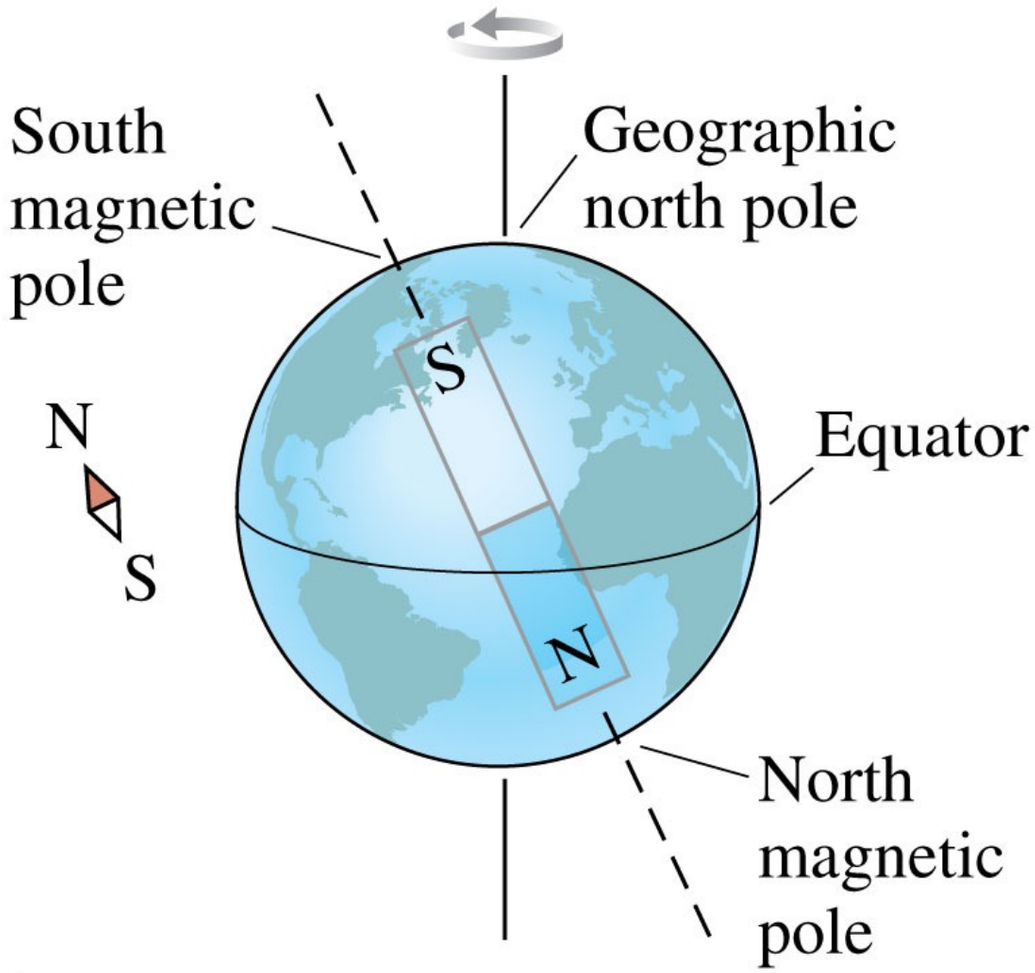
PH 220

Lance Nelson

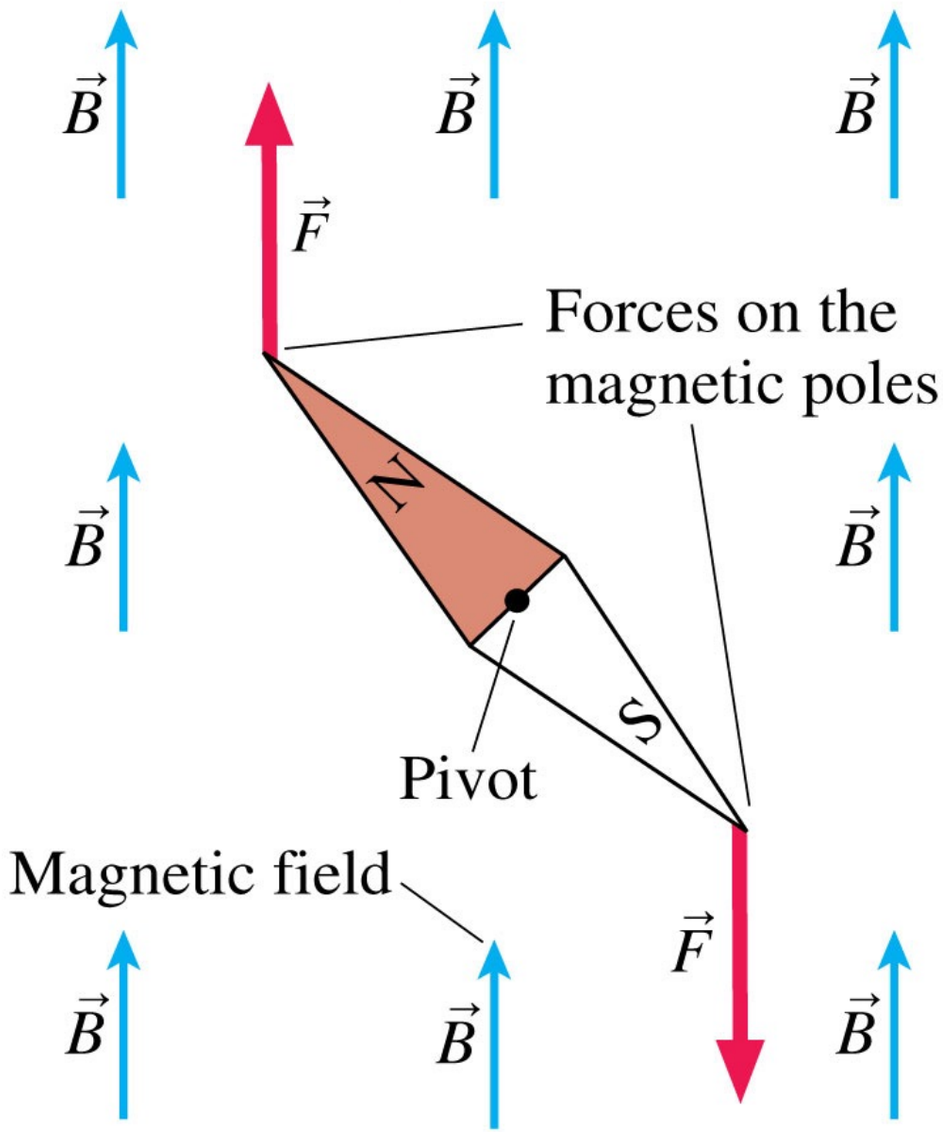
Curious observations about magnets

1. A compass needle rotates when a magnet is brought close.
2. Some materials are attracted to magnets and others are not.
3. If an object is attracted to one end of a magnet, it is also attracted to the other.
4. If I cut a magnet in half, I get two magnets, each with a north and a south pole.
5. A magnet does not have the same effect on an electroscope that the charged rod did.

