
PH 220
Lance Nelson

Coulomb's Law
Electric Field
Electric Potential Energy
Flux
Gauss's Law
Electric Potential
Conductors in Equilibrium

Point Charges
Continuous Charge distributions
Dipoles

$$\rho = \frac{Q}{V} \qquad \lambda = \frac{Q}{L}$$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$\Phi_e = \int \vec{E} \cdot d\vec{A}$$

$$U = \frac{kq_1q_2}{r}$$

$$U = Vq$$

$$V = \frac{kq}{r}$$

$$dV = \frac{kdq}{r}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$dE = \frac{kdq}{r^2}$$

$$\eta = \frac{Q}{A}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\vec{F} = \vec{E}q$$

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1

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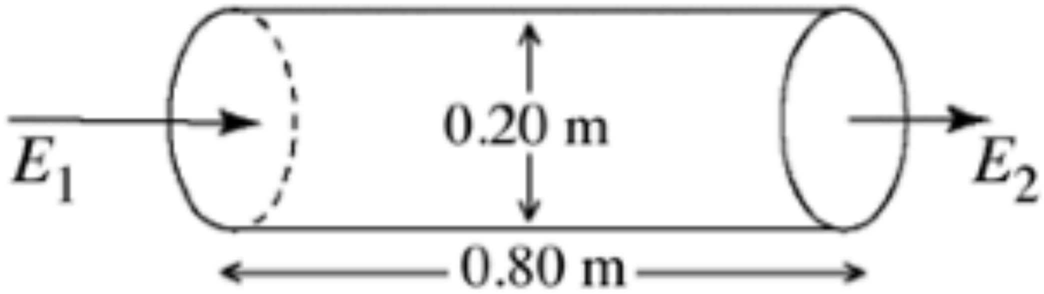
# Gauss's Law

A nonuniform electric field is directed along the x-axis at all points in space. The magnitude of the field varies with $x$, but not with $y$ or $z$. The axis of a cylindrical surface, $0.8$ m long and $0.2$ m in diameter, is aligned parallel to the x-axis, as shown in the figure. -5.4 nC of charge is enclosed in the cylindrical surface. If the electric field $E_1$ has magnitude of 25,442 N/C , what is the magnitude of $E_2$? ( $\epsilon_0 = 8.85 \times 10^{-12}$ C$^2$/N · m$^2$ )

Note: Contrary to what the image below portrays, $E_2$ may, or may not be larger in strength than $E_1$.



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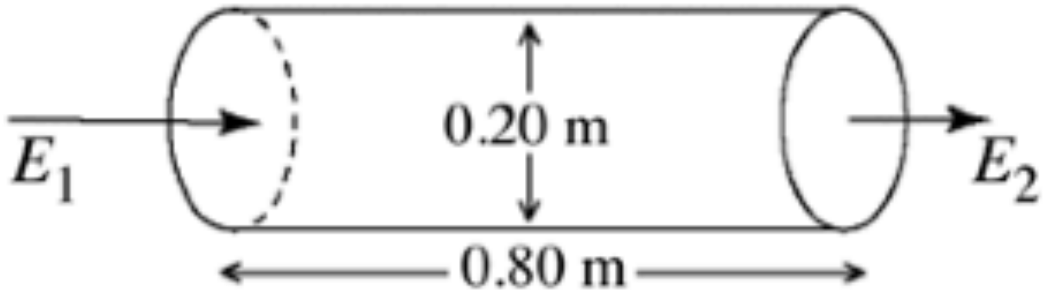
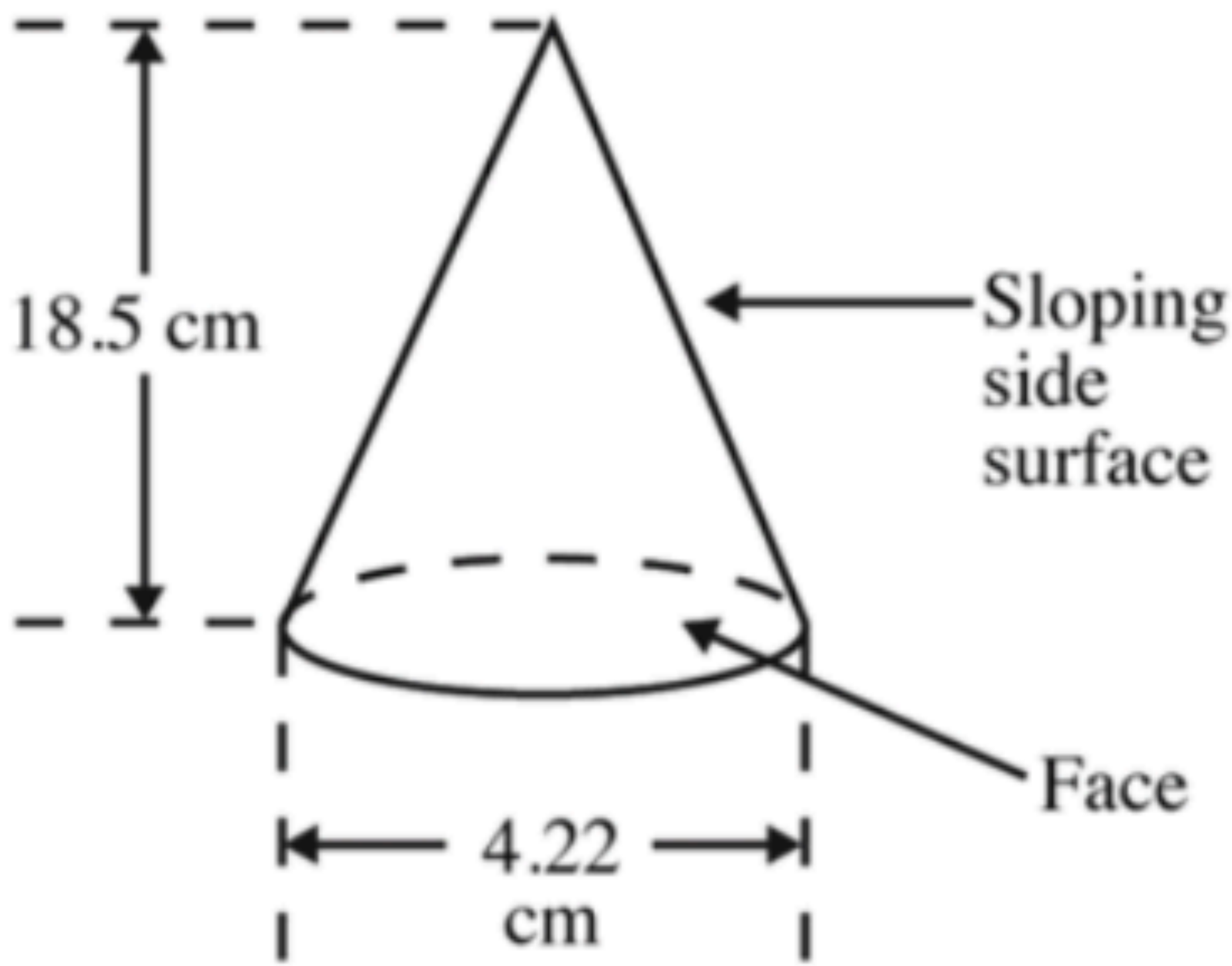



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

# Gauss's Law

A cone is resting on a tabletop as shown in the figure with its face horizontal. A **uniform** electric field of magnitude 2,949 N/C points **vertically downward.** How much electric flux passes through the sloping side surface area of the cone?



