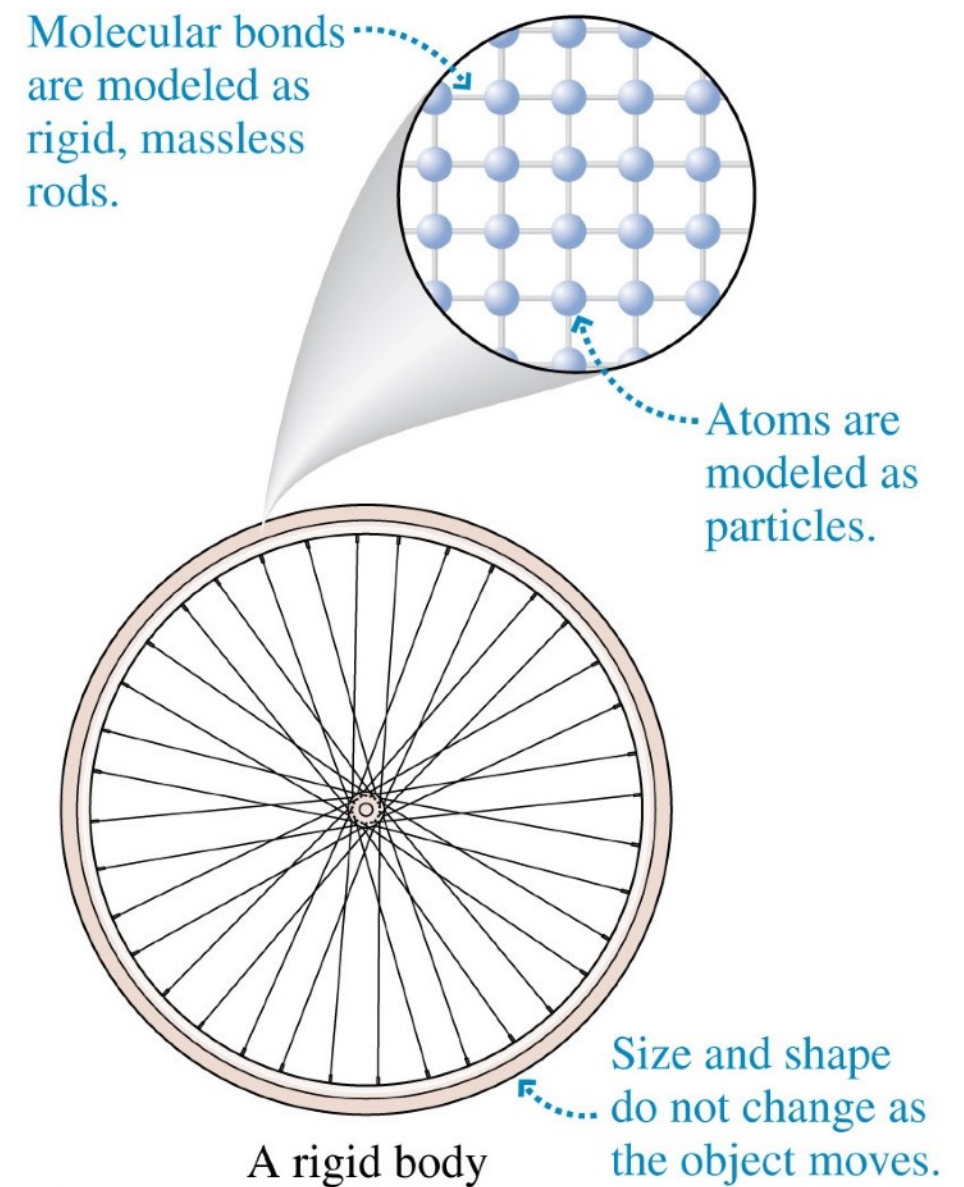
12.6

12.8

12.50 (calculus)

# Rigid-Body Model

A **rigid body** is an extended object whose size and shape do not change as it moves.



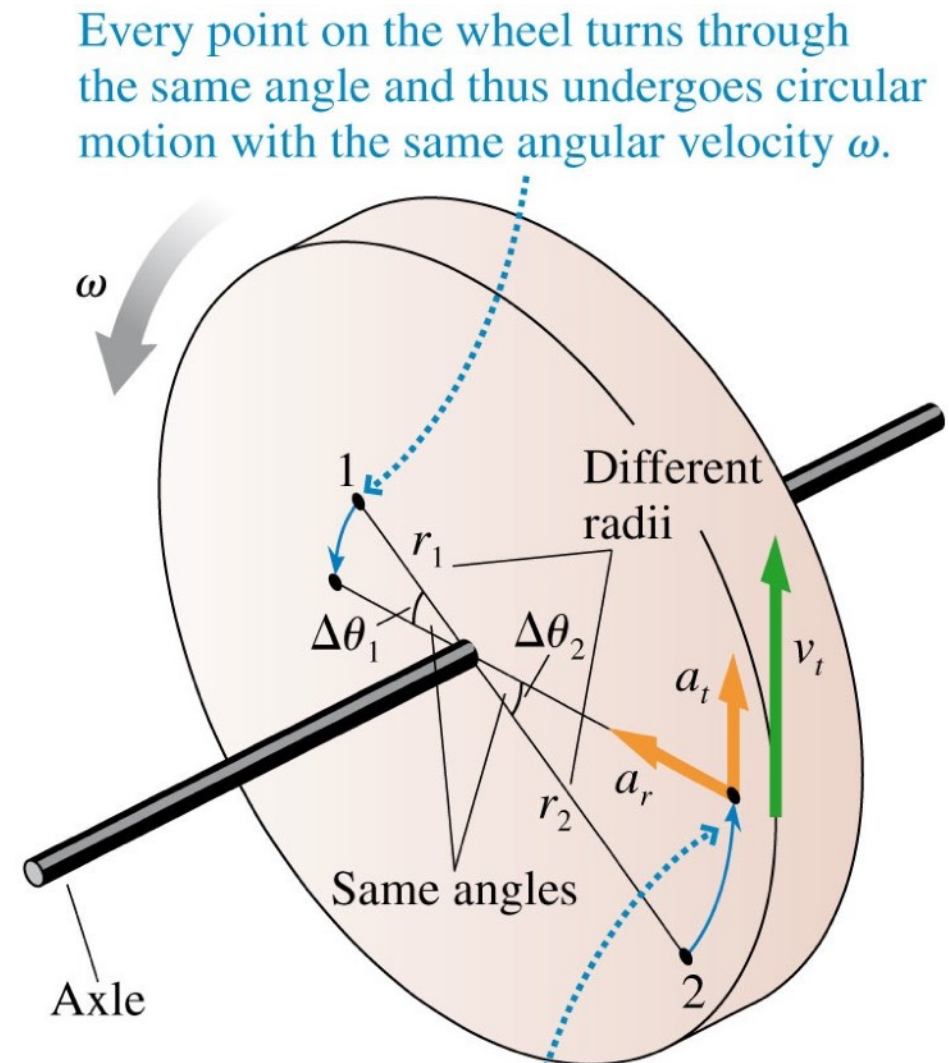
# Rotational motion review

Recall that angular velocity is

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

All points on a rotating rigid body have the same  $\omega$  and the same  $\alpha$ .



All points on the wheel have a tangential velocity and a radial (centripetal) acceleration. They also have a tangential acceleration if the wheel has angular acceleration.