Geomagnetism



5×10^{-5} T

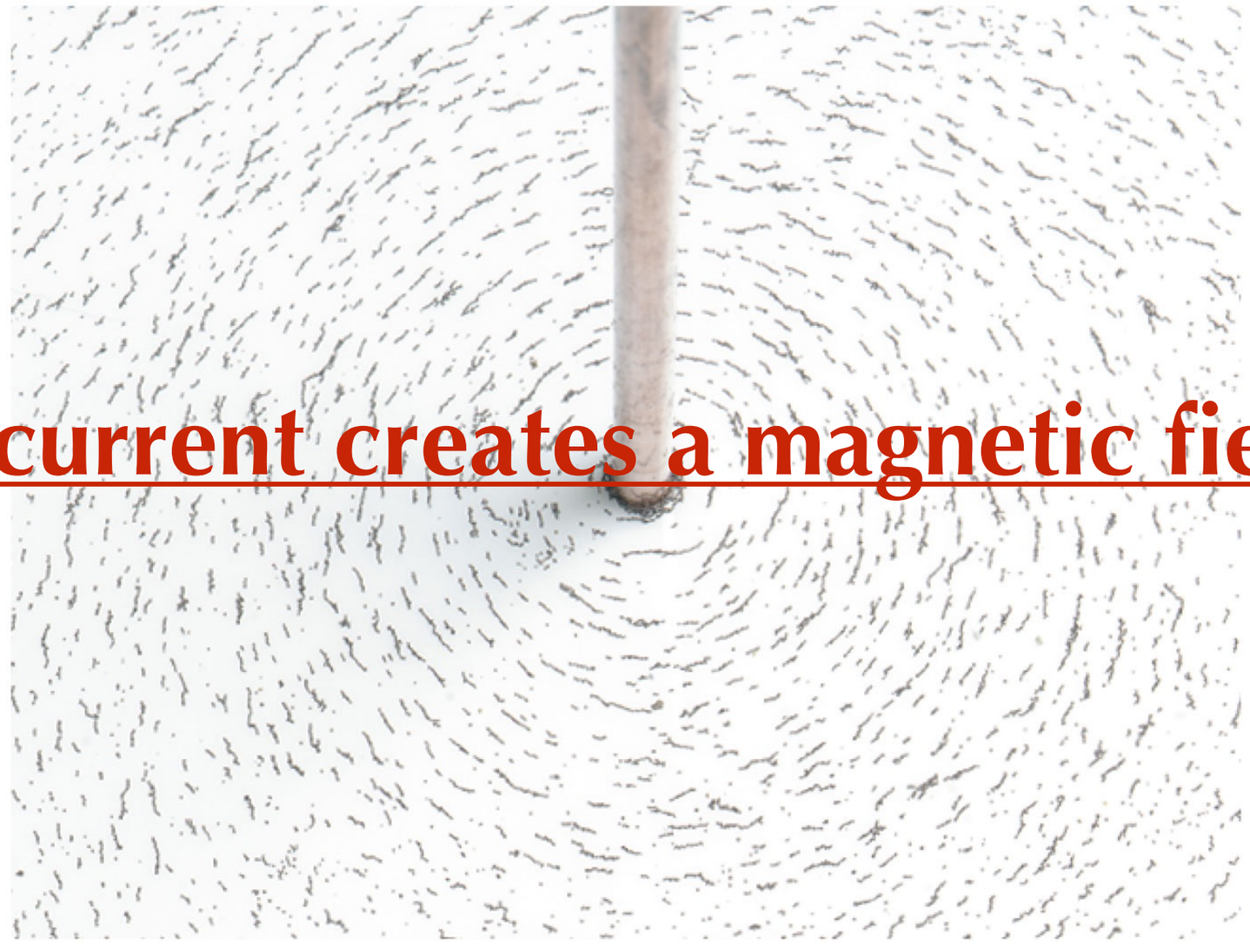

$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{F}$
Forces on the magnetic poles
N
S
Pivot
Magnetic field
$\vec{F}$

What causes magnetism

Current Loop Demo

A current creates a magnetic field

What causes magnetism



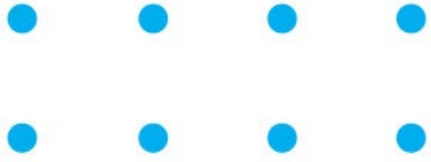
A current creates a magnetic field

A three-dimensional perspective

… but 2D pictures are easier to draw.

Vectors into page

Current into page

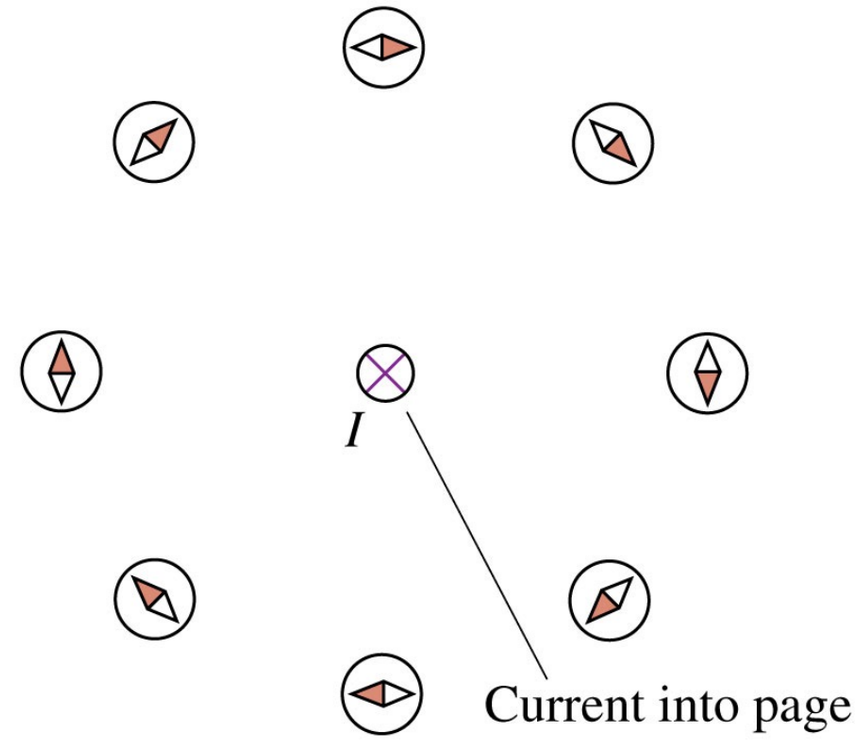


Vectors out of page

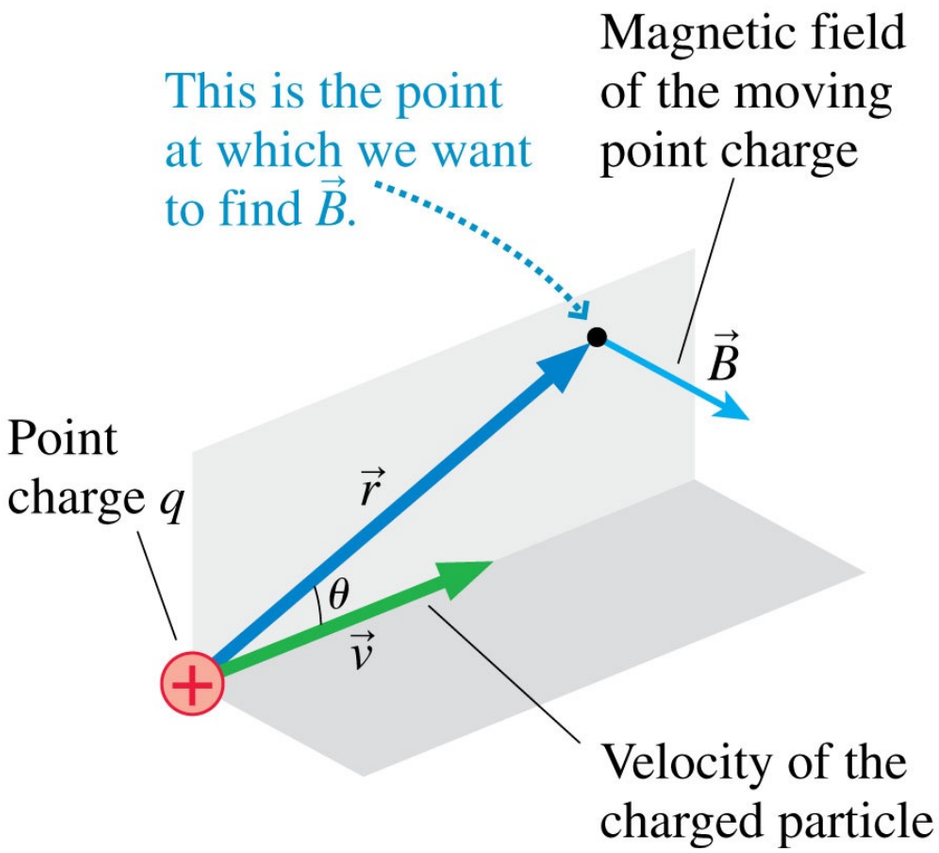
Current out of page

Right-hand rule

Question: If I know the direction of the current, how can I determine the direction of the magnetic field created by that current.



