



PH 220

Lance Nelson

Answer the following questions with your neighbor.

What is the electric field?

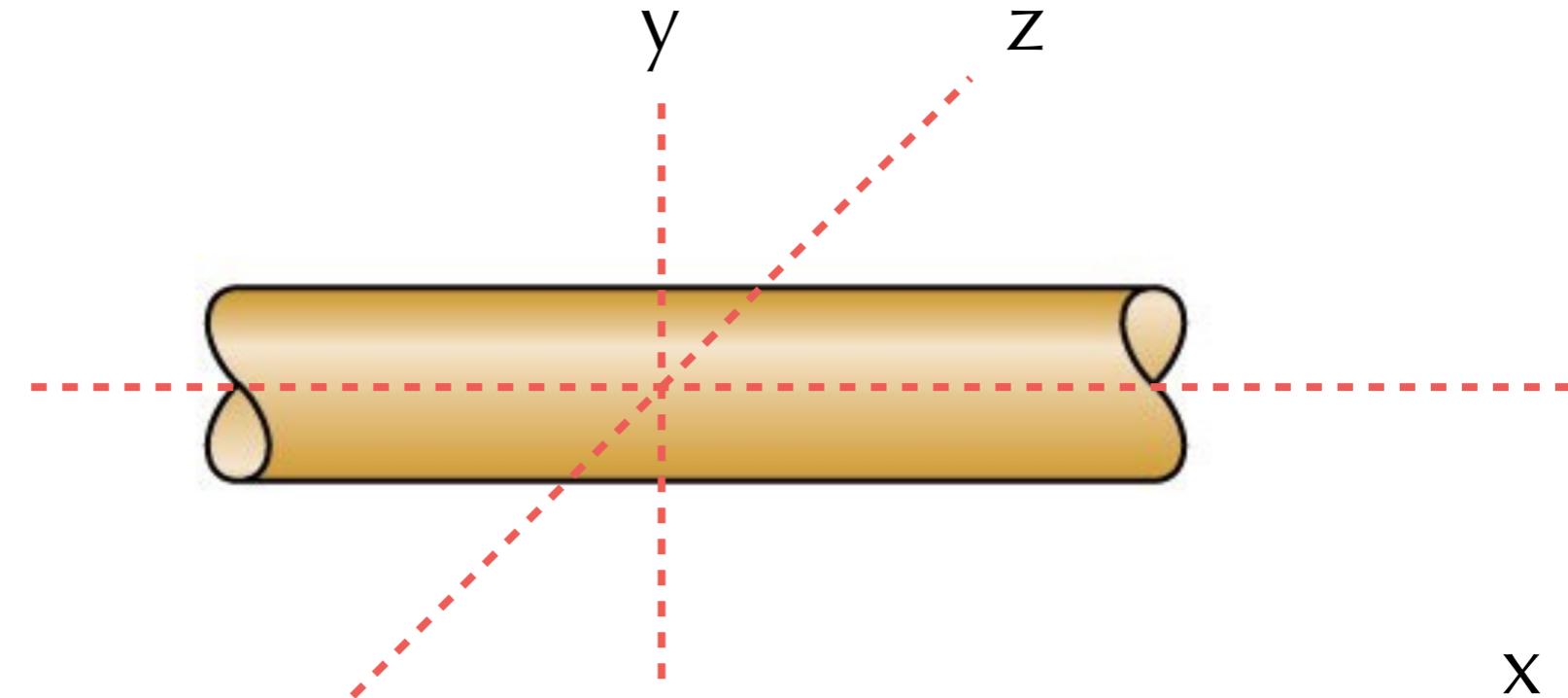
How do you calculate the electric field for
point charges?
continuous charge distributions?

How do you calculate the force on an object
placed in an electric field?

Symmetry

Question #14

What transformation **does not** leave the rod unchanged?

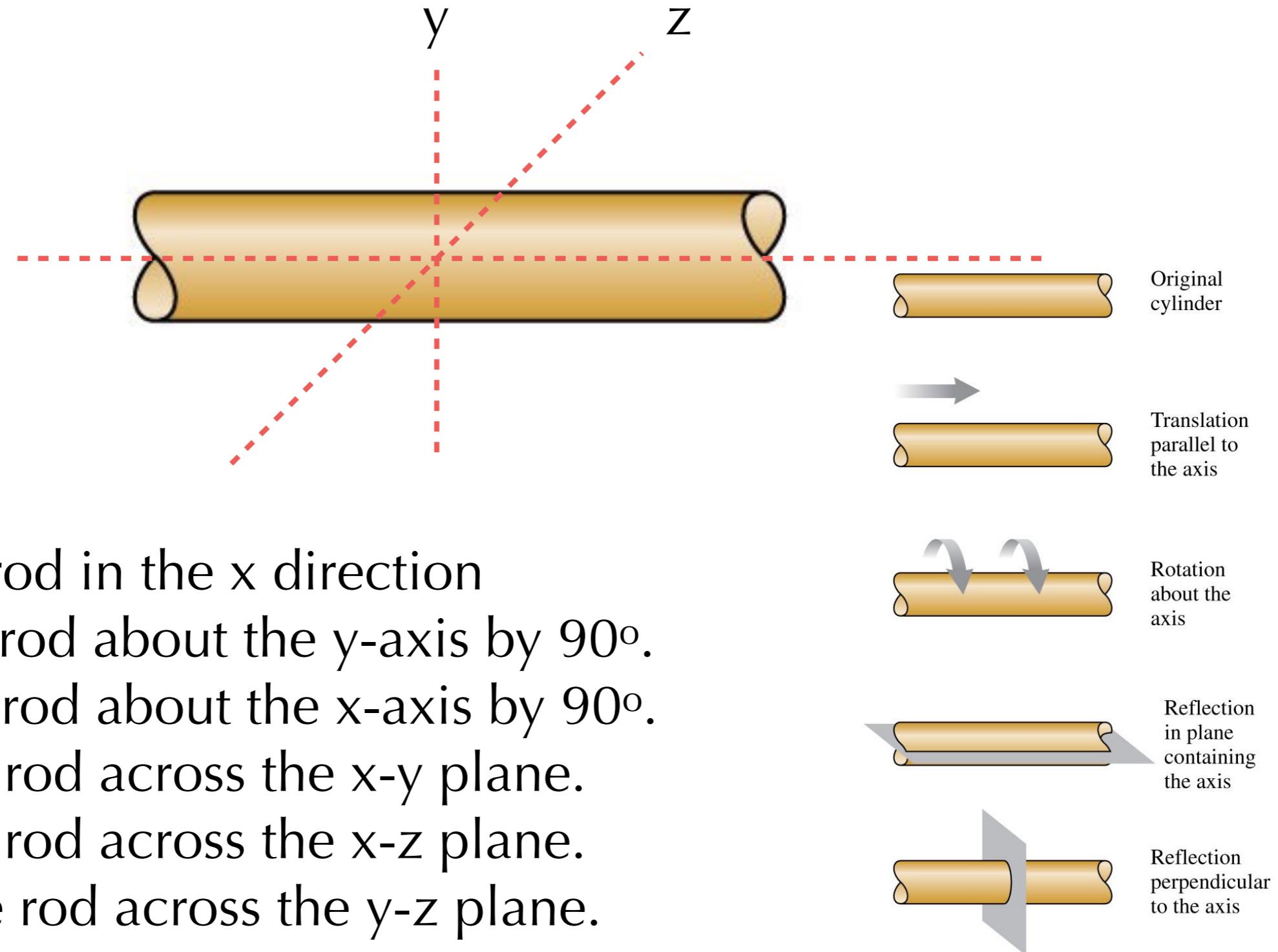


- A: Move the rod in the x direction
- B: Rotate the rod about the y-axis by 90° .
- C: Rotate the rod about the x-axis by 90° .
- E: Reflect the rod across the x-y plane.
- F: Reflect the rod across the x-z plane.
- G: Reflect the rod across the y-z plane.

Symmetry

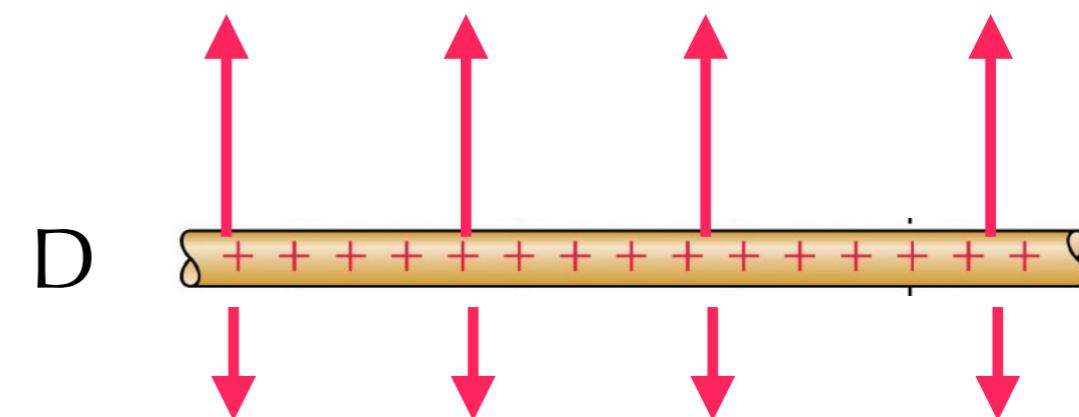
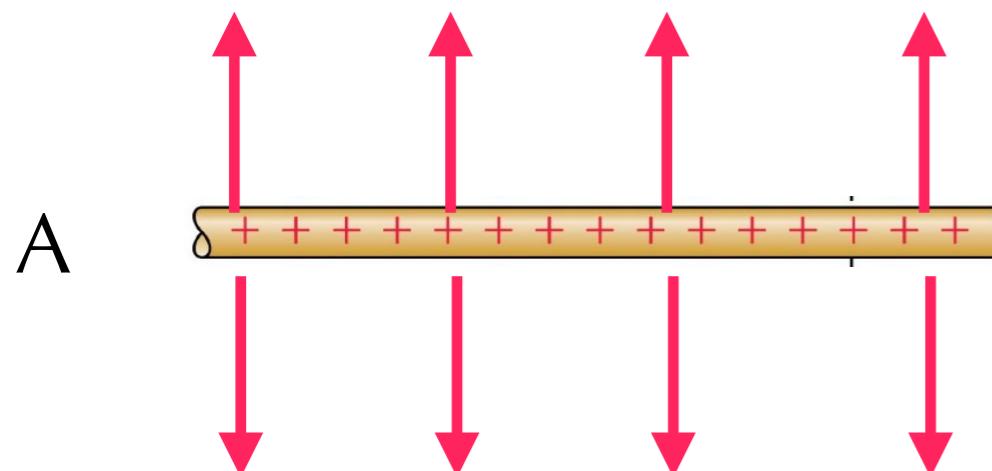
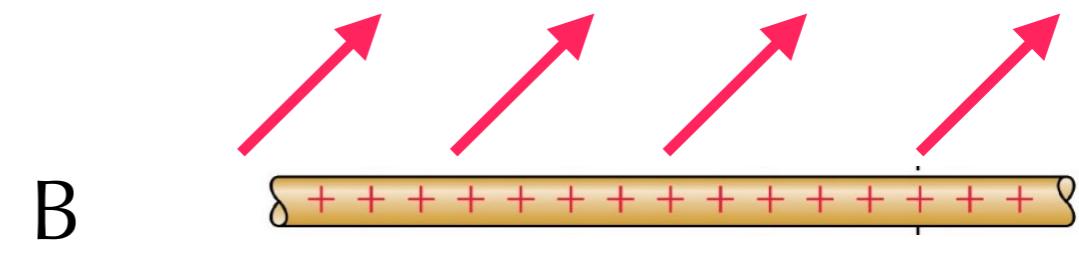
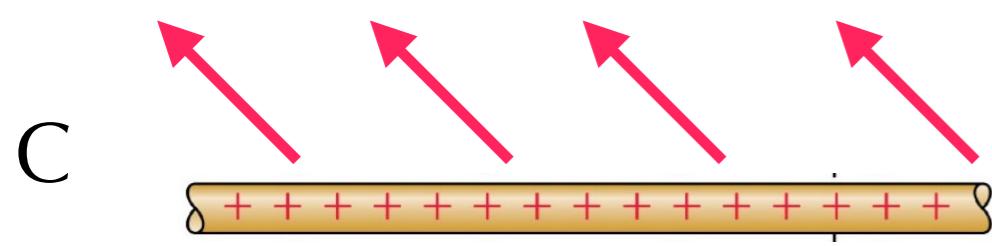
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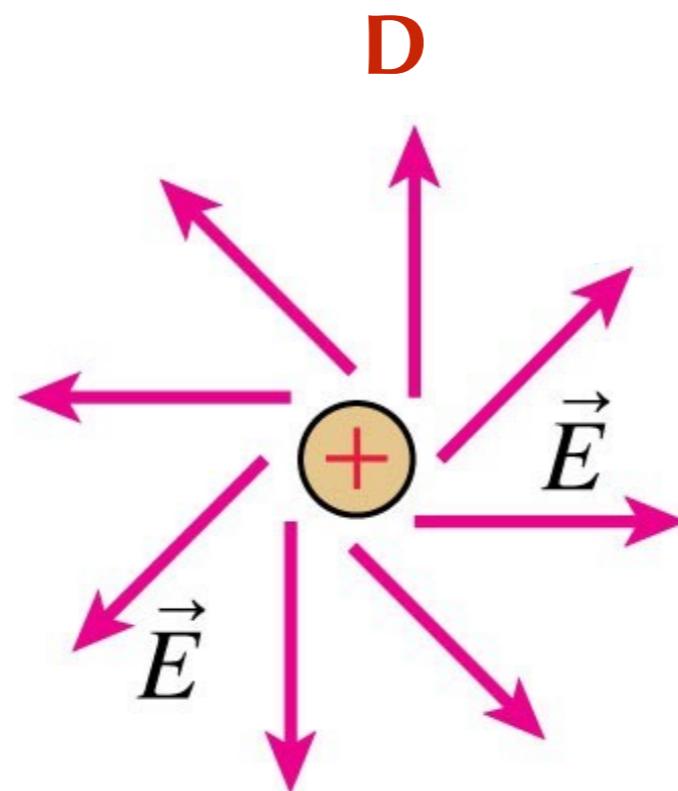
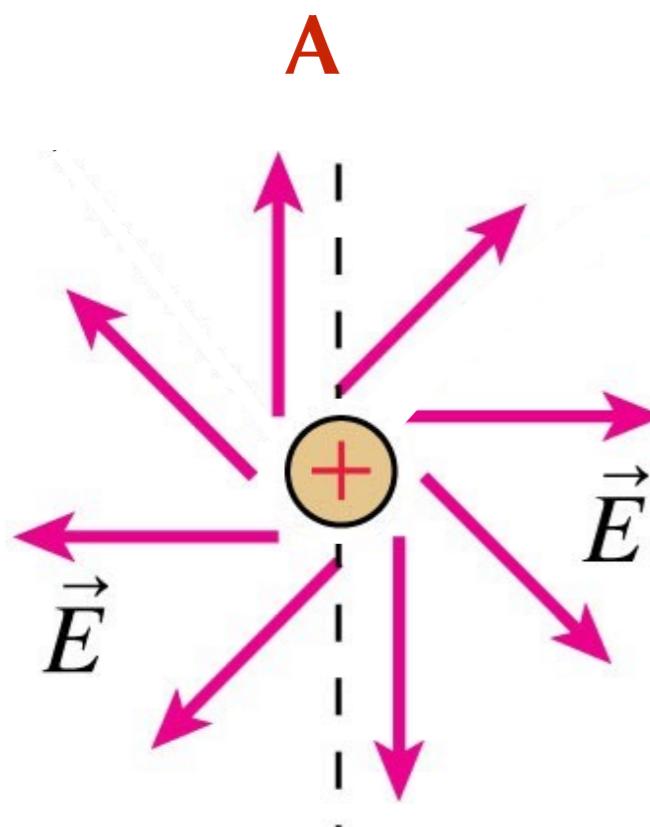
Question #15

Based on the symmetry of the rod only, which electric field is possible.



Question #16

An end view of a very long rod is shown below. Which electric field(s) are not possible?

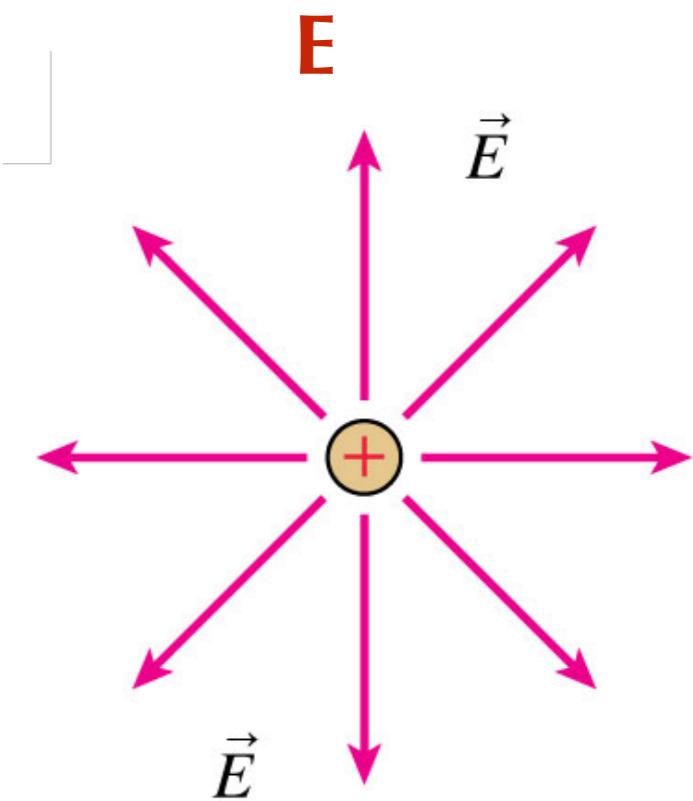
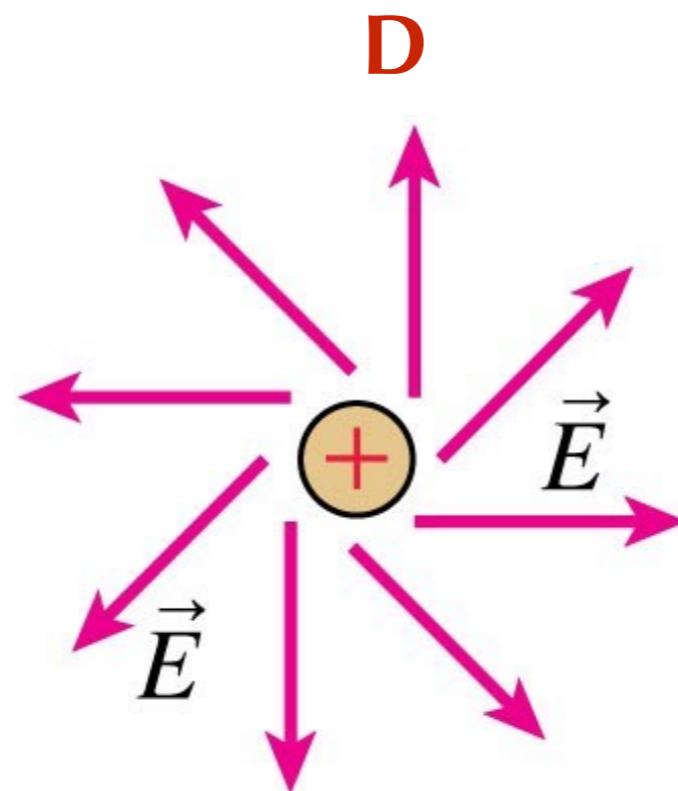
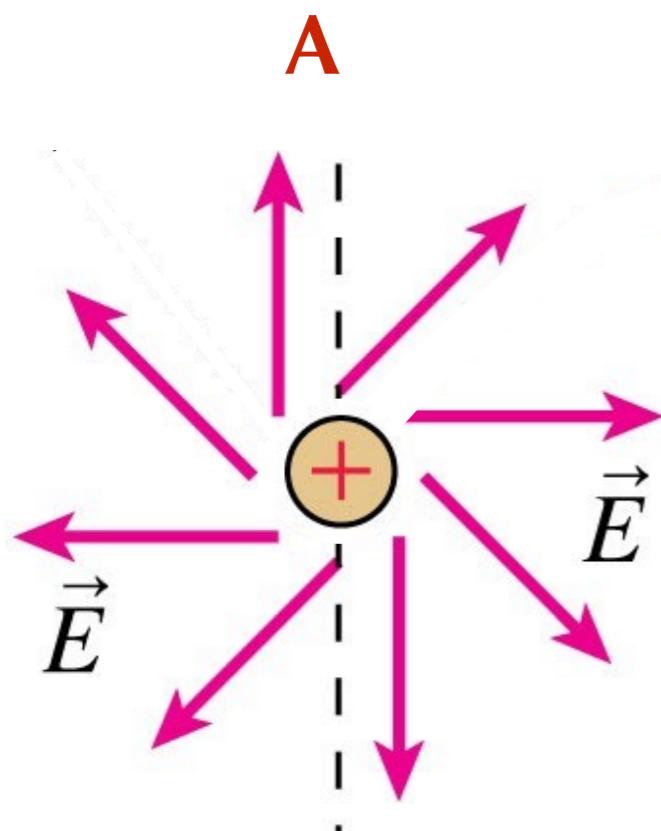