# Rotational Motion Review

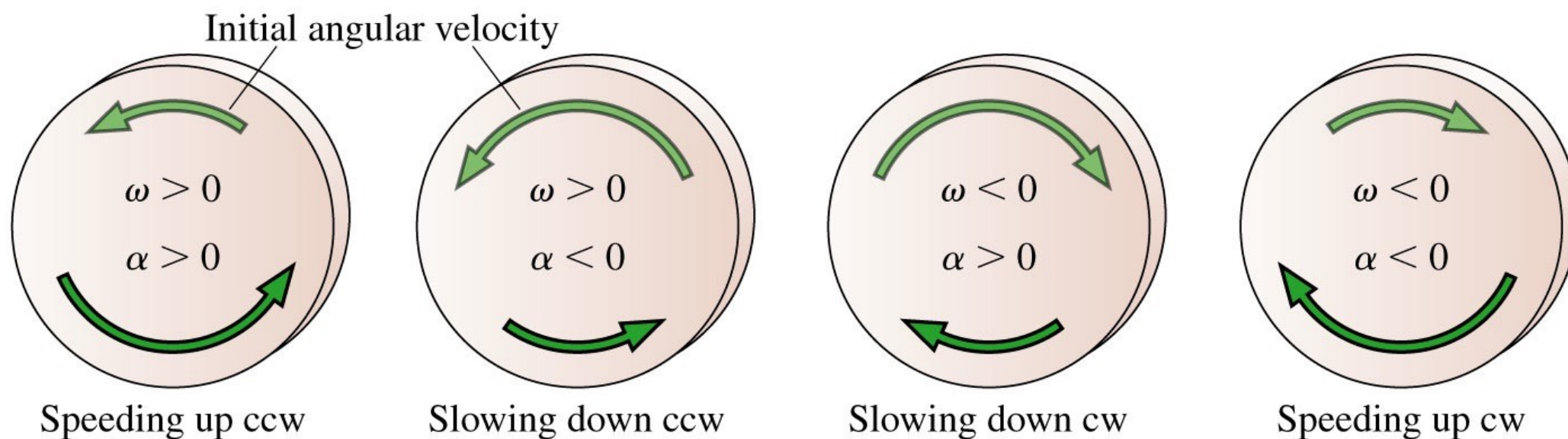
Rotational kinematics for constant angular acceleration

$$\omega_f = \omega_i + \alpha \Delta t$$

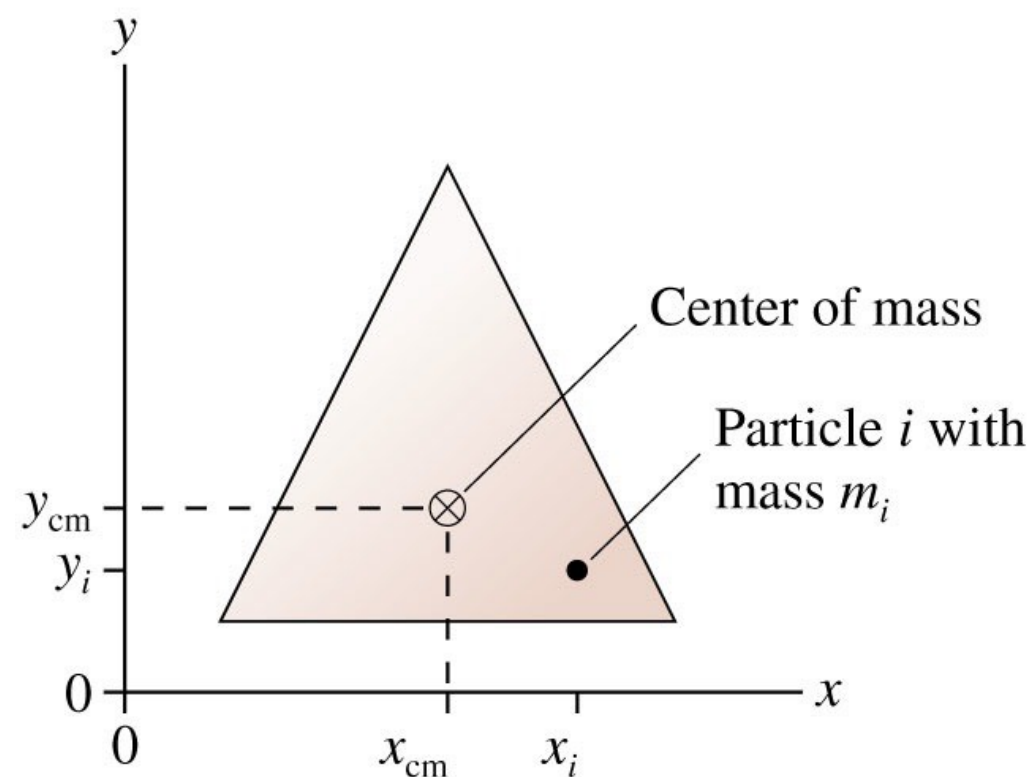
$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$$

The signs of angular velocity and angular acceleration.



# Finding the Center of Mass

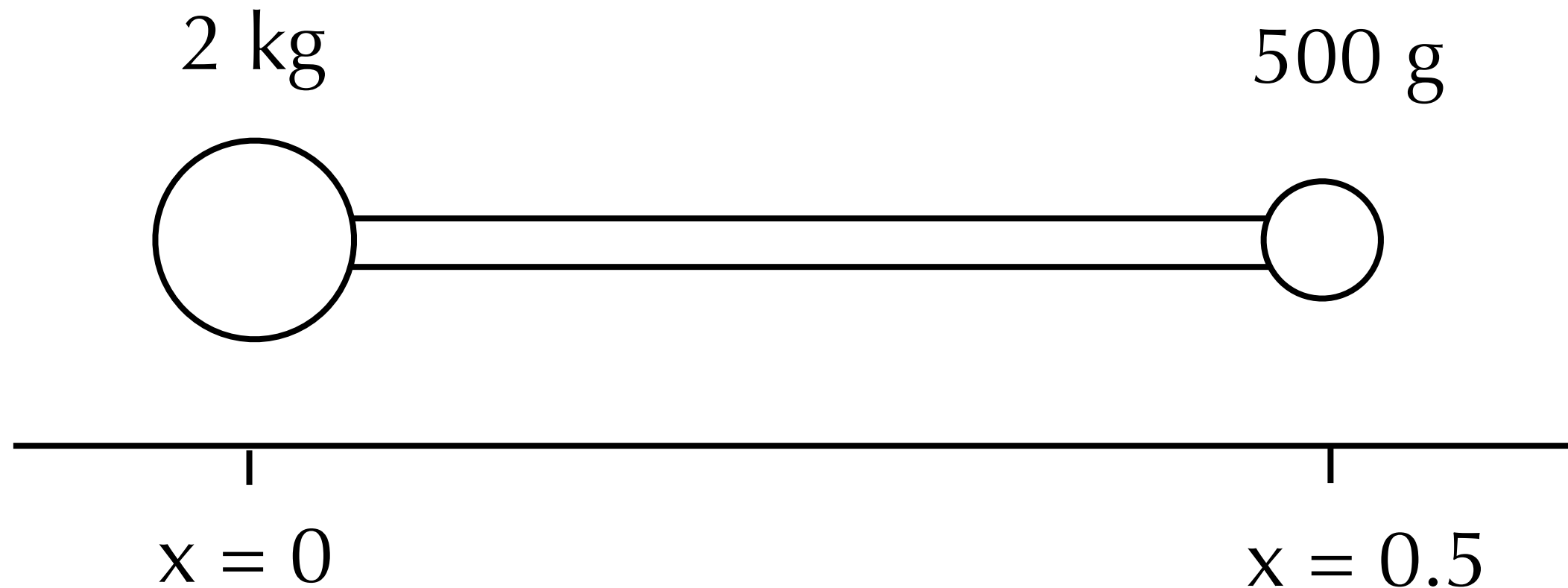


$$x_{\text{cm}} = \frac{1}{M} \sum_i m_i x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$$y_{\text{cm}} = \frac{1}{M} \sum_i m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