Biot-Savart Law



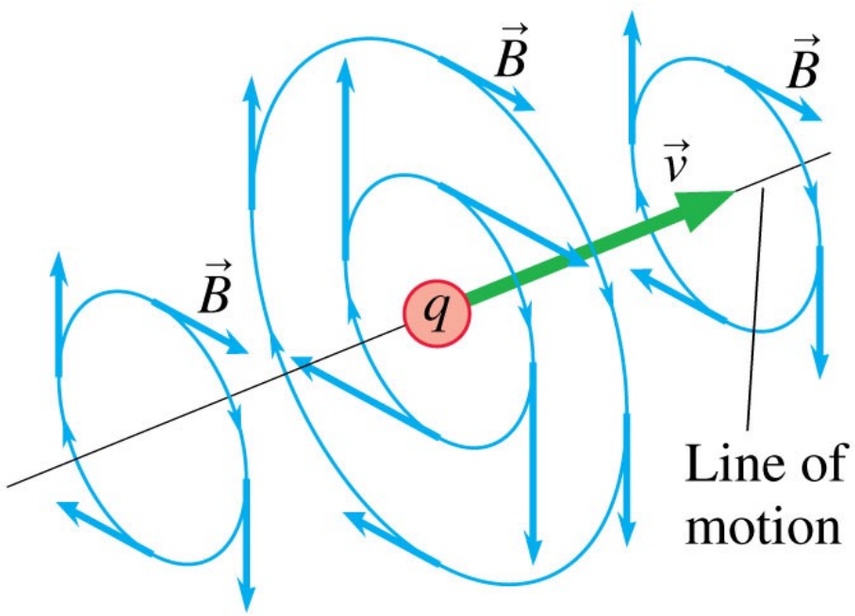
The SI unit of magnetic field strength is the tesla, abbreviated as T:

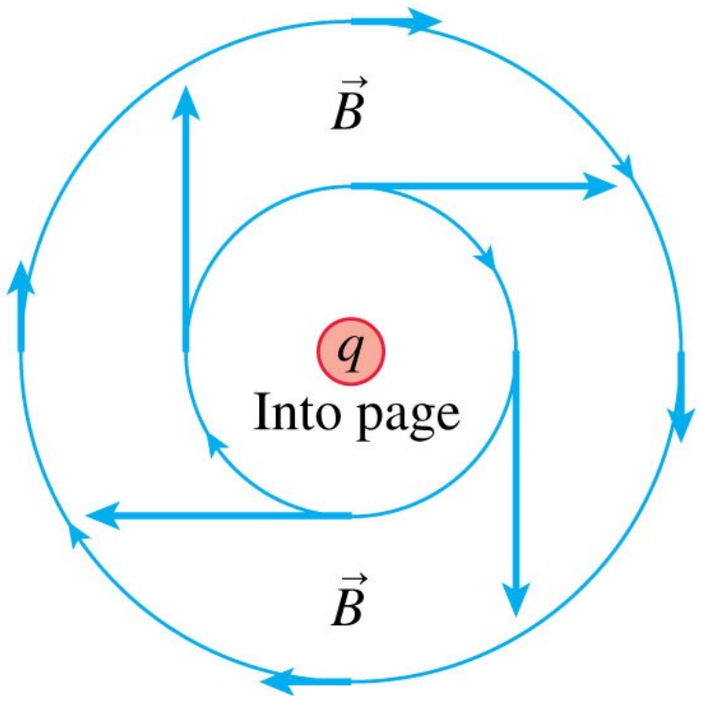
1 tesla = 1 T = 1 N/A m

$$\vec{B}_{\text{point charge}} = \left(\frac{\mu_0}{4\pi} \frac{qv \sin\theta}{r^2}, \text{direction given by the right-hand rule} \right)$$

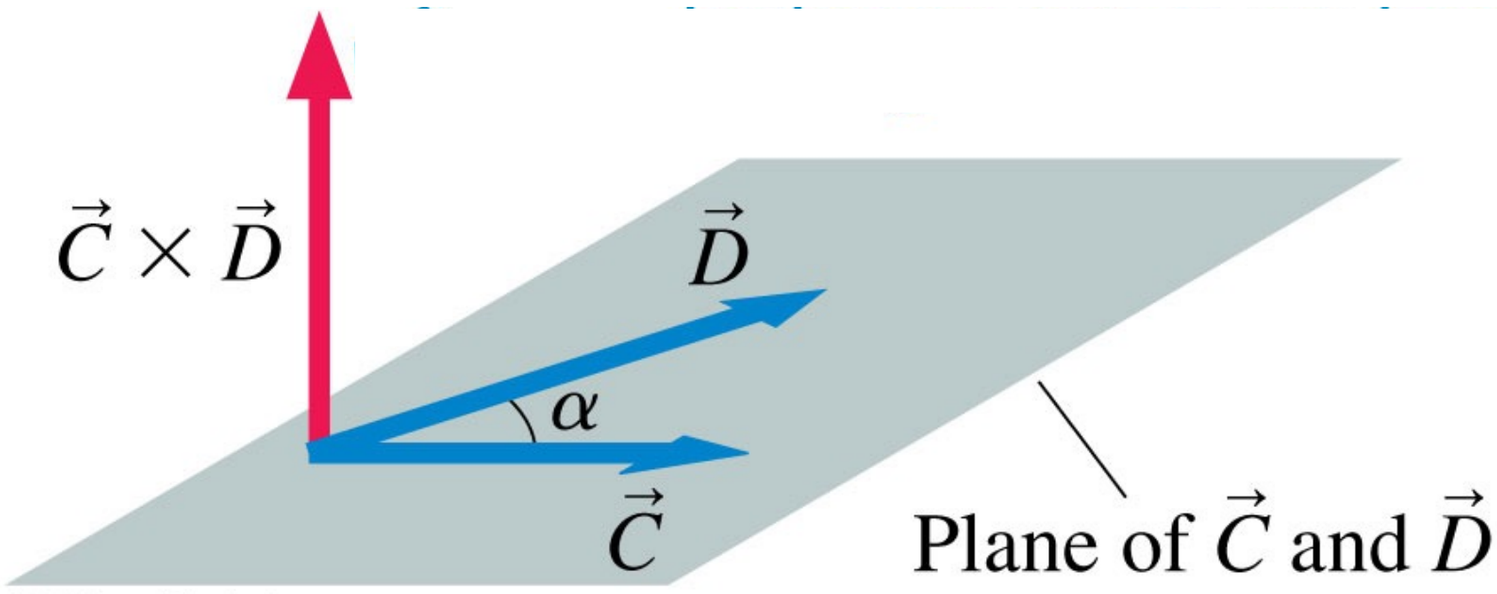
The constant μ_0 in the Biot-Savart law is called the **permeability constant:**

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T m/A} = 1.257 \times 10^{-6}\ \text{T m/A}$$


$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{v}$
q
Line of motion

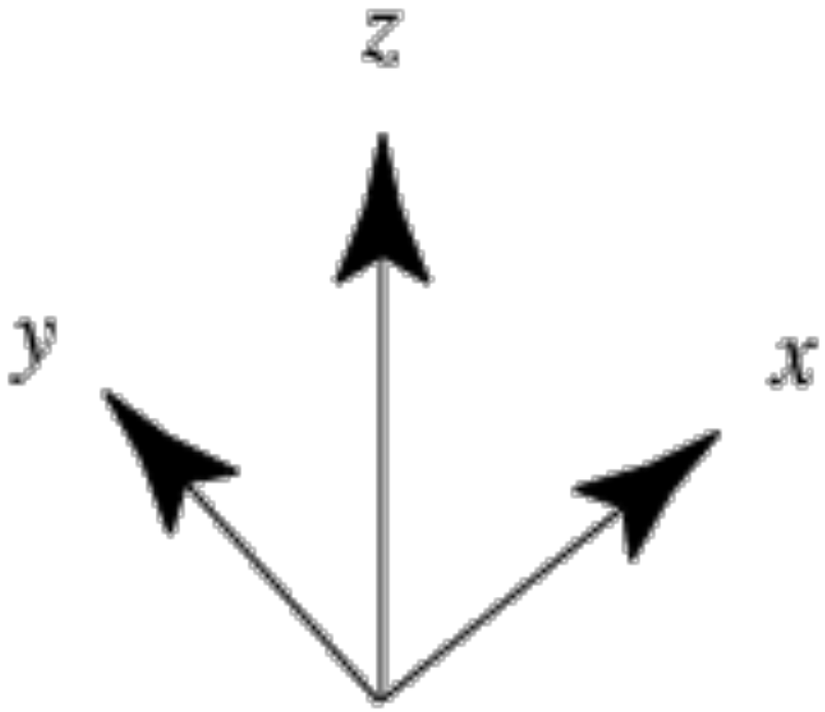

$\vec{B}$
q
Into page
$\vec{B}$

Cross Product



$\vec{C} \times \vec{D}$ = (CD sin α, direction given by the right-hand rule)

Cross Product



$$\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{B} = -3\hat{i} + 1\hat{j} - 6\hat{k}$$

$$\vec{A} \times \vec{B} =$$

A $9\hat{i} + 21\hat{j} - \hat{k}$

B $-15\hat{i} - 2\hat{j} - 18\hat{k}$

C $-9\hat{i} - 21\hat{j}$

D 42

$$\vec{B}_{\text{point charge}} = \left(\frac{\mu_0}{4\pi} \frac{qv \sin\theta}{r^2}, \text{direction given by the right-hand rule} \right)$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$ (Magnetic field of a moving point charge)

Let's try a harder problem

Steps:

a) Write down $\vec{v}$ in component form?

