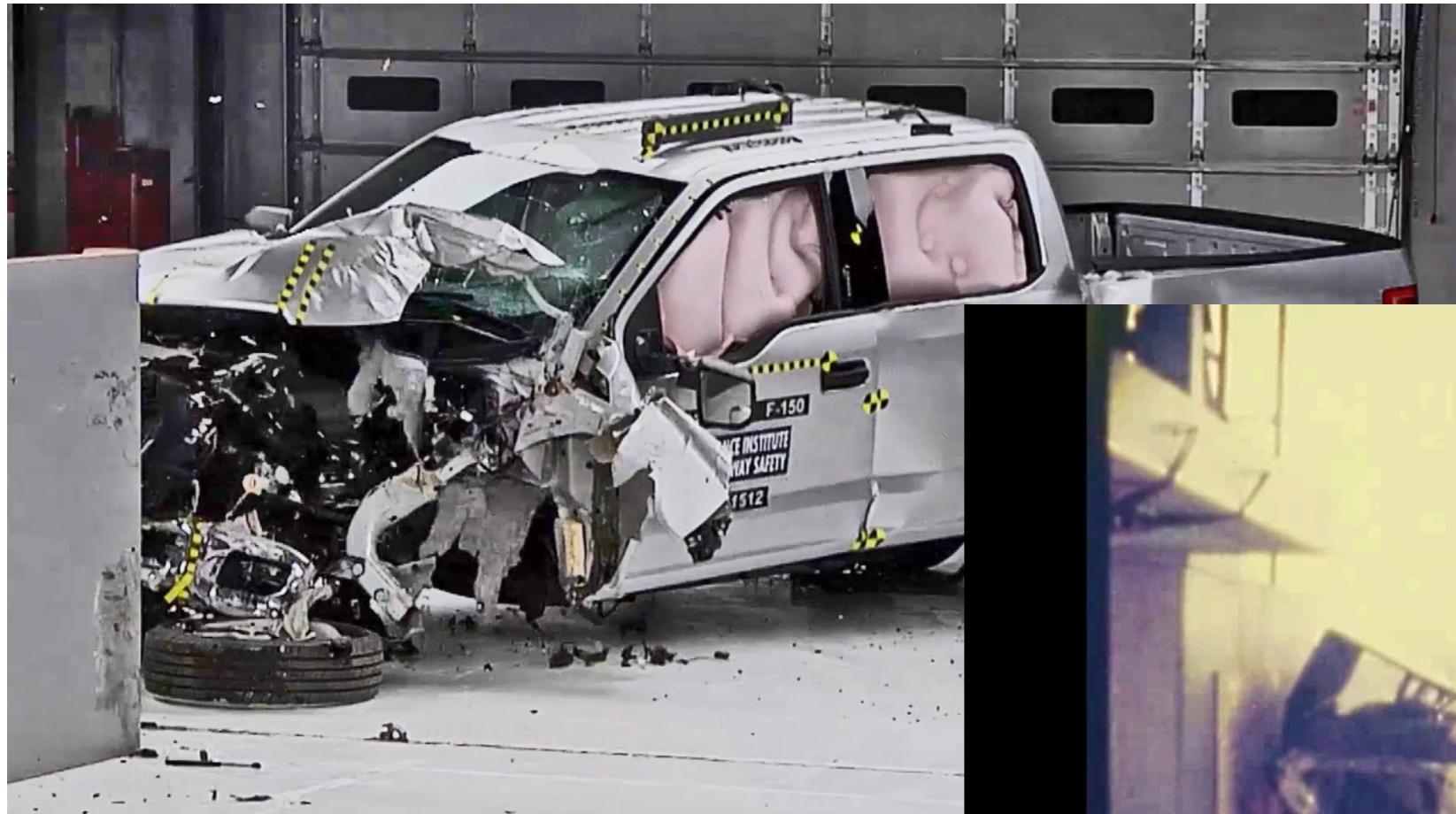


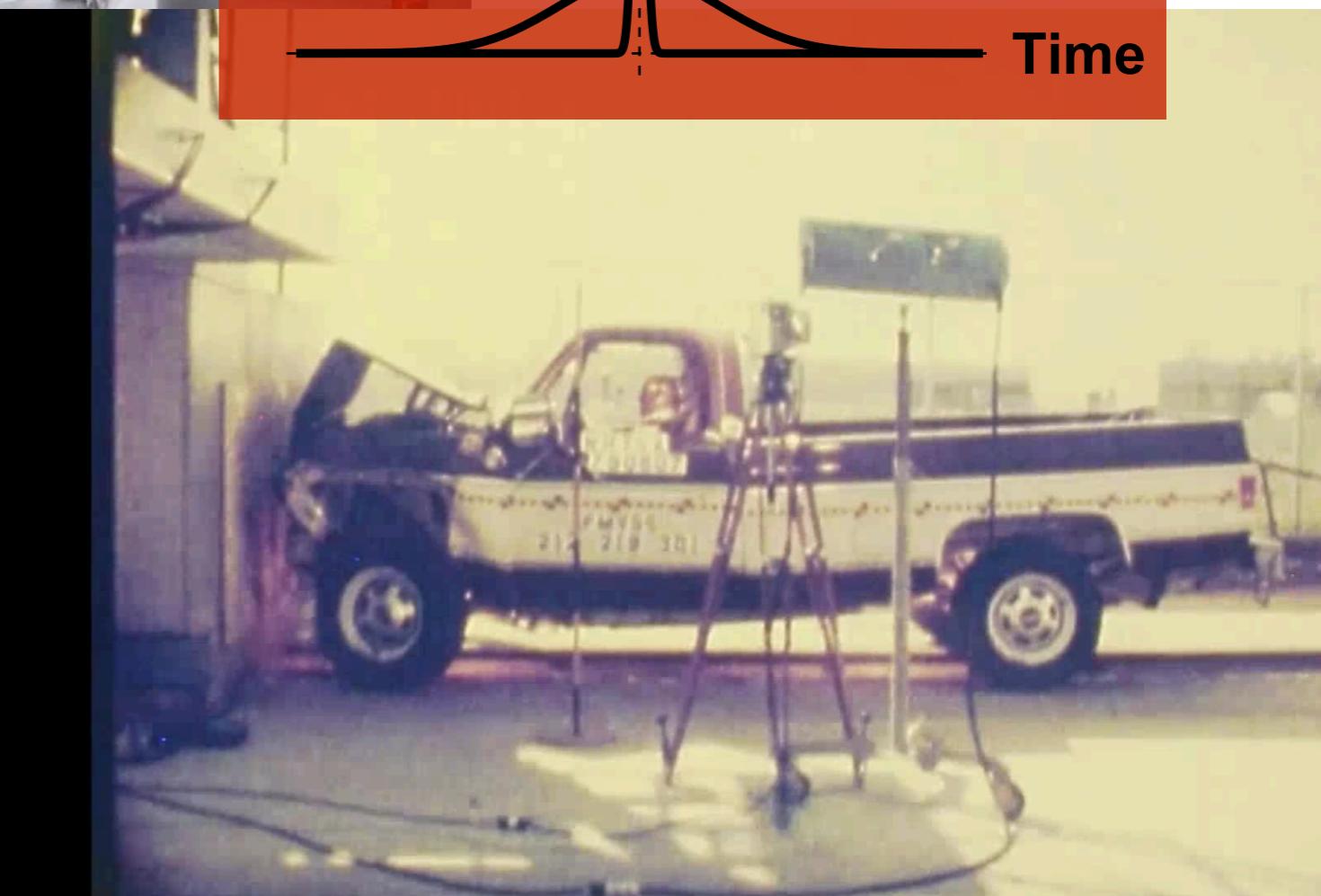
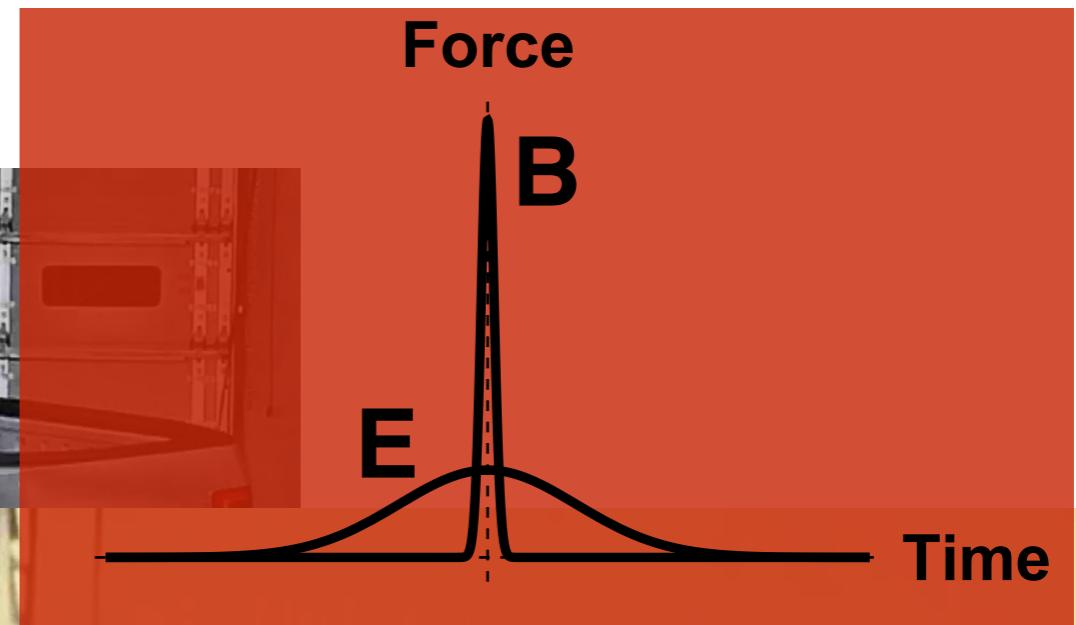
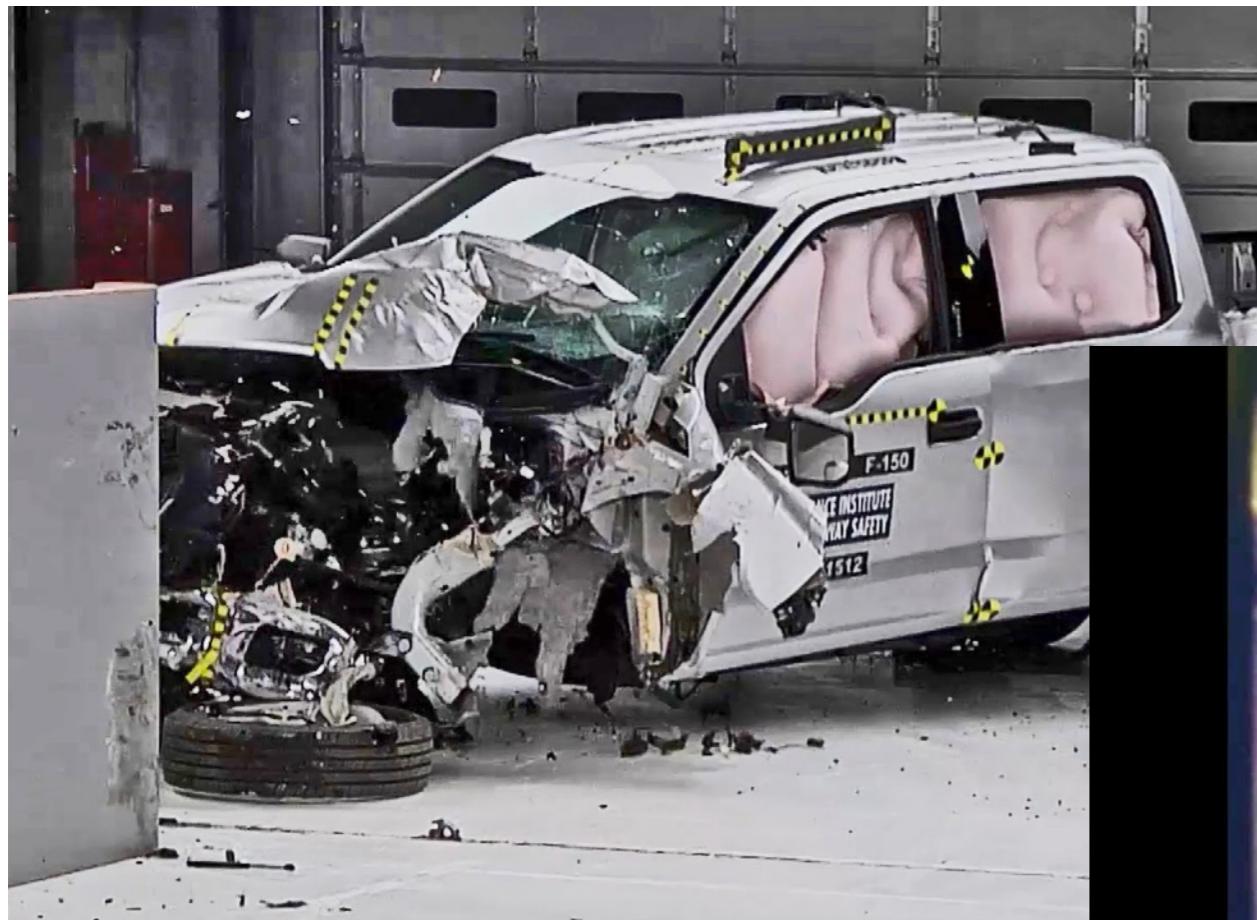
Question #43

We are living in the 21st century. Why can't we design a car that does not get demolished in a wreck? Don't we have stronger materials?



