

Scientific Computing

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1 Python and Jupyter Notebooks

1.1 Python

Python is a computer programming language available on all major platforms (Mac, Windows, Linux). Python is a scripting language which means that the computer interprets and runs your code at the moment you run it. In contrast, with a compiled language like C the code must first be converted into binary before it can run (called “compiling” the code). There are pros and cons to both types of languages. The on-the-fly interpretation of Python makes it quick and easy to write code and provides fast results for simple calculations. When codes become longer and more complex, on-the-fly interpretation becomes less efficient and execution time will be much slower than it would be with a compiled language. The pros and cons flip for a compiled language; writing code in a compiled language can be cumbersome and slow, but the execution time is typically much faster. Out of necessity, most programmers become proficient in both types of languages. Python(or another interpreted language) is used to “toy around” with your problem and build familiarity. As the complexity of the code increases the user is then forced to transition to a compiled language to get the needed speed. This is the famous “two language” problem and there is a new programming language designed to eliminate this problem by combining the pros from both into one language. (The name of the language is [Julia](#))

Python is free, open source software and is maintained by the non-profit Python software foundation. This is great because it means that you will always have free access to the Python language regardless of what organization or university you are affiliated with. You’ll never have to worry about not being able to use your Python code without paying for it. Another benefit of open source languages is that all of the codes developed by other people are available for anyone to inspect, modify, and use. This allows anyone to review another’s code to ensure that it does what they say it does, or to modify it to do something else. One last benefit that comes with an open source language is the community of Python users available to answer questions and provide instruction to the beginner. Answers to most questions about python are readily available on tutorial or forum websites.

1.2 Installing Software

The first step is to install the software (if you haven't already). The most convenient way to install Python and also get many of the commonly-used libraries is to use an installer. I recommend [Anaconda](#). When installing the software be sure to choose Python 3 since this is the current version. By default, Anaconda will install a suit of softwares and libraries that are commonly used. If you want to install other Python libraries, open the Anaconda-Navigator (green circle icon) and select the **Environment** tab on the left. Select **Not Installed** from the pull-down to see all of the libraries that are available to be installed. To install a library, check the box next to it and click **Apply**. Anaconda will take care of the rest.

To Do:

1. Install Anaconda
2. Check to see if the library “numpy” is installed. If not, install it.

1.3 Jupyter Notebooks

A Jupyter notebook is an electronic document designed to support interactive data processing, analysis, and visualization in an easily shared format. A Jupyter notebook can contain live code, math equations, explanatory text, and the output of codes (numbers, plots, graphics, etc..). To launch a Jupyter notebook, first open Anaconda-Navigator (green circle icon) and click the **Launch** button under JupyterLab. Jupyter can also be launched from the command line by typing `jupyter-lab`. The jupyter notebook will launch in your default web browser, but it is not a website. From here you can select an already existing Jupyter notebook, denoted by the orange icons and the .ipynb extension, or create a new notebook by clicking **New** from the **File** menu.

To Do:

1. On iLearn, find the module entitled “Jupyter Notebooks” and download the file “Intro.ipynb”.
2. Launch JupyterLab as explained above.
3. Open “Intro.ipynb” that you downloaded in step 1 and continue reading this book in the jupyter notebook.

1.3.1 Notebook Structure

There are two types of “cells” in a Jupyter notebook: code cells and text cells (also called Markdown cell). Code cells contain “live” Python code that can be run inside of the notebook with any output appearing directly below it. Markdown cells are designed to contain

explanatory information about what is happening inside of the code cells. They can contain text, math equations, and images. Markdown cells support markdown, html, and Latex (for generating pretty math equations).

Both markdown and code cells can be executed by either selecting **Run Selected Cells** in the **Run** menu, by clicking the **Play** icon at the top of the notebook, or by using the **Shift-Return** shortcut when your cursor is in the desired cell.

1.3.2 Navigating Jupyter Notebooks

Navigating a Jupyter notebook is fairly straightforward but there are a few handy shortcuts/hotkeys that will make navigation quicker and your workflow more efficient. When working in a Jupyter notebook, you are always operating in one of two modes: edit mode or navigate mode. In edit mode you can make modifications to the text or code in a cell and in navigate mode you can add/delete cells and modify the cell type. If you can see a blinking cursor in one of the cells you are in edit mode. Otherwise you are in navigate mode. To exit edit mode, simply press the **esc** key and you will enter navigate mode. To exit navigate mode, simply press the **enter** key and you will enter edit mode for the cell you were focused on. (You can also double click on a cell with your mouse to enter edit mode.) The **shift + enter** key sequence will “execute” a cell and produce the associated output. For text cells, executing just means to render the text in a nicely formatted fashion. “Executing” a code cell will actually execute the code block contained in the cell. You also enter navigate mode every time you execute a cell using the **shift + enter** key sequence. A summary of these shortcuts is given below:

- Up/down arrows - Navigate to different cells in the notebook.
- Y - turns a text cell into a code cell.
- M - turns a code cell into a text cell.
- A - inserts a new cell above the current cell.
- B - inserts a new cell below the current cell.
- X - deletes the current cell.
- **enter** - enters edit mode.
- **shift + enter** - execute a cell.
- **esc** - enter navigate mode.

You should take some time now to practice these shortcut keys until you become good at navigating a jupyter notebook. Consider attempting the following actions using the shortcuts above:

1. Add a cell below this one.
2. Turn the cell into a code cell (observe the distinct appearance of code cells).
3. Type the following code into the cell:

- `print("I did it!")`

4. Execute the cell using **shift + enter**. Observe the output.
5. Delete the cell.
6. Enter edit mode for this text cell.
7. Add a sentence of your choice at the end of the cell.
8. “Execute” the cell and observe the new output.
9. Remove the sentence to restore the cell to its previous state.

1.4 Tips for studying Jupyter notebooks

1.4.1 Print Statements

Jupyter notebooks in this class will be a nice mix of text cells (explanation) and code cells (examples). You will soon learn that code cells produce no output unless you explicitly tell them to using a **print** statement (similar to the one you used above). When you encounter a code cell, you should feel free to make modifications and additions to the cell until you fully understand how the code works.

1.4.2 Comments

Comments are a way to describe what each section of code does and makes it easier for you and others to understand the code. It may seem clear what each section of code does as you write it, but after a week, month or longer, it is unlikely to be obvious. Paul Wilson of the University of Wisconsin at Madison is quoted as saying, “Your closest collaborator is you six months ago, but you don’t reply to emails.” Comment your code now so that you are not confused later.

There are several ways to add comments to your code:

1. Use **#** to start a comment. Everything on that line the follows will be ignored.
2. For longer comments that will span several lines, use triple double quotes to begin and end the comment (**"""**)

The cell below illustrates these two ways to make comments:

```
# Speed of light in a vacuum
c = 3e8

v = 300 # Speed of sound in air

"""
The variables below are the initial conditions for a cannon
```

```
launching a ball at a 30 degree angle with an initial speed of  
50 m/s. The initial height of the cannon ball is 1000 m  
"""
```

```
v = 50  
theta = 30  
h_i = 1000
```

To Do:

1. Execute the code block below and verify that no output is produced.
2. Add print statements that help you see the result of the calculation.
3. Add simple comments next to each line explaining the code.

```
a = 2  
b = 3  
c = a**b
```

```
import subprocess  
import sys
```

```
def install(package):  
    subprocess.check_call([sys.executable, "-m", "pip", "install", package])  
install("numpy")
```

Part I

Basic Python

2 Variables and Numbers

Jupyter Notebook

2.1 Variables

When performing mathematical operations, it is often desirable to store the values in *variables* for later use instead of manually typing them back in each time you need to use them. This will reduce effort because small changes to variables can automatically propagate through your calculations.

Attaching a value to a variable is called *assignment* and is performed using the equal sign (=), as demonstrated in the cell below:

```
a = 5.0
b = 3
c = a + b
```

2.1.1 Variable naming convention

There are some rules for allowed variable names in Python. They are as follows:

1. Variable names must begin with a letter or an underscore (_)
2. Variables names must only contain letters, numbers, and underscores.
3. Variable names cannot contain spaces.
4. Variables names cannot be a word reserved by Python for something else. These words are:

	Python	reserved	words	
and	as	assert	break	class
continue	def	del	elif	else
except	False	finally	for	from
global	if	import	in	is
lambda	None	nonlocal	not	or
pass	raise	return	True	try

	Python	reserved	words
why	with	yield	

The cell below contains some allowed variable names and some that are not allowed.

To Do:

1. Determine which variable names are allowed and which are not in the cell below.
2. What does Python do if you try to define a variable using a name that is not allowed?

```
my1variable = 3
1stvariables = 2
a big constant = 3
a_big_constant = 1e8
```

It is also a good practice to make variable names meaningful. For example, in the cell below we calculate $E = mc^2$ using two choices for variable assignments. In one case, it is easy to determine what the calculation is and in the other it isn't.

```
# Good Variable Names
mass_kg = 1.6
light_speed = 3.0e8
E = mass_kg * light_speed**2

# Poor Variable Names
a = 1.6
b = 3.0e8
c = a * b**2
```

2.2 Numbers: Integers and Floats

There are two types of numbers in Python - floats and integers. *Floats*, short for “floating point numbers,” are values with decimals in them. They may be either whole or non-whole numbers such as 3.0 or 1.2, but there is always a decimal point. *Integers* are whole numbers with no decimal point such as 2 or 53.

Mathematical operations that only use integers *and* evaluate to a whole number will generate an integer (except for division). All other situations will generate a float. See the example cell below.

```
a = 24
b = 6
d = 0.3
e = a + b # Produces an integer.
f = a + d # Produces a float
g = a * b # Produces a ???
h = a / b # Produces a ???
```

To Do:

1. For each of the mathematical operations above guess what type of number the result will be.
2. Use `print` statements to verify your guesses and formulate a general rule that you can rely on. (Tip, the `type()` function will tell you what kind of number a variable is.)

Integers and floats can be inter-converted to each other using the `int()` and `float()` functions.

```
int(3.0)
float(4)
```

4.0

The distinction between floats and ints is often a minor detail. Occasionally, a function will require that an argument be a float or an int but usually you won't have to worry about which one you use.

Below you will find some other common mathematical operations that can be performed on numerical variables.

```
a = 20
b = 10
c = a + b
d = a/b
r = a//b
r = a % b
e = a * b
f = c**4
```

To Do:

1. Use print statements to investigate what each operation does.
2. Guess what type of number you expect the result to produce (int or float) and then check yourself?
3. Add comments next to each line (Use # to start a comment) explaining that operation.

2.2.1 Augmented Assignment

Augmented assignment is a shortened way to make a simple modification to a variable. For example, if we want to increase the value of a variable by 10, one way to do it would be like this.

```
a = 5  
a = a + 10
```

This is certainly not difficult, but it does involve typing the variable twice which becomes cumbersome as your variable name gets longer. Alternatively, we can accomplish the same thing with the += operator.

```
a = 5  
a += 10
```

Augmented assignment can be used with addition, subtraction, multiplication, and division as shown in the code cell below.

```
a = 7  
a += 3  
a -= 1  
a *= 4  
a /= 3
```

To Do:

1. Predict what the final result of **a** will be in the code cell above.
2. Add an appropriately-place **print** statement to see if you were correct.
3. If you were wrong, pow-wow with your neighbor until you understand.

2.2.2 Compound Assignment

At the beginning of a program or calculation, it is often necessary to define a set of variables. Each variable may get its own line of code, but if there are a lot of variables, this can begin to clutter your code a little. An alternative is to assign multiple variables on a single line. In the code below, we assign the atomic mass of the first three elements.

```
H, He, Li = 1.01, 4.00, 5.39
```

To Do:

1. Use print statements to verify that each variable was assigned its own value.
2. Add assignments for the atomic masses of the next three elements on the periodic table.

2.2.3 Large numbers

Sometimes you find yourself working with large numbers in your calculation. Maybe your calculation involves the use of ten billion, which has 10 zeros in it. It can be difficult to look at all of those zeros with no commas to help break it up. In those cases, you can use an underscore (_) in place of the comma, as shown below.

```
myLargeNumber = 10000000000 # This is tough to look at.  
myLargeNumber = 10_000_000_000 # This is easy to read  
  
myLargeFloat = 5000000.6 # This is tough to read  
myLargeFloat = 5_000_000.6 # This is easy to read
```

2.2.4 Very Large Numbers

If your number is very large or very small (20–30 zeros), you would probably rather not have to type all of the zeros at all, even if you can break it up with the underscores. For example, the Boltzmann constant, which comes up in thermodynamics, has a value equal to

$$1.38 \times 10^{-23}$$

We can avoid typing all those zeros by using scientific notation when defining the variable. (see example below) This is super handy for very large and very small number. (Numbers of both variety show up frequently in physics!)


```
kB = 1.38e-23
```

2.2.5 Python functions

In addition to basic mathematical functions, python contains several mathematical *functions*. As in mathematics, a function has a name (e.g. f) and the arguments are places inside of the parenthesis after the name. The *argument* is any value or piece of information fed into the function. In the case below, f requires a single argument x .

$$f(x)$$

In the cell below, you will find several useful math equations.

```
abs(-5.5)
float(2)
int(5.6)
print(1.26e-6)
round(-5.51)
str(3.2)
```

1.26e-06

'3.2'

In addition to Python's native collection of mathematical functions, there is also a `math` module with more mathematical functions. Think of a module as an add-on or tool pack for Python just like a library. The `math` module comes with every installation of python and can be *imported* (i.e. activated) using the `import math` command. After the module has been imported, any function in the module is called using `math.function()` where `function` is the name of the function. Here is a list of commonly-used function inside the `math` module:

```
import math
math.sqrt(4)
math.ceil(4.3)
math.cos(1.5)
math.sin(1.5)
math.tan(3.14)
math.asin(1)
math.acos(1/2)
math.atan(2)
```

```
math.degrees(6.28)
math.e
math.exp(5)
math.factorial(4)
math.log(200)
math.log10(1000)
math.radians(360)
math.pi
math.pow(2,8)
```

256.0

To Do:

1. Use print statements to figure out what each function in the code cell above does. Pay special attention to trigonometric function. Do these functions expect the argument to be in radians or degrees?
2. Add comments to remind yourself for later.

There are other ways to import functions from modules. If you only want to use a single function inside the module, you can selectively import it using **from**, as shown below.

```
from math import radians
radians(4)
```

0.06981317007977318

2.3 Flash Cards

1. What are the rules for naming variables in Python?
2. When will a mathematical calculation produce a float?
3. When will a mathematical calculation produce an int?
4. What is the best way to increase/decrease the value of a variable by a multiplicative factor (multiply it by a number)?
5. What is the best way to increase/decrease the value of a variable by an additive constant (add a number to it)?
6. When assigning large numbers to variables, what character can replace the “,” to separate the numbers into groups of three?
7. How do you use scientific notation to assign a very large number to a variable?

8. When using a trig function ($\sin \theta$, $\cos \theta$, $\tan \theta$), the number that is passed in must have what units?
9. What does the `math.radians` function do?
10. Where can you find King Benjamin's address to his people?

2.4 Exercises

1. Calculate the distance from the point $(-48, 56)$ to the point $(23, 81)$ 1- using the `math.hypot()` function and then 2- using the following distance equation:

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

Print out the result and check your answer with your neighbor's answer.

2. Solve the quadratic equation using the quadratic function below for $a = 1$, $b = 2$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Print out the result and check your answer with a classmate.

3. (Solar Mass) The mass of the sun can be calculated using the following formula:

$$M_{\text{sun}} = \frac{4\pi^2(1 \text{ AU})^3}{G(1 \text{ yr})^2}$$

The unit AU is called an astronomical unit of length and is defined to be the average distance between the sun and earth.

$$1 \text{ AU} = 1.58 \times 10^{-5} \text{ light years}$$

where $1 \text{ lightyear} = 9.5 \times 10^{12} \text{ km}$. The constant G is called the gravitational constant and has the value:

$$G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$$

Calculate the mass of the sun and display your result using a print statement. You should find a value of $M_{\text{sun}} = 2.01 \times 10^{30} \text{ kg}$

4. (Projectile Motion) The range of a projectile is given by the equation:

$$d = \frac{2v^2 \cos \theta \sin \theta}{g}$$

or the equivalent expression:

$$d = \frac{v^2 \sin 2\theta}{g}$$

where $g = 9.8 \text{ m/s}^2$, θ is the launch angle, and v is the launch speed.

1. Using a launch angle of 60° and a launch speed of 40 m/s, verify that both expressions above give the same result for the range.
2. Now pick one of the equations above and use trial and error to determine the angle that gives maximum range.

$v = 40$
 $g = 9.8$

5. (Rydberg's constant) Rydberg's constant (R_∞) is a physical constant in Rydberg's formula which was used to predict the spectrum of light emitted by a heavy atom. Rydberg's constant is given by:

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^3 c}$$

where

- $m_e = 9.109 \times 10^{-31}$ kg is the mass of an electron.
- $e = 1.602 \times 10^{-19}$ C is the charge of an electron/proton.
- $\epsilon_0 = 8.854 \times 10^{-12}$ C V⁻¹ m⁻¹ is the electrical constant.
- $h = 6.626 \times 10^{-34}$ J Hz is the Planck constant.
- $c = 3 \times 10^8$ m/s is the speed of light.

These constants show up all over in physics. In the cell below write some code that assigns the constants to variables and then use the variables to calculate Rydberg's constant. Use a print statement to display the result. The result should be: $R_\infty = 10961656.2162$ (in m⁻¹)

6. In Einstein's special theory of relativity, the momentum of an object with mass m (in kg) and velocity v (in m/s) is given by:

$$p = mv\gamma$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

with $c = 3 \times 10^8$ m/s

1. Calculate the momentum of an object with mass $m = 0.14$ kg and speed $v = 50$ m/s. Then compare to the classical expression for momentum: $p = mv$.
2. Now calculate the momentum of an object with mass $m = 0.14$ kg and whose speed is $\frac{1}{4}$ the speed of light. Repeat the comparison to the classical value.
3. Repeat for the following speeds: $\frac{1}{2}$ the speed of light, $\frac{3}{4}$ the speed of light, and $\frac{7}{8}$ the speed of light. Repeat the comparison to the classical value.

3 Strings

Jupyter Notebook

Another commonly-used type of data is a string of characters known simply as *strings*. Strings can contain a variety of characters including letters, numbers, and symbols.

3.1 Creating Strings

Strings are created by placing the sequence of characters in single (or double) quotes.

```
text = "some text"
```

Strings can also be created by converting a float or an integer into a string using the `str()` function.

```
text = str(4.5)
```

One common error made when working with strings is to attempt to perform math with them. Python will not perform math with strings because it sees them as a series of characters and nothing more. In the cell below, we attempt to perform math with some strings.

```
a = "4"  
b = "2"  
c = 5  
  
d = a + b  
e = a * b  
f = b * c
```

To Do:

1. Use print statements in the cell above to determine what happens when you add two strings together. (Ancillary question: Can you subtract strings?)
2. Use print statements in the cell above to determine what happens when you multiply two strings.

3. Use print statements in the cell above to determine what happens when you multiply a string and an integer.
4. What do you think will happen if you multiplied a string and a float? Guess and then see if you're right.
5. Add comments for you to refer back to later.

If you want to know the length of a string, you can use the `len()` function

```
text = "some text"  
len(text)
```

9

The length of the string above is 9 because a space is a valid character.

3.2 Displaying text and numbers together

You have been using `print()` statements quite a lot lately (hopefully) but you probably haven't printed text and numbers together. To display both text and numbers in the same message, there are several options. The first is to just put multiple variables into the `print()` function, separating them with commas.

```
g = 9.8  
unit = "m/s^2"  
print(g,unit)
```

9.8 m/s^2

Another option is to convert the number to a string and then “add” it to the other string. This creates a single string as an argument to the `print()` function.

```
g = 9.8  
unit = "m/s^2"  
print(str(g) + unit)
```

9.8m/s^2

Notice the lack of space between the number and the unit. “Adding” the two strings smashed them together exactly as they were, no spaces added. You can insert multiple numbers into a string using something called “f”-strings. (short for formatted strings). To construct an f-string, simply place an “f” in front of the string. Anytime you want to insert a number in your string, enclose it in curly braces.

```
v = 5.0
c = 3e8
print(f"The speed of light is {c} and the speed of my car is {v}")
```

The speed of light is 300000000.0 and the speed of my car is 5.0

That’s a clever way to insert a numerical value into a string, but the value of the speed of light is still displaying too many digits. To specify how the number should be formatted place a : after the variable name followed by a formatting tag.

```
v1 = 5.0
v2 = 8.3
c = 2.998e8
n = 2

print(f"There are {n:d} cars traveling side by side. One car is traveling at {v1:4.2f} m/
```

There are 2 cars traveling side by side. One car is traveling at 5.00 m/s and the other is t

The structure of the stuff inside of the curly braces is `{variable:formatcode}`; `variable` holds the value to be displayed and `formatcode` indicates how the variable should be formatted when it is printed. The `f` in `:4.2f` indicates that the variable should be displayed as a float and the `4.2` indicates that four spaces should be allocated to display the number and no more than 2 numbers after the decimal should be displayed. A selection of some commonly-used format codes is given below.

Table 3.1: A summary of common format codes.

format code	explanation
<code>{variable}</code>	Use the default format for the data type.
<code>{variable:4d}</code>	Display as an integer, allocating 4 spaces for it.
<code>{variable:.4f}</code>	Display as a float, with four numbers after the decimal being displayed.

format code	explanation
<code>{variable:8.4f}</code>	Display as a float, allocating 8 total spaces and 4 numbers after the decimal place.
<code>{variable:8.4e}</code>	Display using scientific notation, allocating 8 total spaces and 4 numbers after the decimal place.
<code>{variable:6s}</code>	Display as a string, allocating 6 total spaces for it. If the string is longer than 6 spaces, it will display the entire string with no extra white space. If the string is shorter than 6 spaces, it will pad the string with whitespace until it is 6 spaces long.

To Do:

1. Modify the print statement above so that the float variables are given 8 total spaces with only 1 number after the decimal being displayed.
2. Modify the print statement above so that the speed of light is displayed with 3 numbers after the decimal place.

3.3 Indexing and Slicing

Accessing a piece (or slice) of a string is a common task in scientific computing. Often you will import data into Python from a text file and need to extract a portion of the file for later use in calculations. Indexing allows the user to extract a single element, or character, from a string. The key detail about indexing in Python is that *indices start from zero*. That means that the first character is index zero, the second character is index 1, and so on. For example, maybe a string contains the following amino acid sequence ‘MSLFKIRMPE’. For this example, the indices are as follows:

Characters	M	S	L	F	K	I	R	M	P	E
Index	0	1	2	3	4	5	6	7	8	9

To access a single character from a string, place the desired index in square brackets after the name of the string.

```
seq = "MSLFKIRMPE"
seq[0]
```


'M'

To access the last character in a string you could do

```
seq = "MSLFKIRMPI"  
seq[len(seq) - 1]
```

'I'

But this seems overly cumbersome. An easier approach is to index backwards. The string can be reverse indexed from the last character to the first using negative indices, starting with -1 as the last character.

```
seq = "MSLFKIRMPE"  
seq[-1]
```

'E'

To Do:

1. Access the 5th character in the peptide sequence above.
2. Access the character that is 3rd from the end in the peptide sequence above.