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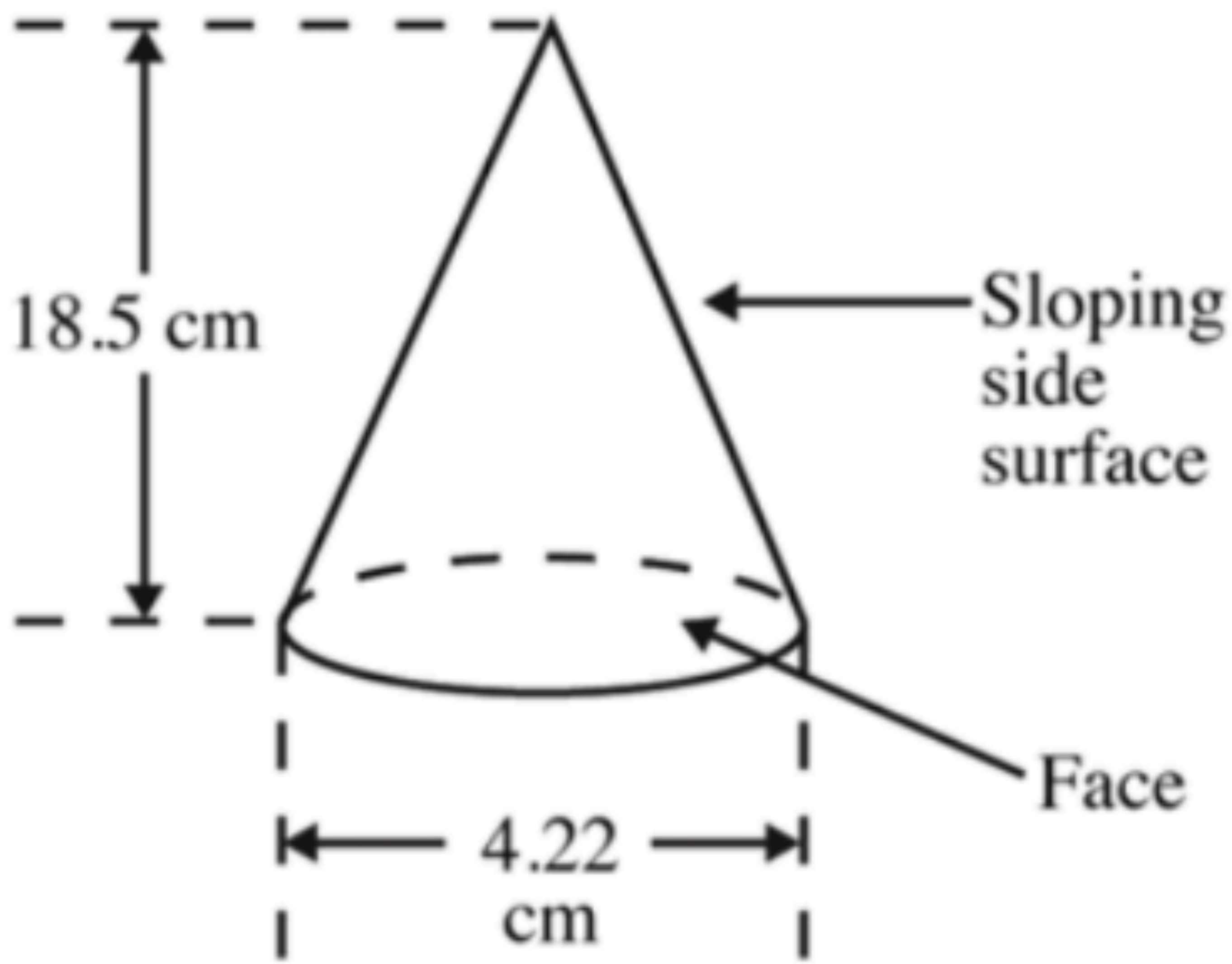


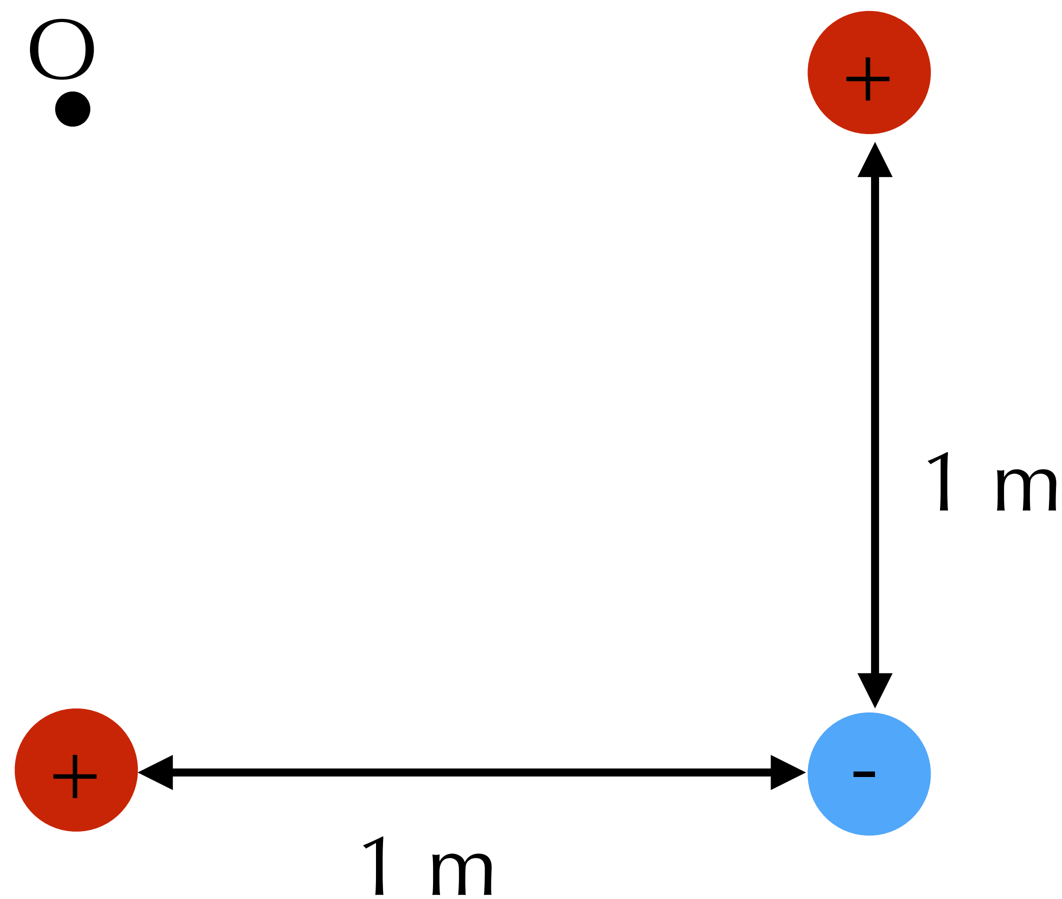

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\rm in}}{\epsilon_0}$$

# Electric Potential of Point charge

Make a sketch of the electric potential of these three point charges

$$V_{\text{elec}} = \frac{Kq_1q_2}{x}$$

$$V = \frac{kq}{r}$$

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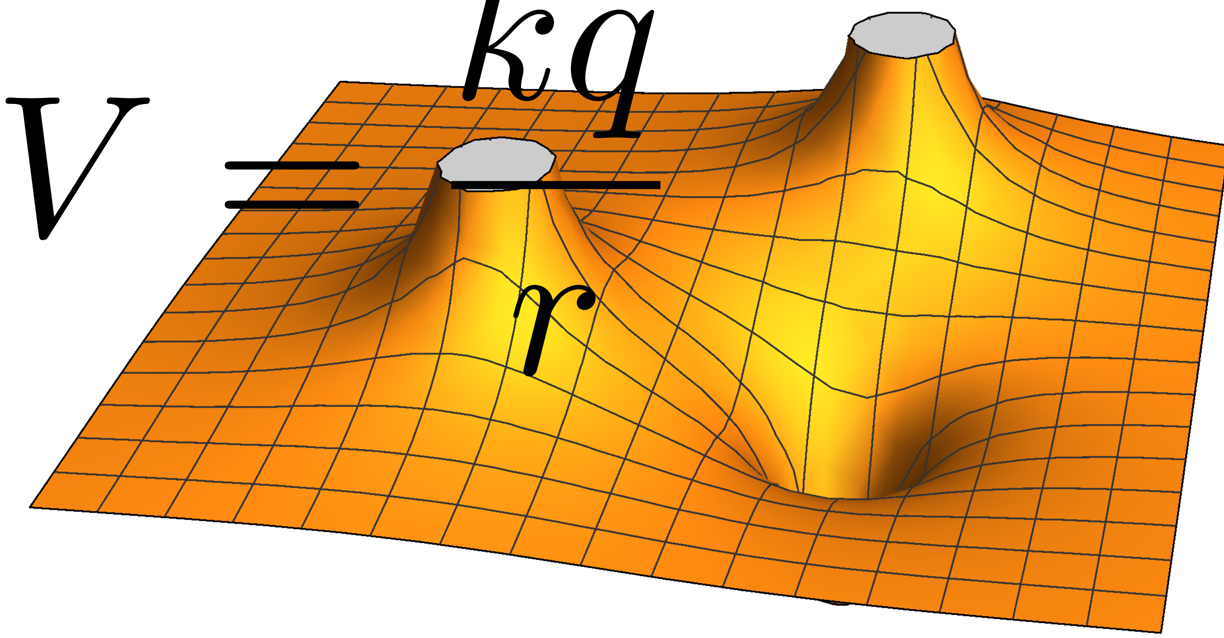