Question #43

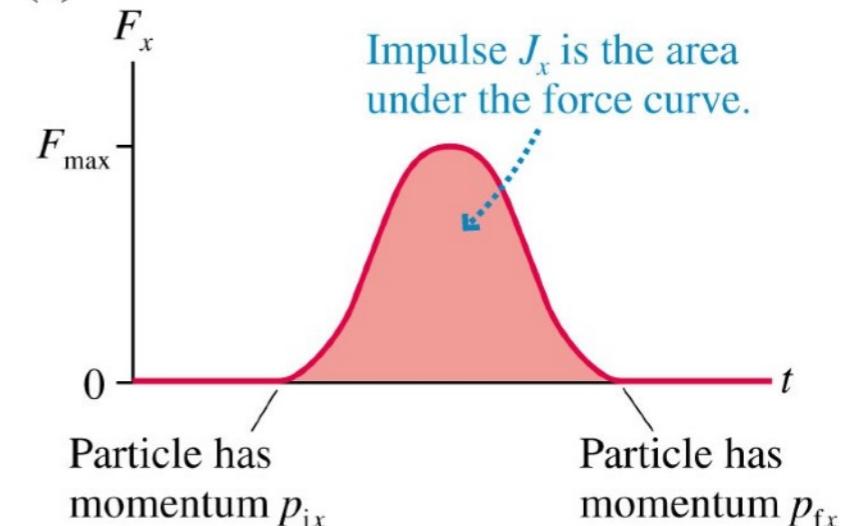
Which force vs. time graph corresponds to the crash involving the modern Ford pickup?



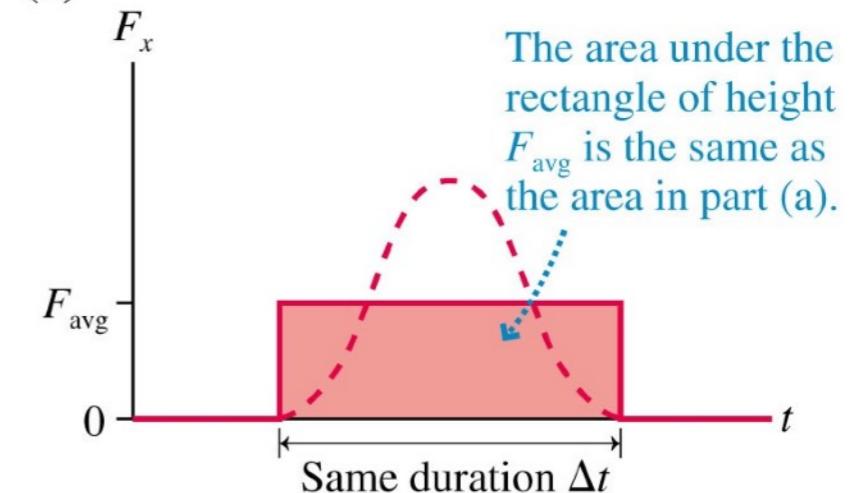
Impulse

Newton's second law can be formulated in terms of momentum rather than acceleration

(a)



(b)

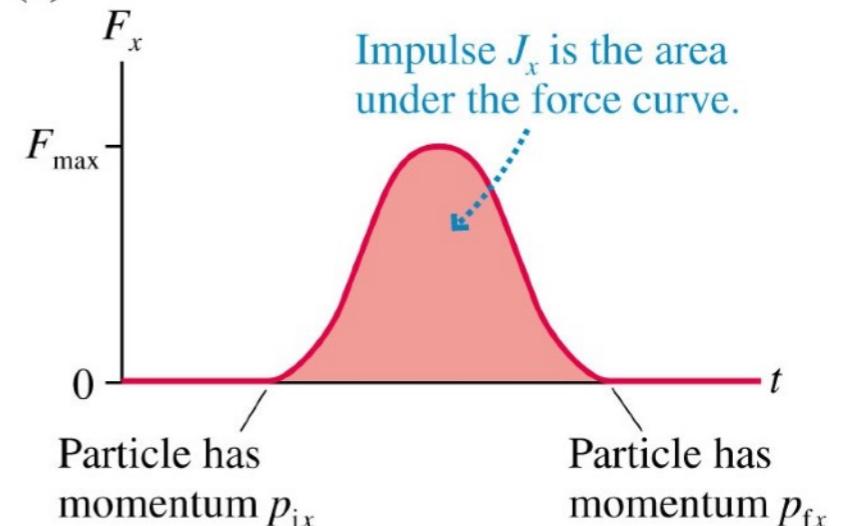


Impulse

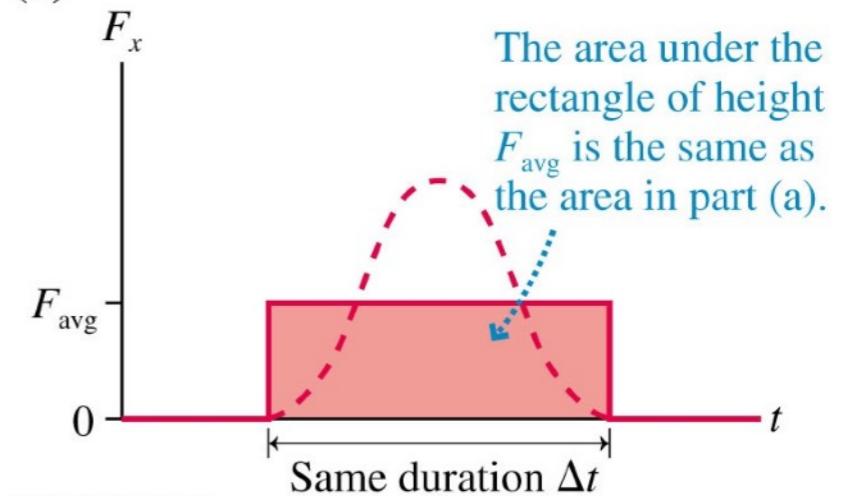
Newton's second law can be formulated in terms of momentum rather than acceleration

$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

(a)



(b)

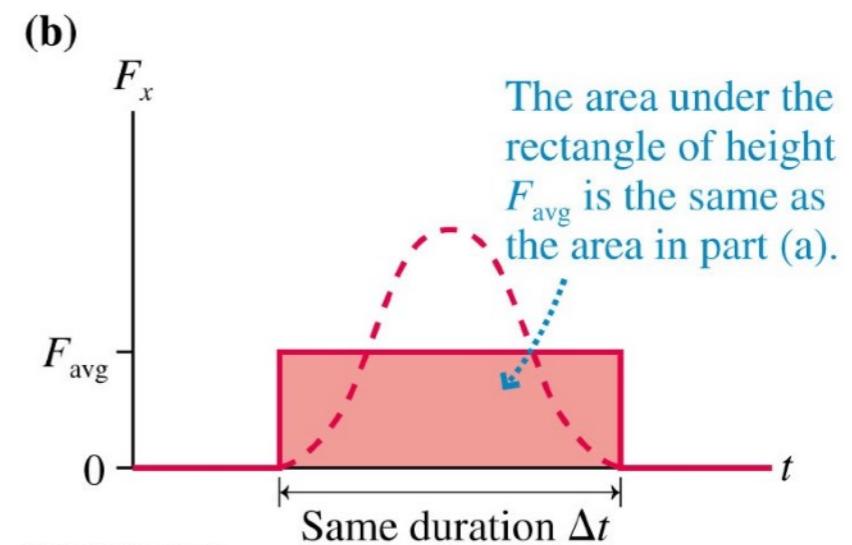
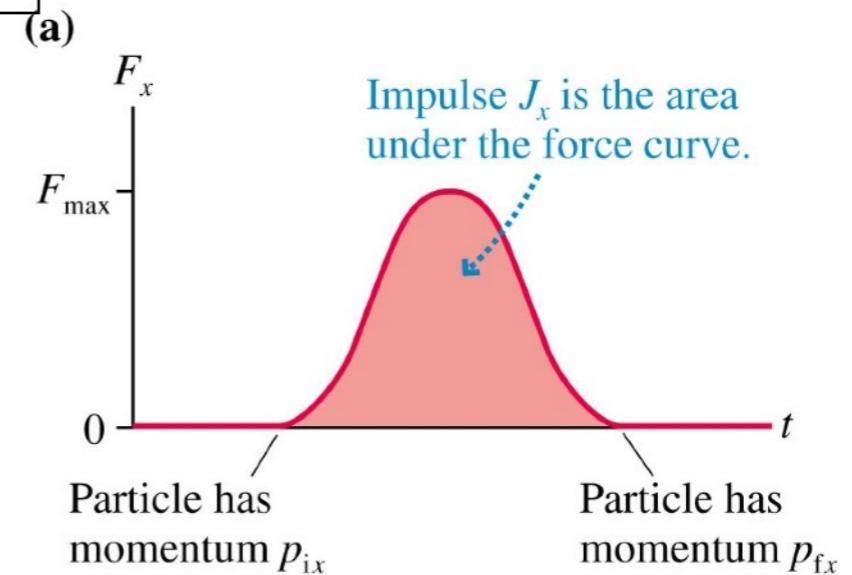


Impulse

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$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

$$\Delta p_x = p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x(t) dt$$



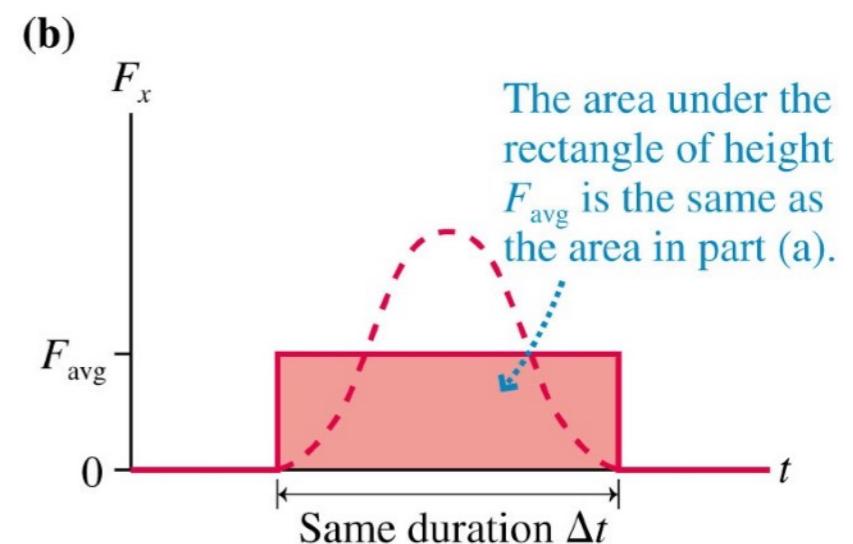
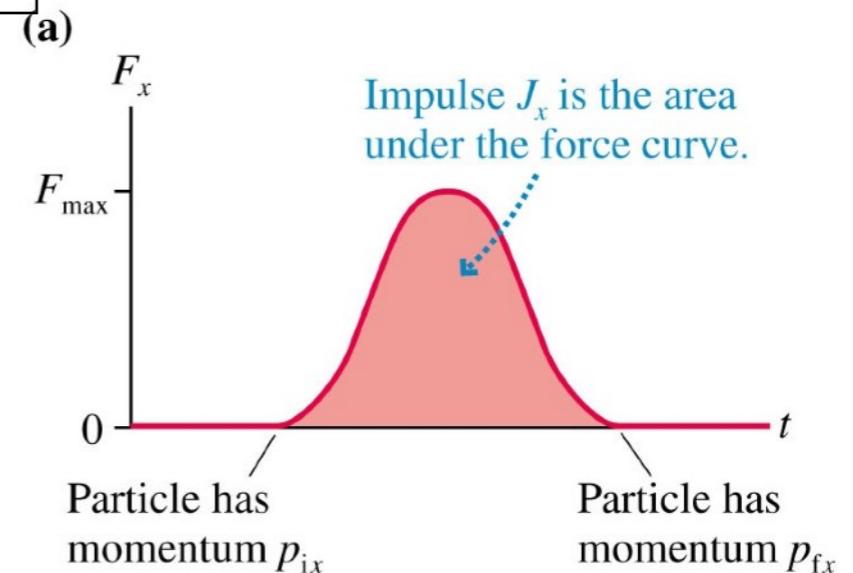
Impulse