# What is the center of mass for this system?

## Question #1



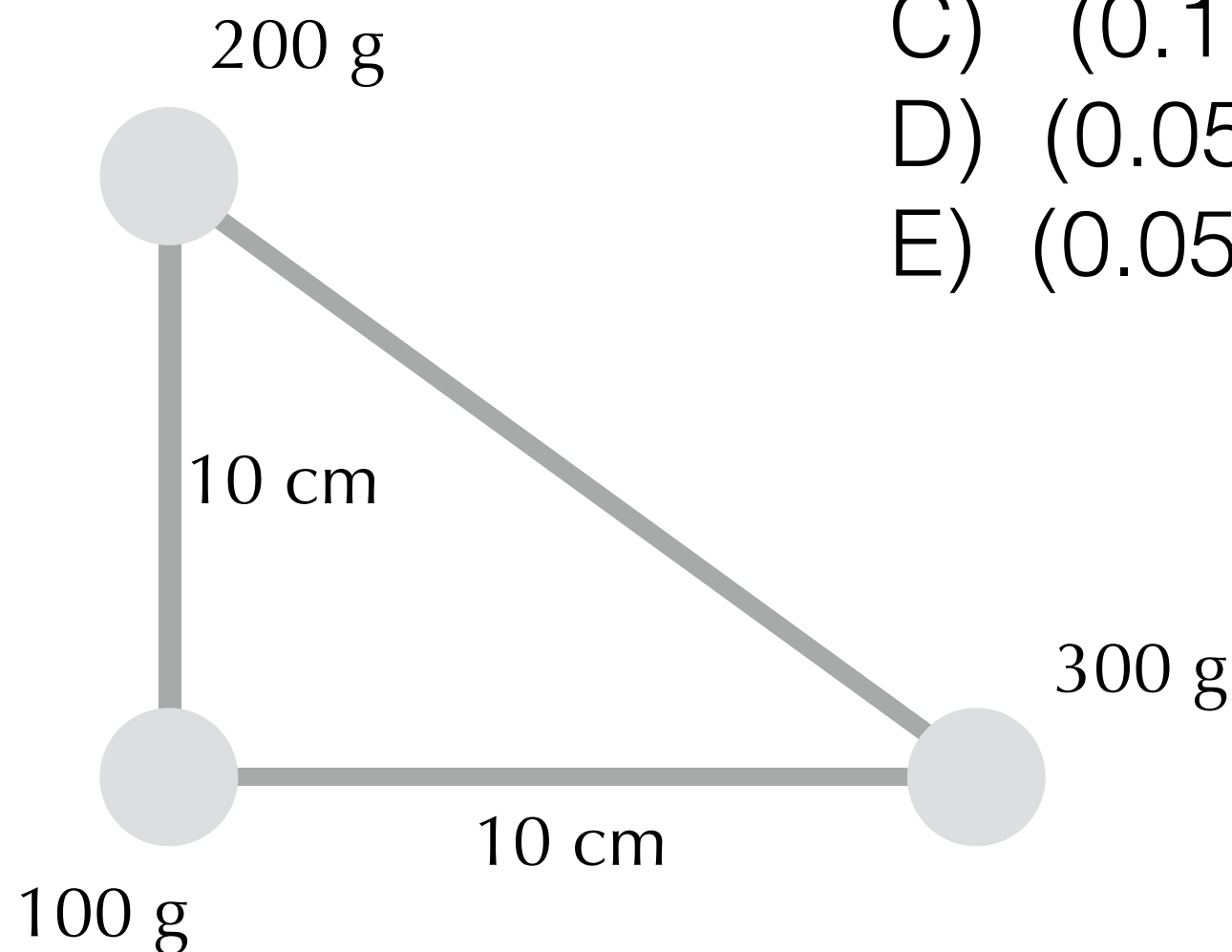
- A) 0.05 m
- B) 0.25 m
- C) 0.1 m
- D) 0.5 m
- E) 0.75 m



# Two-Dimensional Problem

## Question #2

The three balls shown are connected by massless, rigid rods. What are the coordinates of the center of mass?



A)  $(0.05, 0.05) \text{ m}$

B)  $(0.1, 0.03) \text{ m}$

C)  $(0.1, 0.05) \text{ m}$

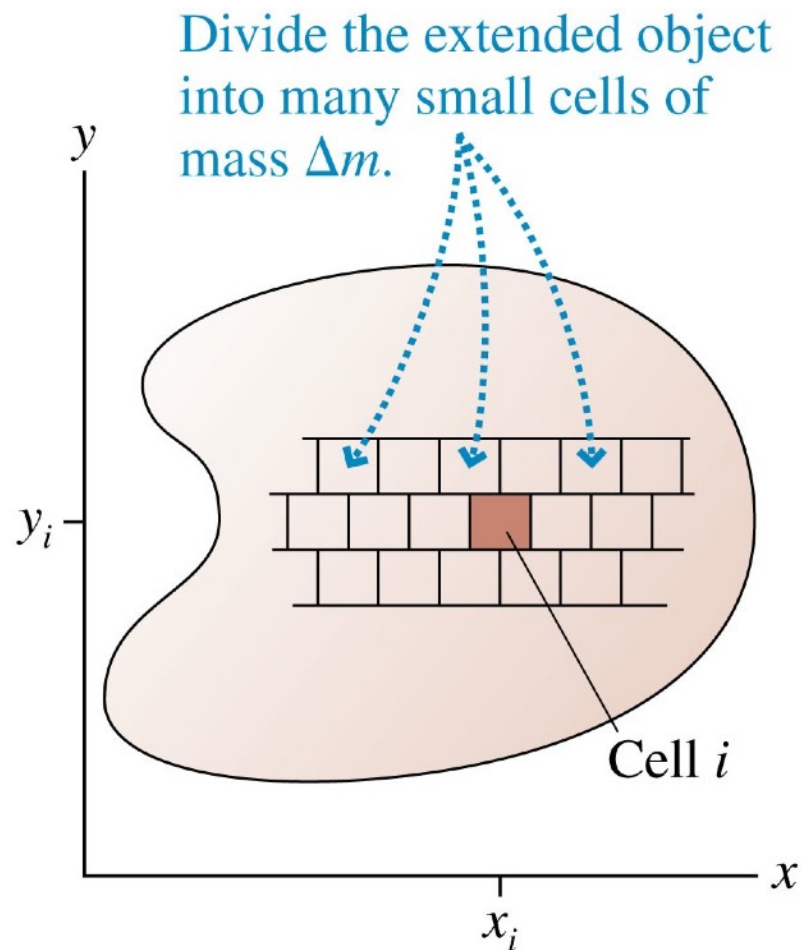
D)  $(0.05, 0.075) \text{ m}$

E)  $(0.05, 0.03) \text{ m}$

# Center of Mass of a solid object

$$x_{\text{cm}} = \frac{1}{M} \int x \, dm$$

$$y_{\text{cm}} = \frac{1}{M} \int y \, dm$$



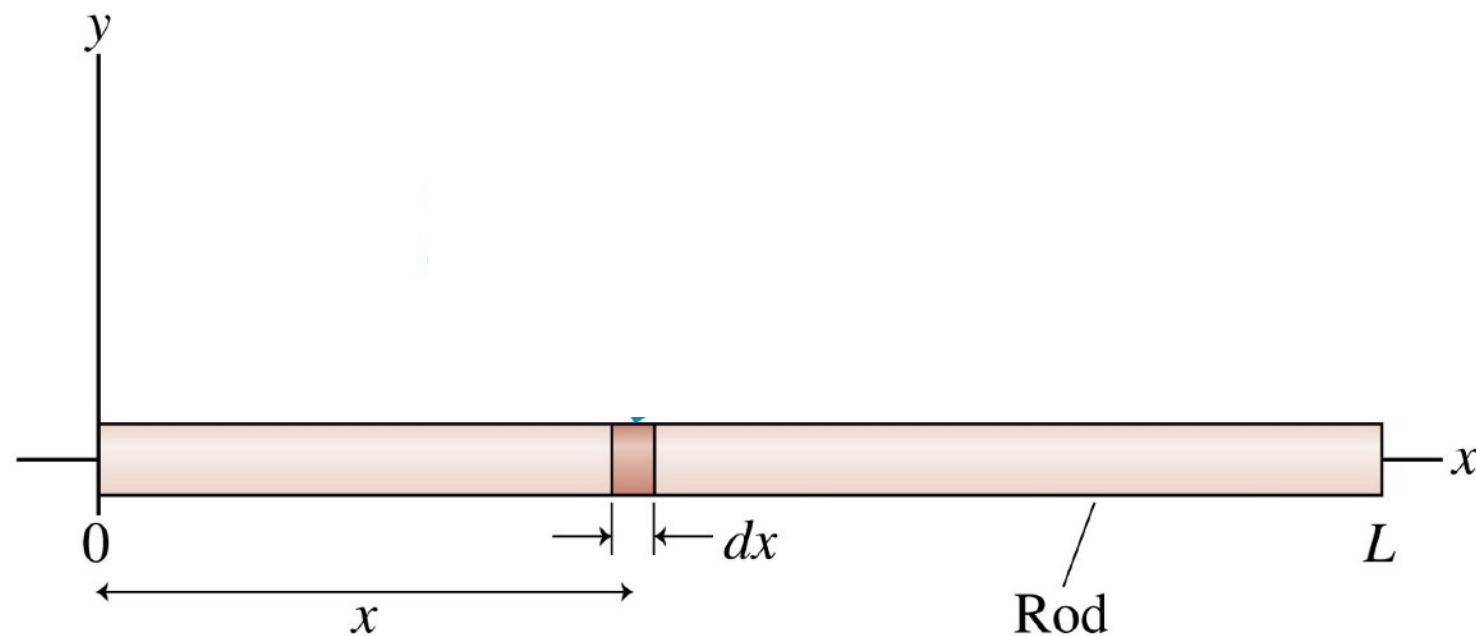
To do these integrals:

- $dm$  must be replaced with expressions involving  $dx$  and  $dy$
- Integration limits must be set