Indexing only provides a single character, but it is common to want a series of characters from a string. *Slicing* allows us to grab a section of a string. Slicing is performed by specifying start and stop indices separated by a colon in the square brackets. One important detail worth mentioning: the character at the starting index is included in the slice while the character located at the final index *is not* included in the slice.

```
seq = "MSLFKIRMPE"  
seq[0:5]
```

'MSLFK'

Looking at the string, you notice that the character at location 5 (I) has been excluded from the slice. You can leave off the first number when slicing and the slice will start at the beginning of the string.

```
seq = "MSLFKIRMPE"  
seq[:5]
```

```
'MSLFK'
```

You can also use negative indices when slicing. This is especially helpful when you want to grab the last few characters in a string.

```
file = "data.txt"  
ext = file[-3:]
```

Finally, we can adjust the step size in the slice. That is, we can ask for every other character in the string by setting a step size of 2. The structure of the slice is [start,stop,step].

```
seq = "MSLFKIRMPE"  
seq[0:8:2]
```

```
'MLKR'
```

You can omit the start and stop indices and Python will assume that you are slicing the entire string.

```
seq = "MSLFKIRMPE"  
seq[::2]
```

```
'MLKRP'
```

3.4 String Methods

A *method* is a function that works only with a specific type of object. String methods only work on strings, and they don't work on other types of objects, like floats or ints. If it helps you, you can just think of a method as a function.

One example of a string method is the `capitalize()` function which returns a string with the first letter capitalized. To use a method (referred to as *calling* the method), the method name is appended to the variable you want it to operate on. For example, below is an Albert Einstein quote that needs capitalized.

```
quote = "i want to know God's thoughts. The rest are details."  
quote.capitalize()  
print(quote)
```

```
i want to know God's thoughts. The rest are details.
```

Notice that the original variable (`quote`) remains unchanged. This particular method does not change the value of the original string but rather returns a capitalized version of it. If we want to save the capitalized version, we can assign it to a new variable, or overwrite the original.

```
quote = "i want to know God's thoughts. The rest are details"
quote = quote.capitalize()
print(quote)
```

```
I want to know god's thoughts. the rest are details
```

In the cell below you will find a list of commonly-used string methods.

```
a = "spdfgssfpvgg"
a.capitalize()
a.center(10)
a.count("s")
a.find("d")
a.isalnum()
a.isalpha()
a.isdigit()
a.lstrip("s")
a.rstrip("g")
a.split("s")
a.startswith("s")
a.endswith("p")
```

False

To Do:

1. Use well-placed print statements to determine what each string method does.
2. Add comments next to each method for future reference.

3.5 Flash Cards

1. What is the index value of the sixth character in a string?
2. What happens when you “add” two strings together?
3. What happens when you multiply two strings together?

4. What happens when you multiply a string by an integer?
5. What happens when you multiply a string by a float?
6. What function will tell you how many characters are in a string?
7. Give two ways that you could slice out the **first 4 characters in a string**.
8. Give two ways that you could slice out the **last 4 characters in a string**.
9. What string method would you use to see if a string contains only numbers and characters, but no symbols?
10. What does the string method `split()` do?
11. What does the string method `find()` do?
12. Explain how to print a sentence with a variable inserted in the middle?
13. What format code should be used to display a variable as a **float with 8 total spaces and 2 numbers after the decimal**?
14. What format code should be used to display a variable in **scientific notation with 6 total spaces and 2 numbers after the decimal**?
15. Where is the parable of the wheat and the tares found?

3.6 Exercises

1. In the cell below you will find a string containing the name of a PNG file. (i.e. name always ends in “.png”).
 - i) Remove the “.png” extension from the string **using a string method**.
 - ii) Remove the “.png” extension from the string **using slicing**.

```
file = "physics.png"
```

2. (Lightning Strike) The dielectric strength of air determines how likely it is for lightning to strike and this value varies with the humidity (moisture content) of the air. In the cell below, you will find a string that contains the dielectric strength of air for three different humidity levels. **Use string methods** to extract the three dielectric constant values. Then calculate
 1. the difference between the largest and smallest values.
 2. The average of the three values.

```
dielectricBreakdownAir = "dry:15847,moderate:8523,wet:7864"
```

3. Below you will find a string containing all experimentally known alloys (compounds formed by combining two or more elements on the periodic table) represented using the elements first discovered to exhibit that crystal structure (or “prototypes”).
 1. Use string methods to find the number of alloys containing Aluminum (symbol: Al).

2. Is there a crystal structure with the designation “CaC₂” (Calcium Carbide) in the list? How about “Fe₂Ag”?
3. Use string methods to calculate the percentage of the alloys listed that have gold in them (symbol Au).

```
alloys = "Cu,W,Mg,betaSn,Indium,alphaAs,alphaHg,Cu,AuCu,CuPt,AB2,MoPt2,AB2,AB3,AB3,A2B2,AB
```

4 Boolean Variables and Conditionals

[Jupyter Notebook](#)

4.1 Boolean Variables

We have learned about three types of variables in Python: ints, floats, and strings. Another variable type is a boolean, which can be one of two values: **True** or **False**. You can assign a boolean variable in the same way that you assign numbers or string, using =

```
myBool = True
```

True *must be* capitalized so don't try **true** or it won't be a boolean

```
myBool = true
```

4.1.1 Boolean Logic

Often you will want to check to see if some condition is true. For example, maybe you want to know if the radius of a certain satellite's orbit is bigger or smaller than Mercury's orbit. To perform this check, there are several boolean operators that will return **True** or **False**. Take note of the boolean operators shown in the cell below along with the comments added to explain what they do.

```
r1 = 3.5e8
r2 = 2.7e6

r1 > r2 # Is r1 greater than r2
r1 < r2 # Is r1 less than r2
r1 >= r2 # Is r1 greater than or equal to r2
r1 <= r2 # Is r1 less than or equal to r2
r1 != r2 # Is r1 not equal to r2
r1 == r2 # Is r1 equal to r2
```

False

A word of caution about comparing Python floats using `==` is in order here. Python floats are stored as a 53-digit, base-2 binary number(that's a mouthful). If you're interested in what that means, we can talk more. If you're not that interested, just know that when you define a float in Python, the number that is stored in the computer is not *exactly* the number that you think it is. This can cause problems when you are comparing two numbers that you think should be equal but actually aren't equal in the computer.¹ The code below illustrates this problem.

```
a = 0.1
b = 3 * a
c = 0.3
print(b==c) # Are they the same number? You would think they would
            # be right?
print(" {:.5f} ".format(b))      # They sure look the same.
print(" {:.5f} ".format(c))      # They sure look the same.
print(" {:.45f} ".format(b))    #b--- out to 45 decimal places
print(" {:.45f} ".format(c))    #c--- out to 45 decimal places
```

False

```
0.30000
0.30000
0.300000000000000044408920985006261616945266724
0.299999999999999988897769753748434595763683319
```

The first two print statements display the value of `b` and `c` to five decimal places. The second two print statements force Python to display the value of `b` and `c` out to 45 decimal places. Notice that the true value of `b` is not *exactly* equal to 0.3. This is why the statement `print(b==c)` returns `False`. The take home message here is that comparing two floats to see if they are equal **is always a bad idea**. A better way to check to see if two floats are equal (or close enough that we can say they are equal) is to check if the absolute value of their difference is very small, like this:

```
a = 0.1
b = 3 * a
c = 0.3
print(abs(b - c) < 1e-10)
```

True

¹There is a library called `Decimal` that will fix a lot of these problems.

4.1.2 Compound Comparisons (Logical Operators)

Comparisons like those shown above can be chained together to make compound comparisons using the **and**, **or**, and **not** operators.

Operator	Description
and	Tests for both being True
or	Tests for either being True
not	Tests for False

The **and** operator requires both inputs to be **True** in order to return **True** while the **or** operator requires only one input to be **True** in order to evaluate at **True**. The **not** operator is different in that it only takes a single input value and returns **True** if and only if the input is **False**. It is a test for **False**.

Truth tables are a good way to visualize the output from compound comparisons.

p	q	p and q	p or q
True	True	True	True
True	False	False	True
False	True	False	True
False	False	False	False

As a simple example, suppose you know the density (ρ) and speed of light (c) for two materials and you'd like to know if both values are bigger for material 1 or material 2.

To Do:

1. Predict the output for each compound comparison given below. Add your guess as a comment.
2. Now add appropriately-placed print statements to check your guesses.
3. Modify your guess as needed and discuss any questions with a neighbor.

```
c1 = 2.5e8
c2 = 2.48e8
1 = 450
2 = 580

c1 > c2 and 1 > 2
c1 < c2 and 1 < 2
c1 > c2 and 1 < 2
```



```

c1 < c2 and 1 > 2

c1 > c2 and not 1 > 2
c1 < c2 and not 1 < 2

c1 > c2 or 1 > 2
c1 < c2 or 1 < 2

```

True

4.1.3 Tests for Inclusion

You can check for inclusion using the Python `in` operator. This provides an easy way to see if a character (or word) is present in a long string. Let's say you have a long string that contains the names of Jupyter's moons (there are 79 of them!!) and you want to see if a certain moon is included in the list. The `in` statement lets us quickly test to see if it is in the list. (see example below)

```

jupytermoons = "Metis,Adrastea,Amalthea,Thebe,Io,Europa,Ganymede,Callisto,Themisto,Leda,Hi
"Cyllene" in jupytermoons

```

True

4.2 Conditions

Conditions allow the user to specify if and when certain lines or blocks of code are executed. Specifically, when a condition is true, the block of *indented* code directly below it will run.

4.2.1 if statement

The `if` statement is used to control when a block of code runs. Its usage is shown below ending in a colon and the block of code below indented with *four spaces*. Using the **Tab** key will also produce four spaces.

```

jupytermoons = "Metis,Adrastea,Amalthea,Thebe,Io,Europa,Ganymede,Callisto,Themisto,Leda,Hi
if "Cyllene" in jupytermoons:

```

```

    found = True
    print("Found Cyllene in the list")

if "Matis" in jupytermoons:
    found = True
    print("Found Matis in the list")

```

Found Cyllene in the list

If the boolean statement after `if` is true, the indented code below it will run. If the statement is false, Python just skips the indented lines below.

4.2.2 else Statment

Sometimes there will be an alternate block of code that you want to run if the `if` statement evaluates to `False`. The `else` statement is used to specify this block of code, as shown below.

```

jupytermoons = "Metis,Adrastea,Amalthea,Thebe,Io,Europa,Ganymede,Callisto,Themisto,Leda,Hi

if "Cyllene" in jupytermoons:
    found = True
    print("Found Cyllene in the string")
else:
    found = False
    print("Did not find Cyllene in the string")

if "Matis" in jupytermoons:
    found = True
    print("Found Matis in the string")
else:
    found = False
    print("Did not find Matis in the string")

```

Found Cyllene in the string
Did not find Matis in the string

Notice that the `else` statement must be followed by a colon and the block of code to be executed is indented, just as in the `if` block.

There is an additional statement called the `elif` statement, short for “else if”, which is used to add extra conditions below the initial `if` statement. The block of code below the `elif` statement only runs if the `if` statement is false and the `elif` statement is true. An example is given below.

```
jupytermoons = "Metis,Adrastea,Amalthea,Thebe,Io,Europa,Ganymede,Callisto,Themisto,Leda,Hi

if "Matis" in jupytermoons:
    foundMatis = True
    print("Found Matis in the string")
elif "Cyllene" in jupytermoons:
    foundCyllene = True
    print("Found Cyllene in the string.")
else:
    foundCyl = False
    foundMatis = False
    print("Did not find Cyllene or Matis in the string")
```

Found Cyllene in the string.

It is worth noting that `else` statements are not required. If you leave the `else` statement off and the `if` statement is false, no code block will execute.

4.3 Flash Cards

1. How do you check to see if one value is **greater than or equal to** another one?
2. Which boolean operators should **never** be used to compare floats?
3. What is the correct way to determine if two floats are equal to each other (or very close)?
4. When will the statement `p and q` evaluate to `True`?
5. When will the statement `p or q` evaluate to `True`?
6. How do you check to see if a certain sequence of characters is present in a string?
7. When constructing an `if` block how does Python know what code to execute if the condition is `True`?
8. Recite Helaman 5:12.

4.4 Exercises

1. (Escape Velocity) The escape velocity is the speed needed for an object to completely escape the gravitational pull of a planet. The expression for escape velocity is

$$v_{\text{escape}} = \sqrt{\frac{GM_p}{r}}$$

where $G = 6.67 \times 10^{-11}$, M_p is the mass of the planet you are trying to escape from, and r is the radius of the planet.

1. Construct an **if-else** code block that checks to see if a guessed speed is bigger or smaller than the escape velocity of Jupiter. The code block should print an appropriate message about the guessed velocity. Jupiter has a mass of $M = 1.90 \times 10^{27}$ kg and a radius of $r = 6.99 \times 10^7$ meters.
2. Modify your guess until both branches of your **if-else** trigger.
3. Repeat for Earth. Earth has a mass of $M = 5.98 \times 10^{24}$ kg and a radius of $r = 6.37 \times 10^6$ meters.

```
G = 6.67e-11
```

2. (Thermoelectrics) A thermoelectric is a material that converts thermal energy into electrical energy and much research is devoted to finding new materials that exhibit this behavior. Good thermoelectrics tend to be materials with high electrical conductivity ($\sigma > 7 \times 10^5 \Omega^{-1}\text{m}^{-1}$) **and** low thermal conductivity ($\kappa < 2.0 \text{ W/ m K}$). Write an **if-else** code block that checks to see if a material's electrical conductivity is above the threshold **and** its thermal conductivity is below the threshold. Then choose some values for the conductivities and test your **if-else** block.

```
sigma = 0      # Electrical Conductivity
kappa = 0      # Thermal Conductivity
```

3. Alloys are often represented by a string containing the elements in the alloy and the relative number of atoms for each element. For example, "Cu2Au" (Copper-2-Gold) means that the alloy is made of copper and gold and there are two copper atoms for every gold atom. Write some python code that calculates the total number of atoms in an alloy. The code should include a print statement to display the result. Then check your code on the alloys in the table below. You can assume that all elements have two-character symbols even though that isn't true; we won't consider any one-character symbols in this problem.

Hints:

1. The possible lengths for the strings are 4 (one atom per element), 5 (one atom for one element and more for the other), or 6 (more than one atom for both elements).

2. My code for this problem was about 15 lines long and it required a nested `if-else` block (an `if-else` inside of an `if` block).
3. You may find the `isdigit()` function to be helpful. Do you remember how to use it and what it does?

Alloy				
"Cr3Si"	"Al2Cu"	"NbPd3"	"CaIn2"	"Al3Zr4"

```
alloy = "CaIn2"
```

4. The quadratic equation has two solutions and they can be positive, negative, or complex. Often the complex and/or negative solutions are not relevant/meaningful. Write an `if/elif/else` block of code that checks what type of solutions will result and only displays the real, positive solution(s) if they exist. If there is no positive solution, the code should display a message indicating such. Then check your code with the values given below. As a reminder, here is the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

`a = 2, b = 3, c = 8` (no real solutions)

`a = 1, b = -3, c = 2` (two positive solutions)

`a = 1, b = -3, c = -2` (one positive solution and one negative solution)

```
# Python code here.
```

5. (Triangle Classification) There is a simple criteria to determine whether a triangle can be formed from a set of specified side lengths. The criteria states that the sum of any two sides must be larger than the third side. This criteria must be satisfied for all possible combinations of two sides if the triangle is to be valid. If the triangle is valid, the angles can be found by first using the law of cosines to find the first two angles

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

and then choosing the third angle so that the sum of the three angles add up to 180°.

1. Write some code that checks to see if a valid triangle can be formed from the given side lengths.

2. If a triangle can be formed, use the law of cosines to calculate the triangle's angles. Print the angles in degrees. Check your code for several choices of side lengths to verify that it is working.
3. All valid triangles are classified according to the largest angle present (acute, obtuse, and right) and also according to their side lengths (scalene - no sides equal, isosceles - two sides equal and equilateral - all sides equal). Modify your code to determine the classification of a triangle (i.e. obtuse scalene) and print that classification to screen.

```
from numpy import arccos,pi
```

```
#a = # Side length 1  
#b = # Side length 1  
#c = # Side length 1
```

5 Lists and Tuples

Jupyter Notebook

Up to this point, we have only worked with single numerical values or strings (multiple characters). Often we will want to work with collections of values (perhaps the orbital period of all of the planets in our solar system.) and it will be quite inconvenient to store each value into its own variable. Instead, the values can be stored in a list or tuple. Lists and tuples are both collections of *elements*, like numbers or strings. The key difference between them is that tuples are *immutable*, which means they cannot be modified after their initial creation. On the other hand, lists are *mutable*, or able to be modified.

5.1 Creating Lists

The easiest way to create a list is by putting the list elements inside of square brackets. Below, we create a list containing the masses of all of the planets in our solar system.

```
mass = [1.8986e27, 5.6846e26, 10.243e25, 8.6810e25, 5.9736e24, 4.8685e24, 6.4185e23, 3.3022e23]
```

Note that square brackets (`[]`) must be used when creating the list. If you accidentally use parenthesis (`()`)¹ or curly brackets (`{}`)² you'll end up creating something other than a list.

Lists can contain any type of data and the type of data doesn't have to be the same for all of the elements. Below, we create a list of the electron configurations for the first 10 elements on the periodic table.

```
electrons = ["1s1", "1s2", "1s2-2s1", "1s2-2s2", "1s2-2s2-2p1", "1s2-2s2-2p2", "1s2-2s2-2p3", "1s2-2s2-2p3-3s1", "1s2-2s2-2p3-3s2", "1s2-2s2-2p3-3s2-3p1"]
```

Lists can contain mixed data types. Below we construct a list of the electrical conductivities of three metals.

```
conductivity = ["Gold", 4.10e7, "Copper", 5.96e7, "Aluminum", 3.5e7]
```

¹Use parenthesis to create a tuple, which is just like a list but cannot be modified.

²Use curly brackets to create a dictionary, which is like a list but can be indexed on any data type, not just integers.

If the elements of a list are also lists, we call it a *nested* list.

```
conductivity = [[1,2,3],[4,5,6],[7,8,9]]
```

5.2 Indexing and Slicing Lists

Indexing is used to access individual elements of a list, and it is similar to indexing strings. The index is the position of the desired element in the list and the *index numbering starts at zero*. Accessing an element of a list is done by placing the numerical index of the element we want in square brackets behind the list name. For example, if we want the electron configuration of the first element in our list from above, we use `electrons[0]` and the electron configuration for the second element would be `electrons[1]` and so on. Just as with string, negative indices can be used to access list elements counting from the back of the list forward.

```
electrons = ["1s1", "1s2", "1s2-2s1", "1s2-2s2", "1s2-2s2-2p1", "1s2-2s2-2p2", "1s2-2s2-2p3", "1s2-2s2-2p3-3s1", "1s2-2s2-2p3-3s2", "1s2-2s2-2p3-3s3", "1s2-2s2-2p3-3d1", "1s2-2s2-2p3-3d2", "1s2-2s2-2p3-3d3", "1s2-2s2-2p3-3d4", "1s2-2s2-2p3-3d5", "1s2-2s2-2p3-3d6", "1s2-2s2-2p3-3d7", "1s2-2s2-2p3-3d8", "1s2-2s2-2p3-3d9", "1s2-2s2-2p3-3d10", "1s2-2s2-2p3-3d11", "1s2-2s2-2p3-3d12", "1s2-2s2-2p3-3d13", "1s2-2s2-2p3-3d14", "1s2-2s2-2p3-3d15", "1s2-2s2-2p3-3d16", "1s2-2s2-2p3-3d17", "1s2-2s2-2p3-3d18", "1s2-2s2-2p3-3d19", "1s2-2s2-2p3-3d20", "1s2-2s2-2p3-3d21", "1s2-2s2-2p3-3d22", "1s2-2s2-2p3-3d23", "1s2-2s2-2p3-3d24", "1s2-2s2-2p3-3d25", "1s2-2s2-2p3-3d26", "1s2-2s2-2p3-3d27", "1s2-2s2-2p3-3d28", "1s2-2s2-2p3-3d29", "1s2-2s2-2p3-3d30", "1s2-2s2-2p3-3d31", "1s2-2s2-2p3-3d32", "1s2-2s2-2p3-3d33", "1s2-2s2-2p3-3d34", "1s2-2s2-2p3-3d35", "1s2-2s2-2p3-3d36", "1s2-2s2-2p3-3d37", "1s2-2s2-2p3-3d38", "1s2-2s2-2p3-3d39", "1s2-2s2-2p3-3d40", "1s2-2s2-2p3-3d41", "1s2-2s2-2p3-3d42", "1s2-2s2-2p3-3d43", "1s2-2s2-2p3-3d44", "1s2-2s2-2p3-3d45", "1s2-2s2-2p3-3d46", "1s2-2s2-2p3-3d47", "1s2-2s2-2p3-3d48", "1s2-2s2-2p3-3d49", "1s2-2s2-2p3-3d50", "1s2-2s2-2p3-3d51", "1s2-2s2-2p3-3d52", "1s2-2s2-2p3-3d53", "1s2-2s2-2p3-3d54", "1s2-2s2-2p3-3d55", "1s2-2s2-2p3-3d56", "1s2-2s2-2p3-3d57", "1s2-2s2-2p3-3d58", "1s2-2s2-2p3-3d59", "1s2-2s2-2p3-3d60", "1s2-2s2-2p3-3d61", "1s2-2s2-2p3-3d62", "1s2-2s2-2p3-3d63", "1s2-2s2-2p3-3d64", "1s2-2s2-2p3-3d65", "1s2-2s2-2p3-3d66", "1s2-2s2-2p3-3d67", "1s2-2s2-2p3-3d68", "1s2-2s2-2p3-3d69", "1s2-2s2-2p3-3d70", "1s2-2s2-2p3-3d71", "1s2-2s2-2p3-3d72", "1s2-2s2-2p3-3d73", "1s2-2s2-2p3-3d74", "1s2-2s2-2p3-3d75", "1s2-2s2-2p3-3d76", "1s2-2s2-2p3-3d77", "1s2-2s2-2p3-3d78", "1s2-2s2-2p3-3d79", "1s2-2s2-2p3-3d80", "1s2-2s2-2p3-3d81", "1s2-2s2-2p3-3d82", "1s2-2s2-2p3-3d83", "1s2-2s2-2p3-3d84", "1s2-2s2-2p3-3d85", "1s2-2s2-2p3-3d86", "1s2-2s2-2p3-3d87", "1s2-2s2-2p3-3d88", "1s2-2s2-2p3-3d89", "1s2-2s2-2p3-3d90", "1s2-2s2-2p3-3d91", "1s2-2s2-2p3-3d92", "1s2-2s2-2p3-3d93", "1s2-2s2-2p3-3d94", "1s2-2s2-2p3-3d95", "1s2-2s2-2p3-3d96", "1s2-2s2-2p3-3d97", "1s2-2s2-2p3-3d98", "1s2-2s2-2p3-3d99", "1s2-2s2-2p3-3d100"]

electrons[1]
electrons[-2]
electrons[5]
electrons[-1]
```

To Do:

1. Predict the output for each of the indexes performed in the cell above.
2. Use print statements to check your answers.
3. Discuss any misunderstandings with a peer.