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$$\Delta p_x = p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x(t) dt$$

$$\text{Impulse} = J_x \equiv \int_{t_i}^{t_f} F_x(t) dt$$

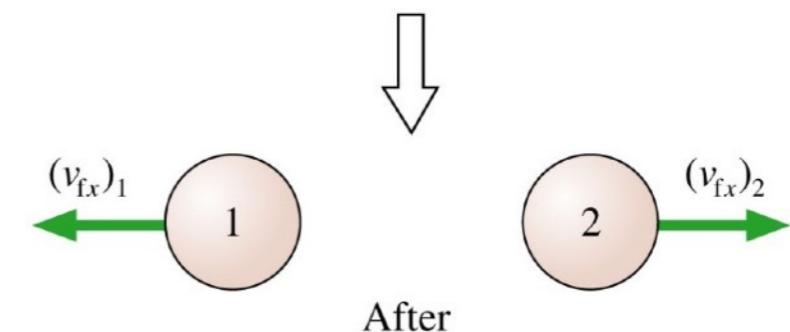
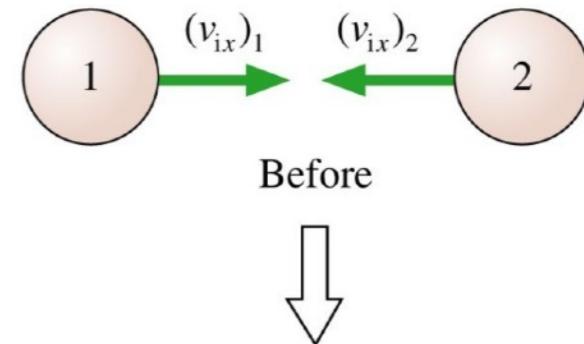


Conservation of momentum Question #1

How does the impulse on ball 1 compare to the impulse on ball 2?

- a) Impulse on ball 1 is greater than impulse on ball 2.
- b) Impulse on ball 2 is greater than impulse on ball 1
- c) Impulses are equal.

$$\text{Impulse} = J_x \equiv \int_{t_i}^{t_f} F_x(t) dt$$

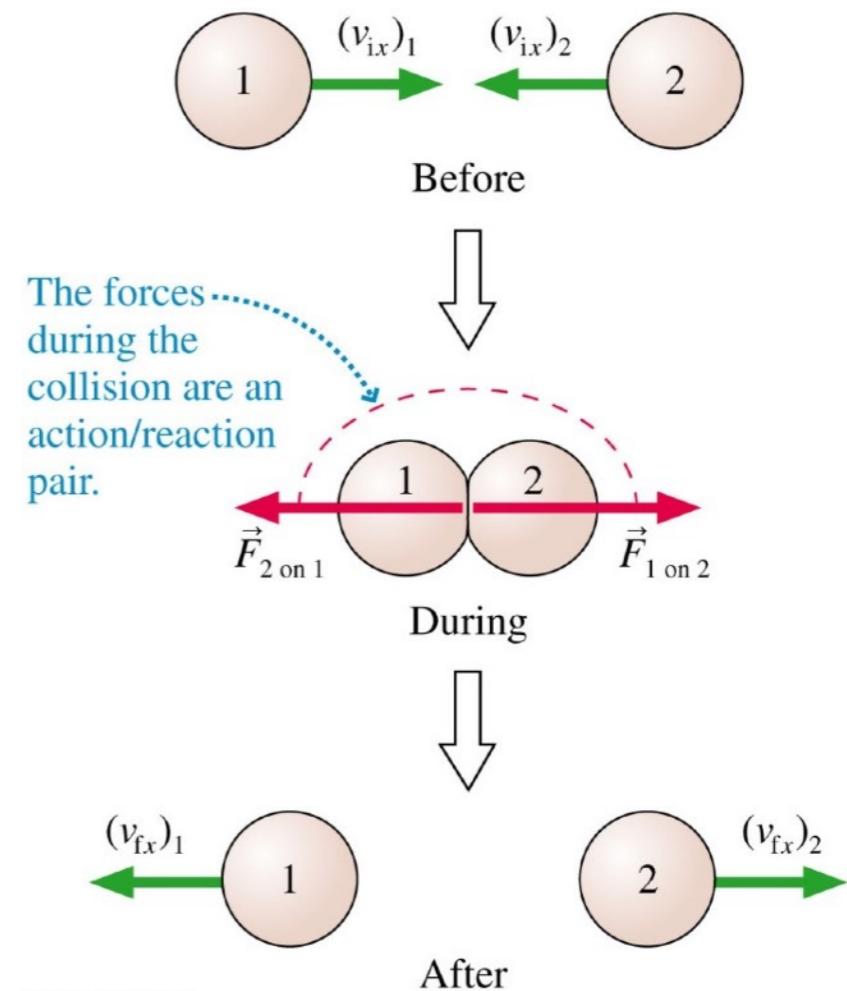


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Conservation of momentum Question #1

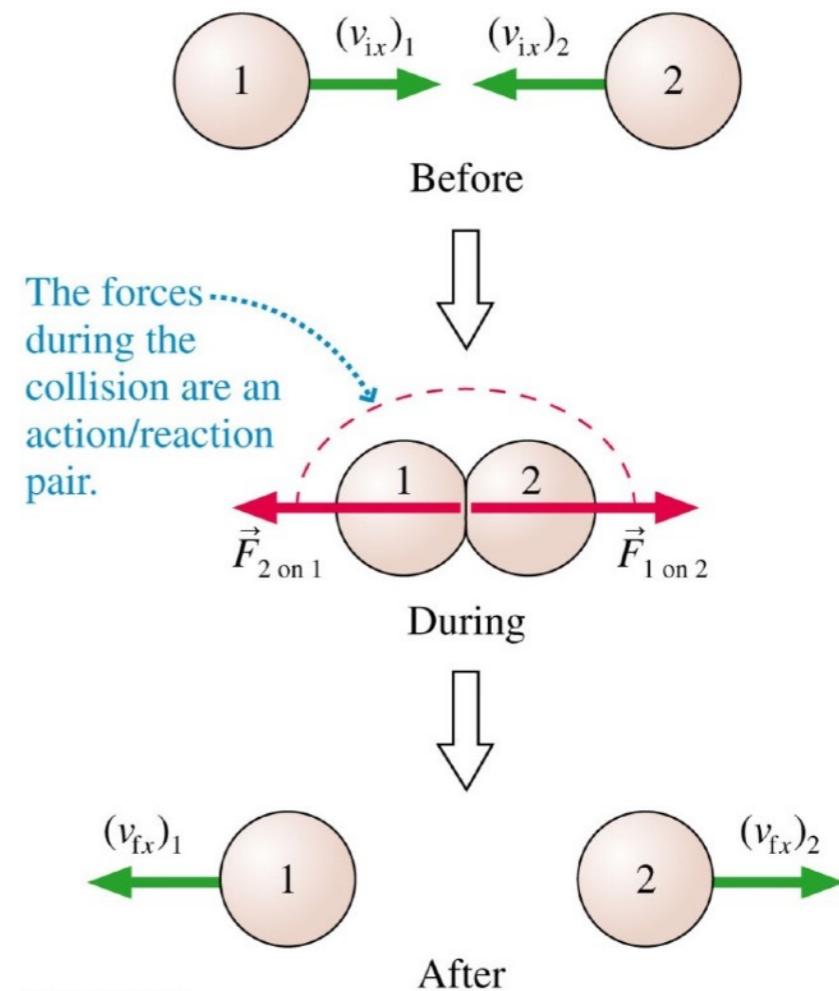
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Conservation Law

$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2$$

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Conservation of momentum Question #1

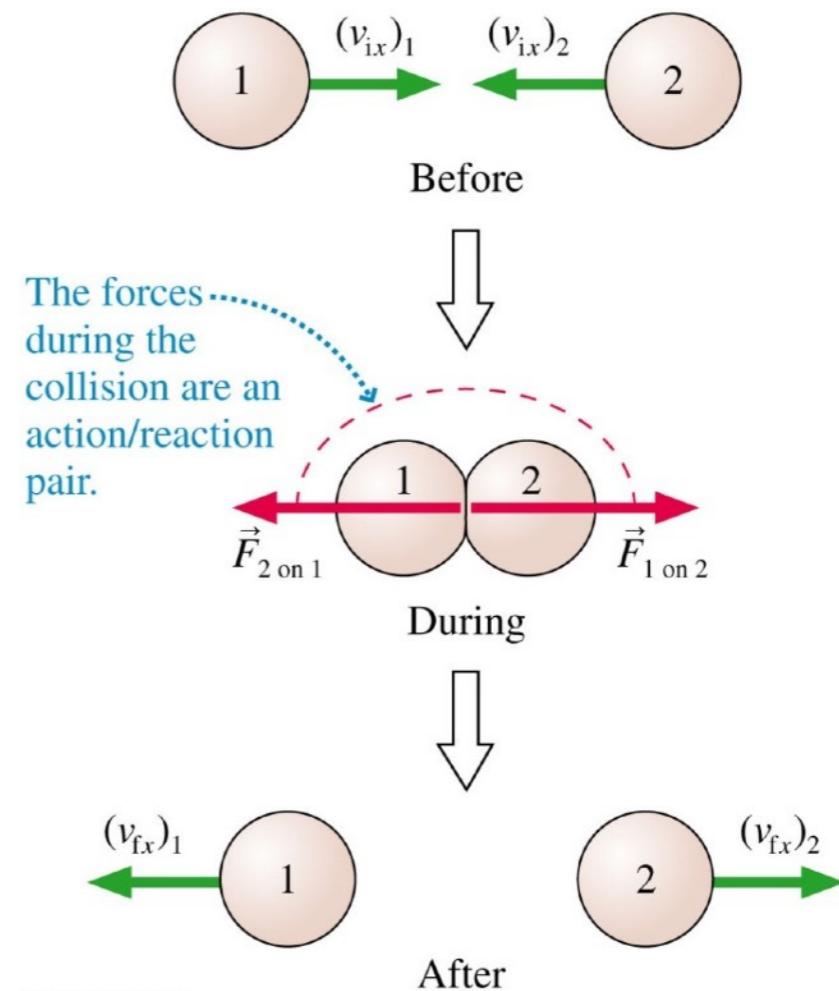
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Example

Question #2

A train car moves to the right with initial speed v_i . It collides with a stationary train car of equal mass. After the collision the two cars are stuck together. What is the train cars' final velocity?