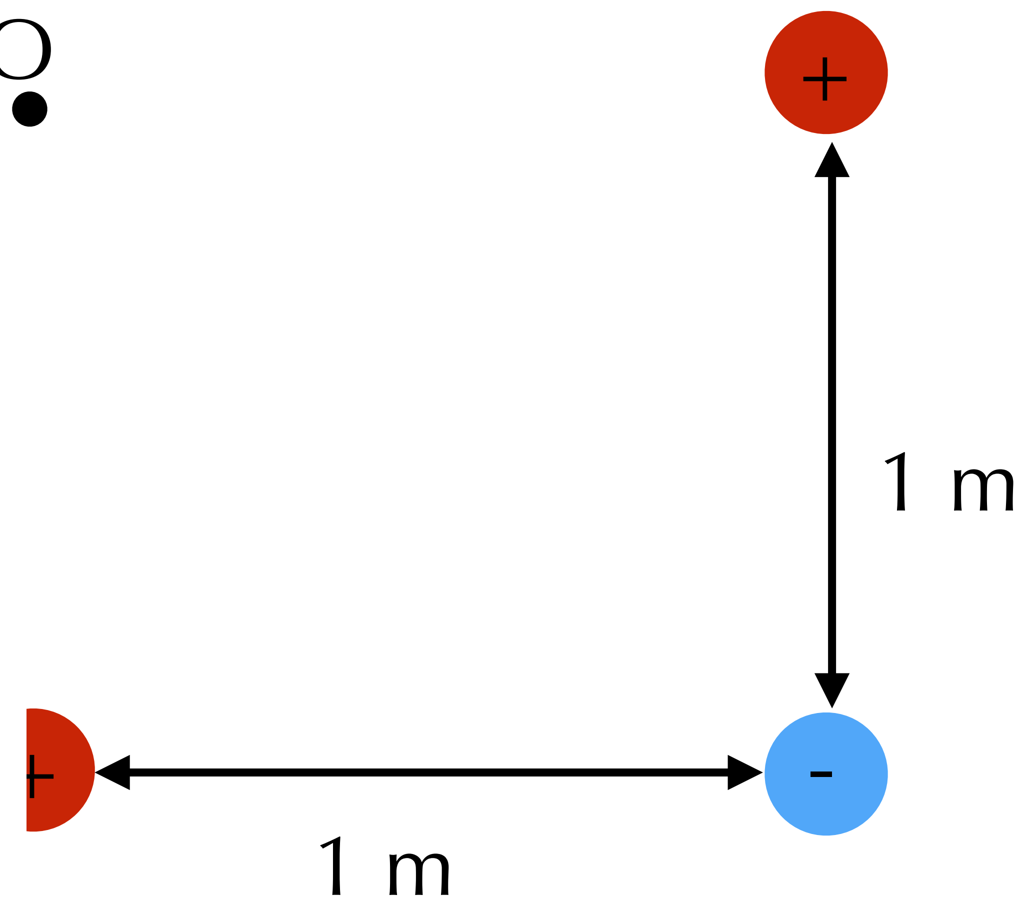

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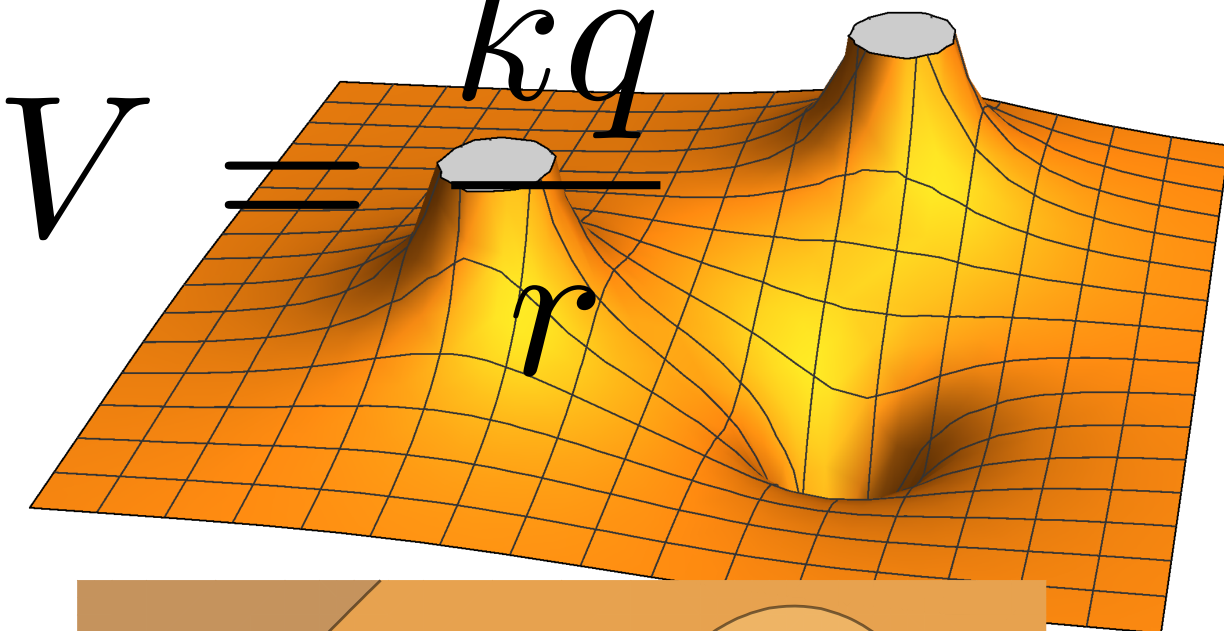

