Outcomes:

- Analyze symmetry of charge distributions to determine direction of electric field.
- Assemble integrals for finding electric fields of continuous charge distributions.

Activities:

- Line charge on symmetry axis
- Line charge off symmetry axis
- Ring on symmetry axis
- Disk on symmetry axis
- Disk off symmetry axis (hard integral)


Lance Nelson

"…..I learned from the scriptures that my conduct and my attitude on the Sabbath constituted a sign between me and my Heavenly Father. With that understanding, I no longer needed lists of dos and don'ts. When I had to make a decision whether or not an activity was appropriate for the Sabbath, I simply asked myself, "What sign do I want to give to God?" That question made my choices about the Sabbath day crystal clear."

Elder Russel M. Nelson

April 2015 General Conference

Steps/tips when setting up integrals

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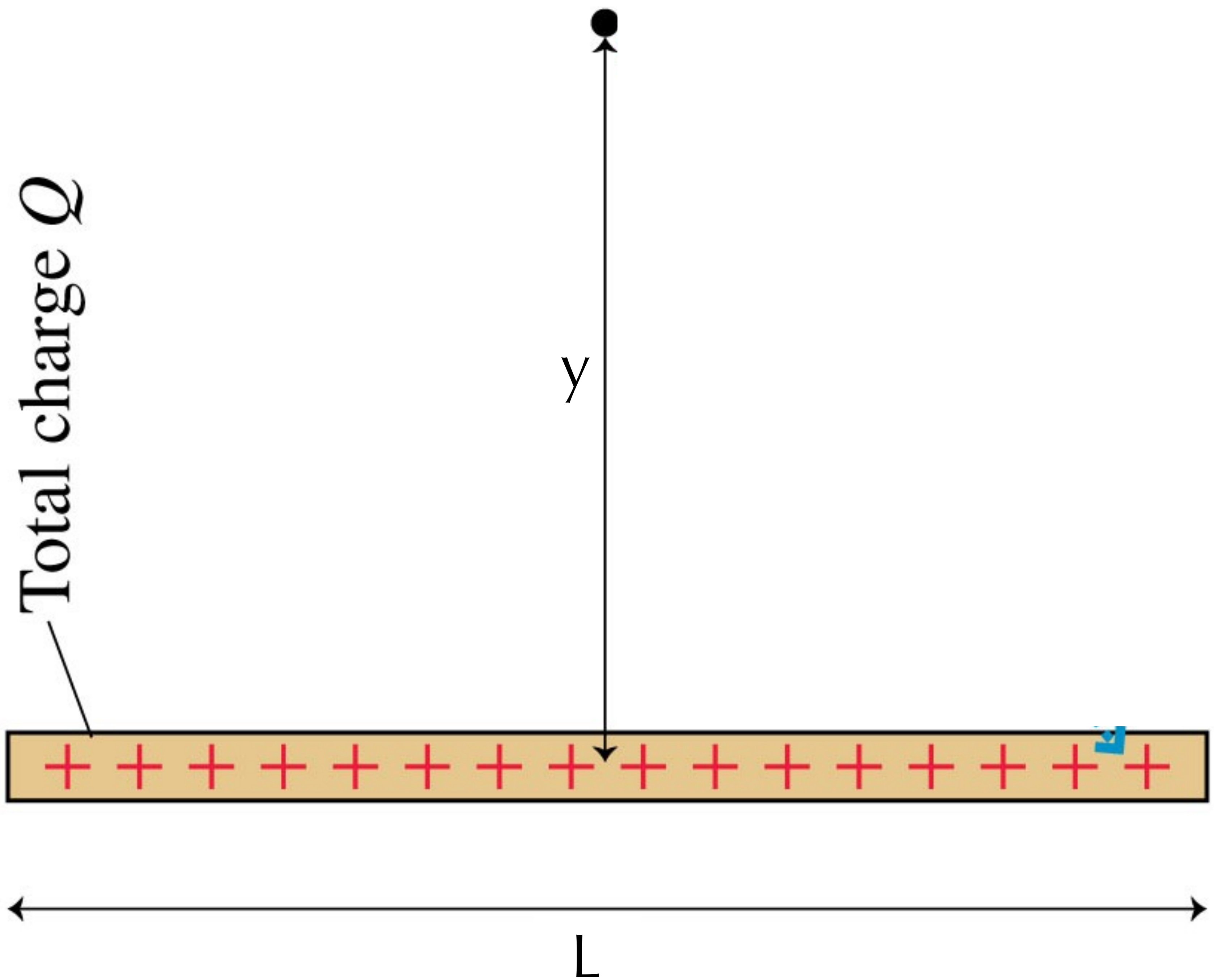
- Write “r” in terms of spatial variables in the problem (some of which may be integration variables)
- Write dQ in terms of spatial differentials.
- One integral for each component.

What is the E field due to this line of charge?

$$dE_y = \frac{k\frac{Q}{L}ydx}{(x^2+y^2)^{3/2}}$$



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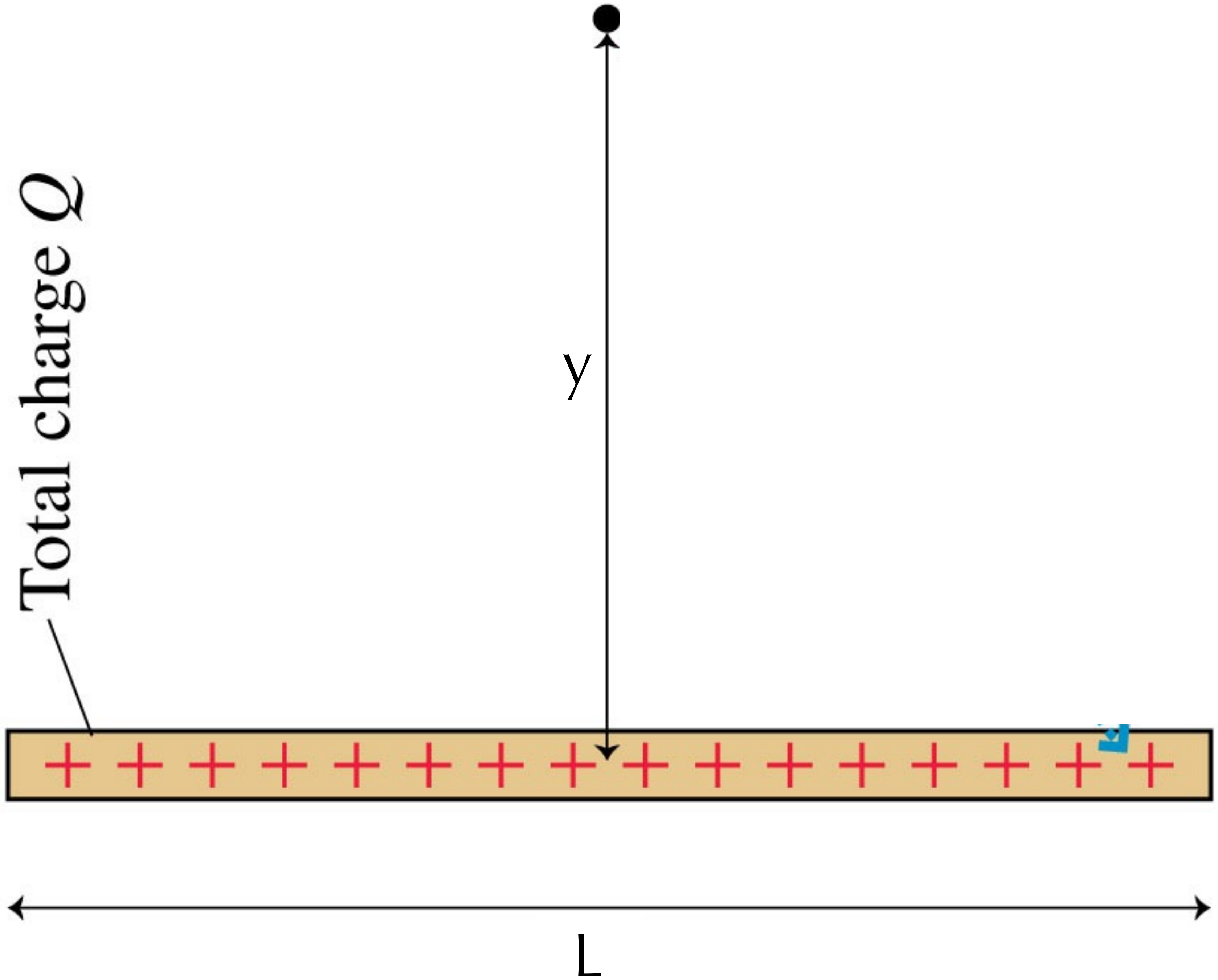

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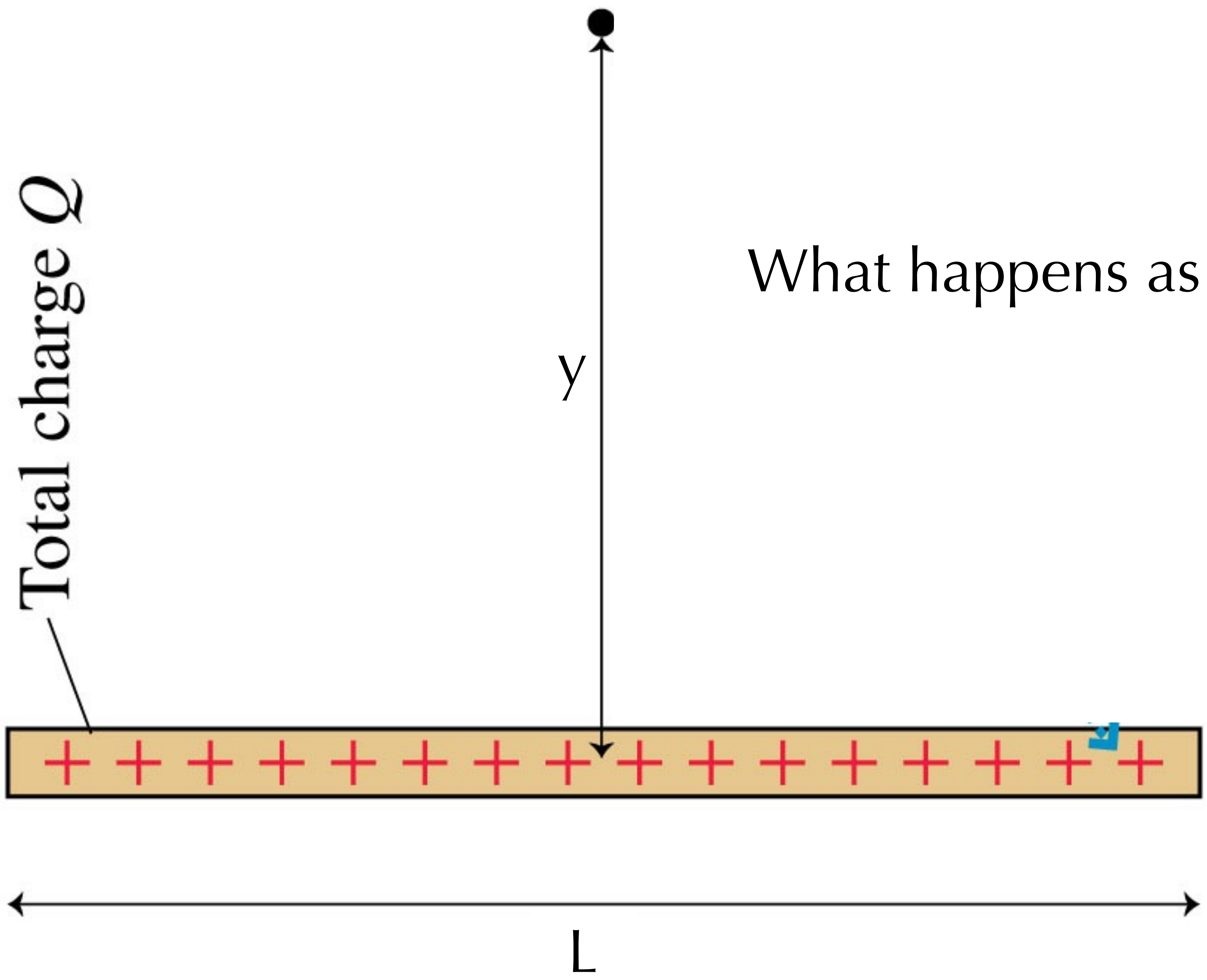

