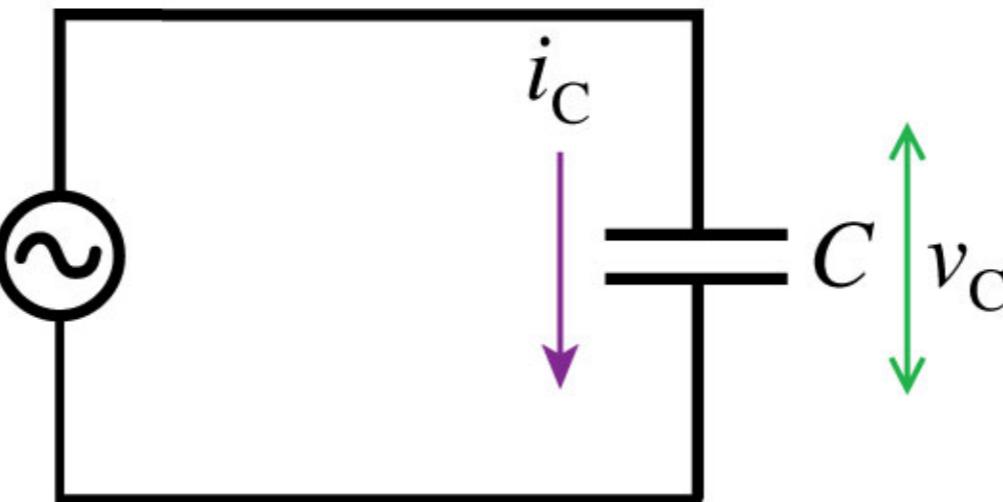




PH 220

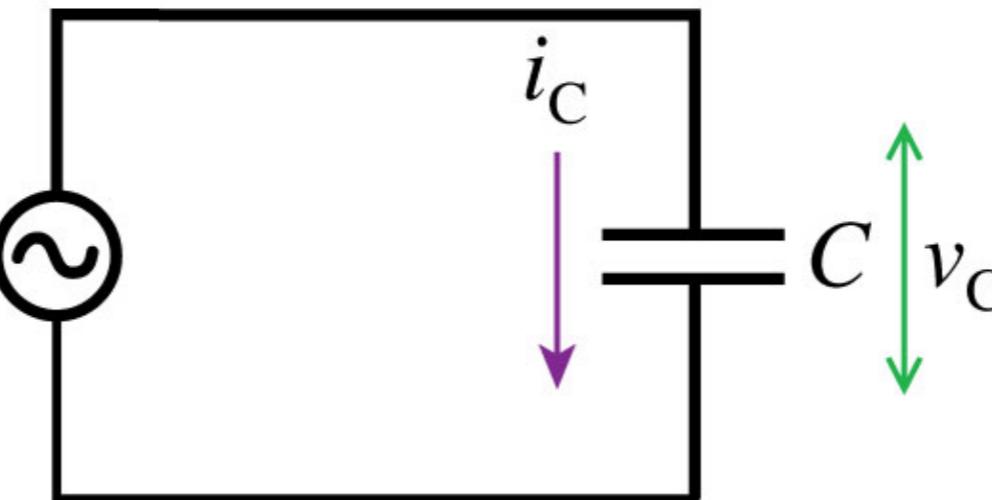
Lance Nelson

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

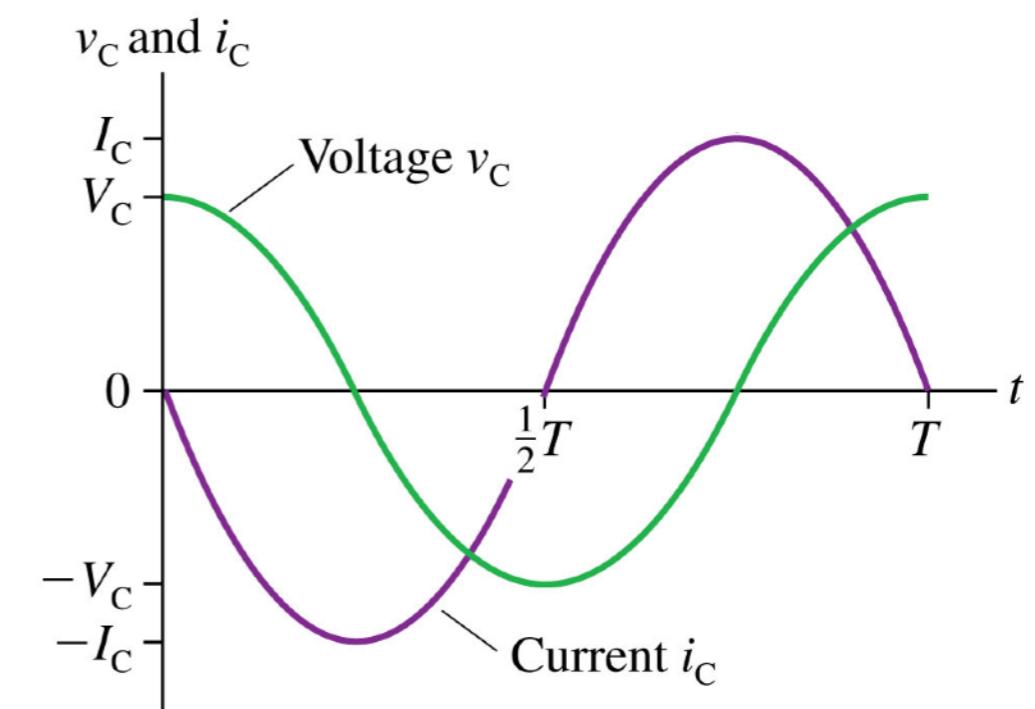
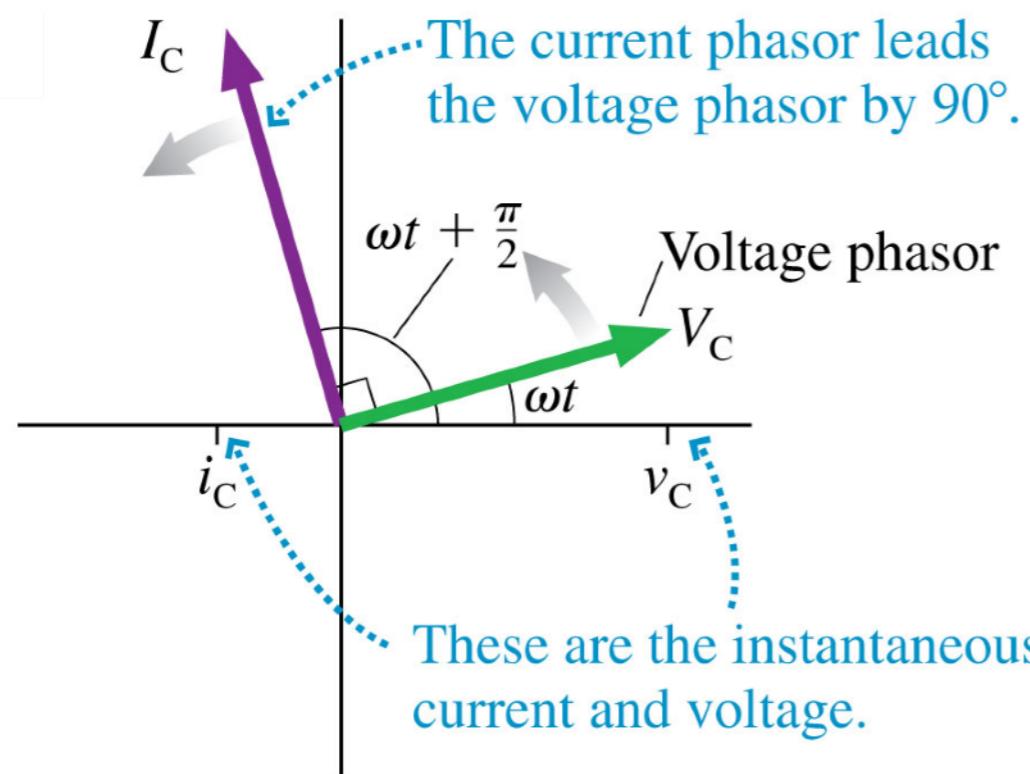


Draw the phasor diagram for this circuit.
Explain to your neighbor why it is the way it is!

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$



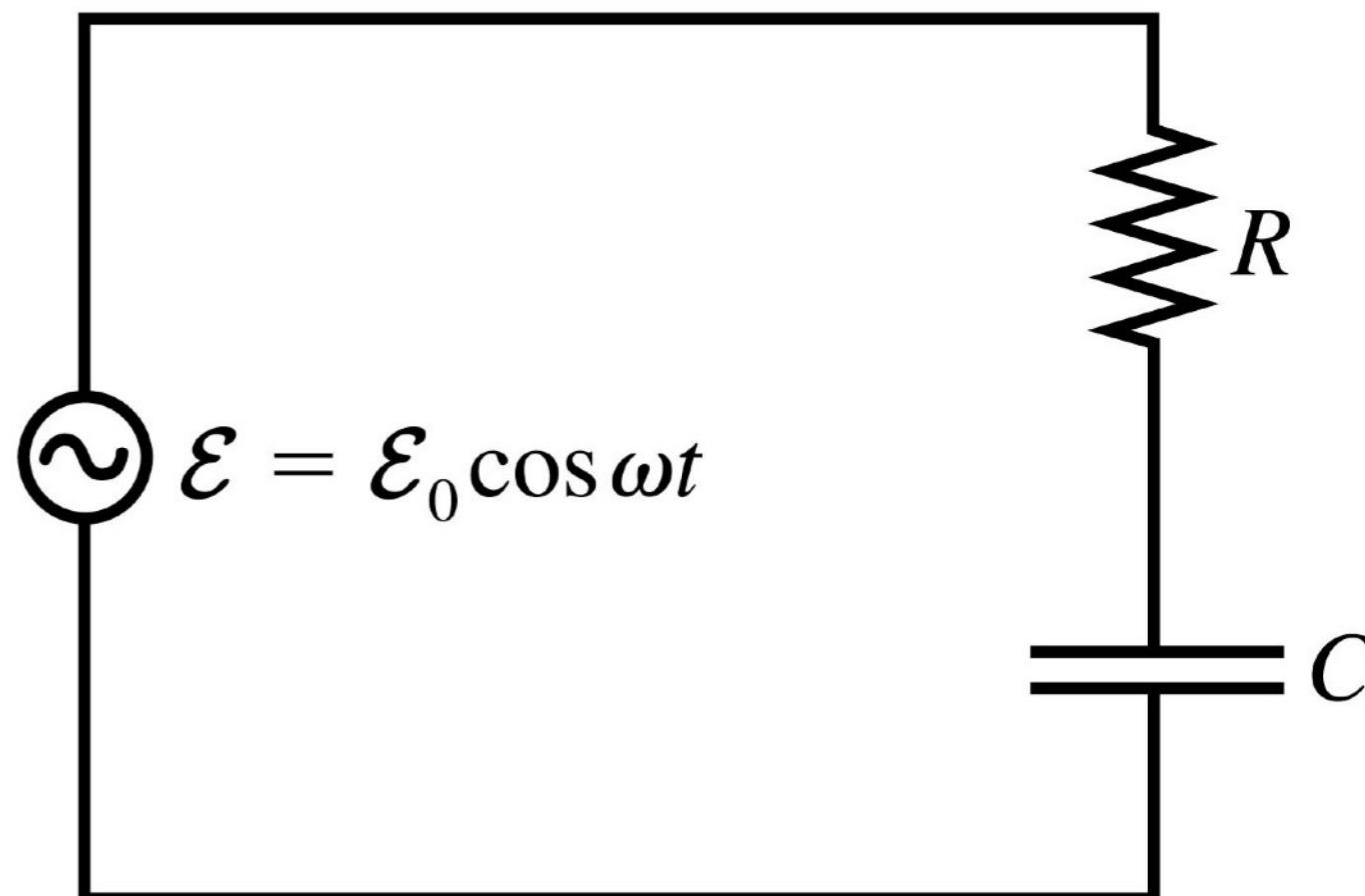
Draw the phasor diagram for this circuit.
Explain to your neighbor why it is the way it is!



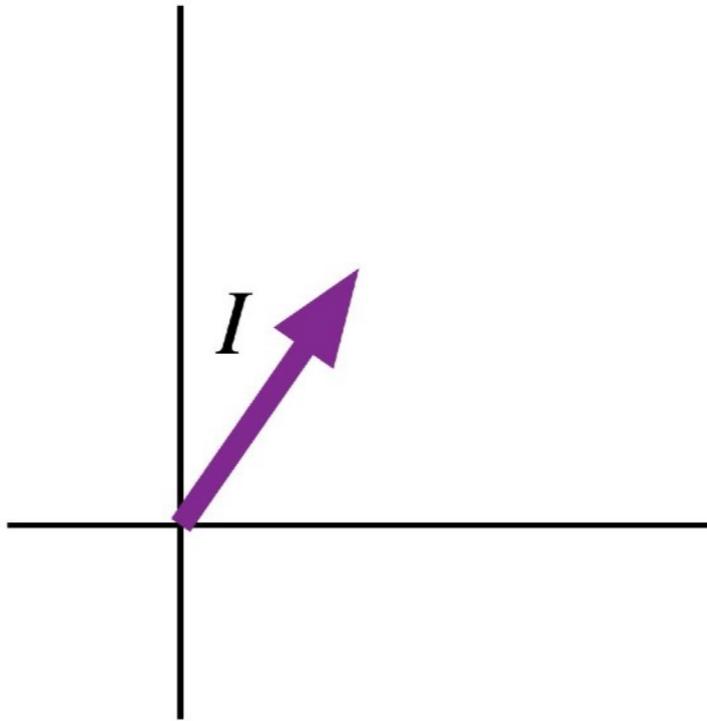
Question #37

Does $V_R + V_C = \mathcal{E}_0$?

- C. Yes.
- D. No.
- E. Can't tell without knowing ω .

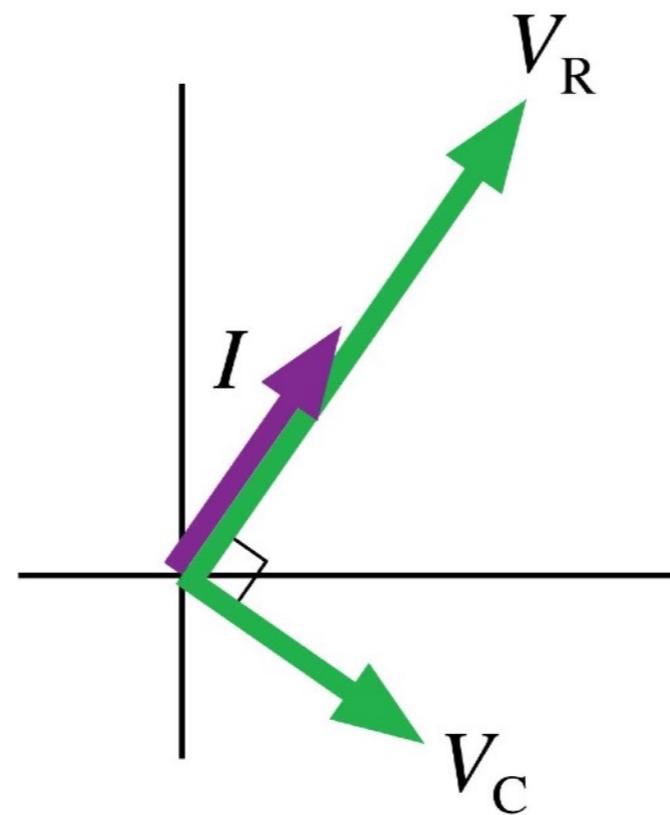


Phasor Diagram



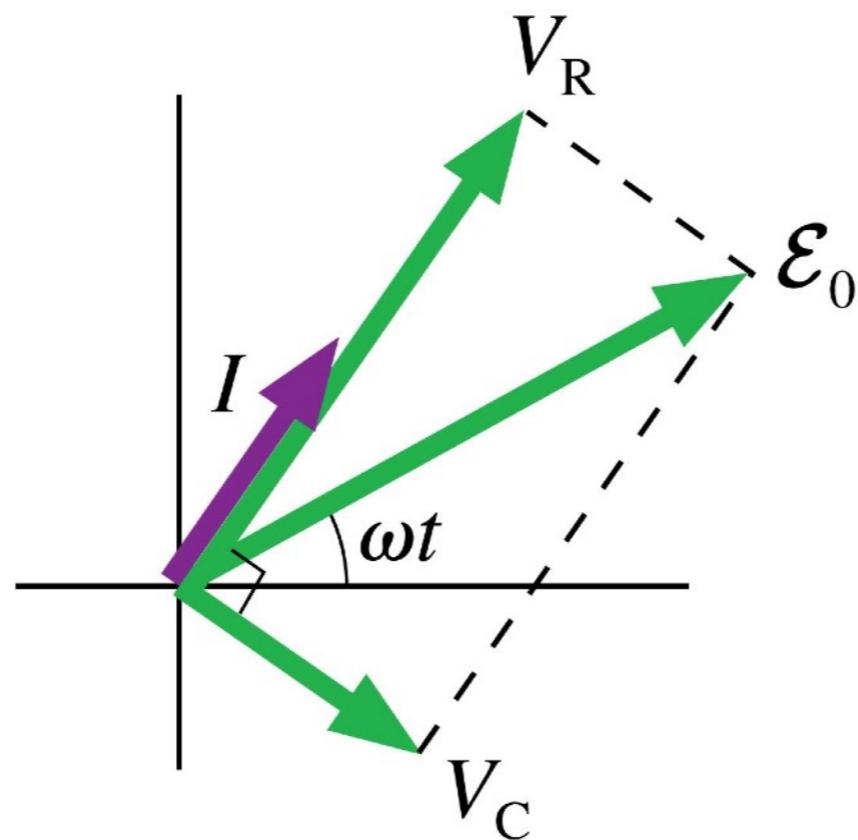
Draw the resistor voltage phasor and the capacitor voltage phasor

Phasor Diagram



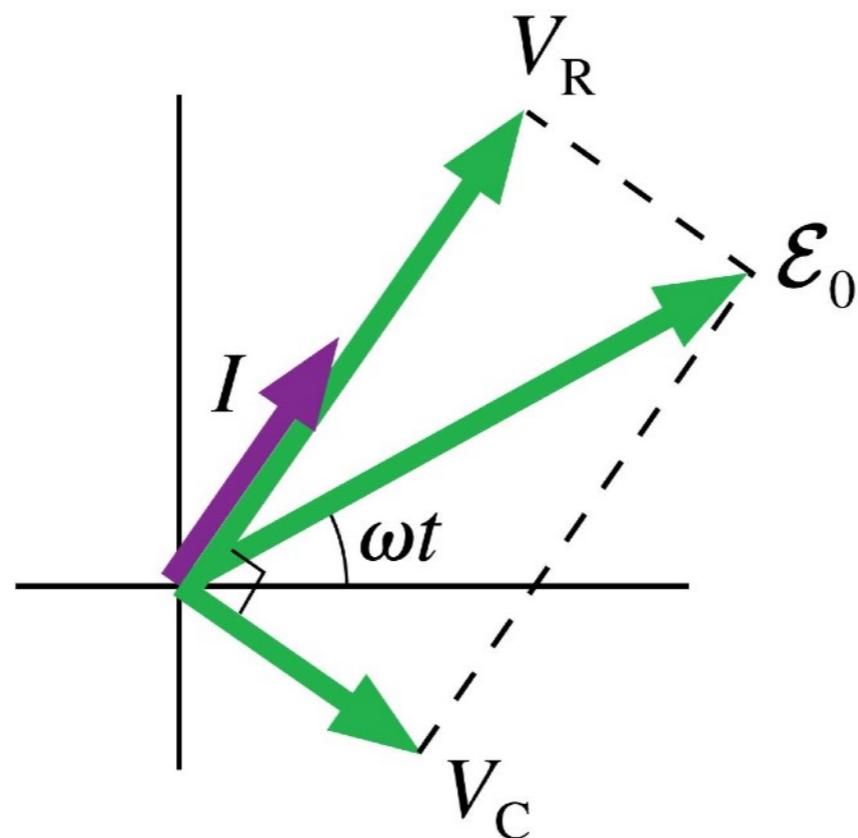
Draw the resistor voltage phasor and the capacitor voltage phasor