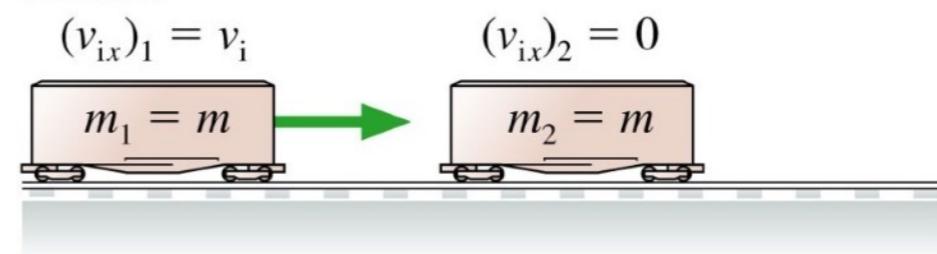
b) $v_f = v_i$

d) $v_f = 4v_i$

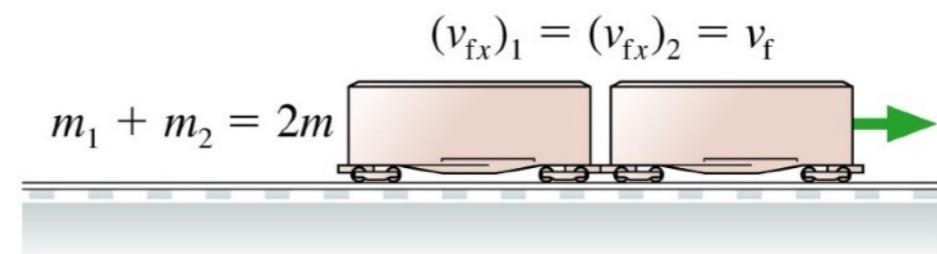
c) $v_f = 2v_i$

e) $v_f = \frac{1}{2}v_i$

Before:



After:



Example

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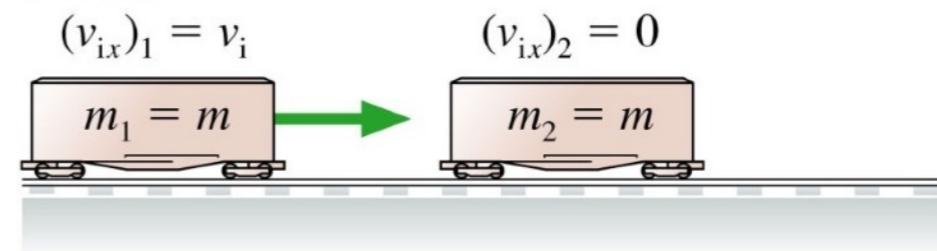
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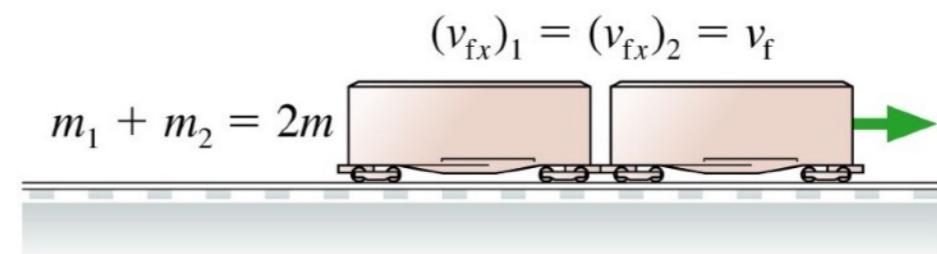
e) $v_f = \frac{1}{2}v_i$

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

Before:



After:



Example

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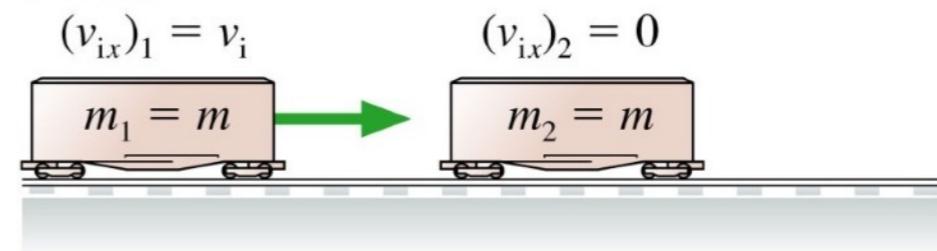
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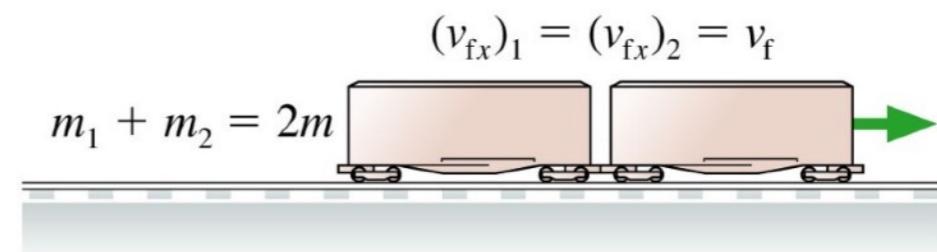
$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

$$2mv_f = mv_i + 0$$

Before:



After:



Example

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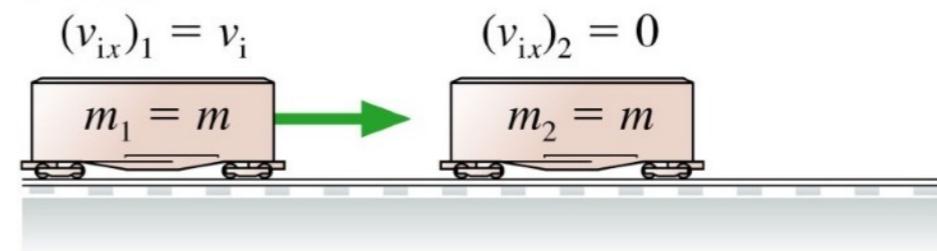
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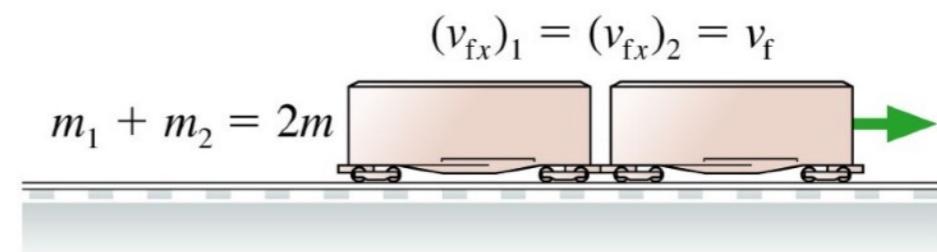
$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

$$2mv_f = mv_i + 0$$

Before:



After:



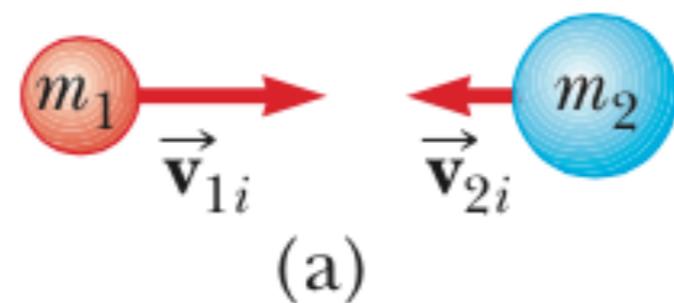
$$v_f = \frac{1}{2}v_i$$

Law of conservation of momentum

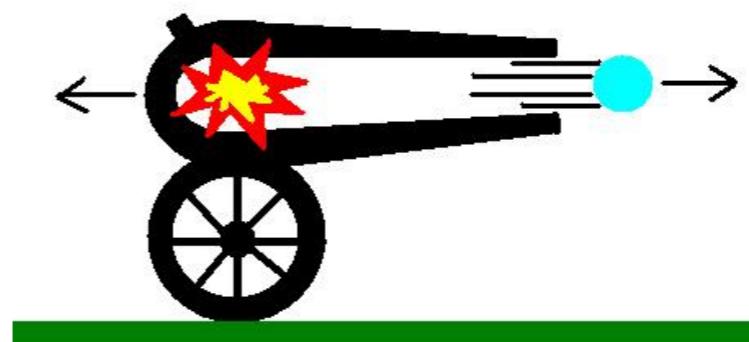
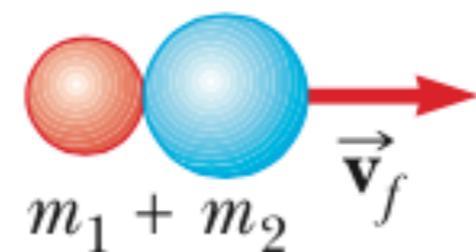
Describe how conservation of momentum applies in the following situations.



Before collision

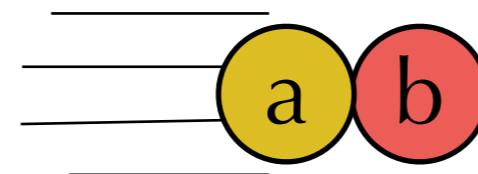
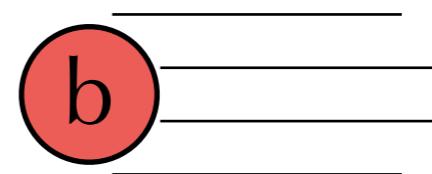
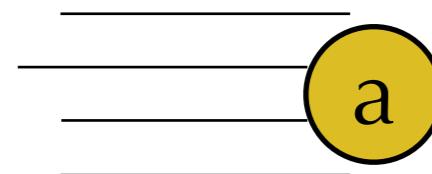


After collision



Question #3

Two balls are moving towards each other and collide. After the collision they stick together and move to the right. Which ball had the larger **speed** before the collision?



- a. No way of knowing without knowing the masses.
- b. ball a
- c. ball b
- d. they had the same speed

[Collision applet](#)

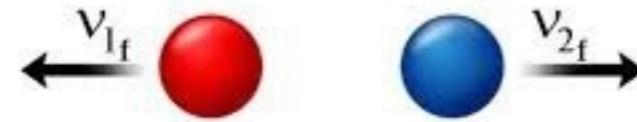
Which is an inelastic collision. Question #4

A

Before collision

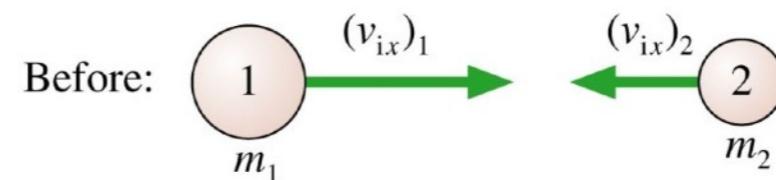


After collision

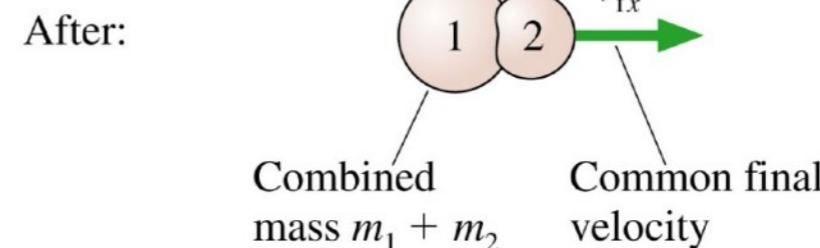


B

Two objects approach and collide.

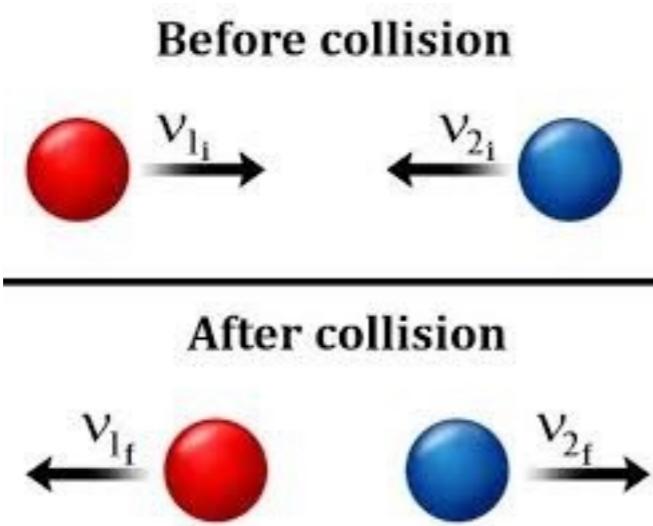


They stick and move together.