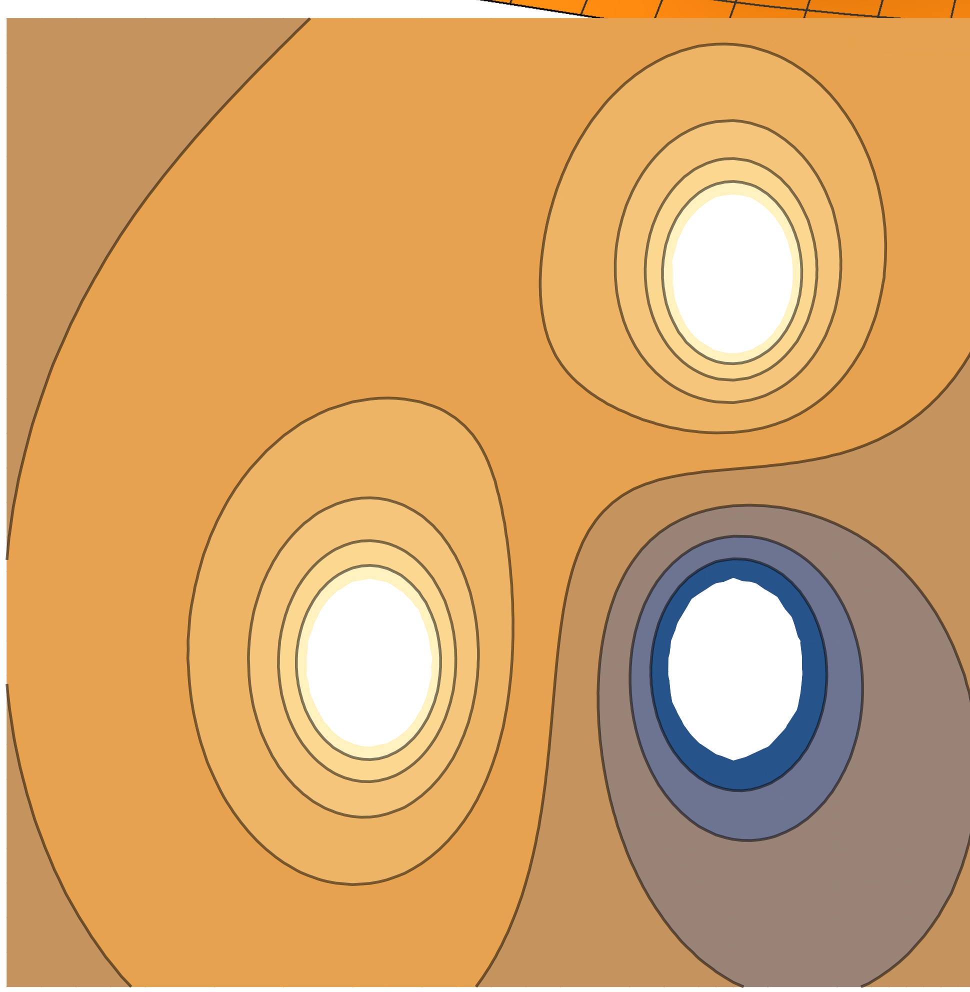

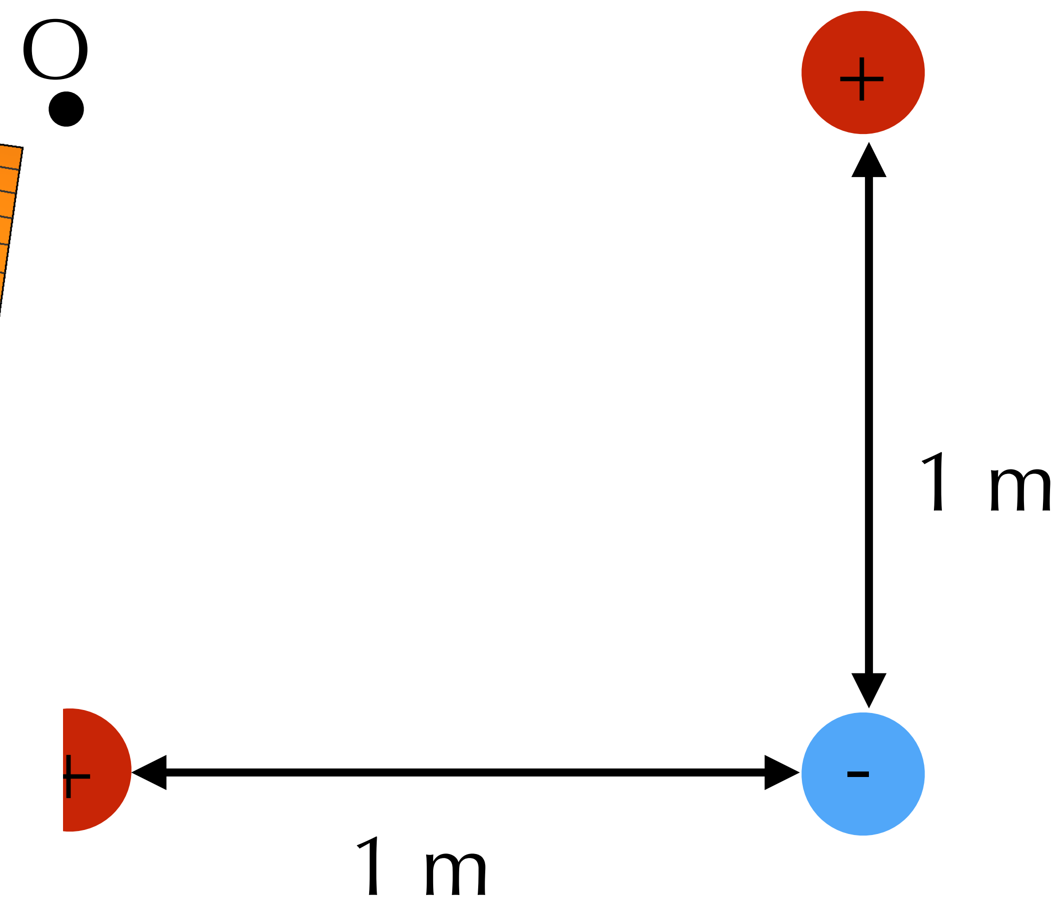

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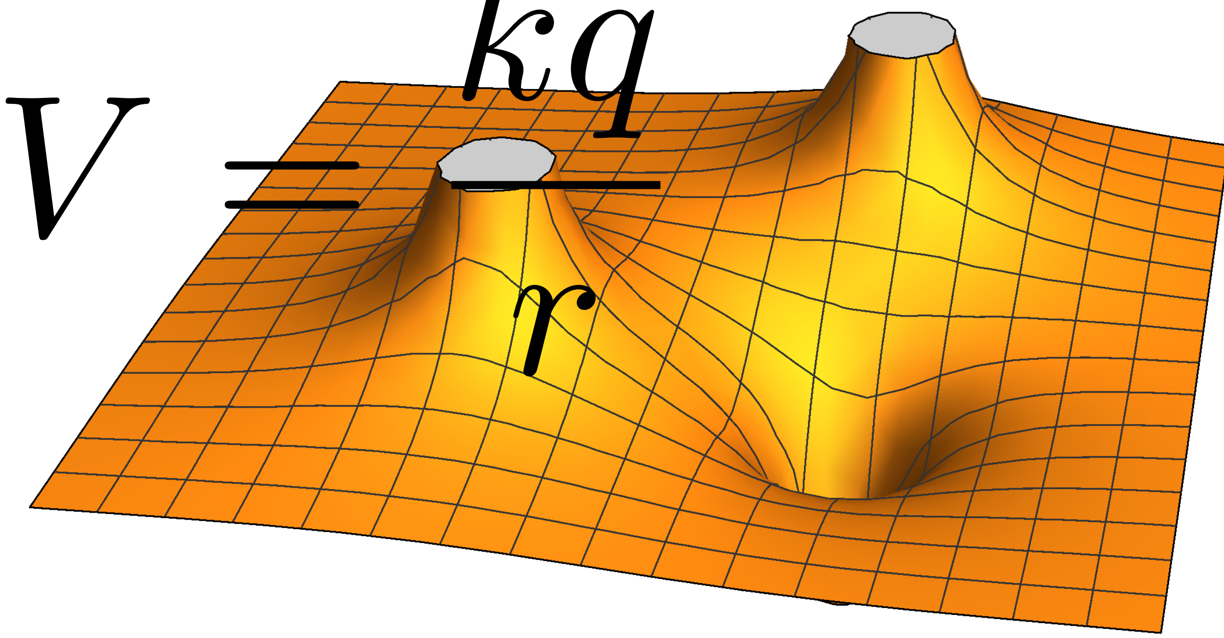

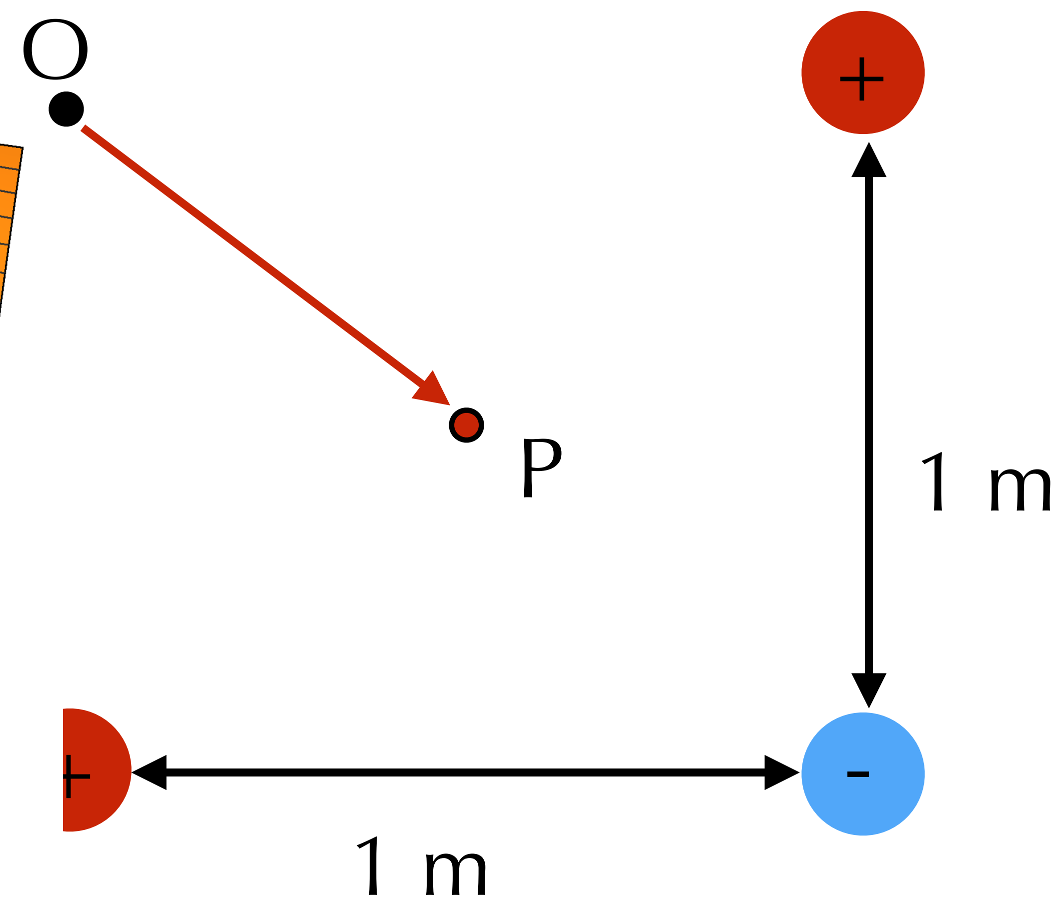


Find the potential difference between point O and point P.