Phasor Diagram



Draw the resistor voltage phasor and the capacitor voltage phasor

Phasor Diagram



Draw the resistor voltage phasor and the capacitor voltage phasor

$$\mathcal{E}_0^2 = V_R^2 + V_C^2$$

$$\mathcal{E}_0^2 = V_R^2 + V_C^2$$

-
- Fill in the details and solve for I
-

$$\mathcal{E}_0^2 = V_R^2 + V_C^2$$

-
- Fill in the details and solve for I
-

$$\begin{aligned}\mathcal{E}_0^2 &= V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2 \\ &= (R^2 + 1/\omega^2 C^2)I^2\end{aligned}$$

$$\mathcal{E}_0^2 = V_R^2 + V_C^2$$

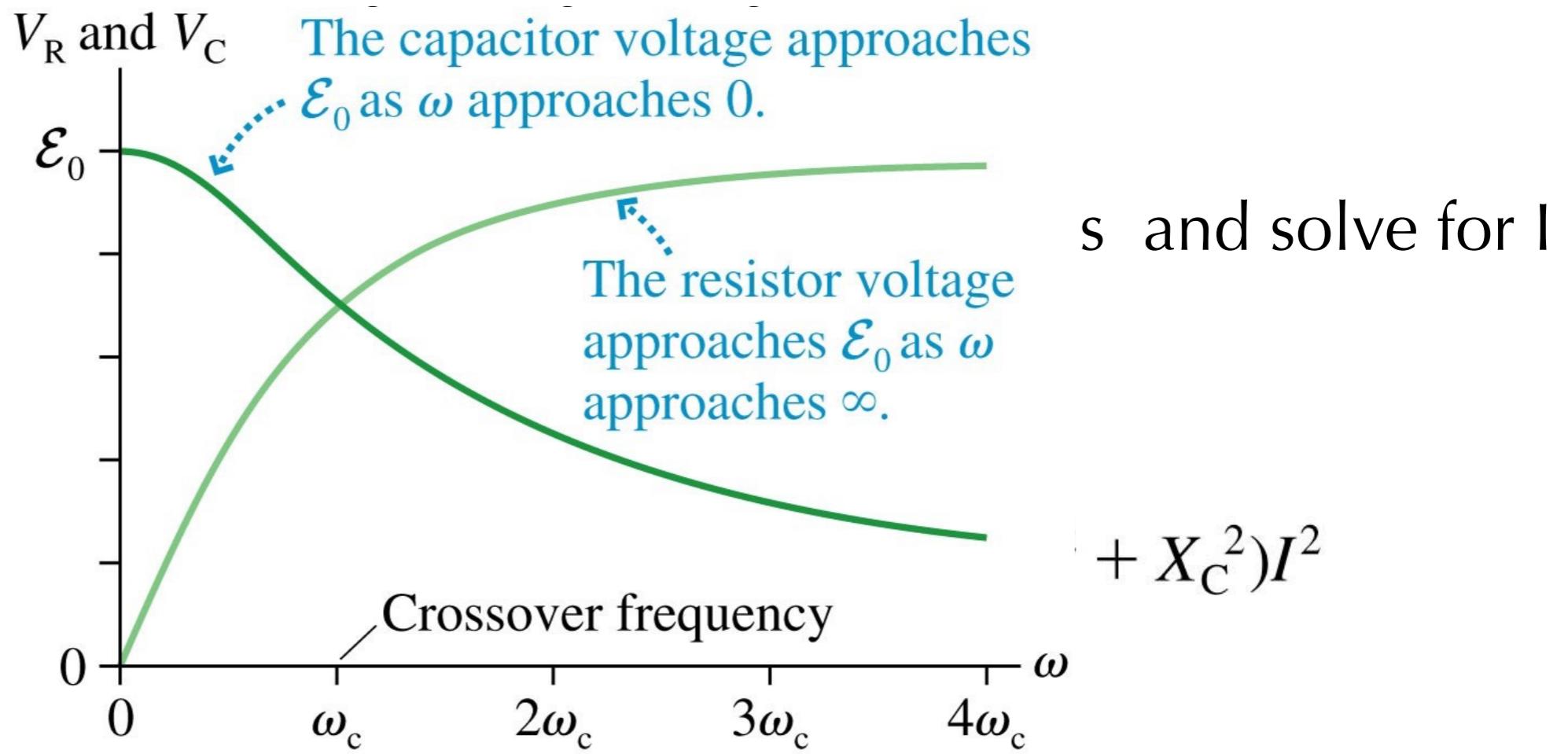
-
- Fill in the details and solve for I
-

$$\begin{aligned}\mathcal{E}_0^2 &= V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2 \\ &= (R^2 + 1/\omega^2 C^2)I^2\end{aligned}$$

$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 / \omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

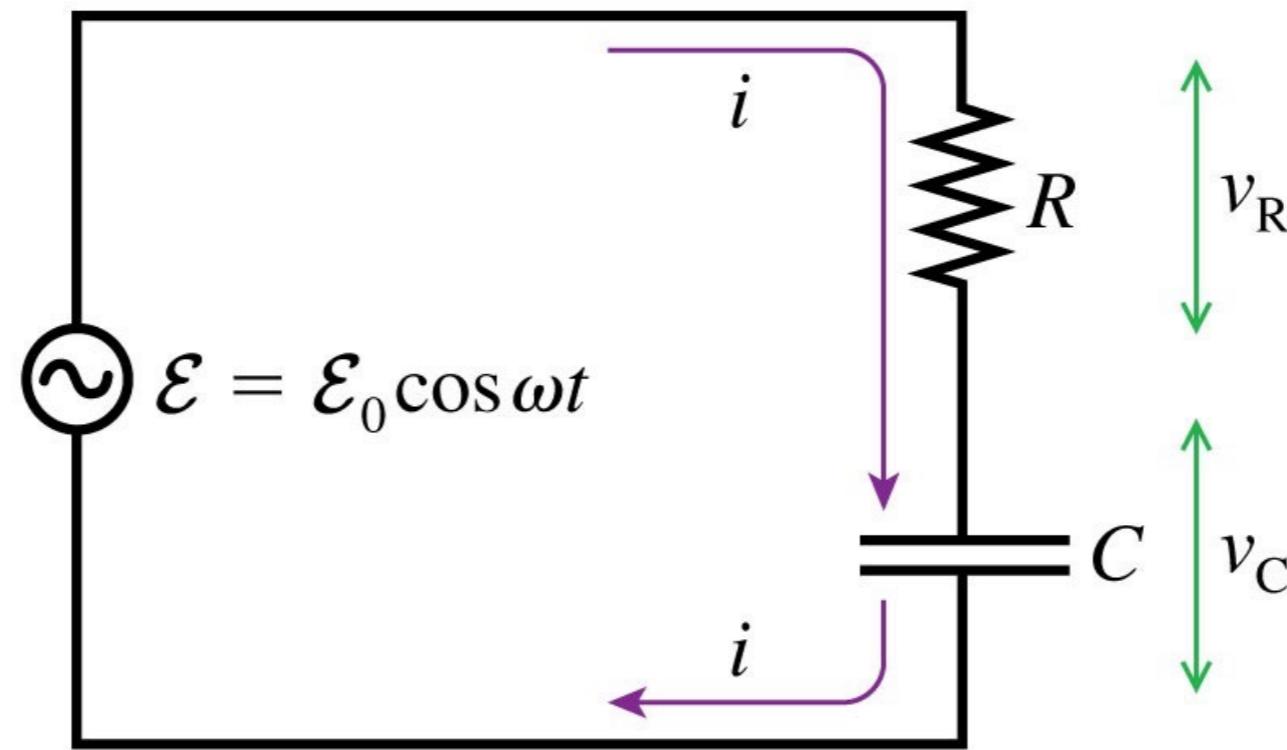
$$\omega_c = \frac{1}{RC}$$



$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

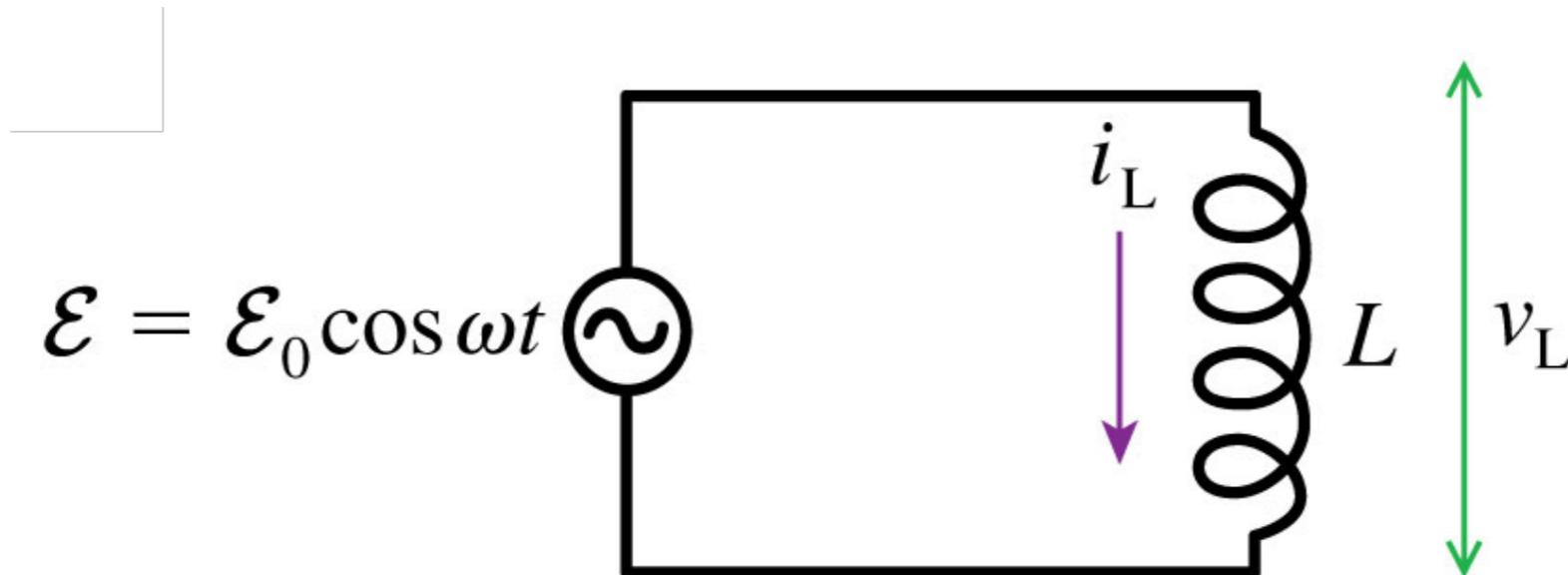
$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 / \omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

AC RC circuit



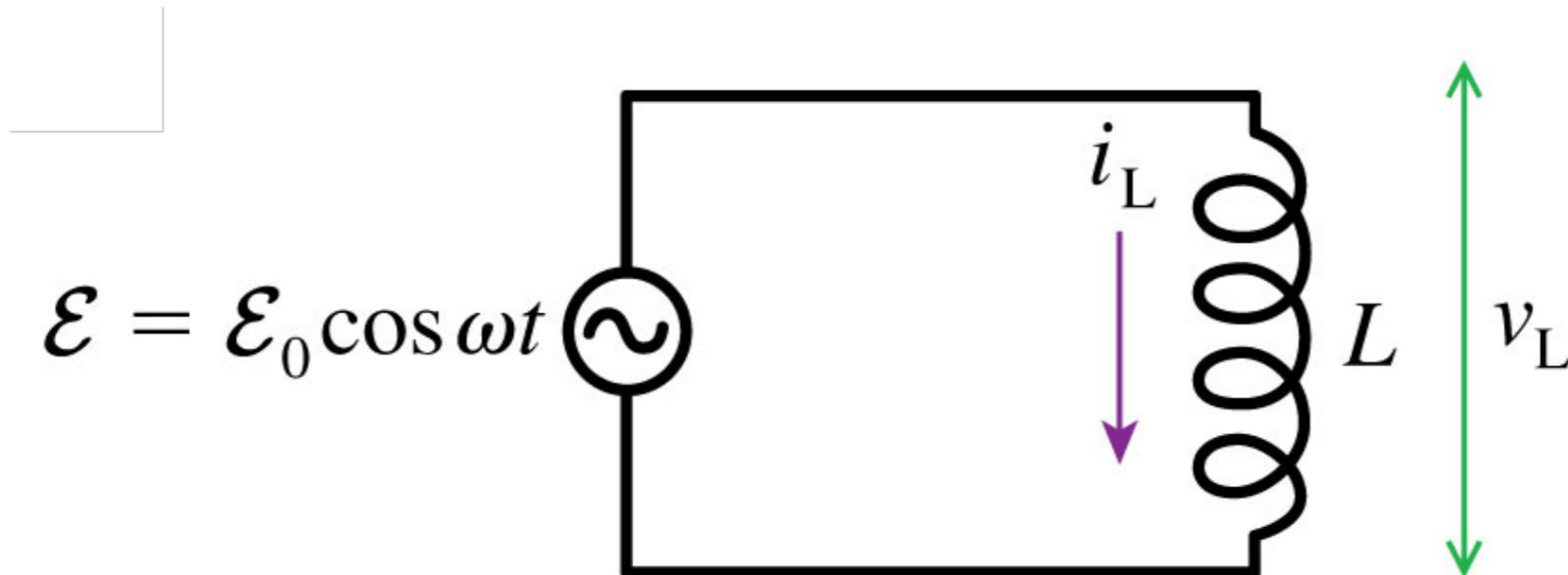
If the source frequency is high(low), will the peak voltage across the resistor (capacitor) be high or low?

Inductor Circuits



- Set these two expressions equal to each other.
- Accumulate time on one side and current on the other.
- Integrate both sides.

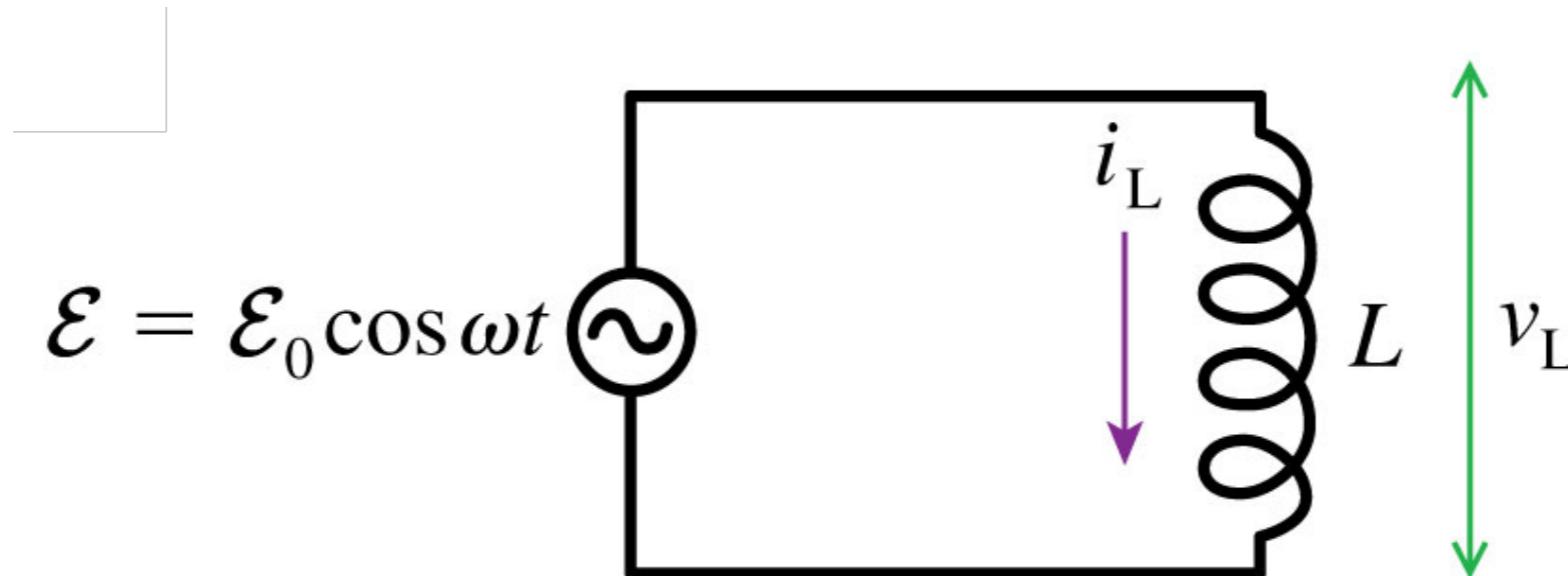
Inductor Circuits



$$v_L = L \frac{di_L}{dt}$$

- Set these two expressions equal to each other.
- Accumulate time on one side and current on the other.
- Integrate both sides.

Inductor Circuits



$$v_L = L \frac{di_L}{dt}$$

$$v_L = V_L \cos \omega t$$

- Set these two expressions equal to each other.
- Accumulate time on one side and current on the other.
- Integrate both sides.

$$i_L = \frac{V_L}{L} \int \cos \omega t dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos \left(\omega t - \frac{\pi}{2} \right)$$

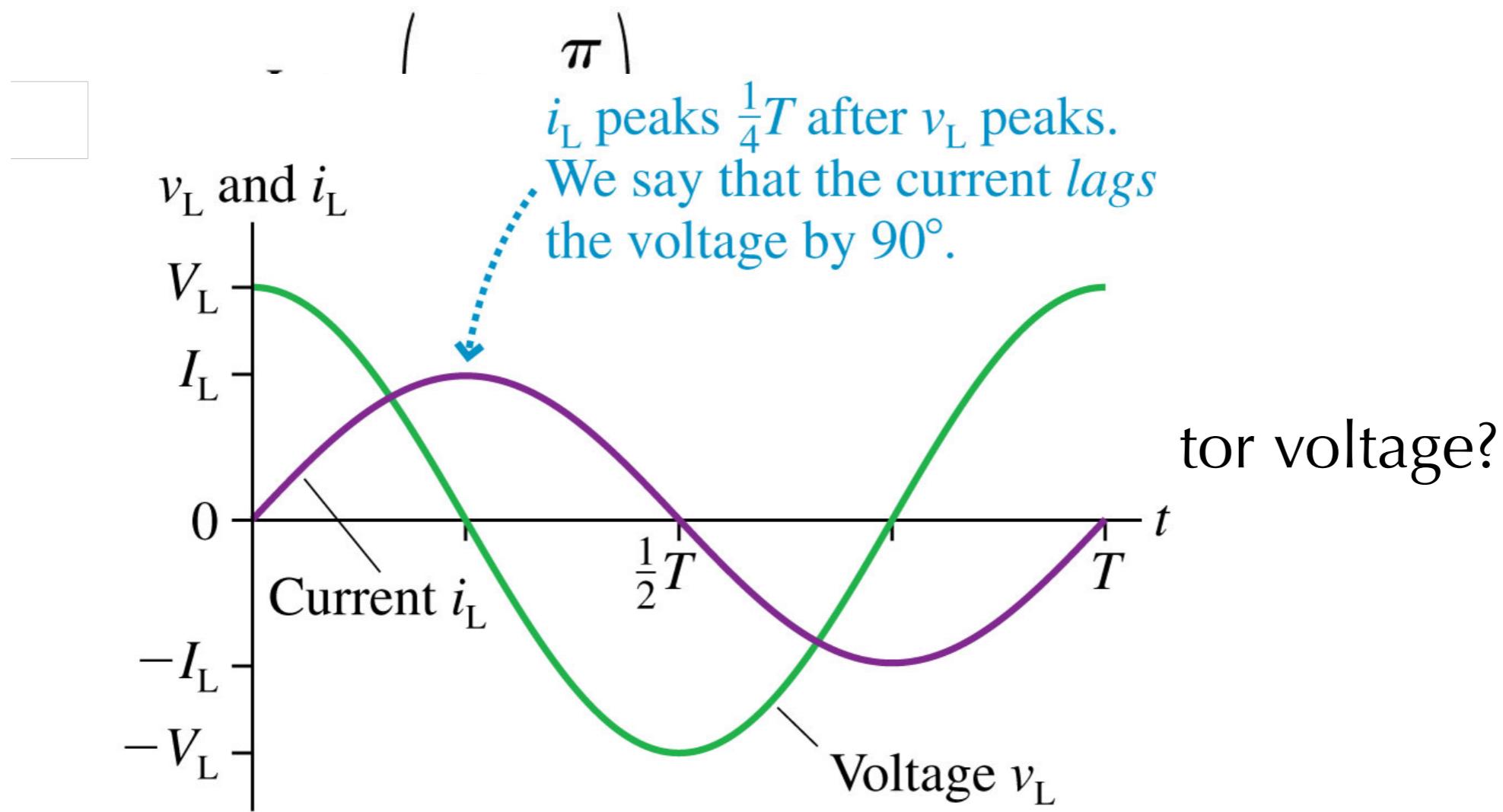
$$= I_L \cos \left(\omega t - \frac{\pi}{2} \right)$$

The inductor's current _____ the inductor voltage?