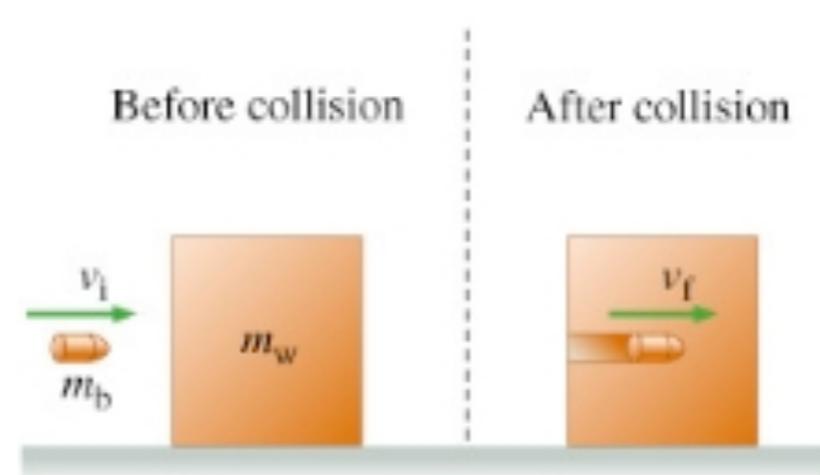
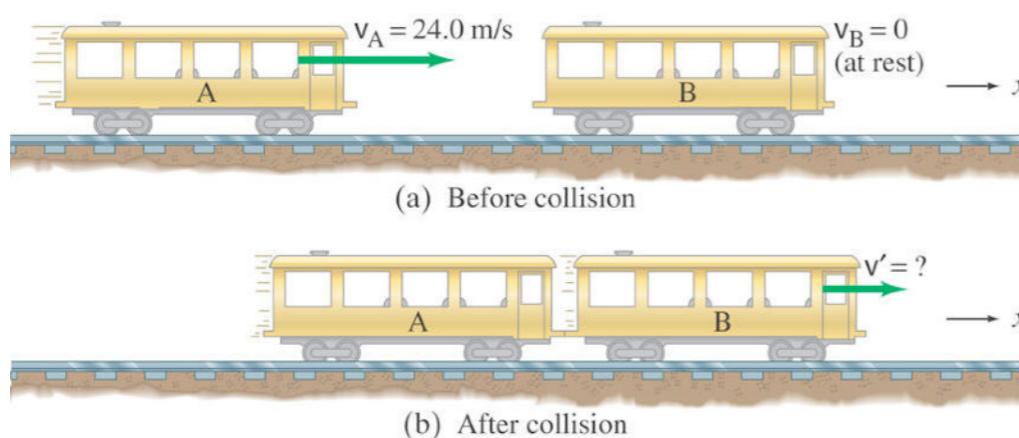
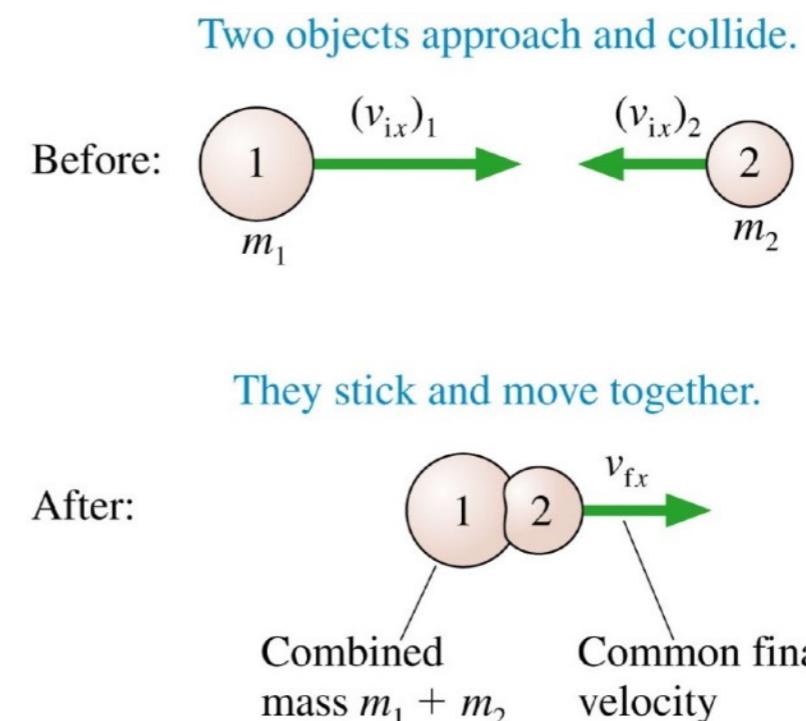


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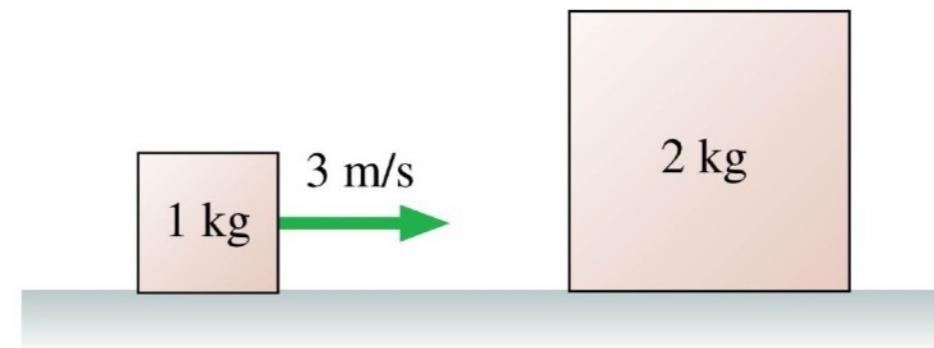


B



Question #5

The 1 kg box is sliding along a frictionless surface. It collides with and sticks to the 2 kg box. Afterward, the speed of the two boxes is

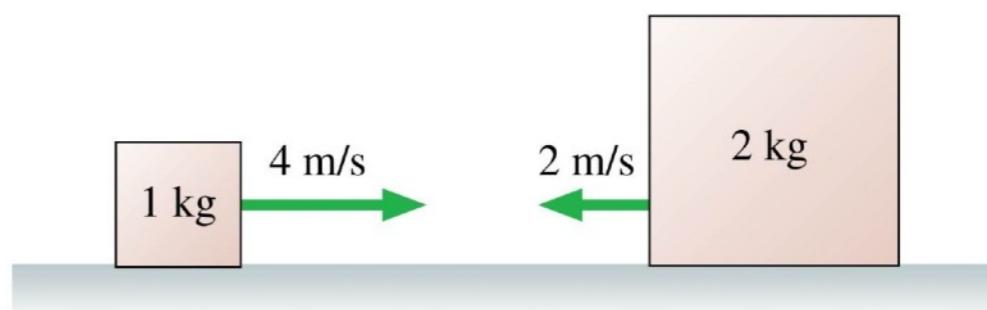


- a. There's not enough information to tell.
- b. 2 m/s.
- c. 0 m/s.
- d. 3 m/s.
- e. 1 m/s.

Question #6

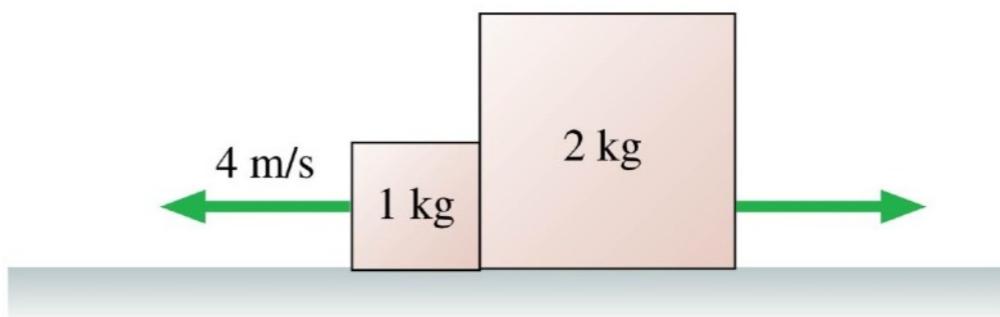
The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is

- a. 1 m/s to the right.
- b. 1 m/s to the left.
- c. 2 m/s to the left.
- d. 0 m/s, at rest.
- e. 2 m/s to the right.



Question #7

The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2 kg box going?



- A. 4 m/s.
- B. 2 m/s.
- C. 1 m/s.
- D. 8 m/s.
- E. There's not enough information to tell.

Try another one

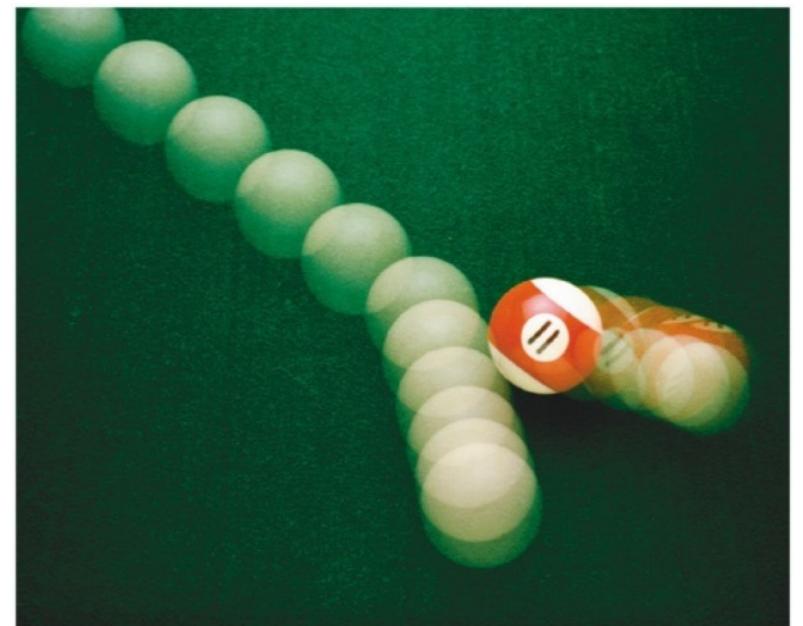
Two ice skaters, with masses 50 kg and 75 kg, are at the center of a 60-m-diameter circular rink. The skaters push off against each other and glide to opposite edges of the rink. If the heavier skater reaches the edge in 20 s, how long does the other skater take to reach the edge?

Momentum in two dimensions

- The total momentum is a vector sum of the momenta of the individual particles.
- Momentum is conserved only if each component of is conserved:

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \dots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \dots$$

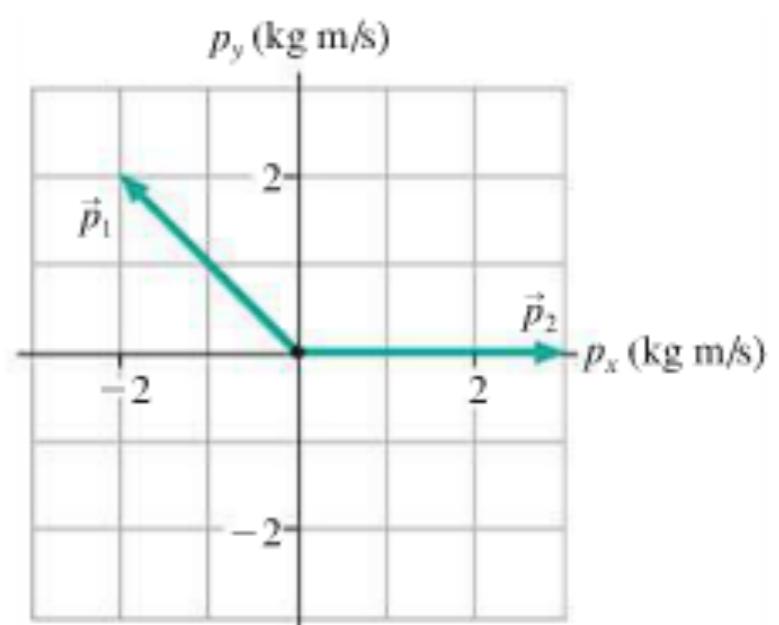
$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \dots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \dots$$