Accessing elements of nested lists requires multiple sets of parenthesis.

```
conductivity = [[1,2,3],[4,5,6],[7,8,9]]

x = conductivity[1][0]
y = conductivity[0][2]
```

To Do:

1. Predict the output for each of the indexes performed in the cell above.
2. Use print statements to check your answers.
3. Discuss any misunderstandings with a peer.

Since lists are *mutable*, we can modify the value of an element in a list using the = operator.


```
conductivity = ["Gold",4.10e7,"Copper",5.96e7,"Aluminum",3.5e7]

conductivity[1] = 4.15e7
conductivity
```

```
['Gold', 41500000.0, 'Copper', 59600000.0, 'Aluminum', 35000000.0]
```

To Do:

1. Use a print statement to verify that the `conductivity` list was indeed modified as expected.
2. Modify the third element to be “Platinum” and the fourth to be the conductivity of Platinum (`9.43e6`).

Multiple list elements can be retrieved at once (called *slicing*) by including the start and stop indices separated by a colon: `[start:stop:step]`. A convention that occurs throughout python is that the first index is included in the slice but the second is *not*. (i.e. `[included:excluded: step]`) Default values for the start location, stop location, and step sizes will be used if these values are omitted. Below we give some examples of slicing.

```
electrons = ["1s1", "1s2", "1s2-2s1", "1s2-2s2", "1s2-2s2-2p1", "1s2-2s2-2p2", "1s2-2s2-2p3", "1s2-2s2-2p4"]

electrons[1:]
electrons[1:3]
electrons[:3]
electrons[1:8:2]
electrons[5:2:-1]
```

To Do:

1. Predict the output for the five slices in the cell above.
2. Use print statements to check your answers.
3. Discuss any misunderstandings with a peer.
4. Now use list slicing to extract every third element of the list in the cell above starting at the beginning.

5.2.1 Tests for Inclusion

Just as with strings, the `in` operator can be used with lists to determine if a list element is present. Suppose you have a list of all the known radioactive elements on the periodic table and you’d like to know if Iridium is in the list. The `in` statement lets us quickly test to see if it is in the list. (see example below)

```
radioactiveElements = ["Technetium","Promethium","Polonium","Astatine","Radon","Francium",  
"Iridium" in radioactiveElements
```

False

The `in` operator will work with numerical data as well.

```
numbers = [5,6,3,1,2]  
4 in numbers
```

False

5.2.2 List Methods

Lists have a collection of methods (or functions) for accomplishing routine tasks. Some of the more common list methods are given below. All of the methods given will modify the original list (except `copy()`). As a reminder, methods only work on the object type that they were designed for (lists in this case) and they are called by appending the method name to the variable you want it to operate on. (i.e. `myList.clear()`)

Method	Description
<code>append(element)</code>	Adds a single element to the end of the list.
<code>clear()</code>	Removes all elements from a list.
<code>copy()</code>	Creates an independent copy of the list.
<code>count(element)</code>	Counts the number of occurrences of <code>element</code> in the list.
<code>extend(elements)</code>	Adds multiple elements to the end of the list.
<code>index(element)</code>	Returns the index of the <i>first occurrence</i> of <code>element</code> .
<code>insert(index,element)</code>	Inserts the given <code>element</code> at the specified <code>index</code> .
<code>pop(index)</code>	Removes and returns the element given at <code>index</code> . If no index is provided, it defaults to the last element.
<code>remove(element)</code>	Removes the first occurrence of <code>element</code> in the list.
<code>reverse()</code>	Reverses the order of the entire list.
<code>sort()</code>	Sorts the list in place. ³

³It modifies the original list. In contrast, the function `sorted()` will leave the original list unchanged.

To Do:

In the cell below you will find a list containing the speed of sound for several materials and at different temperatures. (in meters/second) Use the list methods in the table above to perform the following:

1. The speed of sound in Aluminum is 6420 m/s. Add two elements to the end of this list: the string “Aluminum” and its speed of sound. Do it two ways, first with **append** and then with **extend**.
2. Remove the first two elements from this list.
3. Insert the removed elements so they appear just after the entry for Helium.
4. Find the index location for “Ethyl Alcohol”.
5. Use the **remove** method to remove the entry for “Granite” and its associated speed.
6. Use the **extend** method to append an entry for the speed of sound for “Steel” which is 5790 m/s.

```
a = ["Air 0C",331,"Air 30C",343, "Helium 0C",970,"Ethyl Alcohol",1170,"Water 20C",1480, "G
```

5.2.3 Built-in functions for Lists

Python has several built-in functions that will work with lists. Functions are called by placing the arguments to the function in parenthesis and prepending the name of the function to the parenthesis. Here are a few common functions that are used with lists: (the **zip** and **enumerate** functions will be explained in greater detail next chapter.)

Method	Description
<code>len(list)</code>	Returns the number of elements in the list.
<code>max(list)</code>	Finds the largest element in the list.
<code>min(list)</code>	Finds the smallest element in the list.
<code>sum(list)</code>	Returns the sum of the elements in the list. ⁴
<code>listOne + listTwo</code>	Joins <code>listOne</code> with <code>listTwo</code> to form one list.
<code>2 * list</code>	Repeat <code>list</code> and concatenate to itself, making the list twice as long.
<code>del list[index]</code>	Deletes list elements located at index. Can also specify a range of values.
<code>zip(listOne,listTwo)</code>	Zips two lists together. (Creates <code>[(a1,b1),(a2,b2),...]</code> from <code>[a1,a2,a3...]</code> and <code>[b1,b2,b3...]</code>)

⁴Only works if the list contains only ints and floats.


```
a = range(10) # Generate a list of integers up to 10
list(a)
```

```
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

The `range` function can be called with up to three arguments: `range(start,end,stepsize)`. Consistent with indexing, the range includes the start value and excludes the end value. Below we generate a list of integers starting at 5, ending at 100 with a step size of 5.

```
myList = range(5,100,5)
list(myList)
```

```
[5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95]
```

To Do:

Use the range function to generate a list of **even** numbers from 8 to 88. (the list should include 88!)

5.3 Tuples

Tuples are another object type similar to lists except that they are *immutable*... that is to say, they cannot be modified once created. They look similar to lists except that they are created using parenthesis instead of brackets. Because you can't change the elements of a tuple, they are often used so that you don't inadvertently modify (and lose) critical data. You can think of it as locking a file on your computer to avoid inadvertently modifying it and losing the original content.

Below is a tuple containing the Balmer series, which are the wavelengths of visible light emitted by Hydrogen gas. These values are well known and you most likely wouldn't want to make any modifications to this list which makes them a prime candidate for a tuple. Storing them in a tuple will help prevent you from inadvertently modifying the list. Indexing and slicing work exactly the same as with lists and strings, so we can still use the values inside of a tuple to perform simple calculations. There are only two methods associated with tuples: `count(element)` and `index(element)`. Their usage is identical to the list methods.

```
balmer = (383.5384,388.9049,397.0072,410.174,434.047,486.133,656.272,656.2852)
balmer.index(410.174)
len(balmer)
```

You can transform a list into a tuple using the `tuple` function.

```
balmer = [383.5384, 388.9049, 397.0072, 410.174, 434.047, 486.133, 656.272, 656.2852]

balmerTup = tuple(balmer)
```

5.4 Flash Cards

1. What is the difference between a list and a tuple?
2. What is it called when the elements of a list are also lists?
3. What is the index value of the first element in a list?
4. How do you access elements of a list counting from the back of the list forward? (Give an example.)
5. What does it mean to “slice” a list?
6. How do you extract every third element of a list, starting at the second element and stopping at (but including) the 11th element?
7. How can you check to see if an element is present in a list? (Give an example)
8. What does the **range** function do? (Give an example of how to use it.)
9. What happens when you multiply a list and an integer?
10. What happens when you add two lists together?
11. Explain the usage of the **extend** list method.
12. Explain the usage of the **insert** list method.
13. Explain the usage of the **remove** list method.
14. Explain the usage of the **pop** list method.
15. Where is the record of Jesus calming the storm?

5.5 Exercises

1. Use the **range** function and list slicing techniques to generate a list of **every other** multiple of 3 starting at 3 and going up to and including 201.

```
# Python code here.
```

2. The code in the cell below will generate a list of 200 random integers in the range $0 \rightarrow 15$.
 1. Print the list and inspect it to verify that you understand what it contains.

2. Verify that there indeed are 200 numbers in the list (Use a function, don't count them one by one.)
3. How many 7's appear in the list? (Use a function, don't count them by hand)
4. What is the largest and smallest numbers that appear in the list?
5. Find the index of the first occurrence of the number 1.
6. Sort the list and use a print statement to verify that you were successful.

```
from random import randint

data = [randint(0,15) for x in range(200)]
```

3. The cell below contains a string containing some elementary particles in physics, with the names of the particles separated by commas.
 1. Use the `split` method to generate a list of particles.
 2. Use the `len` function to determine how many particles are in the list.
 3. `sort` the list and use a print statement to verify that the sort worked.

```
particles = "up quark,down quark,charm quark,top quark,strange quark,bottom quark,electron"
```

4. (Fibonacci Series) The Fibonacci series is a very famous sequence of numbers that appears repeatedly in nature. In the cell below you will find a list that contains the first seven Fibonacci numbers.
 1. Study the sequence of numbers until you identify the pattern.
 2. Once you know the pattern, use `extend` to add the next 10 Fibonacci numbers to the list.
 3. The Fibonacci sequence has some very interesting properties (listed below). Add some code in the cell below to verify each of them.
 1. The ratio of the last two numbers in the sequence gets closer and closer to the 'Golden Ratio' (1.618...) as the sequence gets longer. Calculate this ratio for short sequences and then increase the length of the sequence and watch the ratio approach 1.618...
 2. The sum of any ten consecutive Fibonacci numbers is divisible by 11. Check this for at least three different length-ten slices.
 3. The sum of the first n Fibonacci numbers is equal to the $(n + 2)$ -th Fibonacci number minus 1. Check this for at least three different values of n .

```
fibonacci = [1,1,2,3,5,8,13]
```

5. (Projectile Motion) Below you will find a nested list containing the x-y coordinates for a home run baseball hit. Answer the following questions about the baseball's trajectory:

1. How many x-y pairs are in the list?
2. How far did the baseball go before it hit the ground.
3. How far had the baseball traveled horizontally when it reached its max height? To answer this question, follow the steps below:
 1. Use list comprehension to extract the x and y coordinates of the baseball into separate lists. Call these lists `x` and `y`.
 2. Use the `index` and `max` functions to find the index corresponding to the maximum value in the list of y-coordinates. This is the max height of the baseball.
 3. Now use the index found in part 2 to find the x-coordinate of the baseball when it is at max height.

0006602652177489621672239859609043800522606740039953553831627807894210083502253750039283000200293006236024002

6 Dictionaries and Sets

[Jupyter Notebook](#)

6.1 Dictionaries

6.1.1 Creating a Dictionary

A dictionary is a multi-element object just like a list or tuple but instead of connecting each element to an integer index value, a keyword is used. Dictionaries allow you to access data without knowing anything about the order of the items. For example, maybe you want to store the orbital radius of all the planets in the solar system. Using a dictionary you can associate the name of the planet with its orbital radius. A string containing the planet's name could serve as the key and the associated radius as the value.

Dictionaries are built using curly braces (`{}`) instead of square brackets (lists) or parentheses (tuples). Each item in the dictionary is a **key:value** pair and the items are separated by commas just like lists. Below is an example

```
orbitalRadii = {"Mercury":0.39, "Venus":0.72, "Earth":1.0, "Mars":1.5,"Jupiter":5.2,"Saturn":9.5}
```

The keys don't have to be strings, but can be any data type. For example, maybe you'd like the keys to be the orbital radii of the planets and the values to be the associated orbital period (in Earth days).

```
radiiandPeriod = {0.39: 88, 0.72: 224.7, 1.0: 365.2, 1.5: 687, 5.2: 4331, 9.5: 10_747, 19.2: 107_556}
```

Accessing an element of a dictionary is done with square brackets (`[]`) just like lists, but instead of placing an integer index the key value is used.

```
orbitalRadii = {"Mercury":0.39, "Venus":0.72, "Earth":1.0, "Mars":1.5,"Jupiter":5.2,"Saturn":9.5}

jupiterRadius = orbitalRadii["Jupiter"]
```

To Do:

1. Use a print statement to verify that the correct orbital radius was extracted.
2. Can you extract the orbital radius of Pluto from the list?

Additional `key:value` pairs can be added to an existing dictionary by calling the key and assigning it to a value

```
orbitalRadii = {"Mercury":0.39, "Venus":0.72, "Earth":1.0, "Mars":1.5,"Jupiter":5.2,"Saturn":9.5}

orbitalRadii["Moon"] = 0.384
```

6.1.2 keys, items and values methods

Three dictionary methods are so heavily used that they are worth mentioning here. The `keys` method will return a list¹ of all keys that are present in the dictionary. The `values` method will return a list of all values present in the list and the `items` method will return a nested list containing all `key:value` pairs.

```
orbitalRadii = {"Mercury":0.39, "Venus":0.72, "Earth":1.0, "Mars":1.5,"Jupiter":5.2,"Saturn":9.5}

vals = orbitalRadii.values()
keys = orbitalRadii.keys()
items = orbitalRadii.items()
```

To Do:

1. Use print statements in the cell above to see what the `keys`, `items`, and `values` methods produce.
2. Ask questions as needed.

6.1.3 Other useful dictionary methods

There are a few other useful dictionary methods worth mentioning and I'll place them in the cell below so that you can investigate what they do.

```
orbitalRadii = {"Mercury":0.39, "Venus":0.72, "Earth":1.0, "Mars":1.5,"Jupiter":5.2,"Saturn":9.5}

radii = {"Moon":0.384, "Venus": 0.71}
orbitalRadii.pop("Mercury")
orbitalRadii.popitem()
```

¹Actually it produces an iterator that can be used in conjunction with a loop. If you want a list you must use the `list` function.

```
oRadii = dict.fromkeys(["Mercury", "Venus", "Earth"])
print(oRadii)
orbitalRadii.update(radii)
```

```
{'Mercury': None, 'Venus': None, 'Earth': None}
```

To Do:

1. Use print statements to determine what each dictionary method in the cell above does.
2. Add comments next to each line explaining the method.

6.2 Sets

A set is another python data type that is used occasionally. They are multi-element objects similar to lists and tuples with one key difference: there can be no repeated elements in a set.

6.2.1 Creating a Set

Sets are created using curly braces ({}) just like dictionaries but instead of placing **key:value** pairs inside, single values are used. Any repeated elements will be automatically deleted when the set is created.

```
mySet = {1,2,4,2,1,3,4}
```

To Do:

1. Predict what the set defined above will contain.
2. Use a print statement to check yourself and adjust your thinking as needed.

The **set** function can be used to turn a list or tuple into a set and any repeated elements will be deleted. This can be a handy way to remove unwanted duplicates from a list.

```
elements = ["H", "He", "Ne", "He", "N", "Ag", "Pt", "Ag"]

mySet = set(elements)

print(mySet)
```

```
{'Pt', 'H', 'He', 'Ne', 'Ag', 'N'}
```

6.2.2 Modifying a set

Elements can be added to a set using the `add` and/or `update` methods. The `add` method will add a single element to the set and `update` will allow you to add more than one element.

```
elements = {"H", "He", "Ne", "He", "N", "Ag", "Pt", "Ag"}

elements.add("Au")

elements.update(["In", "Cu", "Os", "He"])
```

To Do:

1. Predict what the set will look like after the `add` method executes. Then add a print statement to verify that you're correct.
2. What happens if you try to `add` an element that is already in the set?
3. Predict what the set will look like after the `update` method executes. Then add a print statement to verify that you're correct.

Elements can be removed from a set using the `remove` and/or `discard` methods. The difference between these methods is very subtle. They will both remove an element, but the `remove` method will throw an error if the element you are attempting to remove isn't in the set. `discard` won't throw an error even if the element isn't present. The `clear` method will remove all entries from the set.

```
elements = {"H", "He", "Ne", "He", "N", "Ag", "Pt", "Ag"}

elements.remove("H")
elements.discard("He")
elements.discard("Cu")
elements.remove("Cu")
elements.clear()
```

To Do:

Predict what the set will look like after each statement in the cell above. Then add a print statement to verify that you're correct.

6.2.3 Mathematical Set Operations

Sets are a well-known mathematical idea and there are four mathematical operations that are commonly used with sets. They are: union, intersection, difference, and symmetric difference.

We'll investigate them one at a time.

6.2.3.1 Union

The union of two sets is the set of all unique elements between both sets. The union of two sets can be found using the `|` operator.

```
fibonacci = {1,1,2,3,5,8,13,21,34,55,89,147,236}
triangles = {1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190}

union = fibonacci|triangles
```

6.2.3.2 Intersection

The intersection of two sets is the set of elements that are common to both sets. The intersection of two sets can be found using the `&` operator

```
fibonacci = {1,1,2,3,5,8,13,21,34,55,89,147,236}
triangles = {1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190}

intersect = fibonacci&triangles
```

6.2.3.3 Difference and Symmetric Difference

And finally, the difference of two sets is the set of elements that are found in one set but not in the other. The difference of two sets can be found using the `-` operator. Note that $A - B$ will produce a different set than $B - A$. $A - B$ will produce a set with all of the elements from set A that are not in set B . $B - A$ will do just the opposite, forming a set with all the elements from set B that are not in set A . If you want the set of elements that are in A **and** B but not in both (kind of like the opposite of intersection) then you want the symmetric difference operator (\wedge)

```
fibonacci = {1,1,2,3,5,8,13,21,34,55,89,147,236}
triangles = {1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190}

diff = fibonacci - triangles
diffTwo = triangles - fibonacci
symdiff = fibonacci ^ triangles
```

To Do:

1. Predict what the set will look like after each statement in the cells above.
2. Add print statements to check yourself.

As a final point you should note that set elements cannot be extracted using square brackets like you can with lists and tuples. It's just not allowed. However, you can iterate over sets even though there is no guarantee about the order they will come in. We'll see what it means to iterate when we study loops.

```
elements= {"Li","Be","B", "C","N", "O","F", "Ne"}  
  
print(elements[2])
```

6.3 Flashcards

1. Describe all of the ways that you can make a dictionary.
2. Describe all of the ways that you can make a set.
3. How do you add a new **key:value** pair to a dictionary?
4. How do you add a new value to a set?
5. What does the **keys** dictionary method produce?
6. What does the **values** dictionary method produce?
7. What does the **items** dictionary method produce?
8. What does the union operator (**|**) do?
9. What does the intersection operator (**&**) do?
10. What does the difference operator (**-**) do?
11. What does the symmetric difference operator (**^**) do?
12. Where is the record of The Savior's visit to the American continent?

6.4 Exercises

1. (Physical Constants) The field of physics is filled with experimental constants. In fact, you'll use some of them so frequently throughout your undergraduate experience that you'll end up memorizing them. Until then it could be useful to store some of these constants in a dictionary for easy reference. Choose at least 15 constants from the following [website](#) and build a dictionary of them. Make sure that the following constants are in your list: k_B , \hbar , c , e , and ϵ_0 . You are free to choose the names of the keys to be anything that makes good sense to you.

Then use your dictionary to calculate the following values

$$\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2} \text{ (Stefan-Boltzmann constant)}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \text{ (fine-structure constant)}$$

```
# Python code here.
```

2. (Unit Conversions) Unit conversions show up all over in physics and if you don't have the conversion factors memorized you may find yourself constantly looking them up. Construct a dictionary that contains many of the most useful unit conversions, including those shown below. You are free to choose the key names to be whatever makes most sense to you. I've given a few examples in the code cell below to get you started.

1 kilometer = 1000 meters

1 inch = 2.54 centimeters

1 mile = 1.609 kilometers

1 mph = 0.447 m/s

1 u = 1.661×10^{-27} kg

1 electron-volt = 1.602×10^{-19} Joules

1 day = 24 hours

1 radian = 57.3°

1 rev = 2π radians

Once the dictionary is created use it to perform the following conversions:

1. 5 miles to meters.
2. 250° to radians.
3. 562 Joules to electron-volts.
4. 3500 inches to meters.
5. 5.8 m^3 to cm^3 .

```
conversions = {"km-to-m": 1000, "in-to-cm": 2.54 }
```

3. [Here](#) you will find a table containing planetary information. Pick your top three favorite planets and make one dictionary for each planet. The keys in the dictionary should be the row label from the table (or a shortened version of that label) and the item value should be the number in the table. Once the dictionaries are created, verify that you can extract values from them as you would expect.

```
# Python code here.
```

4. [Here](#) you will find a table containing planetary information. Pick a row from the table and make a dictionary that holds the values in that row. The keys to the dictionary should be the planets and the values are the value from the table. Once the dictionary is created, use it to calculate the following:

1. The average of the property that you chose.
2. The largest values.
3. The smallest value.

```
# Python code here.
```

5. In the cell below you will find the first 15 numbers for several important mathematical sequences. Answer the following questions:

1. Find all of the numbers that are in the fibonacci sequence **and** the triangles sequence. (intersection)
2. Find all of the numbers that are in the pentagonal sequence **but not** in the hexagonal sequence. (difference)
3. Find the union of the fibonacci sequence and the hexagonal sequence and report how many numbers there are.
4. Determine how many numbers are in the caterer sequence **but not** in the hexagonal sequence.

```
fibonacci = {1,1,2,3,5,8,13,21,34,55,89,147,236,383,619}
squares= {1,4,9,16,25,36,49,64,81,100,121,144,169,196,225}
triangles = {1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190}
pentagonal = {1, 5, 12, 22, 35, 51, 70, 92, 117, 145, 176, 210, 247, 287, 330}
hexagonal = {1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435}
caterer = {2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, 121}
```


7 Loops

Jupyter Notebook

Loops allow programs to rerun the same block of code multiple times. This might seem like a funny thing to want to do but it turns out that there are many important tasks that are repetitive in nature (perhaps with small changes for each successive repetition). A loop provides a succinct and efficient way to perform tasks of this nature.

7.1 for loops

The `for` loop is probably the most common loop you will encounter and is a good choice when you know beforehand exactly what things you want to loop over. Here is an example of `for` loop that is used to add up the elements of a list.

```
thesum = 0
for i in [3,2,1,9.9]:
    thesum += i
```

This would be equivalent to the following code:

```
thesum = 0

thesum = thesum + 3
thesum = thesum + 2
thesum = thesum + 1
thesum = thesum + 9.9
```

which isn't that much longer than using a loop. However, as the list gets longer and/or the mathematical operations being performed get more complex the second method would get unreasonably long.

The correct language is to say that we are *iterating* over the list `[3,2,1,9.9]`. This means that the loop variable (`i` in this case but you can choose it to be whatever you want) gets assigned the values of the list elements, one by one, until it reaches the end of the list. You can use `for` loops to iterate over any multi-element object like lists or tuples. Python uses

indentation to indicate where the loop ends. In this case there was only one statement inside to loop, but if you wanted more than one each line should be indented.

To Do:

1. Add a `print` statement inside of the loop above to display the value of the variable `thesum`.
2. Predict what the output will be and then run the code and verify that you were correct.
3. Now change the loop variable to be named `physics` and verify that the loop still works as expected.

You can iterate over `range` objects and strings using `for` loops.