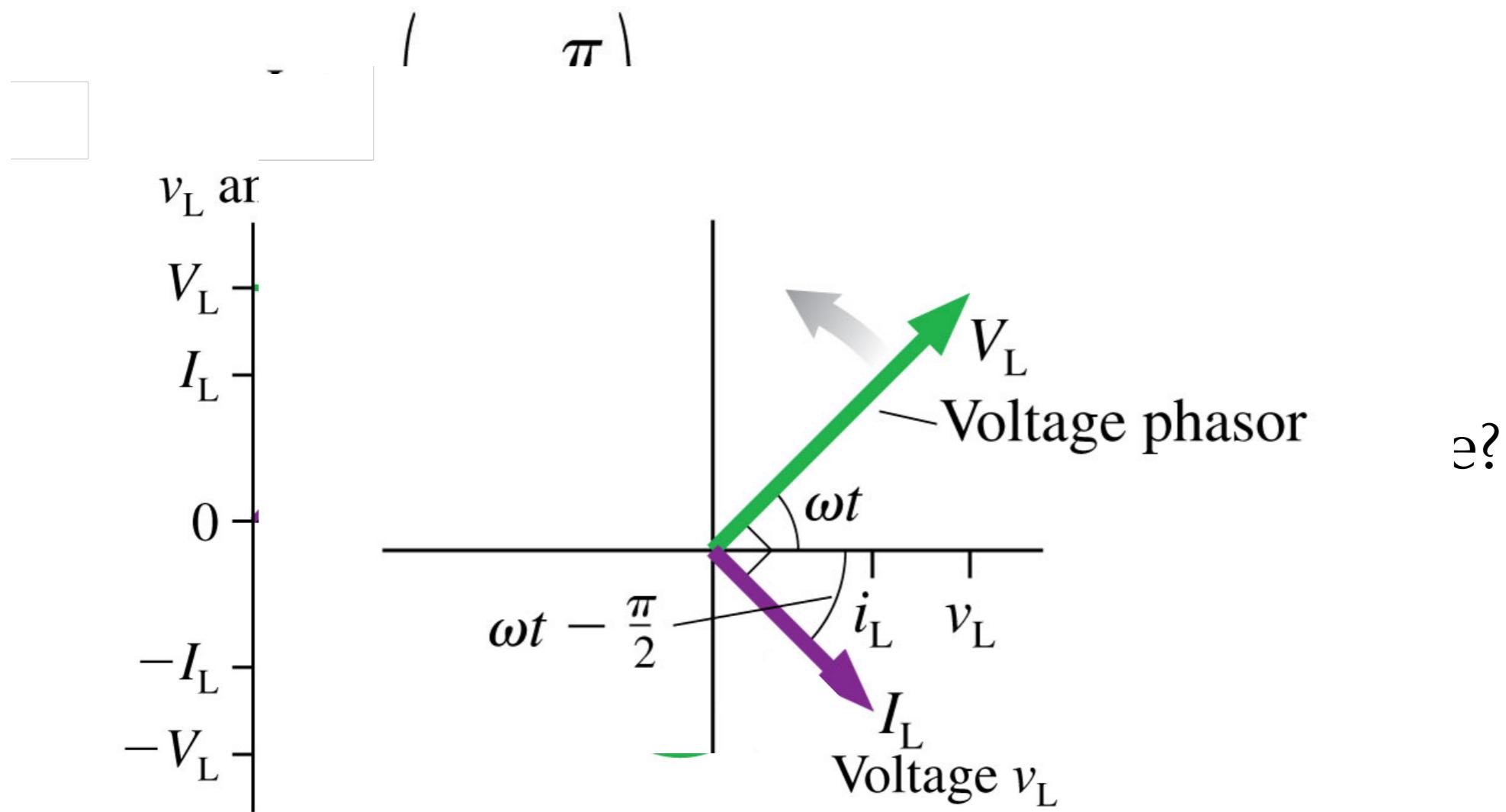
$$i_L = \frac{V_L}{L} \int \cos \omega t dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos \left(\omega t - \frac{\pi}{2} \right)$$
$$= I_L \cos \left(\omega t - \frac{\pi}{2} \right)$$

The inductor's current _____ the inductor voltage?

$$i_L = \frac{V_L}{L} \int \cos \omega t dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos \left(\omega t - \frac{\pi}{2} \right)$$



$$i_L = \frac{V_L}{L} \int \cos \omega t dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos \left(\omega t - \frac{\pi}{2} \right)$$



$$i_L=\frac{V_L}{\omega L}\cos(\omega t-\frac{\pi}{2})$$

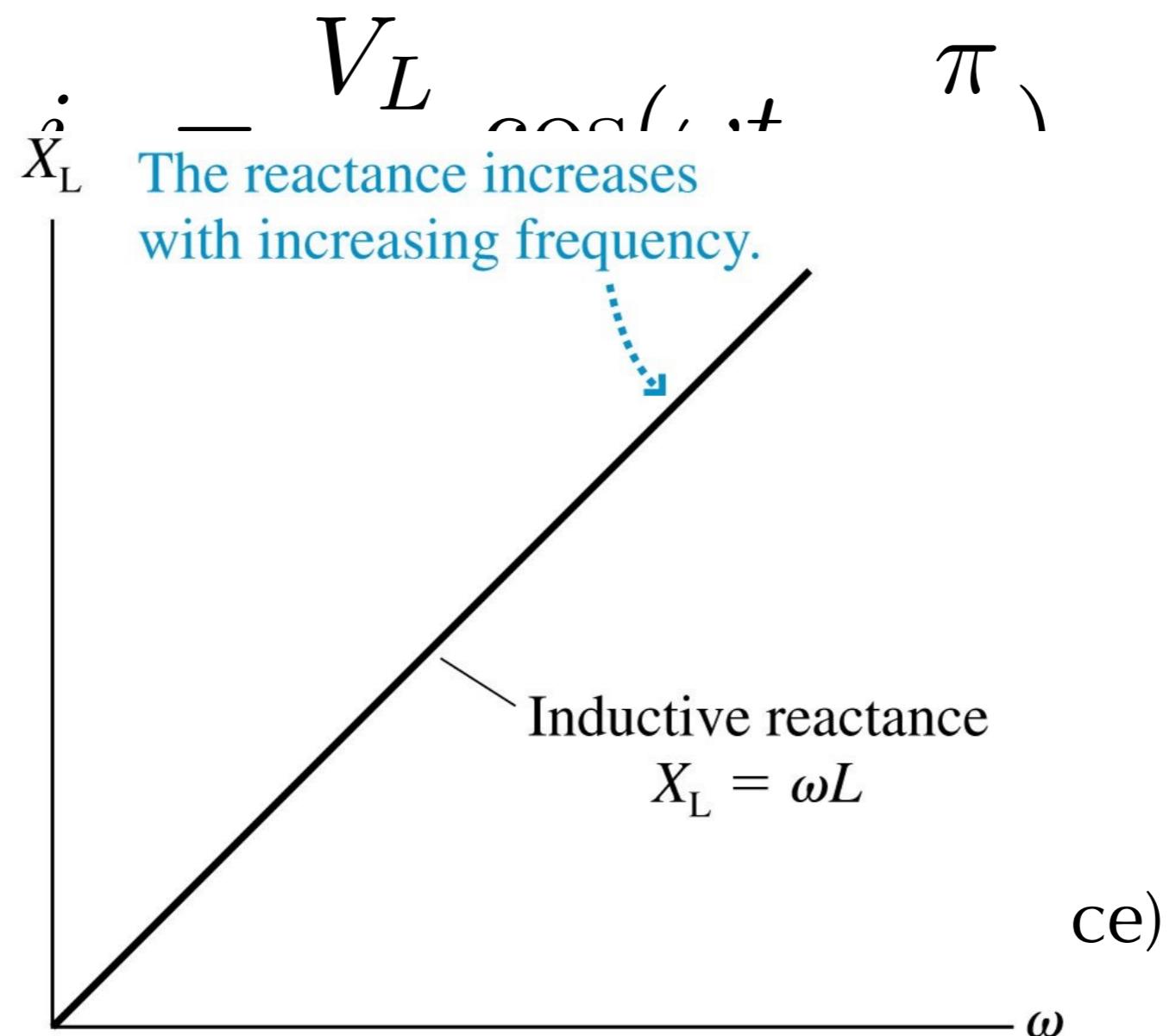
$$i_L = \frac{V_L}{\omega L} \cos(\omega t - \frac{\pi}{2})$$

$$I_L = \frac{V_L}{\omega L} \quad (\text{peak current})$$

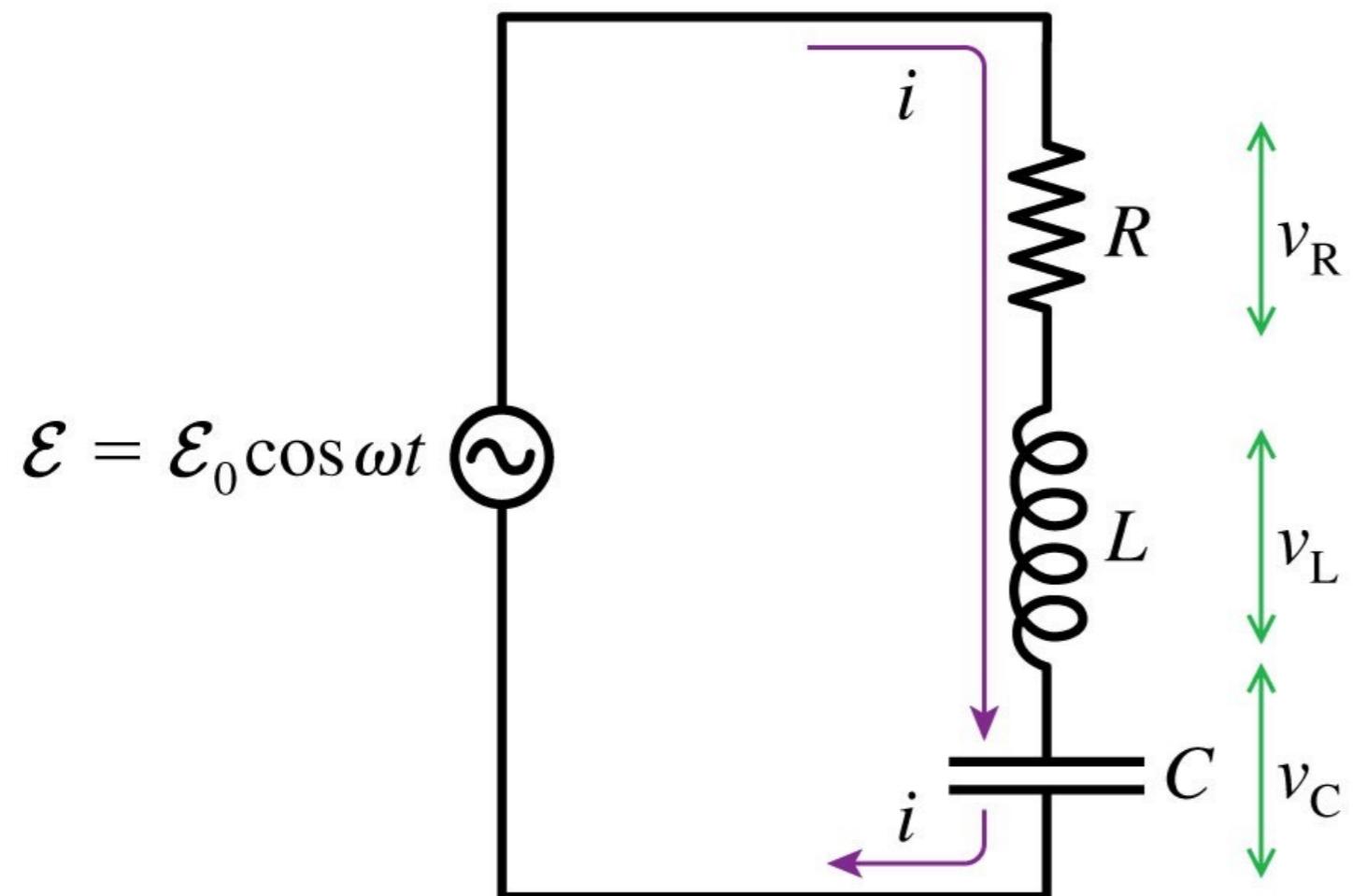
$$i_L = \frac{V_L}{\omega L} \cos(\omega t - \frac{\pi}{2})$$