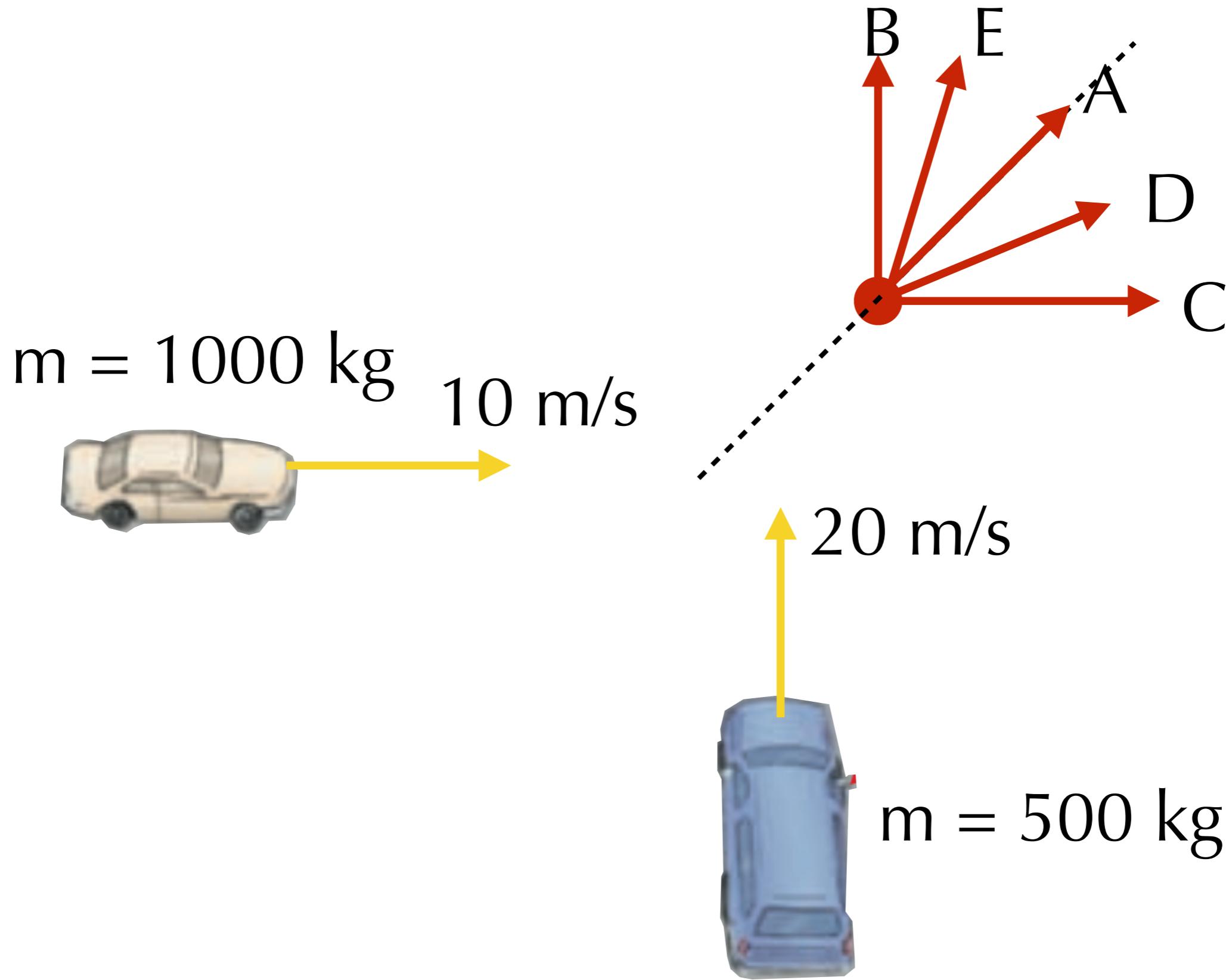
Question #8

An object at rest explodes into three fragments. The figure shows the momentum vectors of two of the fragments. What are p_x and p_y of the third fragment?

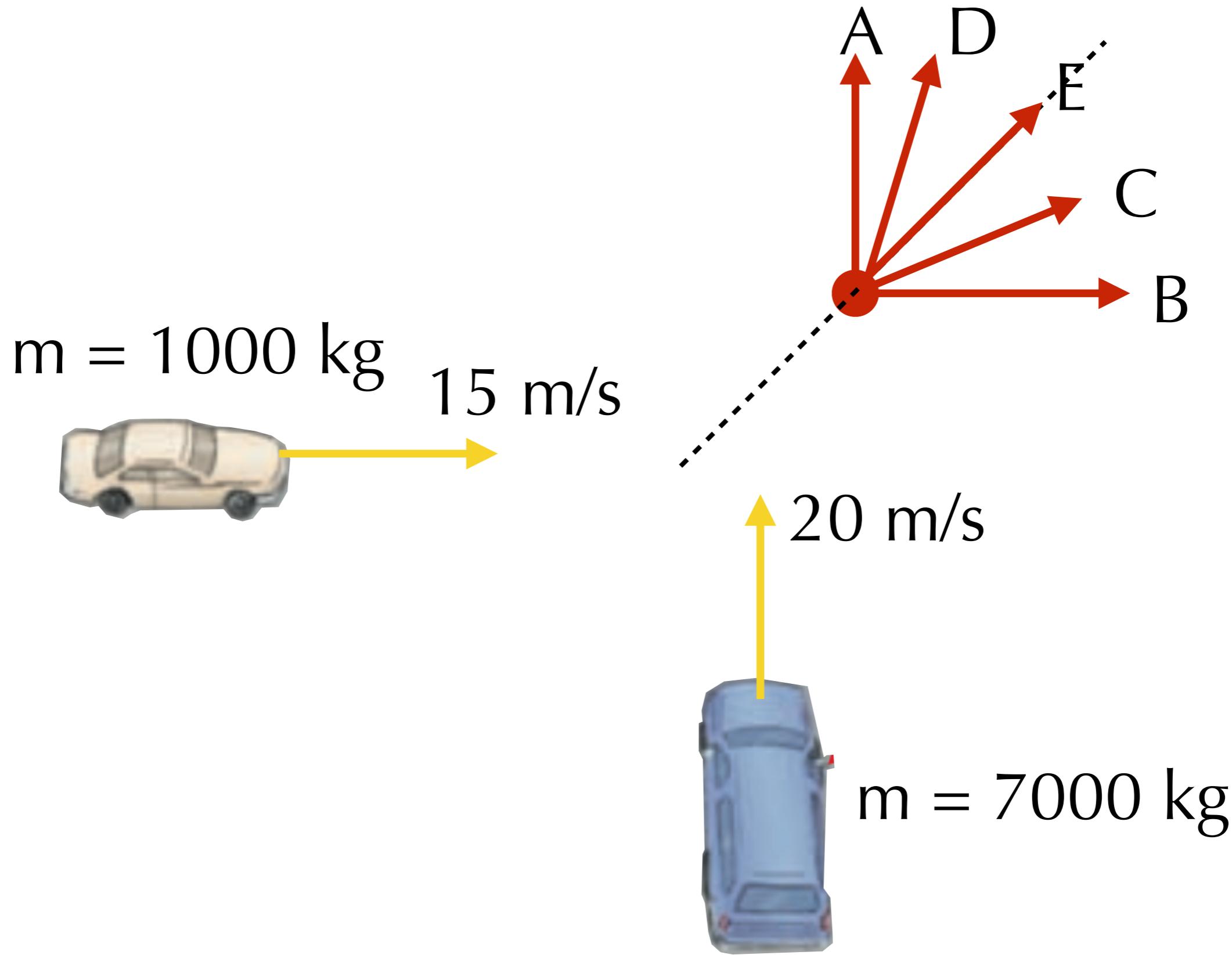
- a. $p_x = -1, p_y = -2$
- b. $p_x = 1, p_y = 2$
- c. $p_x = -2, p_y = -1$
- d. $p_x = 2, p_y = 3$
- e. $p_x = 2, p_y = 1$



Question #9



Question #10



Problem

A 2100 kg truck is traveling east through an intersection at 2.0 m/s when it is hit simultaneously by two cars. One car is a 1200 kg compact traveling north at 5.0 m/s. The other is a 1500 kg midsized traveling east at 10 m/s. The three vehicles become entangled and slide as one body. What are their speed and direction just after the collision

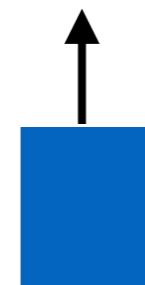
$$v = 10 \frac{m}{s}$$



$$v = 2 \frac{m}{s}$$



$$v = 5 \frac{m}{s}$$



Recommended Problems

One-dimensional cons of momentum

11.20

11.50

11.53

11.54

Two-dimensional cons of momentum

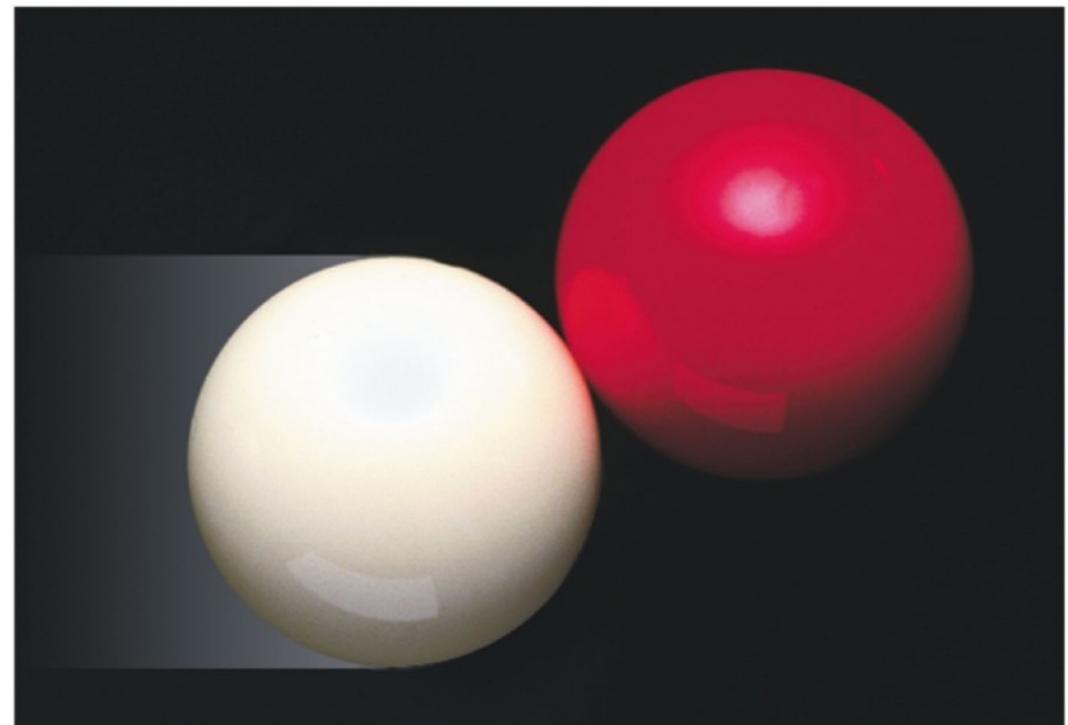
11.33

11.71

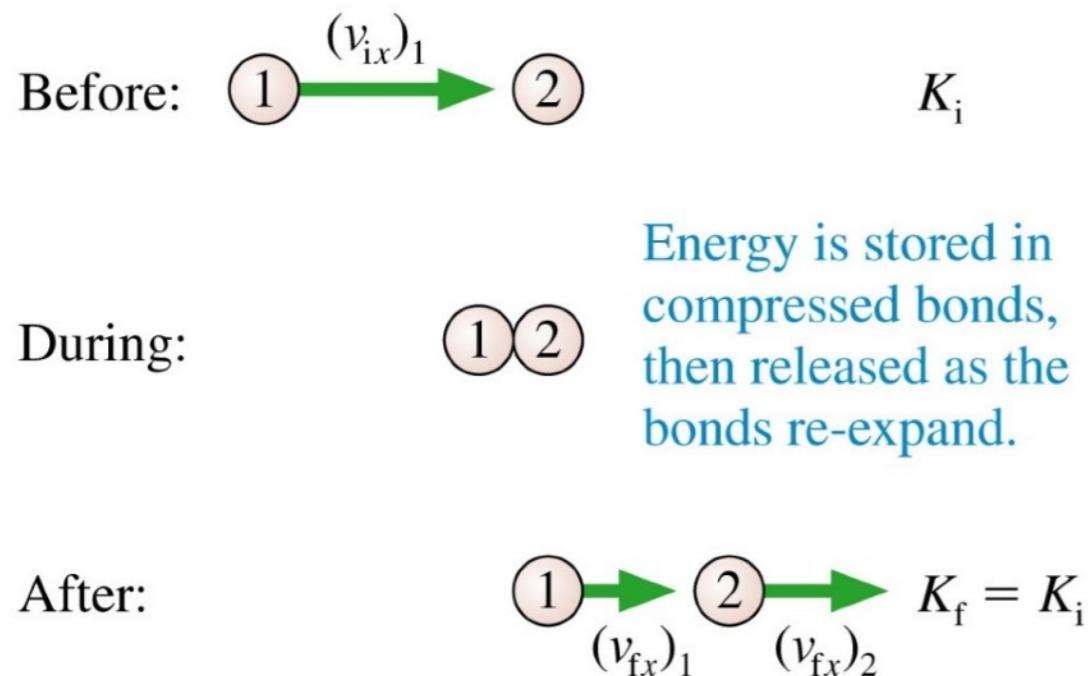
Elastic Collisions

For most collisions, some of the mechanical energy is dissipated inside the objects as thermal energy

A collision in which mechanical energy is conserved is a **perfectly elastic collision**