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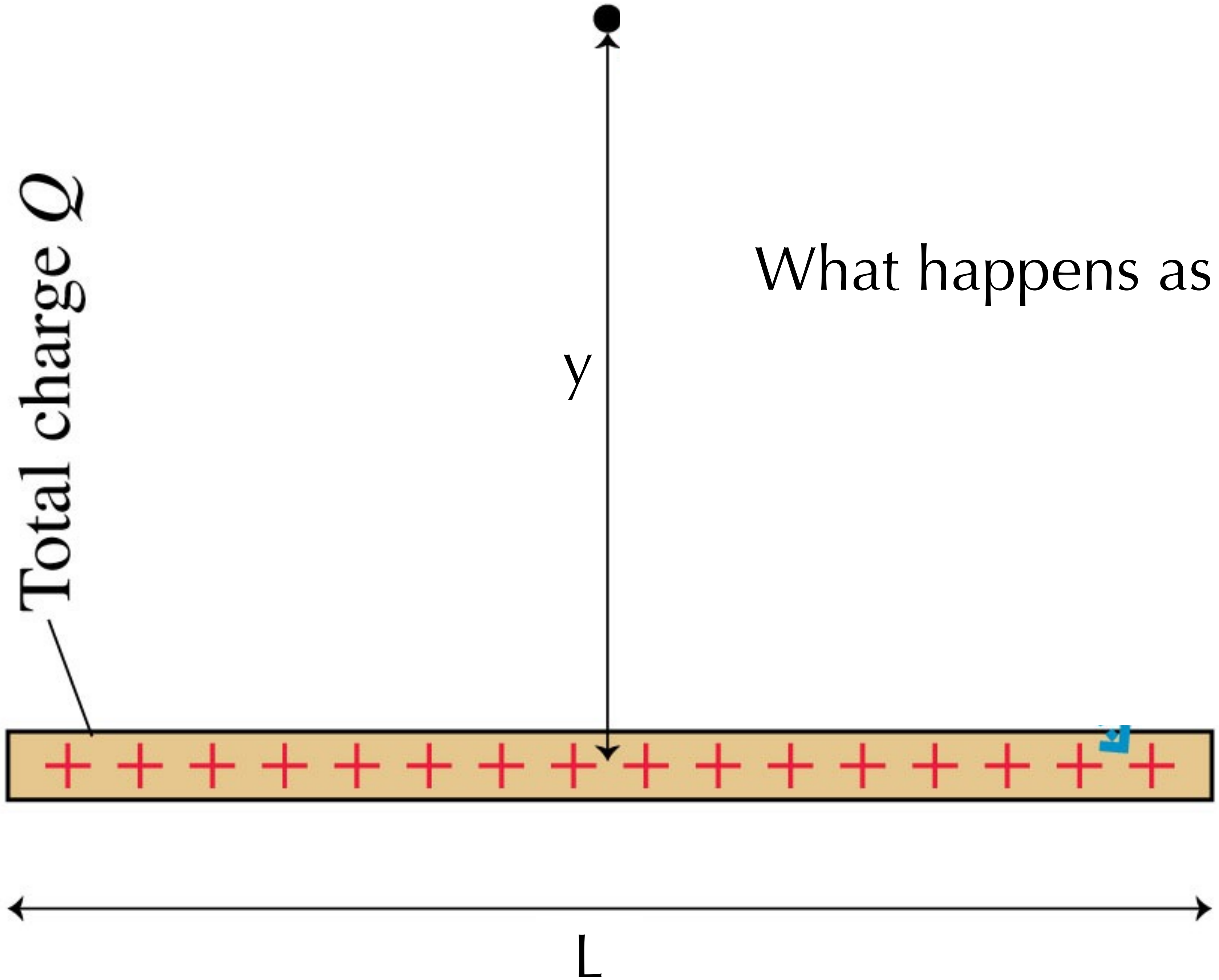
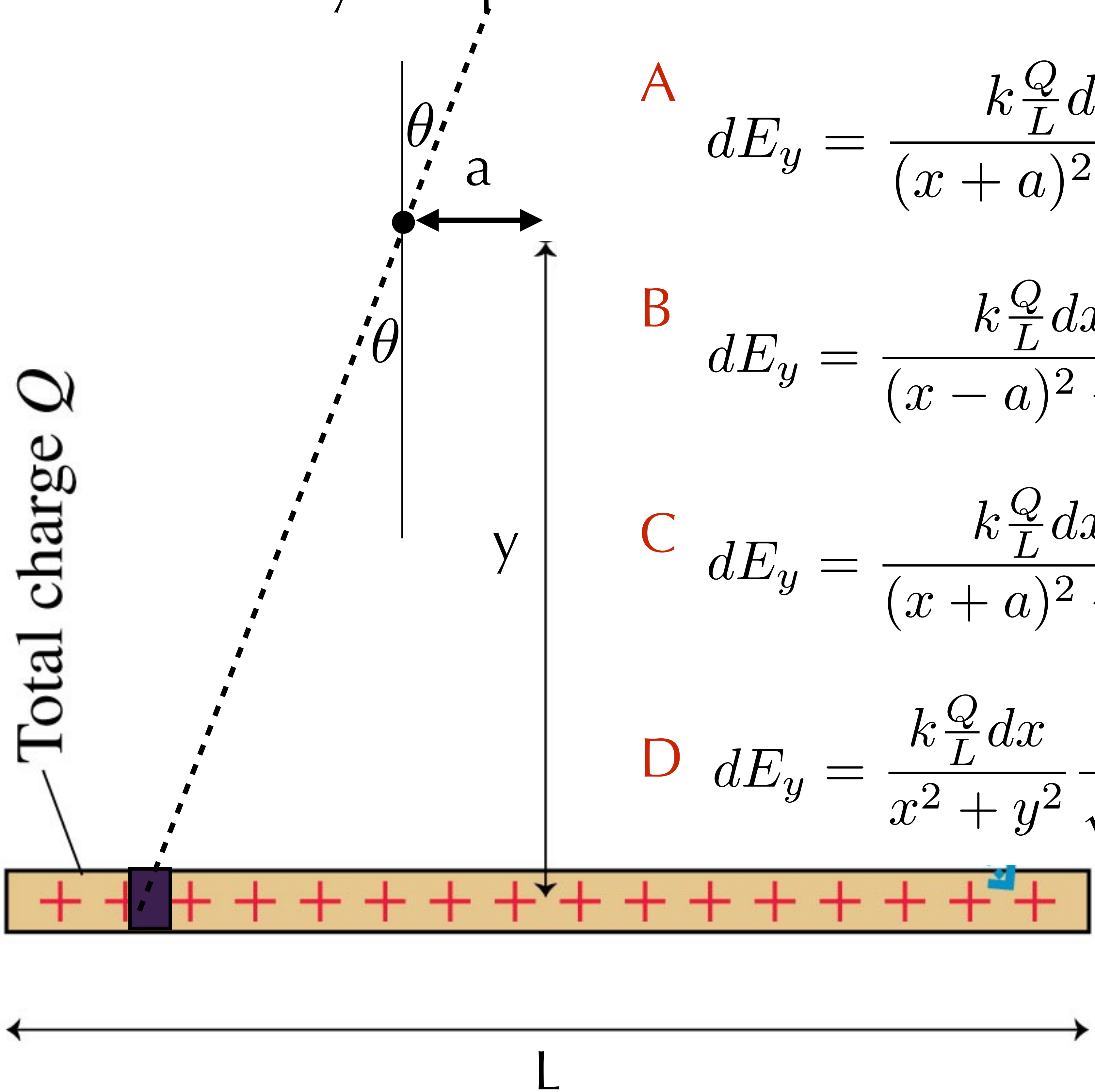


What happens as $L \to \infty$

$$E_{\text{rod}} = \frac{2k\lambda}{d}$$

Which is the y-component of the electric field due to the shaded region?