E

- B: A and D
- C: A and E

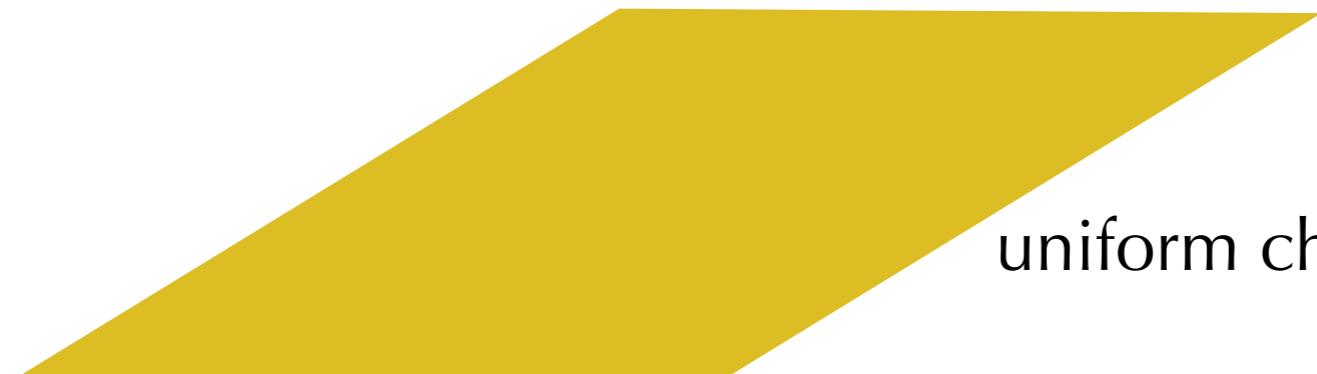
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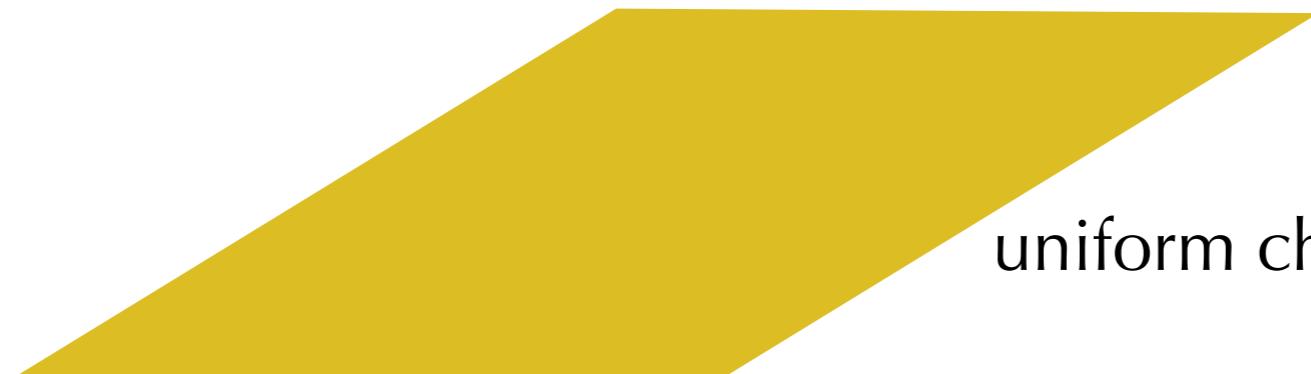
- B: A and D
- C: A and E

Based on symmetry, what must the E field look like?



uniform charge distribution

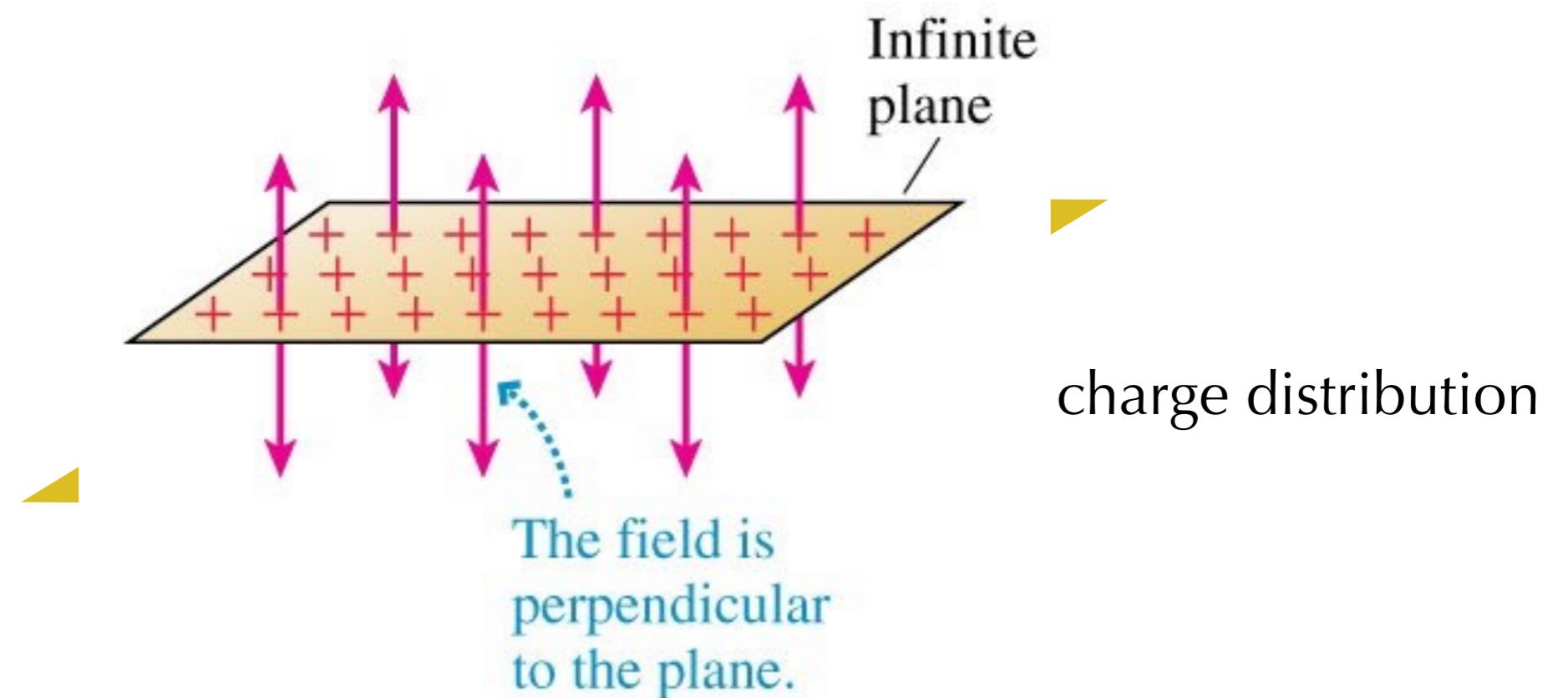
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uniform charge distribution

- Translation parallel to the plane.
- Rotation about any line perpendicular to the plane.
- Reflection in the plane.

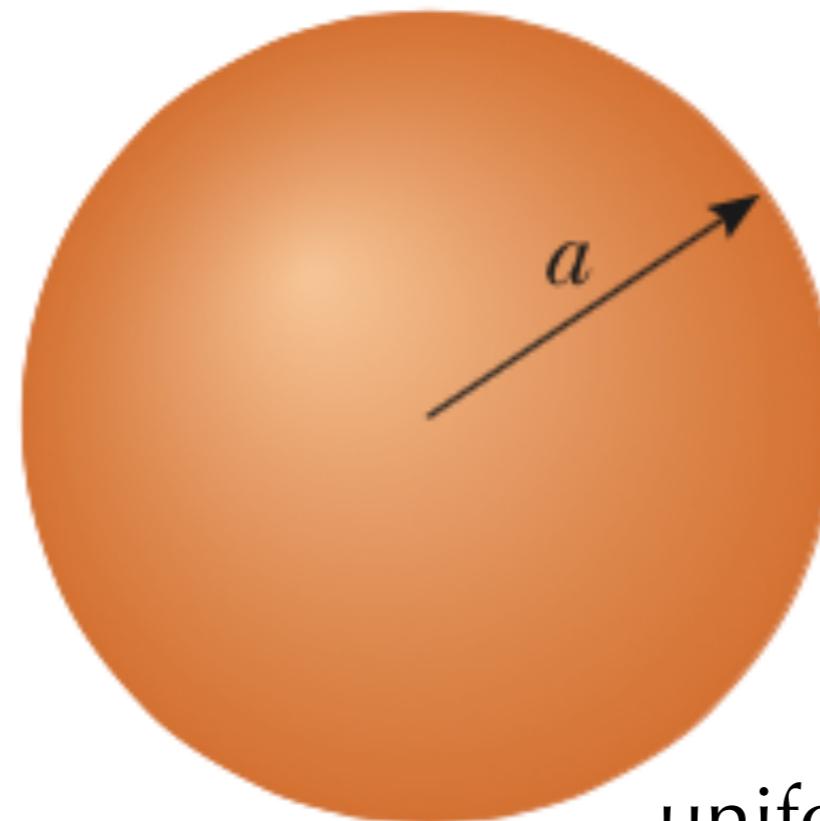
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Sphere

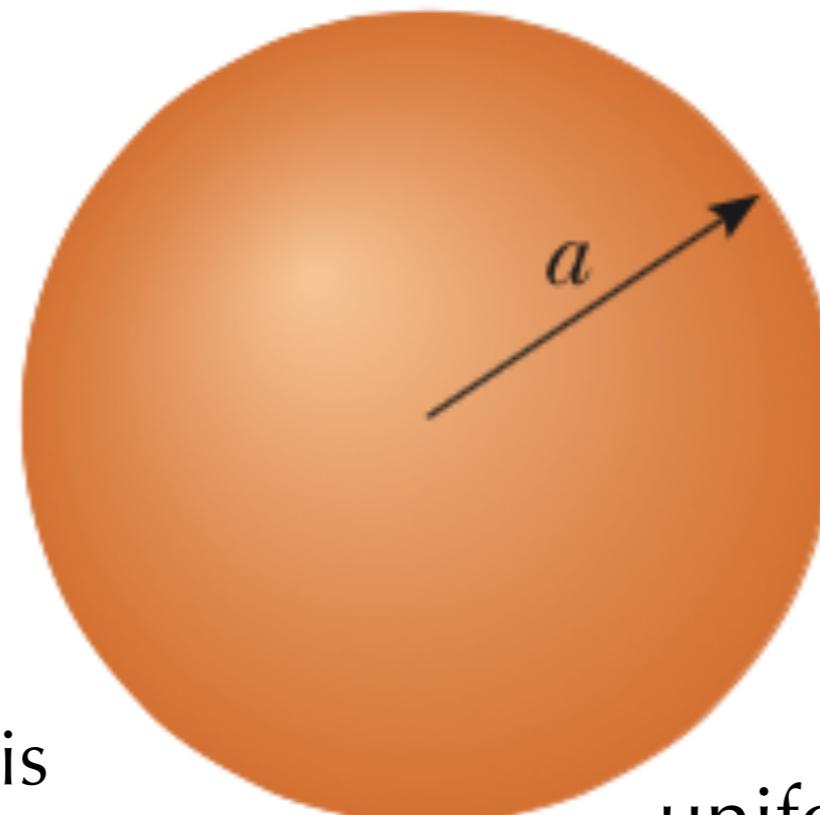
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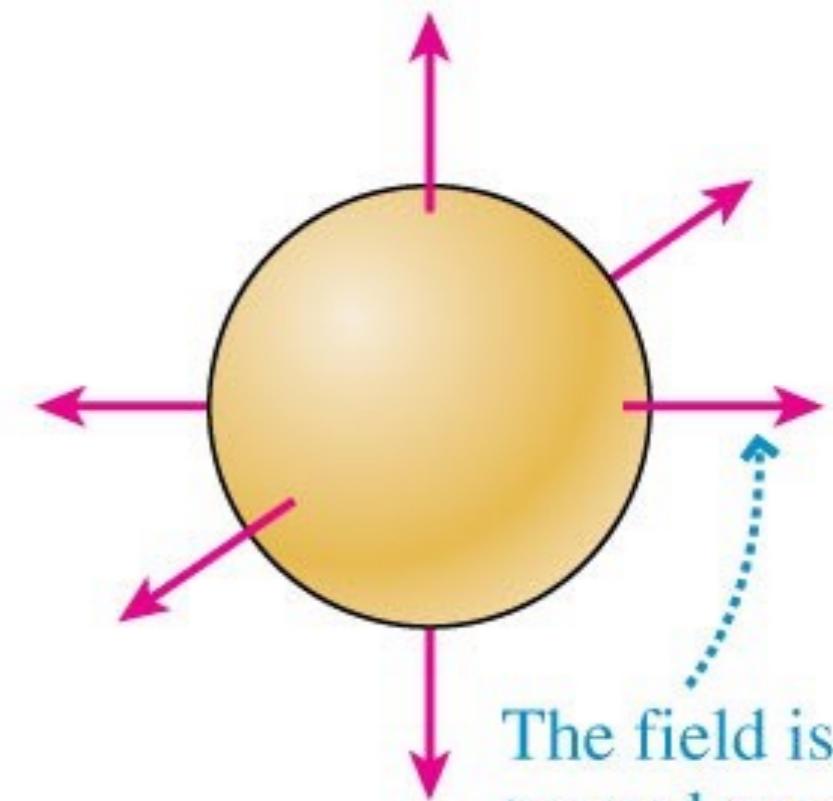
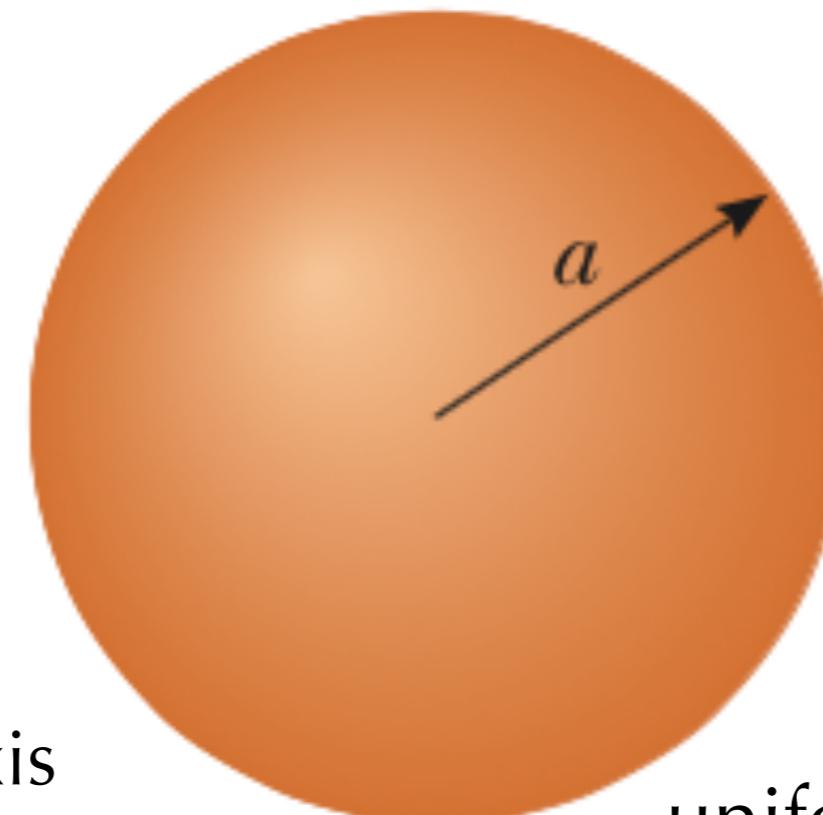


- Rotation about any axis which passes through the center point.
- Reflection in any plane containing the center point.

uniform charge distribution

Sphere

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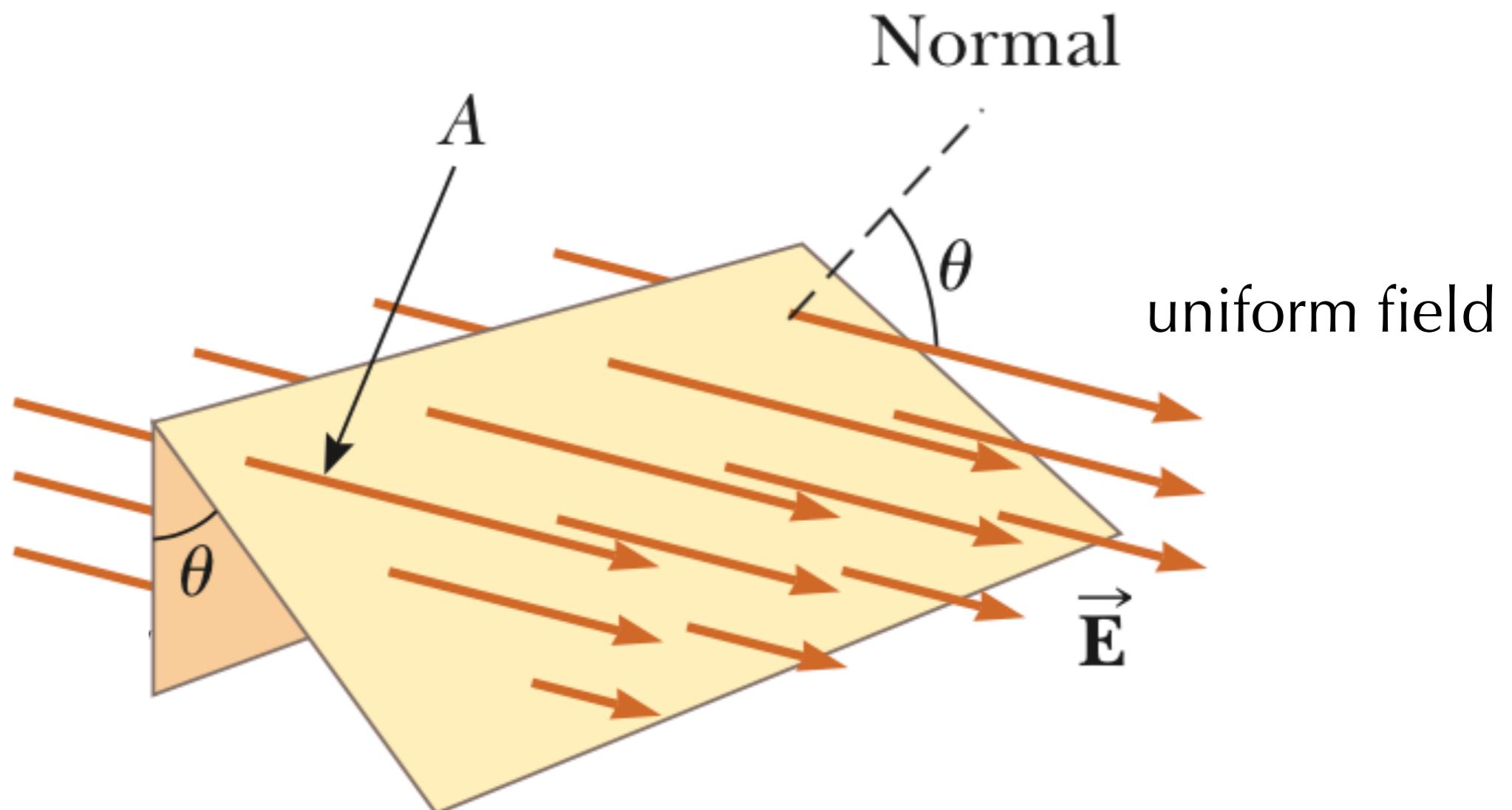