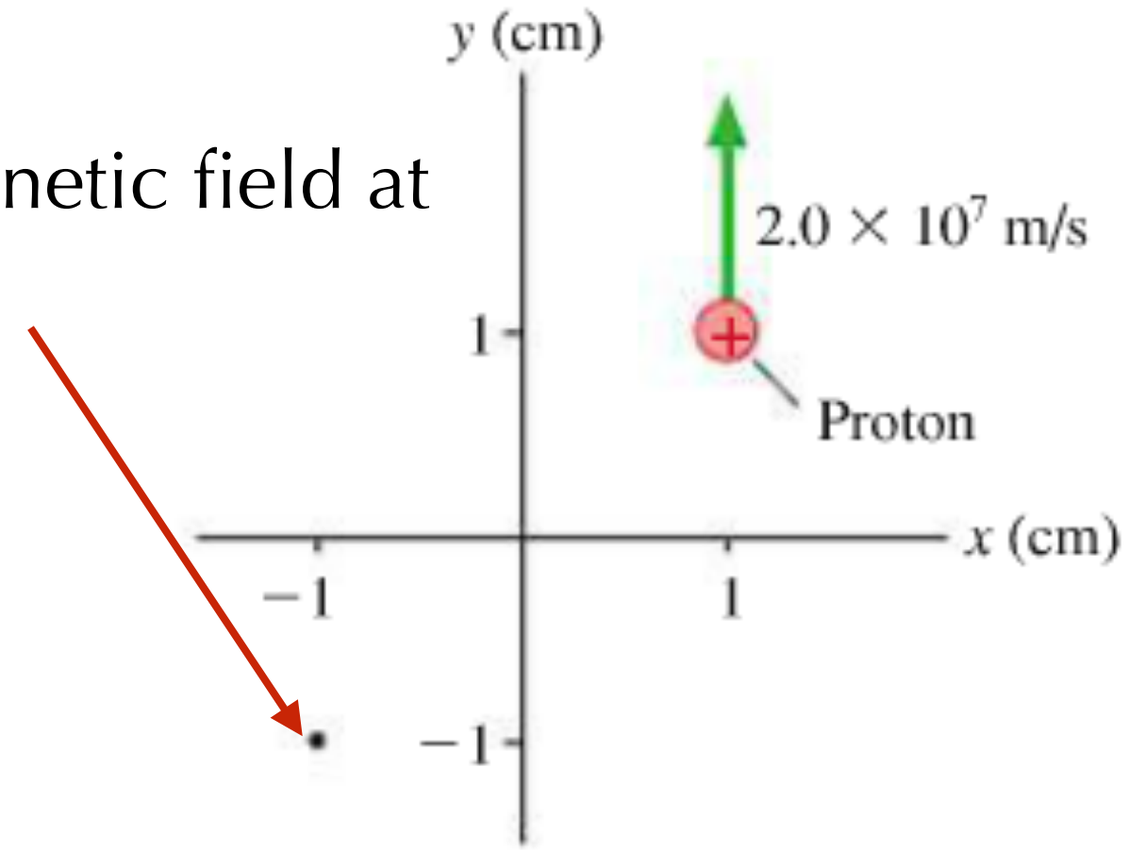
b) Write down $\hat{r}$ in component form?

c) Find the cross product $\vec{v} \times \hat{r}$?

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$\mu_0 = 1.257 \times 10^{-6}$ T m/A

What is the magnetic field at the dot?



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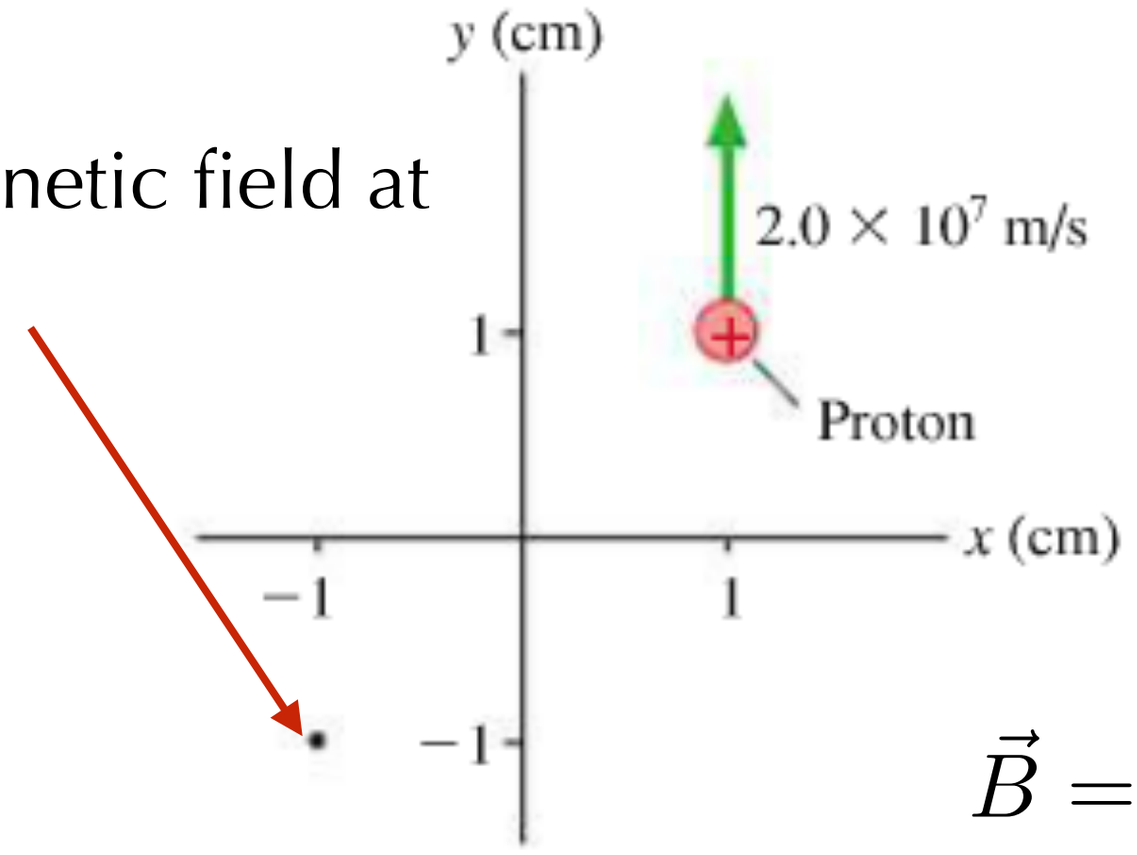

$$\vec{B} = 2.83 \times 10^{-16} \hat{k} \text{ T}$$

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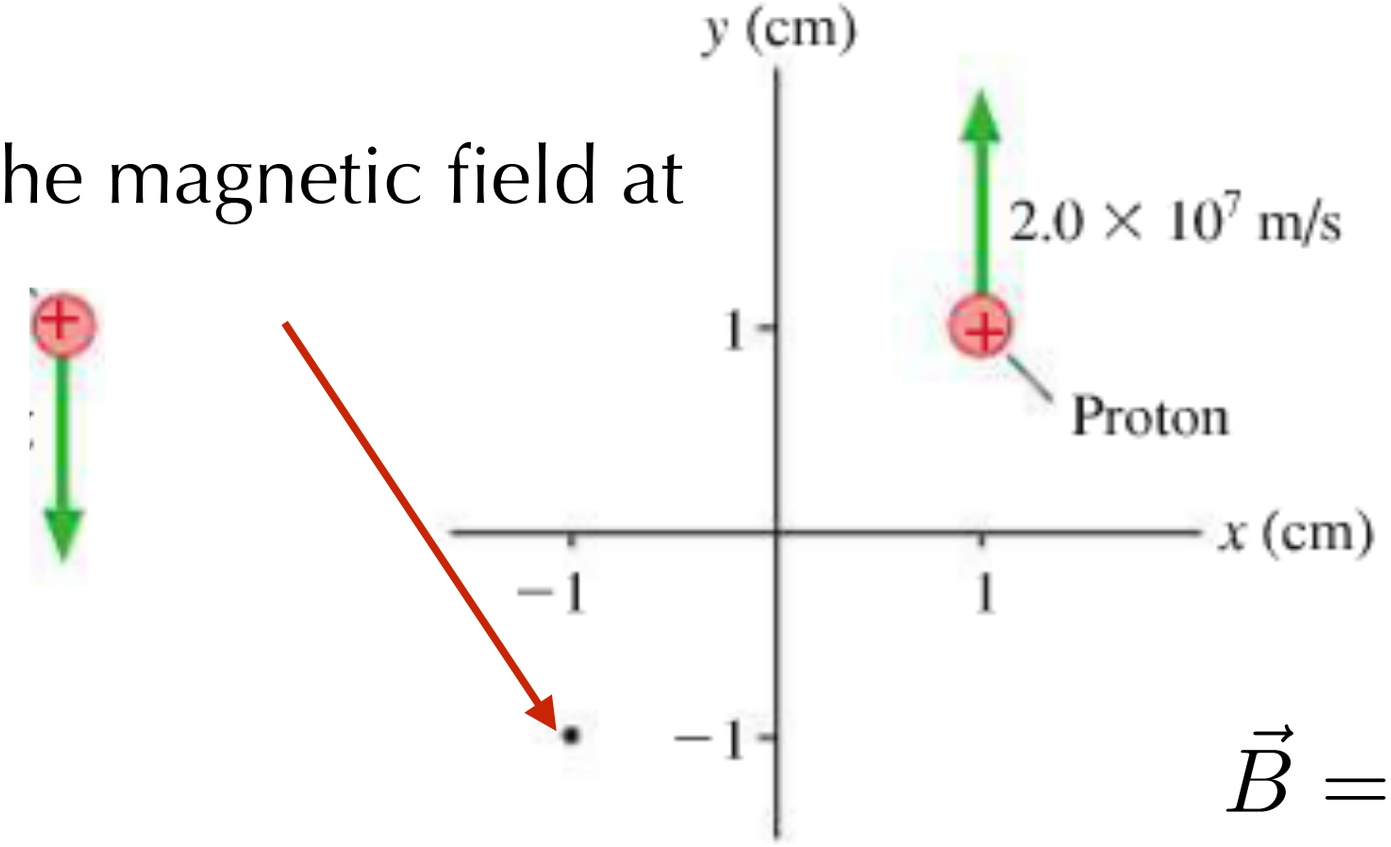
<u>Does adding another moving proton add to or cancel the magnetic field?</u>