A $$dE_y = \frac{k\frac{Q}{L}dx}{(x+a)^2+y^2}\frac{y}{\sqrt{(x+a)^2+y^2}}$$

B $$dE_y = \frac{k\frac{Q}{L}dx}{(x-a)^2+y^2}\frac{y}{\sqrt{(x-a)^2+y^2}}$$

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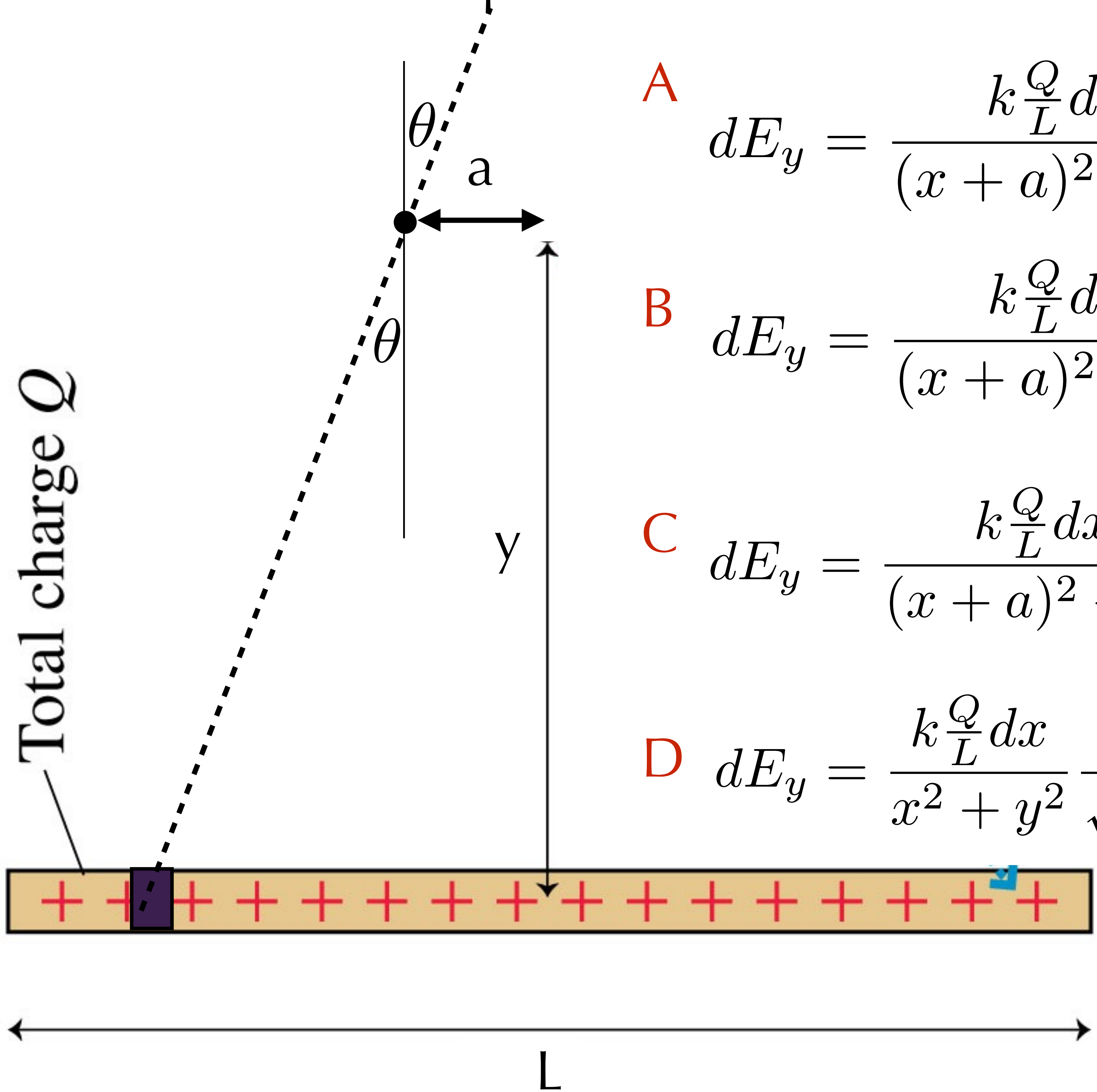
Which is the x-component of the electric field due to the shaded region?



A $$dE_y = \frac{k\frac{Q}{L}dx}{(x+a)^2+y^2}\frac{y}{\sqrt{(x+a)^2+y^2}}$$

B $$dE_y = \frac{k\frac{Q}{L}dx}{(x+a)^2+y^2}\frac{-(x+a)}{\sqrt{(x+a)^2+y^2}}$$

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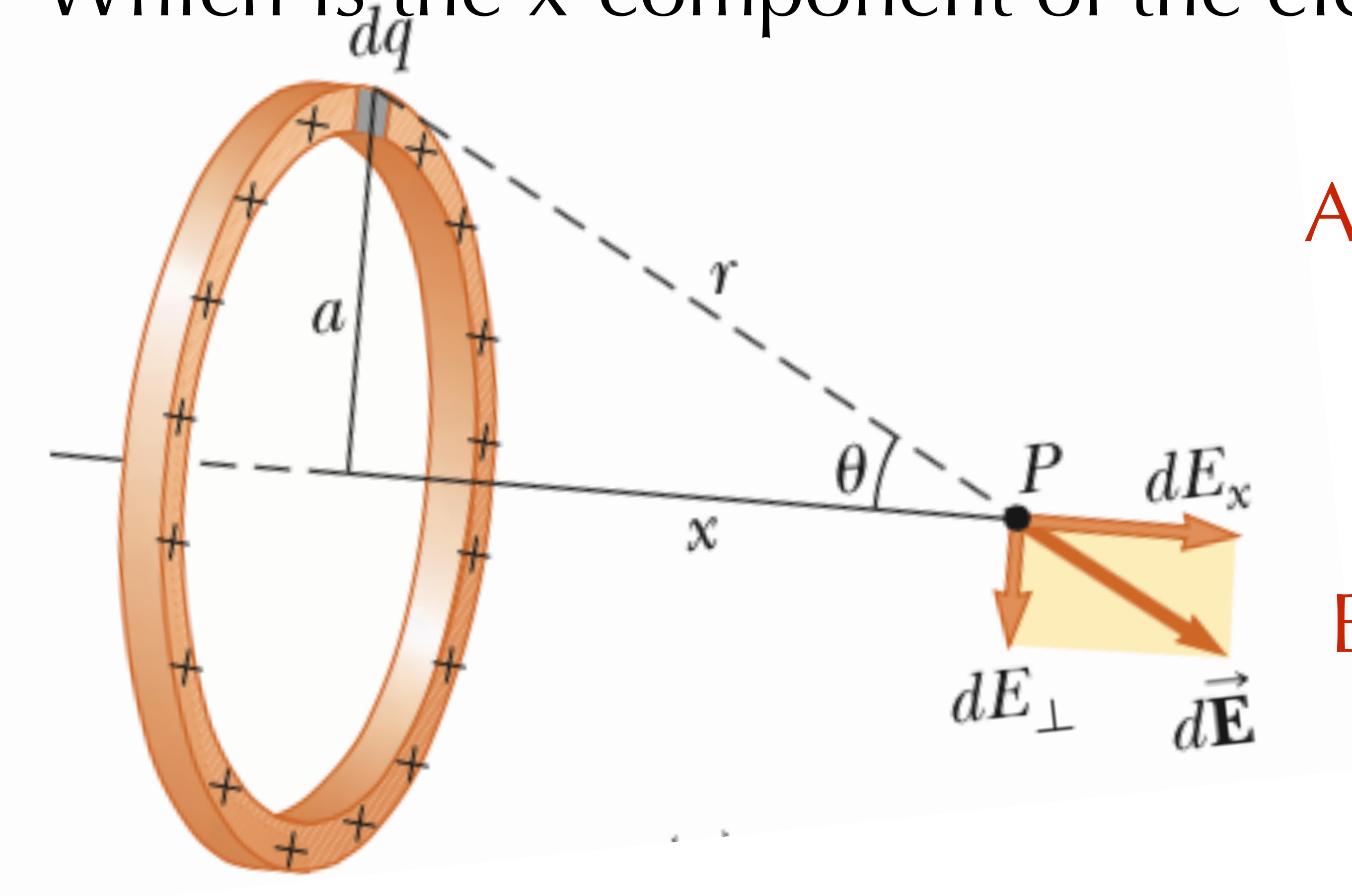

A $$dE_x = \frac{k\frac{Q}{2\pi a}a\,d\theta}{a^2+x^2}\,\frac{a}{\sqrt{a^2+x^2}}$$

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Ring of charge

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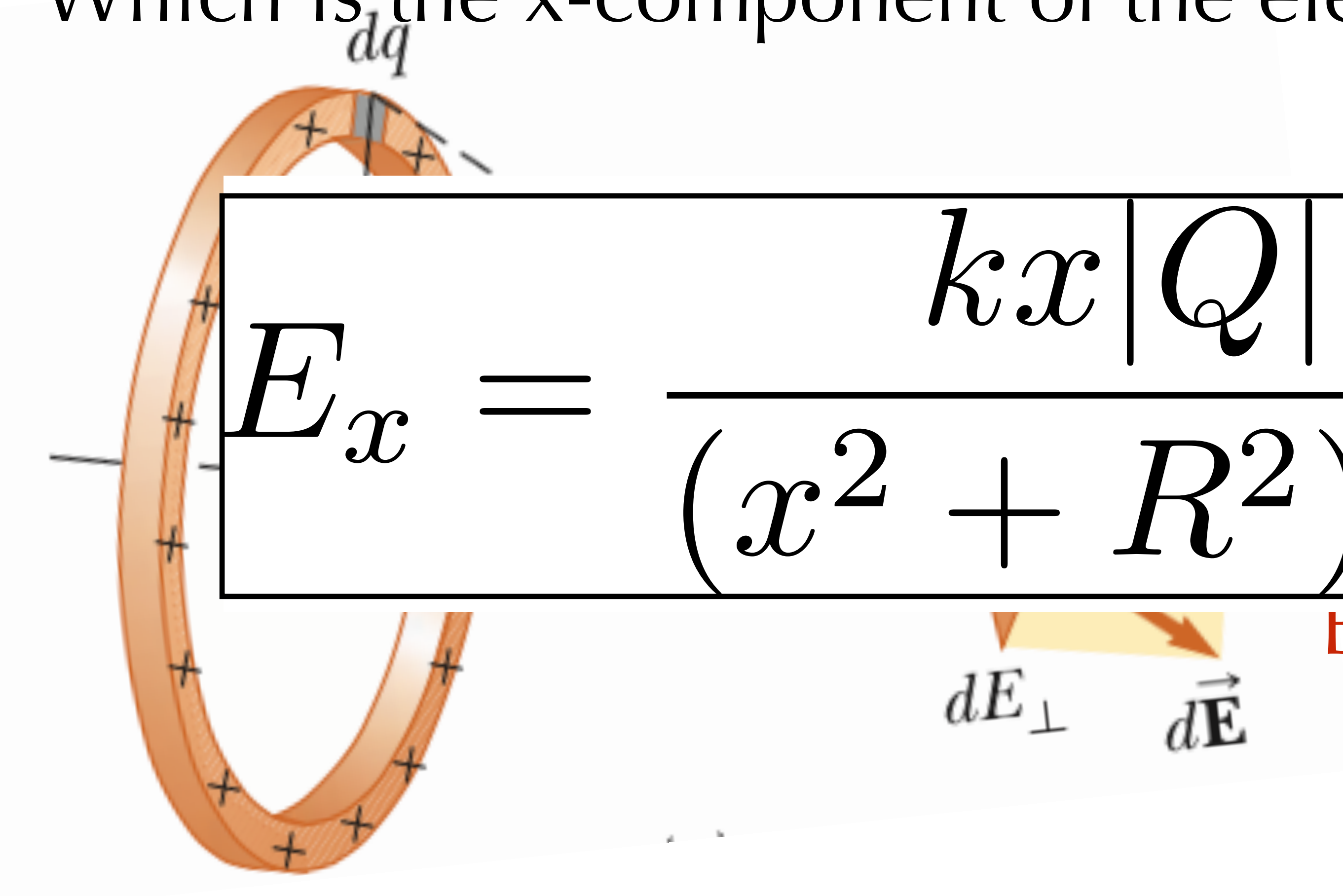