```
for i in ['Physics', 'is', 'so', 'fun']: # Iterate over a list of strings
    print(i)
```

```
Physics
is
so
fun
```

```
for i in range(5,50,3): #Generates a list from 5 -> 50 with a step size of 3
    print(i)
```

```
5
8
11
14
17
20
23
26
29
32
35
38
41
44
47
```

These examples are so simple that you might wonder when a loop might actually be useful to you. Let's see if we can build a loop to calculate the following sum

$$\sum_{n=1}^{1000} \frac{1}{n^2} \quad (7.1)$$

```
theSum = 0
for n in range(1,1000):
    theSum = theSum + 1/n**2
print(theSum)
```

1.6439335666815615

Here, `n` is being assigned the values 1,2,3,4...1000, one by one, until it gets all the way to 1000. Each time through the loop, `n` is different and the expression `1/n**2` evaluates to a new value. The variable `theSum` is updated each time through to be the running total of all calculations performed thus far. Here's another example of a loop used to calculate the value of 20!:

```
theProduct = 1
for n in range(1,21):
    theProduct = theProduct * n #Multiply theProduct by n
print(theProduct)
```

2432902008176640000

Remember that the range function creates a list starting at 1, going up to 21 but not including it. The math library has a function called `factorial` that does the same thing. Let's use it to check our answer:

```
from math import factorial
factorial(20)
```

2432902008176640000

7.1.1 Boolean Logic Inside Loops

Often when using loops, we only want a block of code to execute when some condition is satisfied. We can use boolean logic inside of the loop to accomplish this. For example, let's write a loop to compute the following sum:

$$\sum_{\frac{n}{5} \in \text{Int and } \frac{n}{3} \in \text{Int}} \frac{1}{n^2}$$

which is similar to the one we did above, but this time we only want to include terms where n is a perfect multiple of both 5 and 3. To check to see if n is a perfect multiple of a number we can calculate the modulo (remainder after division) using the % operator and check that it is equal to zero.

```
theSum = 0
for n in range(1,1000):
    if n % 5 == 0 and n % 3 == 0:
        theSum = theSum + 1/n**2
print(theSum)
```

0.007243985583159138

To Do:

Perform the following modifications to the loop above.

1. Increase the upper bound of the sum to go up to and include 5000.
2. Only include the terms where n is a multiple of 5 and 3 **or** is a multiple of 7.
3. Replace the statement that updates `theSum` to its shorthand version.

7.1.2 Zipping and Enumerating

There are times when it is necessary to iterate over two lists simultaneously. For example, let us say that we have a list of atomic numbers (AN) and a list of approximate atomic masses (mass) of the most abundant isotopes for the first six elements on the periodic table.

```
AN = [1,2,3,4,5,6]
mass = [1,4,7,9,11,12]
```

If we want to calculate the number of neutrons in each isotope, we need to subtract each atomic number from the atomic mass. To accomplish this, it would be nice to iterate over both lists simultaneously

7.1.2.1 Zipping

The simplest way to iterate over two lists simultaneously is to combine both lists into a single, iterable object and iterate over it once. The `zip` function does just that by merging two lists or tuples into a nested list

```
AN = [1,2,3,4,5,6]
mass = [1,4,7,9,11,12]

zipped = zip(AN,mass)

for pair in zipped:
    print(pair[1] - pair[0])
```

```
0
2
4
5
6
6
```

The `zip` objects are “single use” so you can’t reuse `zipped` in a later loop. If the two lists being zipped are not the same length, `zip` stops zipping when it reaches the end of the shorter list.

To Do:

1. Print the variable `zipped` and inspect closely. Was the output what you expected?
2. Now do `print(tuple(zipped))` and inspect closely. Draw a conclusion.
3. Add a few more entries to the list named `AN`, but don’t add the corresponding entries to the other list. Now the lists being zipped aren’t the same length. Inspect the output of the print statement to determine what `zip` does in this scenario.

7.1.2.2 Enumeration

A close relative to `zip` is `enumerate` which zips a list to *the index value for that list* (read that last statement again). It also returns a “single use” object that can be iterated over.

```

AN = [1,2,3,4,5,6]
mass = [1,4,7,9,11,12]

enum = enumerate(mass)

for idx,val in enum:
    print(val - AN[idx])

```

0
2
4
5
6
6

To Do:

1. Repeat the previous To-Dos for the cell above.

7.1.3 List Comprehension

It is fairly common to use a `for` loop to populate a list with a sequence of numbers.

```

myList = []

for i in range(10):
    myList.append(i**2)

print(myList)

```

[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]

This entire process can be compressed down into a single line by expressing the `for` loop in square brackets. This is known as *list comprehension*. The code below will generate the list as above.

```

myList = [i**2 for i in range(10)]
print(myList)

```

[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]

List comprehension can take a little time to get used to but it is well worth it. It saves both time and space and makes code less cluttered. You can even add boolean expressions to your conditionals for further control of the final result.

```
myList = [i**2 for i in range(10) if i %2 == 0] # Include only the evens.
print(myList)
```

[0, 4, 16, 36, 64]

To Do:

Think of your favorite mathematical function and modify the code above to generate a few samples from it.

7.2 while Loops

Logic can be combined with loops using something called a **while** loop. A **while** loop is a good choice when you don't know beforehand exactly how many iterations of the loop will be executed but rather want the loop to continue to execute until some condition is met. As an example, notice that in Equation 7.1, the terms in the sum get progressively smaller as n gets bigger. It doesn't make sense to continue adding to the sum once the terms get very small. Let's compute this sum by looping until the fraction $\frac{1}{n^2}$ become smaller than 1×10^{-2} .

```
term = 1 # Load the first term in the sum
s = term # Initialize the sum
n = 1 # Set a counter
while term > 1e-2: # Loop while term is bigger than 1e-2
    n = n + 1 #Add 1 to n so that it will count: 2,3,4,5
    term = 1./n**2 # Calculate the next term to add
    s = s + term # Add 1/n^2 to the running total
```

This loop will continue to execute until `term < 1e-2`. Note that unlike the **for** loop, here you have to do your own counting if you need to know how many iterations have been performed. Be careful about what value `n` starts at and when it is incremented (`n = n + 1`). Also notice that `term` must be assigned prior to the start of the loop. If it wasn't the loop's first logical test would fail and the loop wouldn't execute at all.

To Do:

1. Decrease the threshold on the termination condition and observe any changes in the final result and how many more iterations are performed.

2. After toying around with it for a while pick a termination condition that you feel will produce a result that is accurate.

`while` loops should be used with caution because you can easily write a *faulty termination condition* and inadvertently write a loop that runs forever. This happens because your termination condition was never met. An example of this is given below.

Warning: Do not execute the code block below!!

```
x = 0

while x != 10:
    x = x + 3
print("Done")
```

The loop above is intended to end after a few iterations when the value of `x` is equal to 10. However, closer inspection reveals that the value of `x` will never be equal to 10. After the first iteration `x` is equal to 3, then 6,9,12,15 and so on... but never 10. This loop will run forever because the termination condition is never met (`x != 10` never produces a `False`)!! If you choose to use a `while` loop, triple check your termination condition to make sure you haven't made a mental error. Avoiding the use of `!=` or `==` in your termination condition will help too. Use `<=` or `>=` instead.

To Do:

1. Modify the termination condition in the loop above so that it terminates when `x` gets larger than 15.
2. Run the code and verify that you did it correct.

7.3 continue, break, and pass Commands

The `continue`, `break`, and `pass` commands are used to control the flow of code execution in loops. Here is a description of their usage:

Operator	Description
<code>break</code>	Exits a <code>for/while</code> loop.
<code>continue</code>	Skips the remaining loop block and begins the next iteration.
<code>pass</code>	No action; code continues on

The `break` statement is useful when you want to completely stop a loop early. Here is our sum loop rewritten with a `break` statement added to stop the loop after 1000 iterations.


```

term = 1 # Load the first term in the sum
s = term # Initialize the sum
n = 1 # Set a counter
while term > 1e-10: # Loop while term is bigger than 1e-10
    n += 1 #Add 1 to n so that it will count: 2,3,4,5
    term = 1./n**2 # Calculate the next term to add
    s += term # Add 1/n^2 to the running total
    if n > 1000:
        print('This is taking too long. I'm outta here...')
        break

```

This is taking too long. Im outta here...

The `continue` statement is similar to `break` except that instead of stopping the loop, it only stops the current iteration of the loop. All code below the `continue` statement will be skipped and the next iteration will begin. For example, if you wanted to do the sum from equation ?? but only include those terms for which `n` is a multiple of 3, it could be done like this:

```

term = 1 # Load the first term in the sum
s = term # Initialize the sum
n = 1 # Set a counter
while term > 1e-10: # Loop while term is bigger than 1e-10
    n += 1 #Add 1 to n so that it will count: 2,3,4,5
    if n % 3 != 0:
        continue
    term = 1./n**2 # Calculate the next term to add
    s += term # Add 1/n^2 to the running total

```

Now, when the value of `n` is not a multiple of 3, the sum will not be updated and the associated terms are effectively skipped.

Finally, the `pass` statement does nothing. Seriously!! It is merely a place holder for code that has not been written yet. Usually, you'll use the `pass` statement to run and test code without errors due to missing code.

7.4 Flash Cards

1. Explain the basic structure and key elements of a `for` loop.
2. Explain the basic structure and key elements of a `while` loop.
3. What does the `break` statement do?
4. What does the `continue` statement do?

5. What does the `pass` statement do?
6. What does the `zip` function do? Give an example.
7. What does the `enumerate` function do? Give an example.
8. What is list comprehension? Give an example.
9. Where is the account of Jesus walking on the water?

7.5 Exercises

1. Use a loop to construct a list containing the first 1000 numbers that are multiples of both 5 and 3. (The modulo operator `(%)` is a good and efficient way to see if a number is a multiple of another.)

Python code here.

2. Summations appear often in science and mathematics. One such summation is called the Riemann Zeta function and is given by

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n} = \frac{1}{1^n} + \frac{1}{2^n} + \frac{1}{3^n} + \dots$$

1. Use a `for` loop to evaluate $\zeta(2)$ and $\zeta(4)$ and verify that $\zeta(2) = \frac{\pi^2}{6}$ and $\zeta(4) = \frac{\pi^4}{90}$. This summation is called an infinite sum and we surely don't want our loop to continue forever!! Instead, an infinite sum like this one can be approximated with a `for` loop with a large number of iterations. To determine if you have included enough terms, increase the number of terms steadily and watch for the final result to stop changing appreciably.
2. Now repeat part 1 using list comprehension and the `sum` function?
3. Repeat the exercise one final time using a `while` loop. Write the `while` loop so that it stops iterating when the value of $\frac{1}{k^n}$ gets smaller than 1×10^{-6} .

#Python code here

3. In the cell below you will find three lists containing the masses, lengths, and radii for a collection of cylinders. The moment of inertia for each cylinder can be calculated as

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$

1. Use a loop to iterate over these lists and calculate the moment of inertia for each one. Store the calculated values in a new list. You should use the `enumerate` function on this problem.

2. Calculate the largest, smallest, and average of the calculated values

```
from numpy.random import uniform
mass = uniform(3,8,1000)
radius = uniform(0.5,1.2,1000)
length = uniform(0.8,3,1000)
```

4. An object that is dropped/thrown from a high altitude and allowed to fall through the air will experience a drag force that is opposite its motion. Because of that drag force, the velocity of the particle will not increase forever, but instead approach a constant velocity (called terminal velocity). The function that describes the velocity of the particle as a function of time is given by

$$v(t) = -\frac{mg}{c} + \left(\frac{mg}{c} + v_0\right) e^{\frac{-ct}{m}}$$

and the terminal velocity is given by

$$v_t = \sqrt{\frac{mg}{c}}$$

where m is the mass of the projectile, c is the drag constant, $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity, v_0 is the initial velocity of the particle, and t is time.

1. Use a while loop to calculate the velocity of the particle at 0.1 second time intervals, starting at $t = 0$.
2. The loop should terminate when the velocity of the particle achieves 90% of terminal velocity.
3. Use a print statement to display the amount of time that has elapsed before reaching 90% of terminal velocity.
4. Hypothesize about what might happen if you increased or decreased the mass of the object, while leaving all other physical parameters fixed. Then modify your code and see if your guess was right.
5. Hypothesize about what might happen if you increased or decreased the drag constant (c) for the object, while leaving all other physical parameters fixed. Then modify your code and see if your guess was right.

```
from math import exp,sqrt
m = 20
g = 9.8
c = 0.75
v0=0
```

5. In the cell below you will find a dictionary containing the mass(in kg) and diameter of the planets in our solar system. Use a loop to iterate over this dictionary and calculate the acceleration due to gravity at the surface of the planet.

$$g = \frac{GM}{R^2}$$

. Then add this value to the list for each planet. Print the dictionary when you are done to verify that each dictionary entry is a list with three elements.

```
planetData = {"Mercury": [0.33e24, 4.879e6], "Venus": [4.87e24, 1.2104e7], "Earth": [5.97e24, 1.27e7]}
```

(Extra Credit) A transcendental equation is one that cannot be solved analytically (with pencil and paper). Try solving the following equation for x to see what I mean:

$$\frac{\sin x}{x} = 1$$

One numerical method for solving an equation like this involves first rearranging it to look like this:

$$x = \sin x$$

and then using a loop to repeatedly evaluating the right hand side, using the result of the previous evaluation as the input, until subsequent evaluations differ very little (1×10^{-3} would be fine). ¹ The first few iterations of this process might look something like this:

$$(x_{\text{old}})_1 = 2 \text{ (starting guess)}$$

$$(x_{\text{new}})_1 = \sin((x_{\text{old}})_1)$$

$$(x_{\text{old}})_2 = (x_{\text{new}})_1$$

$$(x_{\text{new}})_2 = \sin((x_{\text{old}})_2)$$

$$(x_{\text{old}})_3 = (x_{\text{new}})_2$$

$$(x_{\text{new}})_3 = \sin((x_{\text{old}})_3)$$

1. Use a `while` loop to solve this equation and report the correct value for x .
2. Plug the answer back into the equation to verify that it indeed does satisfy the equation.
3. Report how many iterations it took to solve the problem.
3. Decrease the value in the termination condition and observe how the number of iterations changes.

Hint: You'll have to define two variables: one to hold the old guess and one to hold the updated guess.

¹This is called successive relaxation.

```
xold = 2 # starting guess
```

8 Functions

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We have already been using functions here and there but in this chapter we will introduce them formally and get into the details. A function encapsulates a block of code designed to perform a specific task or set of tasks. To perform the task correctly, most functions require that you provide some information (called arguments) when you call them. To call a function you type the name of the function followed by the needed arguments enclosed in parenthesis `()`.

```
functionName(argument1, argument2, argument3...)
```

The **number and type** of arguments allowed is different for every function. As a first example, let's consider the `print` function, which is the simplest (and most familiar) function that we have used so far.

```
print("print is a function") #Single string argument
print("print", "is", "a", "function") # Multiple string arguments
print(5) # Single integer argument
print(5,4) # Multiple integer argument
print([5,4,2,5]) # Single list argument
print() # No arguments
```

```
print is a function
print is a function
5
5 4
[5, 4, 2, 5]
```

The `print` function is pretty flexible in what it allows for arguments; the arguments can be any type of data (strings, ints, floats, booleans, and even lists) and the number of arguments can be as large as you want. Most functions are a little more strict on what they allow their arguments to be. For example, the `factorial` function only allows one argument and

that argument must be an integer.¹ If you attempt to call the factorial function with a float argument or with more than one argument, the result will be an error.

```
from math import factorial
a = factorial(5.54)
b = factorial(5,3)
```

To Do:

1. Run the code above and take note of the error message that results.
2. Comment out the definition of `a` so you can also see the error message for the definition of `b`.

Python functions generally fall into three groups: functions that come standard with Python (called native functions), functions that you can import into Python, and functions that you write yourself.

8.1 Native Functions

There are a few functions that are always ready to go whenever you run Python. They are included with the programming language. We call these functions native functions. You have already been using some of them, like these

```
myList = [5,6,2,1]
a = len(myList) # 'len' function is native.

b = float(5) # 'float' function is native.

c = str(67.3) # 'str' function is native.
```

The `len`, `float` and `str` functions are all native and they all take a **single argument**. Other native function have been mentioned in previous chapters and others will be mentioned in the future.

8.2 Imported Functions

Many times, you will need to go beyond what Python can do by itself². However, that doesn't mean you have to create everything you need to do from scratch. Most likely, the function that

¹The factorial of a float can be calculated, but the math library is not equipped to handle it.

²For example, Python does not include `sin()` or `cos()` as Native functions.

you need has already been coded. Somebody else created the function and made it available to anyone who wants it. Groups of functions that perform similar tasks are typically bundled together into libraries ready to be imported so that the functions that they contain can be used.

In order to use a function correctly, you'll need to know what information(arguments) the function expects you to give it and what information the function intends to return to you as a result. This information can be found in the library's documentation. Most libraries have great documentation with lists of the included functions, what the functions do, the expected arguments, and examples on how to use the most common ones. You can usually find the library documentation by searching the internet for the library's name plus "Python documentation".

Providing a complete list of all available libraries and function is not really the purpose of this book. Instead, we'll illustrate how to import functions and use them. As you use Python more and more you should get in the habit of searching out the appropriate library to accomplish the task at hand. When faced with a task to accomplish, your first thought should be, "I'll bet somebody has already done that. I'm going to try to find that library."

Functions are imported using the `import` statement. You've already seen how to perform very simple mathematical calculations (5/6,84, etc..), but for more complex mathematical calculations like $\sin(\frac{\pi}{2})$ or $e^{2.5}$, you'll need to import these functions from a library.

```
import math

a = math.sqrt(5.2)
b = math.sin(math.pi)
c = math.e**2.5
```

The `math.` before each function is equivalent to telling Python "Use the `sqrt()` function that you will find in the `math` book I told you to grab." If you just type

```
sqrt(5.2)
```

Python won't know where to find the `sqrt` function and an error message will result. Sometimes the name of the module can be long and typing `module.` every time you want to use one of it's functions can be cumbersome. One way around this is to rename the module to a shorter name using the `as` statement.

```
import math as mt

a = mt.sqrt(5.2)
b = mt.sin(mt.pi)
c = mt.e**2.5
```


Instead of importing an entire module, you can import only a selection of functions from that module using the `from` statement. This can make your code even more succinct by eliminating the `module.` prefix altogether. The trade-off is that it won't be as clear which function belongs to which module.

```
from math import sqrt, sin, pi, e

a = sqrt(5.2)
b = sin(pi)
c = e**2.5
```

All of the functions belonging to a module can be imported at once using `*`.

```
from math import *

a = sqrt(5.2)
b = sin(pi)
c = e**2.5
```

8.3 User-defined Functions

After having programmed for a while, you will notice that certain tasks get repeated frequently. For example, maybe in your research project you need to calculate the force exerted on an atom due to many other nearby atoms. You could copy and paste your force-calculation code every time it was needed, but that would likely result in lots of extra code and become very cumbersome to work with. You can avoid this by creating your own function to calculate the force between any two atoms. Then, every time you need another force calculation, you simply call the function again. You only write the force-calculation part of the code once and then you execute it as many times as you need to.

To create your own function, you first need to name the function. The name should be descriptive of what it does and makes sense to you and anyone else who might use it. The first line of a function definition starts with the `def` statement (short for definition) followed by the name of the function with whatever information, called *arguments*, that needs to be fed into the function enclosed in parenthesis. **The last character in this line must be a colon.** Everything inside the function is indented four spaces and placed directly below the first line.

```
def functionName(arg1,arg2,arg3):
    # Body of Function
    # Body of Function
```

```
# Body of Function
```

As an example, let's construct a function that calculates the distance between two atoms. The function will need to know the location of each atom, which means that there should be two arguments: the xyz coordinates of both atoms passed as a pair of lists or tuples.

```
import math

def distance(coords1, coords2):

    dx = coords1[0] - coords2[0]
    dy = coords1[1] - coords2[1]
    dz = coords1[2] - coords2[2]
    d = math.sqrt(dx**2 + dy**2 + dz**2)
    print(f"The distance is {d:5.4f}.")

distance([1,2,3],[4,5,6])

distance([4.2,9.6,4.8],[2.9,2.6,3.4])
```

The distance is 5.1962.

The distance is 7.2560.

This function works just fine with integers or floats for the coordinates.

8.3.1 The return statement

The `distance` function prints out the value for the distance, but what if we want to use this distance in a subsequent calculation? Maybe we want to calculate the average distance between several pairs of atoms. We can instruct the function to return the final distance using the `return` statement. If the arguments to the function are the inputs, the `return` statement specifies what the output is. Let's modify the function above to include a `return` statement.

```
import math

def distance(coords1, coords2):
    dx = coords1[0] - coords2[0]
    dy = coords1[1] - coords2[1]
    dz = coords1[2] - coords2[2]
    d = math.sqrt(dx**2 + dy**2 + dz**2)
    return d
```

```

distOne = distance([1,2,3],[4,5,6])

distTwo = distance([4.2,9.6,4.8],[2.9,2.6,3.4])

averageDistance = (distOne + distTwo)/2

print(f"The average distance is {averageDistance:5.4f}.")

```

The average distance is 6.2261.

8.3.2 Local vs Global Variable Scope

Variables created inside of a function have *local scope*. This means that they are not accessible outside of the function. In our `distance` function the variables `dx`, `dy`, `dz`, and `d` were all local variables that are used inside the function but have no value outside of it. This is convenient because we don't have to worry about overwriting a variable or using it twice. If someone sends you a function and you want to use it in your code, you don't have to worry about what variable he/she chose to use inside his function; they won't affect your code at all.

```

def distance(coords1,coords2):
    dx = coords1[0] - coords2[0]
    dy = coords1[1] - coords2[1]
    dz = coords1[2] - coords2[2]
    d = math.sqrt(dx**2 + dy**2 + dz**2)
    return d

distOne = distance([1,2,3],[4,5,6])

print(dx) # There is no value associated with this variable outside of the function.

```

The down side to all of this is that you don't have access to function variables unless you pass them out of the function using the `return` statement.

Any variables defined outside of a function is called a *global variable*, which means that Python remembers these assignments from anywhere in your code including inside of functions. Using global variables with the intention to use them inside of functions is usually considered bad form and confusing and is discouraged. One notable exception to this rule are physical constants like $g = 9.8 \text{ m/s}^2$ (acceleration due to gravity on Earth) or $k_B = 1.38 \times 10^{-23}$ (Boltzmann's constant which is used heavily in thermodynamics) because these values will never change and may be used repeatedly. Generally speaking every variable that is used in a function ought to

be either i) passed in as an argument or ii) defined inside of the function. Below is an example of an appropriate use of a global variable.

```
def myFunction(a,b):
    c=a+g # <--- Notice the reference to 'g' here
    d = 3.0 * c
    f = 5.0 * d**4
    return f

#The variable below are global variables.
r = 10
t = 15
g = 9.8 #<--- g defined to be a global variable
result = myFunction(r,t)
```

8.3.3 Positional vs. Keyword Arguments

The function arguments we have been using so far are called *positional arguments* because they are required to be in a specific position inside the parenthesis. To see what I mean consider the example below.