$\mu_0 = 1.257 \times 10^{-6}$ T m/A

What is the magnetic field at the dot?

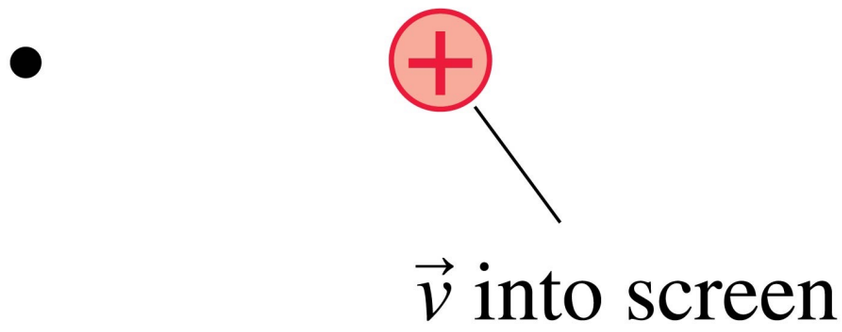


$$\vec{B} = 2.83 \times 10^{-16} \hat{k} \text{ T}$$

Quiz Question

What is the direction of the magnetic field at the position of the dot?

A. Into the screen.
B. Up.
C. Out of the screen.
D. Down.
E. Left.



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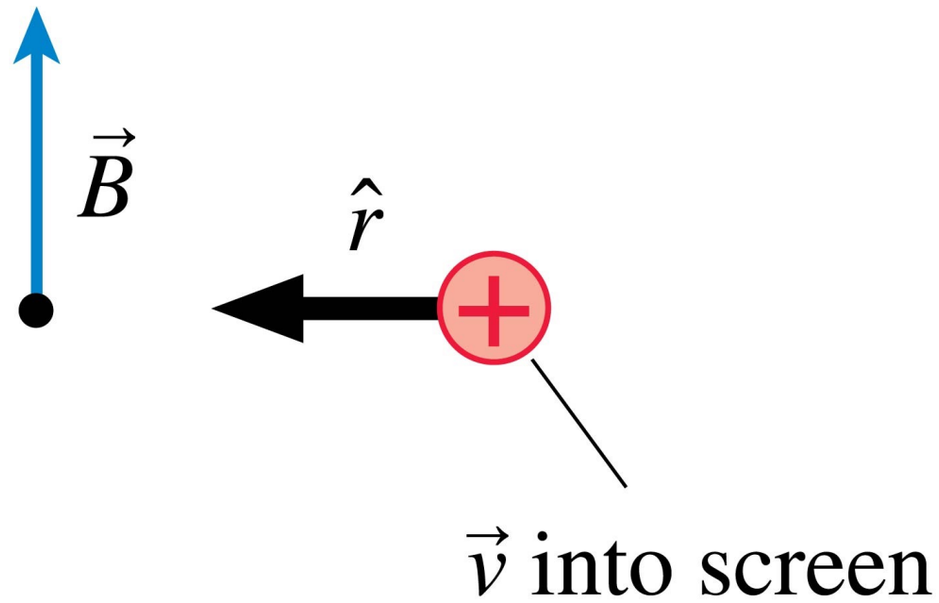
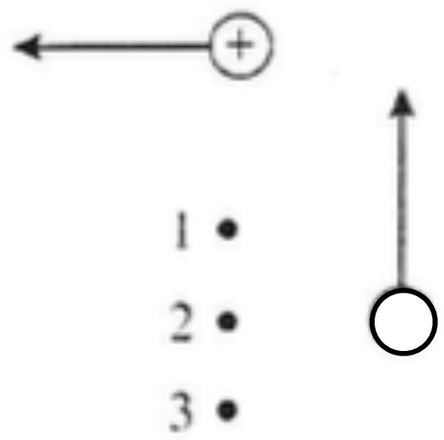


Quiz Question

The magnetic field at point 2 is zero. What is the sign of the charge moving upward?

a) positive
b) negative



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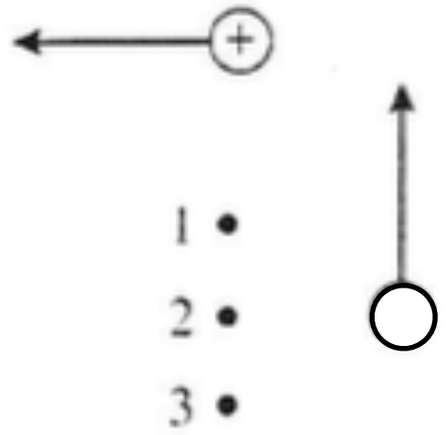
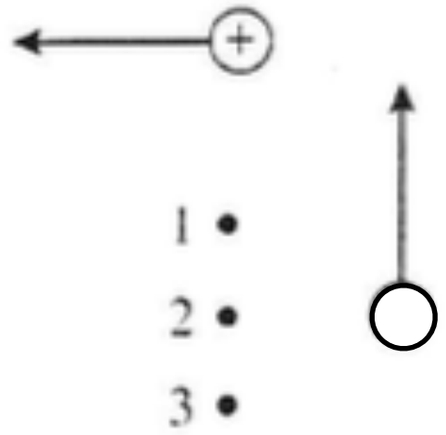
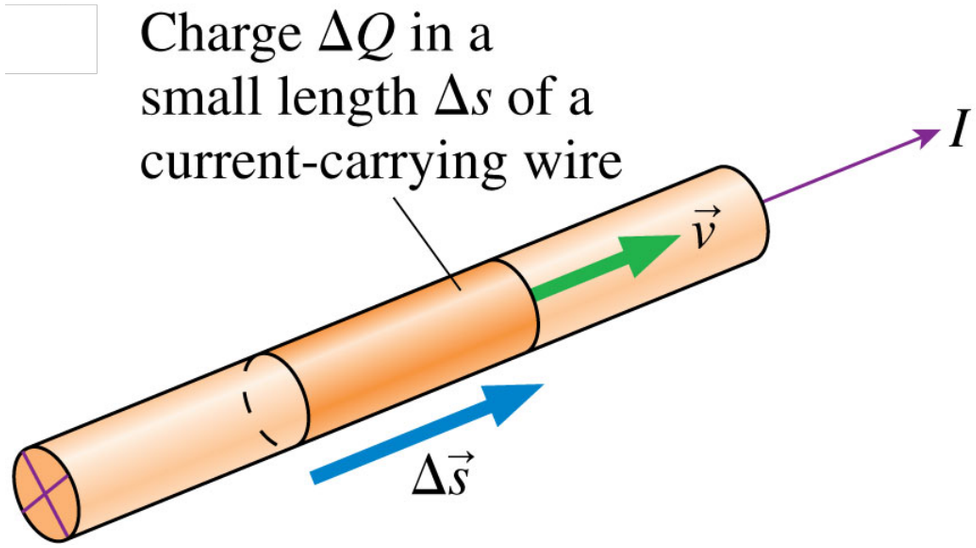