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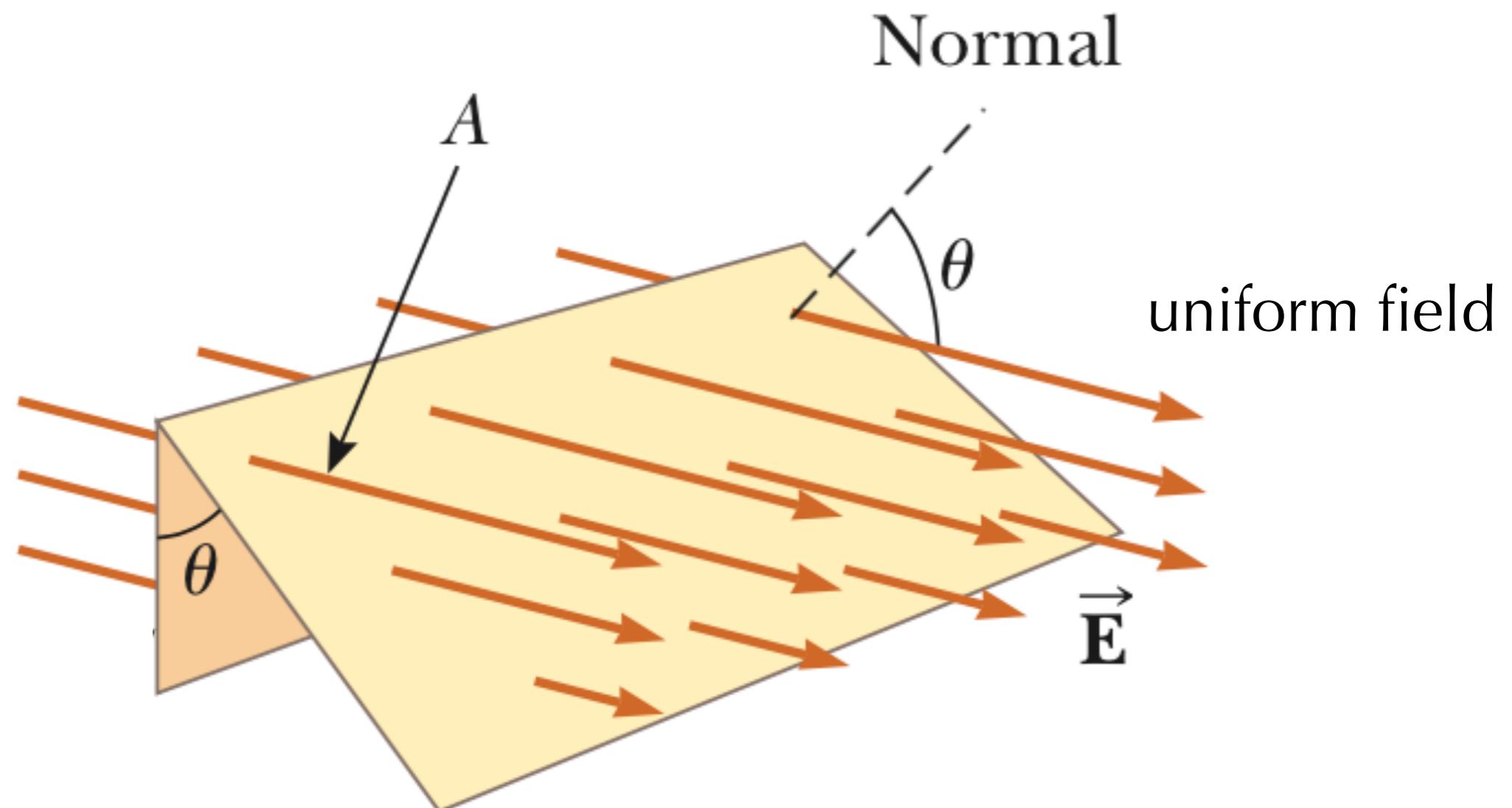
Electric Flux

What is the flux through surface A?



Electric Flux

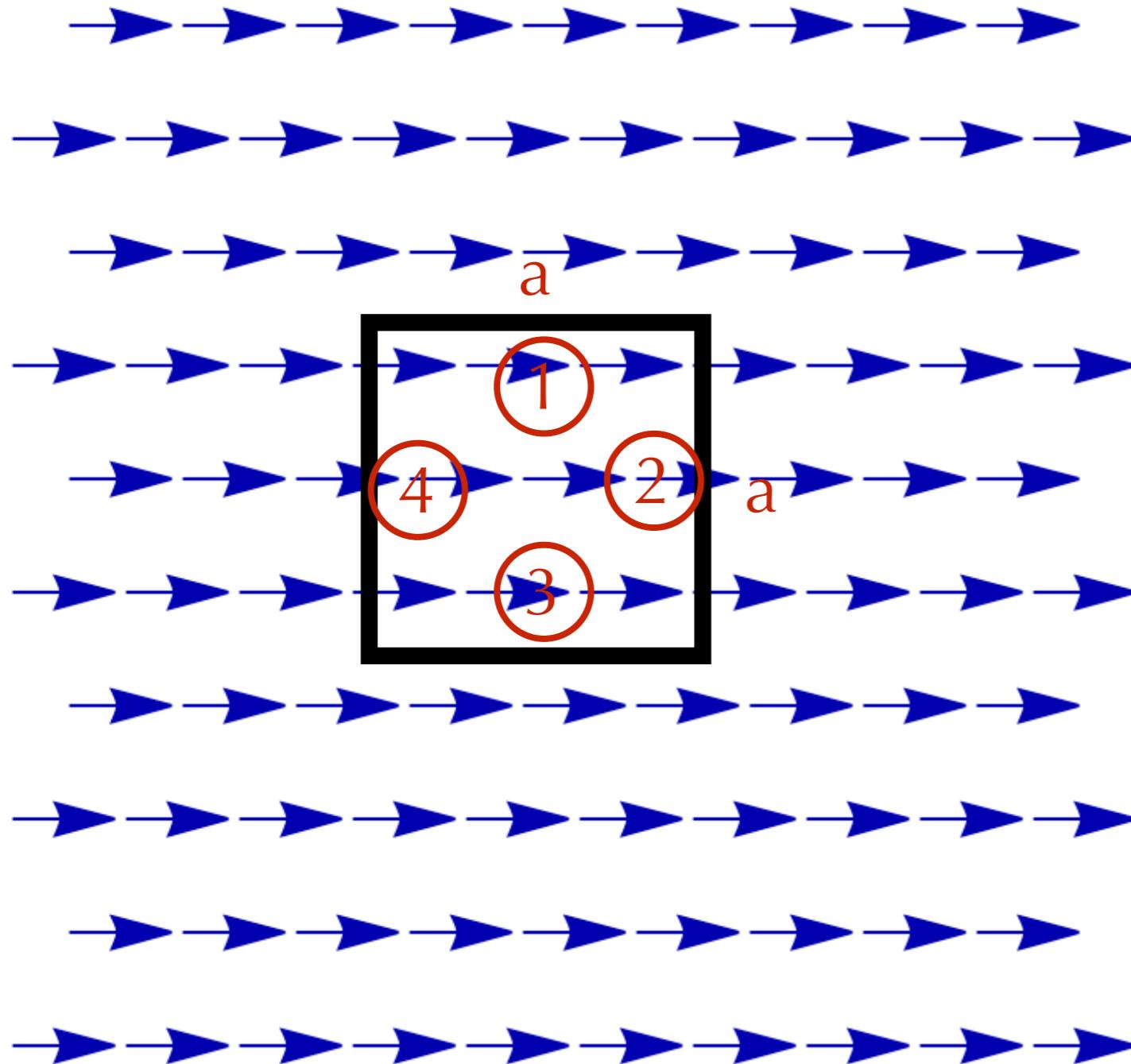
What is the flux through surface A?



$$\Phi = \vec{E} \cdot \vec{A}$$

2D Flux

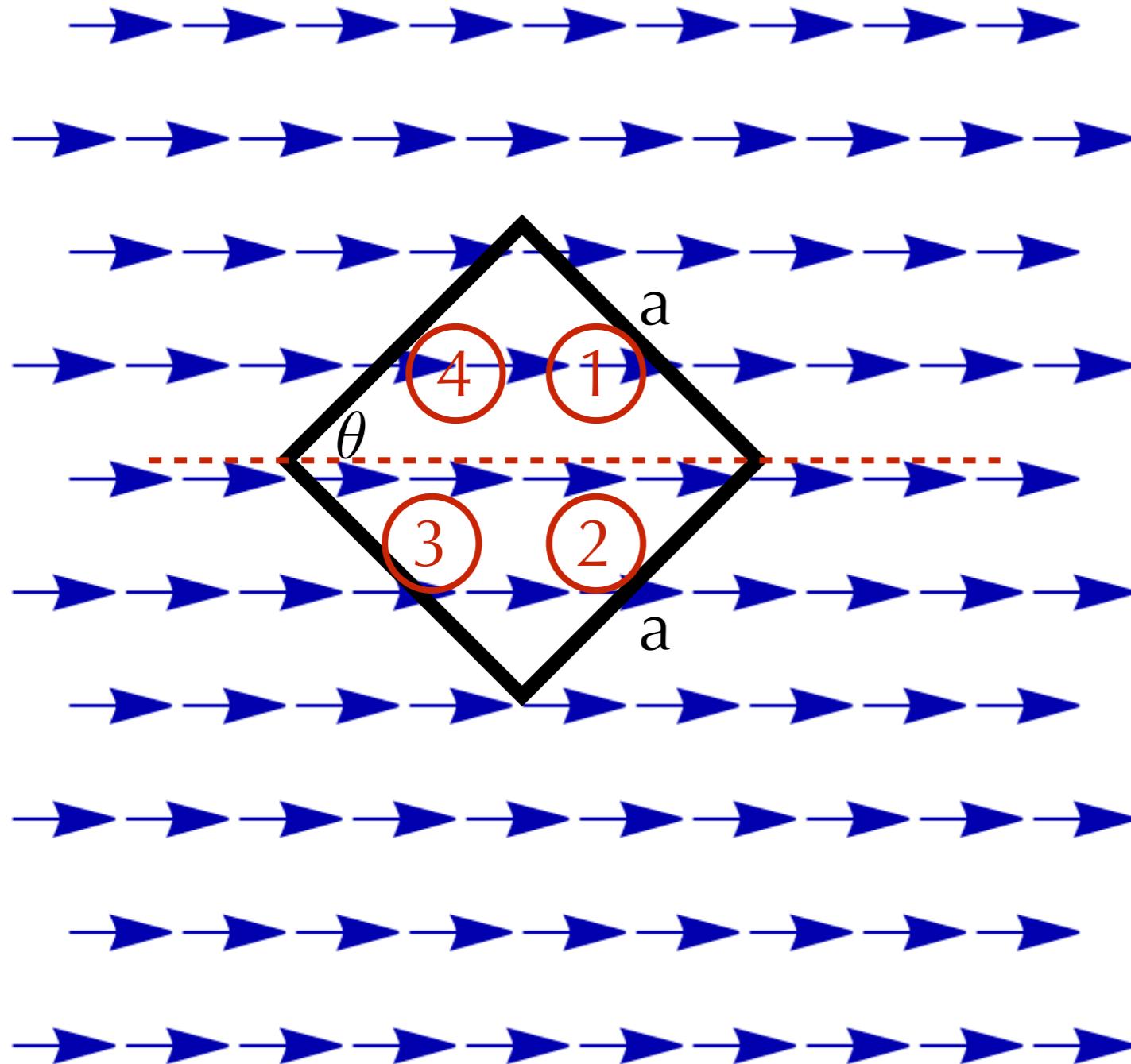
Question #17 Which “surface(s)” have zero flux through them?



- a) 1 and 4
- b) 2 and 3
- c) 4
- d) 2 and 4
- e) 1 and 3

2D Flux

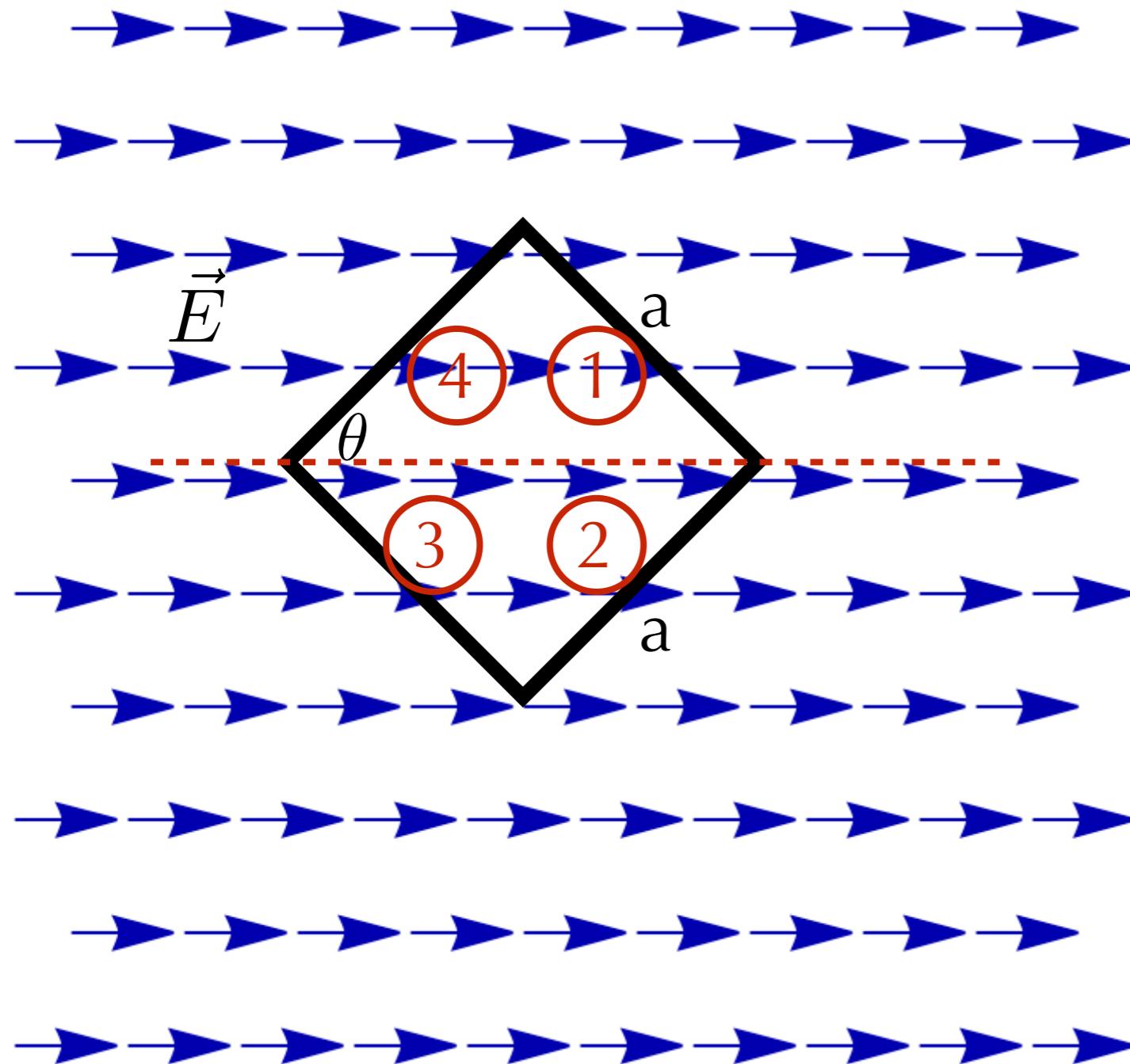
Question #18 Which “surface(s)” have zero flux through them?



- a) 1 and 4
- b) 2 and 3
- c) 1 and 3
- d) 2 and 4
- e) none

2D Flux

Question #19 How would you calculate the flux through surface 4?



A $E \cos \theta$

C $-Ea \sin \theta$

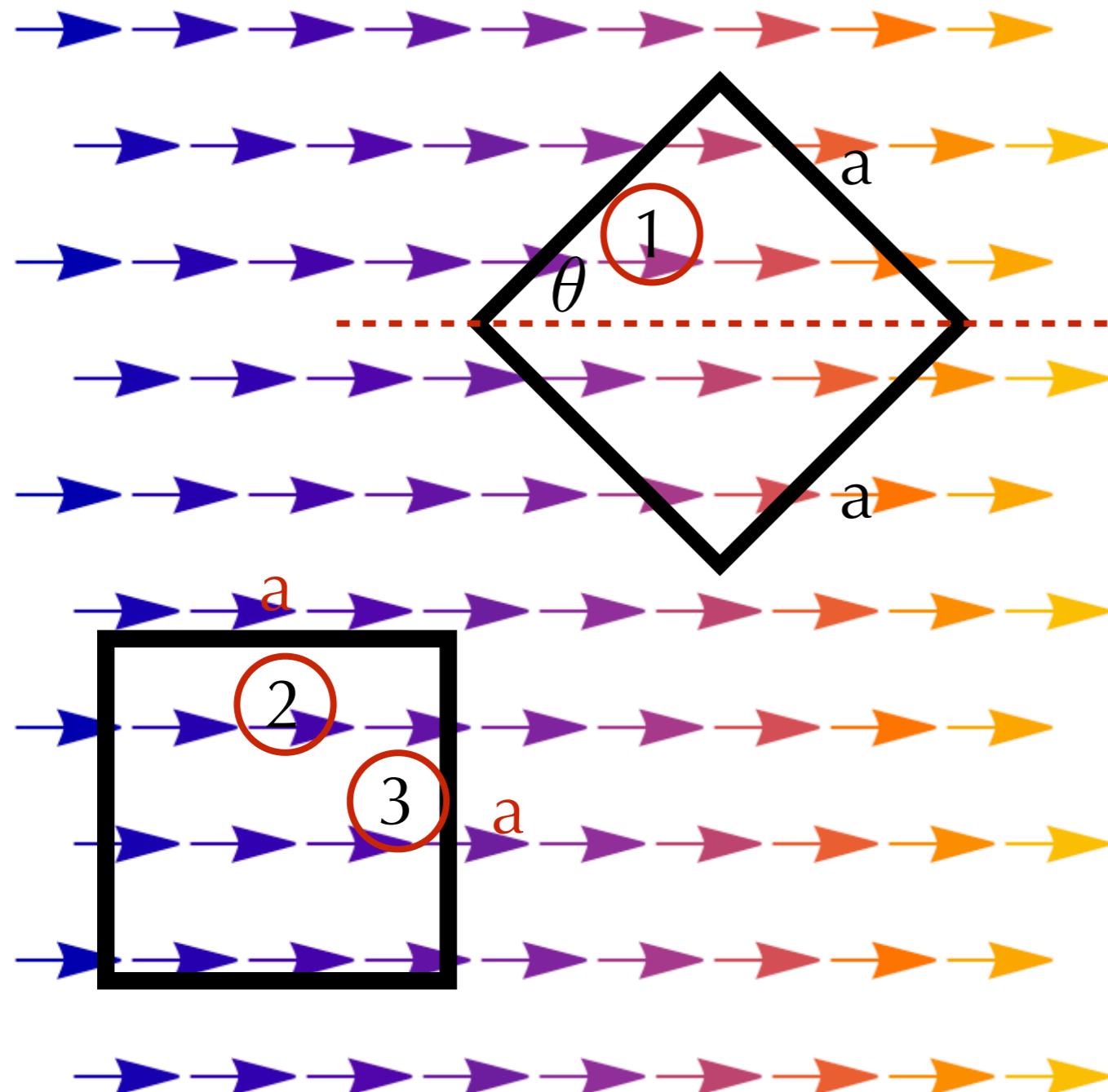
D $Ea \sin \theta$

B $Ea \cos \theta$

E Ea

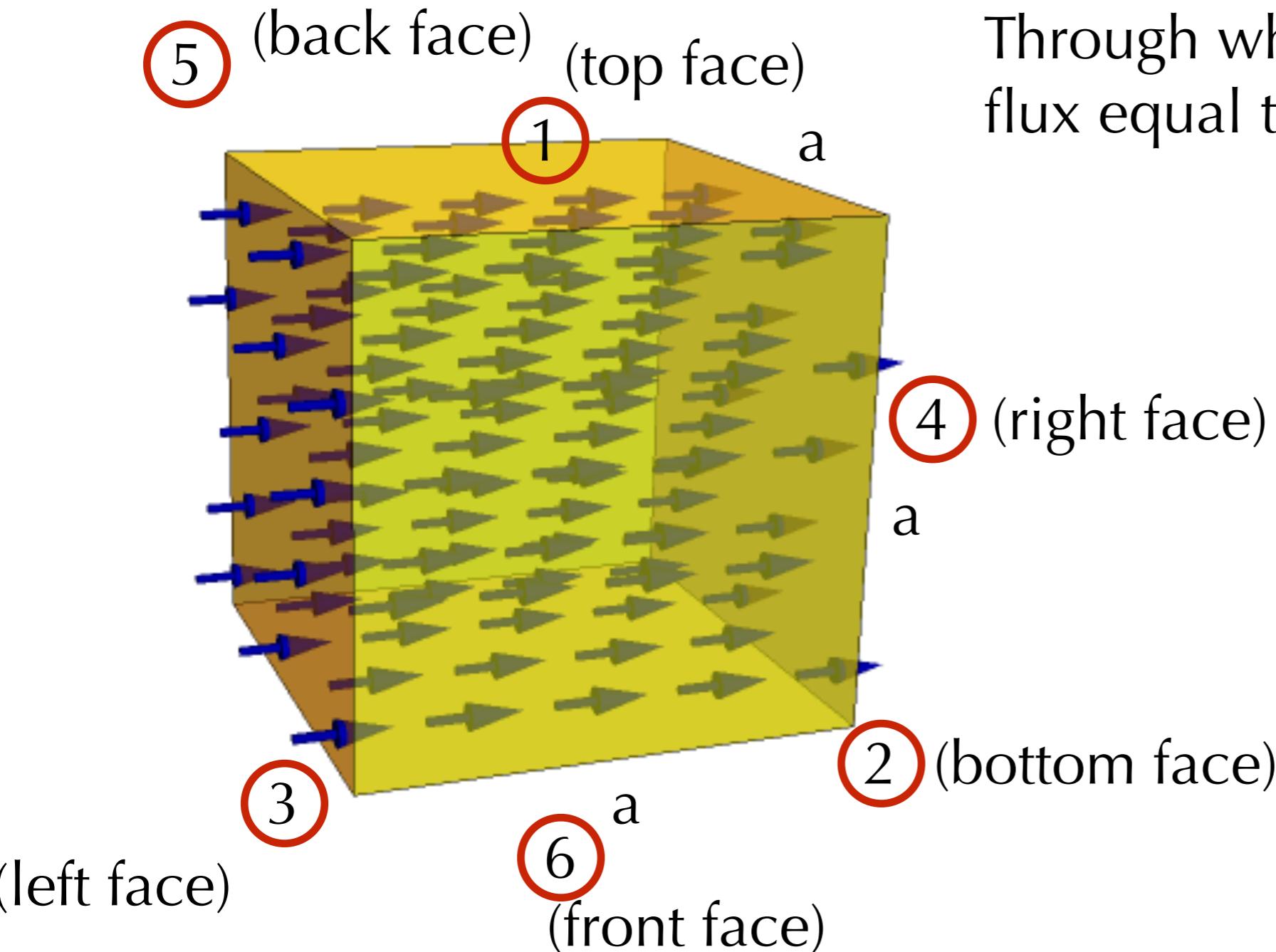
2D Flux

Question #20 Which flux calculation will be most difficult?