```
def example(a,b):
    return a**b

resultOne = example(5,2)
resultTwo = example(2,5)

print(resultOne, resultTwo)
```

25 32

In the first call to `example` the local variable `a` gets assigned to be 5 and the local variable `b` gets assigned 2. In the second call the order of the arguments is switched and the subsequent assignments to `a` and `b` switch with it. This produces a different result from the function. Positional arguments are very common but the user must know what information goes where when calling the function.³

The other type of argument is the *keyword argument*. These arguments are attached to a keyword inside of the parenthesis. The advantage of a keyword argument is that the user does

³This is another reason why you want to choose meaningful variable names for your arguments.

not need to be concerned about the location of the argument as long as it has the proper label.

```
def example(a=1,b=2):  
    return a**b  
  
resultOne = example(a=5,b=2)  
resultTwo = example(b=2,a=5)  
  
print(resultOne, resultTwo)
```

25 25

Another advantage to using keyword arguments is that a default value can be coded into the function. This means that we can call the function with some arguments missing and default values will be used for them. In the example above, the default value of `b` is 2 and if the function is called without specifying a value for that argument, the function will proceed as usual using the default value for `b`.

```
def example(a=1,b=2):  
    return a**b  
  
result = example(a=5)  
  
print(result)
```

25

8.3.4 Splatting

Functions can potentially require dozens of arguments to be passed in which can make calling the function long and difficult to look at. One way to shorten the code is to put all of the arguments in a list or tuple.

```
def func(a,b,c,d):  
  
    return a**b + c/d  
args = (5,4,6,3)
```

```
result = func(*args)

print(result)
```

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8.3.5 Lambda Functions

When the function you want to construct is very simple (one line), there is a shortcut code for making it called a *lambda function*. The benefit is that they occupy less lines of code than the standard `def` functions. A lambda function is defined as shown below with the variable immediately after the `lambda` statement as the independent variable in the function.

```
f = lambda x: x**2

print(f(5))
```

25

As a simple application of lambda functions let's consider a function from the `scipy.integrate` library called `quad` which will perform a numerical integration of a function. The `quad` function takes three arguments: the function to be integrated, and the upper and lower bound on the integral. We can use a lambda function for the first argument rather than using several lines of code to build one in the traditional fashion.

```
from scipy.integrate import quad
from math import sin, pi

quad(lambda x: sin(pi * x)**2, 0, 0.4 )
```

(0.15322553581056808, 1.7011451781741914e-15)

8.4 Flash Cards

1. Describe the basic structure and key elements of a user-defined function.
2. What are function arguments?
3. List all of the ways that a function can be imported from a module.

4. What does the `return` statement do?
5. What is the difference between a local and global variables?
6. What is a keyword argument? Give an example.
7. What is a lambda function? How do you make one?
8. Recite Alma 32:21.

8.5 Exercises

1. A wooden block is placed on a horizontal surface and pushed across the floor with an initial velocity of v_0 . The expression for the position of the block after t seconds have elapsed is

$$x(t) = v_0 t - \frac{1}{2} \mu g t^2$$

where μ is the coefficient of friction and $g = 9.8 \text{ m/s}^2$. You can easily calculate the time it takes for the block to come to rest to be

$$T = \frac{v_0}{\mu g}$$

1. Write a function that takes the coefficient of friction and the initial velocity as arguments and returns the total distance traveled while coming to rest.
2. Call the function for $v_0 = 7.2 \text{ m/s}$ and the following values of $\mu = 0.62, 0.3, 0.45, 0.2$. The answers should be: `[4.266, 8.816, 5.878, 13.224]`.
3. Use list comprehension to put these travel distances into a list.
4. Use the `zip` function to combine the list of friction coefficients with the list of travel distances.
5. Use a `print` statement to verify that you did it correctly.
2. In quantum mechanics you will learn that the energy of an electron can't just be any old value, but can only be certain discrete values. For the hydrogen atom, the allowed energy levels for the electron (in units of electron-volts) are given by

$$E(n) = \frac{-m_e e^3}{32\pi^2 \epsilon_0^2 \hbar^2 n^2}$$

where n must be an integer and the other constants are

- $m_e = 9.109 \times 10^{-31} \text{ kg}$ is the mass of an electron.
- $e = 1.602 \times 10^{-19} \text{ C}$ is the fundamental.
- $\epsilon_0 = 8.854 \times 10^{-12} \text{ C V}^{-1} \text{ m}^{-1}$ is the electrical constant.

- $\hbar = 1.054 \times 10^{-34}$ J s is the reduced Planck constant.
1. Construct a function that takes the quantum number (n) as an argument and returns the energy of the electron in units of electron-volts. All of the fundamental constants in the equation should appear as keyword arguments with default values attached. Call the function for a few values of n to make sure it is working. You should find that $E(1) = -13.6$ eV, $E(2) = -3.4$ eV, $E(3) = -1.5$ eV, etc.
 2. When an electron transitions from a high energy state to a low energy state, it releases a bundle of light (called a “photon”) with an energy that is exactly equal to the energy difference. Use the function you built in part 1 to calculate the energy of a photon that is released when an electron transitions from the $n = 5$ state to the $n = 2$ state.
 3. The wavelength (λ) of the photon can be calculated using the following equation:

$$\lambda = \frac{2\pi\hbar c}{e\Delta E}$$

- . Calculate the wavelength of light released for the transition described in part 2. Go to [this](#) website and determine what color this corresponds to.
4. Use list comprehension together with the function you built in part 1 to build a list containing the 15 lowest-energy levels for the hydrogen atom.

`me = 9.109e-31`

3. Max Planck discovered that **all** objects emit a spectrum of radiation and that the intensity of the spectrum depends on the object’s temperature. The equation for the intensity of the emitted light is

$$I(\lambda) = \frac{2\pi\hbar c^2}{\lambda^5} \frac{1}{e^{\frac{\hbar c}{\lambda k_B T}} - 1}$$

where

- $c = 3.0 \times 10^8$ m/s is the speed of light.
 - $k_B = 1.38 \times 10^{-23}$ m²kg s⁻² K⁻¹ is the Boltzmann constant.
 - $\epsilon_0 = 8.854 \times 10^{-12}$ C V⁻¹ m⁻¹ is the electrical constant.
 - T is temperature in Kelvins.
 - λ is wavelength in meters.
1. Write a function that takes two arguments: the temperature of the object and the wavelength of light. All other physical constants should appear as keyword arguments with appropriate default values attached. The function should return the corresponding light intensity given by the function above.
 2. The temperature of the sun is approximately 5780 K and it emits strongly in the visible region of the spectrum. Use the function you built in part 1 to calculate the intensity of light emitted by the sun at the following wavelengths: [250 nm, 500 nm,

- 750 nm, 1000 nm, 1500 nm] (“nm” is short for nano-meters or $\times 10^{-9}$ meters). Use list comprehension to build a list of these intensities. Comment on the trend that you notice.
3. The coldest temperature ever measured in our solar system was at 25 K at the bottom of a lunar crater during “night” time. That’s -410° F.. Yowza! However, during “day” time, the temperature of the moon is about 400 K. Compute the intensity of daytime light emitted by the moon for the same wavelengths given in part 2. As before, use list comprehension to put these intensities into a list. How do these intensities compare to the sun’s?
 4. The moon emits strongest in the infrared region of the spectrum ($\lambda > 750$ nm) which our eyes are not able to see. The only reason we can see the moon is because it reflects light from the sun. Recalculate the intensity of light from the moon for the following infrared wavelengths: [6000 nm, 7500 nm, 9000 nm, 10000 nm, 12000 nm]. How do these intensities compare to the sun’s light?

Part II

Intermediate Python

9 The numpy module

[Jupyter Notebook](#)

Numpy (short for “numerical python” and pronounced “num”-“pie”) is a popular Python library that is heavily used in the scientific/mathematical community. So much so that numpy is typically included as part of the standard bundle of libraries that comes with your Python installation. The functions inside numpy will allow you to solve problems with less effort and will produce faster-executing code.

9.1 Numpy Arrays

You are already familiar with Python lists but may not have noticed that they are not suitable for mathematical calculations. For example, attempting to multiply a list by a scalar or evaluating a mathematical function like `sin()` on a list will *not* produce a mathematical result or may produce an error. For example, consider the following code.

```
myList = [4,5,7]

newList = 2 * myList
print(newList)
```

```
[4, 5, 7, 4, 5, 7]
```

You probably expected `newList` to be `[8,10,14]` but multiplying a list by a number doesn’t do that. Instead it repeats the list and concatenates it to itself. To multiply each element of a list by a number you must use a `for` loop.

```
myList = [4,5,7]

newList = []
for i in myList:
    newList.append(i* 2)

print(newList)
```

```
[8, 10, 14]
```

but this seems overly cumbersome for such a simple task. Numpy *ndarrays* (short for n-dimensional arrays) or just *arrays* make this task much simpler. Arrays are similar to lists or nested lists except that mathematical operations and **numpy** functions (but not **math** functions) automatically propagate to each element instead of requiring a **for** loop to iterate over it. Because of their power and convenience, arrays are the default object type for any operation performed with NumPy.

9.2 Array Creation

9.2.1 Type Conversion from List

You can create an array from a list using numpy's **array** function. The list that is to be converted is the argument to the **array** function. Mathematical operations can then be performed on the array and that operation will propagate through to all of the elements.

```
from numpy import array

myArray = array([4,5,7])

newArray = 2 * myArray

print(newArray)
```

```
[ 8 10 14]
```

Nested lists, or lists that contain lists as their elements, can be converted to multi-dimensional arrays using the **array** function.

```
from numpy import array
myArray = array([[1,2,3],[4,5,6]])

print(myArray)
```

```
[[1 2 3]
 [4 5 6]]
```

9.2.2 The arange and linspace Functions

Numpy has some sequence-generating functions that generate arrays by specifying start, stop, and step size values similar to the way `range` generates a list. The two most common ones are `arange` and `linspace`. The `arange` function behaves very similar to the native Python `range` function with a few notable exceptions:

1. `arange` produces an array whereas `range` produces a list.
2. The step size for `arange` does not need to be an integer.
3. `range` produces an iterator and `arange` generates a sequence of values immediately.

The arguments to `arange` are similar to `range`

```
arange(start,stop,step)
```

The `linspace` function is related to the `arange` function except that instead of specifying the step size of the sequence, the sequence is generated based on the number of equally-spaced points in the given span of numbers. Additionally, `arange` excludes the stop value while `linspace` includes it. The difference between these two functions is subtle and the use of one over the other often comes down to user preference or convenience.

```
linspace(start,stop,number of points)
```

Below is an example that shows the usage of `linspace` and `arange`.

```
from numpy import linspace,arange

myArray = linspace(0,10,20)
myArray2 = arange(0,10,0.5)
print(myArray)
print(myArray2)
```

```
[ 0.          0.52631579  1.05263158  1.57894737  2.10526316  2.63157895
 3.15789474  3.68421053  4.21052632  4.73684211  5.26315789  5.78947368
 6.31578947  6.84210526  7.36842105  7.89473684  8.42105263  8.94736842
 9.47368421 10.         ]
[0.  0.5 1.  1.5 2.  2.5 3.  3.5 4.  4.5 5.  5.5 6.  6.5 7.  7.5 8.  8.5
 9.  9.5]
```

When using `linspace` you may still want to know what the sequence spacing is. You can request that `linspace` provide this information by adding the optional argument `retstep = True` to the argument list. With this addition, `linspace` not only returns the sequence to you, but also the stepsize.

```
from numpy import linspace, arange

myArray, mydx = linspace(0, 10, 20, retstep= True)
print(mydx)
print(myArray)
```

```
0.5263157894736842
[ 0.          0.52631579  1.05263158  1.57894737  2.10526316  2.63157895
  3.15789474  3.68421053  4.21052632  4.73684211  5.26315789  5.78947368
  6.31578947  6.84210526  7.36842105  7.89473684  8.42105263  8.94736842
  9.47368421 10.         ]
```

Two other useful functions for generating arrays are `zeros` and `ones` which generate arrays populated with exclusively ones or zeros. The functions require shape arguments as a tuple or list to specify the shape of the array.

```
zeros((rows, columns))
```

If the array to be created is only one dimensional, the argument can be a single number instead of a tuple.

```
zeros(n)
```

```
from numpy import zeros, ones

myArray = zeros([3, 4])
myArray2 = ones(5)
print(myArray)
print(myArray2)
```

```
[[0.  0.  0.  0.]
 [0.  0.  0.  0.]
 [0.  0.  0.  0.]]
[1.  1.  1.  1.  1.]
```

Arrays of any constant (not just one or zero) can then be easily generated by performing the needed math on the original array.

```
from numpy import zeros, ones

myArray = zeros([3,4]) + 5
myArray2 = ones(5) * 12
print(myArray)
print(myArray2)
```

```
[[5.  5.  5.  5.]
 [5.  5.  5.  5.]
 [5.  5.  5.  5.]]
[12. 12. 12. 12. 12.]
```

9.2.3 Arrays from Functions

A third approach is to generate an array from a function using the `fromfunction` function which generates an array of values using the array indices as the inputs. This function requires two arguments: the name of the function being used and the shape of the array being generated.

```
fromfunction(function, shape)
```

Let's make a 3 x 3 array where each element is the product of the row and column indices:

```
from numpy import fromfunction

def prod(x,y):
    return x * y

myArray = fromfunction(prod,(3,3))
print(myArray)
```

```
[[0.  0.  0.]
 [0.  1.  2.]
 [0.  2.  4.]]
```

The table below gives a summary of useful functions for creating numpy arrays. The required arguments are also described.

Table 9.1: Common functions for generating arrays

Method	Description
<code>linspace(start,stop,n)</code>	Returns an array of <code>n</code> evenly-spaced points beginning at <code>start</code> and ending at <code>stop</code> .
<code>arange(start,stop,dx)</code>	Returns an array beginning at <code>start</code> , ending at <code>stop</code> with a step size of <code>dx</code> .
<code>empty(dims)</code>	Returns an empty array with dimensions <code>dim</code> .
<code>zeros(dims)</code>	Returns an array of zeros with dimensions <code>dim</code> .
<code>ones(dims)</code>	Returns an array of ones with dimensions <code>dim</code> .
<code>zeros_like(arr)</code>	Returns an array of zeros with dimensions that match the dimensions of <code>arr</code> .
<code>fromfunction(function,dims)</code>	Returns an array of numbers generated by evaluating <code>function</code> on the indices of an array with dimensions <code>dims</code> .
<code>copy(arr)</code>	Creates a copy of array <code>arr</code> .
<code>genfromtext(file)</code>	Reads <code>file</code> and loads the text into an array (file must only contain numbers).

9.3 Accessing and Slicing Arrays

Accessing and slicing arrays can be done in exactly the same way as is done with lists. However, there is some additional functionality for accessing and slicing arrays that do not apply to lists.

9.3.1 One-dimensional Arrays

Elements from a one-dimensional array can be extracted using square brackets (`[]`) just like we have done with lists.

```
from numpy import array

myArray = array([3,4,7,8])
print(myArray[2])
```


9.3.2 Multi-dimensional Arrays

Multi-dimensional array can be indexed in a similar fashion to nested lists, but because we often encounter multi-dimensional arrays there is a shortcut that makes the syntax simpler and more convenient. Let's consider a two-dimensional array as an example. To access the entire second row of the array, provide the row index in square brackets just as with one-dimensional arrays.

```
from numpy import array
myArray = array([[1,2,3],[4,5,6], [7,8,9]])

print(myArray[1])
```

```
[4 5 6]
```

To access the second element *in the second row*, we can add another set of square brackets with the appropriate index inside, just as we did with nested lists. However, for convenience the second set of square brackets can be omitted and the row and column indices can be placed next to each other and separated by a comma.

`array_name[row,column]`

```
from numpy import array
myArray = array([[1,2,3],[4,5,6], [7,8,9]])

print(myArray[1][1]) # This works, but is a bit hard on the eyes
print(myArray[1,1]) # This works also and is easier to look at.
```

```
5
5
```

9.3.3 Accessing Multiple Elements

Multiple elements of an array can be accessed using a list for the index instead of a single number.

```
from numpy import array

myArray = array([1,2,3,4,5,6,7,8,9,10])
```

```
print(myArray[2]) # Extract element 2
print(myArray[ [3,6,9] ]) # Extract elements 3, 6, and 9.
```

```
3
[ 4  7 10]
```

This can even be done with multi-dimensional arrays. If the index is a single list, the corresponding rows will be extract. If the corresponding list of columns is added to the index list, individual elements will be extracted.

```
array_name[[rows]] # Access set of rows
```

```
array_name[[rows], [columns]] # Access set of elements
```

```
from numpy import array
myArray = array([[1,2,3],[4,5,6], [7,8,9]])

print(myArray[[0,1]]) # Extract rows 0 and 1.

print(myArray[[1,2,0],[0,2,2]]) # Extract elements (1,0), (2,2), and (0,2)
```

```
[[1 2 3]
 [4 5 6]]
[4 9 3]
```

9.4 Slicing Arrays

9.4.1 Multi-dimensional Arrays

We've already shown you how to slice a list using the `:` operator. The same can be done with arrays. However, for 2D (and higher) arrays the slicing is more powerful (intuitive). It can be helpful to visualize an array as a matrix, even if it is not being treated that way Mathematically. For example, let's say that you define the following array:

```
from numpy import array
myArray = array([[1,2,3],[4,5,6], [7,8,9]])
```

which can be visualized as the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

To slice out the following 2 x 2 sub matrix:

$$\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix}$$

we could do

```
from numpy import array
myArray = array([[1,2,3],[4,5,6],[7,8,9]])

print(myArray[1:3,1:3])
```

```
[[5 6]
 [8 9]]
```

To include all of the elements in a given dimension, use the `:` alone with no numbers surrounding it.

```
from numpy import array
myArray = array([[1,2,3],[4,5,6],[7,8,9]])

print(myArray[:,1:3]) # Extract all rows with columns 1 and 2
```

```
[[2 3]
 [5 6]
 [8 9]]
```

9.4.2 Boolean Slicing

Boolean operations can be evaluated on arrays to produce corresponding arrays of booleans. The boolean array can then be used to index the original array and extract elements that meet some criteria.

```

from numpy import array

a = array([1,2,3,4,5,6])

boolArray = a > 2

print(boolArray)

print(a[boolArray])

```

```

[False False  True  True  True  True]
[3 4 5 6]

```

This also works on multi-dimensional arrays although the result is always one-dimensional regardless of the shape of the original array.

```

from numpy import array
myArray = array([[1,2,3],[4,5,6],[7,8,9]])

print(myArray[myArray>2]) # Extract elements that are greater than 2.

```

```

[3 4 5 6 7 8 9]

```

9.5 Vectorization and Broadcasting

A major advantage of numpy arrays over lists is that operations *vectorize* across the arrays. This means that mathematical operations propagate through the array instead of requiring a `for` loop. This speeds up the calculation and makes code easier to write and read. Simple mathematical operations like adding, subtracting, etc can be performed on arrays as you would expect and the operation propagates through to all elements.

```

from numpy import array

a = array([1,2,3])
b = array([4,5,6])

c = a + b
d = a**2
e = 2 * b

```

```
f = 2/b
g = a * b

print(c,d,e,f,g)
```

```
[5 7 9] [1 4 9] [ 8 10 12] [0.5          0.4          0.33333333] [ 4 10 18]
```

All of the common mathematical operations that you learned for numbers now apply to arrays. Cool!

9.5.1 Numpy Functions

The numpy library has a massive collection of vectorized mathematical functions and these functions should be used instead of similar functions from other libraries that are not vectorized (like `math`).

```
from numpy import array
from numpy import sqrt as nsqrt
from math import sqrt as mathsqrt

squares = array([1,4,9,16,25])

print(nsqrt(squares))
#print(mathsqrt(squares)) #This will fail because it wasn't a numpy function.
```

```
[1.  2.  3.  4.  5.]
```

9.5.2 Arrays of same Dimensions

If a mathematical operation is performed between two arrays of the same dimensions, the mathematical operation is performed between corresponding elements in the two arrays. For example, if two 2 x 2 arrays are added together, element (0,0) of the first array gets added to the corresponding element in the second and so forth for all elements:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

```

from numpy import array

a = array([[1,2],[3,4]])
b = array([[5,6],[7,8]])

c = a + b # Add the elements of the arrays together.
d = a * b # Multiply the elements of the arrays together.

print(c)
print(d)

```

```

[[ 6  8]
 [10 12]]
[[ 5 12]
 [21 32]]

```

9.5.3 Arrays of Different Dimensions

When a mathematical operation between two arrays of different dimensions is attempted, Python has to figure out how to make them have the same shape before performing the operation. *Broadcasting* refers to the set of rules used for operations like this. To handle arrays with different dimensions, NumPy pads or clones the array with fewer dimensions to make it have the same dimensions as the larger array. For example, what would happen if you attempted this operation:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 2 \end{pmatrix}$$

One array is 2 x 2 and the other is 1 x 2. Before the addition can take place, NumPy clones the smaller array and repeats it until it has the same size as the bigger array.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

```

from numpy import array

a = array([[1,2],[3,4]])
b = array([2,2])
c = a + b

```

```
print(c)
```

```
[[3 4]
 [5 6]]
```

There are some cases where NumPy simply cannot figure out how to broadcast one of the arrays appropriately and an error results. When broadcasting, NumPy must verify that all dimensions are compatible with each other. Two dimensions are compatible when i) they are equal or ii) one of the dimensions is 1. For example, if we tried to perform the following mathematical operation, broadcasting would fail because the first dimension of the first array is 2 and the first dimension of the second array is 3.

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

```
from numpy import array

a = array([[1,2,3],[4,5,6]])
b = array([[1,1,1],[2,2,2],[3,3,3]])
c = a + b
```

but if we attempted

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

the operation would succeed because the first dimension of the first array is 1 and the second dimension of both arrays are 3.

9.5.4 Vectorizing user-defined functions

Standard Python functions are often designed to perform a single calculation rather than iterate over a list to perform many calculations. For example, here is a function to calculate the average acceleration of an object given its final velocity and time of travel.

```
def accel(velocity, time):
    return velocity / time
```

```
print(accel(52.6,5.6))
```

9.392857142857144

Now what if I have a list of many times that I'd like to feed into this function and get an acceleration value for each one.

```
def accel(velocity, time):  
    return velocity / time
```

```
times = [4.6,7.9,3.2,8.5,9.2,4.7]  
print(accel(52.6,times)) # Produces an error because you can't divide by a list
```

An error results here because Python does not know how to divide by a list. We can NumPy-ify this function using a function called **vectorize**. The resulting function will behave just like the other functions from the NumPy library, vectorizing across the list of times.

```
from numpy import vectorize  
def accel(velocity, time):  
    return velocity / time
```

```
times = [4.6,7.9,3.2,8.5,9.2,4.7]  
vaccel = vectorize(accel) # Vectorize the function!  
print(vaccel(52.6,times)) # Succeeds because NumPy knows how to vectorize.
```

[11.43478261 6.65822785 16.4375 6.18823529 5.7173913 11.19148936]

Of course, we also could have just converted our times list into an array and used the original function.