$$\boxed{E_x = \frac{kx|Q|}{(x^2+R^2)^{3/2}}}$$

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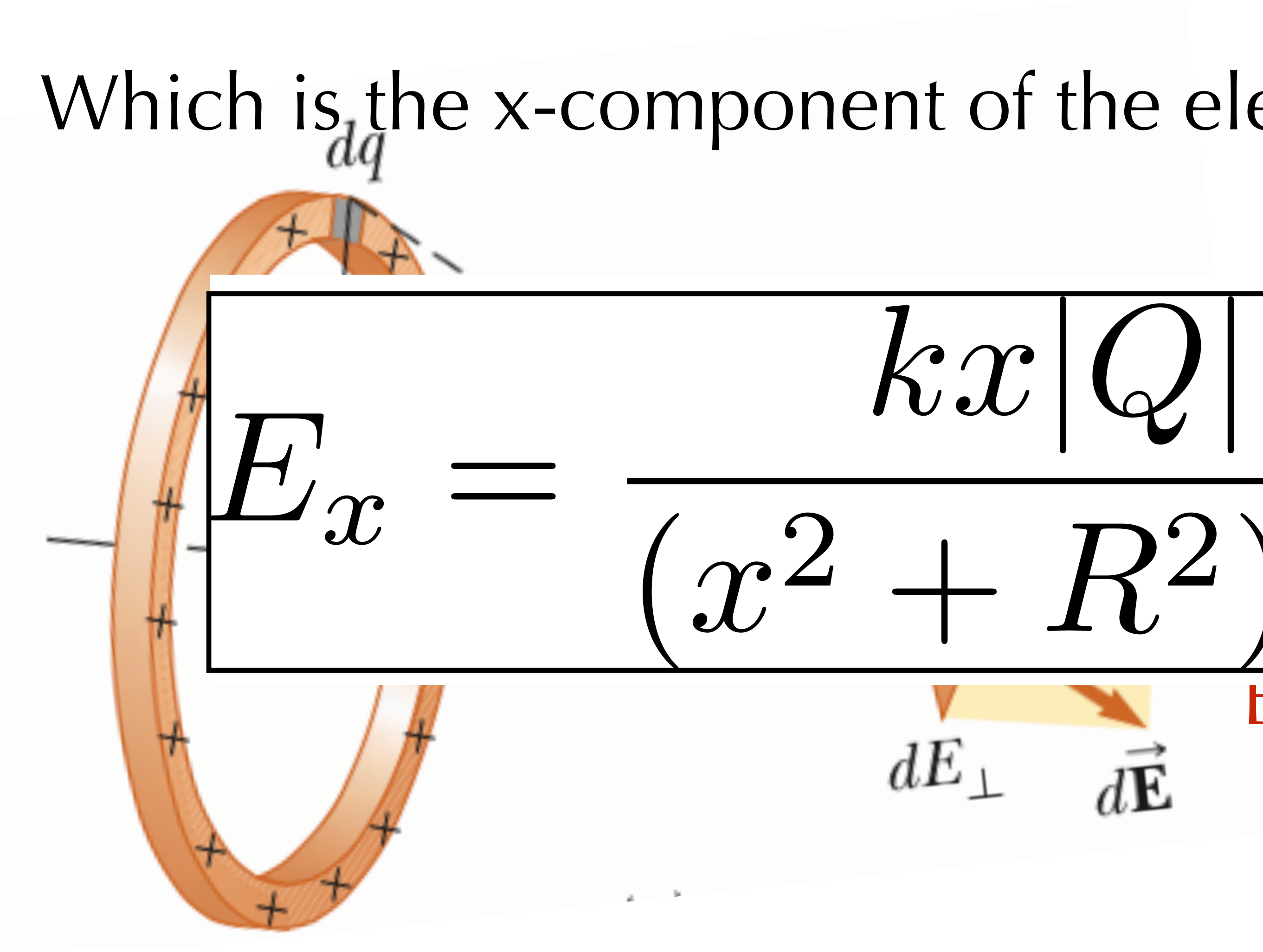

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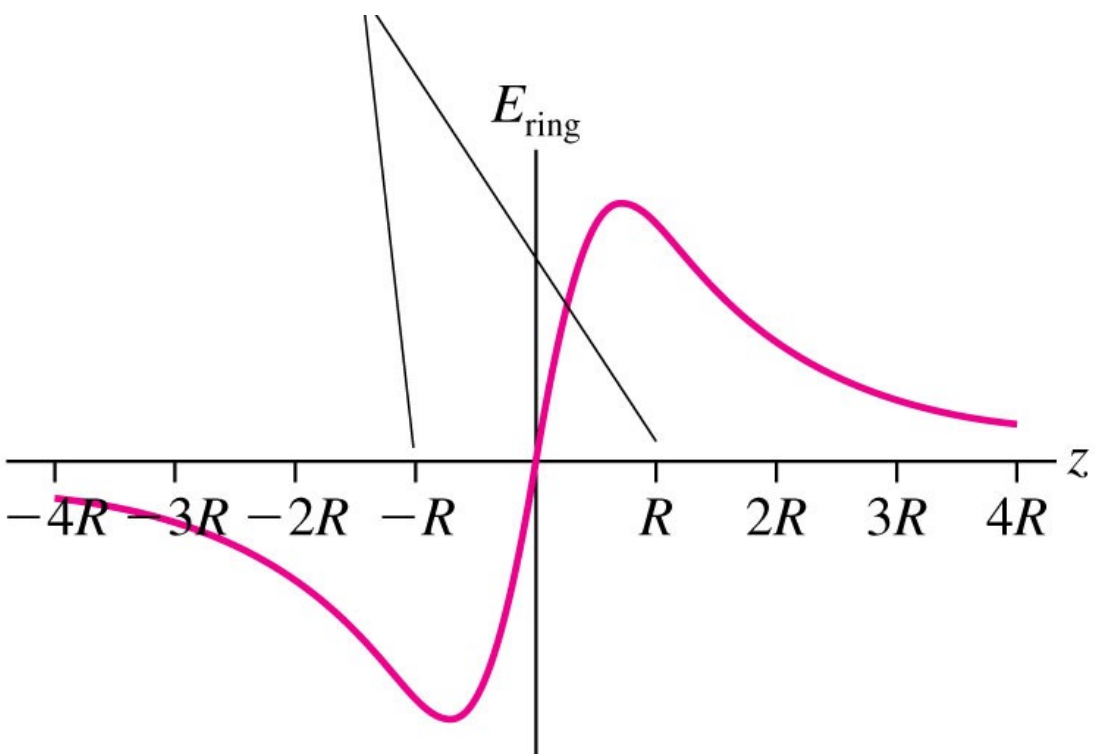

Disk of Charge

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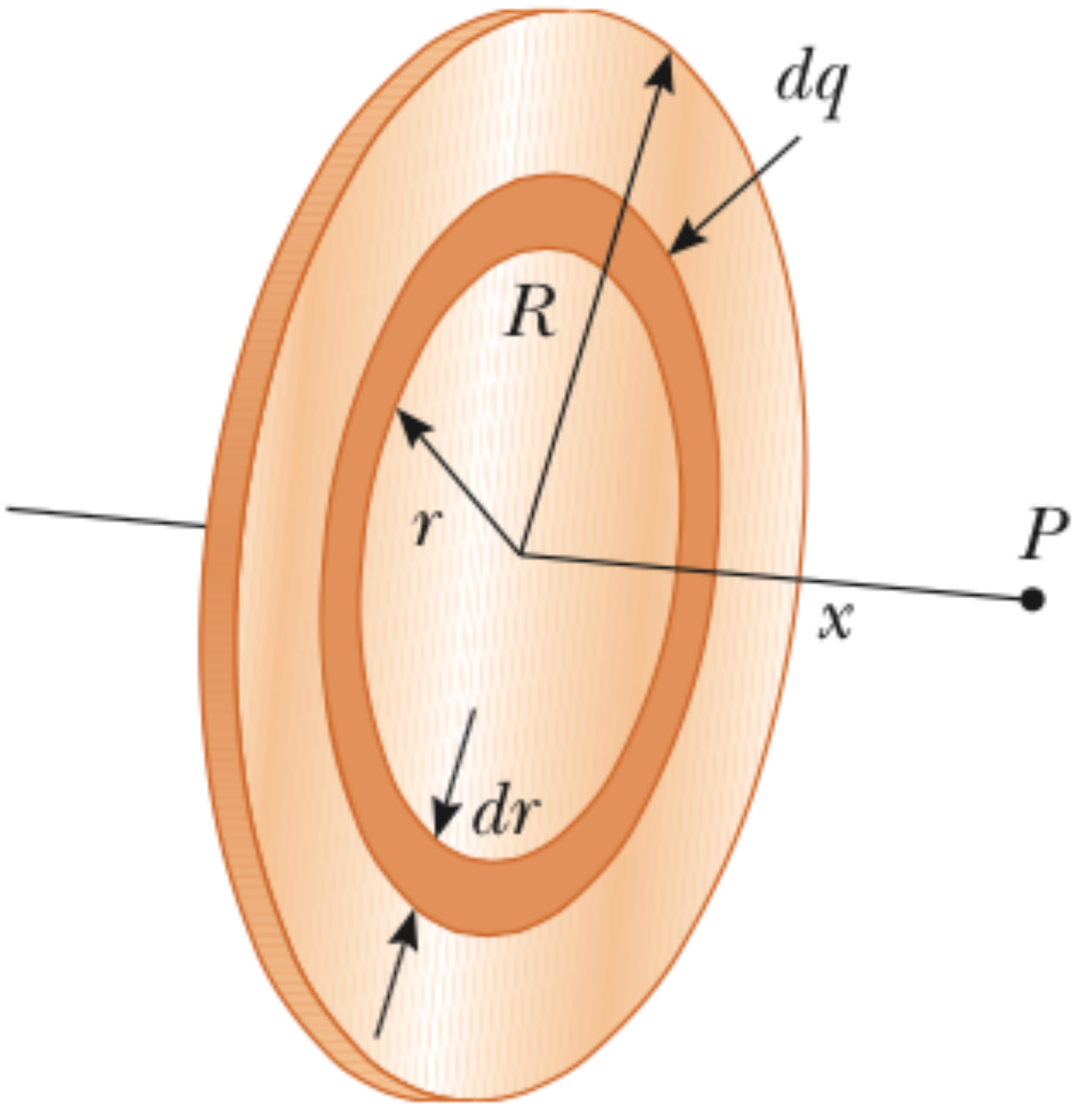