Quiz Question



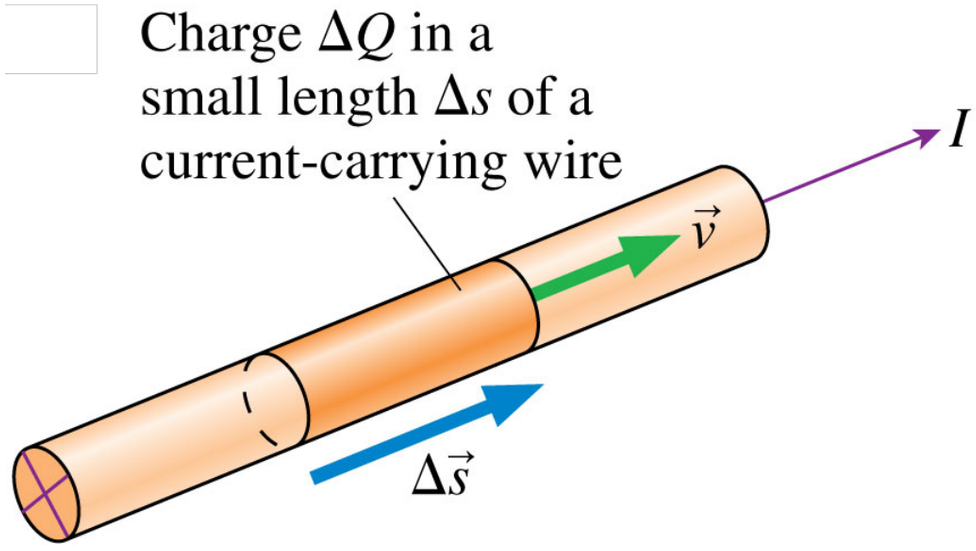
Magnetic Field of a Current



For moving point charge…

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic Field of a Current

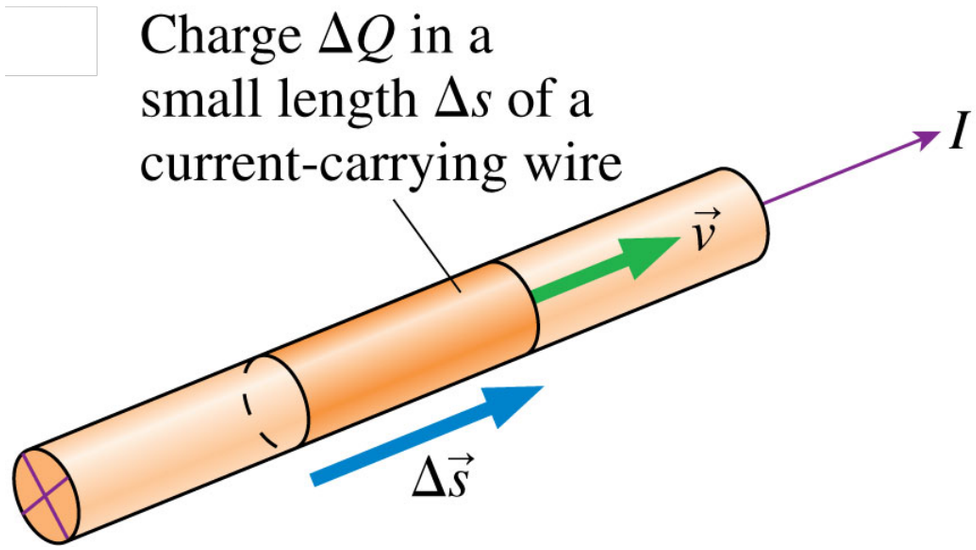


For moving point charge…

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$$(\Delta Q)\vec{v} = \Delta Q\frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\,\Delta\vec{s}$$

Magnetic Field of a Current



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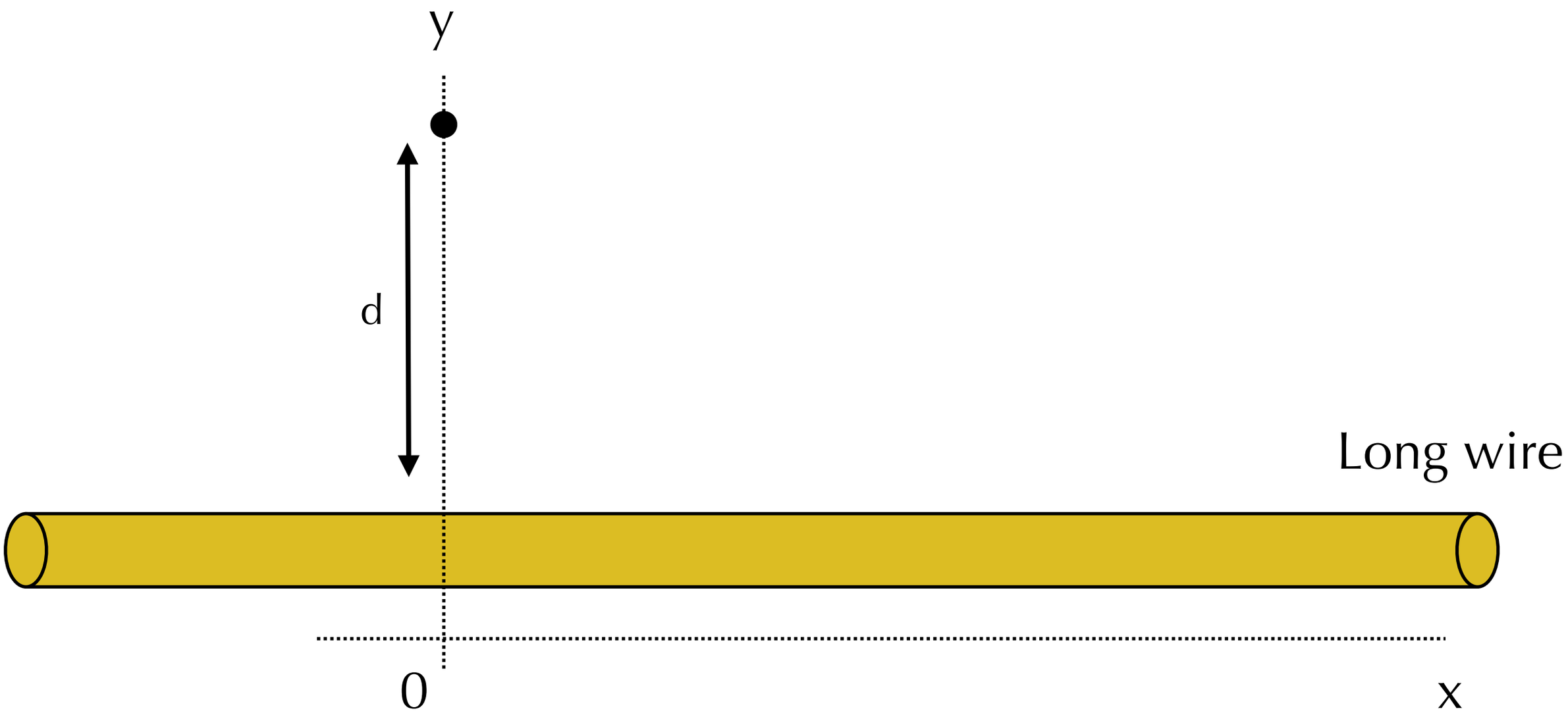
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{\Delta s}\times\hat{r}}{r^2}$$

For small segment of current carrying wire.