```
from numpy import array  
def accel(velocity, time):  
    return velocity / time
```

```
times = array([4.6,7.9,3.2,8.5,9.2,4.7])  
print(accel(52.6,times)) # Succeeds because times is an array not a list.
```

[11.43478261 6.65822785 16.4375 6.18823529 5.7173913 11.19148936]

9.6 Manipulating and Modifying Arrays

A wealth of functions exist to perform routine manipulation tasks on arrays once they are created. Often these tasks will involve changing the number of rows or columns or merging two arrays into one. The *size* and *shape* of an array are the number of elements and dimensions, respectively. These can be determined using the `shape` and `size` methods.

```
from numpy import array

a = array([[1,2,3],[4,5,6]])
print(a.size)
print(a.shape)
```

```
6
(2, 3)
```

9.6.1 Reshaping Arrays

The dimensions of an array can be modified using the `reshape` function. This method maintains the number of elements and the order of elements but repacks them into a different number of rows and columns. Because the number of elements is maintained, the size of the new array has to be the same as the original. Let's see an example.

```
from numpy import array, reshape

a = array([[1,2,3],[4,5,6]])

b = reshape(a,[3,2])

print(a)
print(b)
```

```
[[1 2 3]
 [4 5 6]]
[[1 2]
 [3 4]
 [5 6]]
```

The original array (`a`) was a 2 x 3 and had 6 elements and the reshaped array also has 6 elements but is a 3 x 2. You can start with a one-dimensional array and reshape it to a higher dimensional array.

```

from numpy import linspace, reshape

a = linspace(0,10,12)

b = reshape(a,[3,4])

print(a)
print(b)

```

```

[ 0.          0.90909091  1.81818182  2.72727273  3.63636364  4.54545455
 5.45454545  6.36363636  7.27272727  8.18181818  9.09090909 10.         ]
[[ 0.          0.90909091  1.81818182  2.72727273]
 [ 3.63636364  4.54545455  5.45454545  6.36363636]
 [ 7.27272727  8.18181818  9.09090909 10.         ]]

```

9.6.2 Flattening Arrays

Flattening an array takes a higher-dimensional array and squishes it into a one-dimensional array. You can “flatten” an array with the `flatten` method, but note that `flatten` doesn’t actually modify the original array.

```

from numpy import array

a = array([[1,2,3],[4,5,6]])
a.flatten()

print(a) # 'a' remains unchanged

a = a.flatten() # If you want to change the definition of a, redefine it.
print(a)

```

```

[[1 2 3]
 [4 5 6]]
[1 2 3 4 5 6]

```

9.6.3 Transposing Arrays

Transposing an array rotates it across the diagonal and can be accomplished with the `transpose` function. There is also a shortcut method for this of `array.T` to accomplish the

same thing but just as with `flatten` it does not modify the original array. (neither does `transpose`)

```
from numpy import array, transpose

a = array([[1,2,3],[4,5,6]])
transpose(a)

print(a) # 'a' remains unchanged

a = a.T # If you want to change the definition of a, redefine it.
print(a)
```

```
[[1 2 3]
 [4 5 6]]
[[1 4]
 [2 5]
 [3 6]]
```

9.6.4 Merging Arrays

Several function exist for combining multiple arrays into a single array. We'll give examples for a couple and mention the others in reference tables. The most commonly used functions for this task are `vstack` (vertical stacking) and `hstack` (horizontal stacking). `vstack` will stack the original arrays vertically to create the new array and `hstack` will stack them horizontally. Here are some examples.

```
from numpy import linspace, hstack, vstack

a = linspace(0,10,10)
b = linspace(0,5,10)

c = vstack((a,b))
print(c) # 'a' remains unchanged

d = hstack((a,b))
print(a.T)
print(d)
```

```
[[ 0.          1.11111111  2.22222222  3.33333333  4.44444444  5.55555556
```

```

    6.66666667  7.77777778  8.88888889 10.          ]
[ 0.          0.55555556  1.11111111  1.66666667  2.22222222  2.77777778
 3.33333333  3.88888889  4.44444444  5.          ]]
[ 0.          1.11111111  2.22222222  3.33333333  4.44444444  5.55555556
 6.66666667  7.77777778  8.88888889 10.          ]
[ 0.          1.11111111  2.22222222  3.33333333  4.44444444  5.55555556
 6.66666667  7.77777778  8.88888889 10.          0.          0.55555556
 1.11111111  1.66666667  2.22222222  2.77777778  3.33333333  3.88888889
 4.44444444  5.          ]

```

9.7 Commonly-used Array Methods and Functions

NumPy contains an extensive listing of array methods and functions and it would be impractical to list them all here. However, below you will find some tables of some of the commonly used ones that can serve as a reference.

Table 9.2: Array Attribute Methods and Functions

Method	Description
<code>shape(array)</code> or <code>array.shape</code>	Returns the dimensions of the array.
<code>ndim(array)</code> or <code>array.ndim</code>	Returns the number of dimensions (i.e. a 2D array is 2).
<code>size(array)</code> or <code>array.size</code>	Returns the number of elements in an array.

Table 9.3: Array Modification Methods and Functions

Method	Description
<code>array.flatten()</code>	Returns a flattened view of <code>array</code> .
<code>reshape(array,dims)</code> or <code>array.reshape(dims)</code>	Returns a view of <code>array</code> reshaped into an array with dimensions given by <code>dims</code> .
<code>array.resize(dims)</code>	Modifies <code>array</code> to be a resized array with dimensions <code>dims</code> .
<code>transpose(array)</code> or <code>array.T</code>	Returns a view of the transpose of <code>array</code> .
<code>sort(array)</code>	Returns a view of a sorted version of <code>array</code> .
<code>array.sort()</code>	Modifies <code>array</code> to be sorted.
<code>argsort(array)</code>	Returns index values that will sort <code>array</code> .
<code>array.fill(x)</code>	Modifies <code>array</code> so that all elements are equal to <code>x</code> .
<code>vstack(a,b)</code>	Vertically stack arrays <code>a</code> and <code>b</code> to form the new array.
<code>hstack(a,b)</code>	Horizontally stack arrays <code>a</code> and <code>b</code> to form the new array.

Method	Description
<code>vsplit(array,n)</code>	Splits <code>array</code> vertically into <code>n</code> equal parts.
<code>hsplit(array,n)</code>	Splits <code>array</code> horizontally into <code>n</code> equal parts.
<code>append(array,x)</code>	Returns a view of <code>array</code> with <code>x</code> added to the end of the array.
<code>insert(array,n,x)</code>	Returns a view of <code>array</code> with <code>x</code> inserted at location <code>n</code> .
<code>delete(array,n)</code>	Returns a view of <code>array</code> with element at location <code>n</code> removed.
<code>unique(array)</code>	Returns a view of the unique elements of <code>array</code> .

Table 9.4: Array Measurement Methods and Functions

Method	Description
<code>min(array)</code> or <code>array.min()</code>	Returns the minimum value in an array.
<code>max(array)</code> or <code>array.max()</code>	Returns the maximum value in an array.
<code>argmin(array)</code> or <code>array.argmin()</code>	Returns the location of the minimum value in an array.
<code>argmax(array)</code> or <code>array.argmax()</code>	Returns the location of the maximum value in an array.
<code>fmin(array1,array2)</code>	Returns the minimum between two arrays of the same size. (Arrays are compared element wise.)
<code>fmax(array1,array2)</code>	Returns the maximum between two arrays of the same size. (Arrays are compared element wise.)
<code>mean(array)</code> or <code>array.mean()</code>	Returns the mean of <code>array</code> .
<code>median(array)</code> or <code>array.median()</code>	Returns the median of <code>array</code> .
<code>std(array)</code> or <code>array.std()</code>	Returns the standard deviation of <code>array</code> .
<code>cumprod(array)</code> or <code>array.cumprod()</code>	Returns the cumulative product of <code>array</code> .
<code>cumsum(array)</code> or <code>array.cumsum()</code>	Returns the cumulative sum of <code>array</code> .
<code>sum(array)</code> or <code>array.sum()</code>	Returns the sum of all elements in <code>array</code> .
<code>prod(array)</code> or <code>array.prod()</code>	Returns the product of all elements in <code>array</code> .
<code>floor(array)</code>	Returns the floor (i.e., rounds down) of all elements in <code>array</code> .
<code>ceil(array)</code>	Returns the ceiling (i.e., rounds up) of all elements in <code>array</code> .

9.8 Flashcards Part I

1. In what ways are arrays “better” than lists? (better might be too strong.)

2. How do you create an array from a list?
3. Explain the usage of the `linspace` function.
4. Explain the usage of the `arange` function.
5. Explain the usage of the `zeros_like` function.
6. How do you access a single element from a two-dimensional array?
7. How do you access **multiple** array elements at once in a one-dimensional array?
8. How do you access **multiple** array elements at once in a two-dimensional array?
9. How do you slice a two-dimensional array?
10. What is boolean slicing? Given an example.
11. What is broadcasting?
12. What has President Nelson taught us about the meaning of the word Israel?

9.9 Flashcards Part II

1. How can you determine the shape and size of an array?
2. What does the `reshape` function do? Give an example to show the usage.
3. What does the `flatten` function do? Give an example to show the usage.
4. What does the `transpose` function do? Give an example to show the usage.
5. What does the `vstack` function do? Give an example to show the usage.
6. What does the `hstack` function do? Give an example to show the usage.
7. What do the `argmax` and `argmin` functions do?
8. How many temples does the church have in operation currently?

9.10 Exercises Part I

1. [Here](#) you will find a picture of the unit circle.
 1. Use the `arange` function to generate all of the angles (in degrees) on the unit circle **displayed in blue or black text**.
 2. Using a single line of code, convert all of the angle from part 1 into radians.
 3. Use the `linspace` function to generate all of the angles (in degrees) on the unit circle **displayed in red or black text**.
 4. Using a single line of code, convert all of the angle from part 3 into radians.
 5. Using a single line of code, evaluate `sin()` onto both sets of angles and verify that they agree with the values on the unit circle.
 6. Using a single line of code, evaluate `cos()` onto both sets of angles and verify that they agree with the values on the unit circle.
 7. Using a single line of code, evaluate $\frac{\sin(\theta)}{\cos(\theta)}$ for all of the angles in your arrays. This should be equal to $\tan(\theta)$

Python code here

2. In the cell below you will find three arrays containing the masses, lengths, and radii for a collection of cylinders. The moment of inertia for each cylinder can be calculated as

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$

1. Using a single line of code, calculate the moment of inertia for all of the values in the arrays.
2. Determine the largest, smallest, and average of the calculated values.

```
from numpy.random import uniform
mass = uniform(3,8,1000)
radius = uniform(0.5,1.2,1000)
length = uniform(0.8,3,1000)
```

3. Construct a two-dimensional array with the following entries:

$$\begin{bmatrix} 1 & 5 & 7 & 2 \\ 3 & 9 & 1 & 4 \\ 4 & 2 & 2 & 8 \\ 9 & 1 & 6 & 3 \end{bmatrix}$$

1. Now access the number in the third column and second row. (It's a 1)
 2. Slice the array from columns 2 \rightarrow 4 and rows 1 \rightarrow 3.
4. The following temperatures are prominent on the Fahrenheit scale: [0,32,100,212,451]. The are the

1. Create an array that contains these temperatures.
2. Using a single line of code, convert these temperatures into degrees Celsius using the following formula

$$T_C = \frac{5}{9}(T_F - 32)$$

3. Using a single line of code, convert the temperatures from part 2 into Kelvins using the following formula

$$T_K = T_C + 273.15$$

4. Using a single line of code, convert the temperatures from **part 1** into Kelvins using the following formula

$$T_K = \frac{5}{9}(T_F - 32) + 273.15$$

```
# Python code here.
```

5. The equation below defines the relationship between energy (E) in Joules of a photon and its wavelength (λ) in meters. The h is Plank's constant (6.626×10^{-34} J · s) and c is the speed of light in a vacuum (2.998×10^8 m/s).

$$E = \frac{hc}{\lambda}$$

1. Generate an array of wavelength values of visible light (400 nm \rightarrow 800 nm) in 50 nm increments. (“nm” stands for nanometers or $\times 10^{-9}$ m)
2. Generate a second array containing the energy of each wavelength of light from part 1. Does the energy of a photon increase or decrease with wavelength?

```
# Python code here
```

6. A boat is out at sea with the following location and velocity vectors:

$$x = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \text{ km}$$

$$v = \begin{bmatrix} -13 \\ 25 \end{bmatrix} \text{ m/s}$$

when a gust of wind causes the boat to accelerate for approximately 3 minutes with the following acceleration vector:

$$a = \begin{bmatrix} -8 \\ -5 \end{bmatrix} \text{ m/s}^2$$

We can find the position and velocity vectors of the boat after the wind has died back down with the following equation:

$$\vec{x}_f = \vec{x}_i + \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$$

1. Create arrays containing the initial position, initial velocity, and acceleration of the boat **in SI units**.
2. Using a single line of code, calculate the final position of the boat. (Watch out: You must convert 3 minutes to seconds before performing the calculation.)
3. Using a single line of code, calculate the final velocity of the boat. (Watch out: You must convert 3 minutes to seconds before performing the calculation.)

7. Predict the outcome of the following operations between two arrays. Then test your prediction.

$$\begin{bmatrix} 1 & 8 & 9 \\ 8 & 1 & 9 \\ 1 & 8 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + [1]$$

$$\begin{bmatrix} 1 & 8 & 9 \\ 8 & 1 & 9 \\ 1 & 8 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 & 9 \\ 8 & 1 & 9 \\ 1 & 8 & 1 \end{bmatrix} + [1 \quad 2 \quad 3]$$

Python code here

8. In quantum mechanics you will learn that the allowed energy levels for the harmonic oscillator are given by:

$$E_n = \hbar \sqrt{\frac{k}{m}} \left(n + \frac{1}{2} \right)$$

where $k = 4\pi^2$ and $m = 1$ gram.

1. Generate an array of n values from $1 \rightarrow 10$ (inclusive).
2. Using a single line of code, calculate the first 10 allowed energies for this harmonic oscillator. (Don't forget to convert the mass to kg so everything is in SI units.)
3. Combine these two arrays into a single 11×2 array with the first column containing n values and the second column containing the corresponding energies.

Python code here.

9. Generate an array containing integers $0 \rightarrow 14$ (inclusive)

1. Reshape the array to be a 3×5 array.
2. Transpose the array from part 1 so that it is 5×3 .
3. Make the array from part 2 one-dimensional.

10. The $\cos()$ function can be written as an infinite sum:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

. (As it turns out, any function can be written as a sum like this.)

1. Make an array of integer n values from $0 \rightarrow 100$.
2. Using a single line of code, generate the terms in the sum for $x = \pi$. Note: `scipy.special` has a `factorial` function that will work on arrays.
3. Use `cumsum` to evaluate the sum.
4. Interpret the results and verify that as you include more terms in the sum the approximation to $\cos(x)$ gets better and better.

10 File I/O (Input/Output)

Up to now, we have been working with computer-generated or manually typed data sets. Often in a scientific setting your data will be stored in a file and you will need to read the contents of the file into Python so you can perform an analysis. Most data files are *text* files, but there is a large variety of these that differ mostly in the way the information is formatted. The file extension (i.e., the 3 or 4 letters after the period at the end of a file name) specifies the formatting of the file. For example, a *.csv* file (short for comma-separated values) has commas to separate the information.

10.1 The `os` module

The `os` module allows you to perform standard operations on the files and folders on your computer. When opening files from a python script, the default search path is the directory where the current file is located. If you want to open files located in other directories, you'll need to use the `os` module to navigate there first. The following functions are the most commonly used ones

Function	Description
<code>getcwd()</code>	Short for get current working directory. Returns a string of the current directory.
<code>chdir(path)</code>	Change the current direction to be somewhere else.
<code>listdir()</code>	List all of the files and folders in the current working directory.

In the cell below, we show the usage of these functions.

```
from os import getcwd, chdir, listdir

currentdir = getcwd()
chdir(/path/to/new/directory)
myfiles = listdir()
```

To Do:

Add print statements to the cell above and inspect the result

The `os` module has many, many more functions that do useful things but you will mostly use the functions mentioned above. As you get more experience using your computer at the command line, you will be better equipped to understand the usefulness of the rest of this library.

10.2 Reading Files

10.2.1 Reading Line by Line

The first way to read a file is using a `for` loop to iterate over the file line by line. Admittedly, this is not the most elegant or efficient way to read a file but we present it first because it always works. First, the file is opened using the `open` command. The file should be attached to a variable for later use. Next, the data is read one line at a time using the `readlines()` method. We should use a `for` loop for this. Finally, it is a good idea to close the file when you're finished. Let's see an example for reading in the following file, which will be named `squares.csv`

```
1, 1
2, 4
3, 9
4, 16
5, 25
6, 36
7, 49
```

```
file = open("squares.csv")
for line in file.readlines():
    print(line)

file.close()
```

```
1,1
2,4
3,9
4,16
```

5,25
6,36
7,49
8,64
9,81
10,100

You can see how each line gets read separately and printed off. But this isn't super useful yet because we'd probably like to have the numbers stored in lists for our forthcoming analysis. We can fix this by creating some empty lists and appending the appropriate values as they are read in.

```
numbers = []
squares = []

file = open("squares.csv")
for line in file.readlines():
    numbers.append( int(line.split(',')[0]) )
    squares.append( int(line.split(',')[1]) )

file.close()

print(numbers)
print(squares)
```

```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
[1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

Now the data from the file is saved in Python lists and our analysis can proceed.

One final note: if all you want to do is read your file in as a list of strings (one string for each line), that can be done without a `for` loop using the `readlines` function (rather than `readline`).

```

file = open("squares.csv")
data = file.readlines()
file.close()

print(data)

```

```
['1,1\n', '2,4\n', '3,9\n', '4,16\n', '5,25\n', '6,36\n', '7,49\n', '8,64\n', '9,81\n', '10,100\n']
```

10.2.2 Using NumPy's `genfromtxt` function

If the data file is highly structured (every line looks the same, separator character is consistent across the file, etc) then NumPy's `genfromtxt` function can read the data very efficiently into an array. The `genfromtxt` function requires only one argument (the file name) with another optional argument (`delimiter`) that is typically included to specify the character used to separate the data. Below is an example for using this function to read in the .csv data we have been working with.

```

from numpy import genfromtxt

data = genfromtxt('squares.csv', delimiter = ",")

print(data)

```

```

[[ 1.  1.]
 [ 2.  4.]
 [ 3.  9.]
 [ 4. 16.]
 [ 5. 25.]
 [ 6. 36.]
 [ 7. 49.]
 [ 8. 64.]
 [ 9. 81.]
[10. 100.]]

```

Notice that the data was read into a NumPy array (2D in this case because there were two columns of data), which means that we can easily slice off the individual columns if needed.

```

from numpy import genfromtxt

data = genfromtxt('squares.csv', delimiter = ",")

```

```

numbers = data[:,0]
squares = data[:,1]
print(numbers)
print(squares)

```

```

[ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
[ 1.  4.  9. 16. 25. 36. 49. 64. 81. 100.]

```

Other optional arguments that can be used with the `genfromtxt` function are given in the table below:

Argument	Description
<code>delimiter</code>	The string used to separate value. By default, whitespace acts as the delimiter.
<code>skip_header</code>	The number of lines to skip at the beginning of a file.
<code>skip_footer</code>	The number of lines to skip at the end of a file.
<code>usecols</code>	Specify which columns to read with 0 being the first. For example, <code>usecols = (0,2,5)</code> will read the 1st, 3rd, and 6th columns.
<code>comments</code>	The character used to indicate the start of a comment. Lines beginning with this character will be discarded.

10.3 Writing Files

Writing Python data to file is as simple as is reading a file. Just like when you are reading a file, determining which method to use will be determined by the type of data that you are writing. If your data is strictly numerical information stored in an array, Numpy has a function that will quickly save the data to a file. If your data is rife with inconsistencies, non-numerical data, etc, you'll have to use Python's native `write` function.

10.3.1 Writing Line by Line

Sometimes the data file that you want to write includes some text or other non-numerical data. For example, what if you wanted to write the following data to file:

Planet	Acceleration due to gravity (m/s ²)
Earth	9.8

Planet	Acceleration due to gravity (m/s ²)
Moon	1.6
Mars	3.7
Venus	8.83
Saturn	11.2
Uranus	10.5
Neptune	13.3
Pluto	0.61
Jupiter	24.5
Sun	275

In this case you must open the file you want to write through and write each line of the file one by one. This requires that you use a loop to iterate over the data. An example is given below.

```
planets = ["Earth", "Moon", "Mars", "Venus", "Saturn", "Uranus", "Neptune", "Pluto", "Jupiter", "Sun"]
g=[9.8, 1.6, 3.7, 8.83, 11.2, 10.5, 13.3, 0.61, 24.5, 275]

f= open("planets.txt", "w")

f.write("Planet    g (m/s^2)\n")
f.write("-----\n")

for idx, planet in enumerate(planets):
    f.write(f"{planet:10s}    {g[idx]:5.2f} \n" )

f.close()
```

10.3.2 Using NumPy's savetxt function

If the data is strictly numerical and contains no text, the `savetxt` function is a fast and efficient way to write the data to file. For example, maybe you have a two dimensional array containing various columns of planetary data.

Radius ($\times 10^6$ meters)	Mass ($\times 10^{23}$ kg)	Acceleration due to gravity (m/s ²)
6.37	59.8	9.8
1.74	0.736	1.6
3.38	6.42	3.7
6.07	48.8	8.83
58.2	5680	11.2

Radius ($\times 10^6$ meters)	Mass ($\times 10^{23}$ kg)	Acceleration due to gravity (m/s^2)
23.5	868	10.5
22.7	1030	13.3
1.15	0.131	0.61
69.8	19000	24.5
696	19890000	275

```
from numpy import savetxt

data = [[6.37,59.8,9.8],[1.74,0.736,1.6],[3.38,6.42,3.7],[6.07,48.8,8.83],[58.2,5680,11.2],

savetxt("planetsData.txt",data,fmt = "%5.2e")
```

The `fmt` keyword argument can be added to specify how to format the data. If the data is stored in separate one-dimensional arrays, you can pack them into a single list(tuple) when using the `savetxt` function and it will write each data set to its own line in the file.

```
radius = [6.37,1.74,3.38,6.07,58.2,23.5,22.7,1.15,69.8,696]
mass = [59.8,0.736,6.42,48.8,5680,868,1030,0.131,19000,19890000]
g = [9.8,1.6,3.7,8.83,11.2,10.5,13.3,0.61,24.5,275]

savetxt("planetsDataTwo.txt",(radius,mass,g),fmt = "%5.2e")
```

Other optional arguments that can be used with the `savetxt` function are given in the table below:

Argument	Description
<code>delimiter</code>	String or character to separate columns.
<code>newline</code>	String or character to separate lines.
<code>header</code>	String to be written at the beginning of the file.
<code>footer</code>	String to be written at the end of the file.
<code>comments</code>	String that will be prepended to the header and footer string to mark them as comments.