A $$dE_x = \frac{kx\frac{Q}{\pi R^2}2\pi r dr}{\sqrt{x^2+r^2}}$$

B $$dE_x = \frac{kxQdr}{(x^2+r^2)^{3/2}}$$

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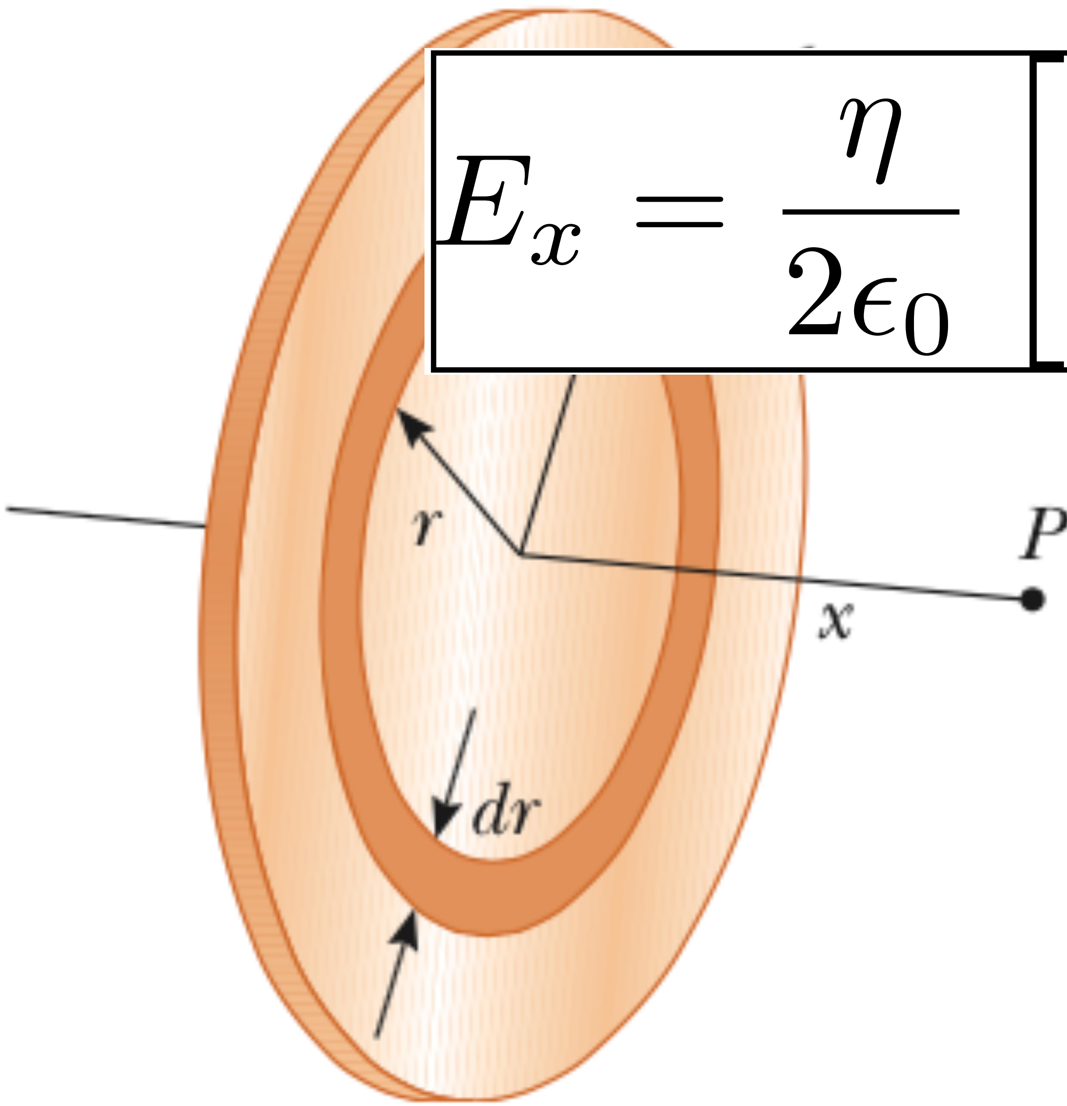

$$\boxed{E_x = \frac{\eta}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]}$$

$$\frac{Qdr}{(x^2+r^2)^{3/2}}$$

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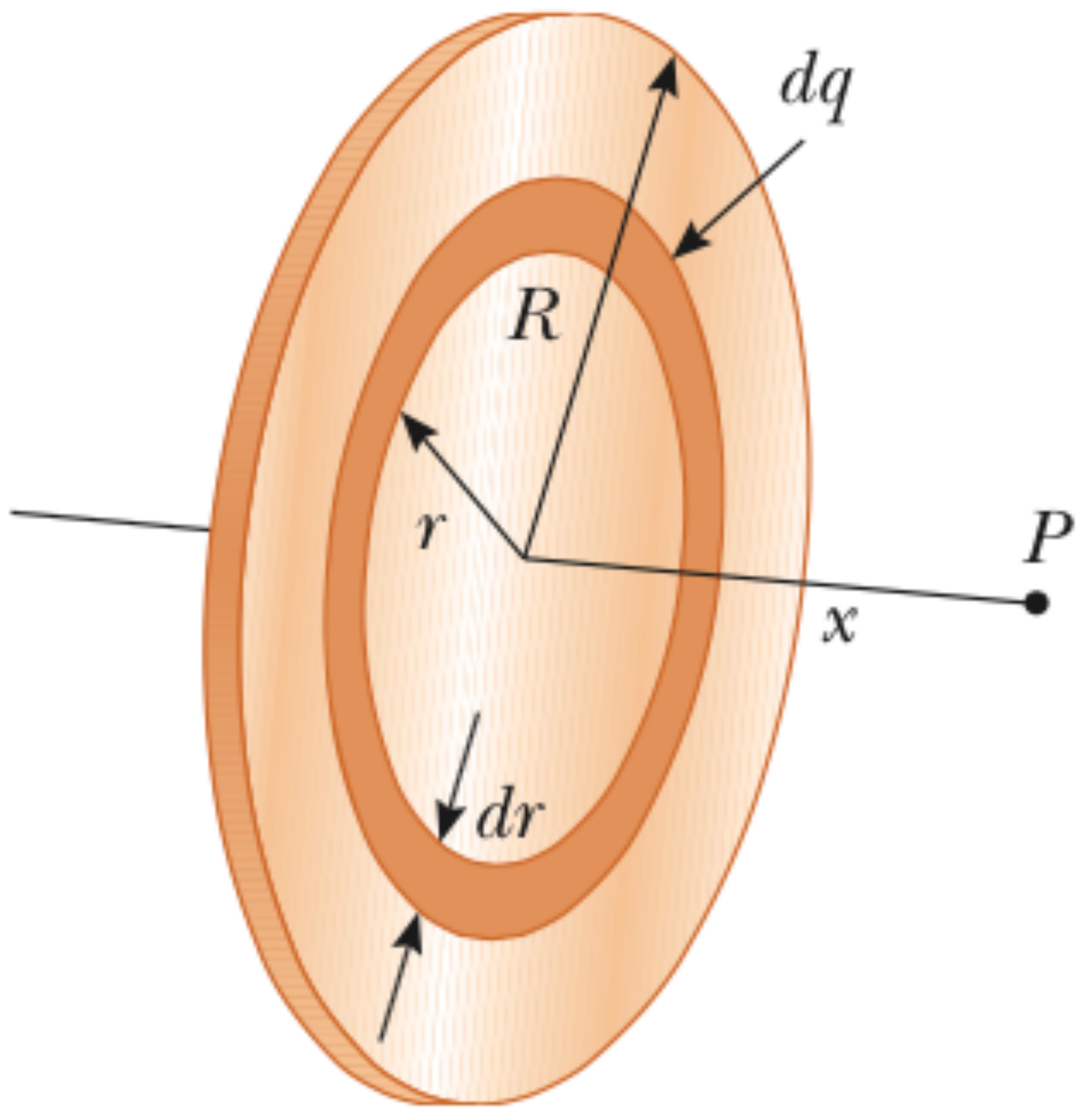
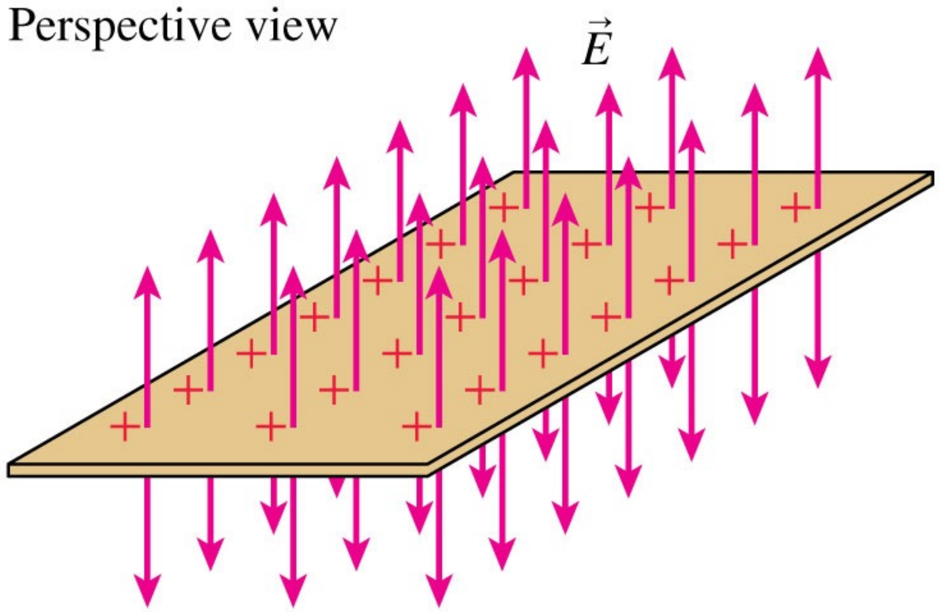
A plane of charge