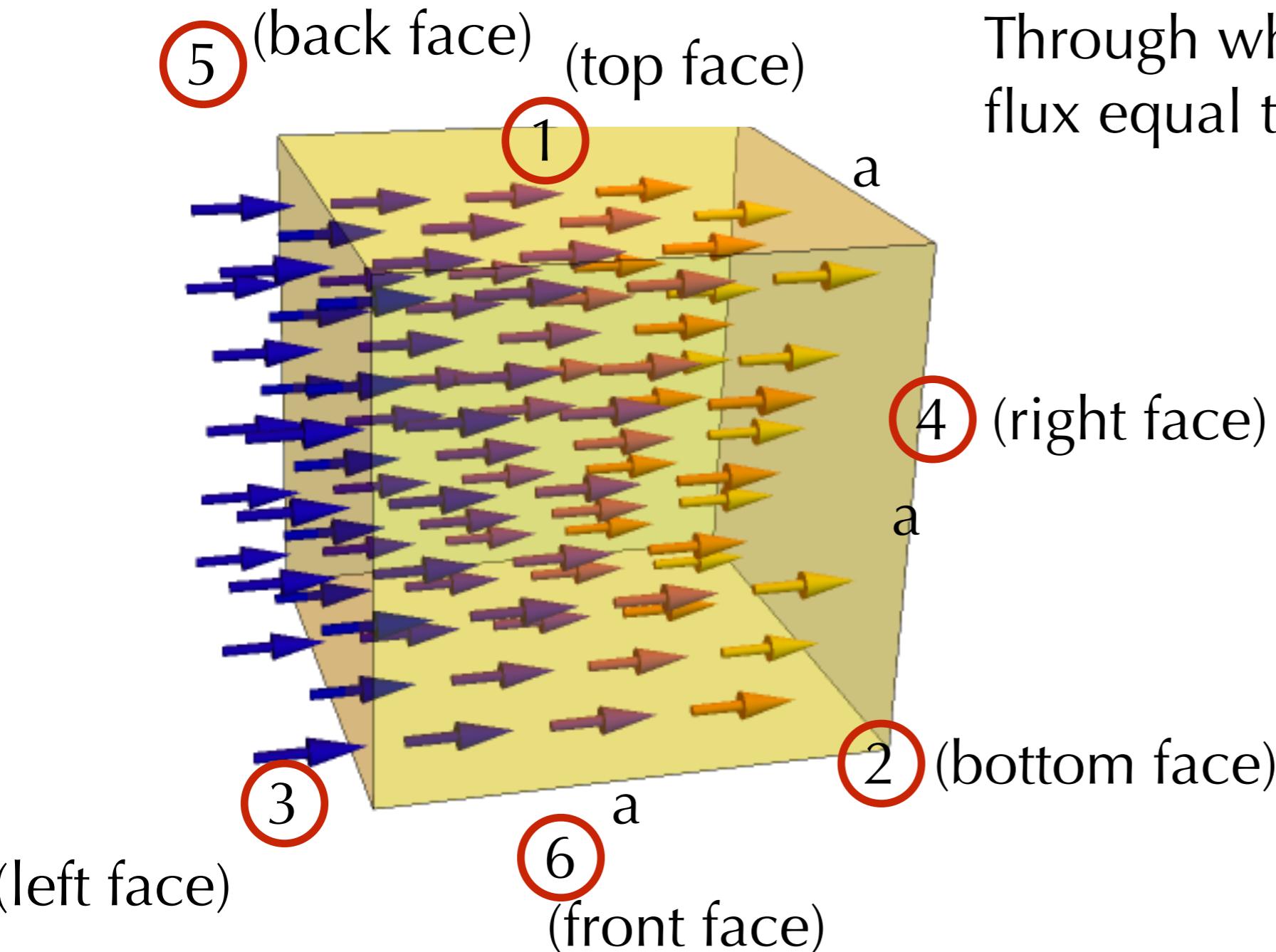
- a) Surface 3
- b) Surface 2
- c) Surface 1
- d) They're all equally hard

3D Flux



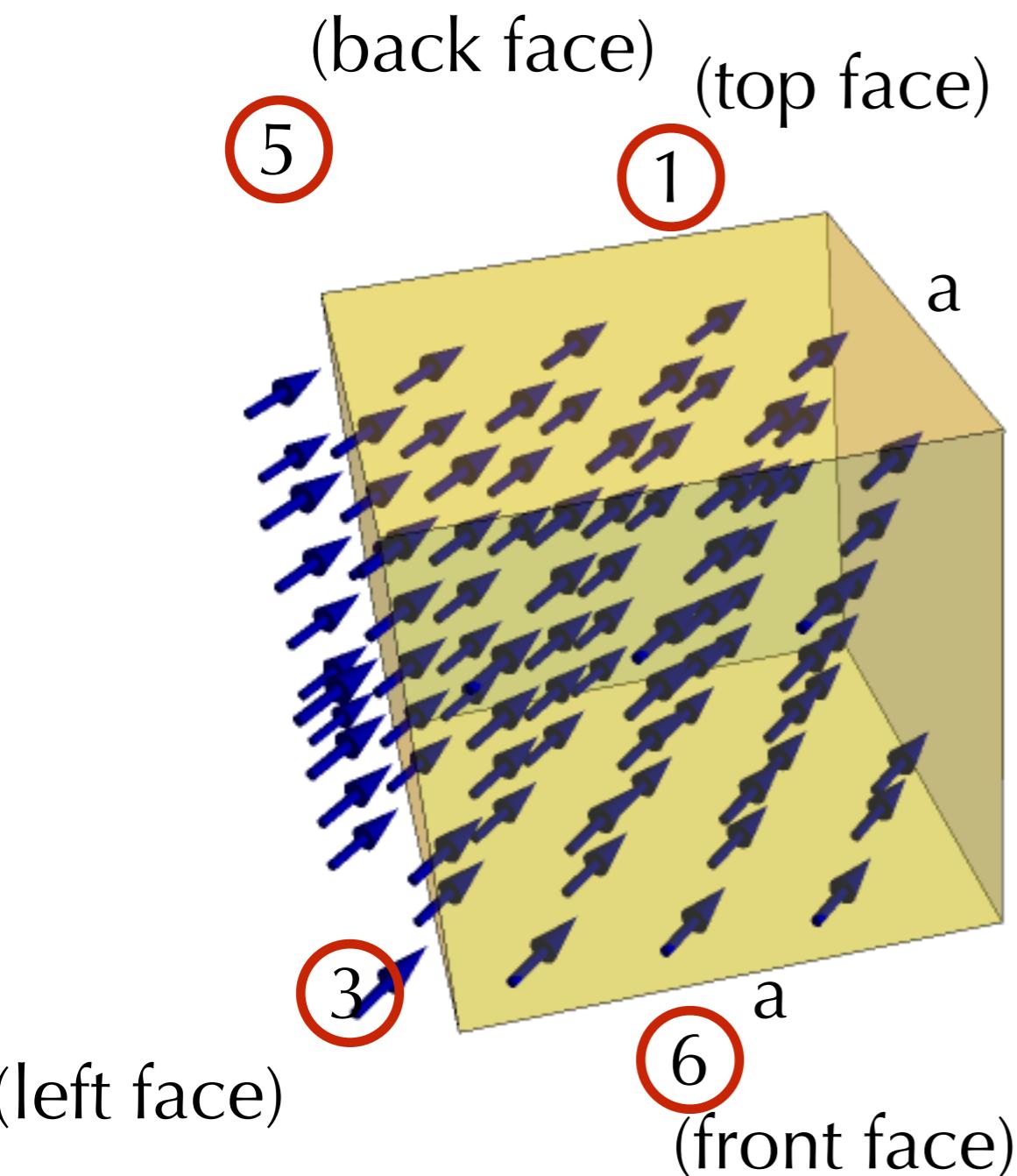
Through which surface(s) is(are) the flux equal to zero?

3D Flux



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3D Flux



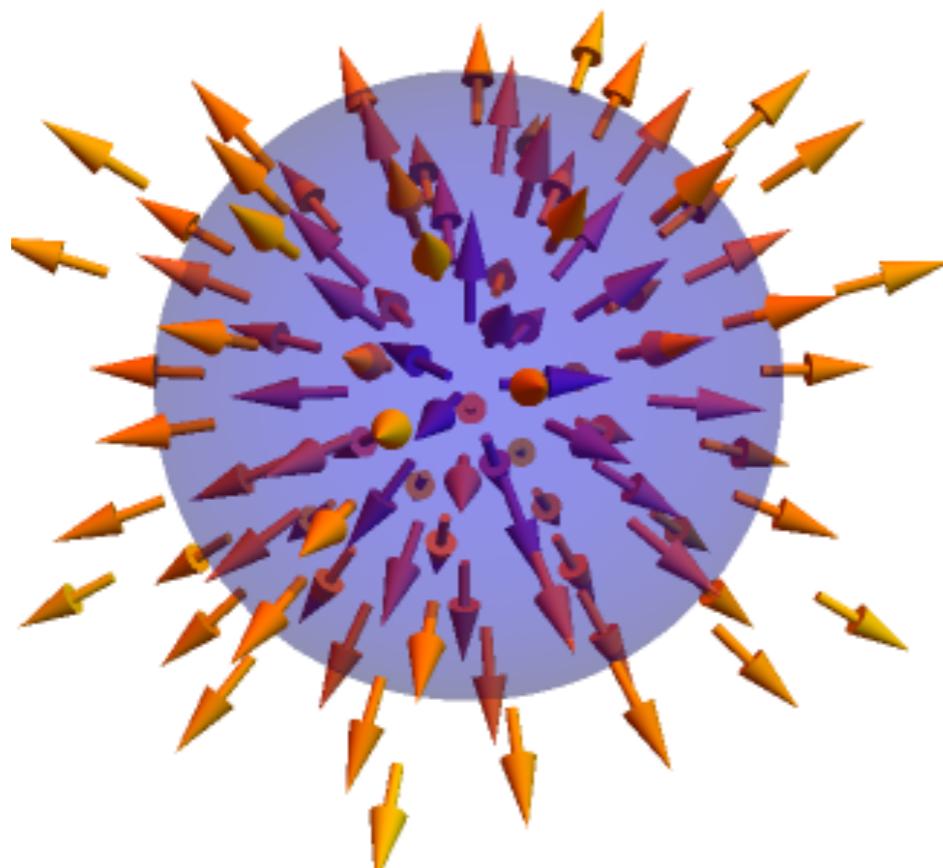
Through which surface(s) is(are) the flux equal to zero?

3D Flux

Question #21

Shown is the electric field due to a point charge. What is the flux through the blue sphere ($r = 0.5$ m).

For ease of calculation, let:



$$q = 5 \text{ C}$$

$$k = 1$$

A $\frac{20}{3}\pi$

C 20π

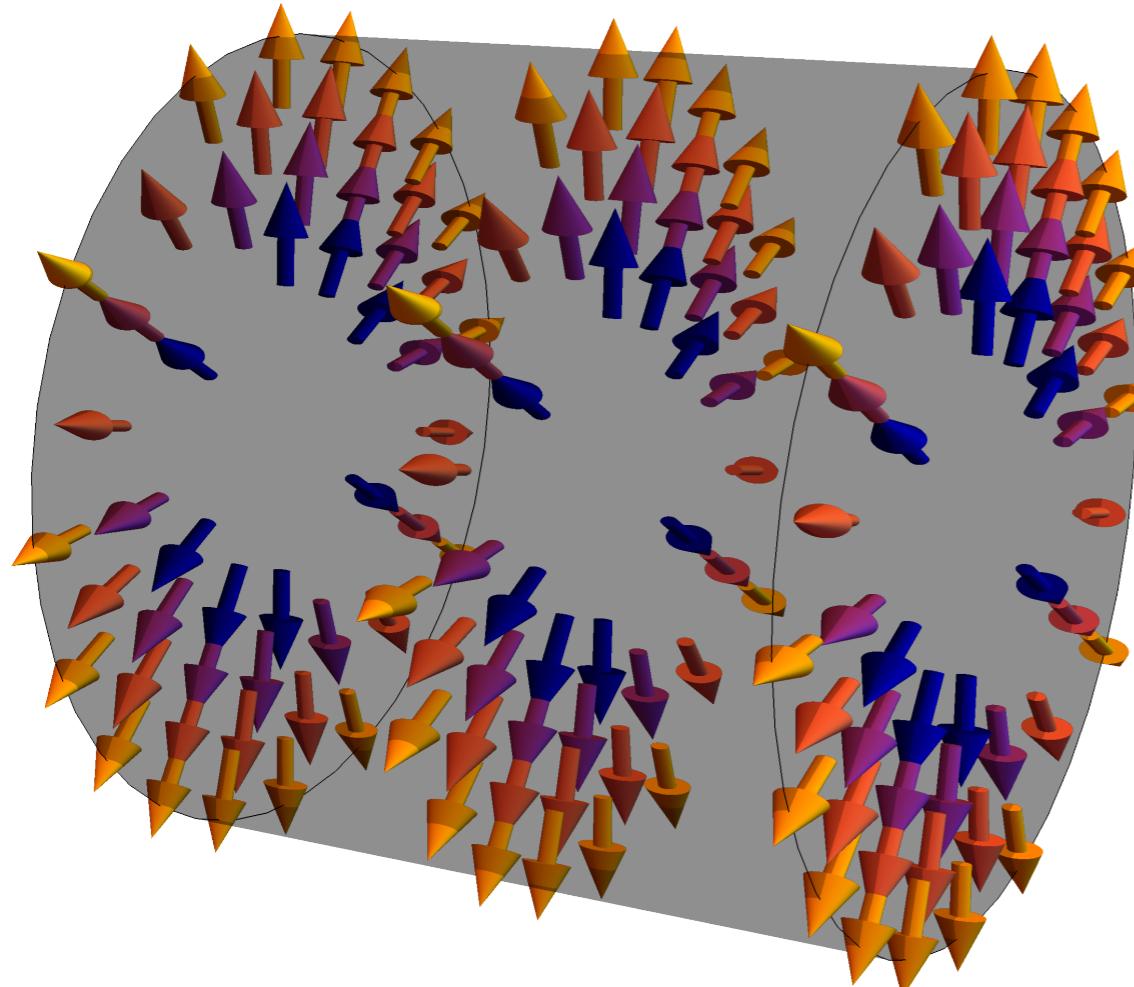
D $\frac{10}{3}\pi$

E 10π

Question #22

3D Flux

Shown is the electric field due to a very long line charge. What is the flux through the gray cylinder ($r = 0.5$ m).



$$E = \frac{2k|\lambda|}{r}$$

For ease of calculation, let:

$$L = 1 \text{ m}$$

$$\lambda = 5 \text{ C/m}$$

$$k = 1$$

A $\frac{20}{3}\pi$

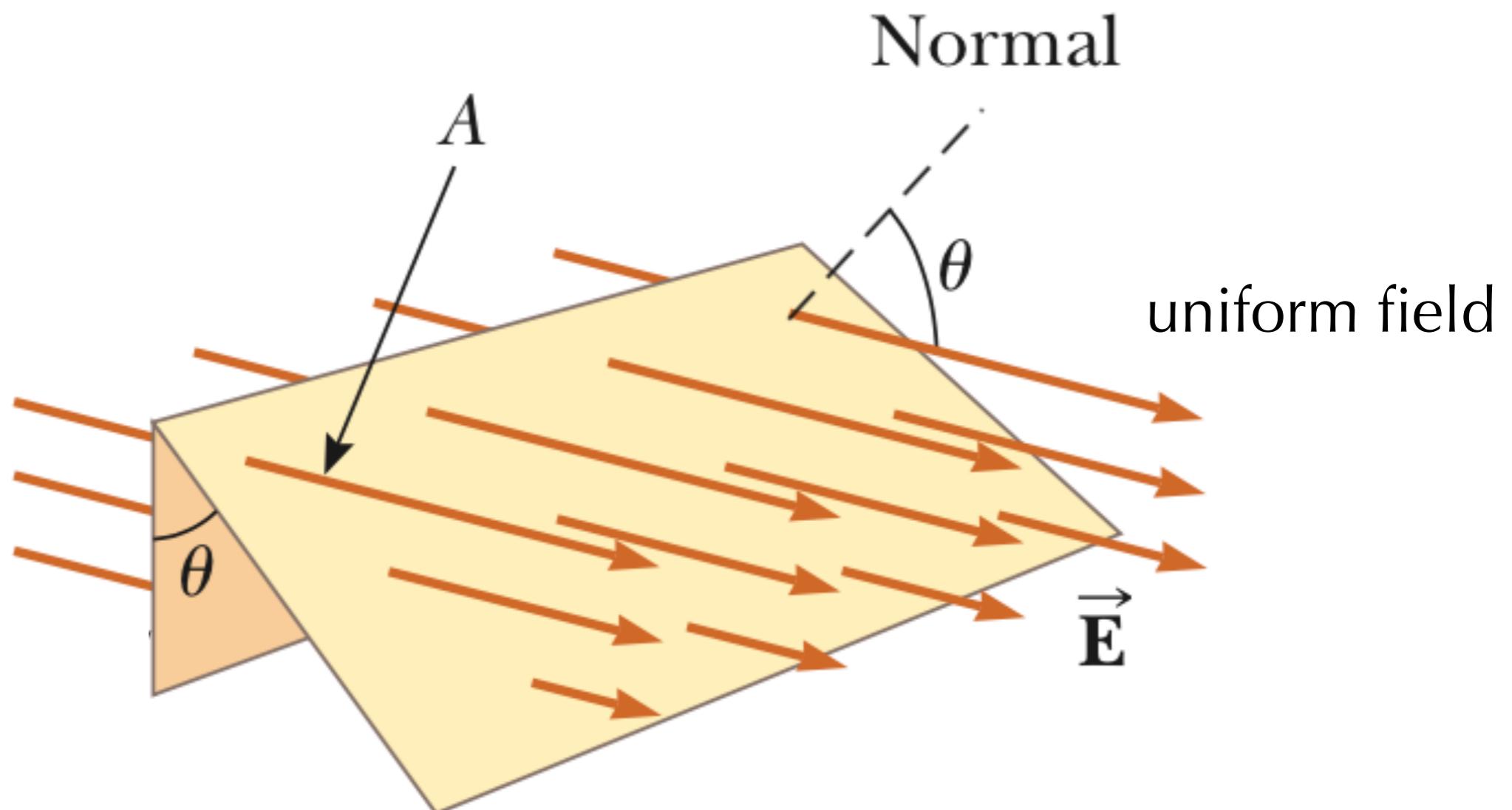
D 20π

B $\frac{10}{3}\pi$

E 10π

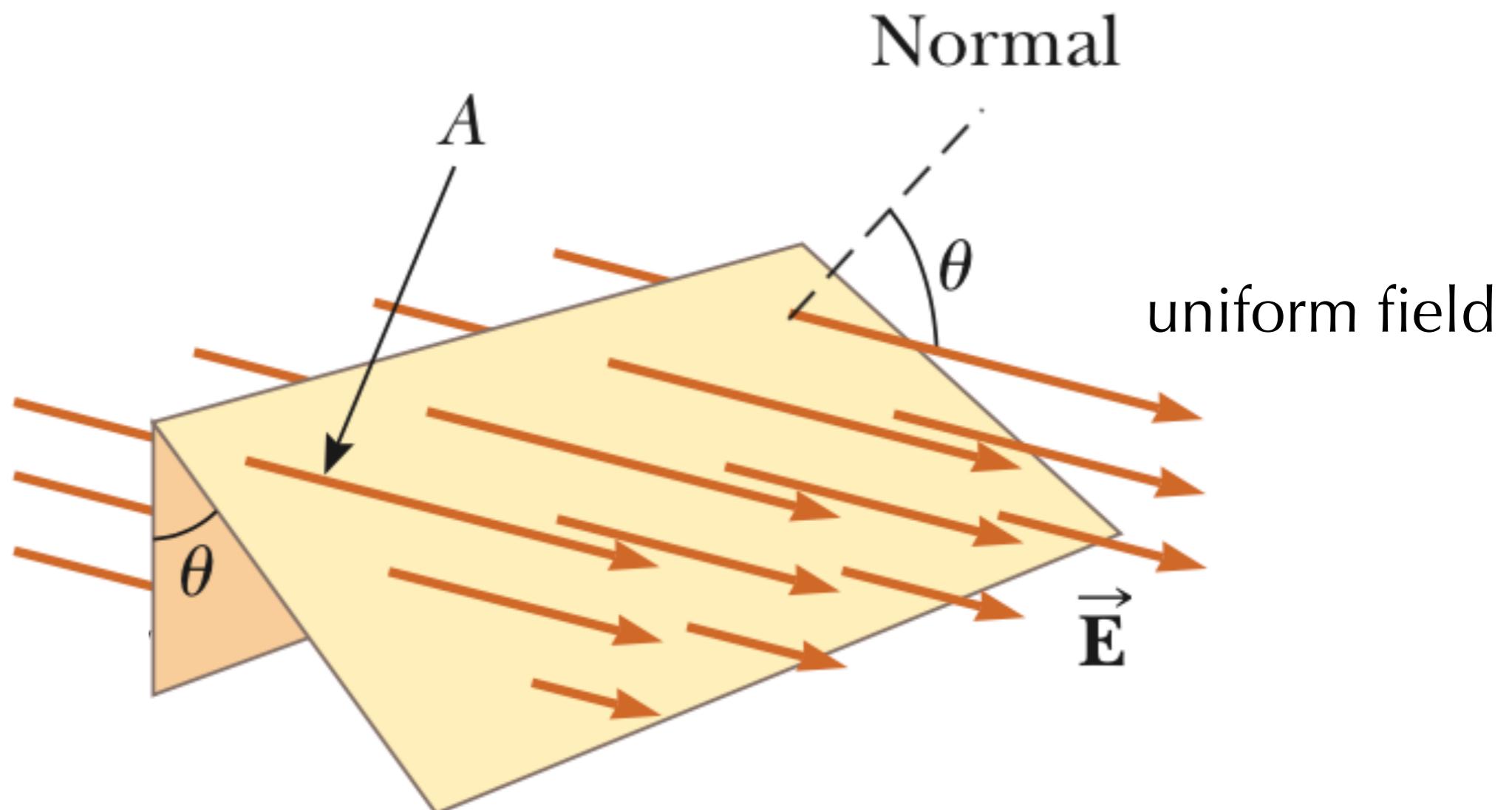
Uniform Field

What is the flux through surface A?



Uniform Field

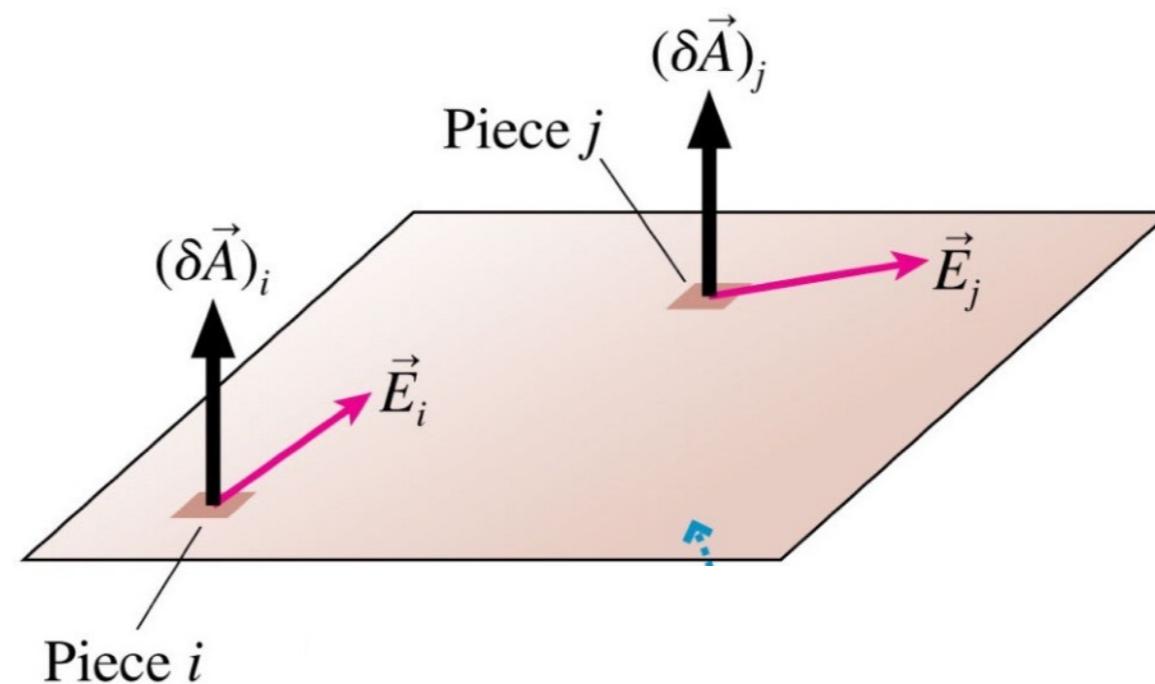
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$$\Phi = \vec{E} \cdot \vec{A}$$

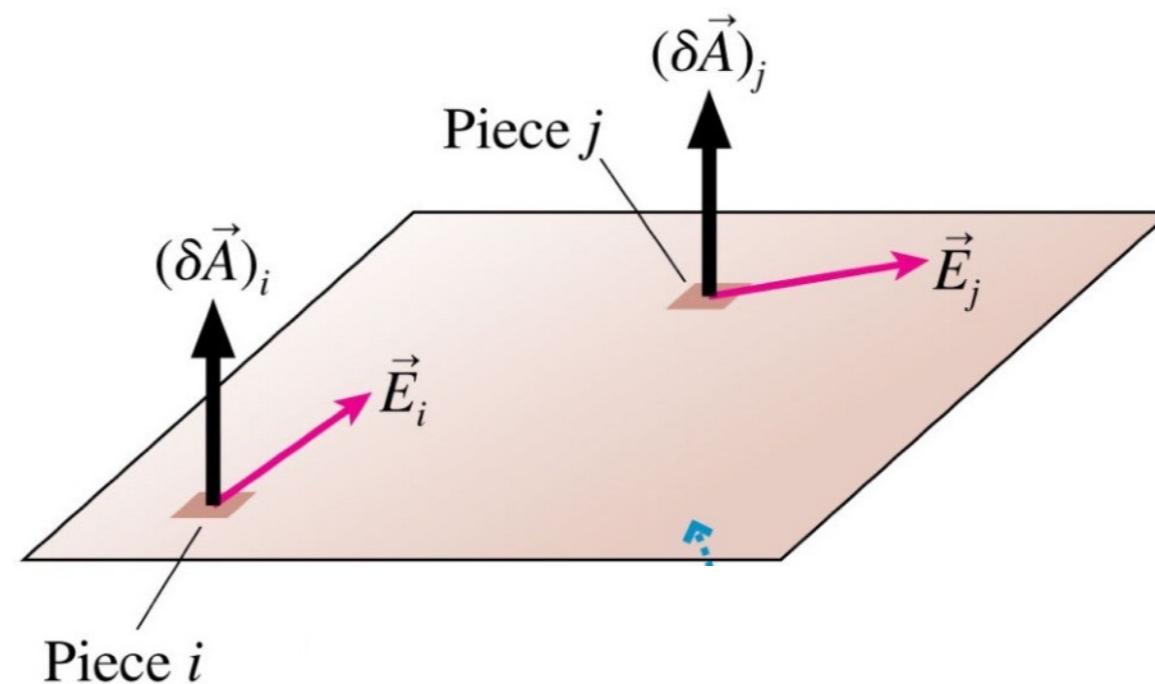
Non uniform field

Talk about how you would calculate the flux if the E field were not constant over the surface.



Non uniform field

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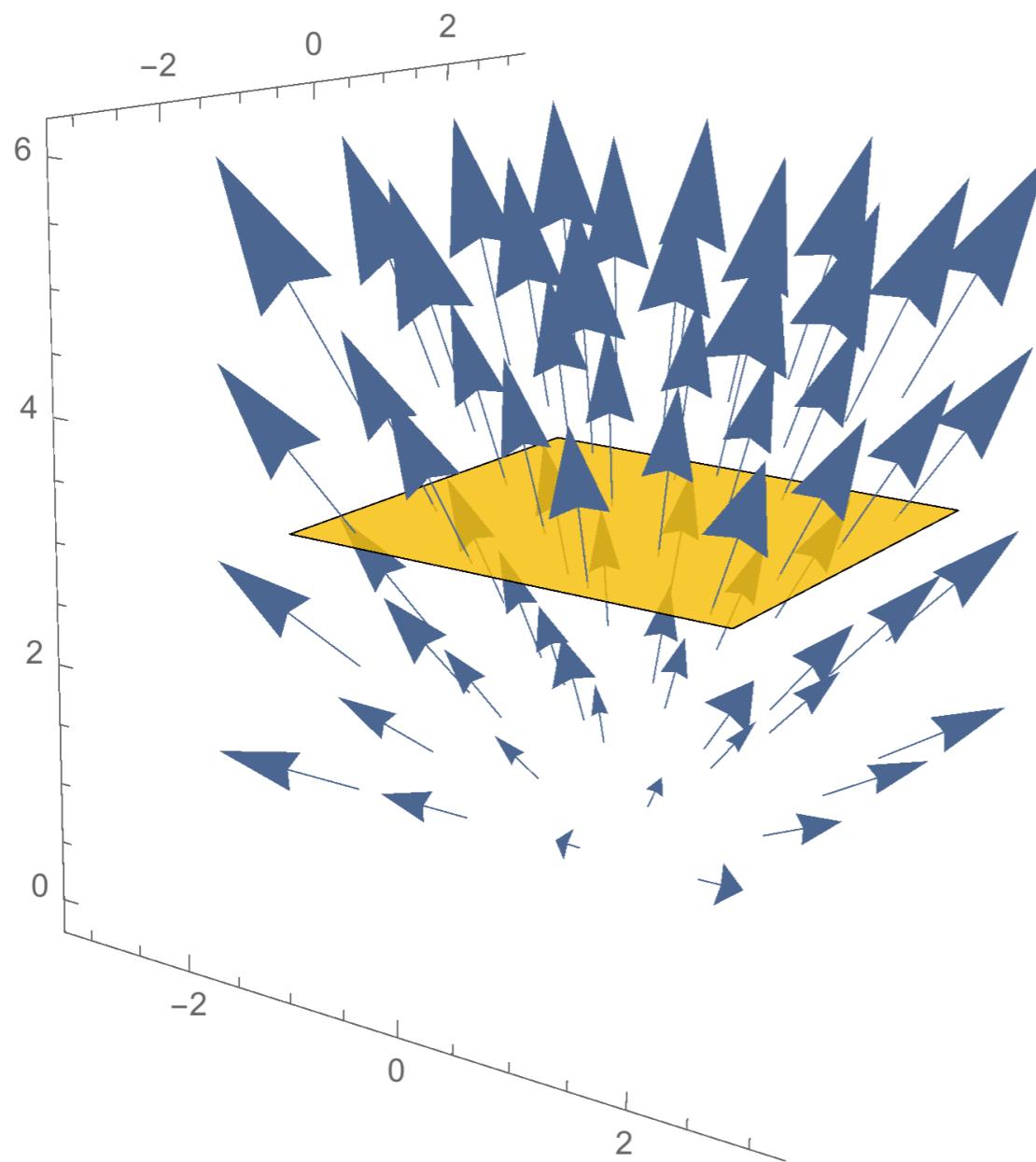


$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$$= \int_{\text{surface}} \vec{E} \cdot \hat{N} dS$$

Example

$$\vec{E} = x\hat{i} + y\hat{j} + z\hat{k}$$

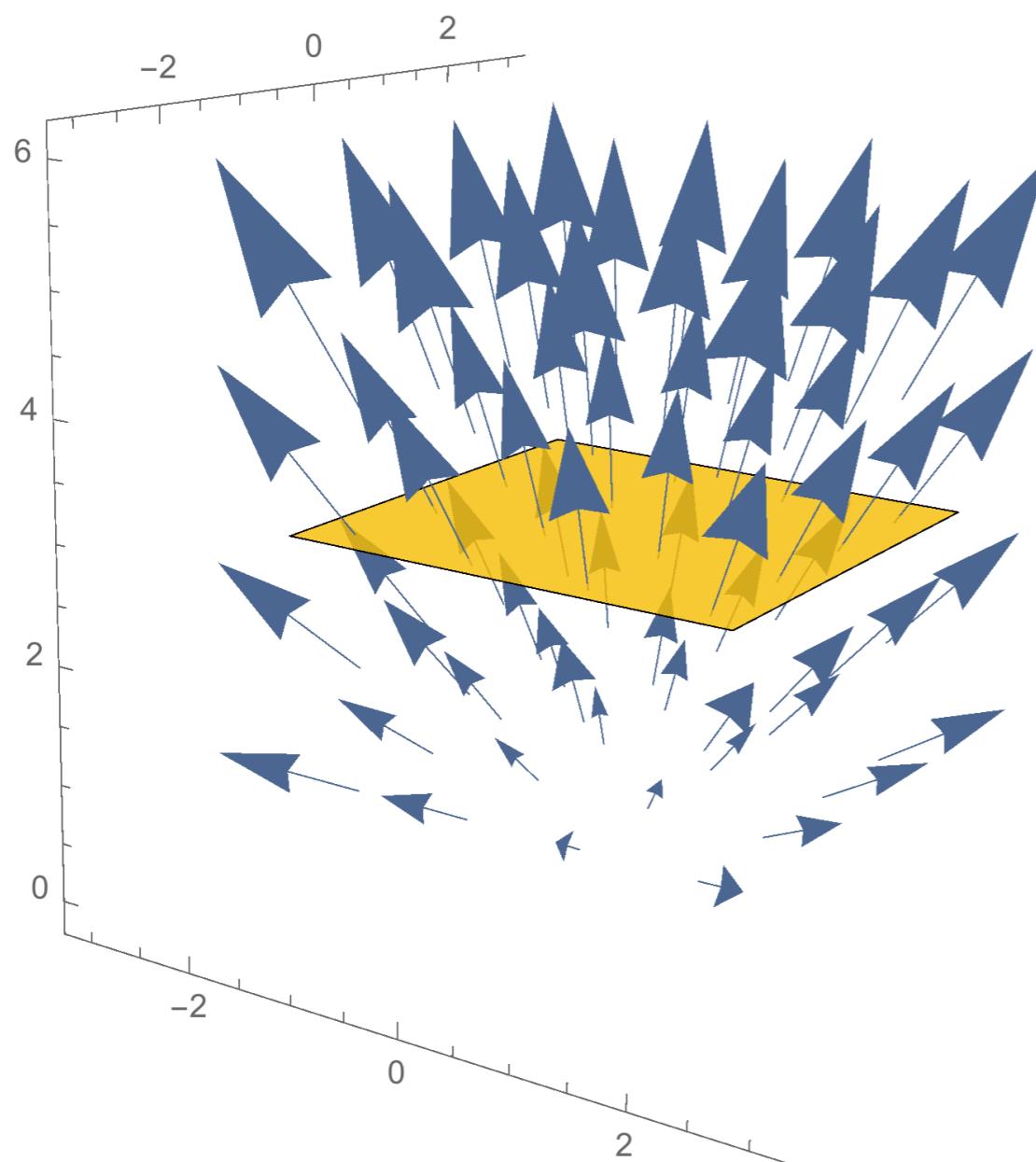


The surface is an origin-centered 4×4 square at a height $z = 3$.

What is the flux?