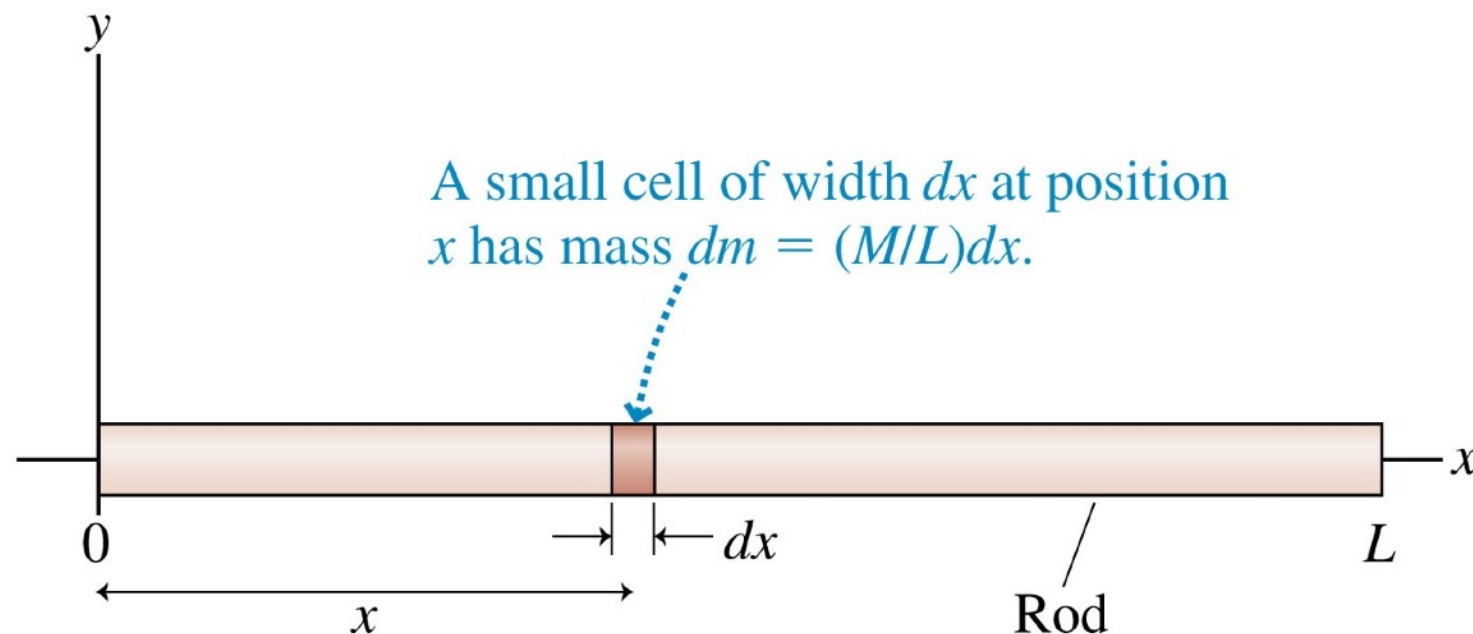
# Example

Find the center of mass of a thin, uniform rod of length  $L$  and mass  $M$ . Use your result to find the tangential acceleration of one tip of a 1.60-m-long rod that rotates about its center of mass with an angular acceleration of  $6.0 \text{ rad/s}^2$ .



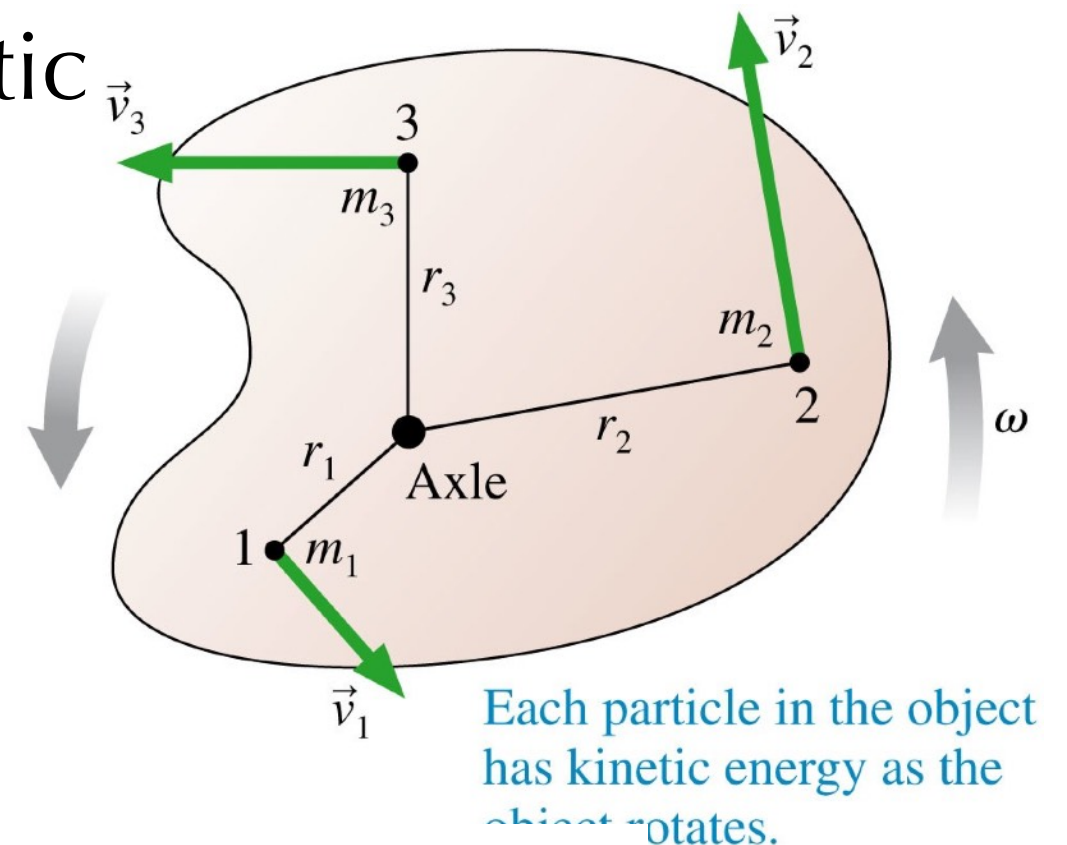
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# Rotational Energy

- When an object rotates, each individual piece of mass has kinetic energy.
- This is called rotational kinetic energy



$$K_{\text{rot}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \dots$$

recall that  $v = r\omega$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \dots = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

# Rotational Energy

Define the object's **moment of inertia**:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

$$K_{\text{rot}} = \frac{1}{2} \left( \sum_i m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