$$I_L = \frac{V_L}{\omega L} \quad (\text{peak current})$$

$$X_L = \omega L \quad (\text{inductive reactance})$$



Series RLC circuit

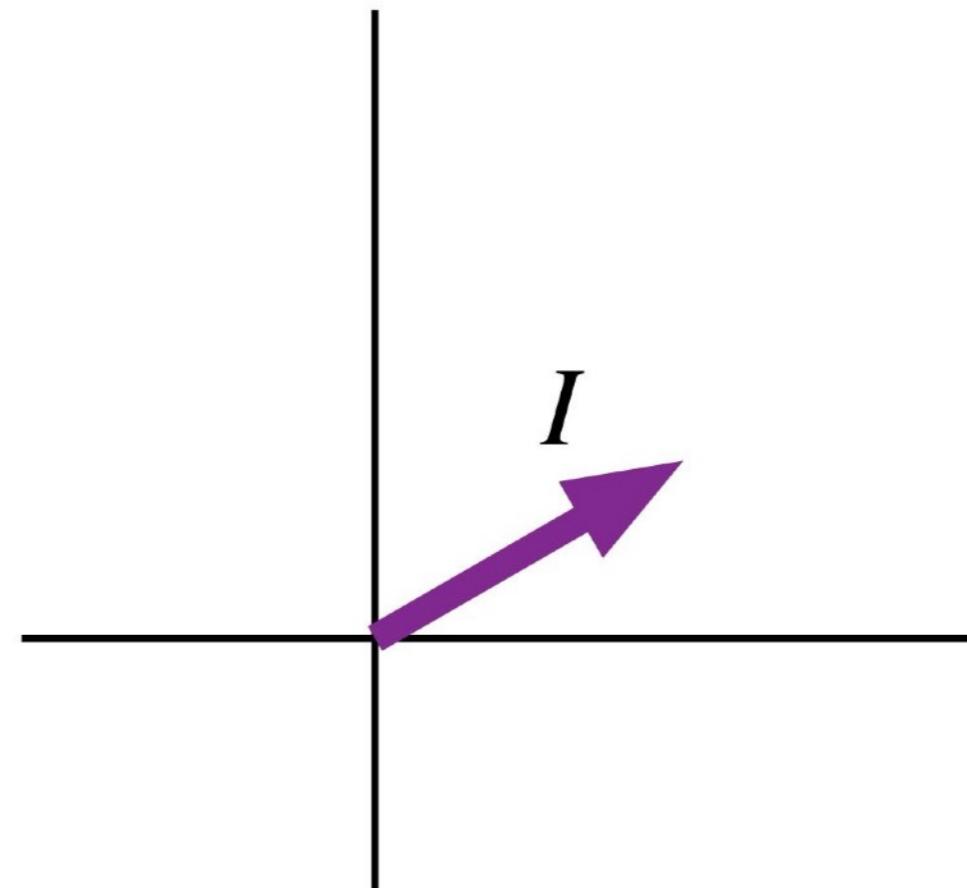


$$i = i_R = i_L = i_C$$

$$\mathcal{E} = v_R + v_L + v_C$$

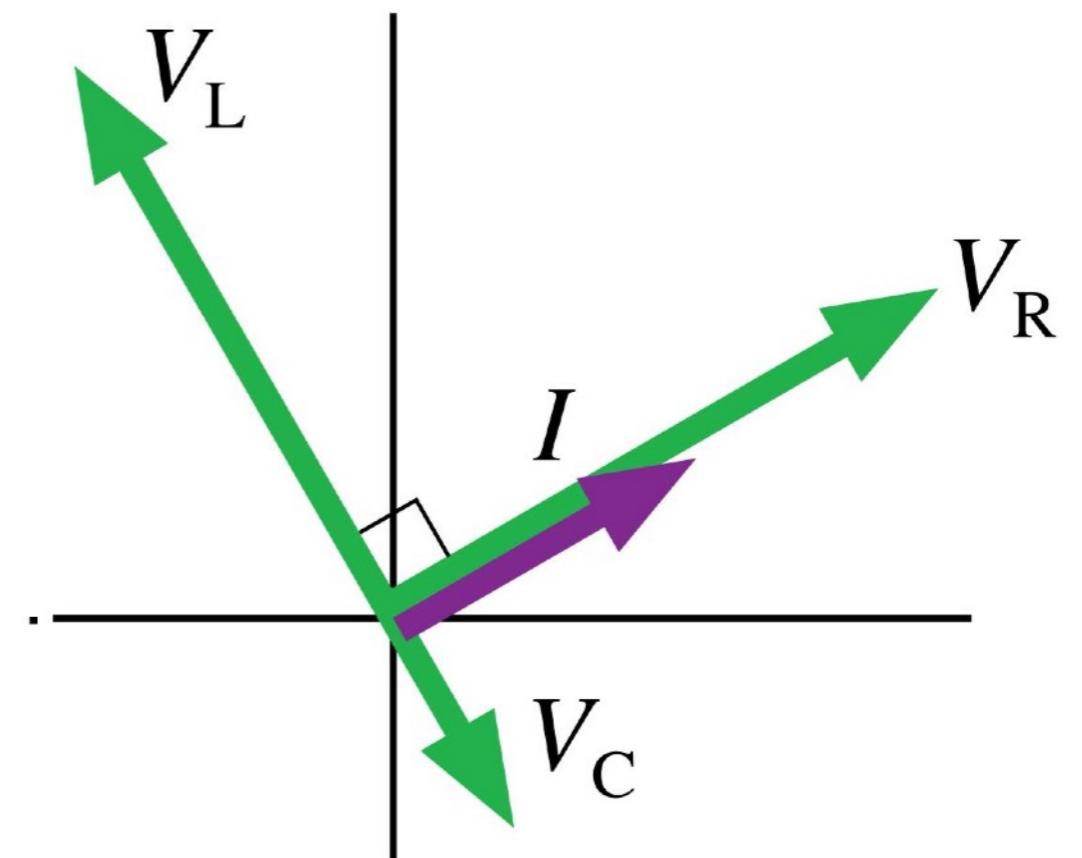
Phasor diagram

$$\mathcal{E} = v_R + v_L + v_C$$



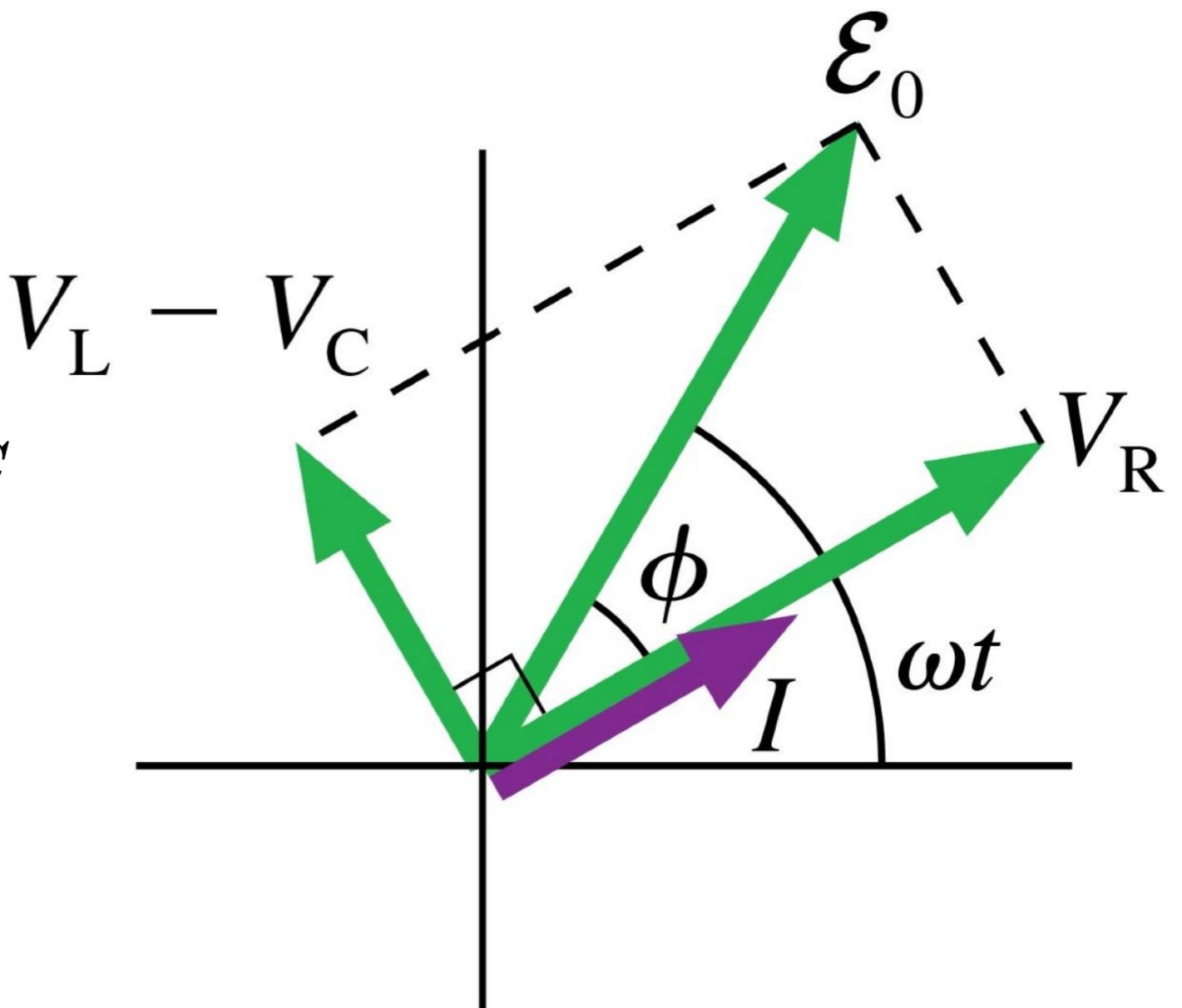
Phasor diagram

$$\mathcal{E} = v_R + v_L + v_C$$



Phasor diagram

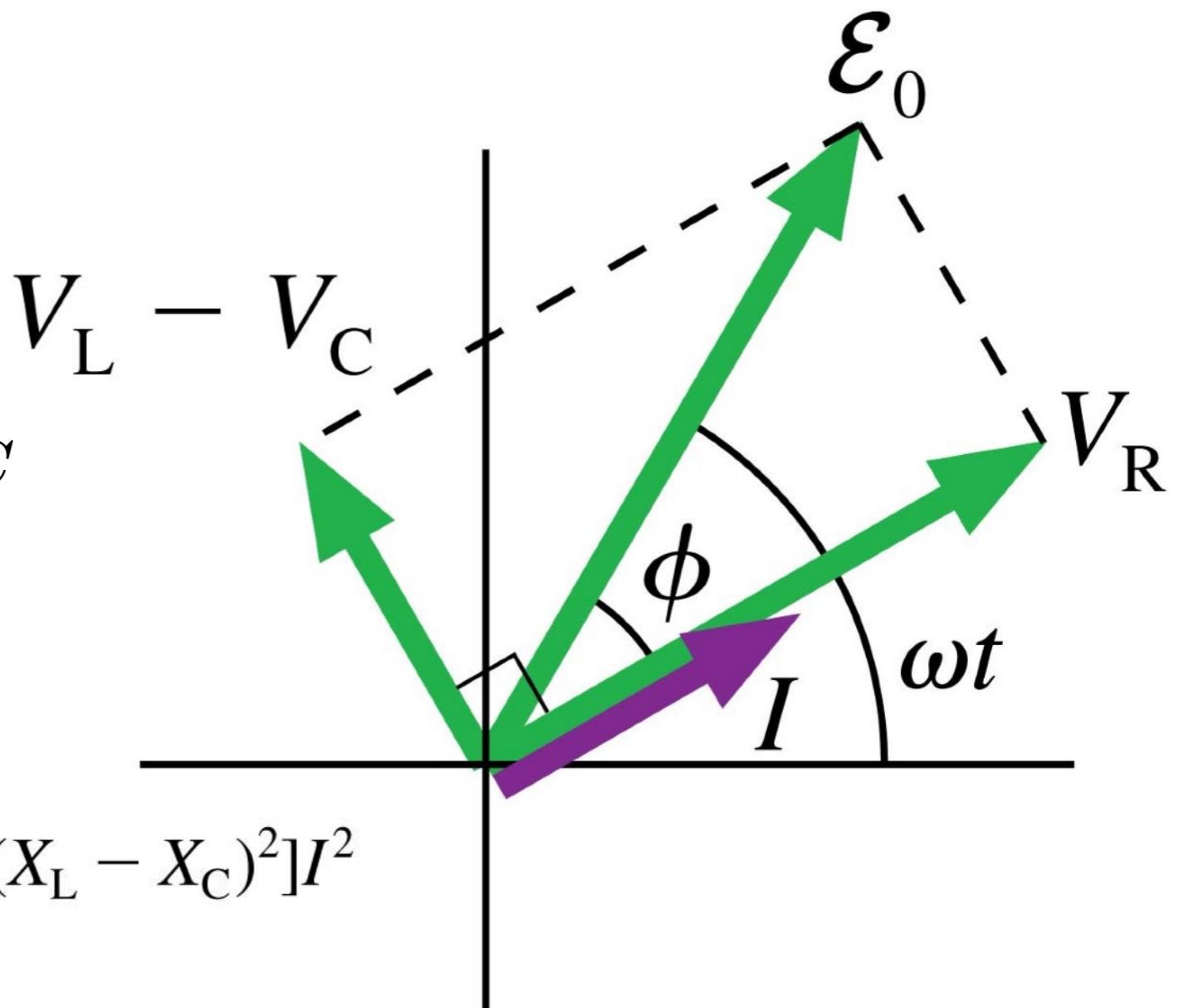
$$\mathcal{E} = v_R + v_L + v_C$$



Mathematica Notebook

Phasor diagram

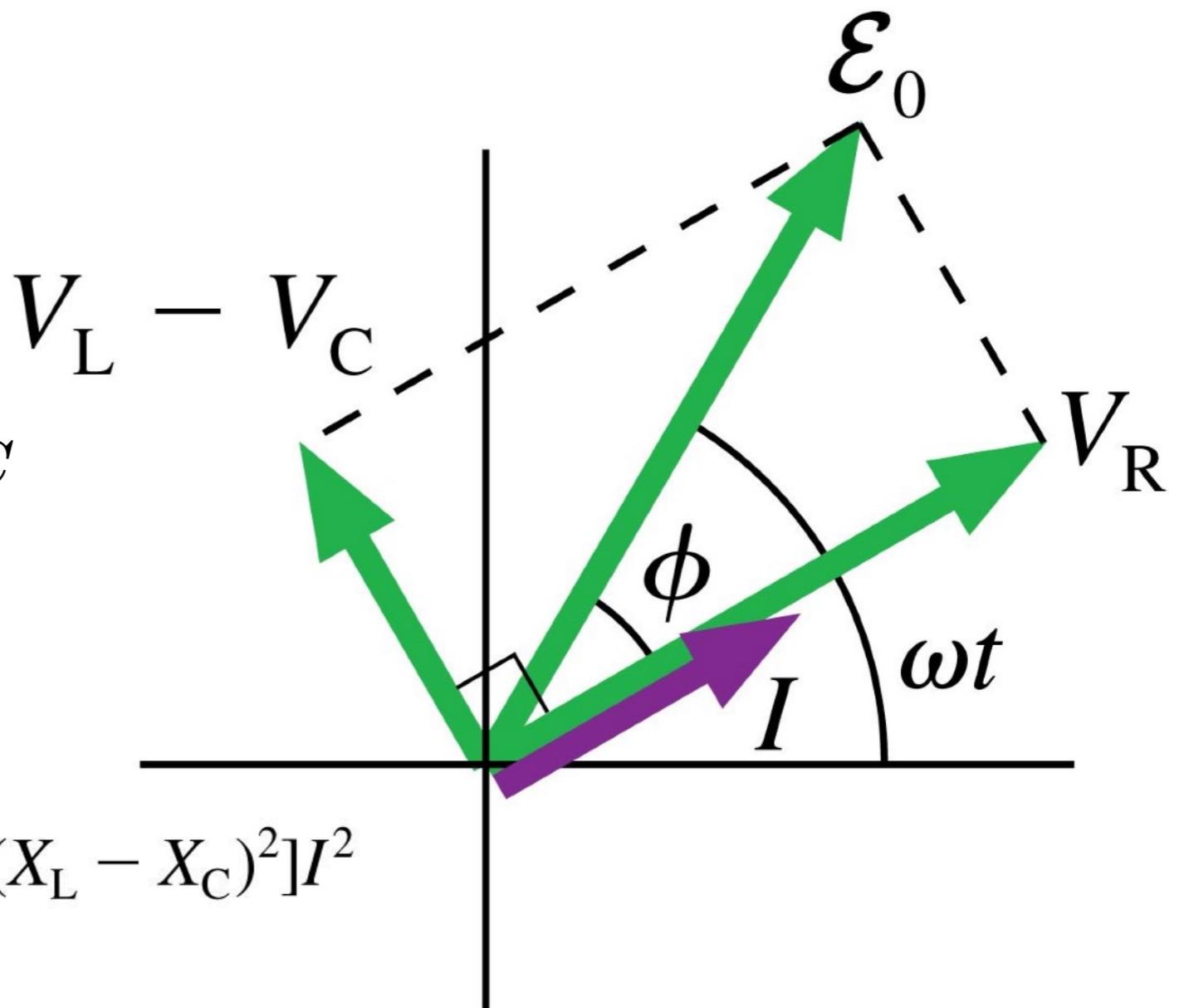
$$\mathcal{E} = v_R + v_L + v_C$$



$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = [R^2 + (X_L - X_C)^2]I^2$$

Phasor diagram

$$\mathcal{E} = v_R + v_L + v_C$$



$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = [R^2 + (X_L - X_C)^2]I^2$$

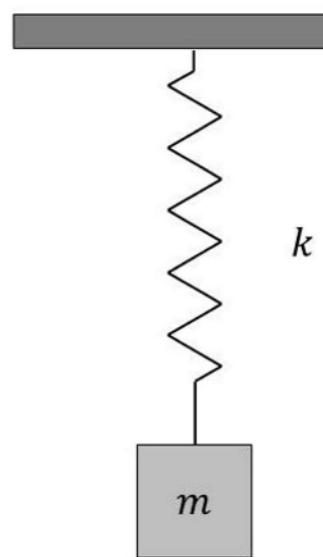
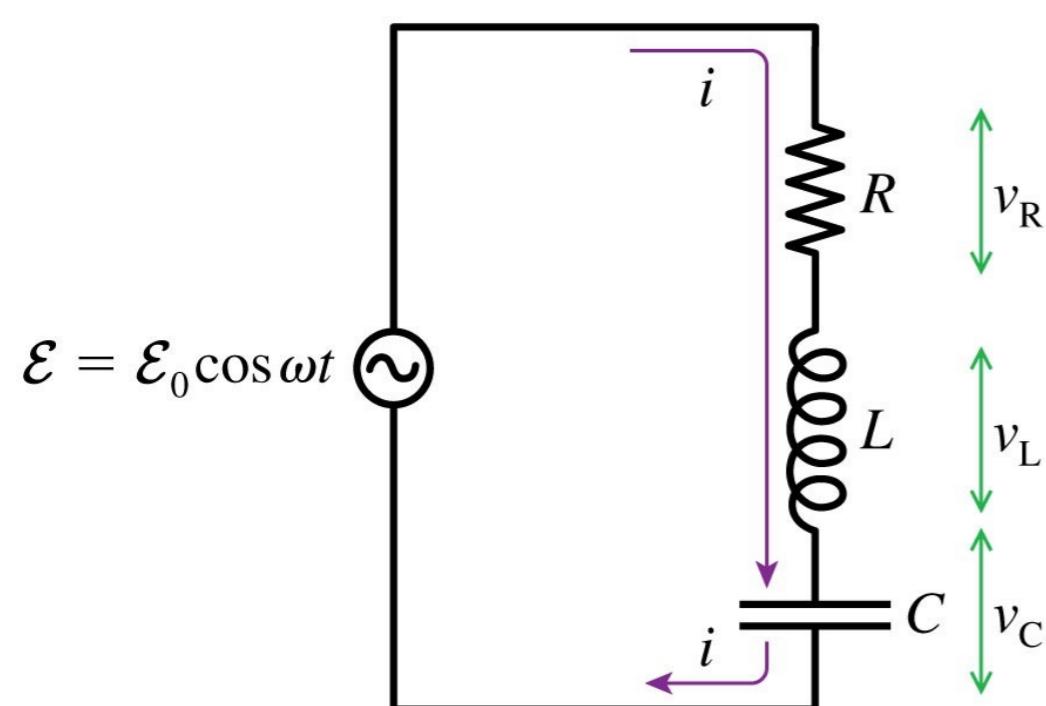
$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Mathematica Notebook

An interesting parallel

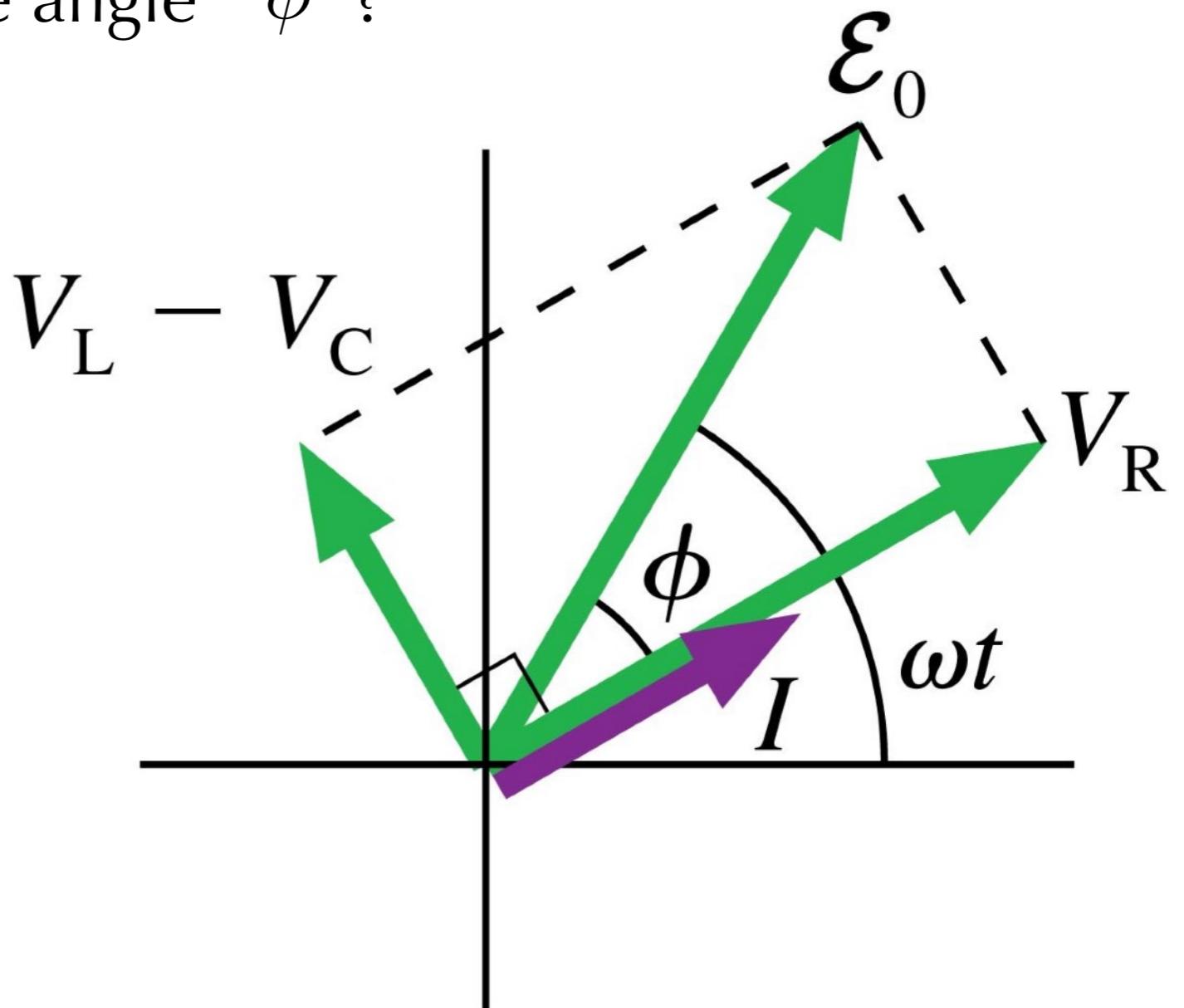
$$-\frac{Q}{C} - IR - \epsilon - L \frac{di}{dt} = 0$$

$$-kx - bv + F = ma$$

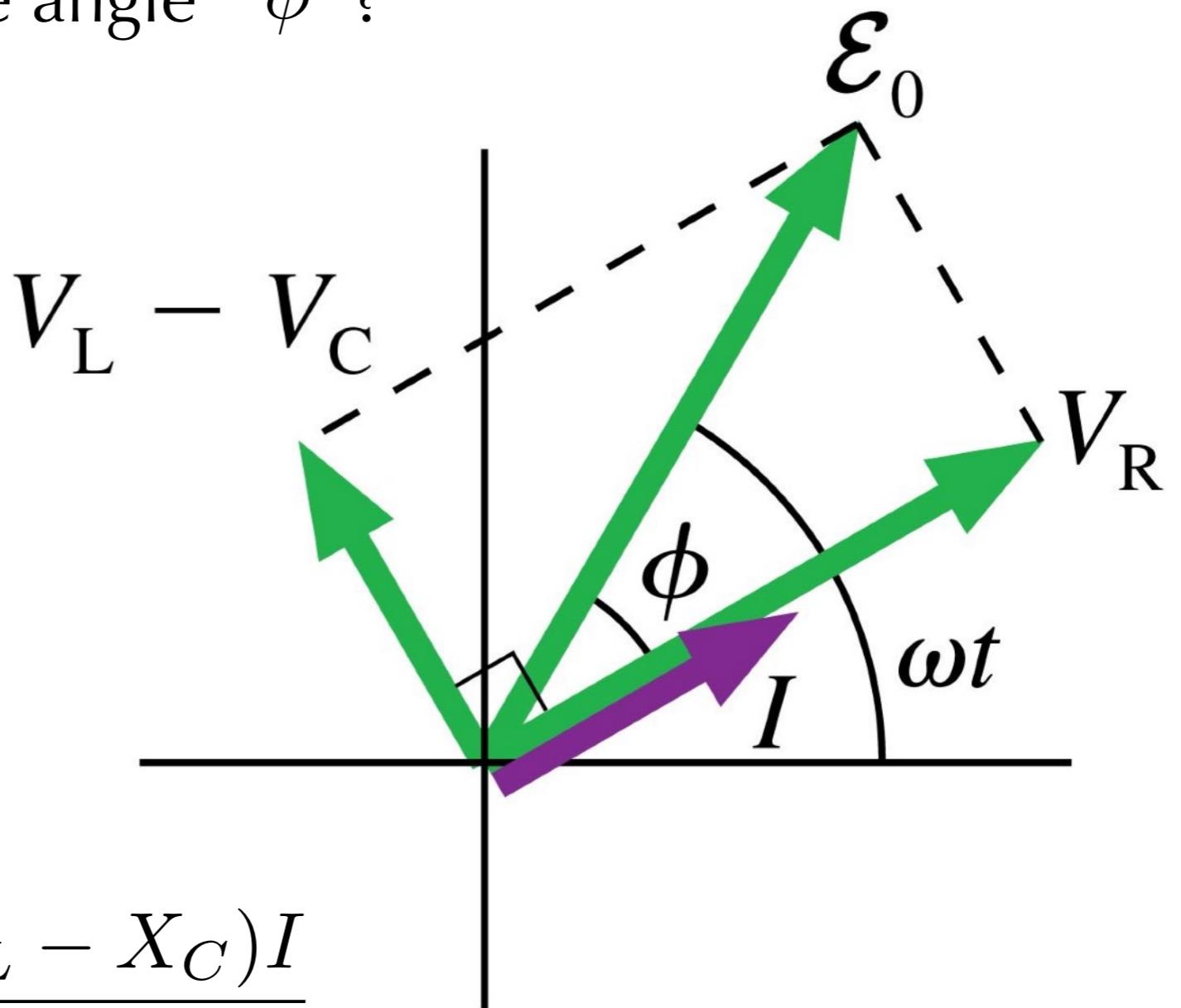


How would you find the angle ϕ ?

How would you find the angle ϕ ?



How would you find the angle ϕ ?



$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{(X_L - X_C)I}{IR}$$

Resonance

[video demo](#)

Resonance

$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

[video demo](#)

Resonance

$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

When is this function a maximum?

[video demo](#)

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$$X_L = X_C$$

[video demo](#)

Resonance

$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

When is this function a maximum?

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

video demo

Resonance

$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

When is this function a maximum?