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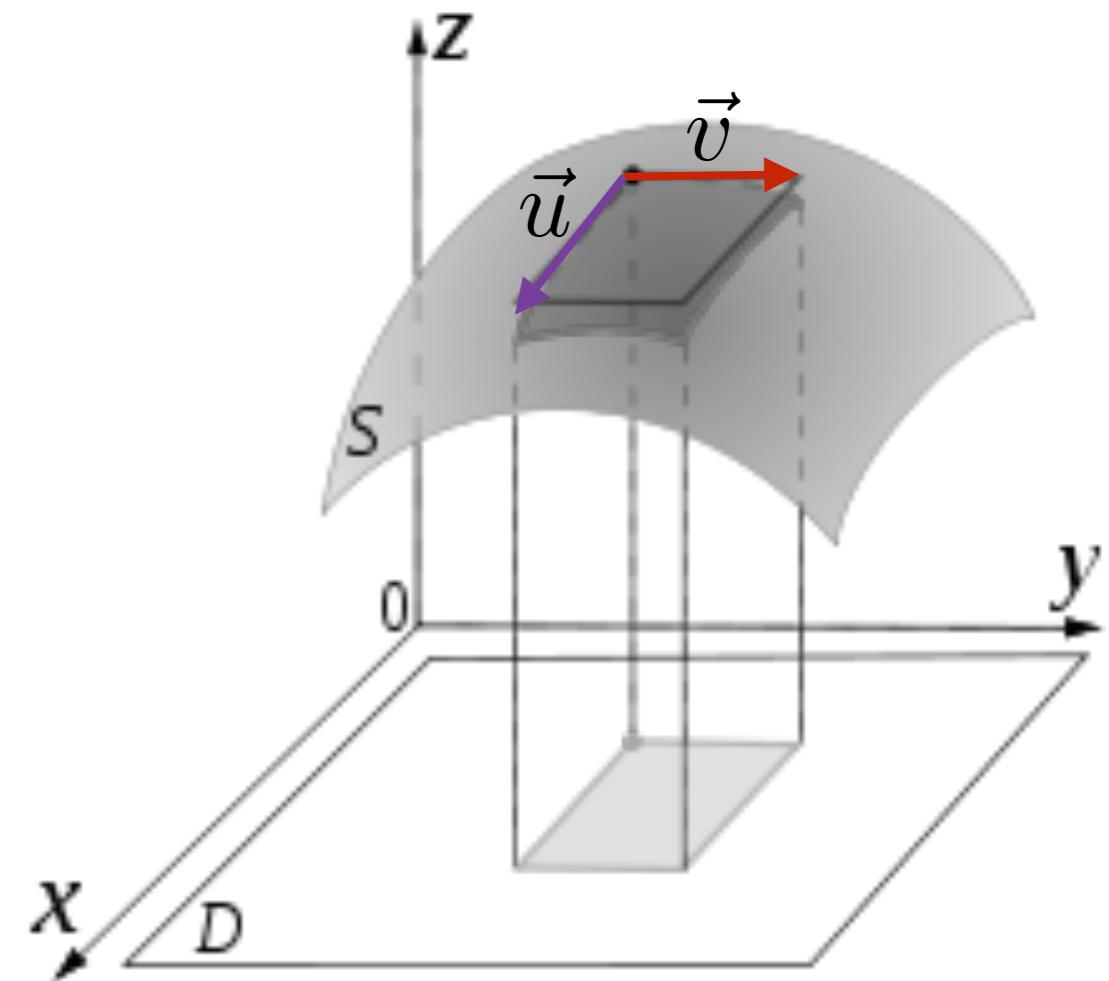


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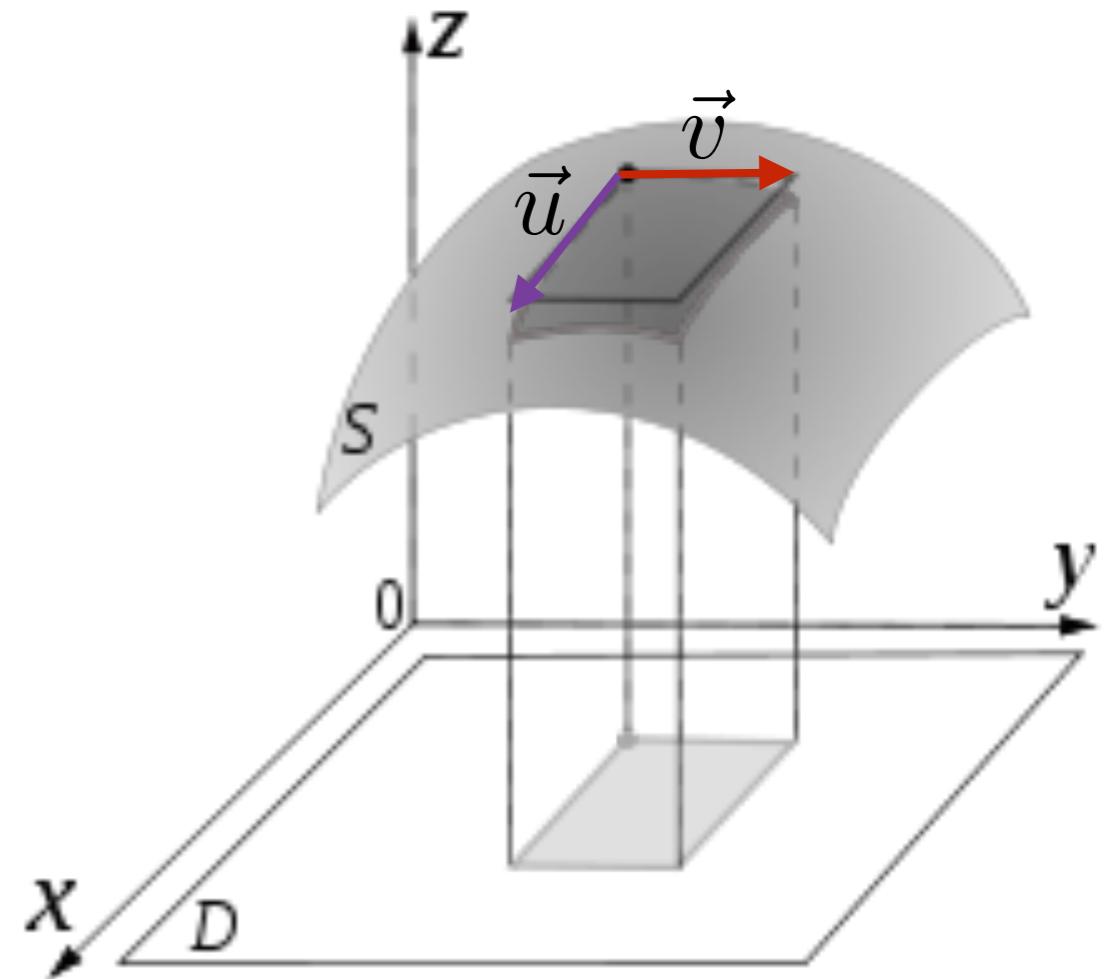
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Harder Flux Integrals (Calculus Review)



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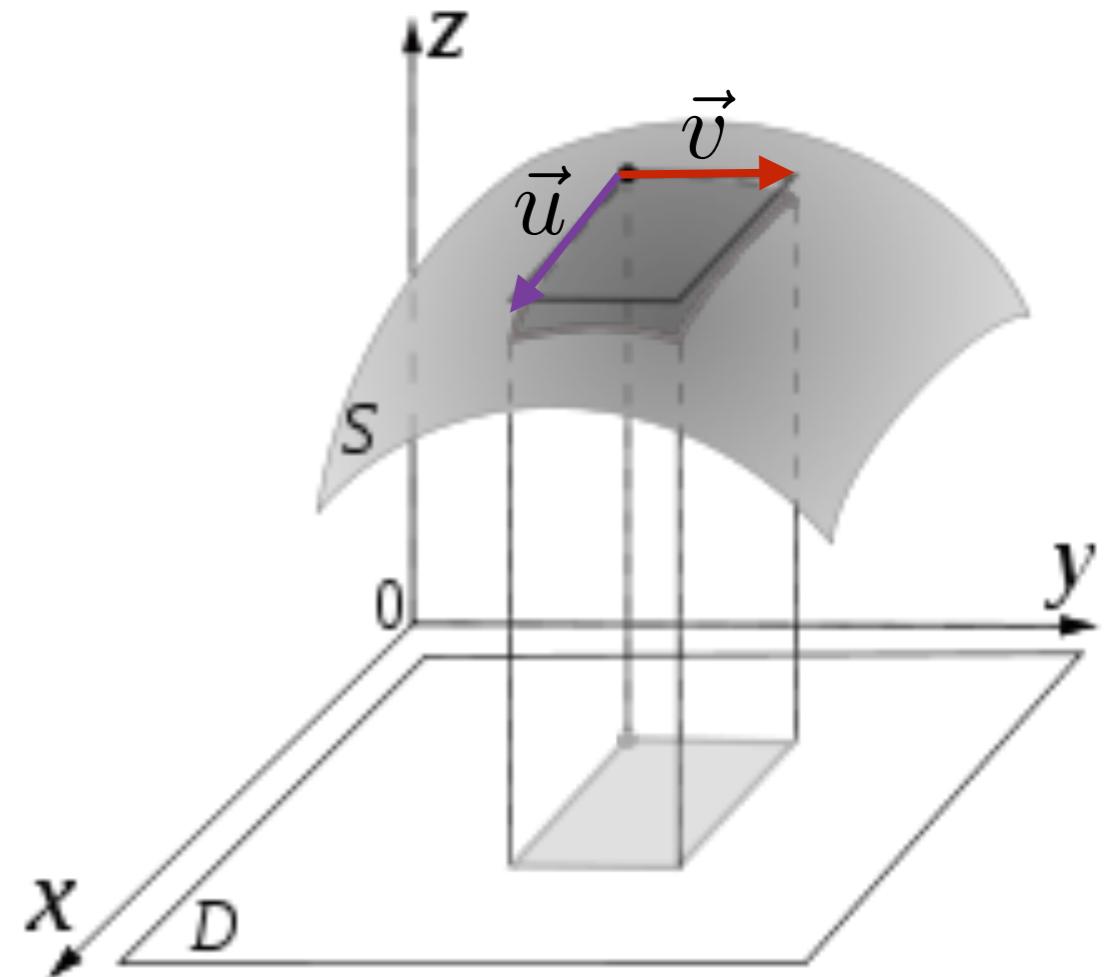
$$\vec{u} = \Delta x \hat{i} + \frac{df}{dx} \Delta x \hat{k}$$



Harder Flux Integrals (Calculus Review)

$$\vec{u} = \Delta x \hat{i} + \frac{df}{dx} \Delta x \hat{k}$$

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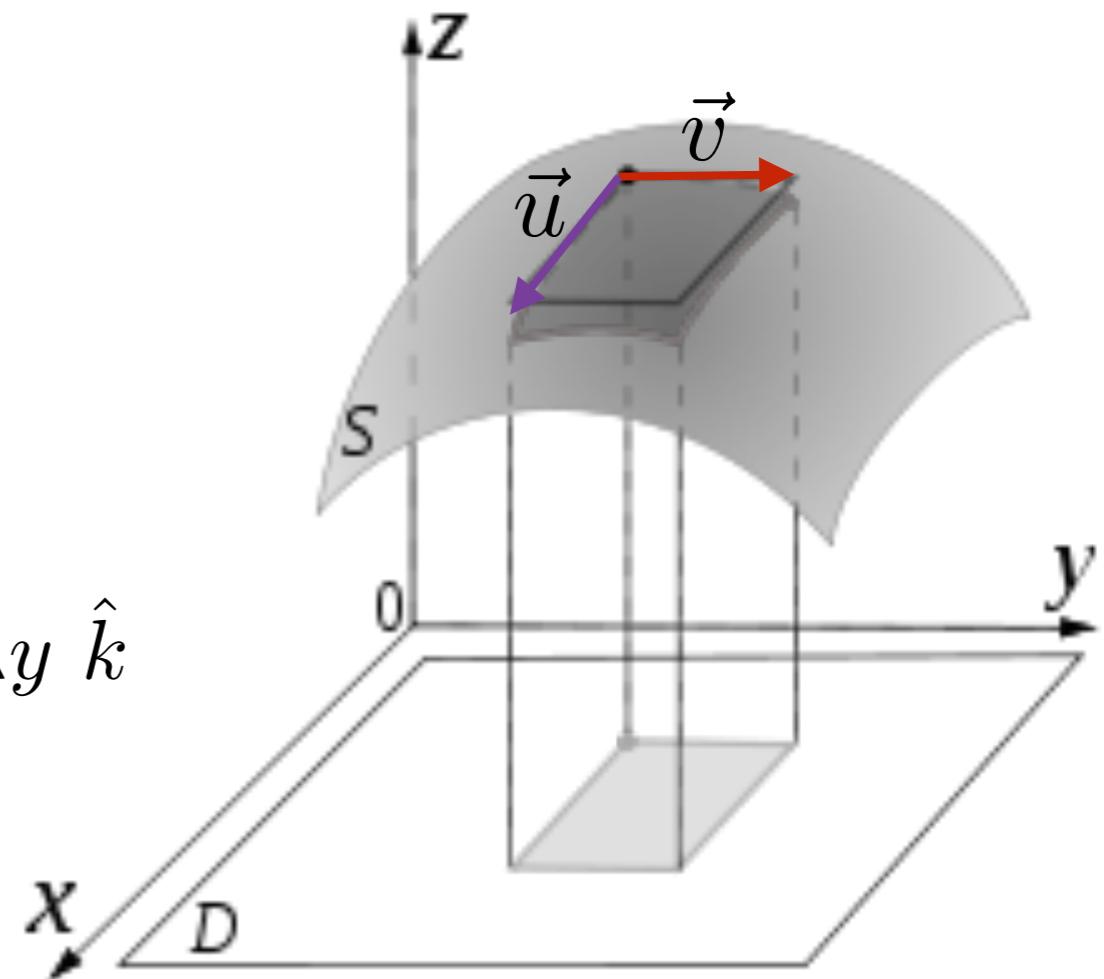


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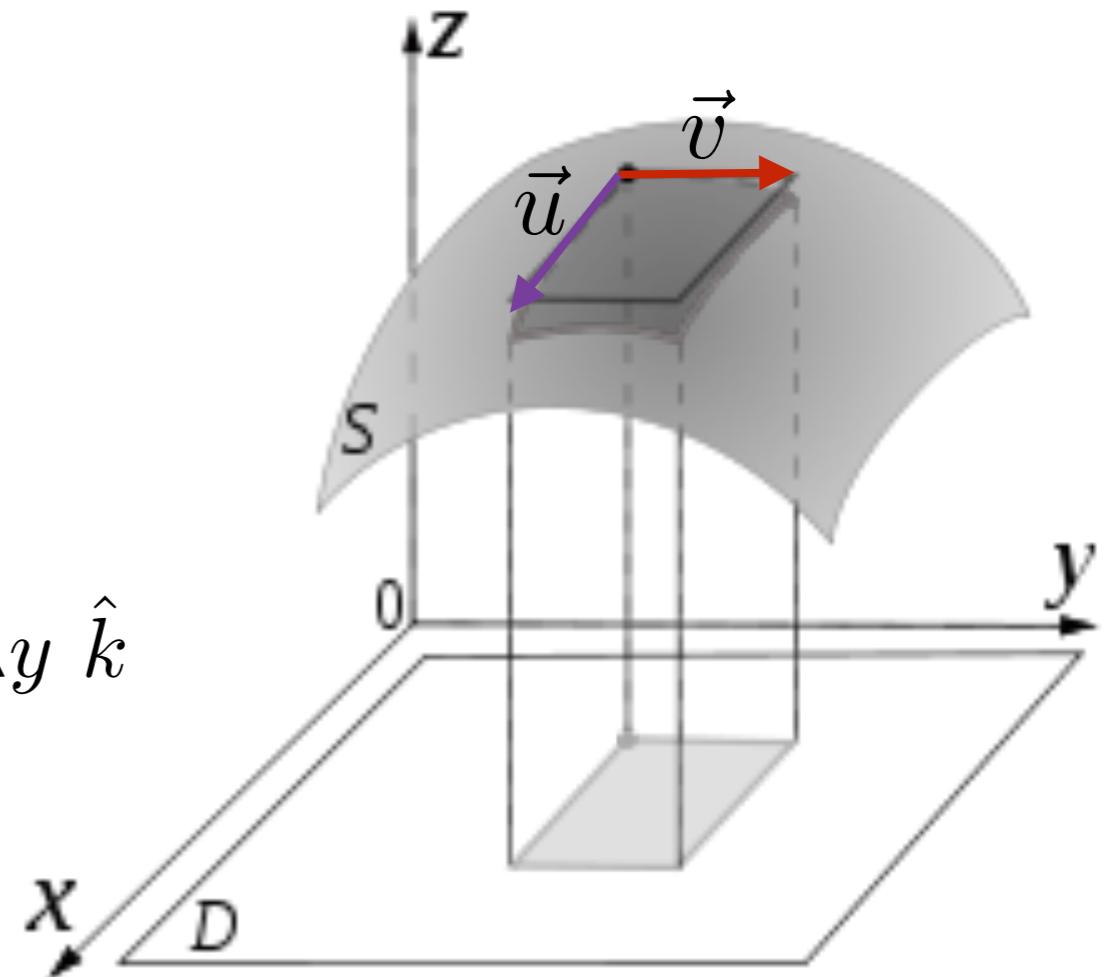
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$$= \left(-\frac{df}{dx} \hat{i} - \frac{df}{dy} \hat{j} + 1 \hat{k} \right) \Delta A$$



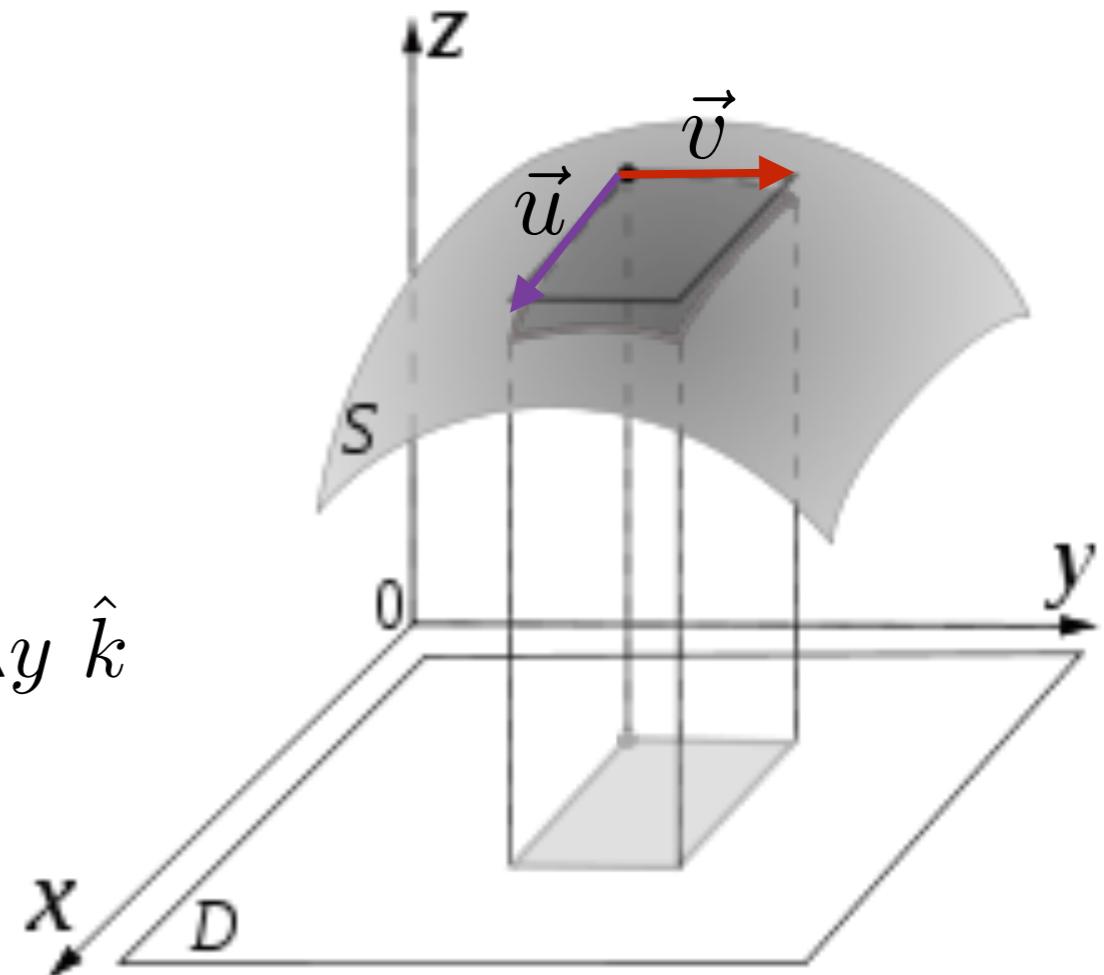
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$$|\vec{u} \times \vec{v}| = \Delta A \sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2 + 1}$$

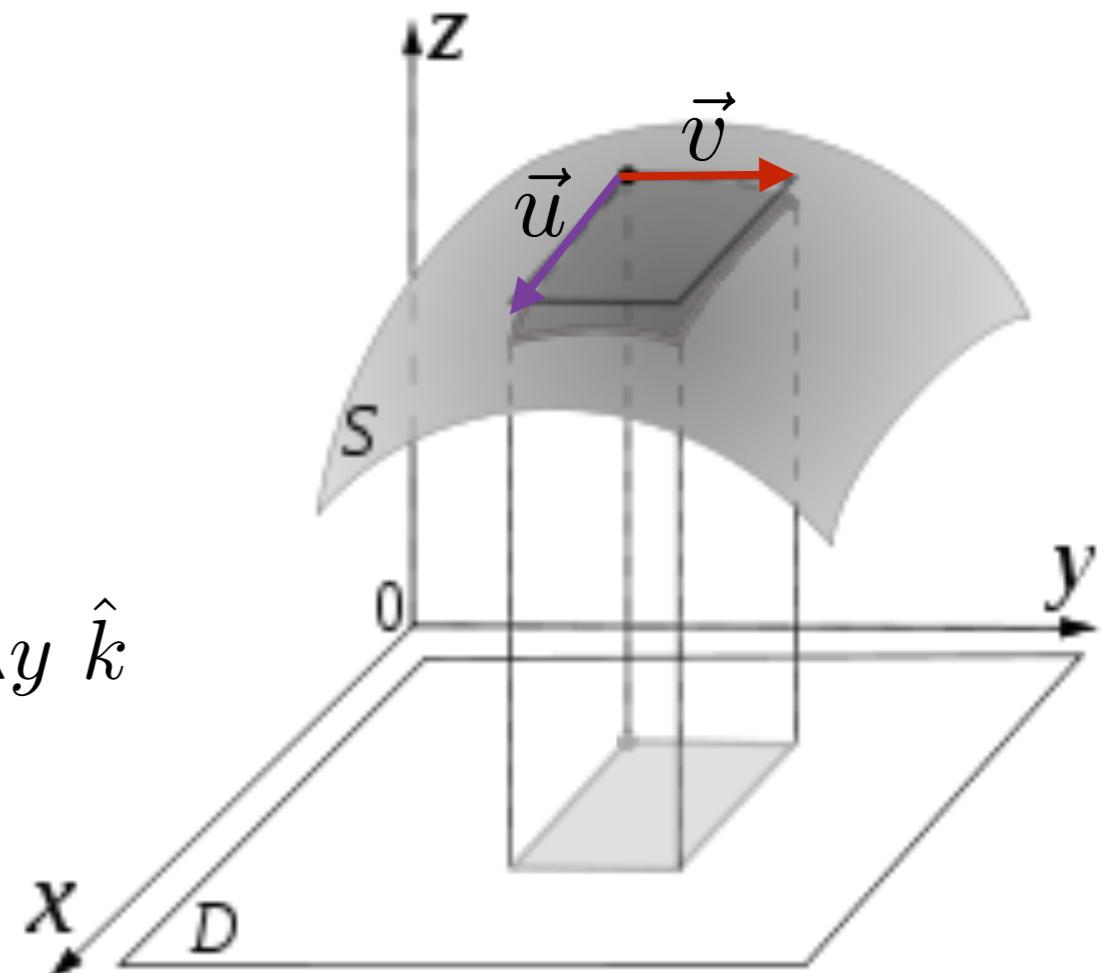
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$$dS = \sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2 + 1} dA$$