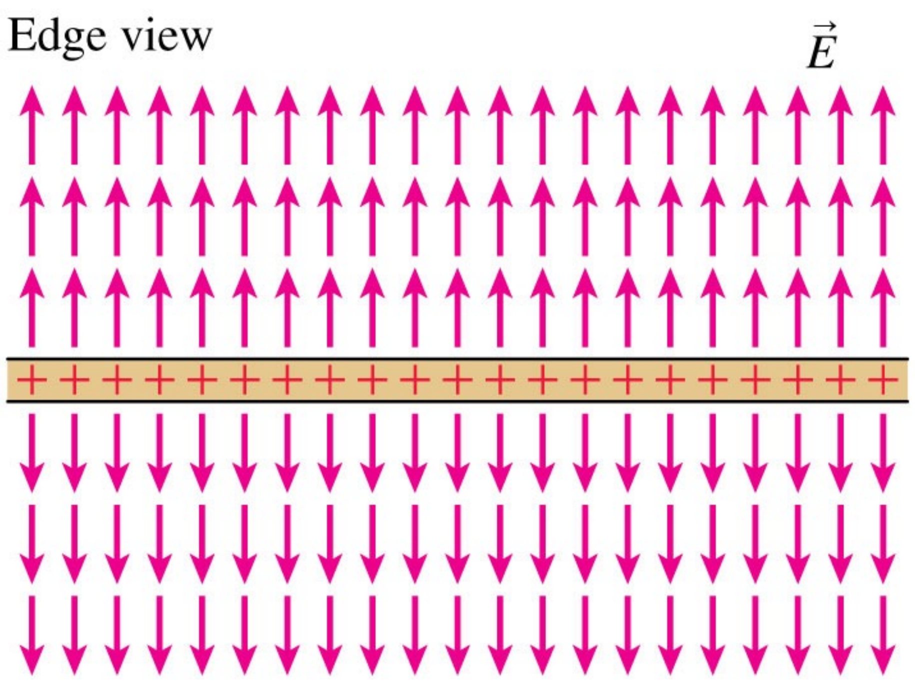


What does the disk become if we let:

$$R \to \infty$$

A plane of charge





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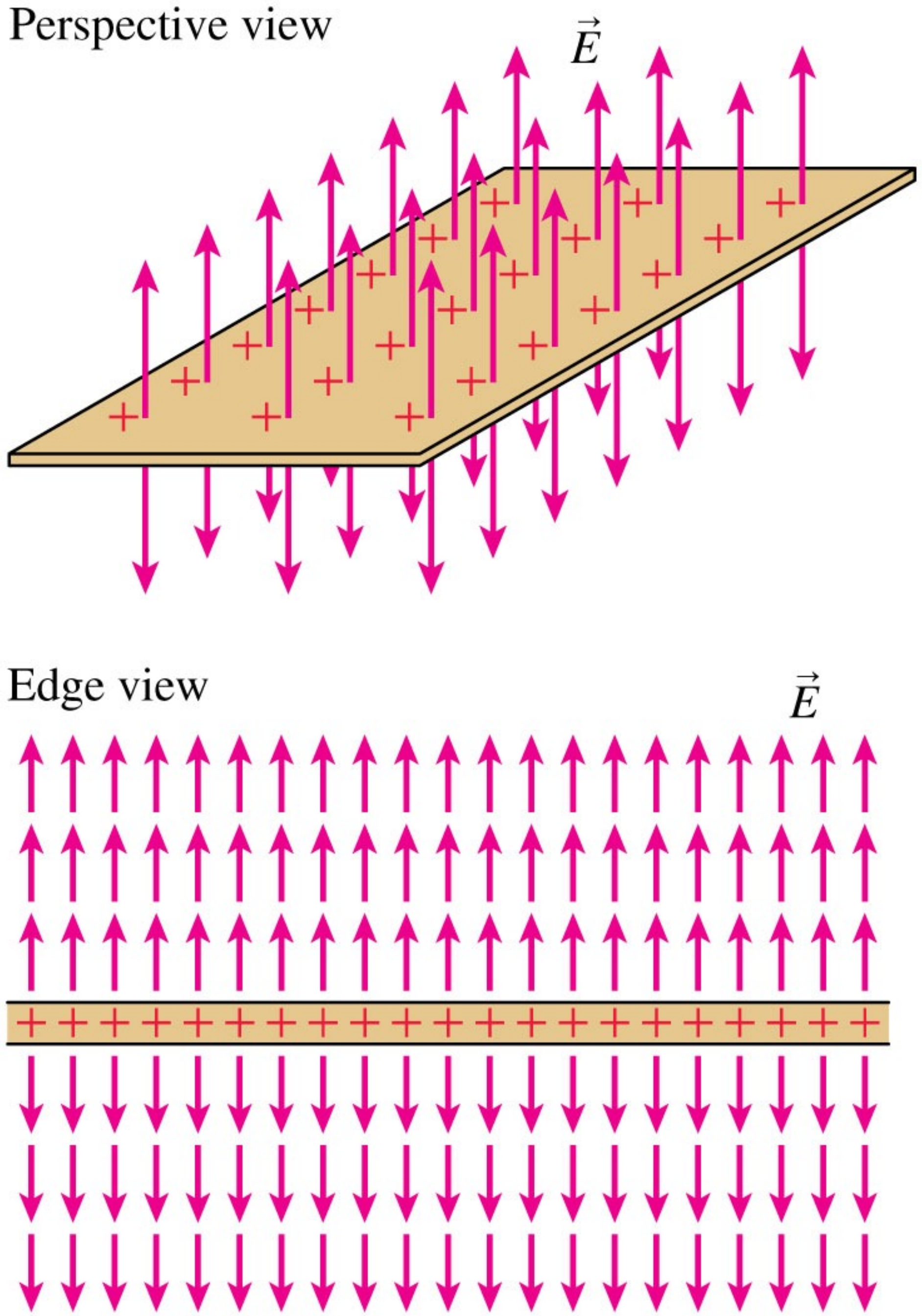

What does the disk become if we let:

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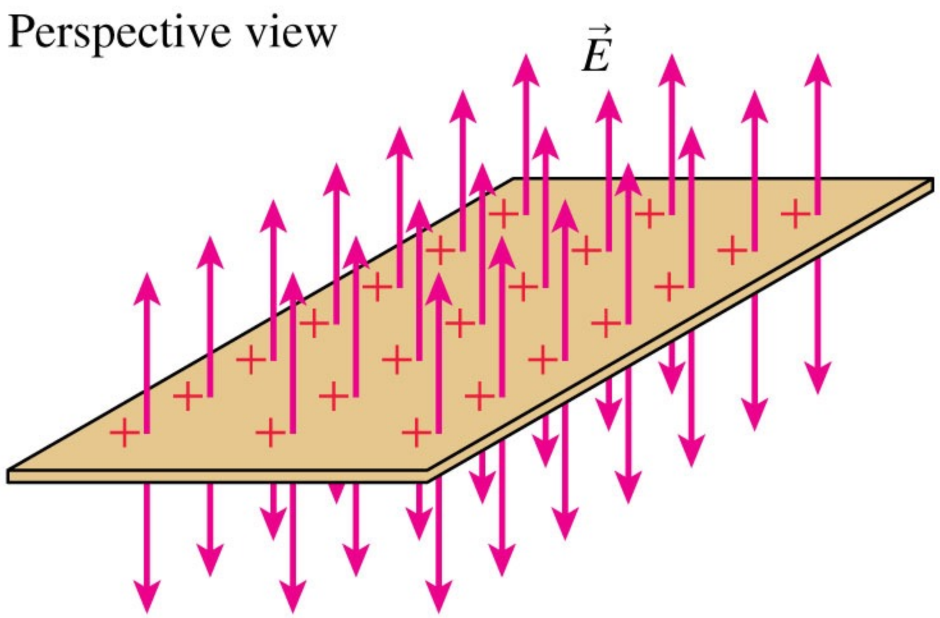
$$E_x = \frac{\eta}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

What does this function become?

A plane of charge



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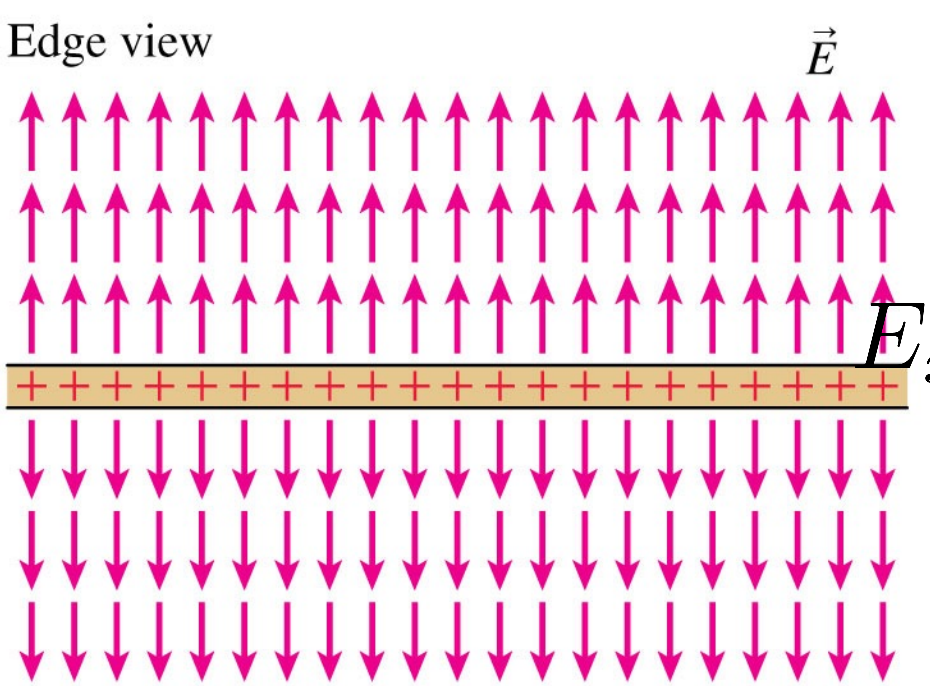

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$$E_x = \frac{\eta}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2\left(\frac{x^2}{R^2} + 1\right)}}\right]$$

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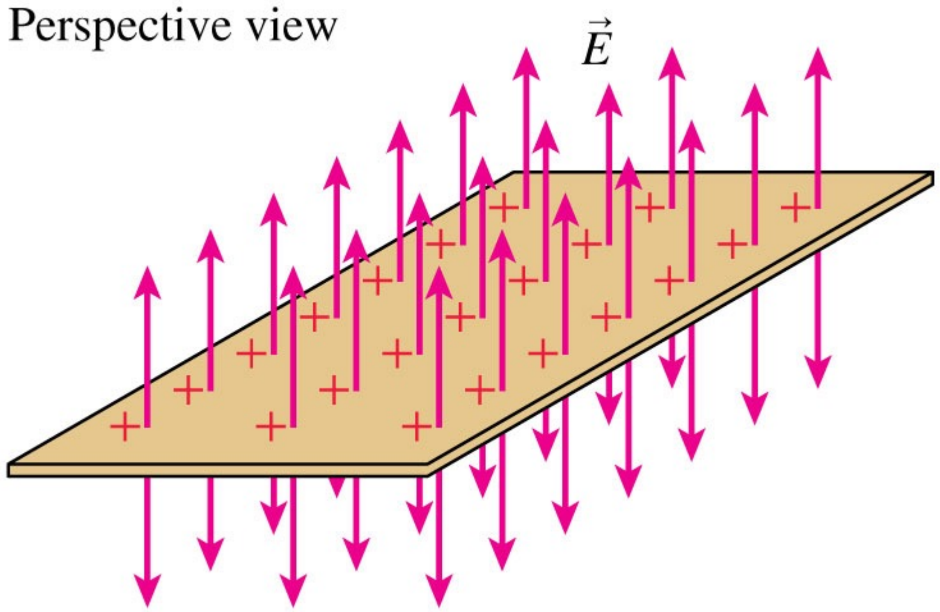