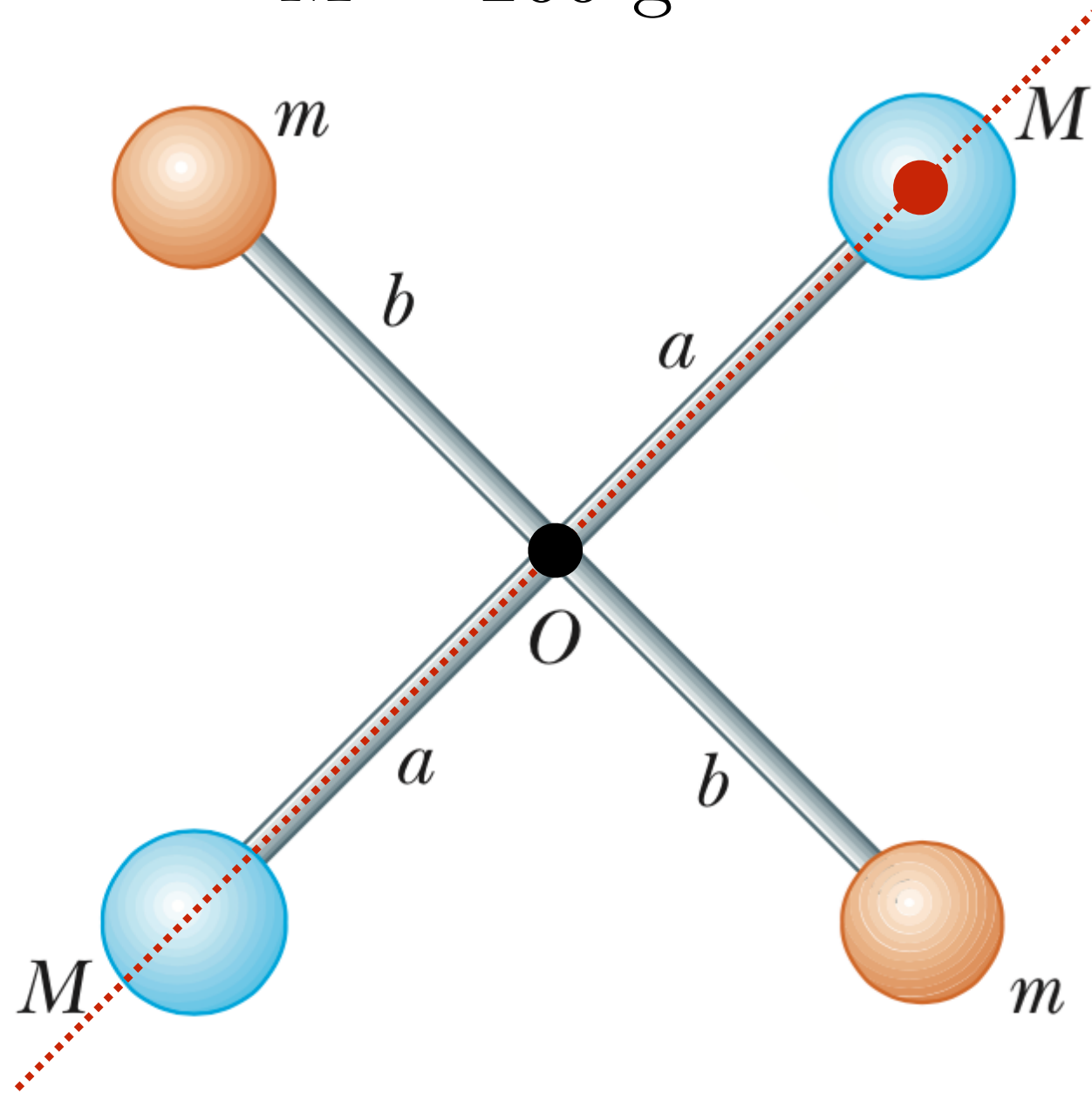
- The units of moment of inertia are kg m<sup>2</sup>.
- **Moment of inertia depends on the axis of rotation.**
- Mass farther from the rotation axis contributes more to the moment of inertia than mass nearer the axis.
- This is *not* a new form of energy, merely the familiar kinetic energy of motion written in a new way.

### Question #3

$$a = 50 \text{ cm} \quad b = 75 \text{ cm} \quad m = 100 \text{ g}$$

$$M = 200 \text{ g}$$

$$I = \sum m r^2$$


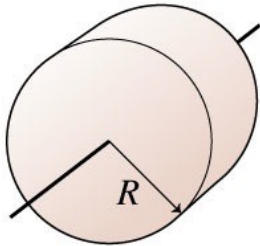
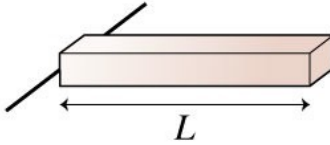
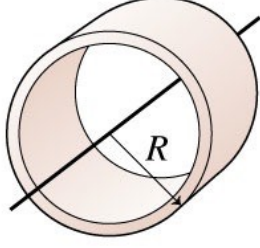
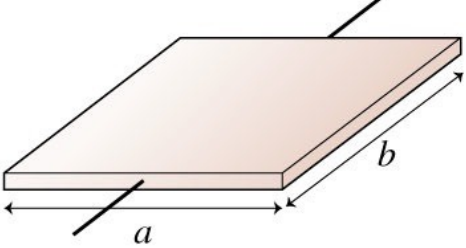
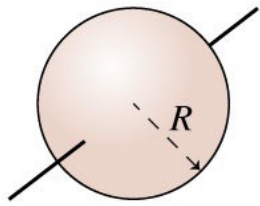
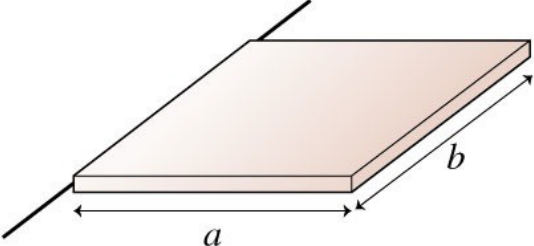
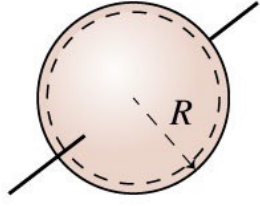


The moment of inertia about which axis will be greatest?

- a) red dot
- b) black dot
- c) red dotted line.

# Calculating the Moment of Inertia

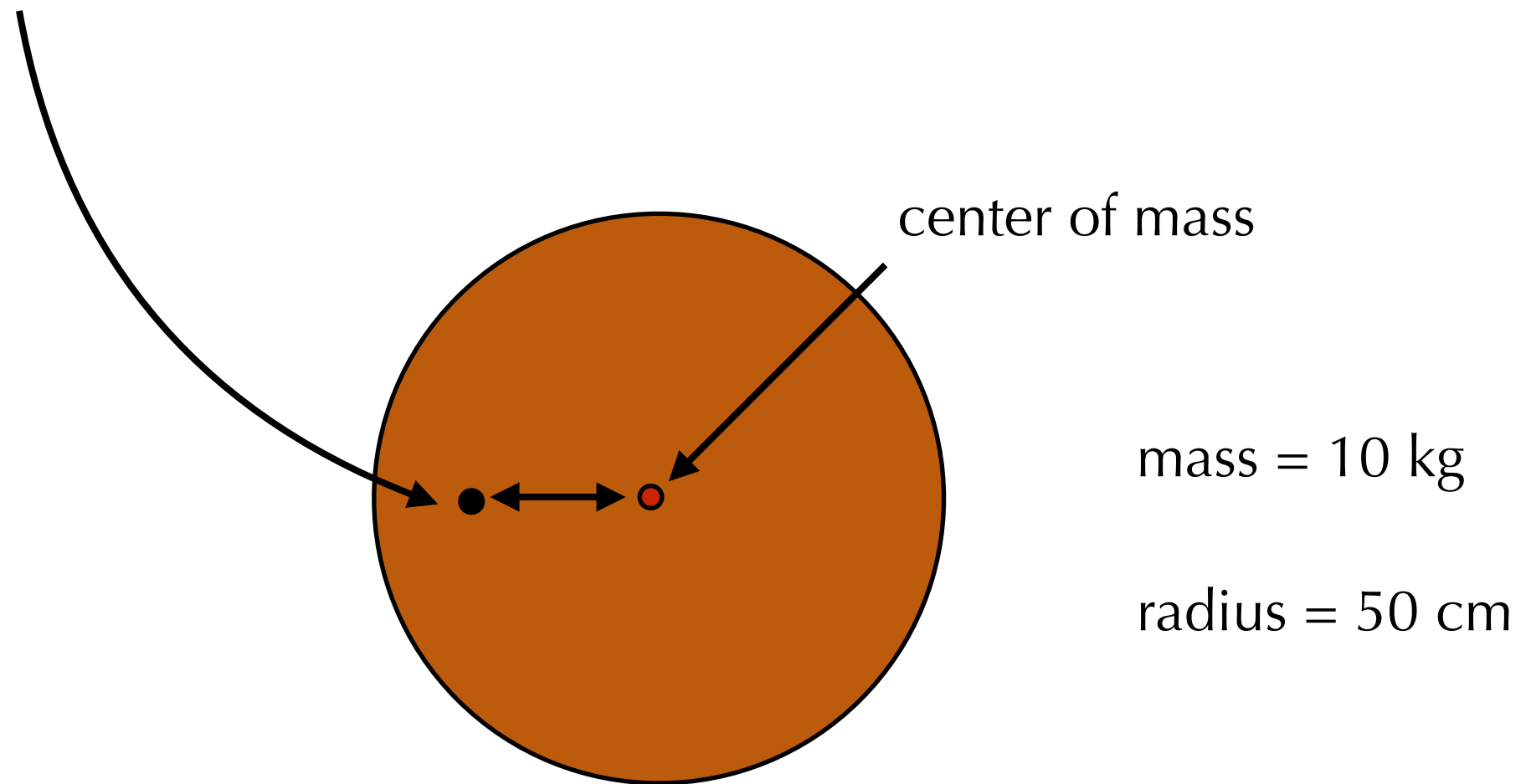
**TABLE 12.2** Moments of inertia of objects with uniform density

Object and axis	Picture	$I$	Object and axis	Picture	$I$
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		$MR^2$
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$