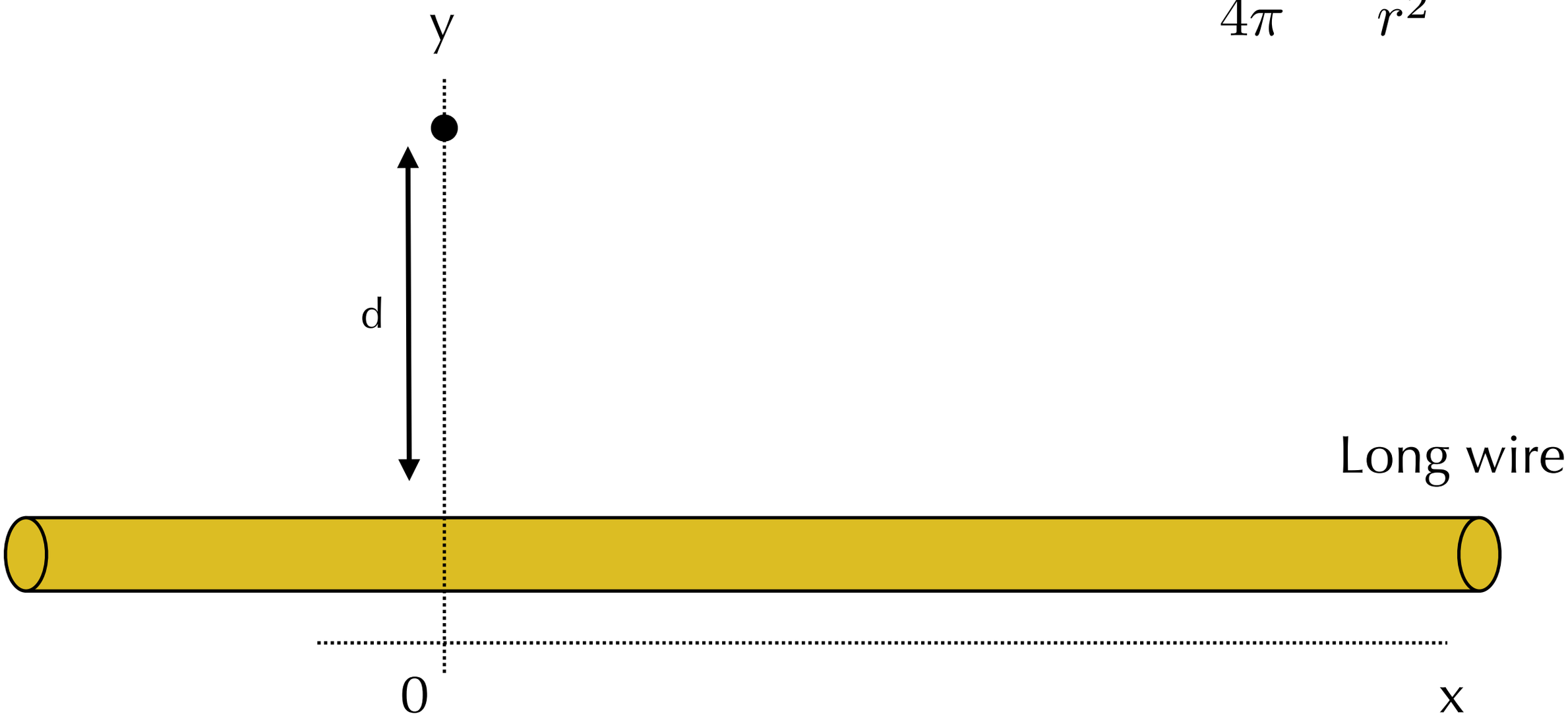
Let's see how to do these integrals!



$$\vec{B} = \frac{\mu_0 I}{2\pi d}$$

Let's see how to do these integrals!

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I\vec{\Delta}s \times \hat{r}}{r^2}$$



$$\vec{B} = \frac{\mu_0 I}{2\pi d}$$

Let's see how to do these integrals!

1 $$\hat{r} = \frac{-x\hat{i} + d\hat{j}}{\sqrt{x^2 + d^2}}$$

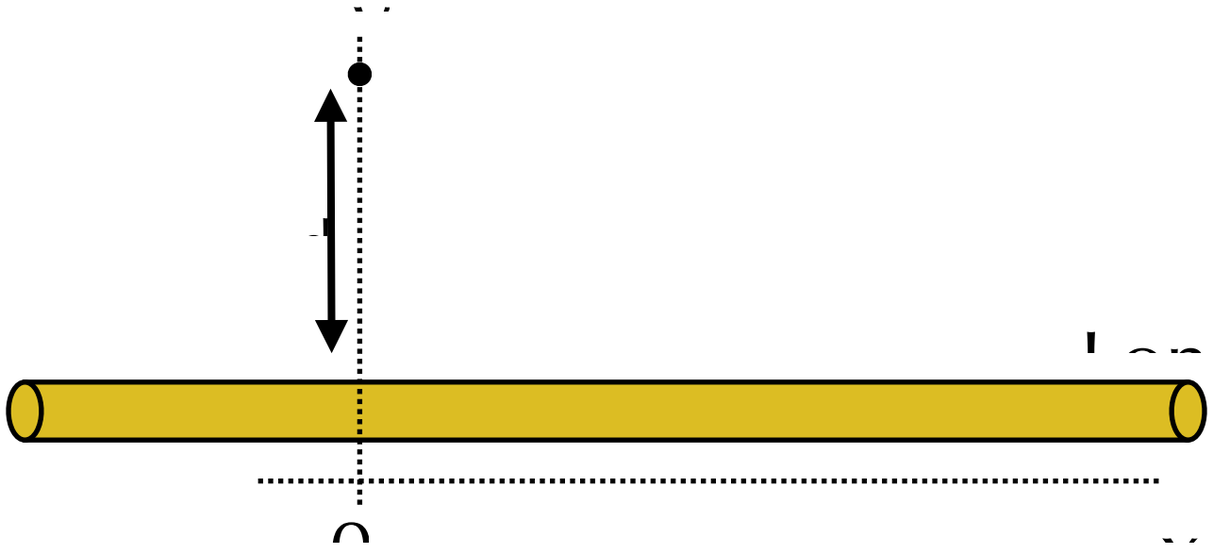
2 $$d\vec{s} = dx\hat{i}$$

3 $$d\vec{s} \times \hat{r} = \frac{ddx}{\sqrt{x^2 + d^2}}$$

4 $$r^2 = x^2 + d^2$$

5 $$\frac{\mu_0 I d}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + d^2)^{3/2}}$$

6 $$\frac{\mu_0 I d}{4\pi} \left| \frac{x}{d^2\sqrt{x^2 + d^2}} \right|_{-\infty}^{\infty}$$



$$\boxed{\frac{\mu_0 I}{4\pi d}}$$

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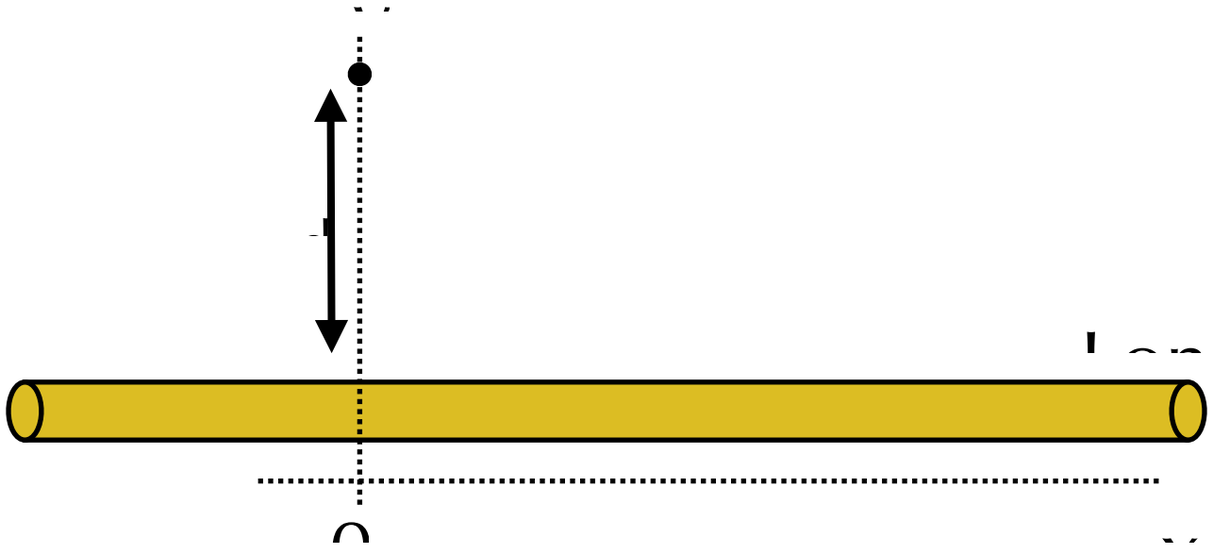