10.4 Flashcards

10.5 Exercises

11 Basic Plotting

Creating plots is an important task in science and engineering. The old adage “A picture is worth a thousand words!” is wrong.... it’s worth way more than that if you do it right. When making plots of functions on a computer it is important to remember that computers don’t plot functions, rather they plot individual points. Only when you connect those points does the image look like the function you are used to seeing. In this chapter we will use a library called `matplotlib` for plotting. More specifically, we will import the `pyplot` function inside of `matplotlib`. It is customary to use `plt` as an alias for `pyplot`.

```
from matplotlib import pyplot as plt
%matplotlib inline
```

The `%matplotlib inline` statement is a Jupyter notebook command. It tells Jupyter to display any plots generated directly in the notebook instead of in a separate window. If you use `matplotlib` in another environment, you should remove this line and instead place `plt.show()` at the plot commands.

11.1 Plotting Functions of a Single Variable

In order to make a plot, `matplotlib` needs lists of the `x` and `y` coordinates for the points that are going to be plotted. If you want to plot a function, the list of `x` coordinates should be chosen to be a dense array of points spanning the function’s domain and the list of `y` coordinates should be the function values at those points. A good choice for generating the `x`-coordinates is either `linspace` or `arange` from NumPy. Let’s consider an example. The function shown below is called the Lennard-Jones equation and it gives the energy of two atoms interacting as a function of separation distance

$$E = 4\sigma \left[\left(\frac{\epsilon}{r} \right)^{12} - \left(\frac{\epsilon}{r} \right)^6 \right]$$

Let’s plot this function from 0.9 to 4.0.

```
from matplotlib import pyplot as plt
%matplotlib inline
```

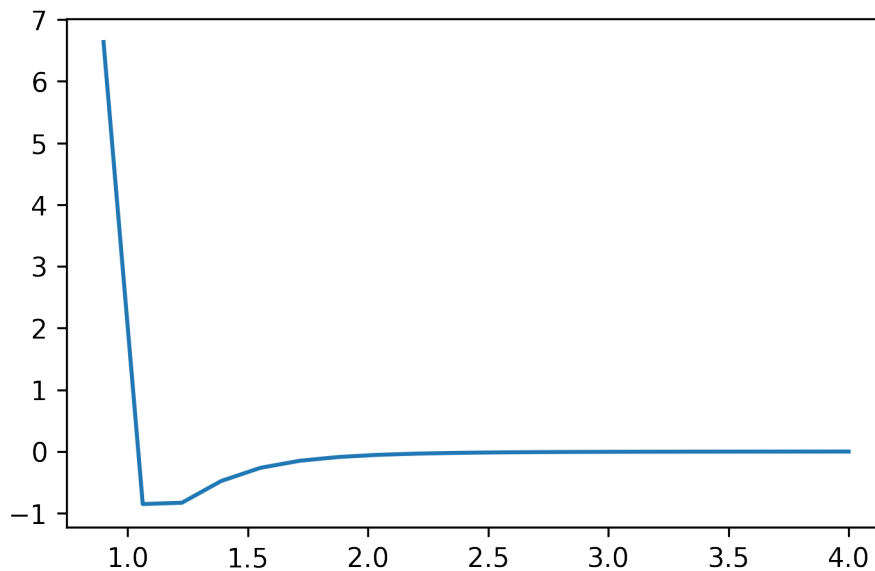
```

from numpy import linspace,sqrt

sigma = 1
epsilon = 1
r = linspace(0.9,4,20)
energy = 4 * sigma* ((epsilon/r)**12 - (epsilon/r)**6)

plt.figure()
plt.plot(r,energy)

```



To Do:

1. You may have noticed that the plot doesn't appear very smooth. Think about what modifications you might make so that the plot is smoother.
2. Run the code to verify that you did it correct.

11.1.1 Linestyles, Markers, and Colors

Three optional arguments can help you control the look of the plot: **marker**, **linestyle**, and **color**. All of these arguments take strings. The **linestyle** argument determines if the line is solid or dashed and what type of dashing to use. The **marker** argument specifies the shape of the plot marker to be used. Below are tables listing possible options for these arguments.

Table 11.1: Common Marker Styles

Argument	Description
<code>o</code>	circle
<code>*</code>	star
<code>p</code>	pentagon
<code>^</code>	triangle
<code>s</code>	square

Table 11.2: Common Line Styles

Argument	Description
<code>-</code>	solid
<code>--</code>	dashed
<code>-.</code>	dash-dot
<code>:</code>	dotted

Table 11.3: Common Colors

Argument	Description
<code>b</code>	blue
<code>r</code>	red
<code>k</code>	black
<code>g</code>	green
<code>m</code>	magenta
<code>c</code>	cyan
<code>y</code>	yellow

Several other keyword arguments exist for helping you customize the look of your plots. They are summarized in the table below.

Table 11.4: A Few Common `plot` keyword arguments

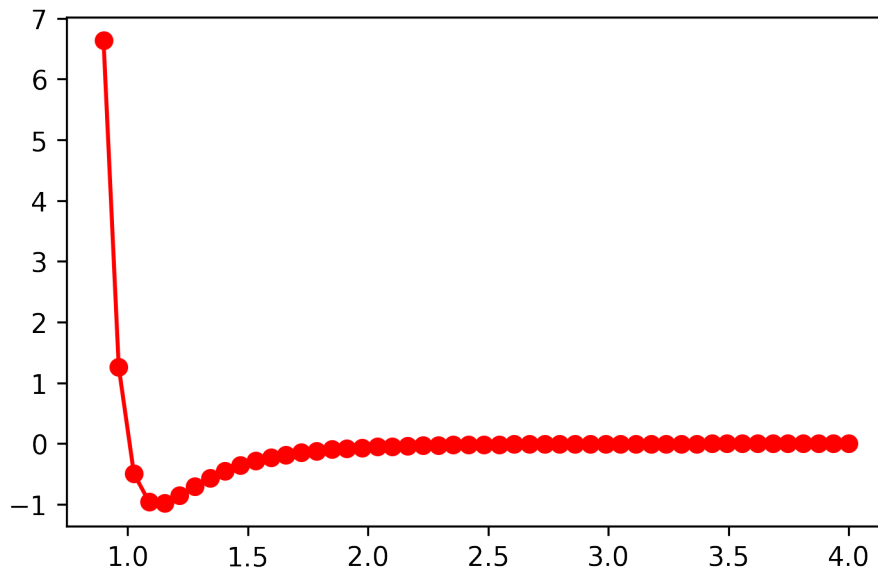
Argument	Description
<code>linestyle</code> or <code>ls</code>	line style
<code>marker</code>	marker shape
<code>linewidth</code> or <code>lw</code>	line width
<code>color</code> or <code>c</code>	line color
<code>markersize</code> or <code>ms</code>	marker size

Here is the plot from above with some of these keyword arguments added.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.9,4,50)
energy = 4 * ((1/r)**12 - (1/r)**6)

plt.figure()
plt.plot(r,energy,linestyle = '-', marker = 'o', color = 'r')
```



11.1.2 Labeling Plots

All good plots have axes labels and a title and you can add them to a matplotlib plot using the `xlabel()`, `ylabel()`, and `title()` functions which are placed on their own line after the plot command.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.9,4,50)
```

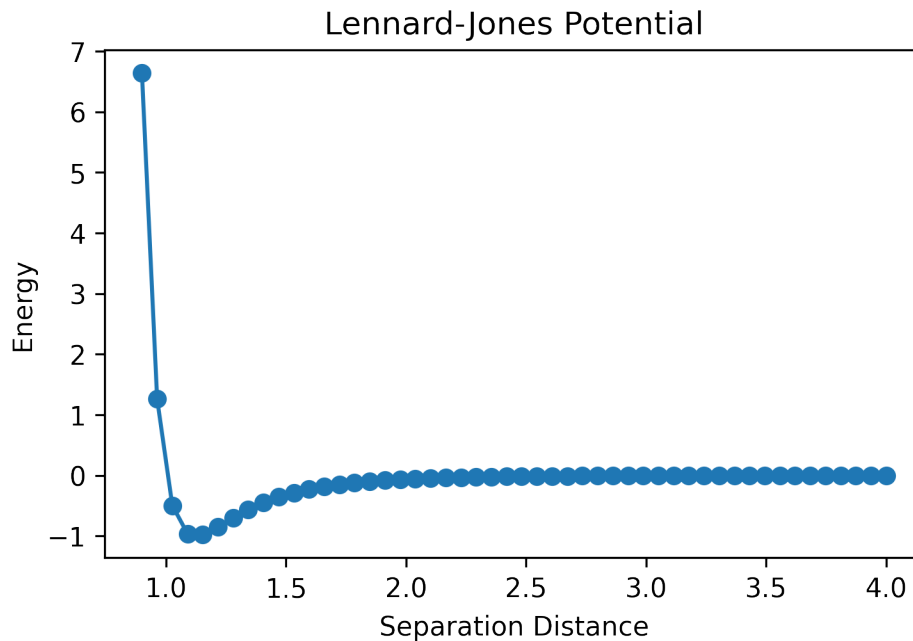
```

energy = 4 * ((1/r)**12 - (1/r)**6)

plt.figure()
plt.plot(r,energy,marker = 'o')
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")

```

```
Text(0.5, 1.0, 'Lennard-Jones Potential')
```



11.1.3 Greek Letters

In physics, Greek letters get used all the time and you may find yourself wanting to use one in a plot title or axes label. This can be accomplished by placing an `r` in front of the title string and then placing the name of the greek variable inside of `$` with a backslash in front of it. This is better illustrated with an example.

```

from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

```

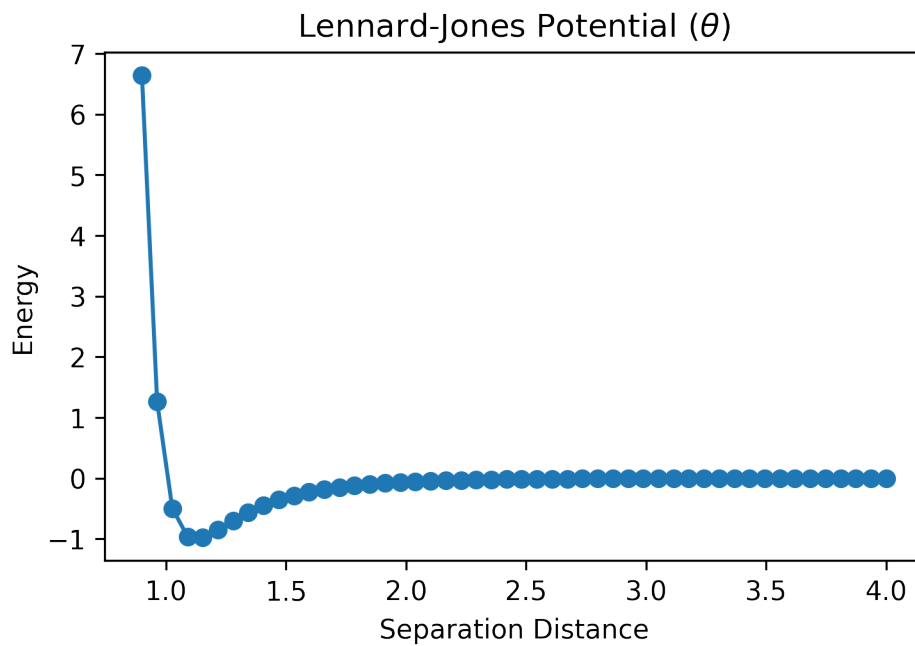
```

r = linspace(0.9,4,50)
energy = 4 * ((1/r)**12 - (1/r)**6)

plt.figure()
plt.plot(r,energy,marker = 'o')
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title(r"Lennard-Jones Potential ( $\theta$ )")

```

```
Text(0.5, 1.0, 'Lennard-Jones Potential ( $\theta$ )')
```



You can subscript any character using the `_` character followed by the subscript. θ_1 can be written as `\theta_1`. If the subscript is more than one character, you'll need to enclose it in curly braces. θ_{12} is written as `\theta_{12}`. Superscripts work the same way only using the `^` character instead of the underscore.

Table 11.5: Lowercase greek letters

Argument	Description
α	<code>\alpha</code>
β	<code>\beta</code>

Argument	Description
γ	<code>\gamma</code>
δ	<code>\delta</code>
ϵ	<code>\epsilon</code>
ϕ	<code>\phi</code>
θ	<code>\theta</code>
κ	<code>\kappa</code>
λ	<code>\lambda</code>
μ	<code>\mu</code>
ν	<code>\nu</code>
π	<code>\pi</code>
ρ	<code>\rho</code>
σ	<code>\sigma</code>
τ	<code>\tau</code>
ξ	<code>\xi</code>
ζ	<code>\zeta</code>

11.1.4 Controlling the Axes

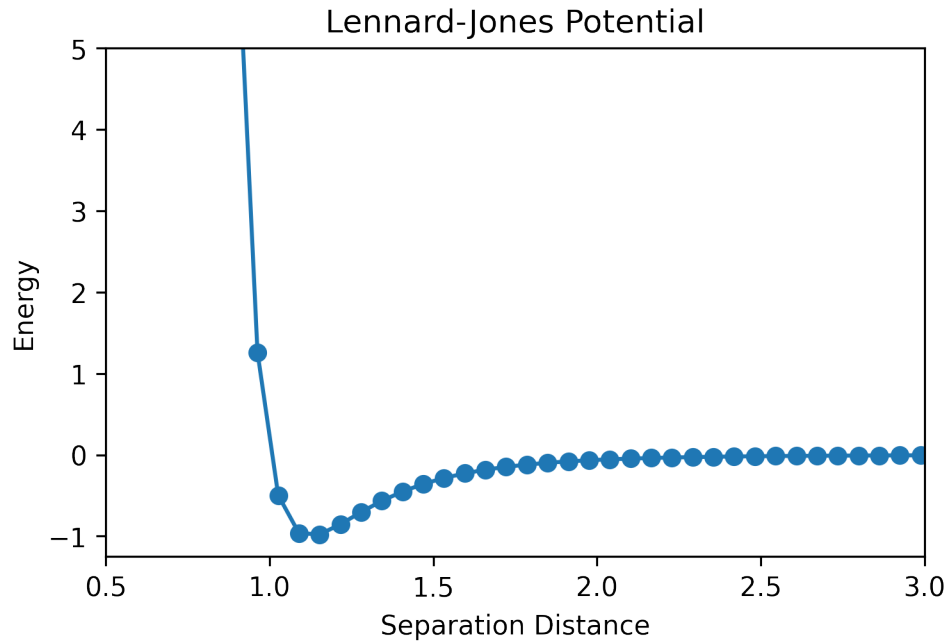
By default, matplotlib will size the plot window to include all of the points. If you want to zoom in or out, you can do so with the `xlim` and `ylim` functions. These functions should be placed after the plot command on their own line just like the label commands.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.5,4,50)
energy = 4 * ((1/r)**12 - (1/r)**6)

plt.figure()
plt.plot(r,energy,marker = 'o')
plt.xlim(0.5,3)
plt.ylim(-1.25,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
```

```
Text(0.5, 1.0, 'Lennard-Jones Potential')
```

11.1.5 Overlaying Plots

Often you will want to plot more than one set of data on the same set of axes. This can be accomplished two ways. The first way is to call the `plot` function twice in the same Jupyter notebook cell. Matplotlib will automatically place the plots on the same figure and scale it appropriately. Below you will find a plot of the Lennard-Jones potential for two choices of the parameters σ and ϵ .

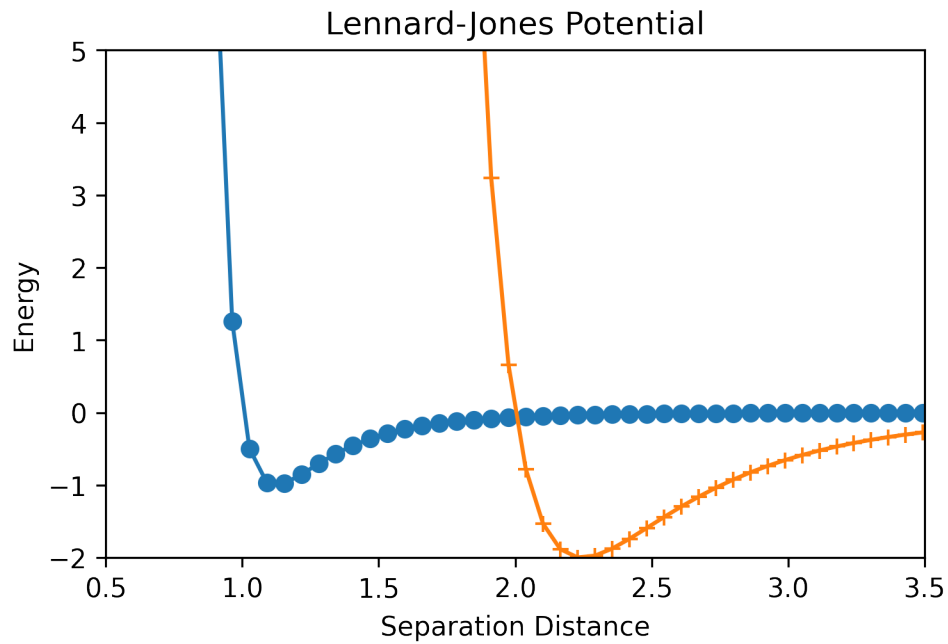
```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.9,4,50)

sigmaOne = 1
epsilonOne = 1
sigmaTwo = 2
epsilonTwo = 2
energyOne = 4 * sigmaOne* ((epsilonOne/r)**12 - (epsilonOne/r)**6)
energyTwo = 4 * sigmaTwo * ((epsilonTwo/r)**12 - (epsilonTwo/r)**6)
```

```
plt.figure()
plt.plot(r,energyOne,marker = 'o')
plt.plot(r,energyTwo,marker = '+')
plt.xlim(0.5,3.5)
plt.ylim(-2,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
```

```
Text(0.5, 1.0, 'Lennard-Jones Potential')
```



The other way to overlay plots is to include both sets of data into a single plot command.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.9,4,50)

sigmaOne = 1
epsilonOne = 1
```

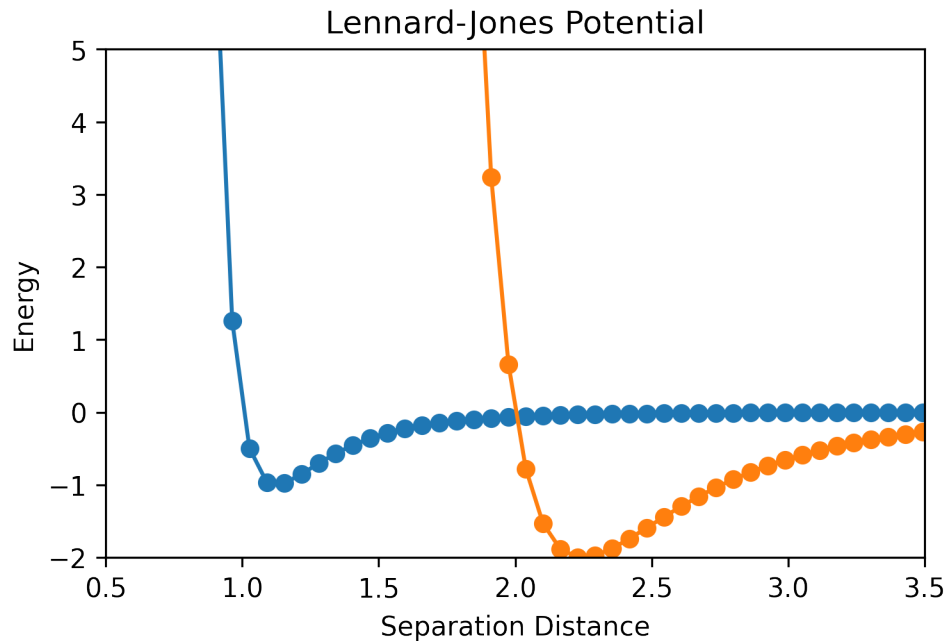
```

sigmaTwo = 2
epsilonTwo = 2
energyOne = 4 * sigmaOne * ((epsilonOne/r)**12 - (epsilonOne/r)**6)
energyTwo = 4 * sigmaTwo * ((epsilonTwo/r)**12 - (epsilonTwo/r)**6)

plt.figure()
plt.plot(r,energyOne,r,energyTwo,marker = 'o')
plt.xlim(0.5,3.5)
plt.ylim(-2,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")

```

```
Text(0.5, 1.0, 'Lennard-Jones Potential')
```



11.2 Other Plot Types

Beyond the line plot that we learned about in the previous section, matplotlib can generate many other types of plots that are very useful in a scientific setting. We'll explore some of them here.