# Example (Parallel Axis Theorem)

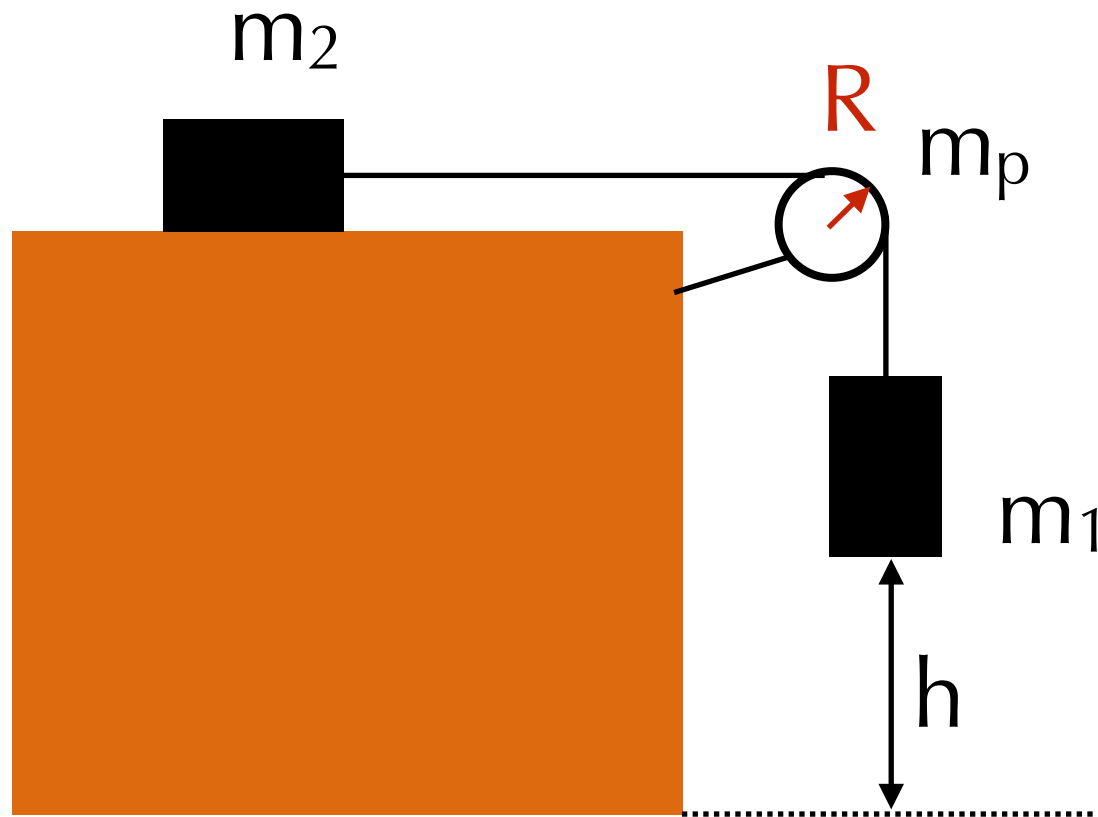
What is the moment of inertia if this disk is rotated about this axis?



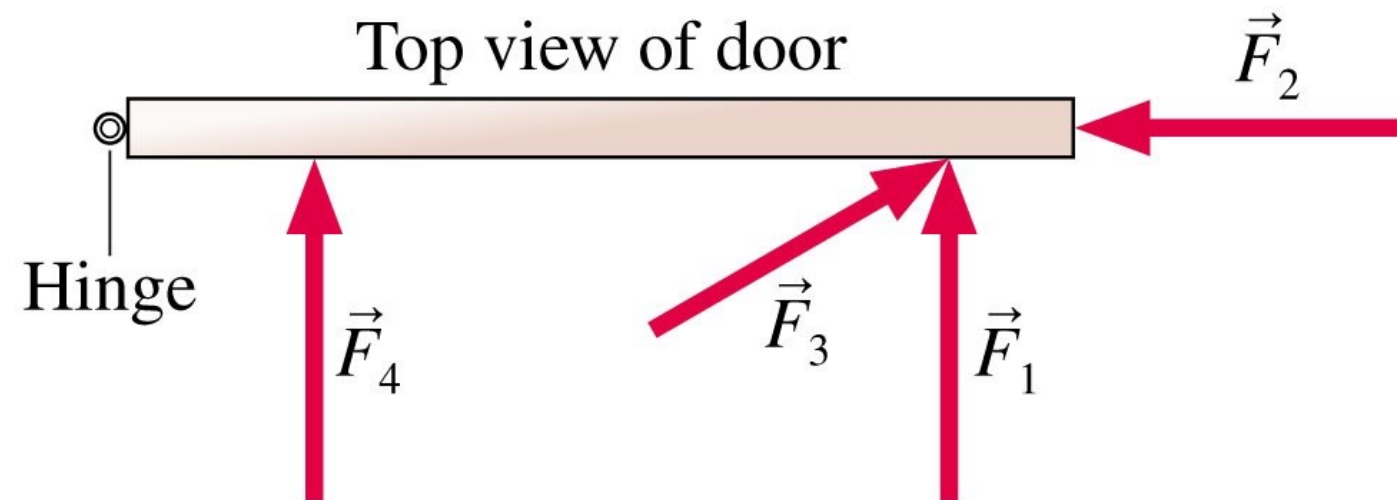
$$I = I_{\text{cm}} + Md^2$$

# Rotational Kinetic Energy

How fast are the block moving just before  $m_1$  hits the ground?



The four forces shown have the same strength. Which force would be most effective in opening the door?

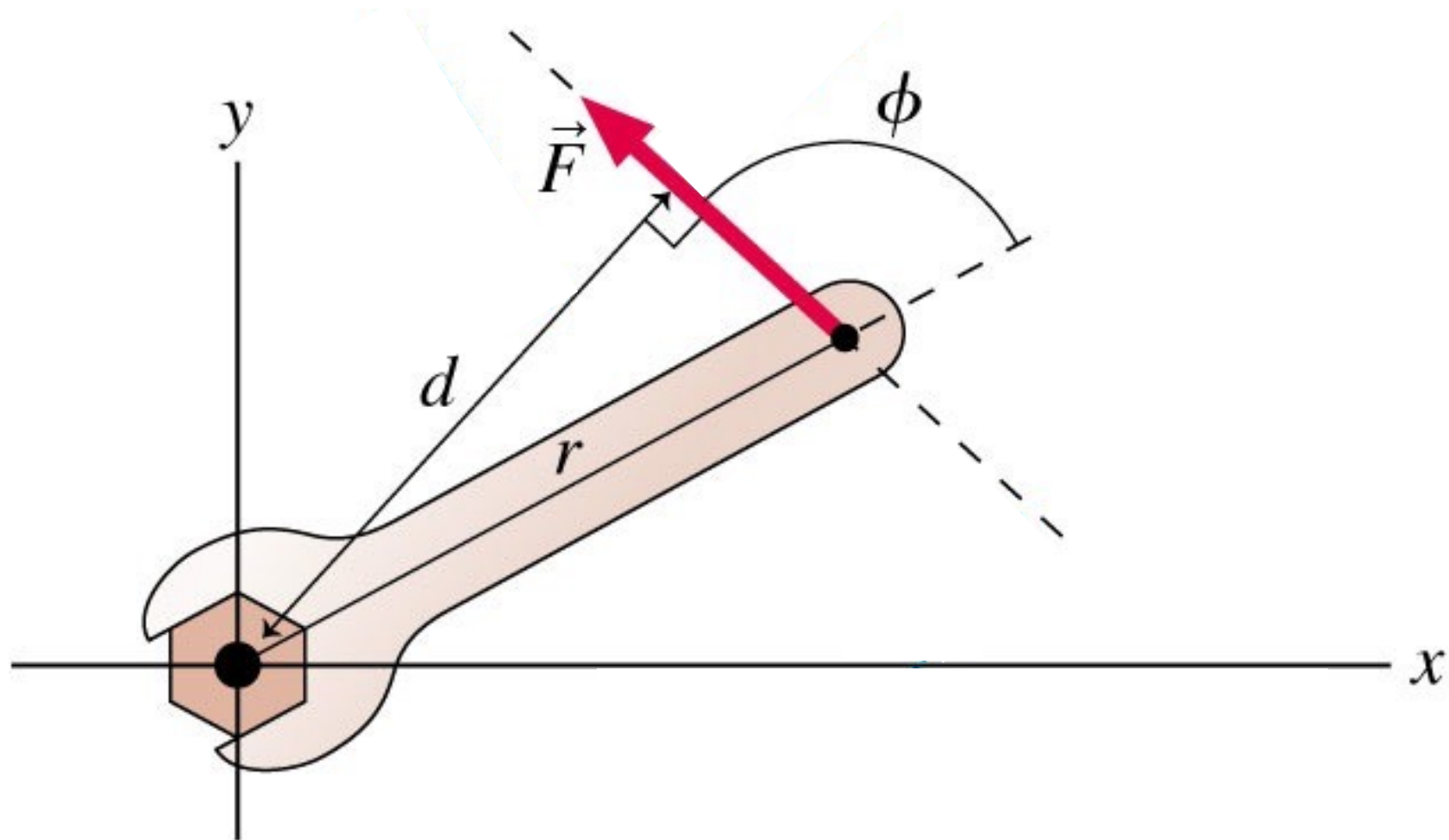


- a. Force  $F_2$ .
- b. Force  $F_1$ .
- c. Force  $F_3$ .
- d. Force  $F_4$ .
- e. Either  $F_1$  or  $F_3$ .