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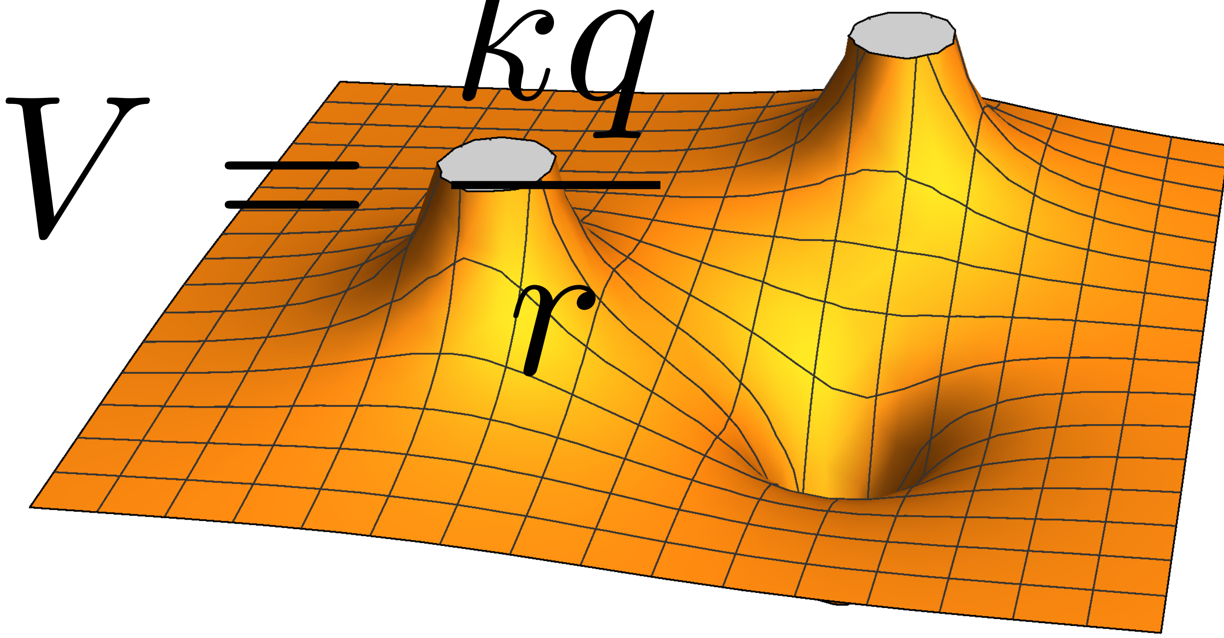
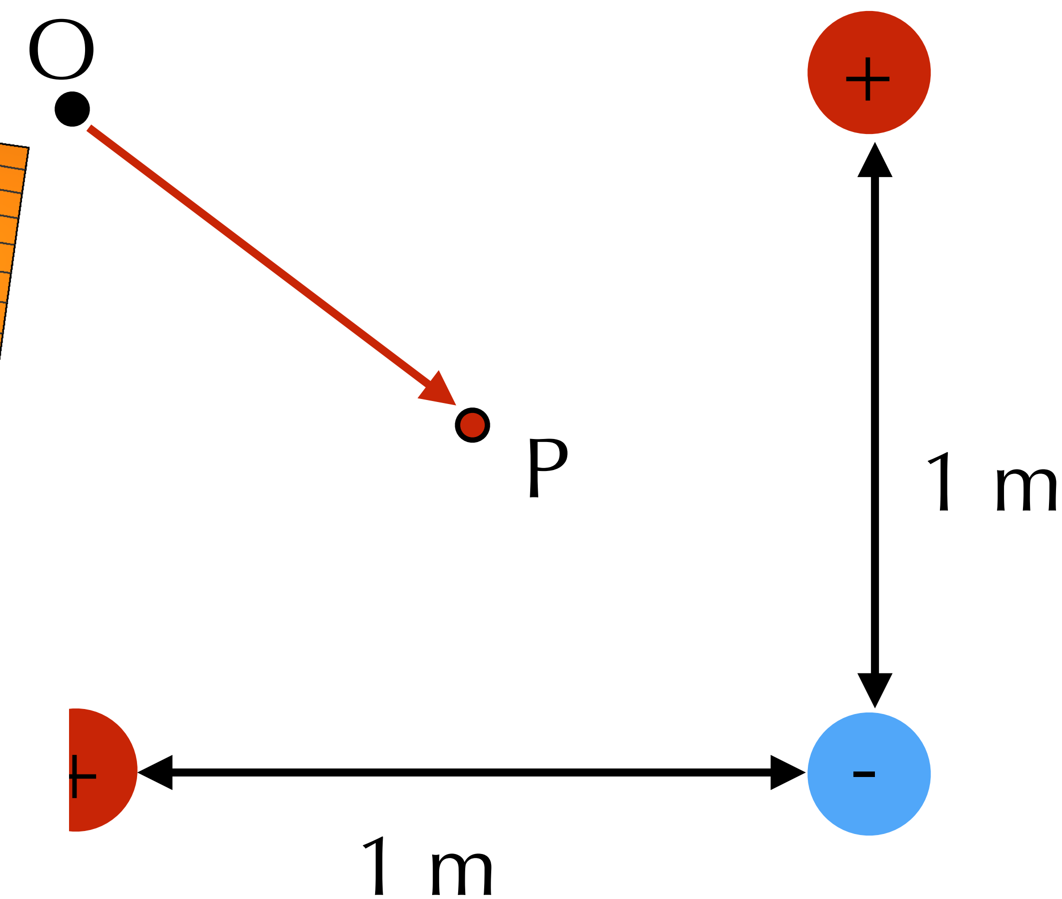




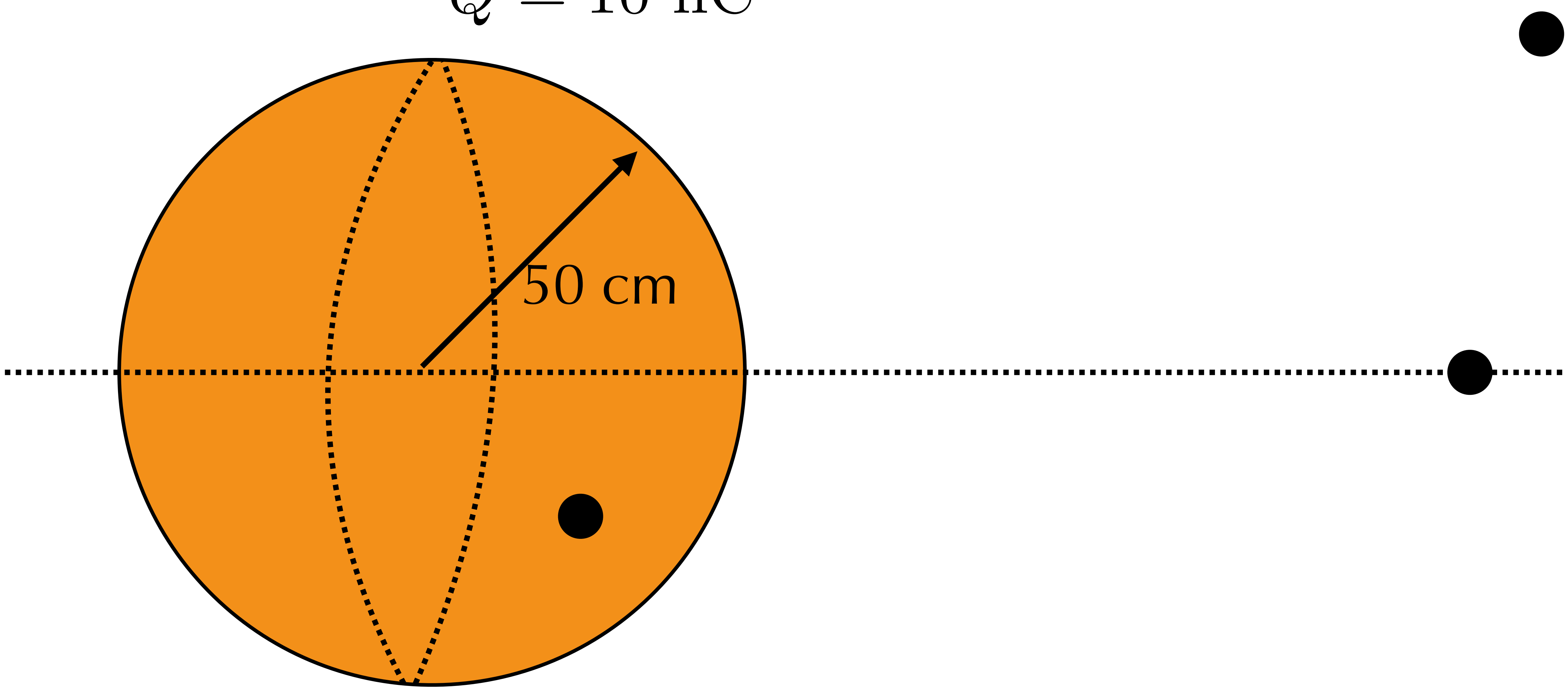

Find the potential difference between point O and point P.