A perfectly elastic collision

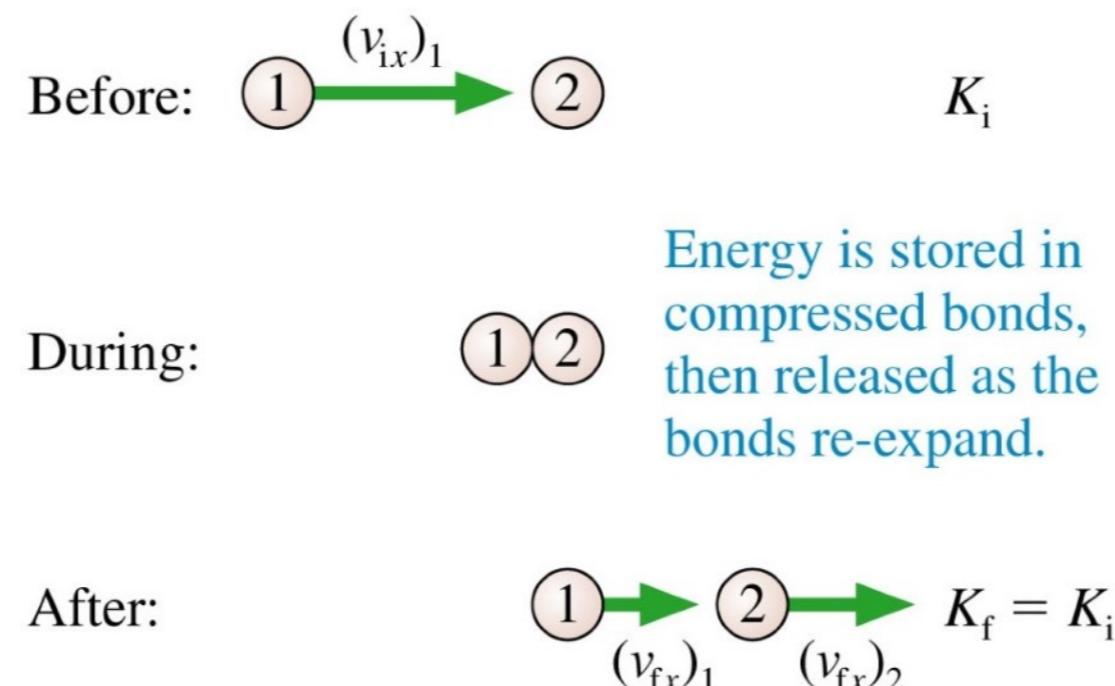


- Momentum is conserved in all isolated collisions.
- In a perfectly elastic collision, the kinetic energy must also be conserved.

momentum conservation: $m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$

energy conservation: $\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

A perfectly elastic collision



Energy is stored in compressed bonds, then released as the bonds re-expand.

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

A perfectly elastic collision: special case 1

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 = m_2$$



$$m_1 = m_2$$

Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

The first ball stops and transfers all of its momentum to the second particle.

A perfectly elastic collision: special case 1

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 = m_2$$



$$(v_{fx})_1 = 0$$



$$(v_{fx})_2 = (v_{ix})_1$$

$m_1 = m_2$
Ball 1 stops. Ball 2 goes forward with $v_{f2} = v_{i1}$.

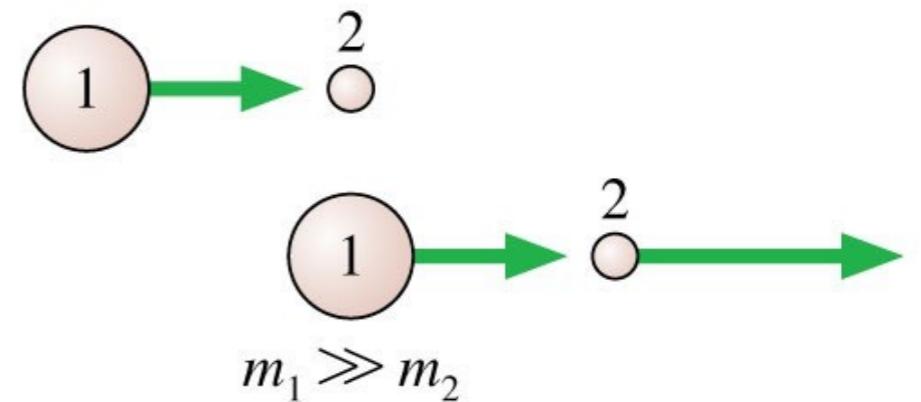
The first ball stops and transfers all of its momentum to the second particle.

A perfectly elastic collision: special case 2

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 \gg m_2$$



Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

A perfectly elastic collision: special case 2

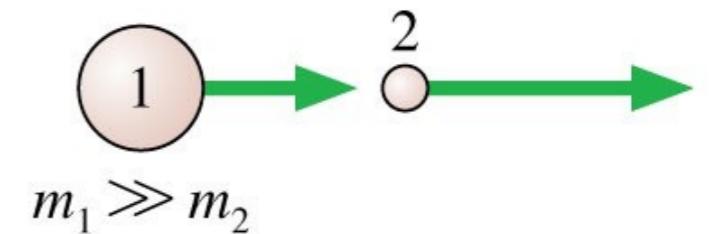
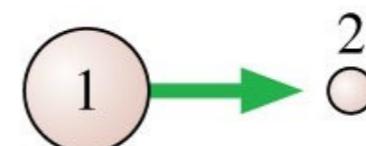
$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 \gg m_2$$

$$(v_{fx})_1 \approx (v_{ix})_1$$

$$(v_{fx})_2 \approx 2(v_{ix})_1$$



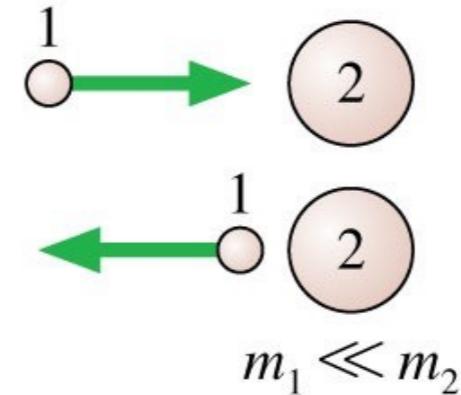
Ball 1 hardly slows down. Ball 2 is knocked forward at $v_{f2} \approx 2v_{i1}$.

A perfectly elastic collision: special case 3

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 \ll m_2$$



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

A perfectly elastic collision: special case 3

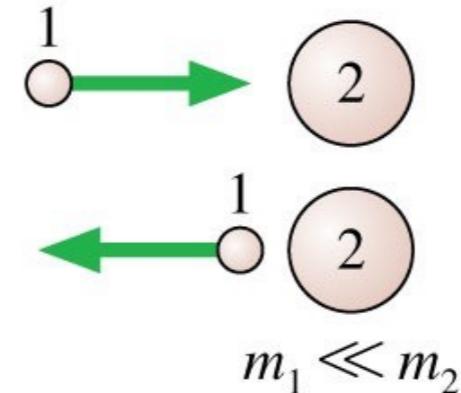
$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

$$m_1 \ll m_2$$

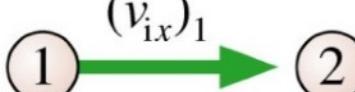
$$(v_{fx})_1 \approx -(v_{ix})_1$$

$$(v_{fx})_2 \approx 0$$



Ball 1 bounces off ball 2 with almost no loss of speed. Ball 2 hardly moves.

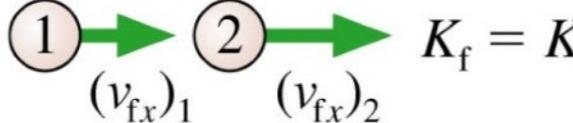
Perfectly elastic collisions: Using reference frames

Before:  $(v_{ix})_1$

K_i

During: 

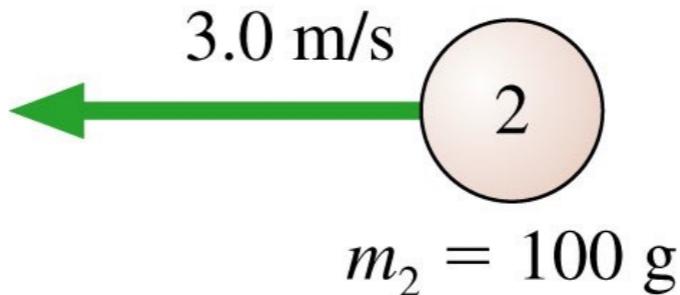
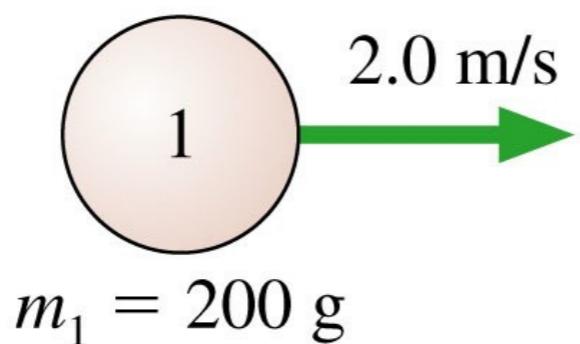
Energy is stored in compressed bonds, then released as the bonds re-expand.

After:  $(v_{fx})_1$ $(v_{fx})_2$ $K_f = K_i$

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1$$

What if....

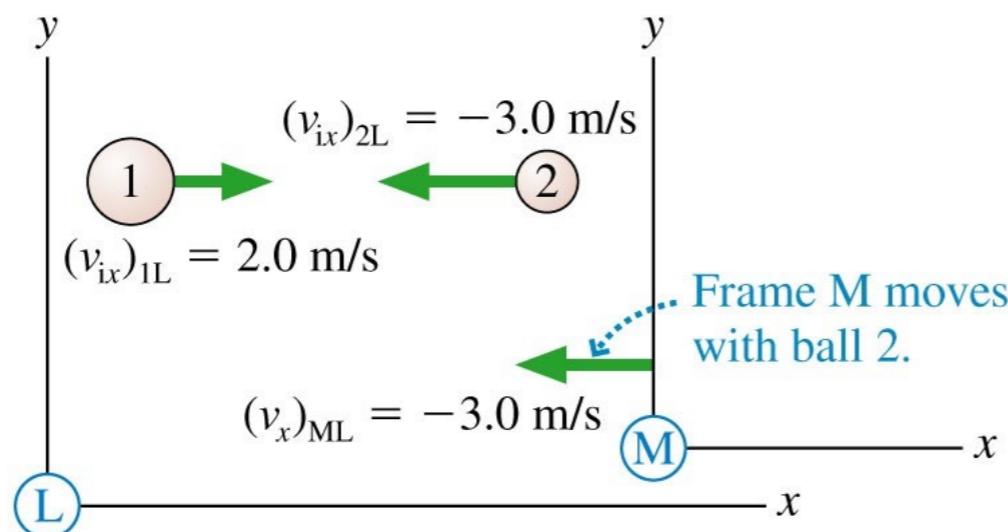


Using Reference Frames: Quick Example

$$(v_{ix})_{1M} = (v_{ix})_{1L} + (v_x)_{LM} = 2.0 \text{ m/s} + 3.0 \text{ m/s} = 5.0 \text{ m/s}$$

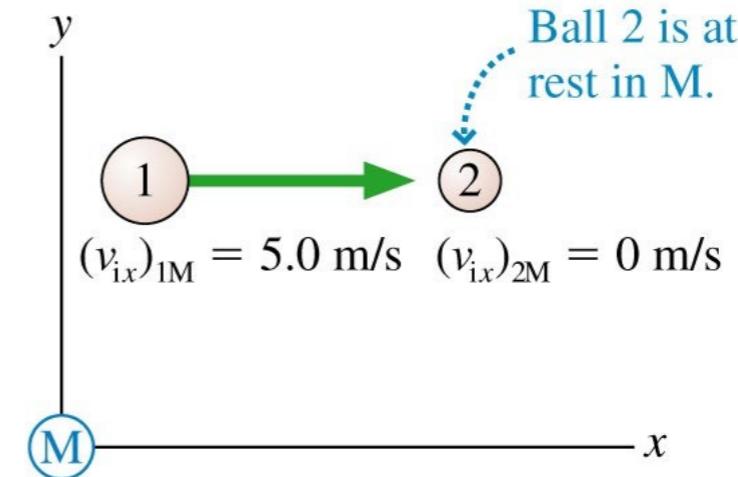
$$(v_{ix})_{2M} = (v_{ix})_{2L} + (v_x)_{LM} = -3.0 \text{ m/s} + 3.0 \text{ m/s} = 0 \text{ m/s}$$

(a)



The collision seen
in the lab frame L.