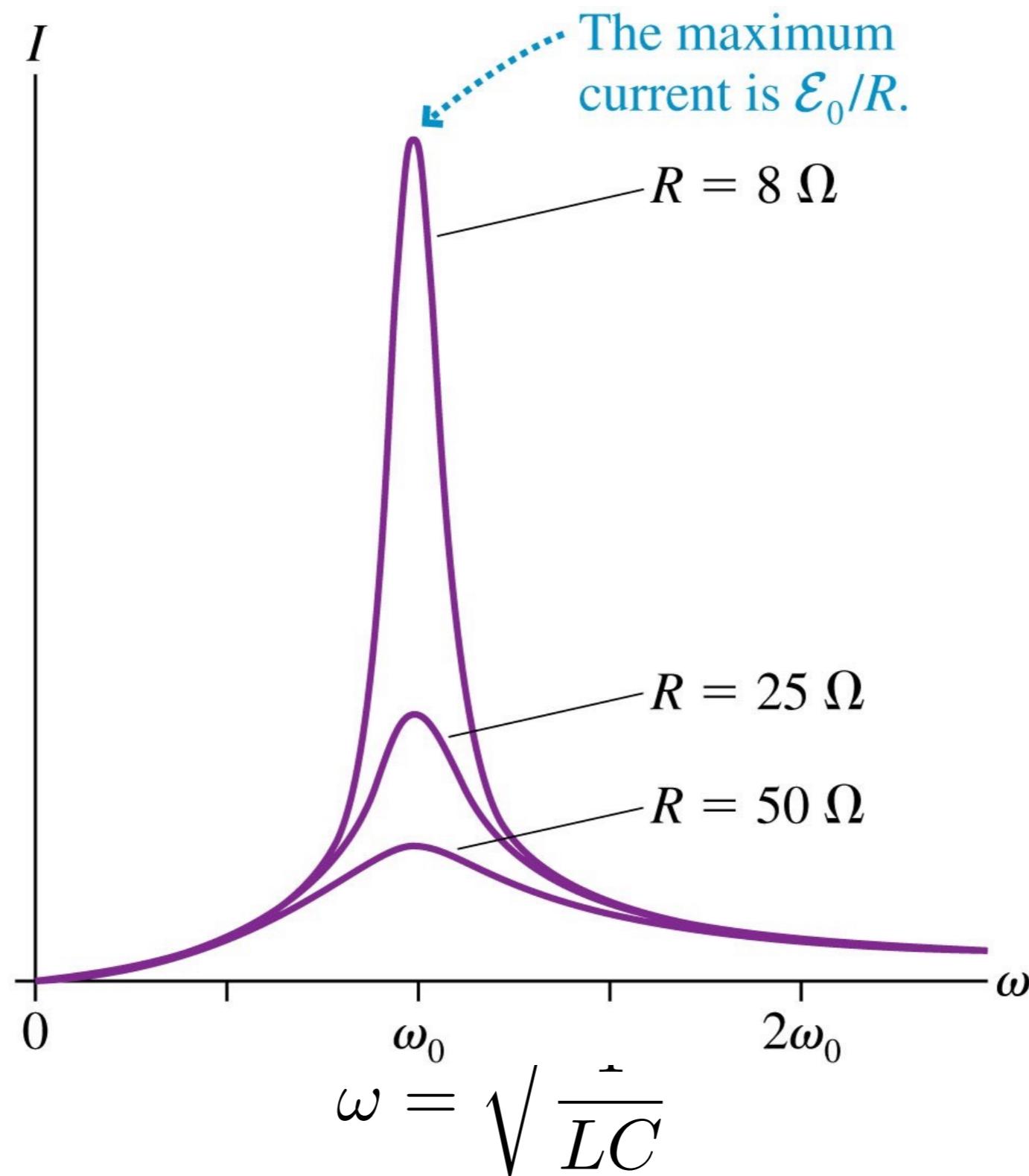
$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \sqrt{\frac{1}{LC}}$$

video demo

Resonance

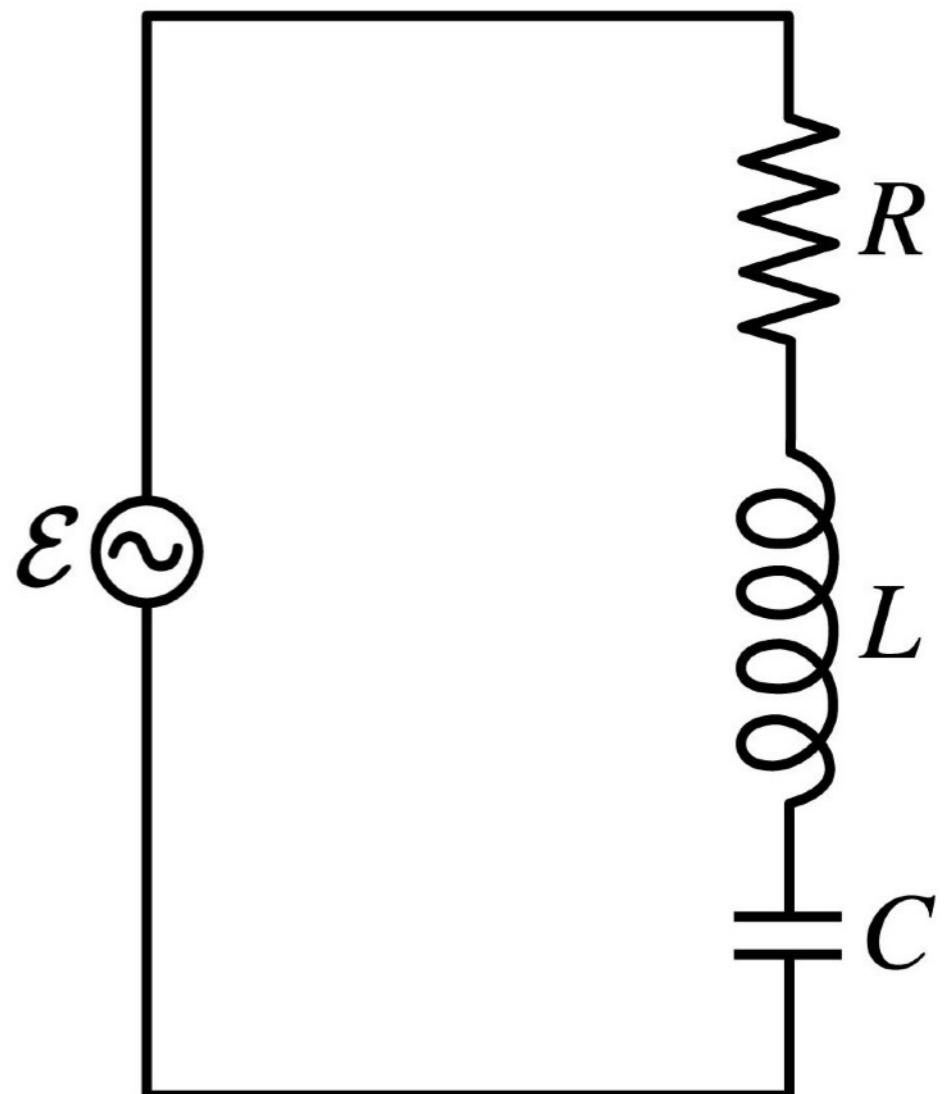


video demo

Question #38

If the value of R is increased, the resonance frequency of this circuit

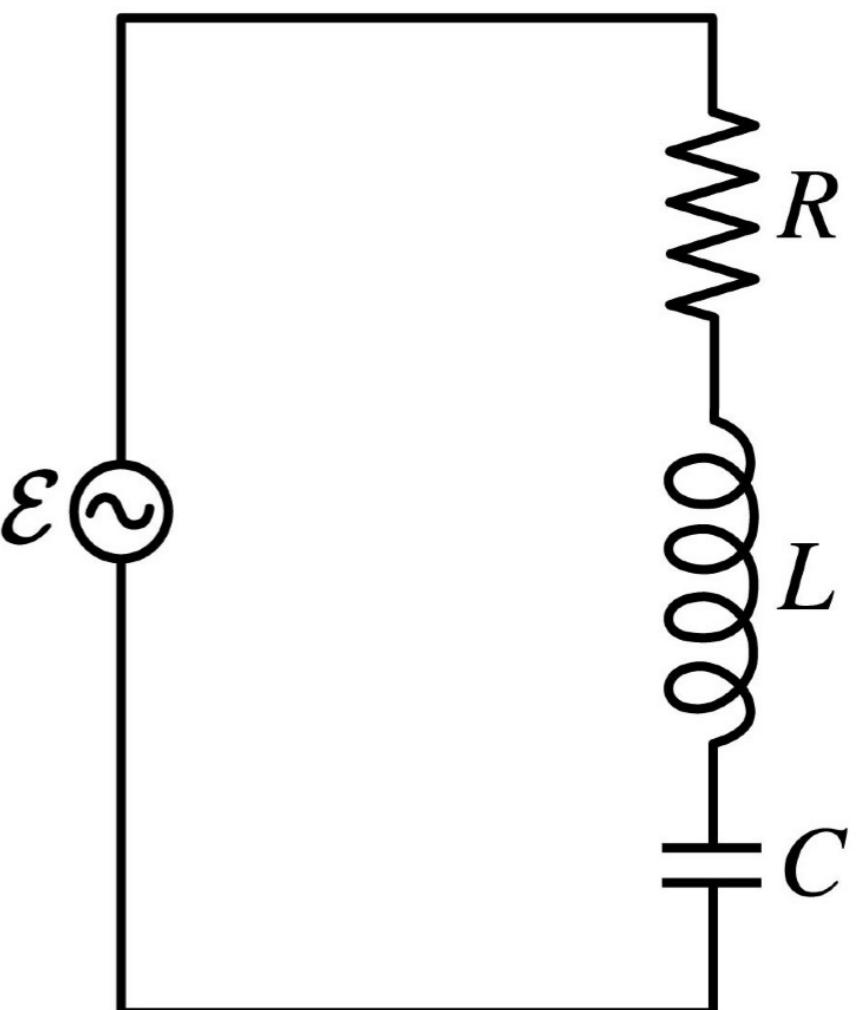
- A. Increases.
- B. Decreases.
- C. Stays the same.



Question #39

The resonance frequency of this circuit is 1000 Hz. To change the resonance frequency to 2000 Hz, replace the capacitor with one having capacitance

- A. $4C$.
- B. Keep it the same
- C. $2C$.
- D. $C/2$.
- E. $C/4$.

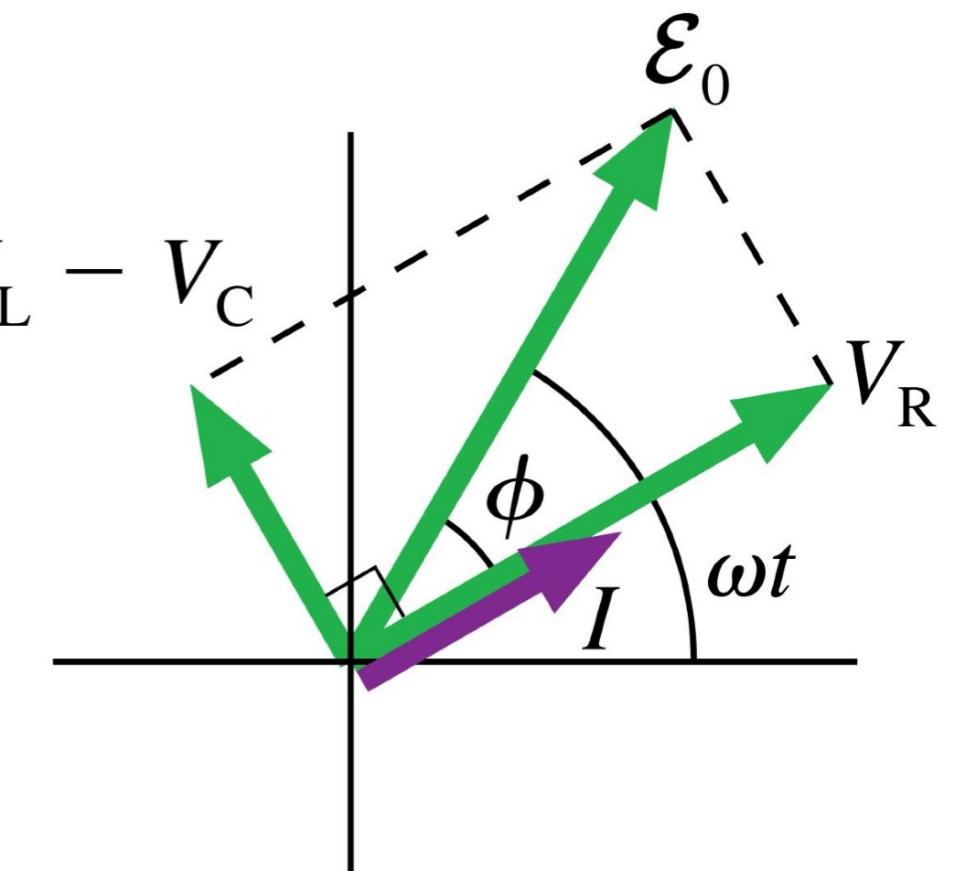


Question #40

The resonance frequency of a series RLC circuit is less than the emf frequency. Does the current lead or lag the emf? Explain

C - Lead

D - Lag

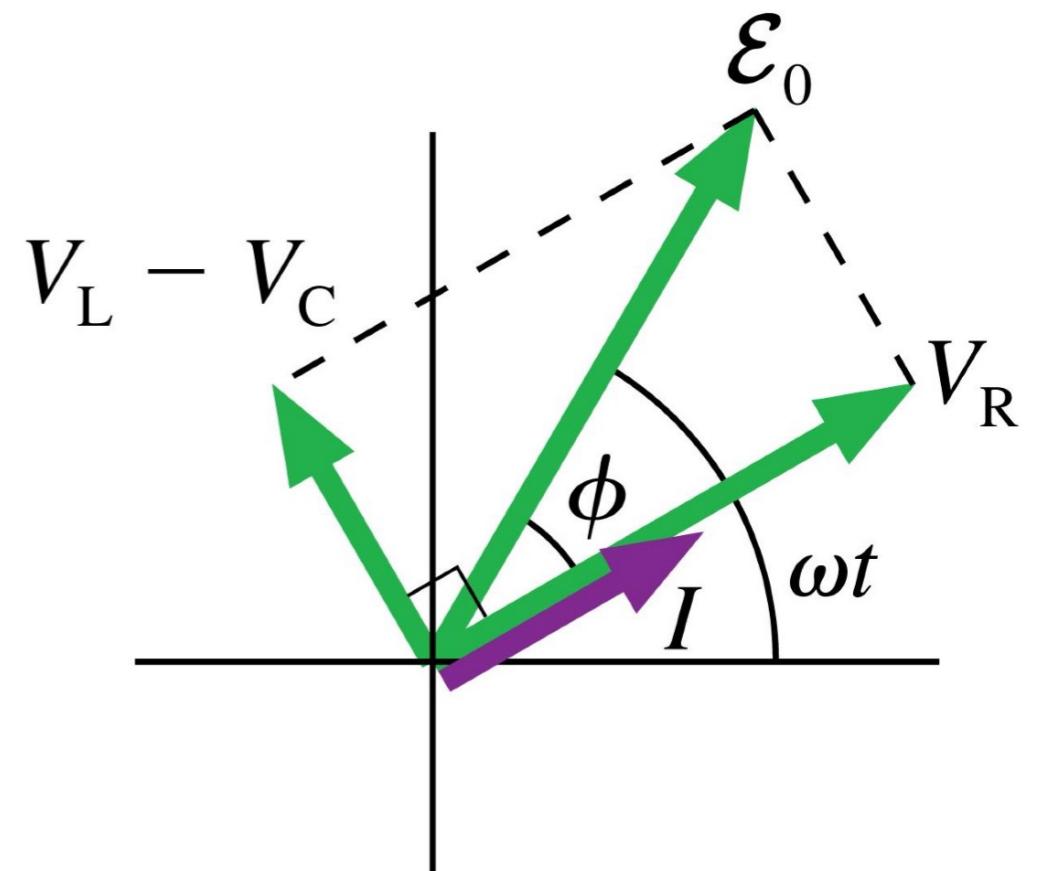


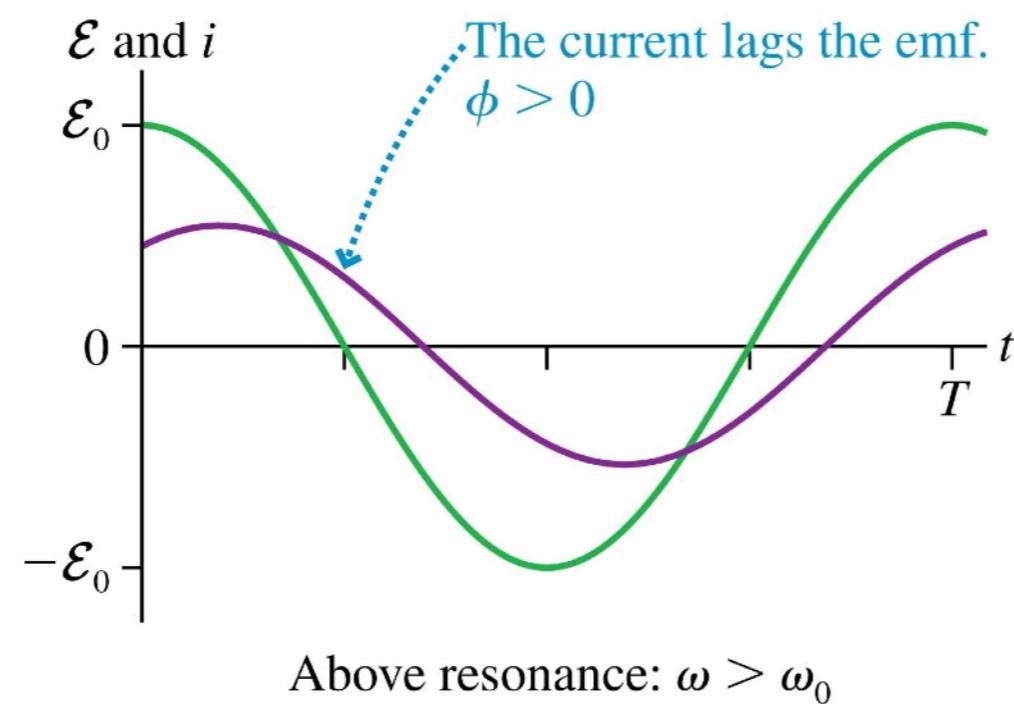
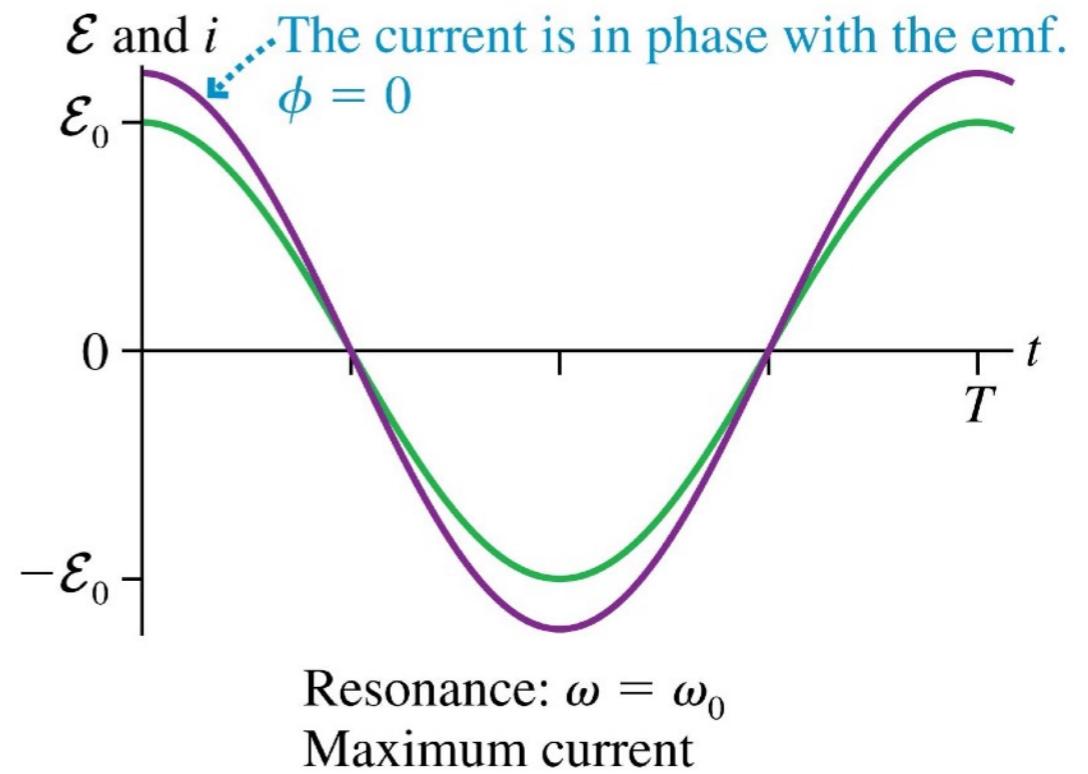
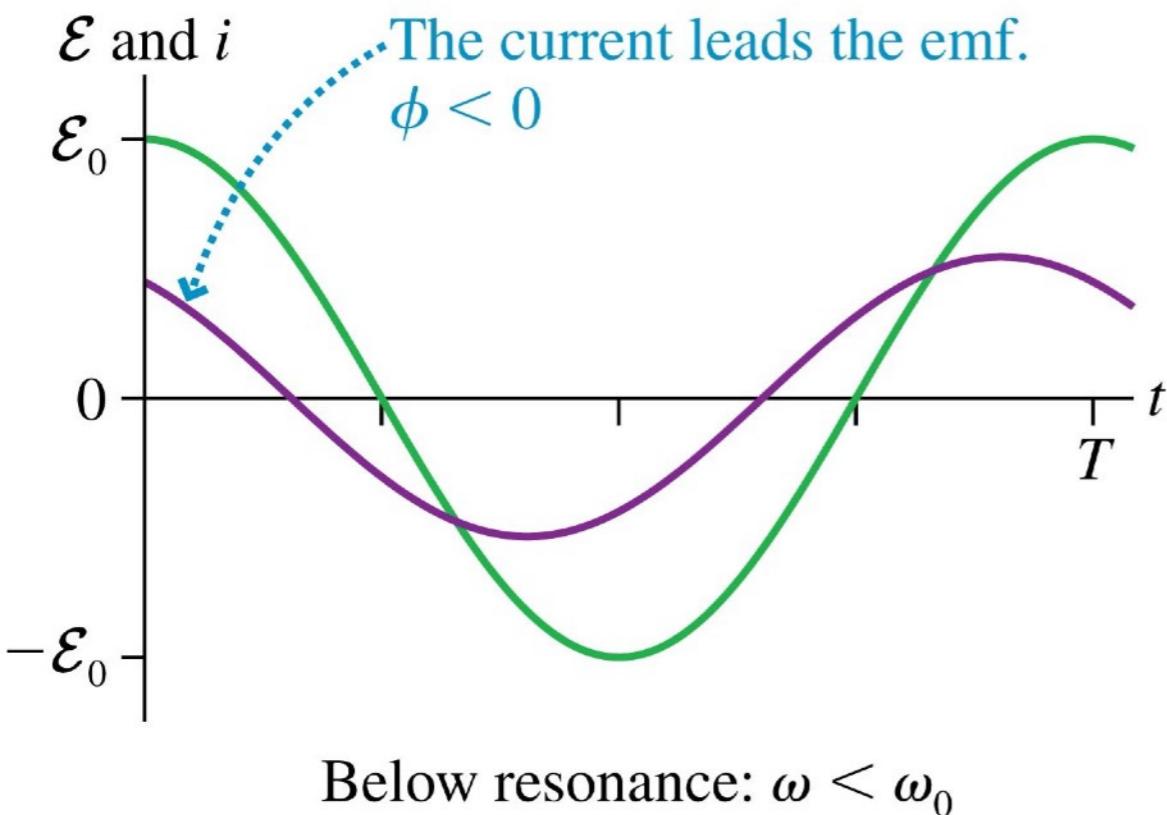
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The resonance frequency of a series RLC circuit is less than the emf frequency. Does the current lead or lag the emf? Explain