If an electron is placed at O and released, how fast will it be going when it reaches point P?

Find electric field at dot.

uniformly charged sphere

$Q = 10\ \mathrm{nC}$




To your neighbor:
Describe your knee-jerk reaction to the problem
(What tool do you reach for?).

Find electric field at dot.

uniformly charged sphere

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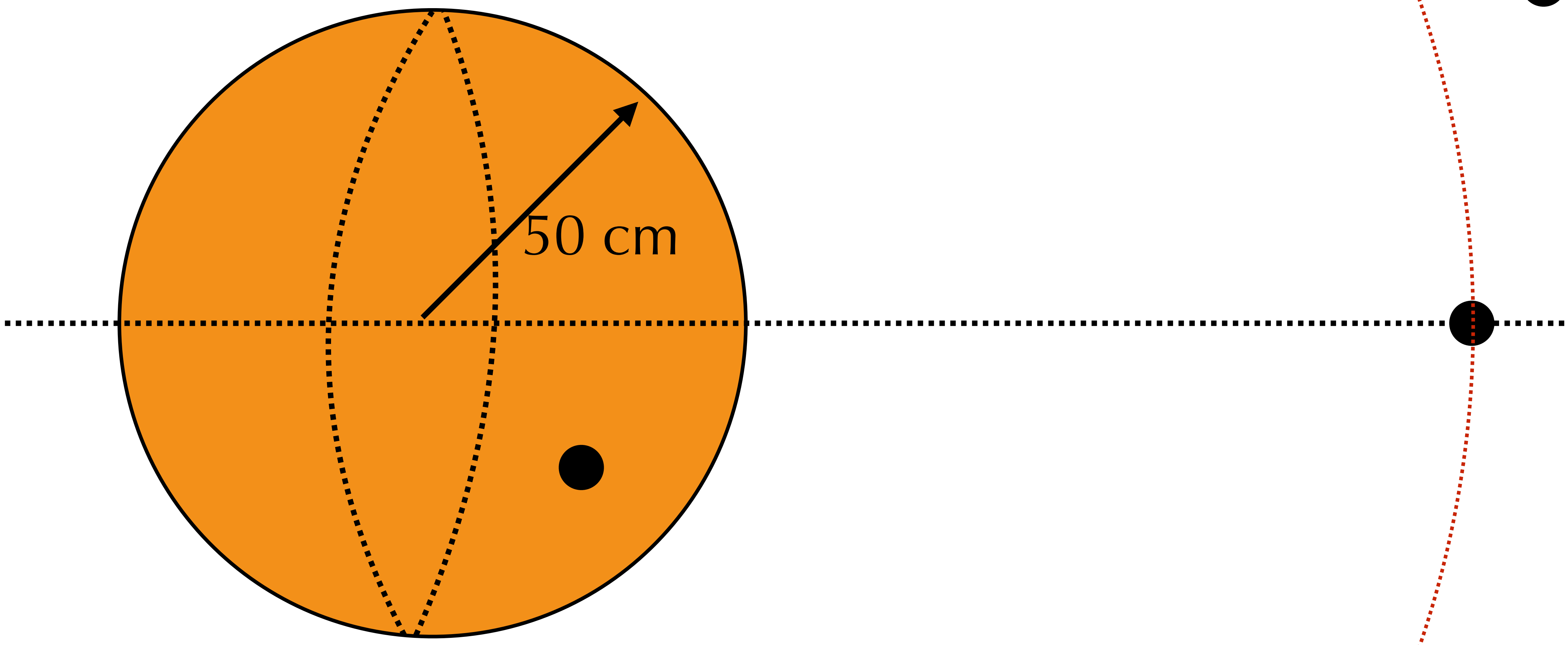


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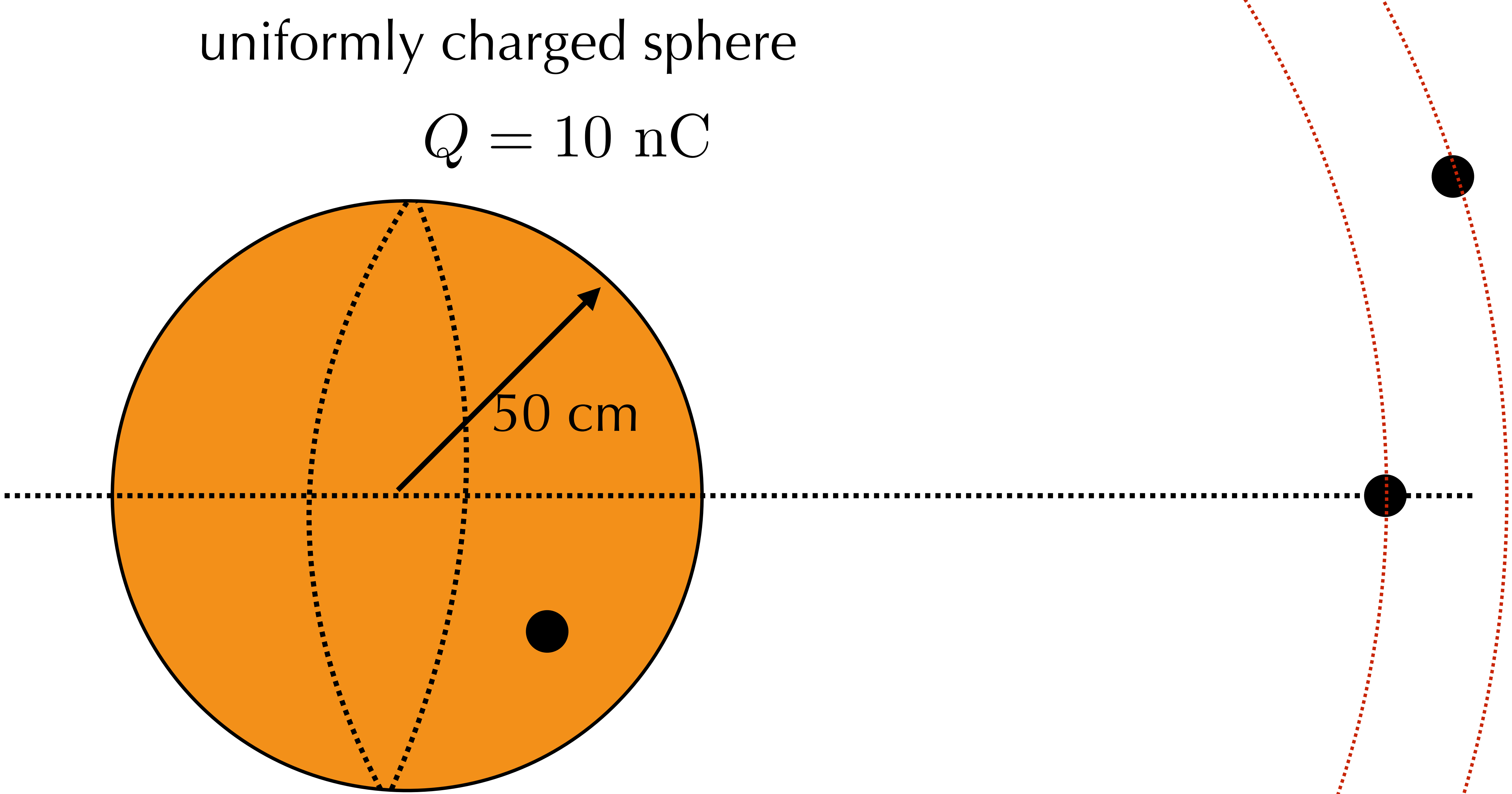