Harder Flux Integrals (Calculus Review)

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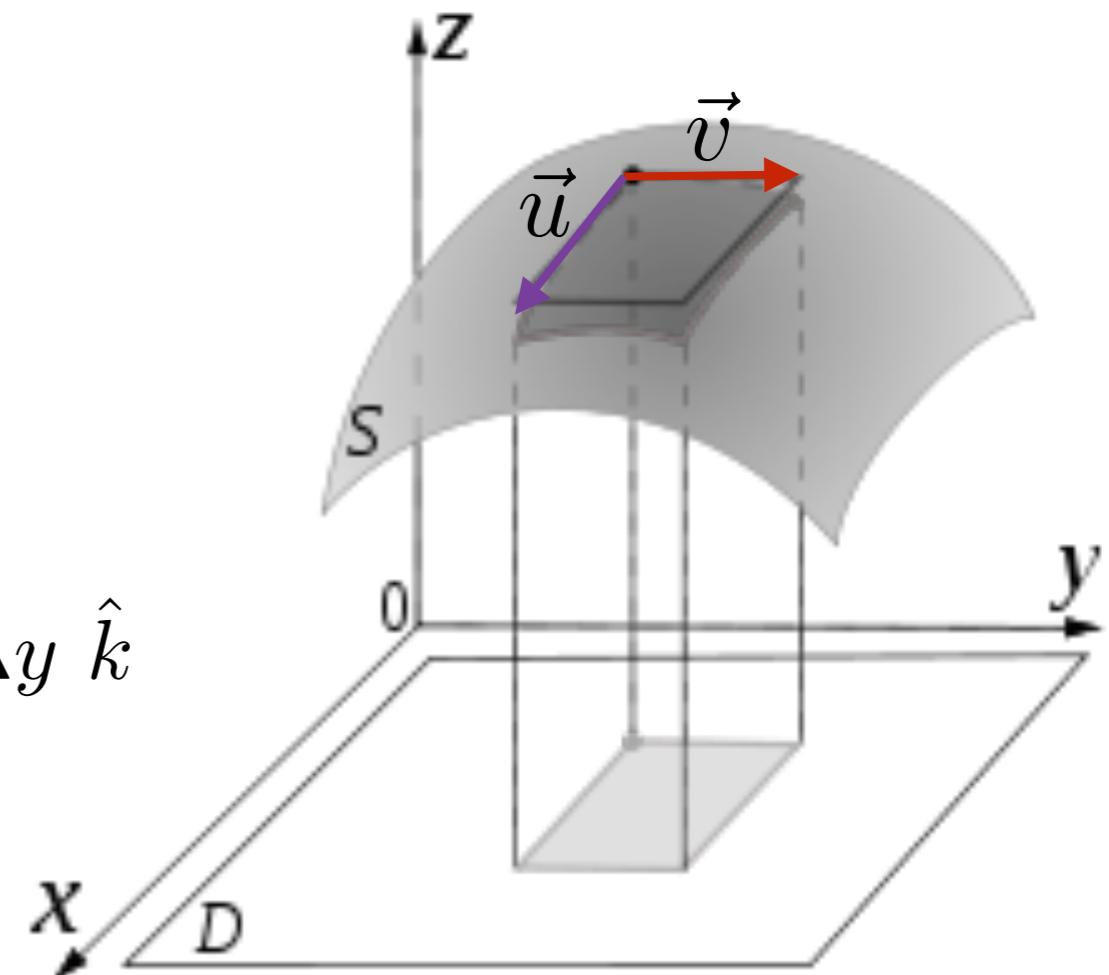
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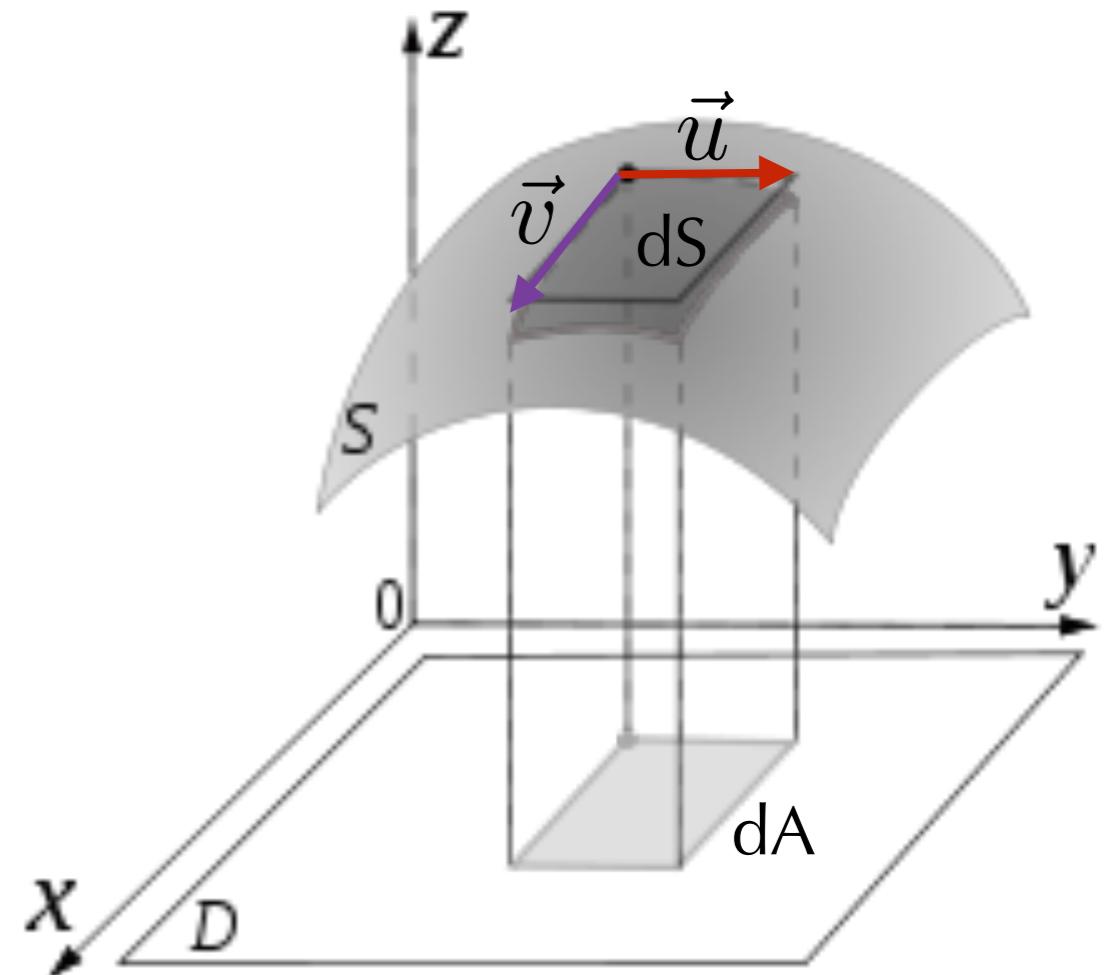
$$\Phi = \int_{\text{surface}} \vec{E} \cdot \hat{N} dS$$

$$\int \vec{E} \cdot \hat{\vec{N}} dS$$

$$\hat{\vec{N}} = \frac{\nabla f(x, y, z)}{|\nabla f(x, y, z)|}$$

$$= \frac{-\frac{df}{dx}\hat{i} - \frac{df}{dy}\hat{j} + 1\hat{k}}{\sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2 + 1}}$$

$$\int \vec{E} \cdot \hat{\vec{N}} dS = \int \vec{E} \cdot \left(-\frac{df}{dx}\hat{i} - \frac{df}{dy}\hat{j} + 1\hat{k}\right) dA$$



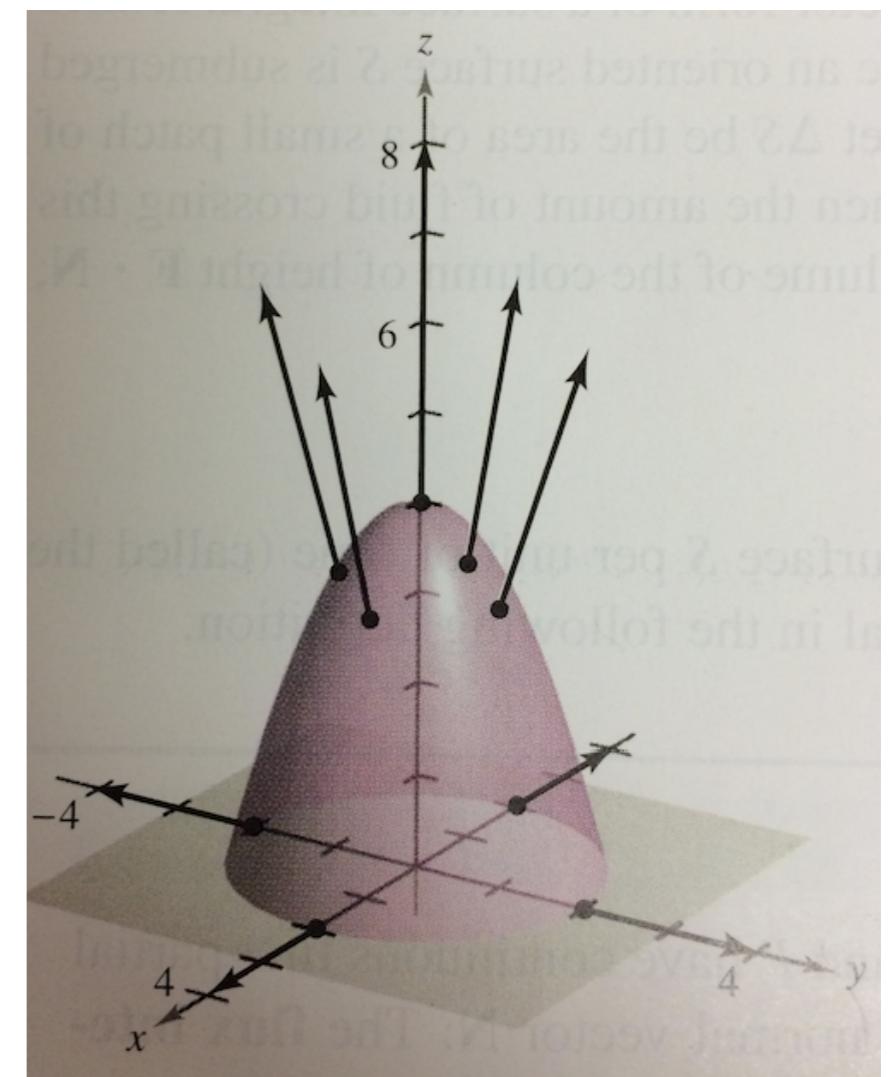
Let S be the portion of the paraboloid

$$z = 4 - x^2 - y^2$$

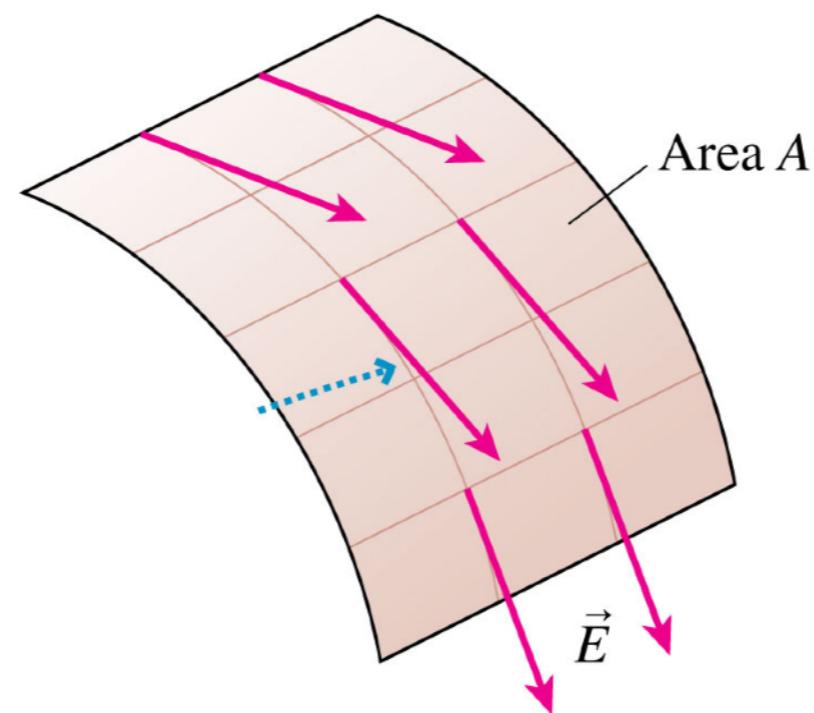
lying above the xy -plane, oriented by an upward unit normal vector, as shown in the figure. The Electric field in the region is given by:

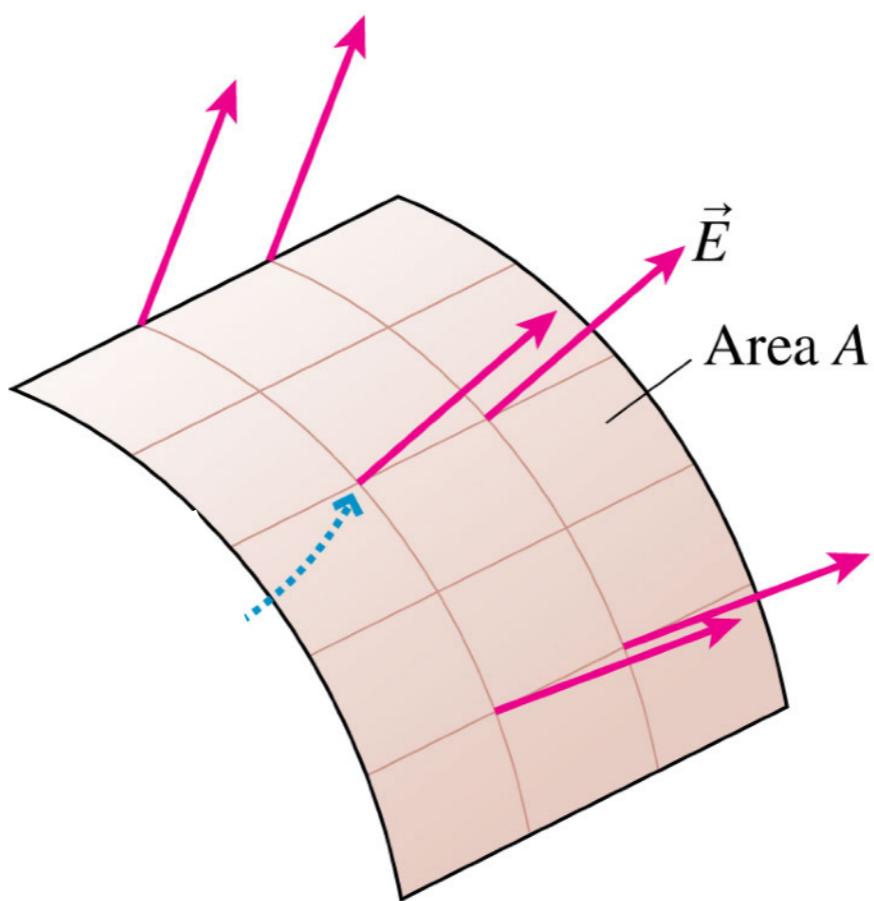
$$\vec{E}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$$

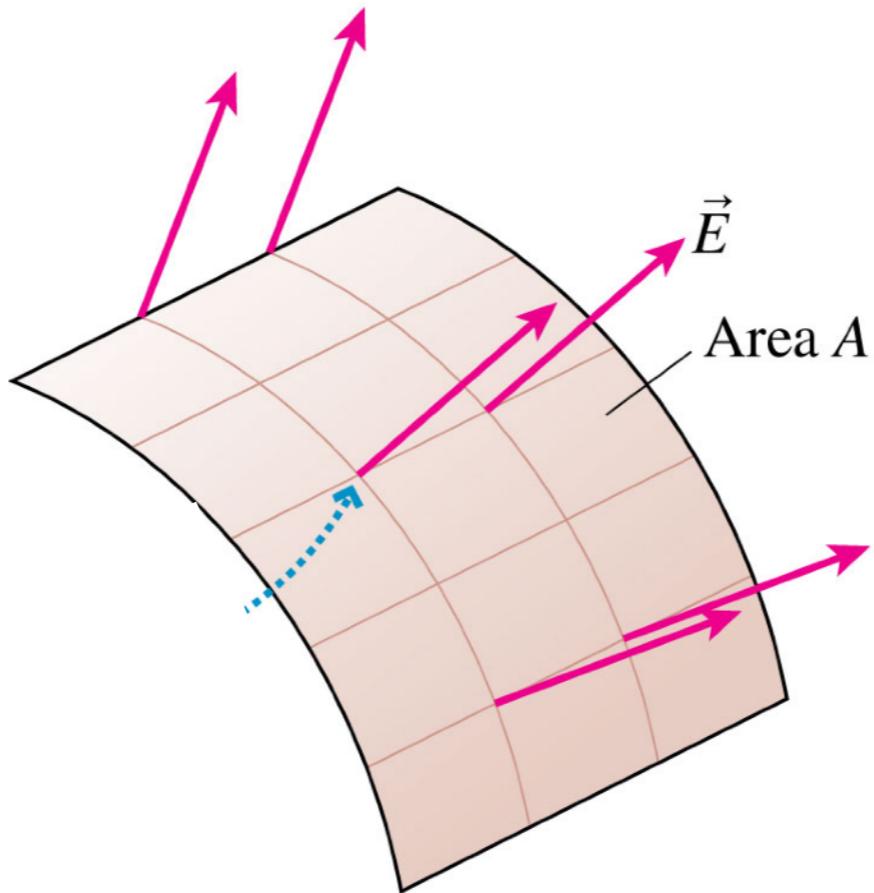
Find the electric flux through surface S



What is the flux?





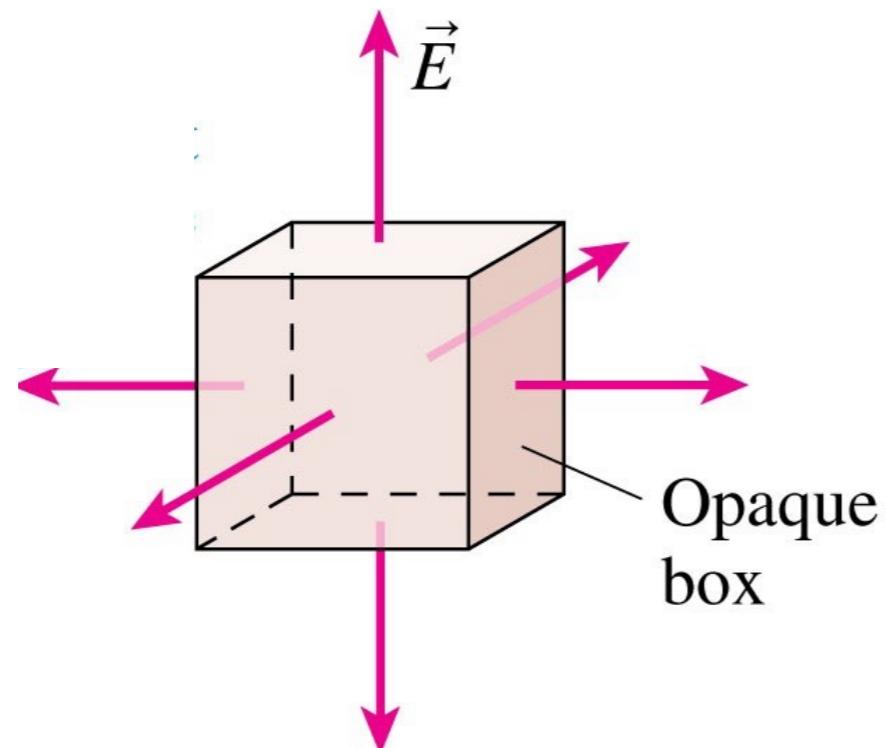


$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E dA = E \int_{\text{surface}} dA = EA$$

Why Flux? What does it even tell us?