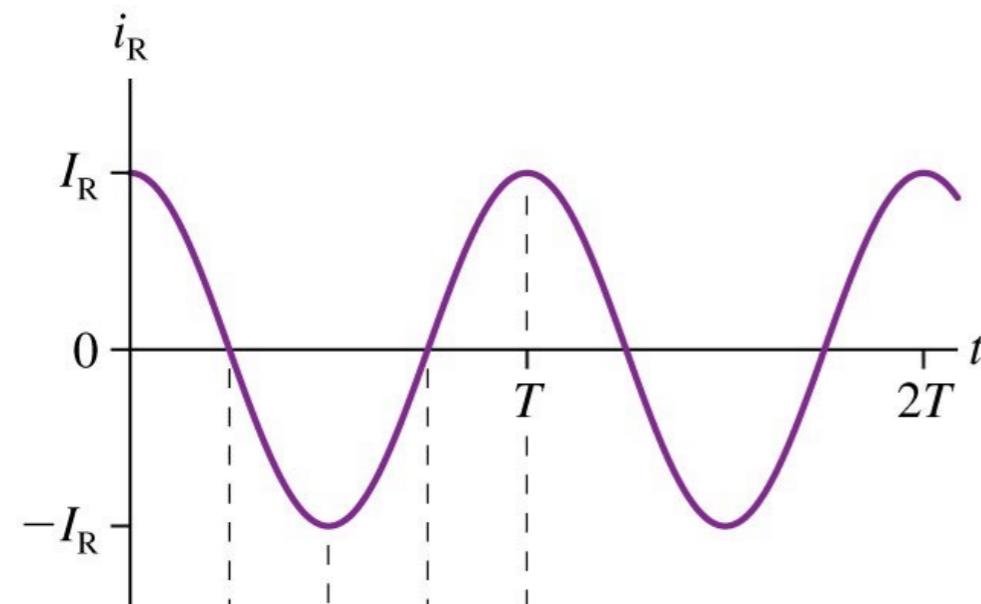
C - Lead
D - Lag

lag



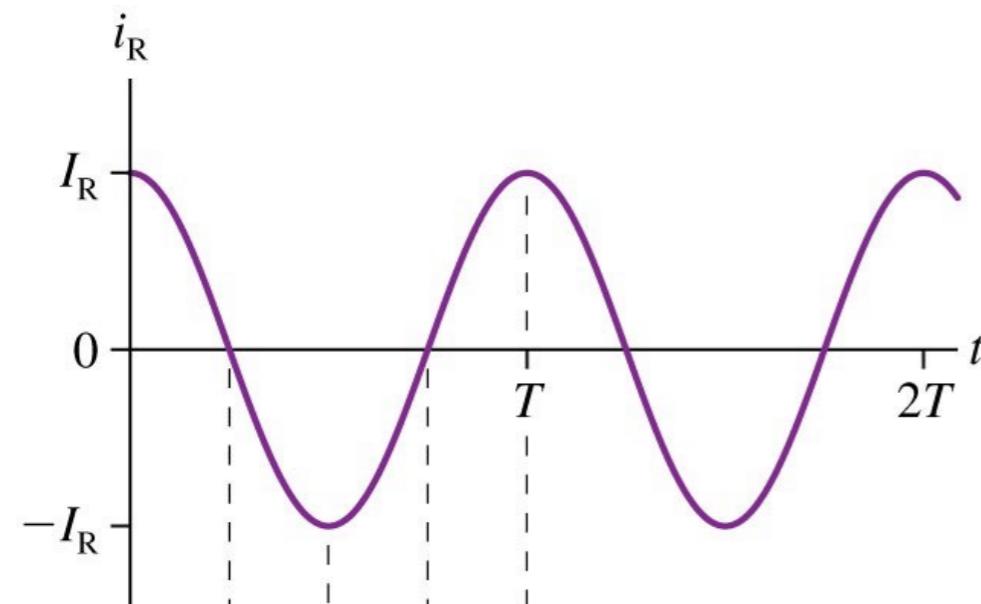


Power dissipated in resistor



$$p_R = i_R^2 R = I_R^2 R \cos^2 \omega t$$

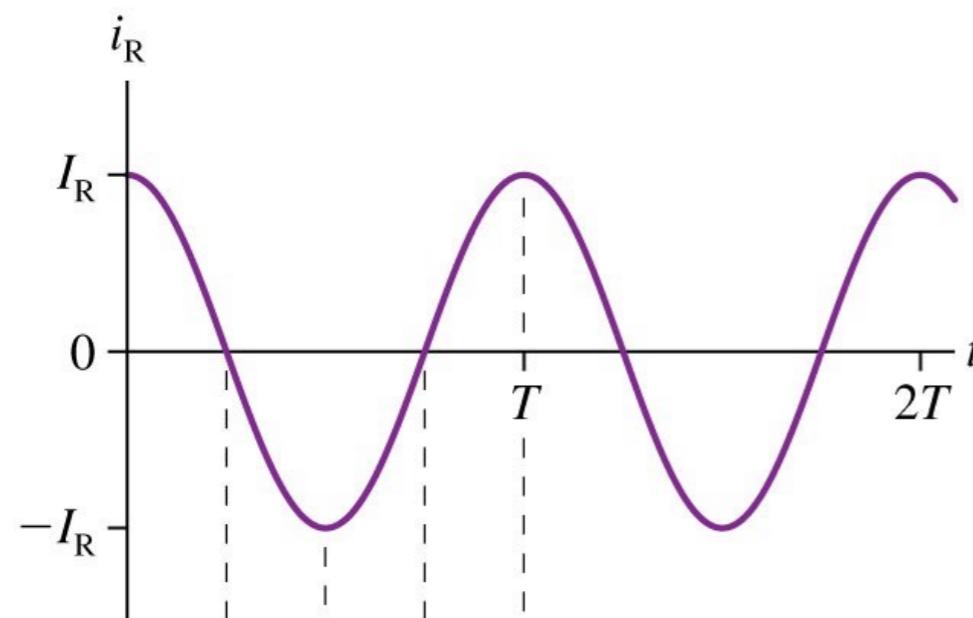
Power dissipated in resistor



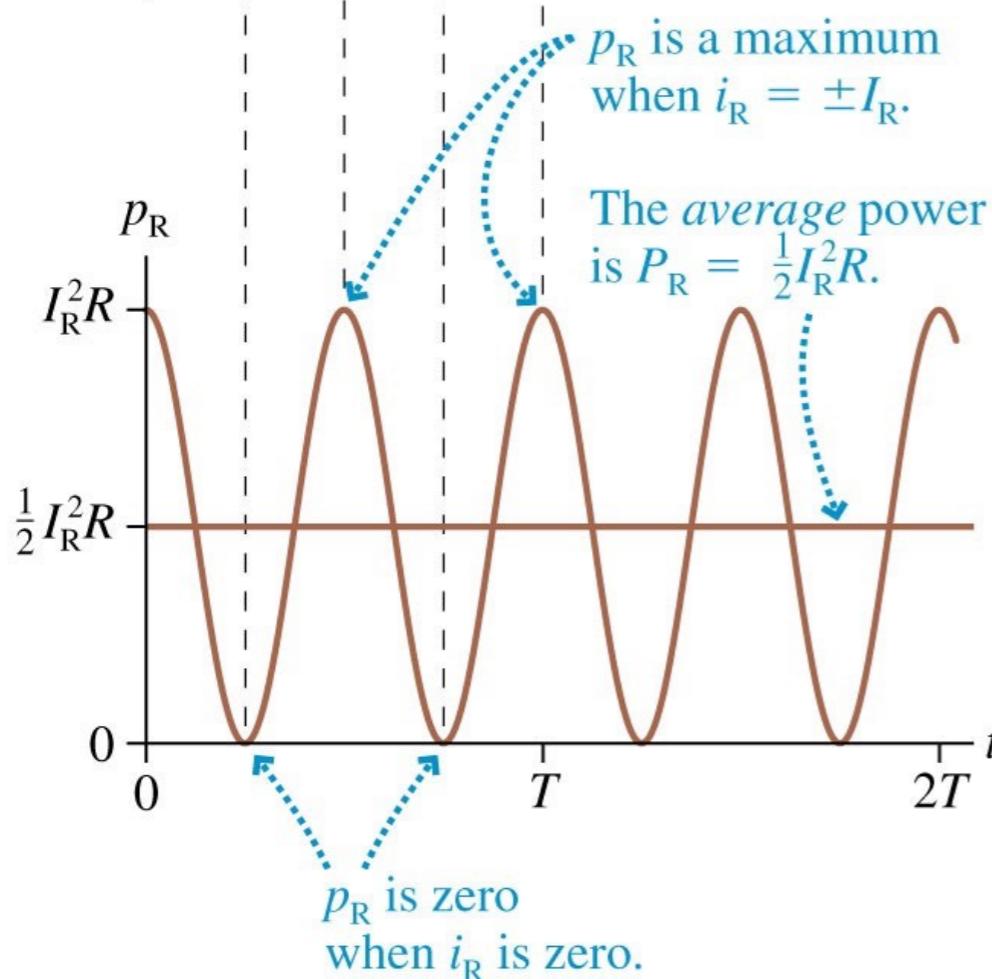
$$p_R = i_R^2 R = I_R^2 R \cos^2 \omega t$$

What does a \cos^2 function look like?

Power dissipated in resistor

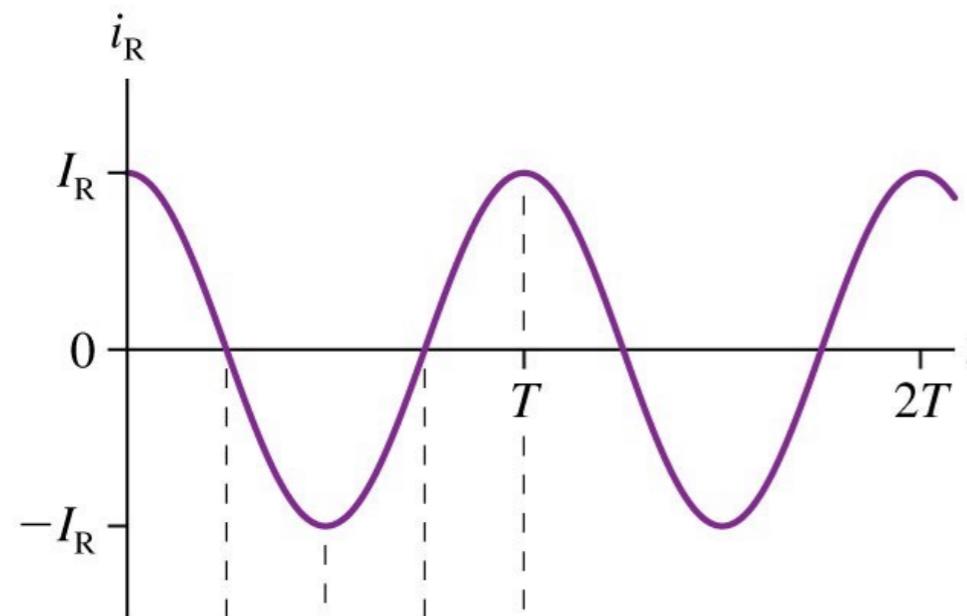


$$p_R = i_R^2 R = I_R^2 R \cos^2 \omega t$$

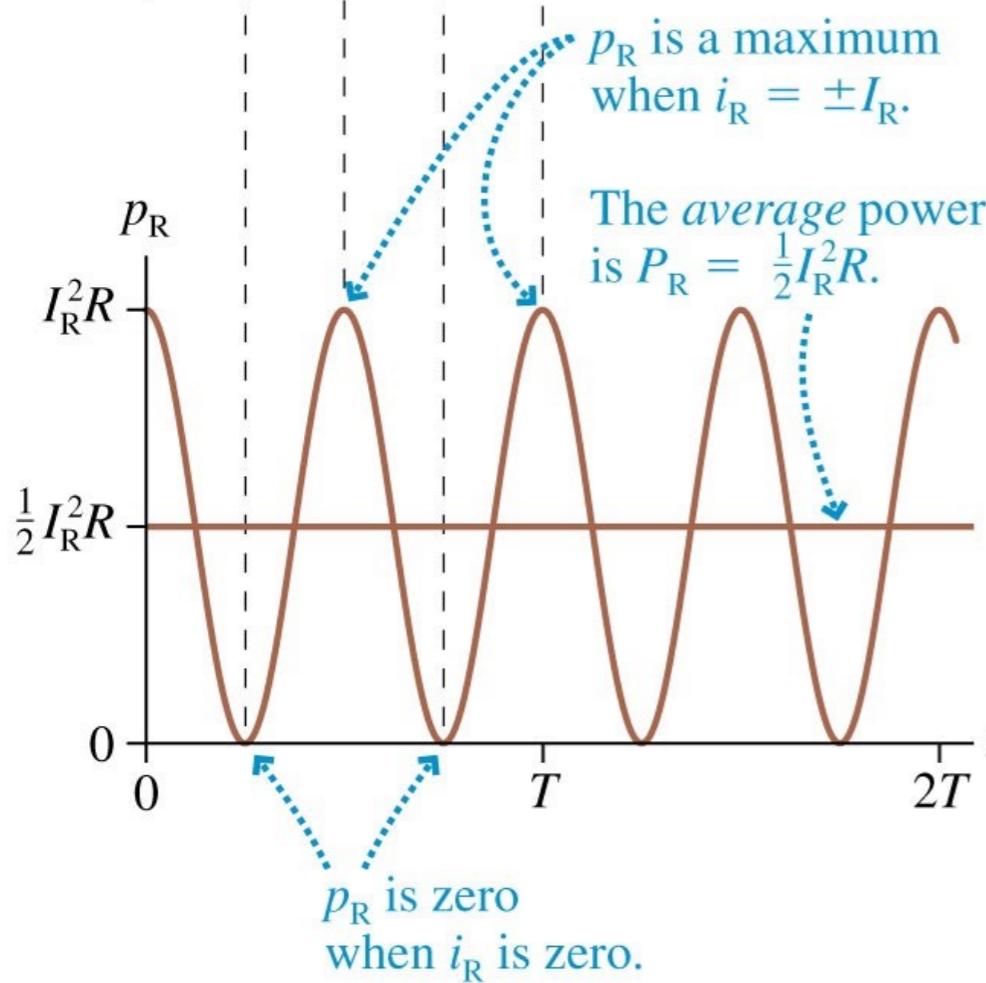


What does a \cos^2 function look like?

Power dissipated in resistor



$$p_R = i_R^2 R = I_R^2 R \cos^2 \omega t$$



What does a \cos^2 function look like?

$$P_{\text{avg}} = \frac{1}{2} I_R^2 R$$

Root Mean Square average

$$I_{\text{rms}} = \frac{I_{\text{R}}}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_{\text{R}}}{\sqrt{2}}$$

2

What do these expressions look like in terms of RMS values?

Root Mean Square average

$$I_{\text{rms}} = \frac{I_{\text{R}}}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_{\text{R}}}{\sqrt{2}}$$

$$P_{\text{avg}} = \frac{1}{2} I_R^2 R$$

What do these expressions look like in terms of RMS values?

Root Mean Square average

$$I_{\text{rms}} = \frac{I_R}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_R}{\sqrt{2}}$$

$$P_{\text{avg}} = \frac{1}{2} I_R^2 R = \frac{V_R^2}{2R}$$

What do these expressions look like in terms of RMS values?

Root Mean Square average

$$I_{\text{rms}} = \frac{I_R}{\sqrt{2}}$$

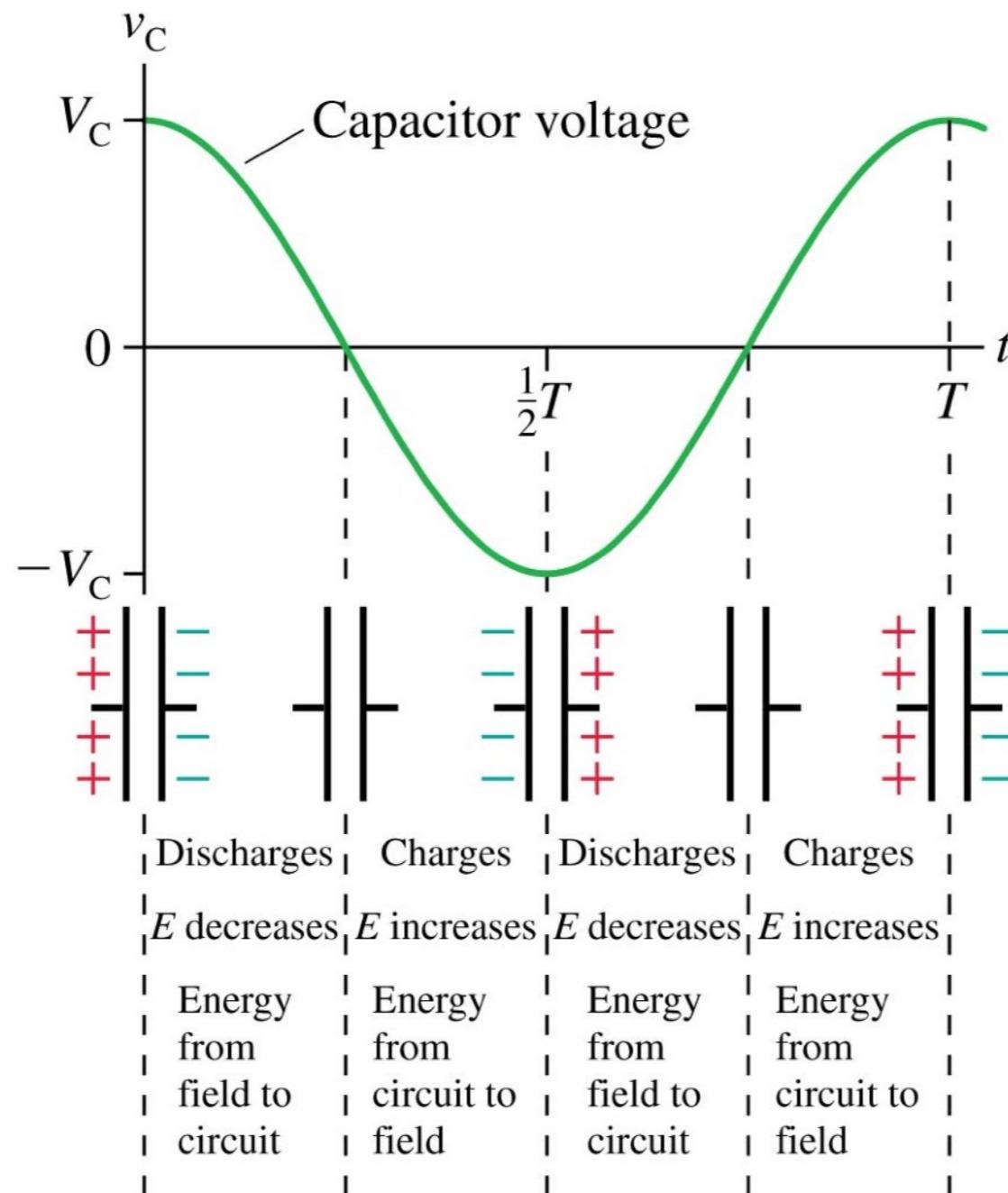
$$V_{\text{rms}} = \frac{V_R}{\sqrt{2}}$$

$$P_{\text{avg}} = \frac{1}{2} I_R^2 R = \frac{V_R^2}{2R}$$

What do these expressions look like in terms of RMS values?