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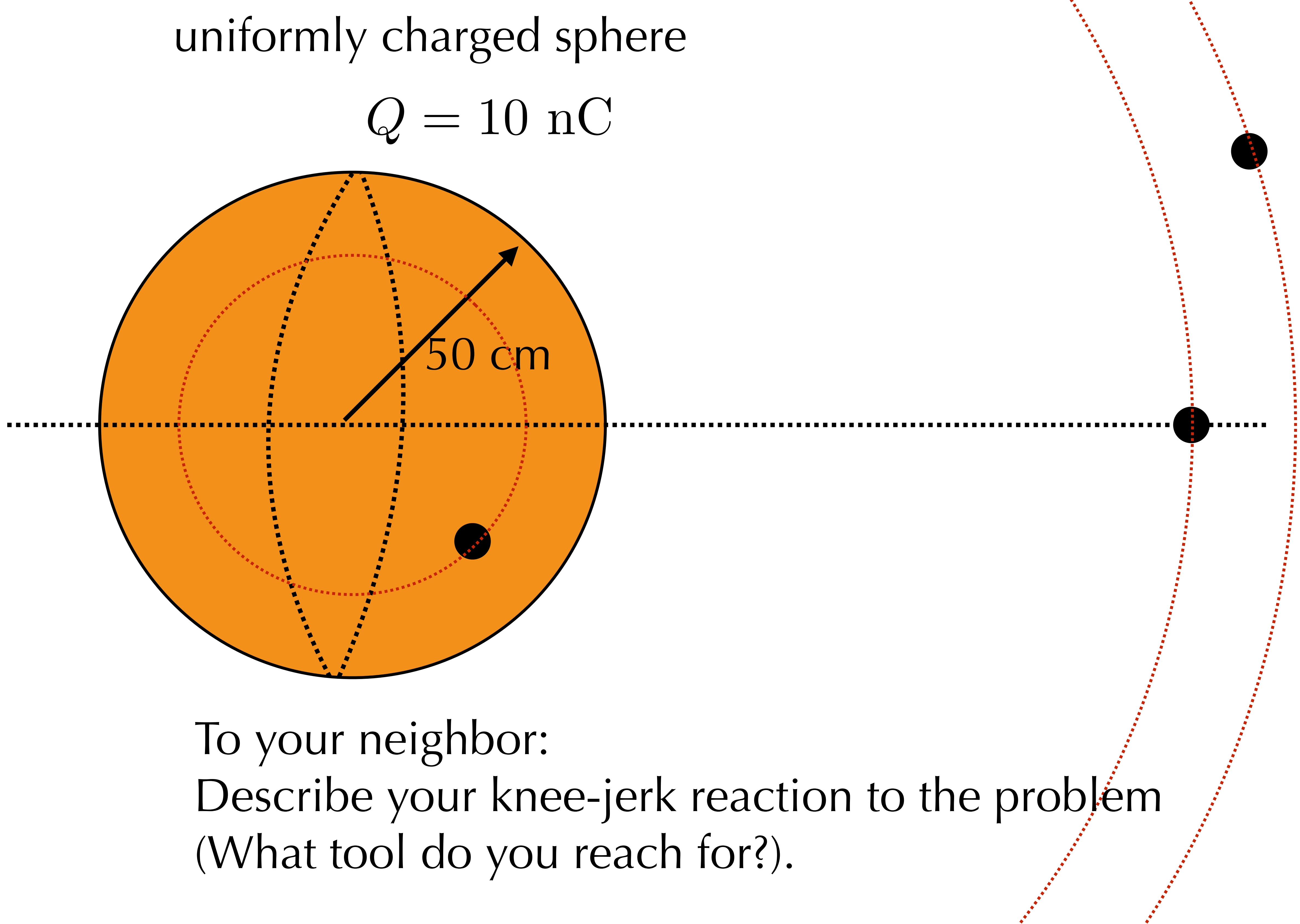


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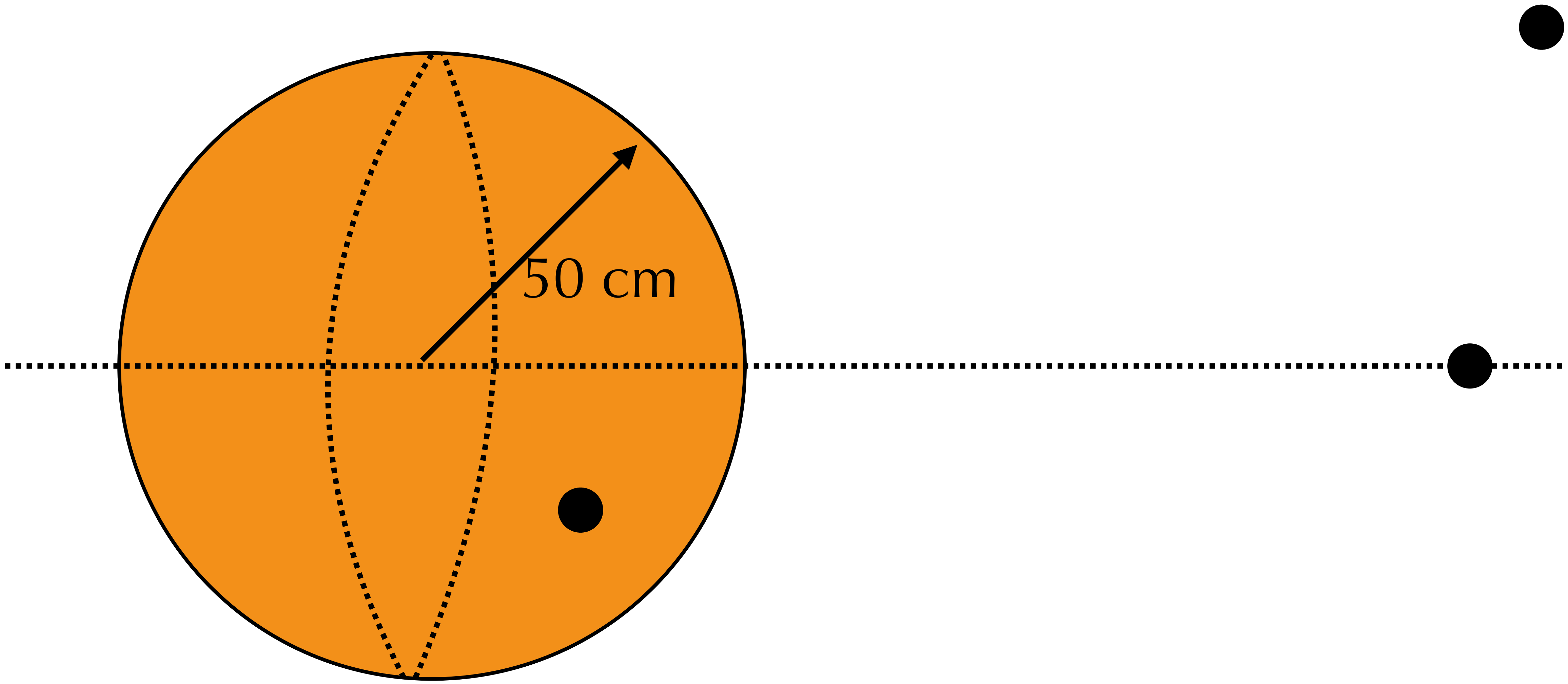
$$\rho(r) = 5\sqrt{r}$$

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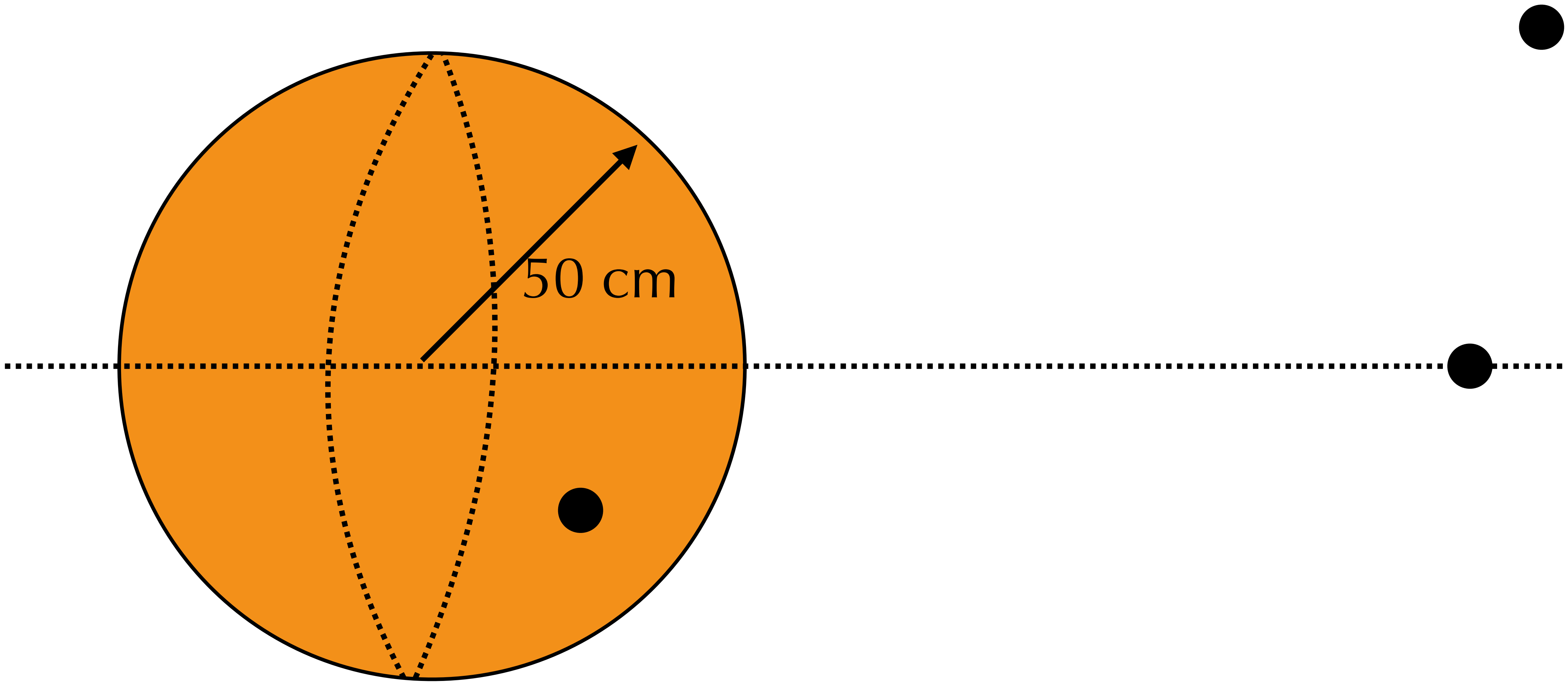
$$\rho(r) = 5\sqrt{r}\cos\theta$$