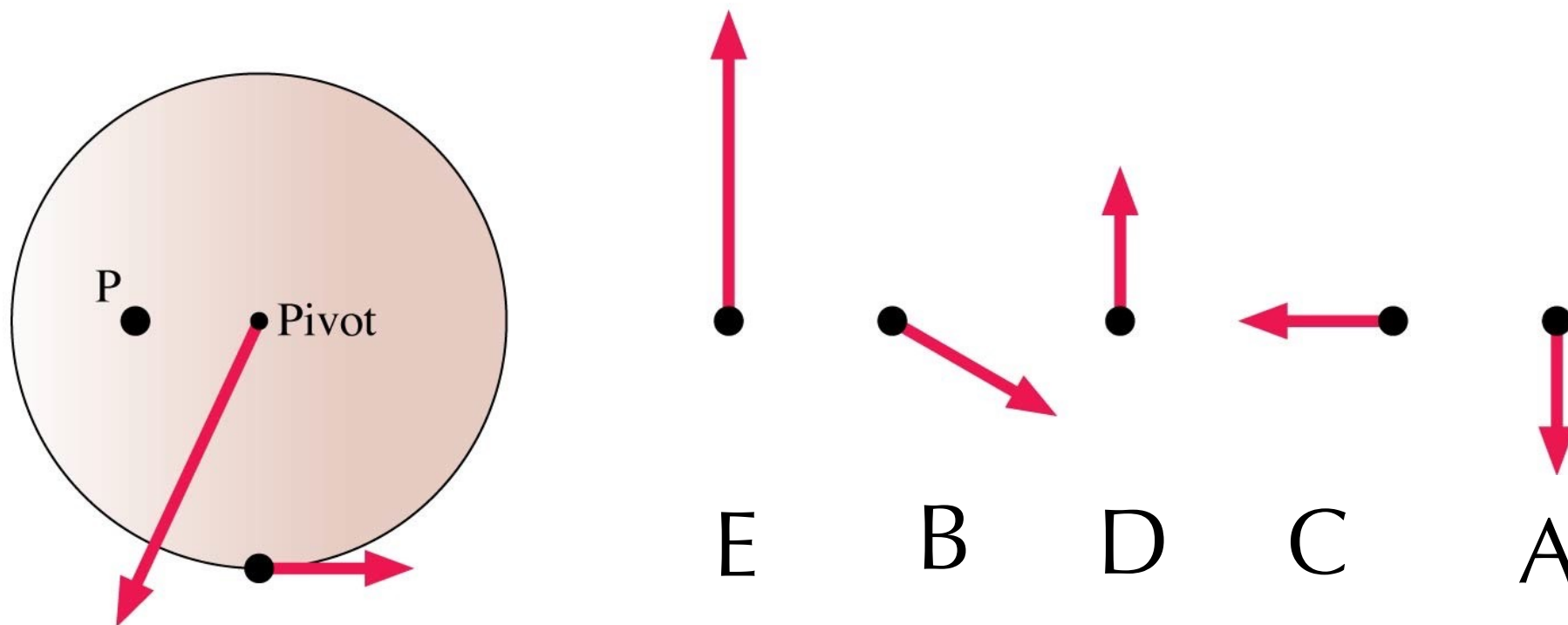
$$\tau = F_{\perp} r$$

or

$$\tau = F r_{\perp}$$



Which third force on the wheel, applied at point P, will make the net torque zero?



# Gravitational Torque

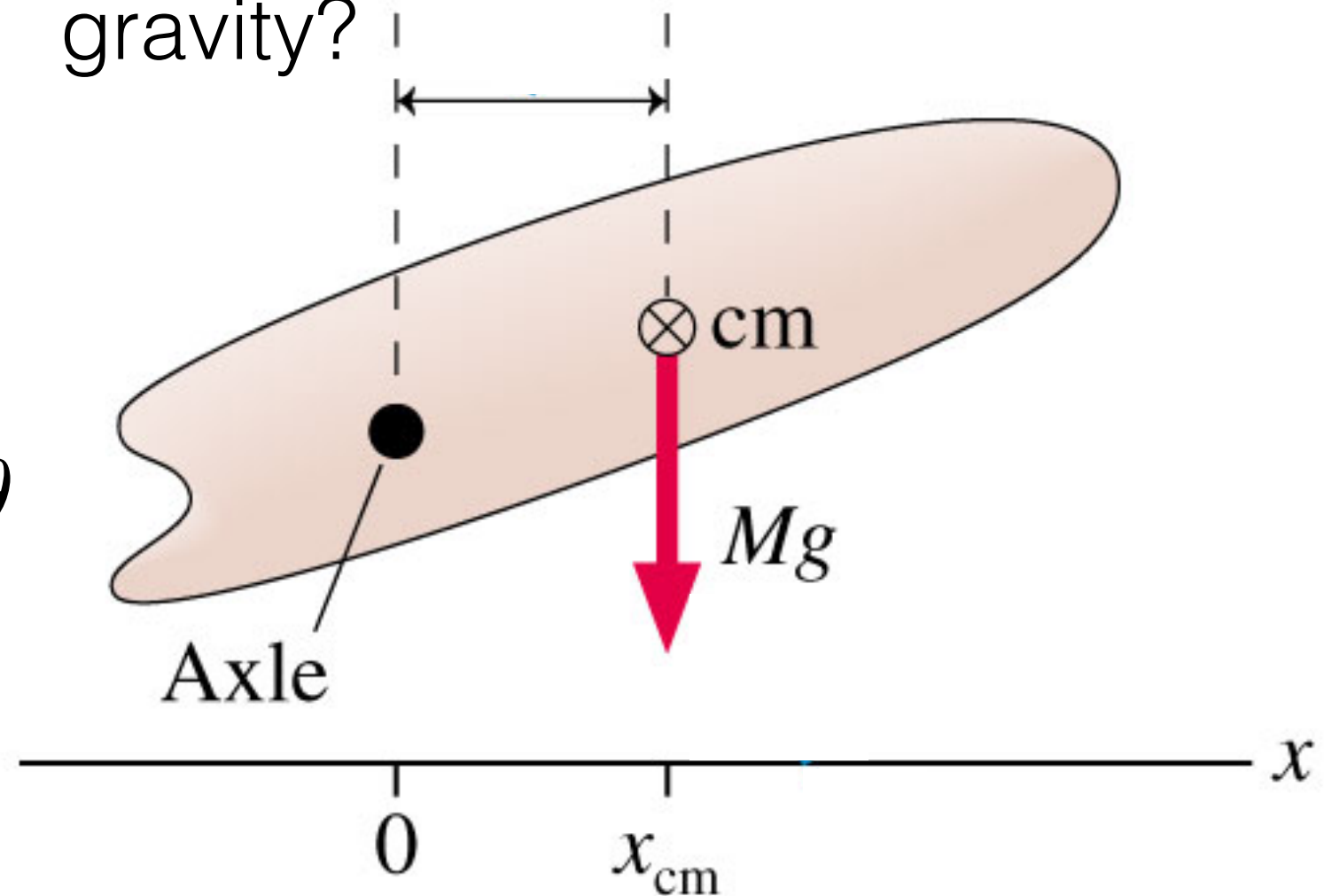
## Question #6

Which is the correct expression for the torque produced by gravity?

D  $\tau_{\text{grav}} = -Mgx_{\text{cm}}$

C  $\tau_{\text{grav}} = -Mgx_{\text{cm}} \sin \theta$

B  $\tau_{\text{grav}} = -Mgx_{\text{cm}} \cos \theta$



When I release the object what will happen?