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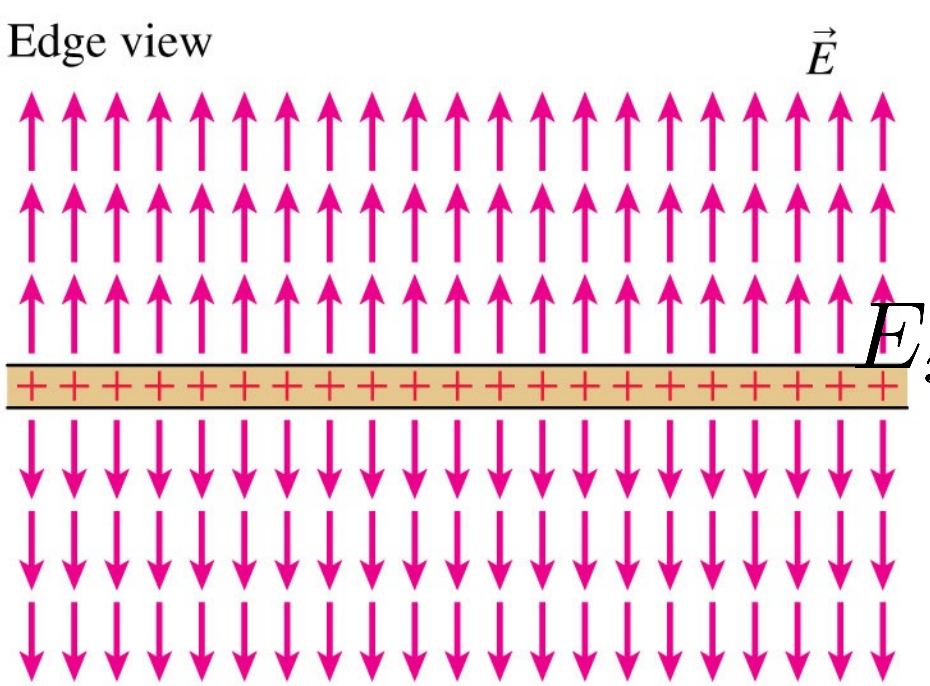

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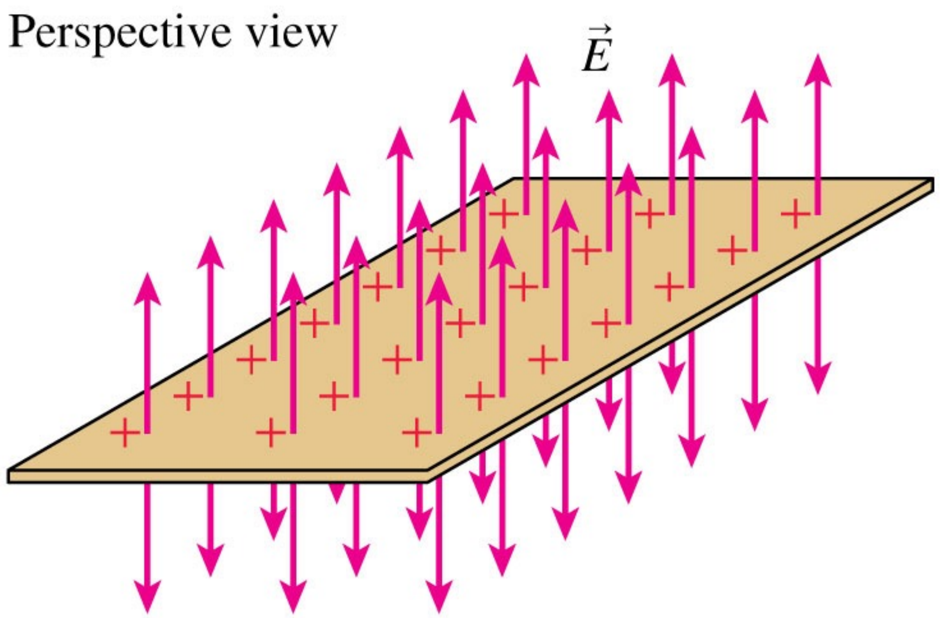

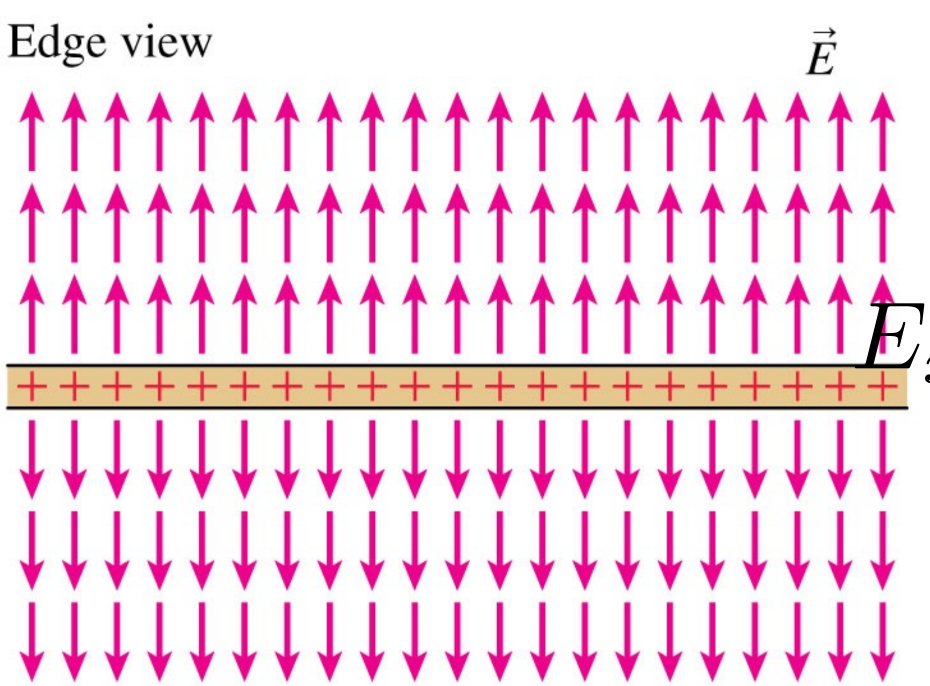

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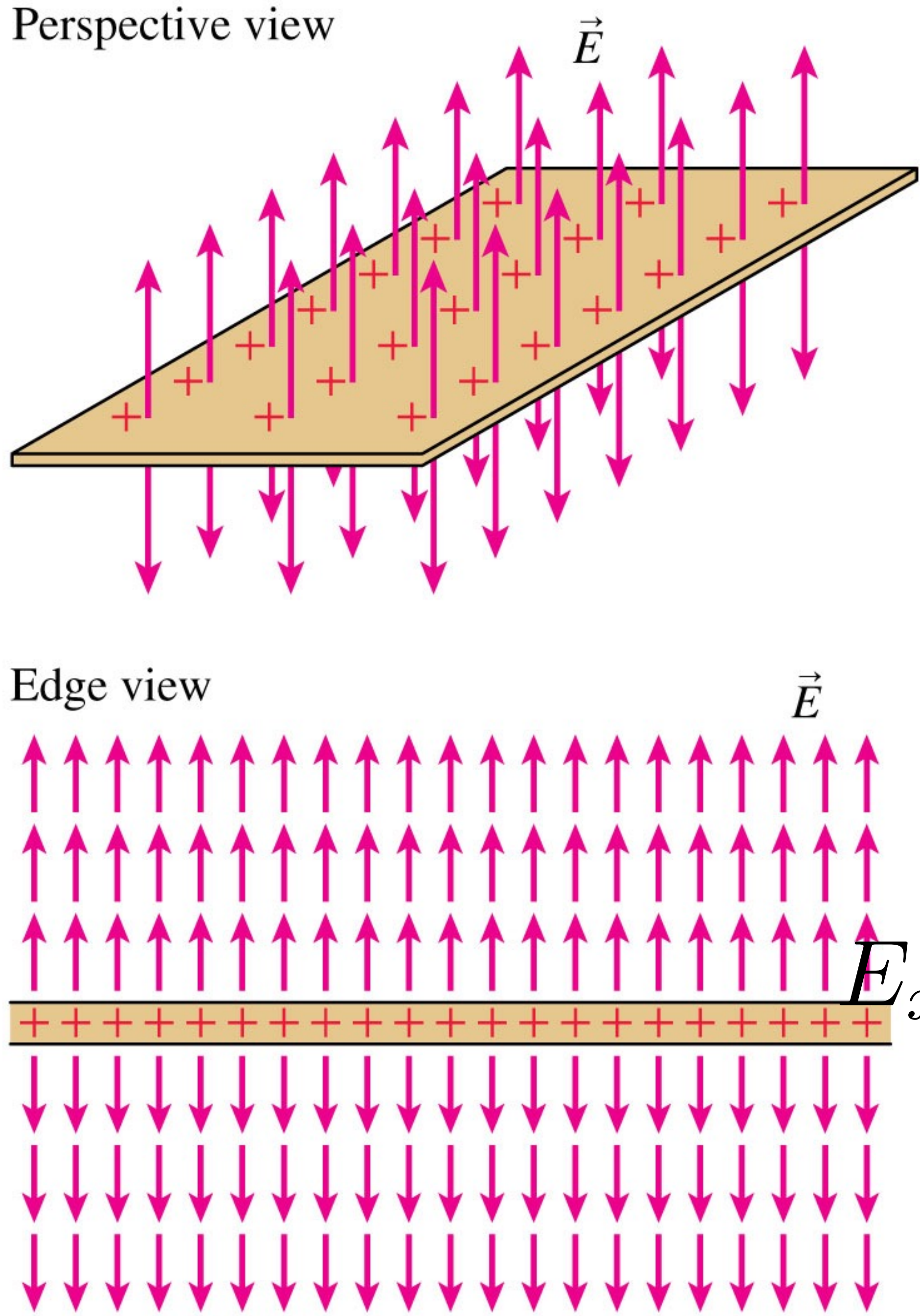