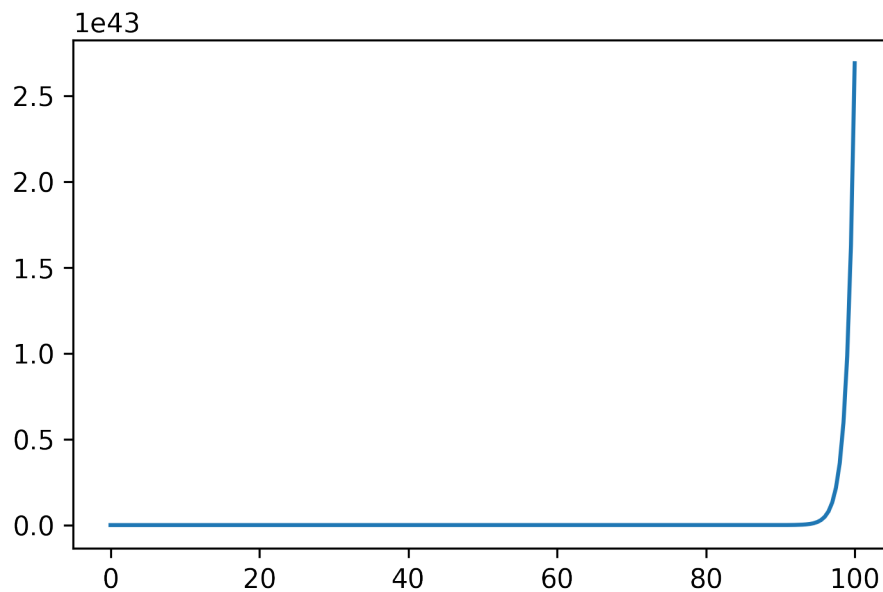
11.2.1 Logarithmic Plots

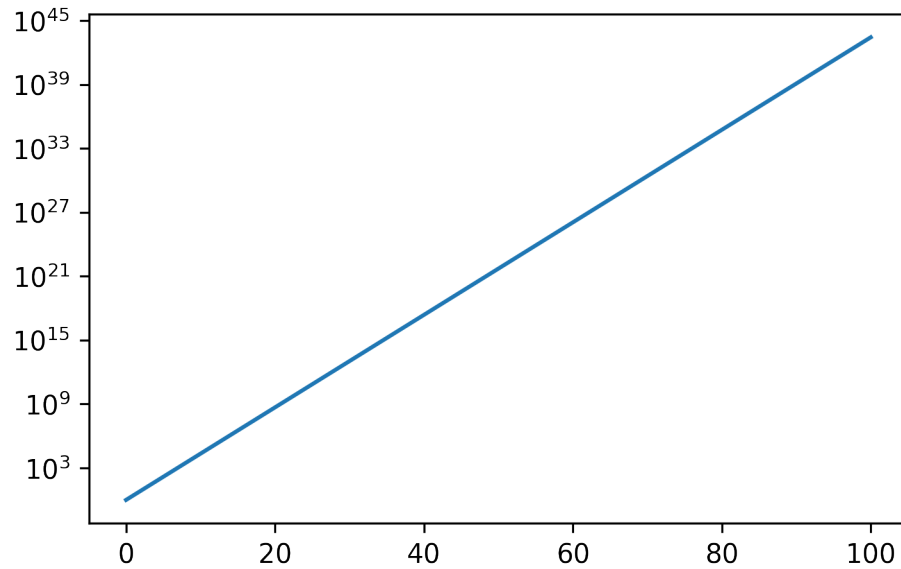
Sometimes the function being plotted increases or decreases by many orders of magnitude and a normal linear plot would not be particularly useful. Logarithmic plots can be made with the use of the `semilogx`, `semilogy`, or `loglog` functions depending on which axes you want to be on a logarithmic scale. Consider the first plot produced below. Notice that it rises from 0 at $x = 0$ to 10^{45} at $x = 100$. Plotting this function with the y-axis scaled logarithmically will smooth out the plot and make it easier to analyze.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,exp

x = linspace(0,100,200)
y = exp(x)

plt.figure()
plt.plot(x,y)
plt.figure()
plt.semilogy(x,y)
```





11.2.2 Bar Plots

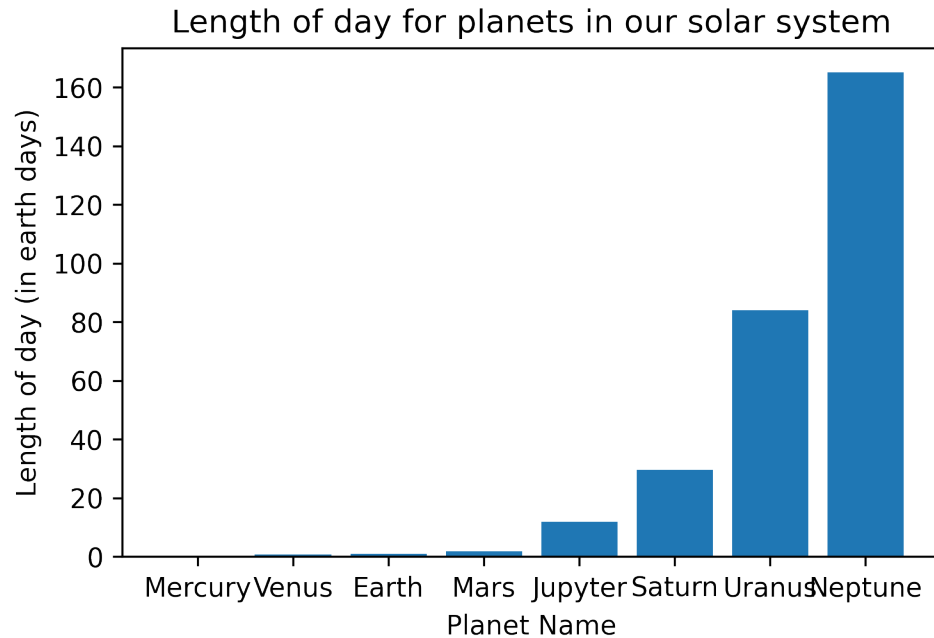
A bar plot and a scatter plot are quite similar except that instead of a plot marker indicating the associated value, the height of the bar represents the value. You can make a bar plot using the `bar` function inside of `pyplot`.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

#d = [5.8e10,1.9e11,1.5e11,2.3e11,7.8e11,1.4e12,2.9e12,4.5e12]
d = [1,2,3,4,5,6,7,8]
T = [0.241,0.615,1,1.88,11.9,29.5,84,165]

plt.figure()
plt.bar(d,T,tick_label = ["Mercury","Venus","Earth","Mars","Jupyter","Saturn","Uranus","Neptune"])
plt.xlabel("Planet Name")
plt.ylabel("Length of day (in earth days)")
plt.title("Length of day for planets in our solar system")
```

```
Text(0.5, 1.0, 'Length of day for planets in our solar system')
```



Notice the optional argument `tick_labels` used to add labels to the bars. If that were left off, the bars would be labeled using the numbers supplied. Other optional arguments that are available for the `bar` command are given below.

Table 11.6: A Few Common `bar` keyword arguments

Argument	Description
<code>width</code>	bar width
<code>color</code>	bar color
<code>xerr</code>	X error bar
<code>yerr</code>	Y error bar
<code>capsize</code>	size of caps on error bars

11.2.3 Errorbar Plots

To make plots with error bars use matplotlib's `errorbar` function. You can choose to add error bars on the x or y axis using the keyword arguments `xerr` and `yerr`.

```
from matplotlib import pyplot as plt
```

```

from numpy import arange

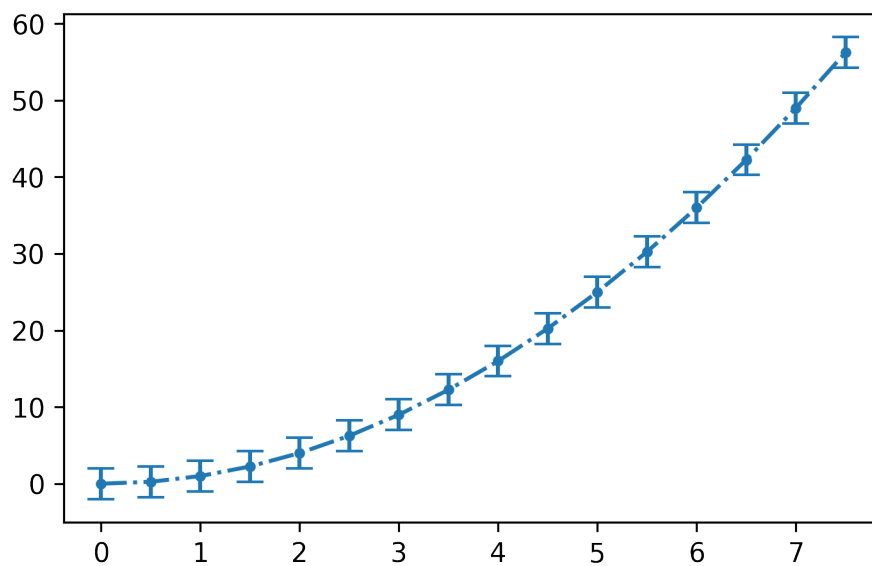
x = arange(0,8,0.5)
y = x**2

x_err = 0.05 # Same error for all points
y_error = 2 # Different error for each point

plt.errorbar(x,y,linestyle = '-.', marker = 'o',markersize = 3,yerr=y_error,capsize = 5)

```

<ErrorbarContainer object of 3 artists>



The `linestyle`, `marker`, and `markersize` arguments work with this plot type also.

11.2.4 Scatter Plots

We have already generated scatter plots using the `plot` function but you can also use the `scatter` command to do the same thing. The only other additional functionality with `scatter` is the ability to specify the color, shape, and size of each plot marker individually.

```

from matplotlib import pyplot as plt
%matplotlib inline

```

```

from numpy import linspace,log

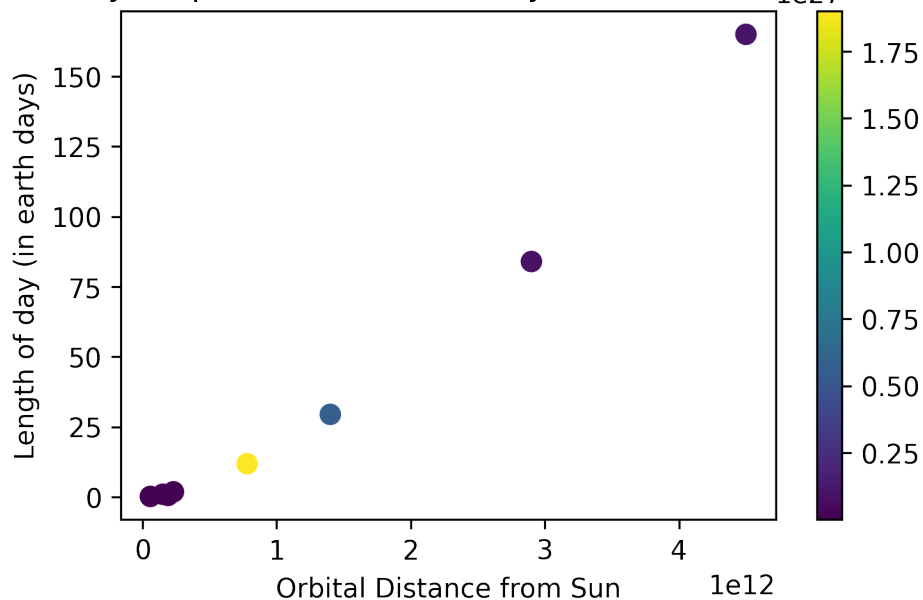
d = [5.8e10,1.9e11,1.5e11,2.3e11,7.8e11,1.4e12,2.9e12,4.5e12]
#d = [1,2,3,4,5,6,7,8]
T = [0.241,0.615,1,1.88,11.9,29.5,84,165]
mass = [3.2e23,4.9e24,6e24,6.4e23,1.9e27,5.7e26,8.7e25,1e26]

plt.figure()
plt.scatter(d,T,c = mass,s= 50)
plt.xlabel("Orbital Distance from Sun")
plt.ylabel("Length of day (in earth days)")
plt.title("Length of day for planets in our solar system (color indicates mass)")
plt.colorbar()

```

<matplotlib.colorbar.Colorbar at 0x11a515be0>

Length of day for planets in our solar system (color indicates mass)



11.2.5 Histograms

Histograms display bars representing the frequency of values in a given data set. Unlike bar plots, the width of the bar is meaningful since each bar represents the number of x-values


```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,log

densities = [0.00009,0.000178,0.00125,0.001251,0.001293,0.001977,0.534,0.810,0.900,0.920,0.934,0.954,0.974,0.984,0.994,1.000]

plt.figure()
plt.hist(densities,bins = 5,edgecolor = 'r')
plt.xlabel("Material Densities")
plt.title(r"Histogram of Material Densities (g/cm$^3$)")
```

A histogram titled "Histogram of Material Densities (g/cm³)" showing the frequency of material densities. The x-axis is labeled "Material Densities" and ranges from 0 to 25 with major ticks every 5 units. The y-axis ranges from 0.0 to 20.0 with major ticks every 2.5 units. The histogram consists of five blue bars with red outlines. The first bar (0-5 g/cm³) has a frequency of 20.0. The second bar (5-10 g/cm³) has a frequency of 5.0. The third bar (10-15 g/cm³) has a frequency of 2.0. The fourth bar (15-20 g/cm³) has a frequency of 1.0. The fifth bar (20-25 g/cm³) has a frequency of 5.0.

Material Density Range (g/cm³)	Frequency
0 - 5	20.0
5 - 10	5.0
10 - 15	2.0
15 - 20	1.0
20 - 25	5.0

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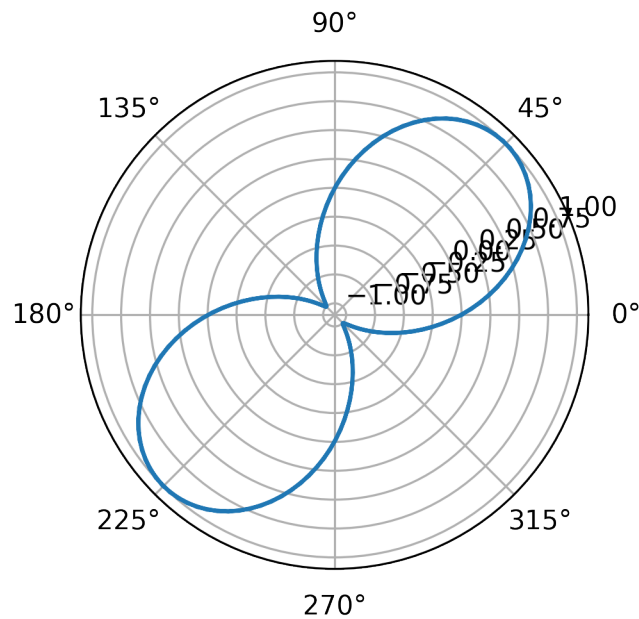

```

from numpy import linspace,pi,sin

theta = linspace(0,4 * pi,150)
r = sin(2* theta)

plt.figure()
plt.polar(theta,r)

```



11.3 Multifigure Plots

To generate multiple, independent plots in the same figure a few more lines of code are necessary to specify how you want the plots arranged. There are two general methods for doing that and we will demonstrate both of them. In the first method, you'll first use the `figure()` command to generate a figure and the `subplot()` function to generate the individual plots in the figure. The `subplot` function takes three arguments.

```
subplot(rows,columns,plot_number)
```

As an example, to generate a figure of two plots side by side.

```

from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

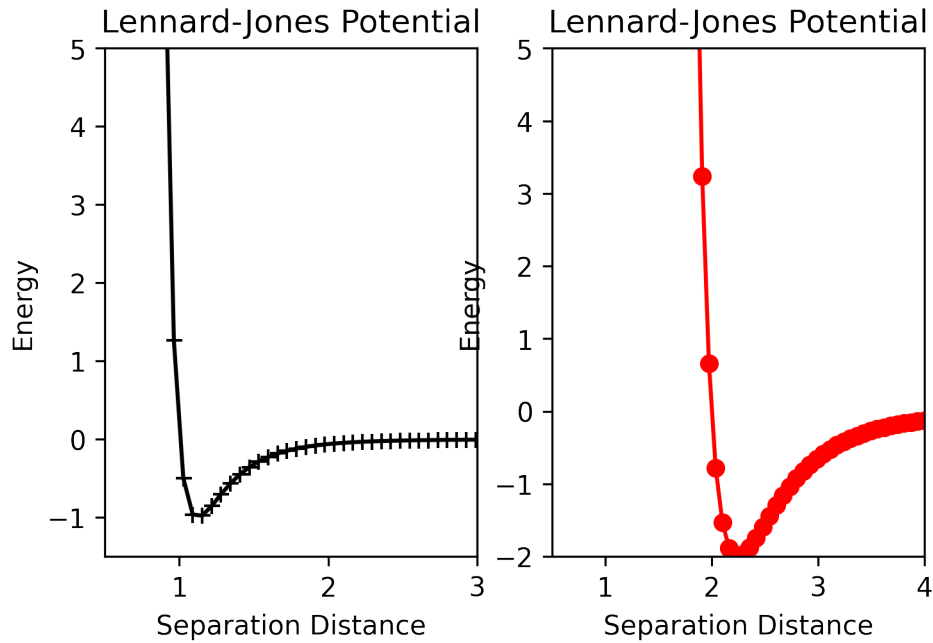
r = linspace(0.9,4,50)

sigmaOne = 1
epsilonOne = 1
sigmaTwo = 2
epsilonTwo = 2
energyOne = 4 * sigmaOne* ((epsilonOne/r)**12 - (epsilonOne/r)**6)
energyTwo = 4 * sigmaTwo * ((epsilonTwo/r)**12 - (epsilonTwo/r)**6)

plt.figure()
plt.subplot(1,2,1)
plt.plot(r,energyOne,marker = '+',color = 'k')
plt.xlim(0.5,3.0)
plt.ylim(-1.5,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
plt.subplot(1,2,2)
plt.plot(r,energyTwo,marker = 'o',color = 'r')
plt.xlim(0.5,4.0)
plt.ylim(-2,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")

```

```
Text(0.5, 1.0, 'Lennard-Jones Potential')
```



Pay close attention to the `subplot` function. `subplot(1,2,1)` will generate a 1 x 2 grid of plots and place the next plot generated at the 1st location. Below is an example of a more advanced array of plots. Pay close attention to the `subplot` functions until you understand.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace,sqrt

r = linspace(0.9,4,50)

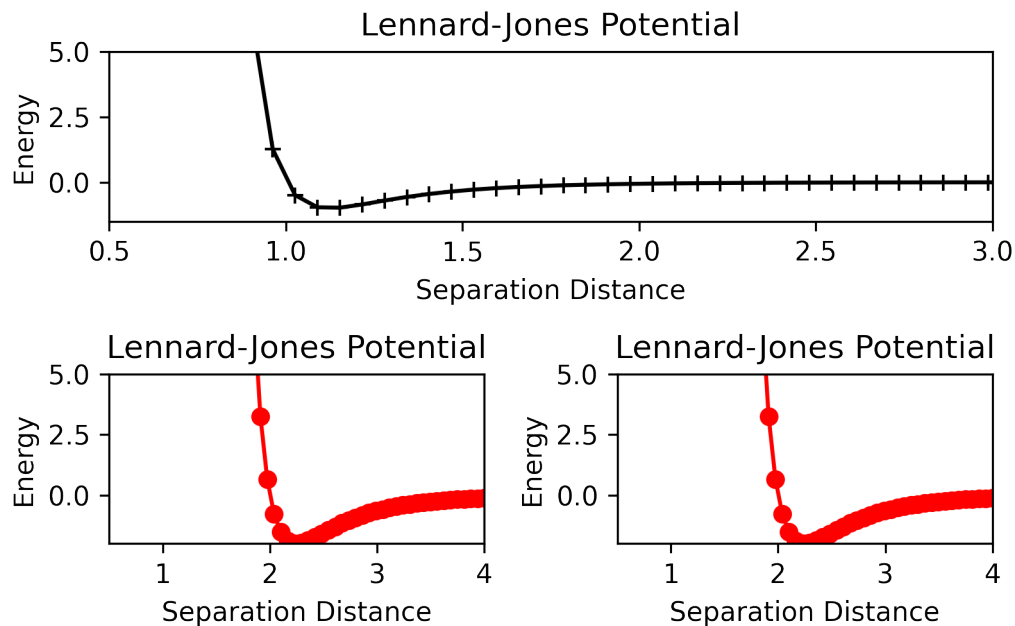
sigmaOne = 1
epsilonOne = 1
sigmaTwo = 2
epsilonTwo = 2
energyOne = 4 * sigmaOne* ((epsilonOne/r)**12 - (epsilonOne/r)**6)
energyTwo = 4 * sigmaTwo * ((epsilonTwo/r)**12 - (epsilonTwo/r)**6)

plt.figure()
plt.subplot(2,1,1)
plt.plot(r,energyOne,marker = '+',color = 'k')
plt.xlim(0.5,3.0)
plt.ylim(-1.5,5)
```

```

plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
plt.subplot(2,2,3)
plt.plot(r,energyTwo,marker = 'o',color = 'r')
plt.xlim(0.5,4.0)
plt.ylim(-2,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
plt.subplot(2,2,4)
plt.plot(r,energyTwo,marker = 'o',color = 'r')
plt.xlim(0.5,4.0)
plt.ylim(-2,5)
plt.xlabel("Separation Distance")
plt.ylabel("Energy")
plt.title("Lennard-Jones Potential")
plt.tight_layout()

```



The other method for generating an array of plots is similar to the first but different functions are used and a little more control over the plots is gained. In this approach, functions to explicitly creates and links subplots, called *axes* are used. Just as before, the **figure** function is

used to generate the overall figure. To create each subplot, the `add_subplot(rows, columns, plot_number)` function is used. The arguments to this function are the same as for `subplot` in the previous method. After the plot object has been created, we can call the `plot` function again, but this time preceded by the axes object that you just created. Here is an example.

```
from matplotlib import pyplot as plt
%matplotlib inline
from numpy import linspace, sqrt

r = linspace(0.9,4,50)

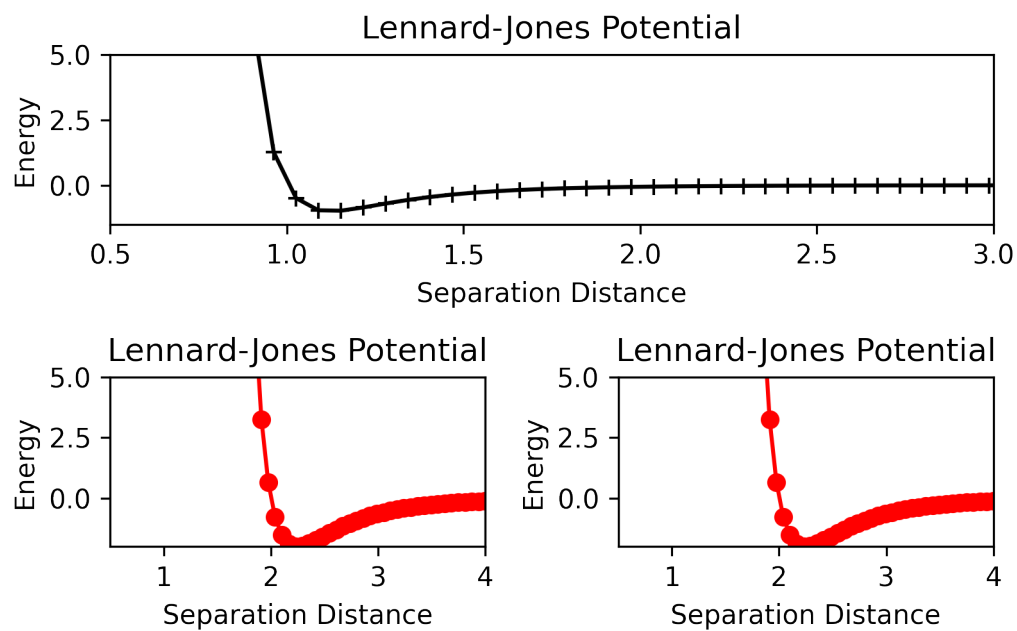
sigmaOne = 1
epsilonOne = 1
sigmaTwo = 2
epsilonTwo = 2
energyOne = 4 * sigmaOne * ((epsilonOne/r)**12 - (epsilonOne/r)**6)
energyTwo = 4 * sigmaTwo * ((epsilonTwo/r)**12 - (epsilonTwo/r)**6)

fig = plt.figure()
ax1 = fig.add_subplot(2,1,1)
ax1.plot(r,energyOne,marker = '+',color = 'k')
ax1.set_xlim(0.5,3.0)
ax1.set_ylim(-1.5,5)
ax1.set_xlabel("Separation Distance")
ax1.set_ylabel("Energy")
ax1.set_title("Lennard-Jones Potential")

ax2= fig.add_subplot(2,2,3)
ax2.plot(r,energyTwo,marker = 'o',color = 'r')
ax2.set_xlim(0.5,4.0)
ax2.set_ylim(-2,5)
ax2.set_xlabel("Separation Distance")
ax2.set_ylabel("Energy")
ax2.set_title("Lennard-Jones Potential")

ax3 = fig.add_subplot(2,2,4)
ax3.plot(r,energyTwo,marker = 'o',color = 'r')
ax3.set_xlim(0.5,4.0)
ax3.set_ylim(-2,5)
ax3.set_xlabel("Separation Distance")
ax3.set_ylabel("Energy")
ax3.set_title("Lennard-Jones Potential")
```

```
plt.tight_layout()
```



Notice the `set_xlim`, `set_ylim`, `set_xlabel` etc methods that were used to customize each individual plot. There are a host of other methods available for further customization. [This website](#) has a comprehensive list of them.

12 Advanced Plotting

13 Pandas

14 The sympy module

Part III

Numerical Methods

15 Extrapolation and Interpolation

16 Function Fitting

17 Numerical Derivatives

18 Numerical Integration

19 Euler's Method