- A) It will oscillate back and forth. B) Nothing. It will stay where you put it.  
C) It will first rotate and then quickly come to rest.



# Gravitational Torque

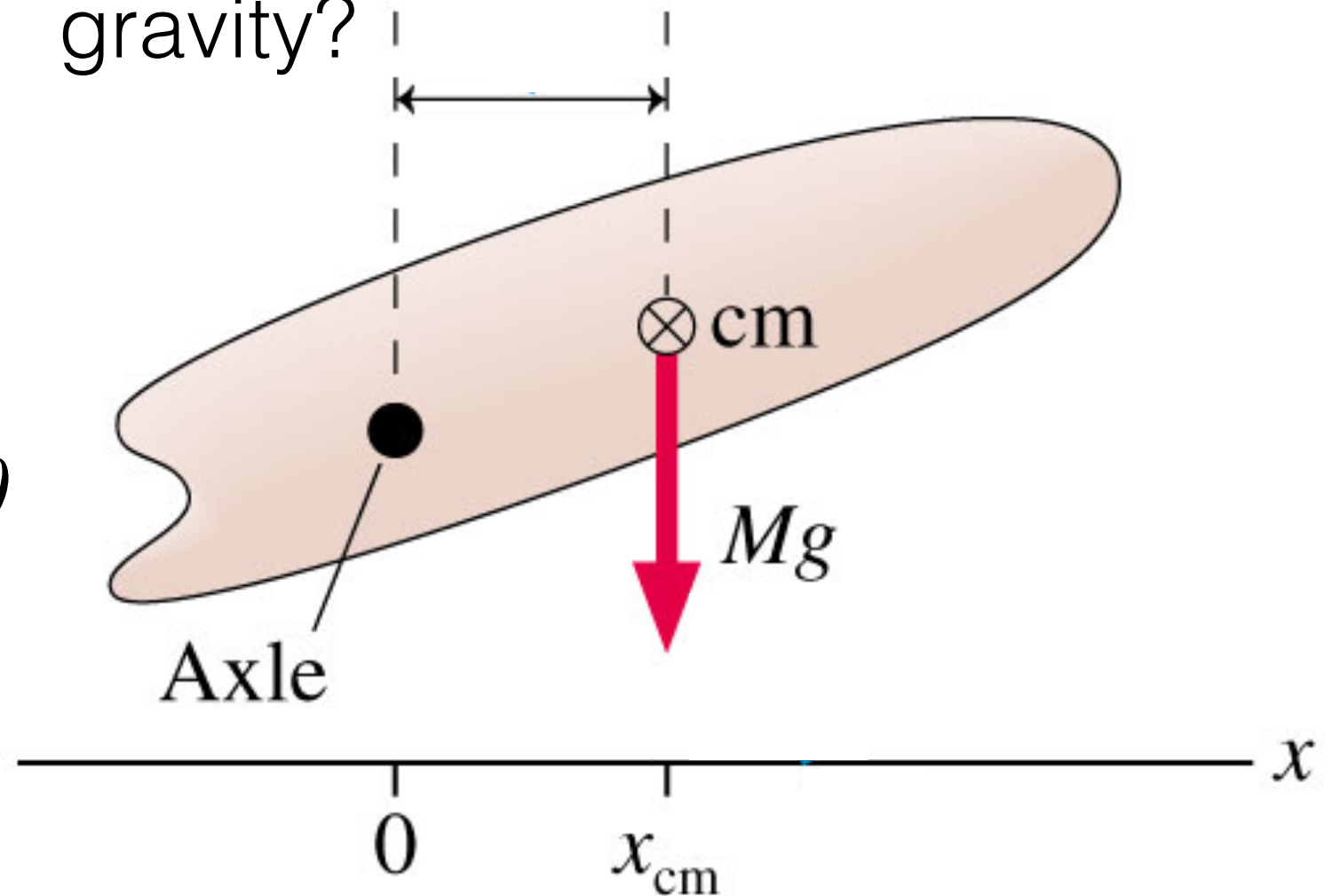
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## Question #7

When I release the object what will happen?

- A) It will oscillate back and forth. B) Nothing. It will stay where you put it.  
C) It will first rotate and then quickly come to rest.

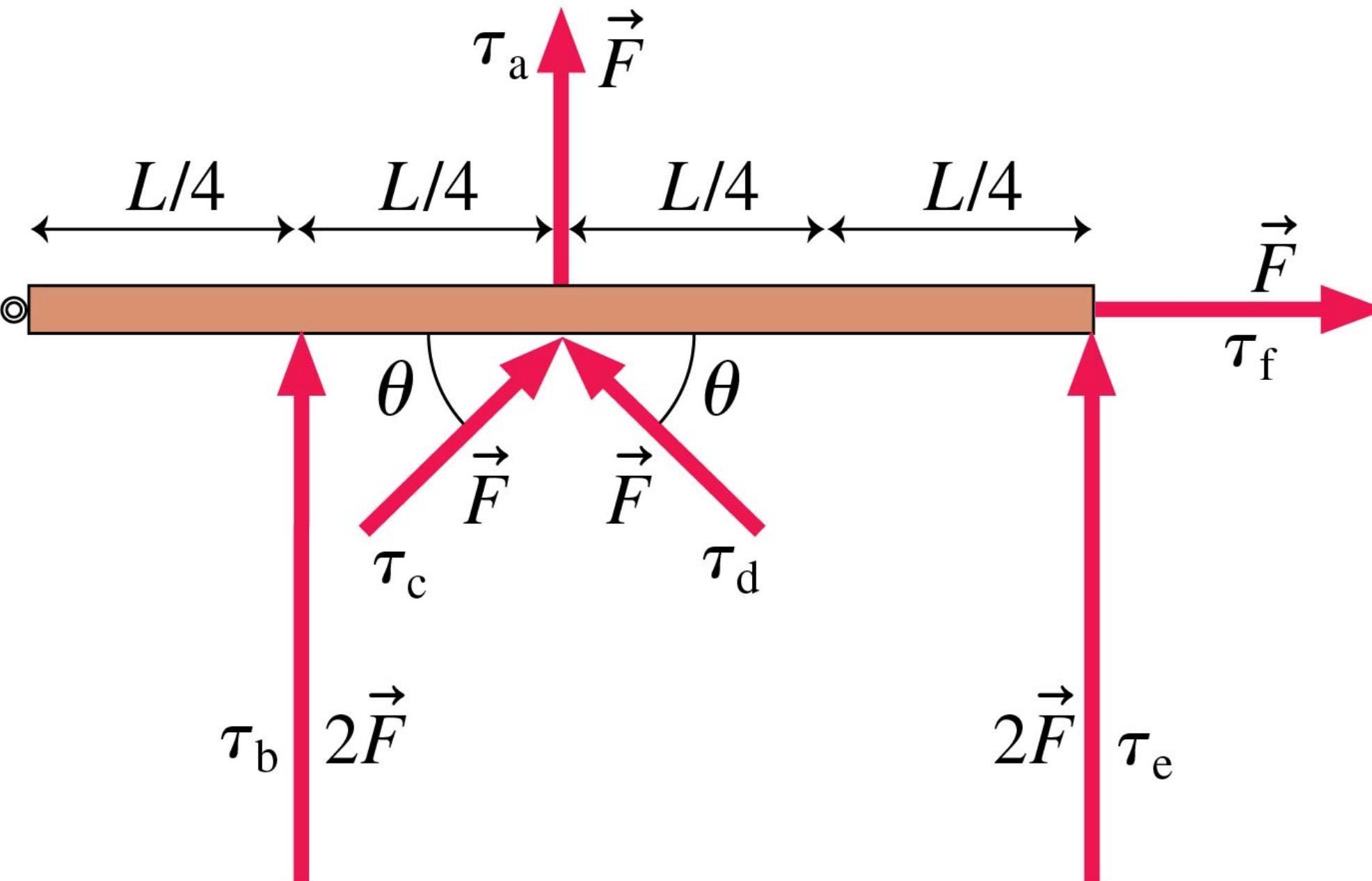
# Question #8

Rank the torques

A)  $e > a = b > c = d > f$

B)  $e = a > b > c > d > f$

C)  $e > b > a > d = c > f$



# Conservation of Energy with Rotational Kinetic Energy

A 1.0-m-long, 200 g rod is hinged at one end and connected to a wall. It is held horizontal, then released. What is the speed of the tip of the rod as it hits the wall?