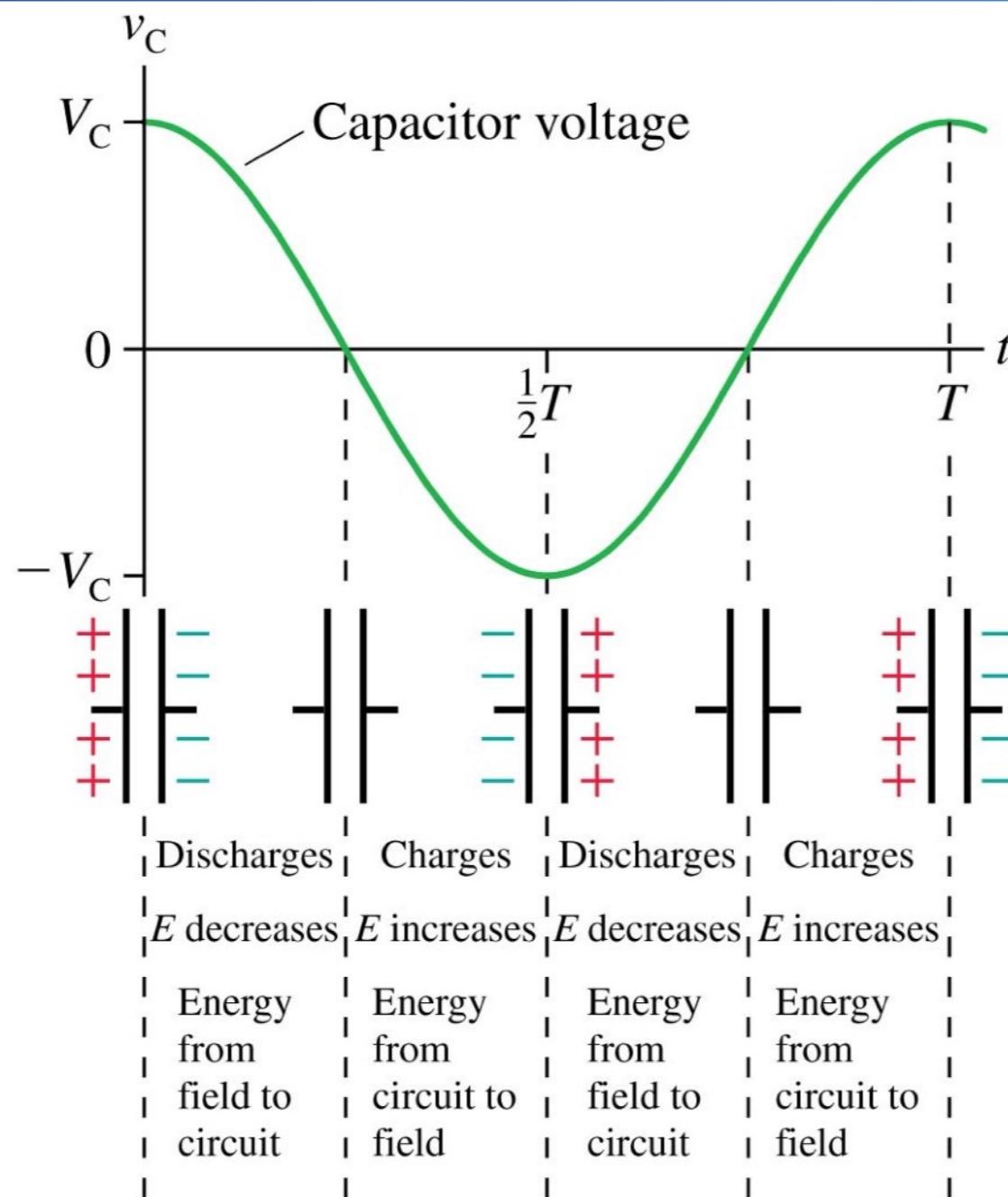
$$P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

Capacitors in AC circuits



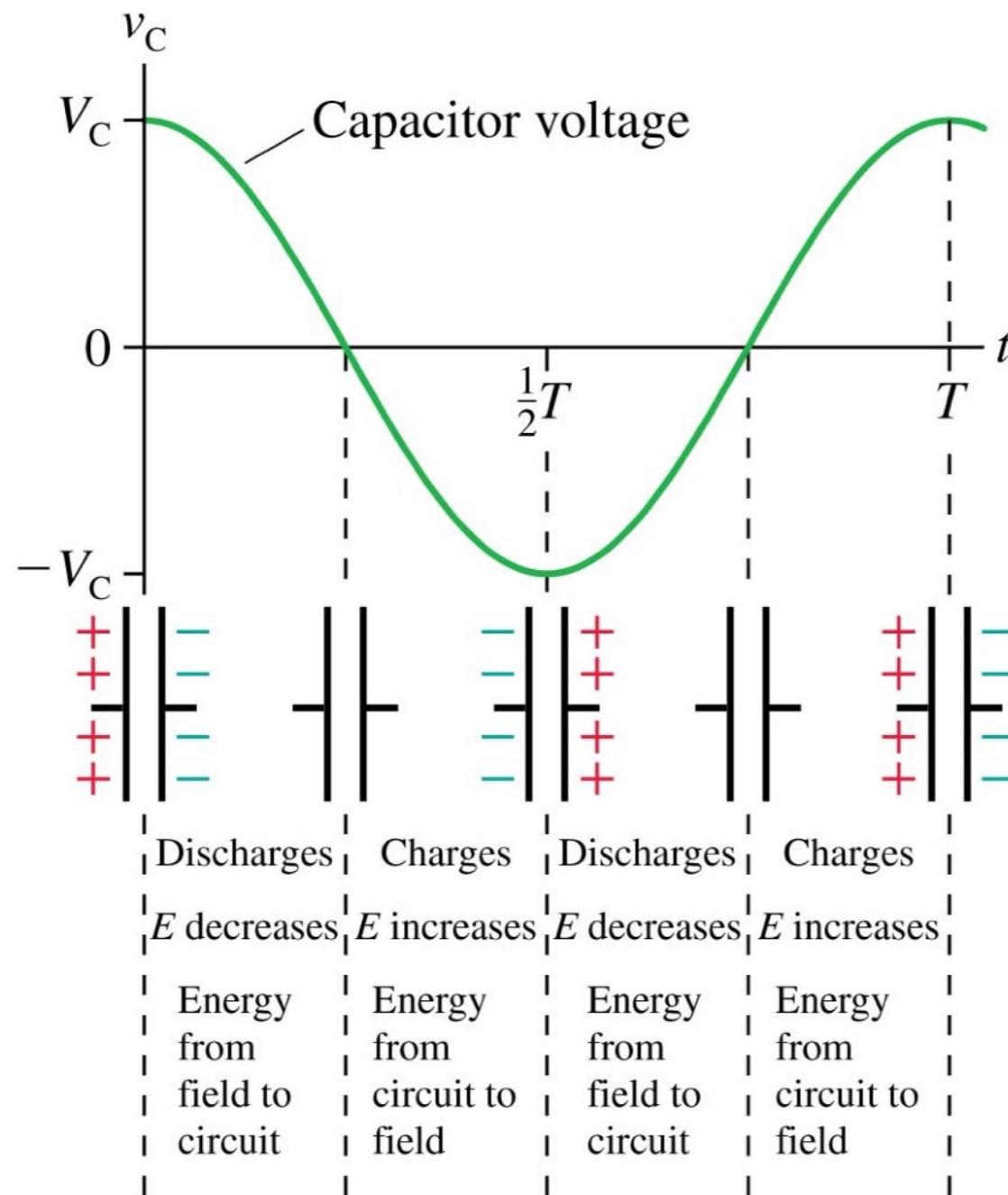
$$p_C = v_C i_C$$

Capacitors in AC circuits



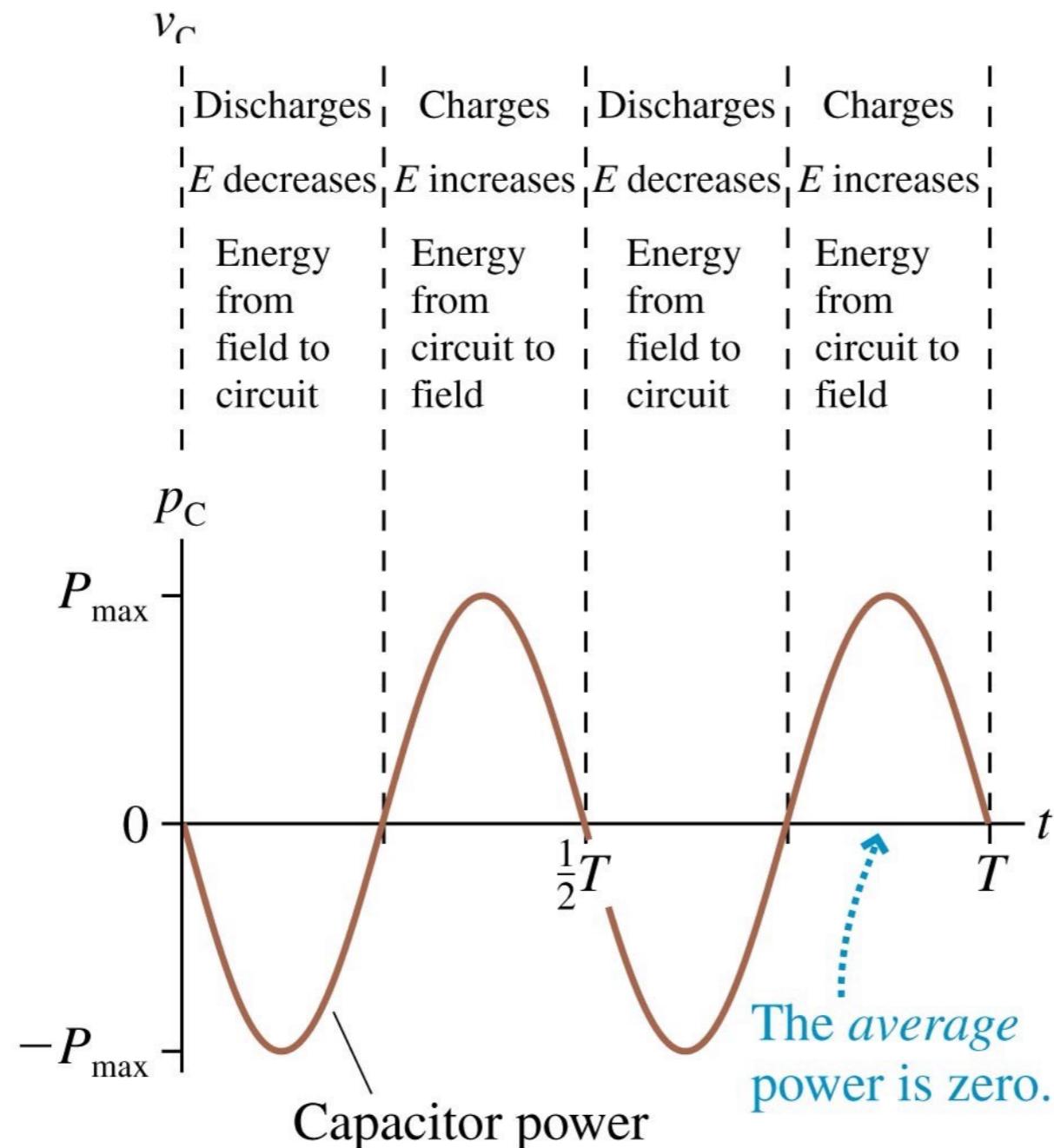
$$p_C = v_C i_C = (V_C \cos \omega t)(-\omega C V_C \sin \omega t)$$

Capacitors in AC circuits



$$p_C = v_C i_C = (V_C \cos \omega t)(-\omega C V_C \sin \omega t) = -\frac{1}{2} \omega C V_C^2 \sin 2\omega t$$

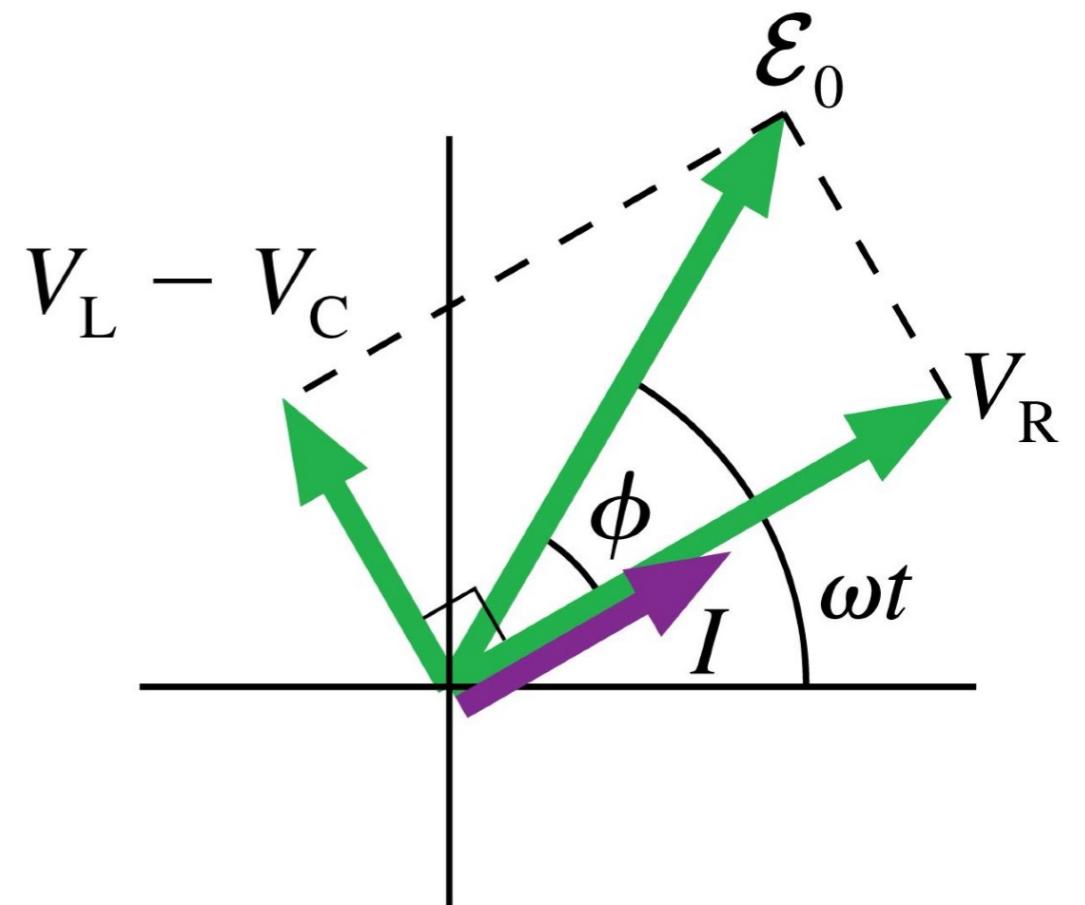
Capacitors in AC circuits



$$p_C = v_C i_C = (V_C \cos \omega t)(-\omega C V_C \sin \omega t) = -\frac{1}{2} \omega C V_C^2 \sin 2\omega t$$

Power delivered by source

$$P_{\text{source}} = i \mathcal{E}$$

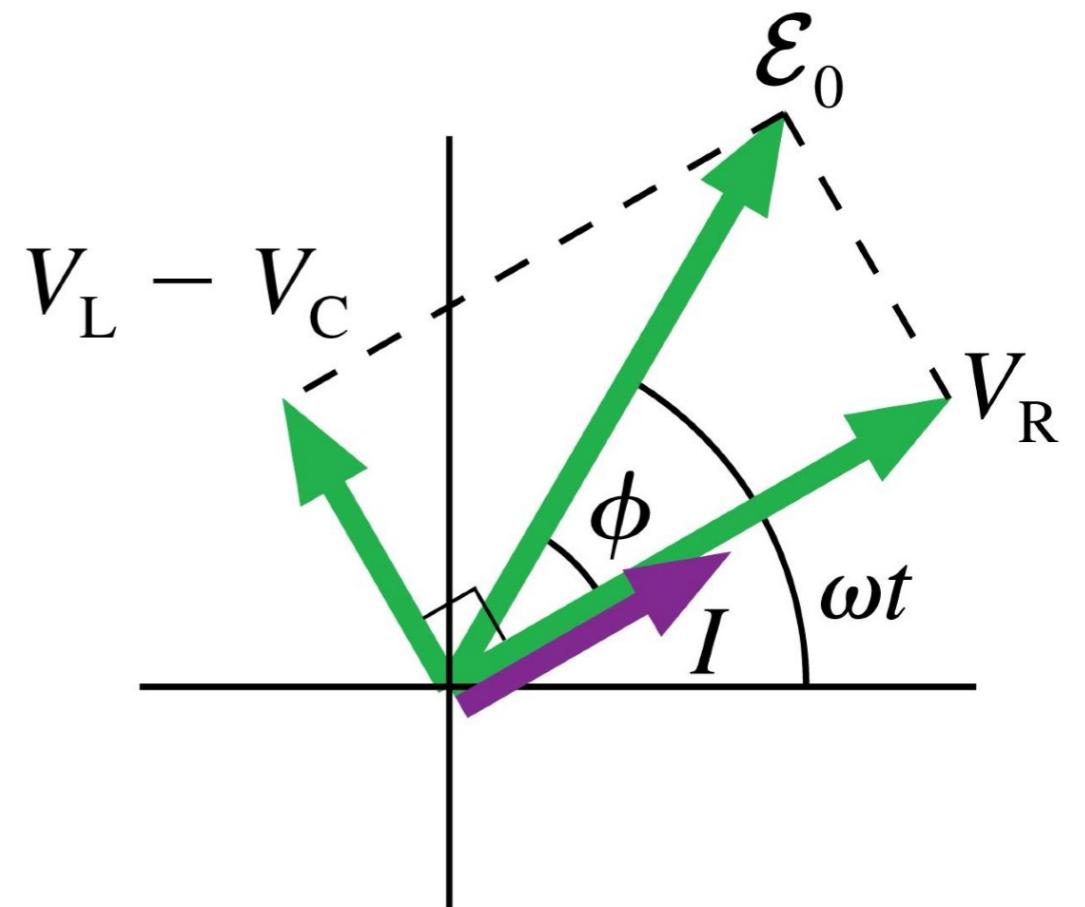


applet

Power delivered by source

$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$



applet

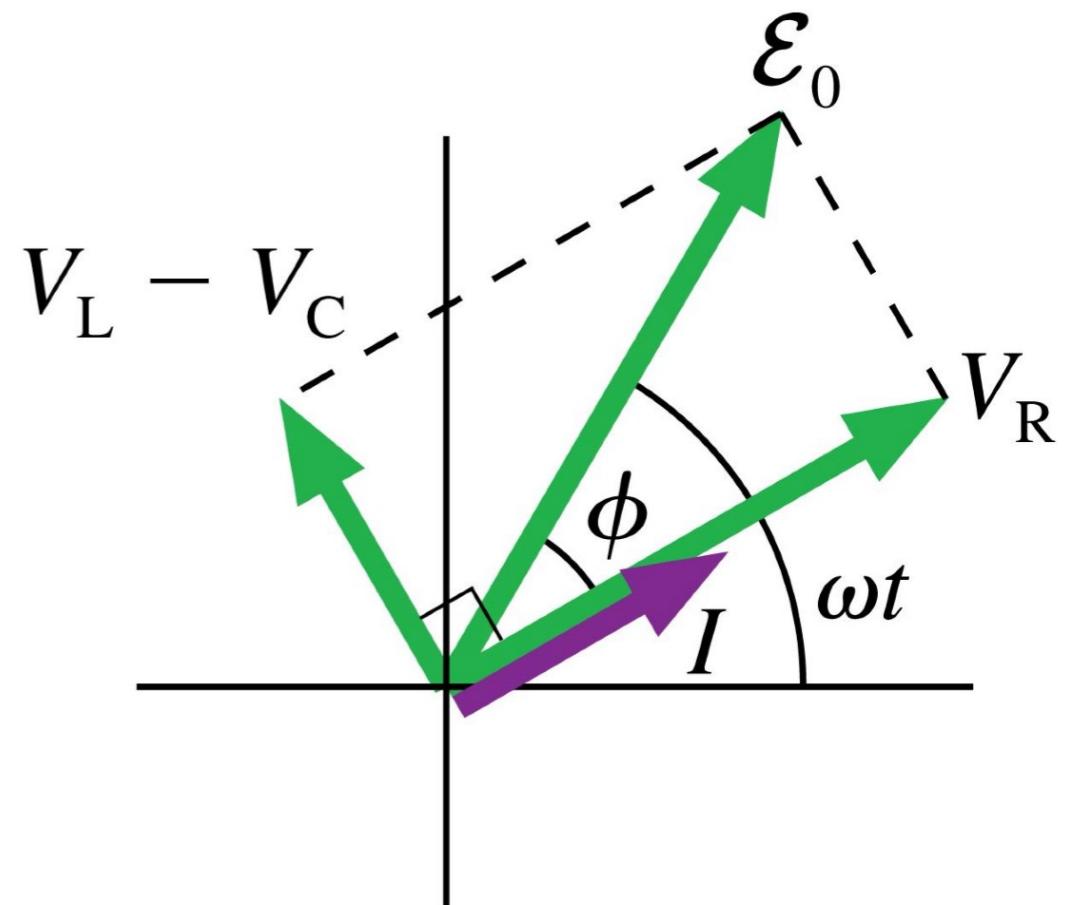
Power delivered by source

$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$

Use this trig identity to re-write this expression

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$



applet

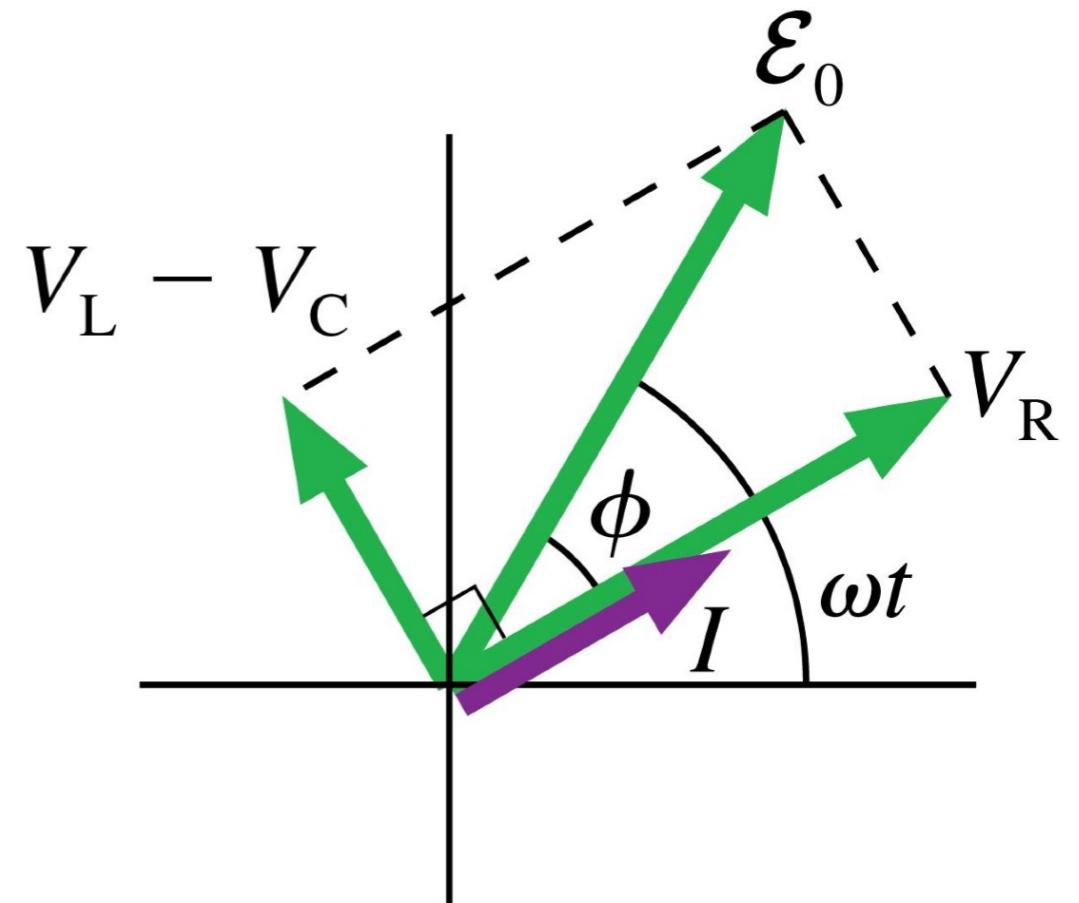
Power delivered by source

$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$

Use this trig identity to re-write this expression

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$



$$= I\mathcal{E}_0 \cos \phi \cos^2 \omega t + I\mathcal{E}_0 \sin \phi \sin \omega t \cos \omega t$$