You can't see inside the box, all you know is there an outward-pointing electric field at each face.

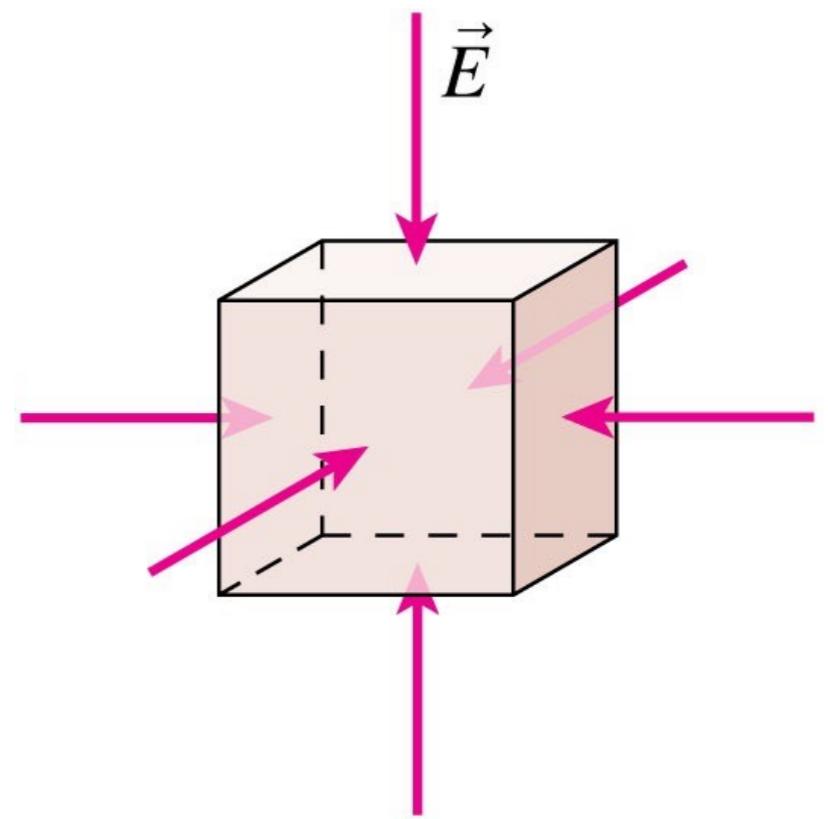
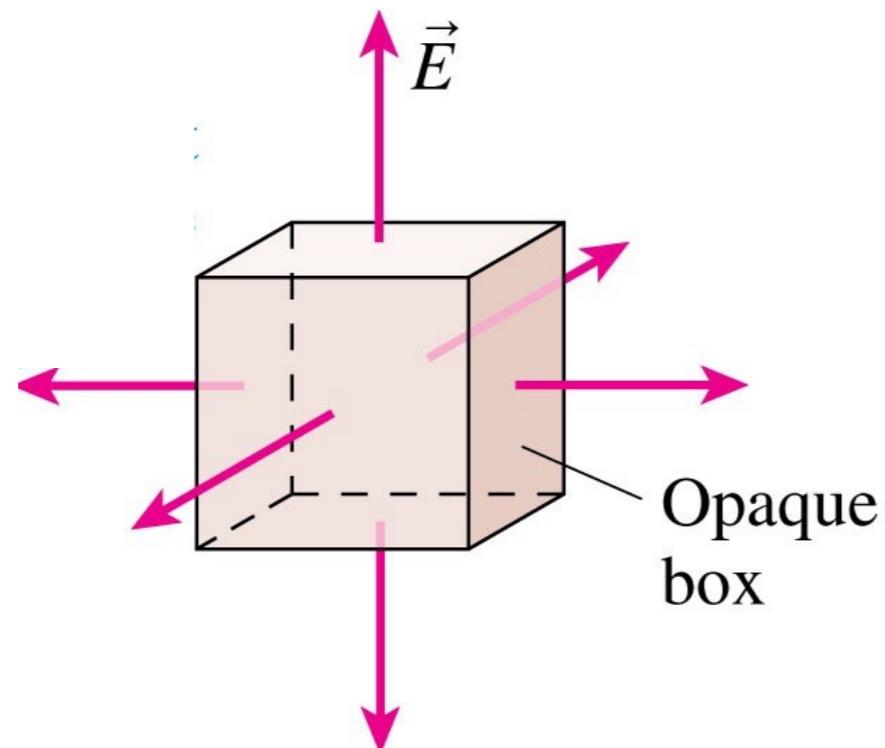
What can you tell about the charge that is inside the box?



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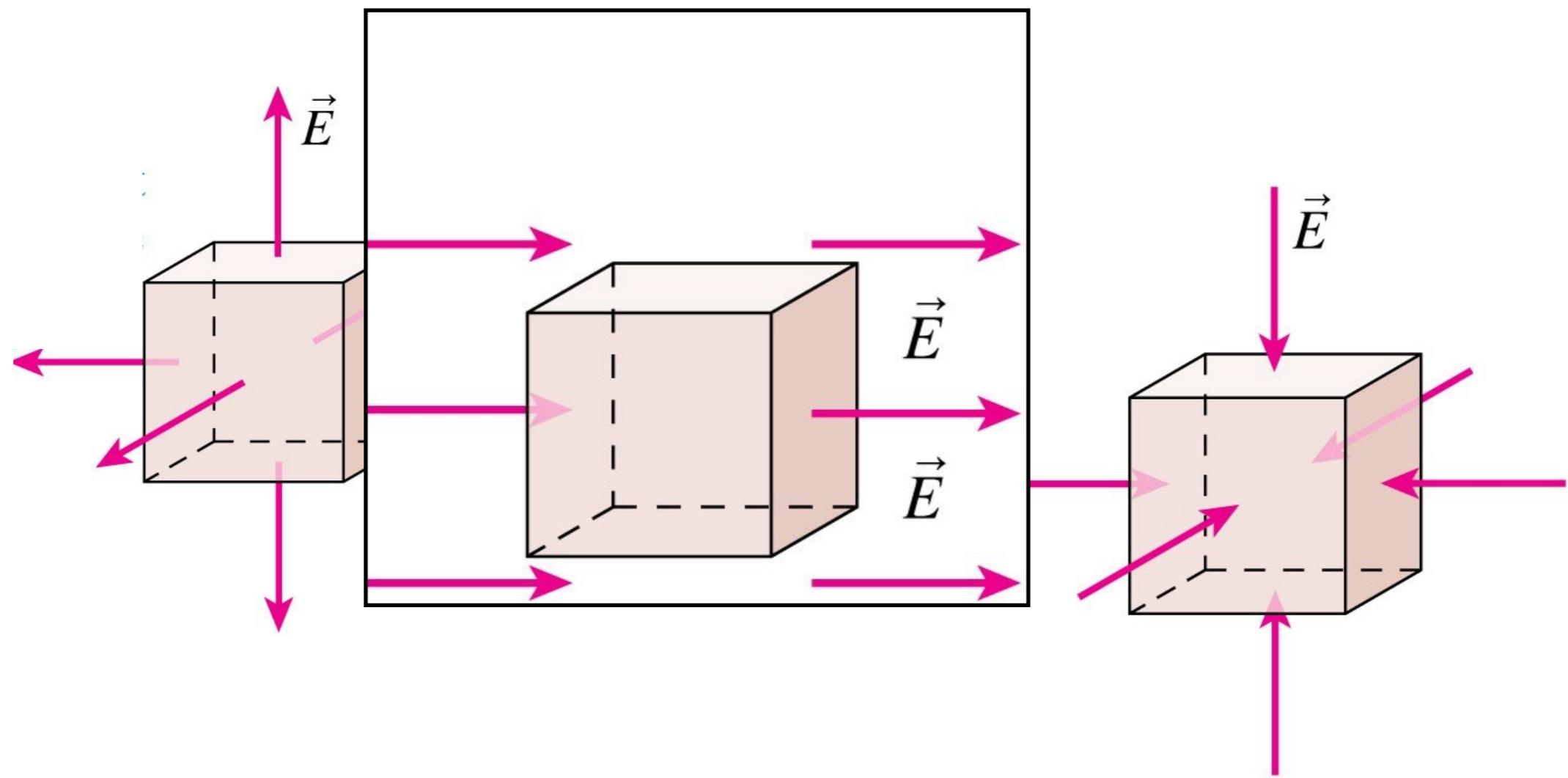
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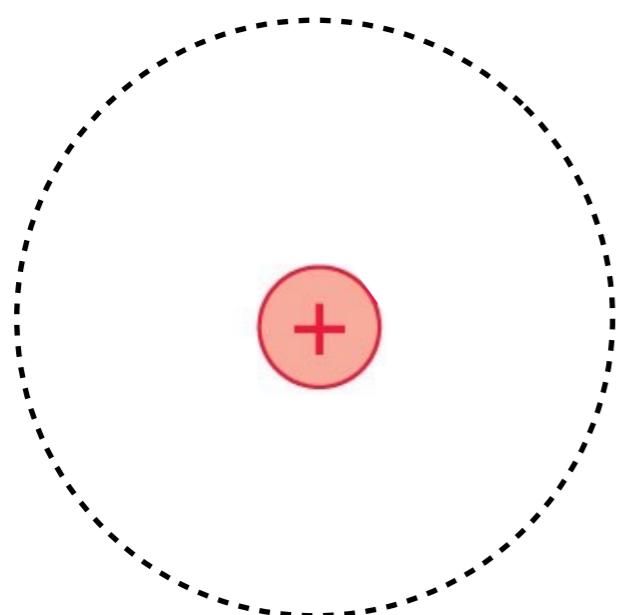
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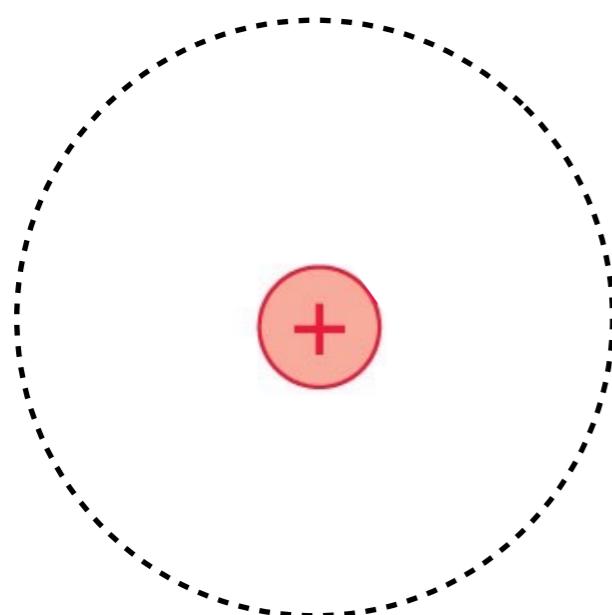


Push a button on your clicker after answering each question.



- A. Draw several electric field lines at the gaussian surface.
- B. Are the magnitudes of these electric field vectors the same or different.
- C. Calculate the electric flux through this surface.
- D. Reduce your expression as far as you can.

Push a button on your clicker after answering each question.



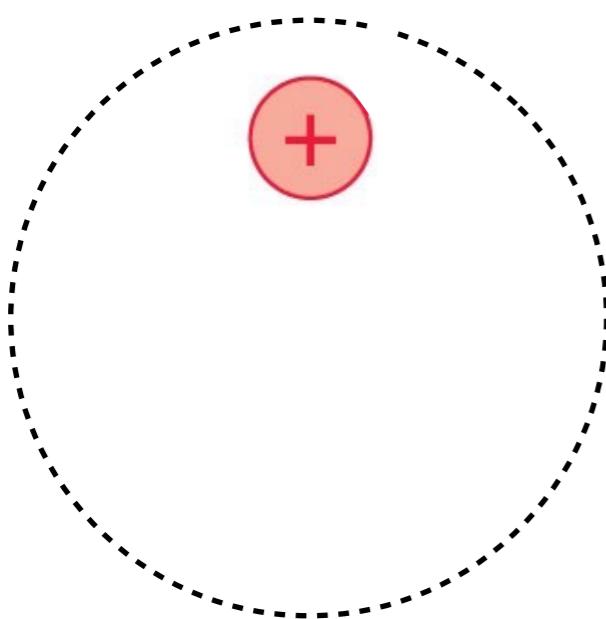
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Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

How/When is Gauss's law helpful

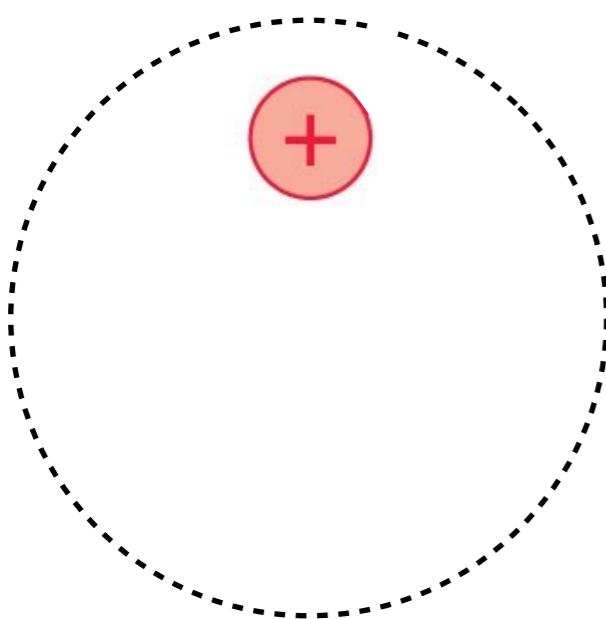
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How/When is Gauss's law helpful

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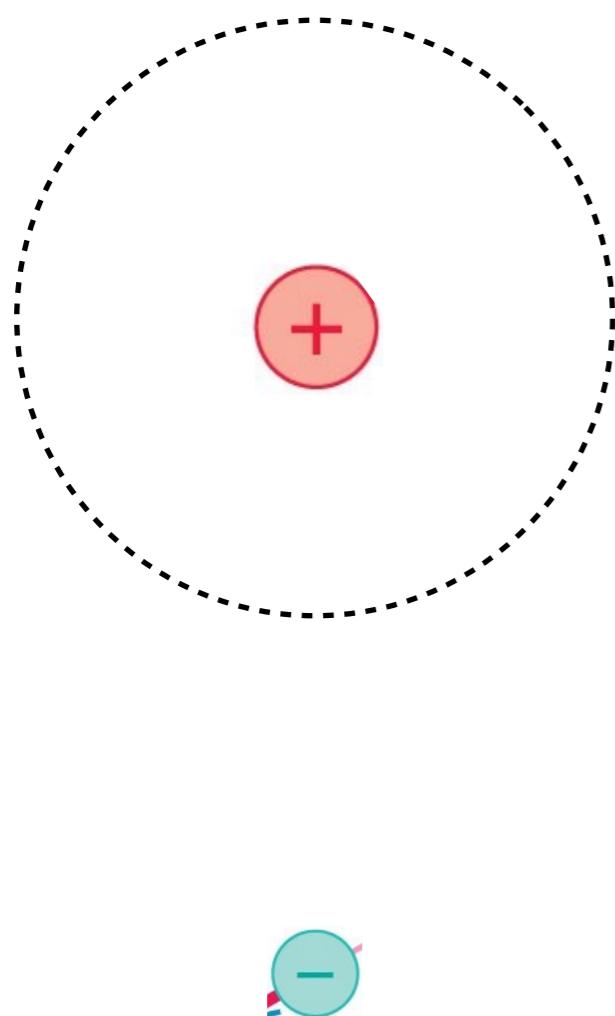
Is Gauss's Law helpful if I choose this surface?



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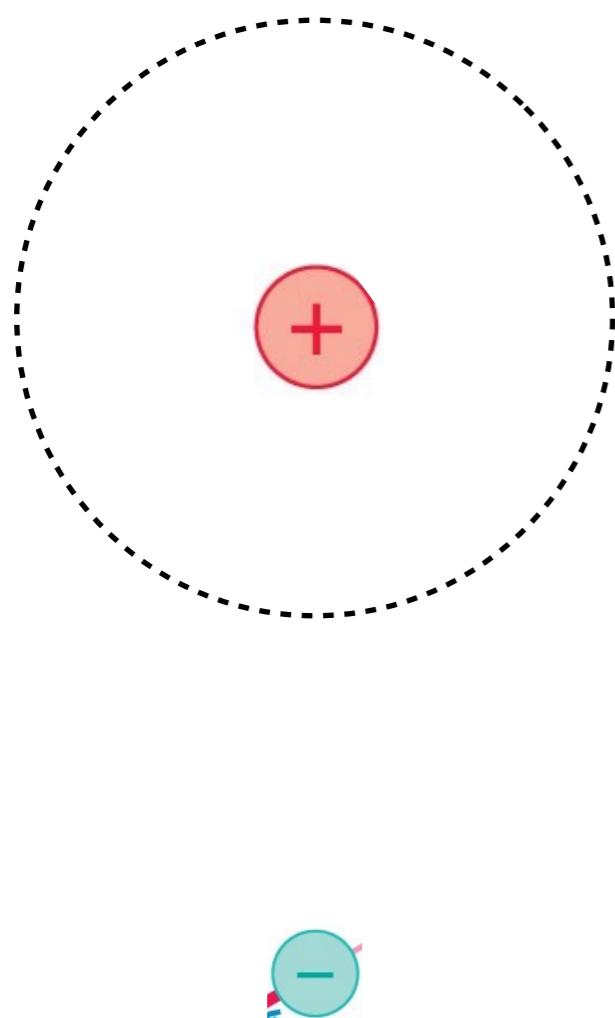


Is Gauss's Law helpful for this charge distribution?

How/When is Gauss's law helpful

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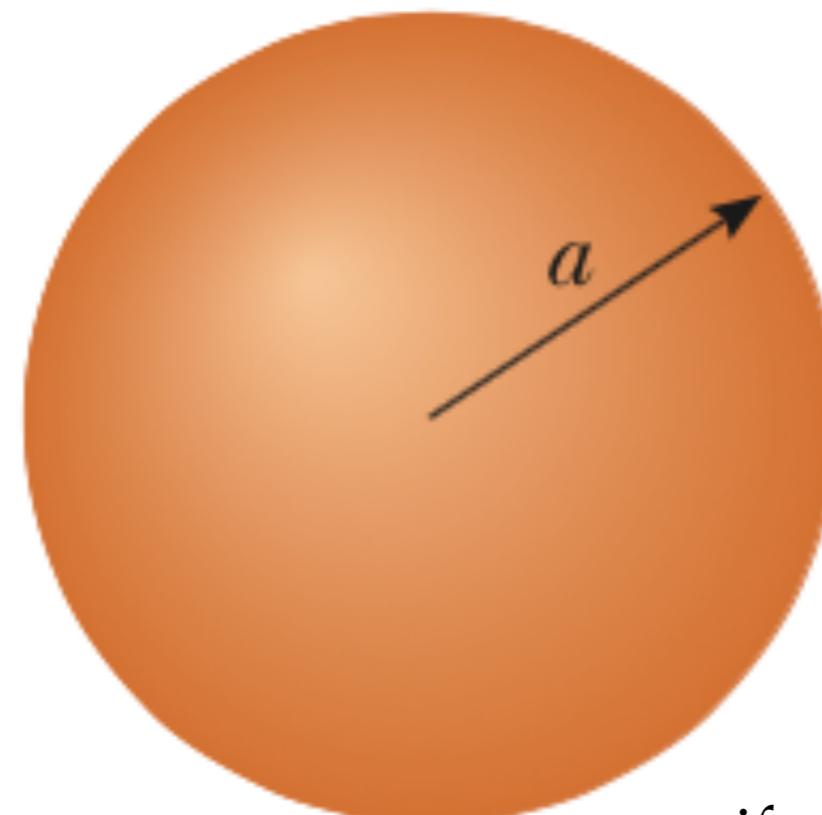


Is Gauss's Law helpful for this charge distribution?

Gauss's Law is most useful when the surface is chosen such that the integral becomes trivial (easy).

What direction must the E field point?

What is the electric field inside and outside of the sphere



uniform charge distribution