$$\frac{\mu_0 I}{4\pi d}$$

Quiz Question

Compared to the magnetic field at point A, the magnetic field at point B is

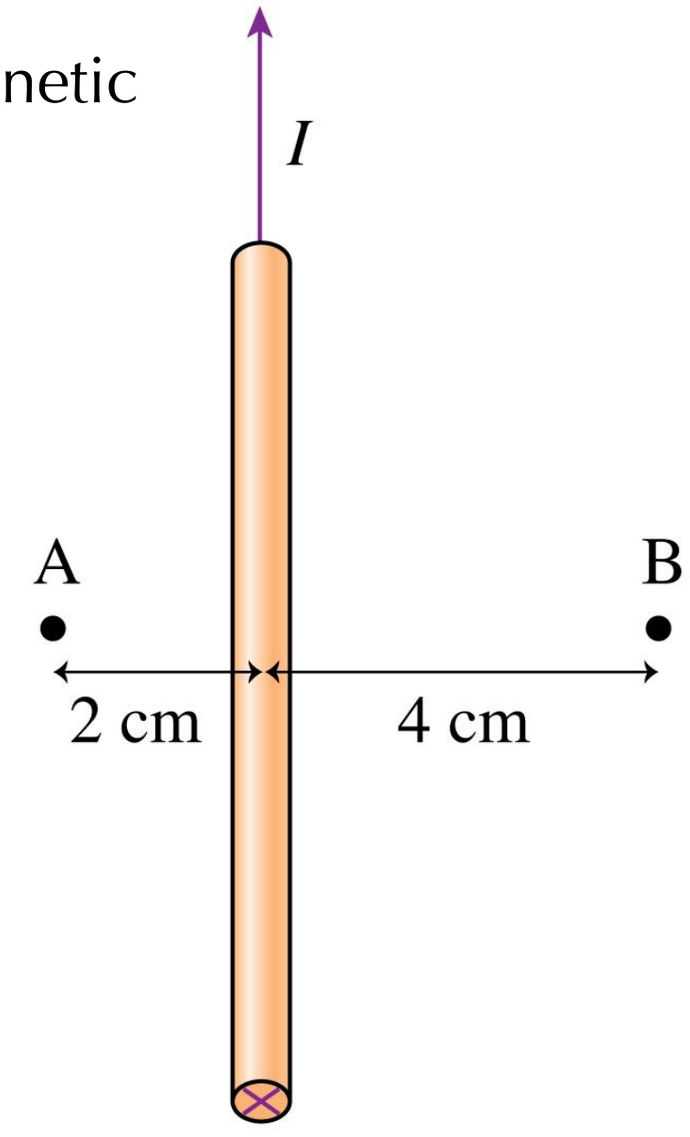
A. Half as strong, same direction.

B. Can't compare without knowing *I*.

C. One-quarter as strong, same direction.

D. One-quarter as strong, opposite direction.

E. Half as strong, opposite direction.



Quiz Question

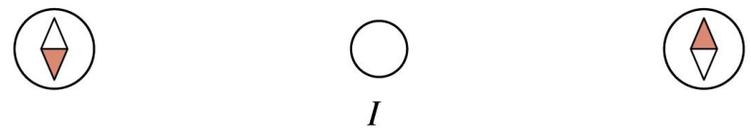
A long, straight wire extends into and out of the screen. The current in the wire is

B. Into the screen.

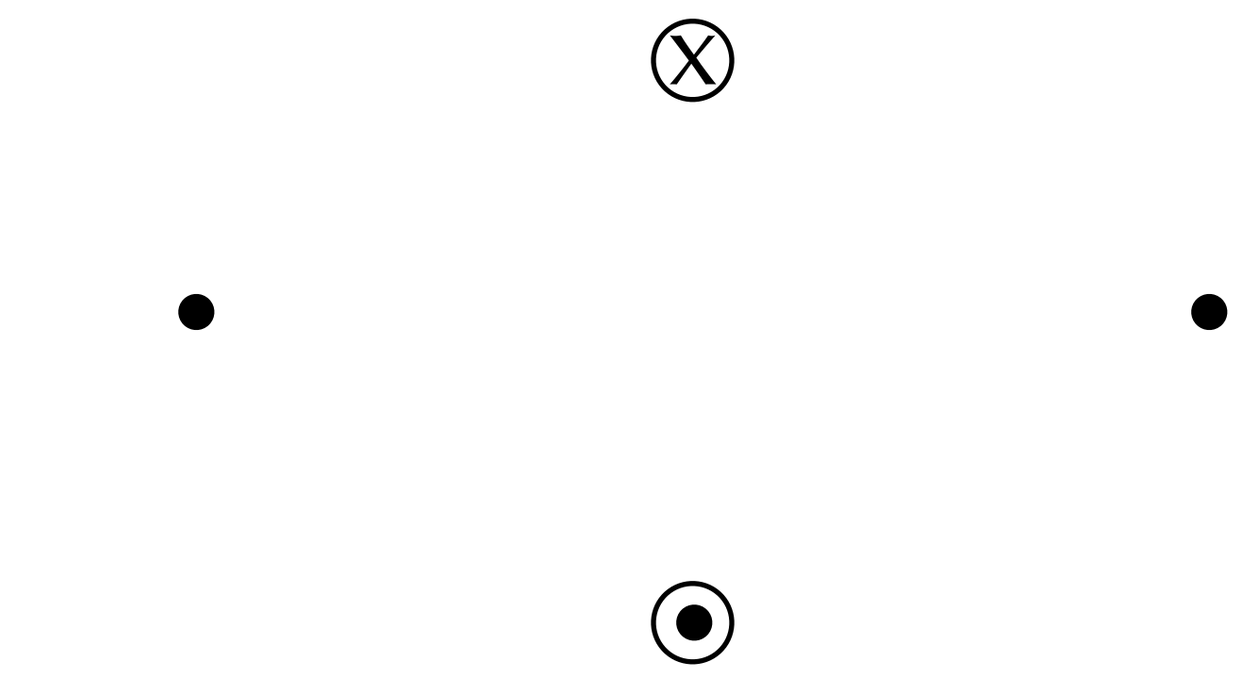
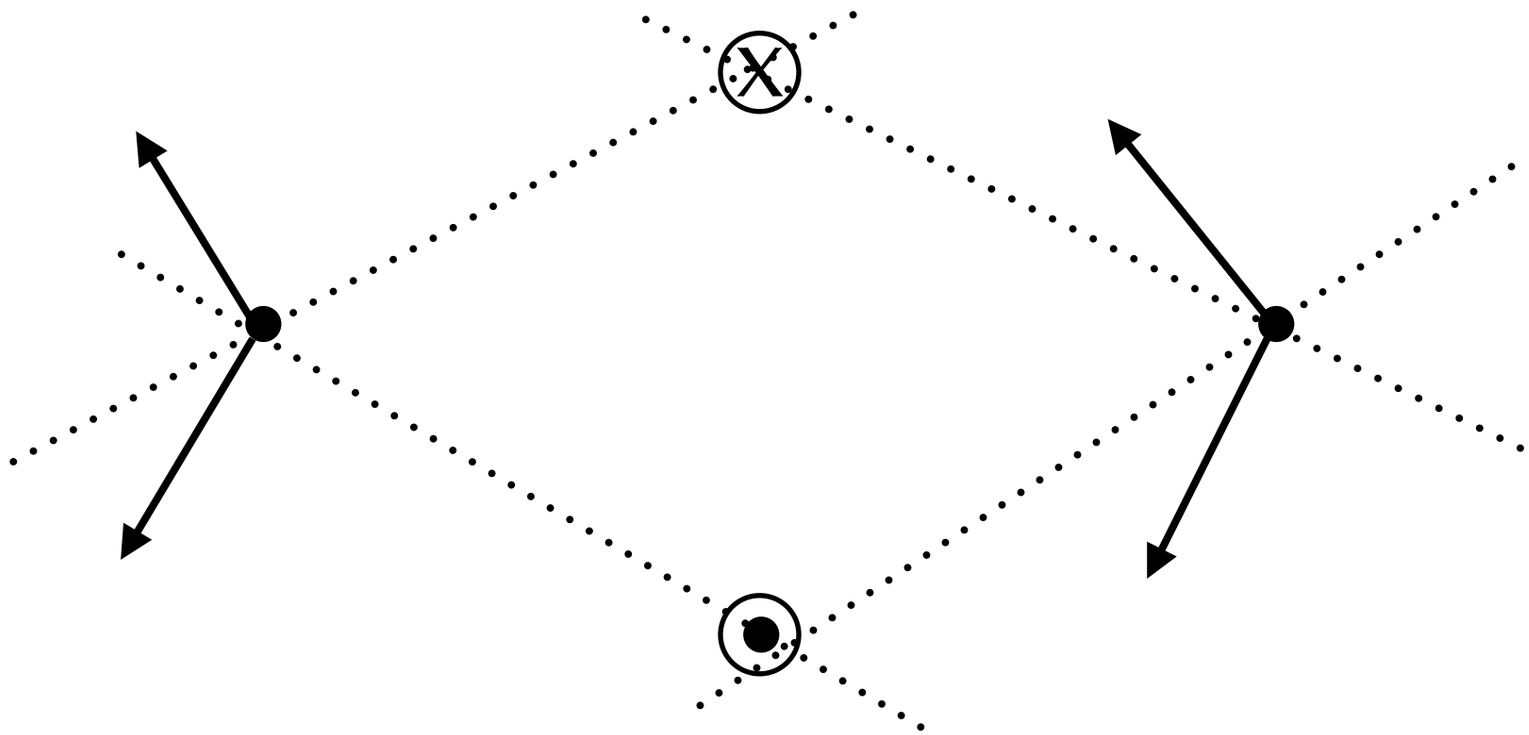
C. Not enough info to tell the direction.

D. Out of the screen.

E. There is no current in the wire.



Draw the net magnetic field vector at both points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