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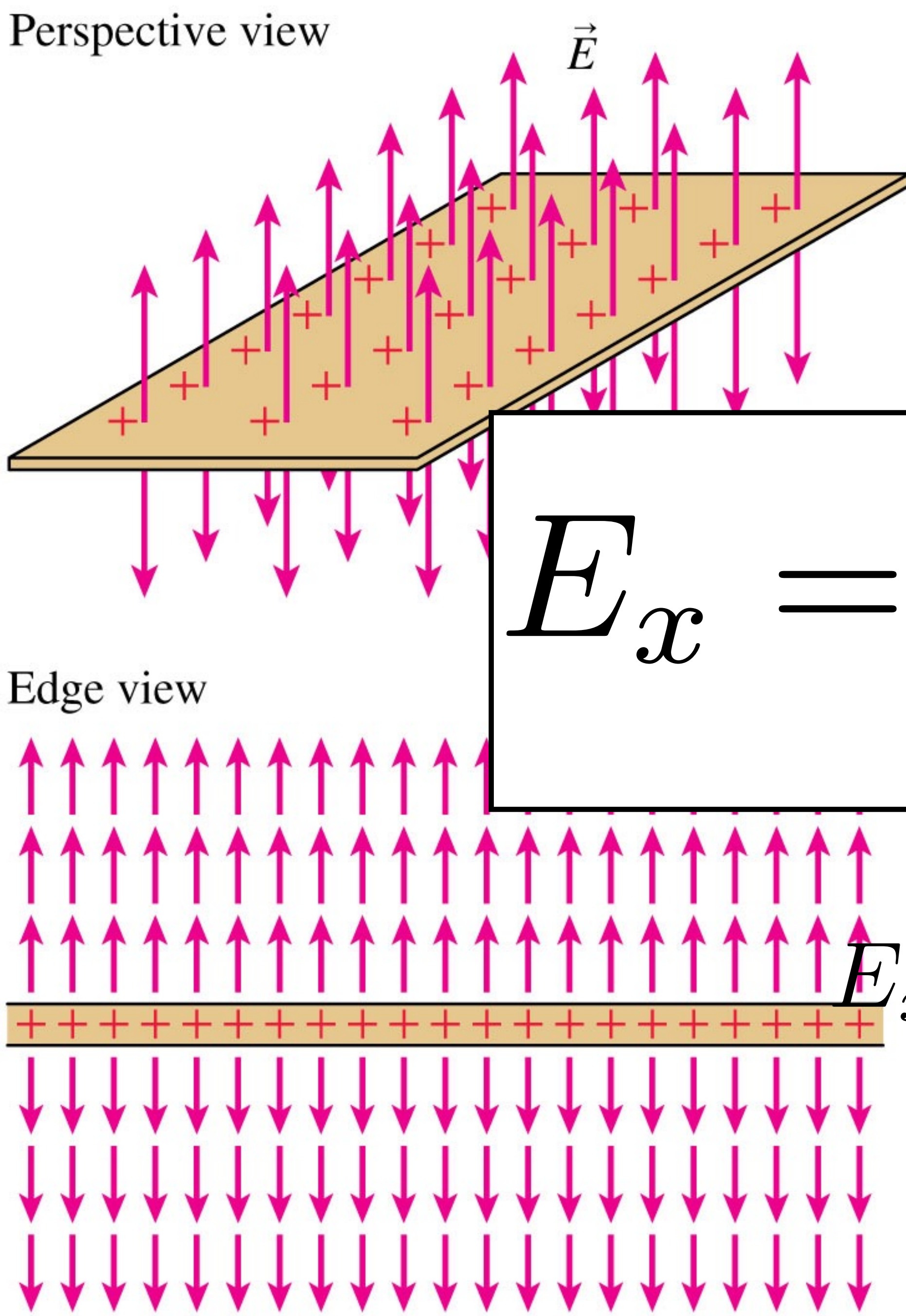

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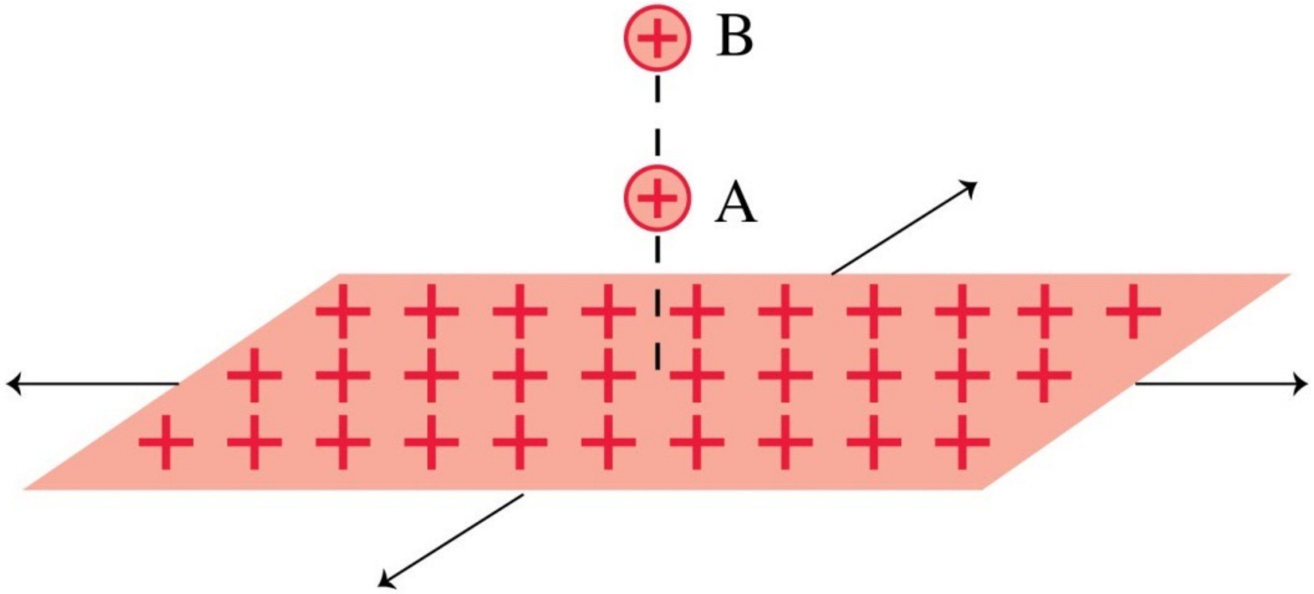
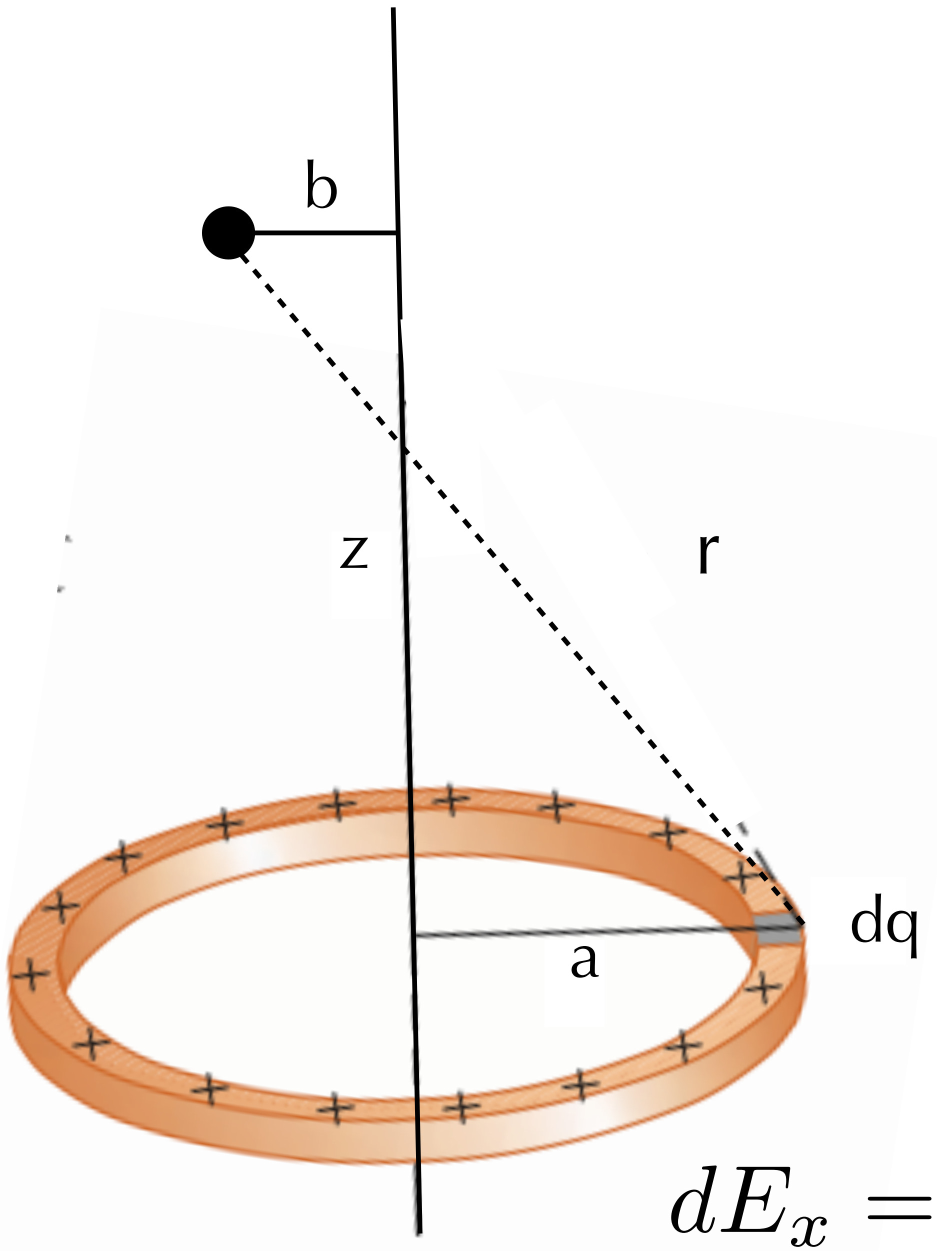
$$\boxed{E_x = \frac{\eta}{2\epsilon_0}}$$

Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?



A. Proton A.
B. Proton B.
C. Both have the same acceleration.

Back to the ring of charge



Let's step off the symmetry axis in one dimension. Which components of the electric field will be nonzero.

Write down dE_x and dE_z.

$$dE_x = k\lambda a \frac{b + a\cos\theta}{\left[(b + a\cos\theta)^2 + (a\sin\theta)^2 + z^2\right]^{3/2}} d\theta$$

$$dE_z = k\lambda a \frac{z}{\left[(b + a\cos\theta)^2 + (a\sin\theta)^2 + z^2\right]^{3/2}} d\theta$$