applet

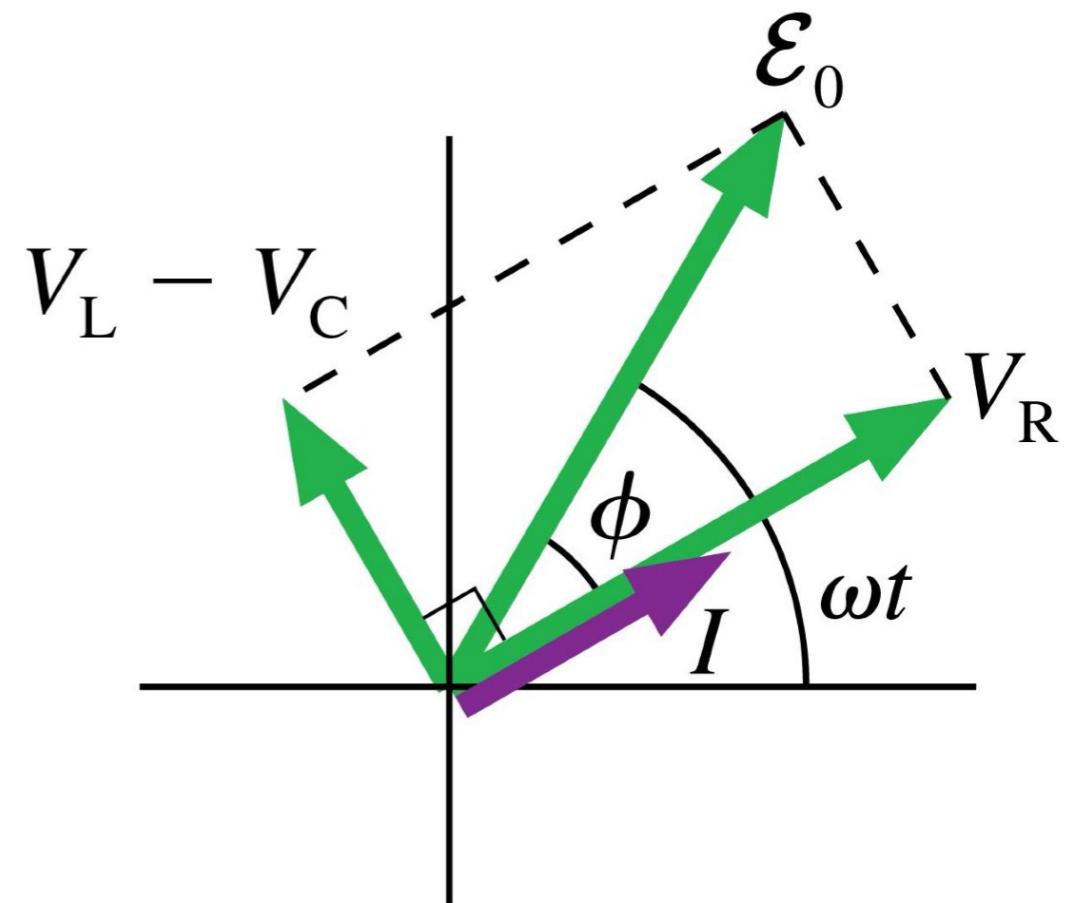
Power delivered by source

$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$

Use this trig identity to re-write this expression

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$



$$= I\mathcal{E}_0 \cos \phi \boxed{\cos^2 \omega t} + I\mathcal{E}_0 \sin \phi \boxed{\sin \omega t \cos \omega t}$$

What are the averages of these functions?

applet

Power delivered by source

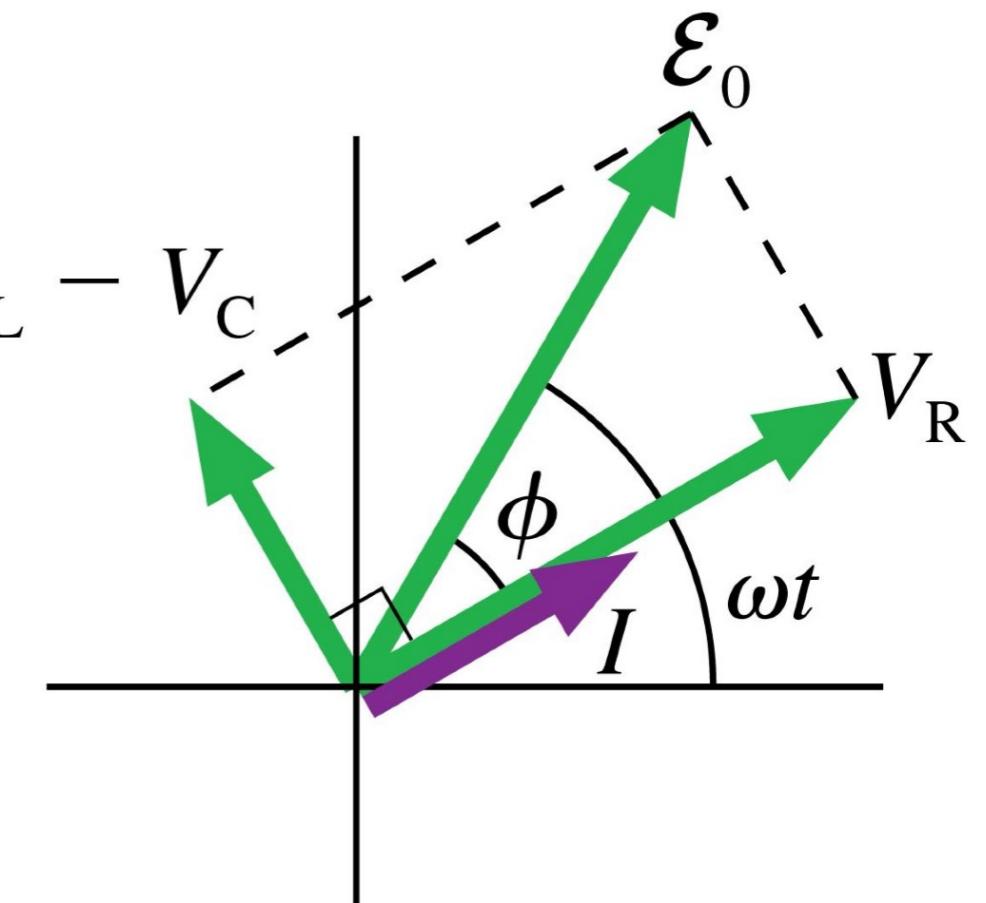
$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$

Use this trig identity to re-write this expression

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$= I\mathcal{E}_0 \cos \phi \boxed{\cos^2 \omega t} + I\mathcal{E}_0 \sin \phi \boxed{\sin \omega t \cos \omega t}$$



What are the averages of these functions?

applet

Power delivered by source

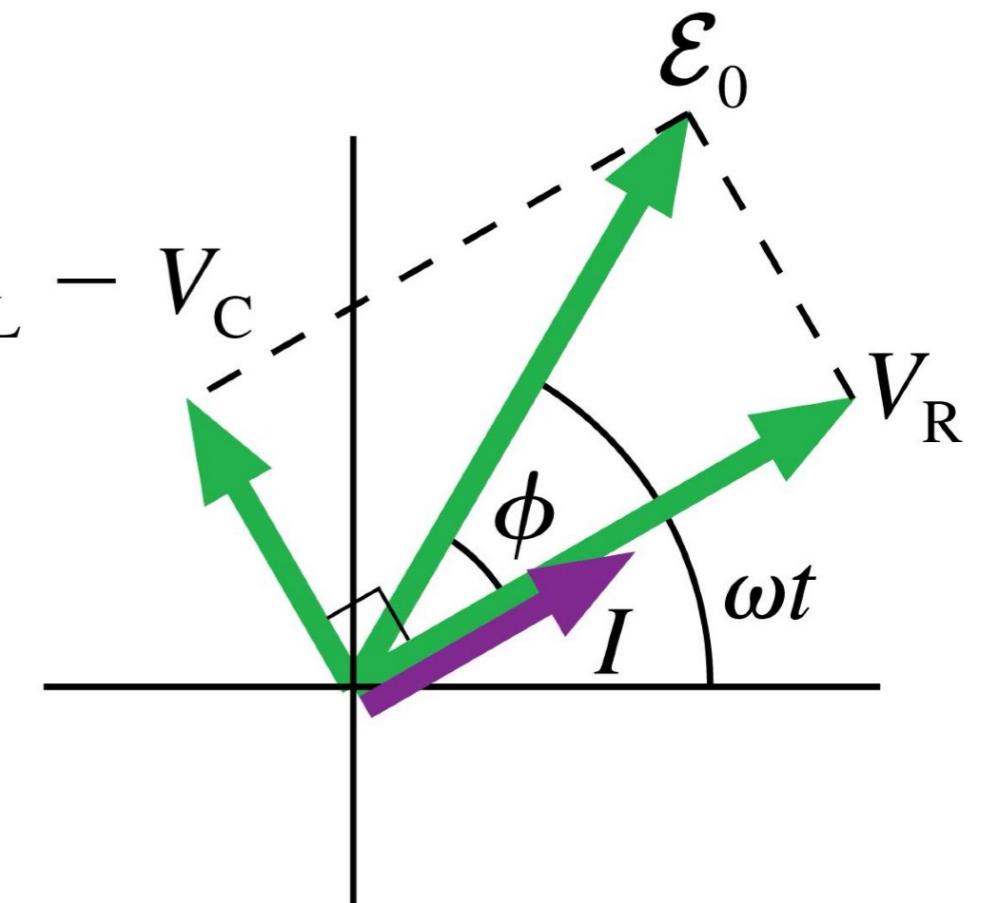
$$P_{\text{source}} = i\mathcal{E}$$

$$= I \cos(\omega t - \phi) \mathcal{E}_0 \cos(\omega t)$$

Use this trig identity to re-write this expression

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$= I\mathcal{E}_0 \cos \phi \boxed{\cos^2 \omega t} + I\mathcal{E}_0 \sin \phi \boxed{\sin \omega t \cos \omega t}$$



What are the averages of these functions?

$$P_{\text{source}} = \frac{1}{2} I\mathcal{E}_0 \cos \phi = I_{\text{rms}} \mathcal{E}_{\text{rms}} \cos \phi$$

applet





$$P_{\text{source}} = \frac{1}{2} I \mathcal{E}_0 \cos \phi = I_{\text{rms}} \mathcal{E}_{\text{rms}} \cos \phi$$

A series RLC circuit consists of a 25 Ohm resistor, a 0.10 H inductor, and a 100 microFarad capacitor. It draws a 2.5 A rms current when attached to a 60 Hz source.

- a) Find the rms value of the emf.
- b) Find the phase angle.
- c) Find the power delivered by the emf.

A series LRC circuit consists of an ac source of amplitude 75.0 V and a variable frequency, a 12.5 microFarad capacitor, a 5.00 milliHenry inductor, and a 35.0 Ohm resistor.

- a) To what angular frequency should the ac source be set so that the current amplitude has its largest value?
- b) What is the value of this current?
- c) Suppose the voltage source frequency is doubled. What is the current amplitude now?
- d) What is the phase angle? (Draw the phasor diagram)

A series LRC circuit consists of an ac source of amplitude 75.0 V and a variable frequency, a 12.5 microFarad capacitor, a 5.00 milliHenry inductor, and a 35.0 Ohm resistor.

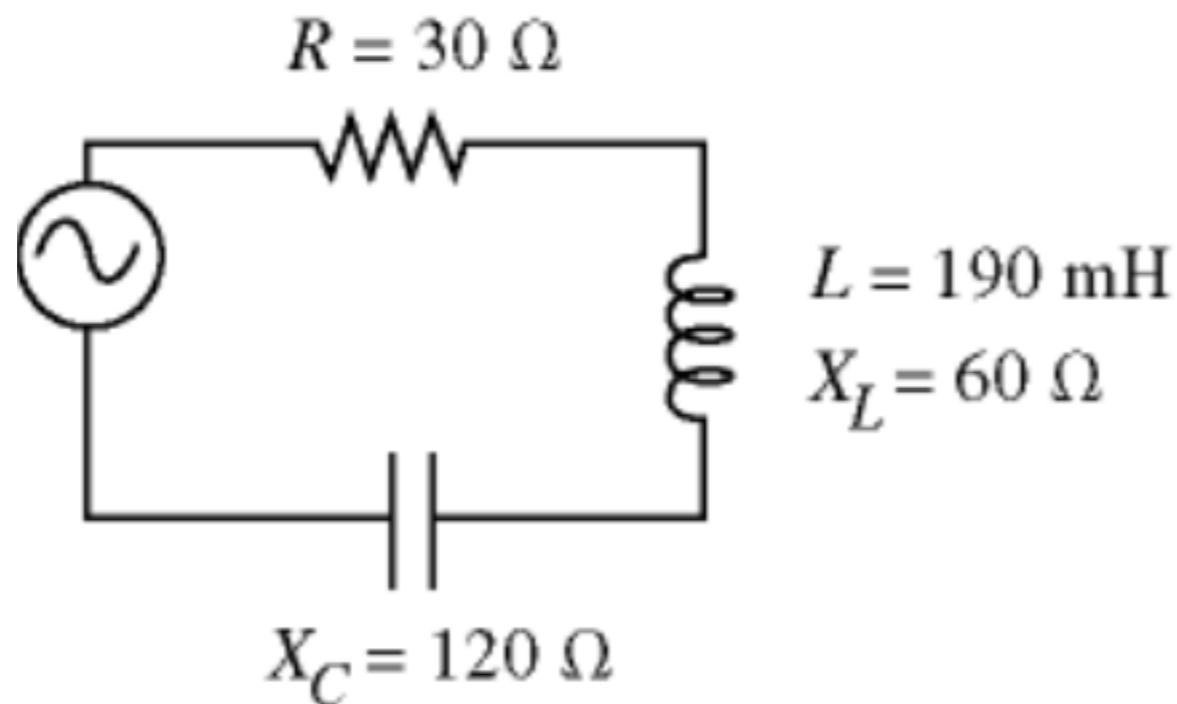
- a) To what angular frequency should the ac source be set so that the current amplitude has its largest value? 4.00×10^3 rads/s
- b) What is the value of this current?
- c) Suppose the voltage source frequency is doubled. What is the current amplitude now?
- d) What is the phase angle? (Draw the phasor diagram)

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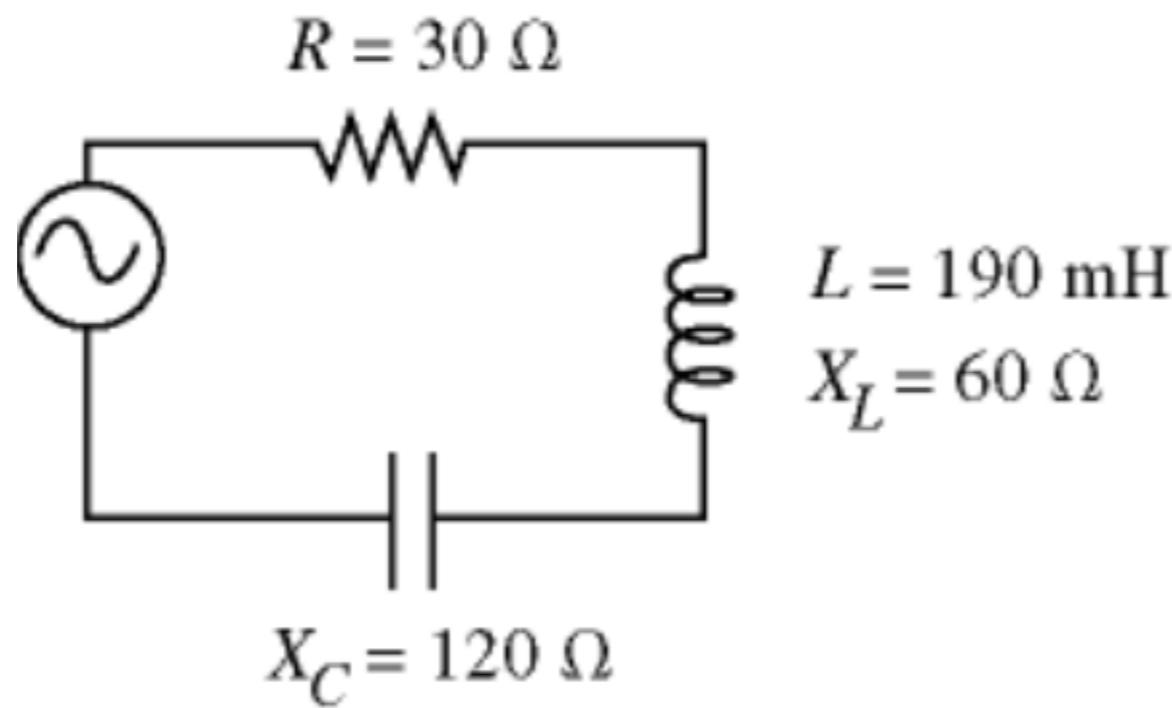
- a) To what angular frequency should the ac source be set so that the current amplitude has its largest value? 4.00×10^3 rads/s
- b) What is the value of this current? 2.14 A
- c) Suppose the voltage source frequency is doubled. What is the current amplitude now?
- d) What is the phase angle? (Draw the phasor diagram)

A series LRC circuit consists of an ac source of amplitude 75.0 V and a variable frequency, a 12.5 microFarad capacitor, a 5.00 milliHenry inductor, and a 35.0 Ohm resistor.

- a) To what angular frequency should the ac source be set so that the current amplitude has its largest value? 4.00×10^3 rads/s
- b) What is the value of this current? 2.14 A
- c) Suppose the voltage source frequency is doubled. What is the current amplitude now? 1.63 A
- d) What is the phase angle? (Draw the phasor diagram)



The rms current is 1.3 A.
What is the max energy
stored in the inductor?



The rms current is 1.3 A.
What is the max energy stored in the inductor?

0.32 J

A 25.0 mH inductor, a 2.00 microF capacitor, and a resistor are connected in series across an ac voltage source at 1000 Hz. If the impedance of the circuit is 200 Ohms, what is the resistance of the resistor

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184 Ohms