(b)



The collision seen
in moving frame M.

Using Reference Frames: Quick Example

- We can use Equations 10.42 to find the post-collision velocities in the moving frame M:

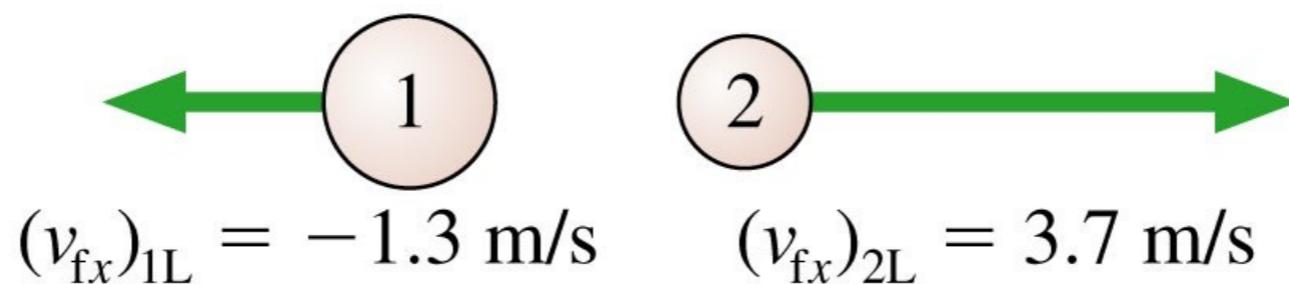
$$(v_{fx})_{1M} = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_{1M} = 1.7 \text{ m/s}$$

$$(v_{fx})_{2M} = \frac{2m_1}{m_1 + m_2} (v_{ix})_{1M} = 6.7 \text{ m/s}$$

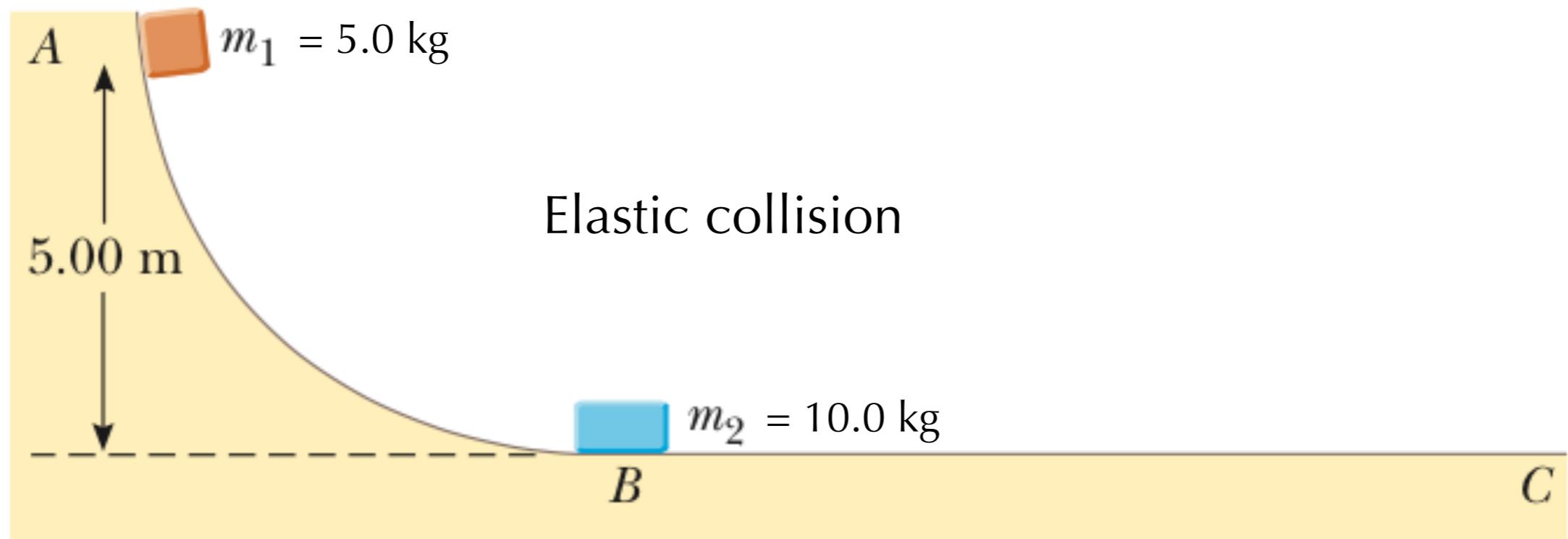
- Transforming back to the lab frame L:

$$(v_{fx})_{1L} = (v_{fx})_{1M} + (v_x)_{ML} = 1.7 \text{ m/s} + (-3.0 \text{ m/s}) = -1.3 \text{ m/s}$$

$$(v_{fx})_{2L} = (v_{fx})_{2M} + (v_x)_{ML} = 6.7 \text{ m/s} + (-3.0 \text{ m/s}) = 3.7 \text{ m/s}$$



What is the max height of m_1 after the collision?

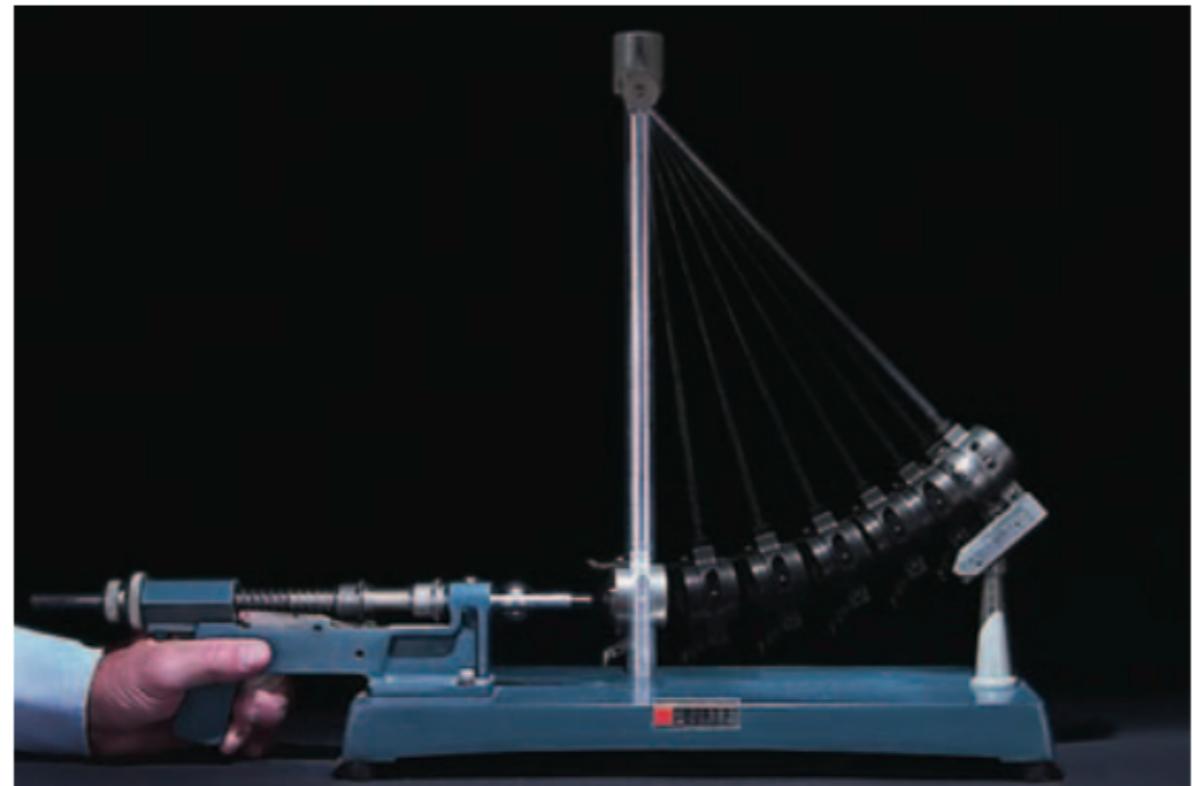
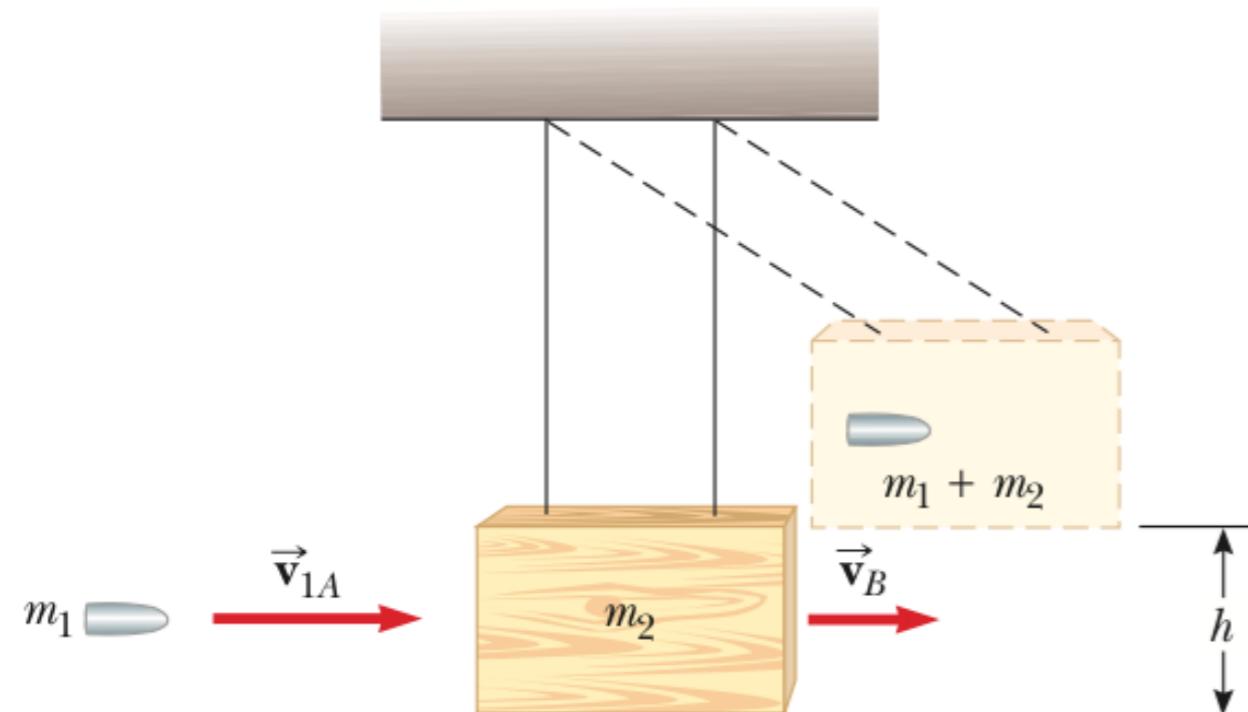


Ballistic Pendulum

Symbolic problem alert

If you knew: m_1 , m_2 , and h , find an expression for v_{1A} , the initial speed of the bullet.

Think about: Which principle applies to which part of the problem?



New Equations (Chapters 10 & 11)

$$\vec{p} = m\vec{v}$$

$$K = \frac{1}{2}mv^2$$

$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{i1}$$

$$J = \int_{t_i}^{t_f} F dt$$

$$U_g = mgy$$

$$J = \Delta p$$

$$F = -k\Delta x$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{i1}$$

$$\vec{P}_i = \vec{P}_f$$

$$U_s = \frac{1}{2}k\Delta x^2$$

Conservation of Energy

New Equations (Chapters 10 & 11)

$$\vec{p} = m\vec{v} \quad \boxed{5}$$

$$K = \frac{1}{2}mv^2 \quad \boxed{9}$$

$$v_{f1} = \frac{m_1 - m_2}{m_1 + m_2} v_{i1} \quad \boxed{7}$$

$$\boxed{8} \quad J = \int_{t_i}^{t_f} F dt$$

$$U_g = mgy \quad \boxed{4}$$

$$J = \Delta p \quad \boxed{2}$$

$$F = -k\Delta x \quad \boxed{10}$$

$$v_{f2} = \frac{2m_1}{m_1 + m_2} v_{i1} \quad \boxed{3}$$

$$\vec{P}_i = \vec{P}_f$$

$$\boxed{1}$$

$$U_s = \frac{1}{2}k\Delta x^2 \quad \boxed{11}$$

Conservation of Energy 6