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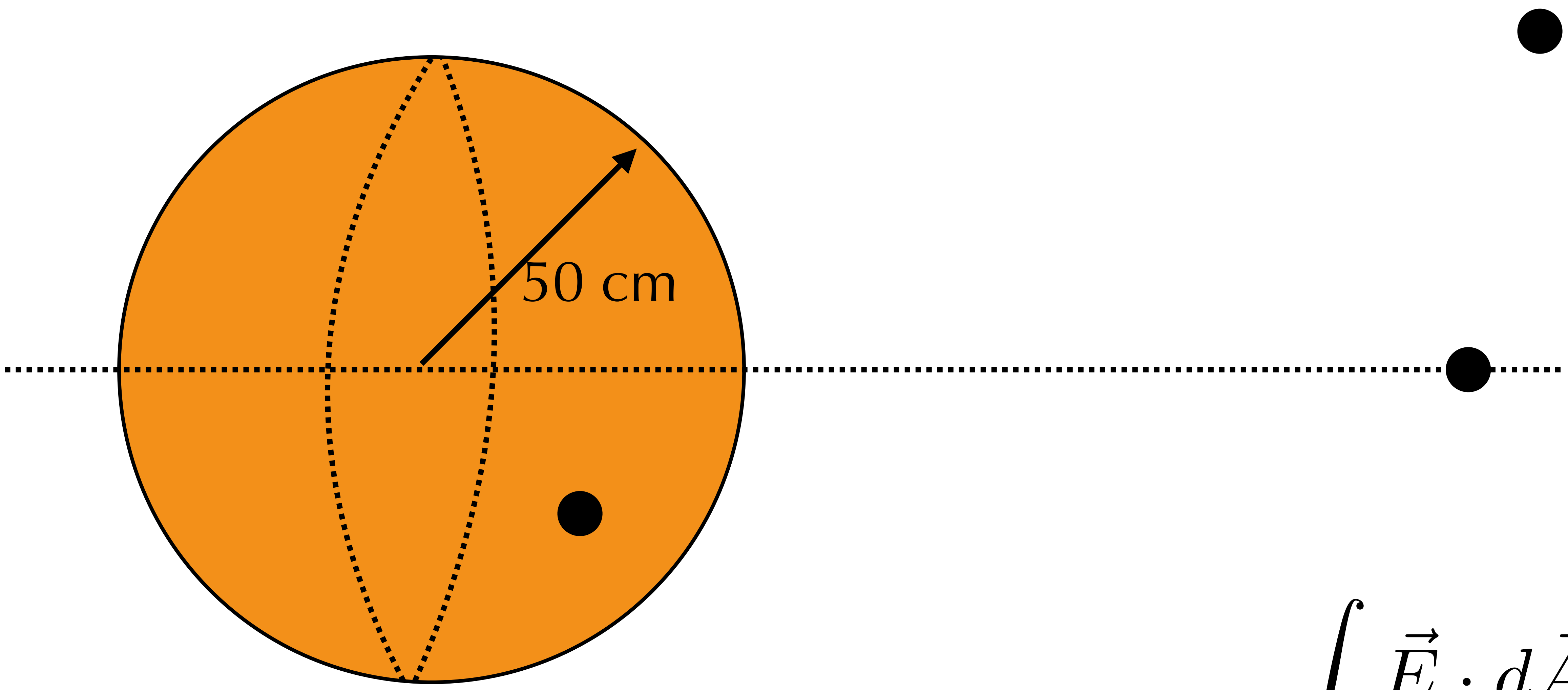


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Describe your knee-jerk reaction to the problem
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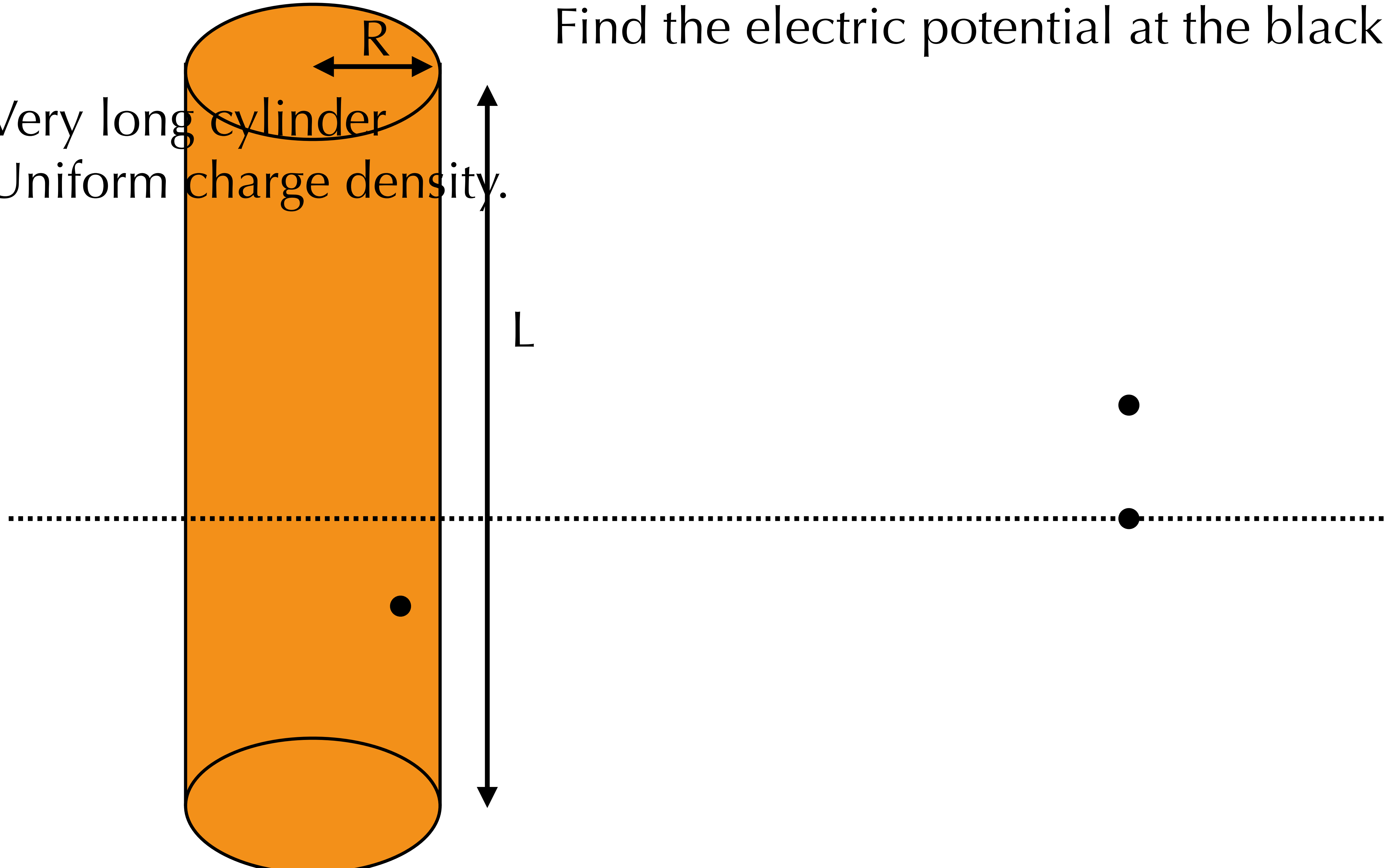
Find the electric potential at the black dots.

Very long cylinder
Uniform charge density.

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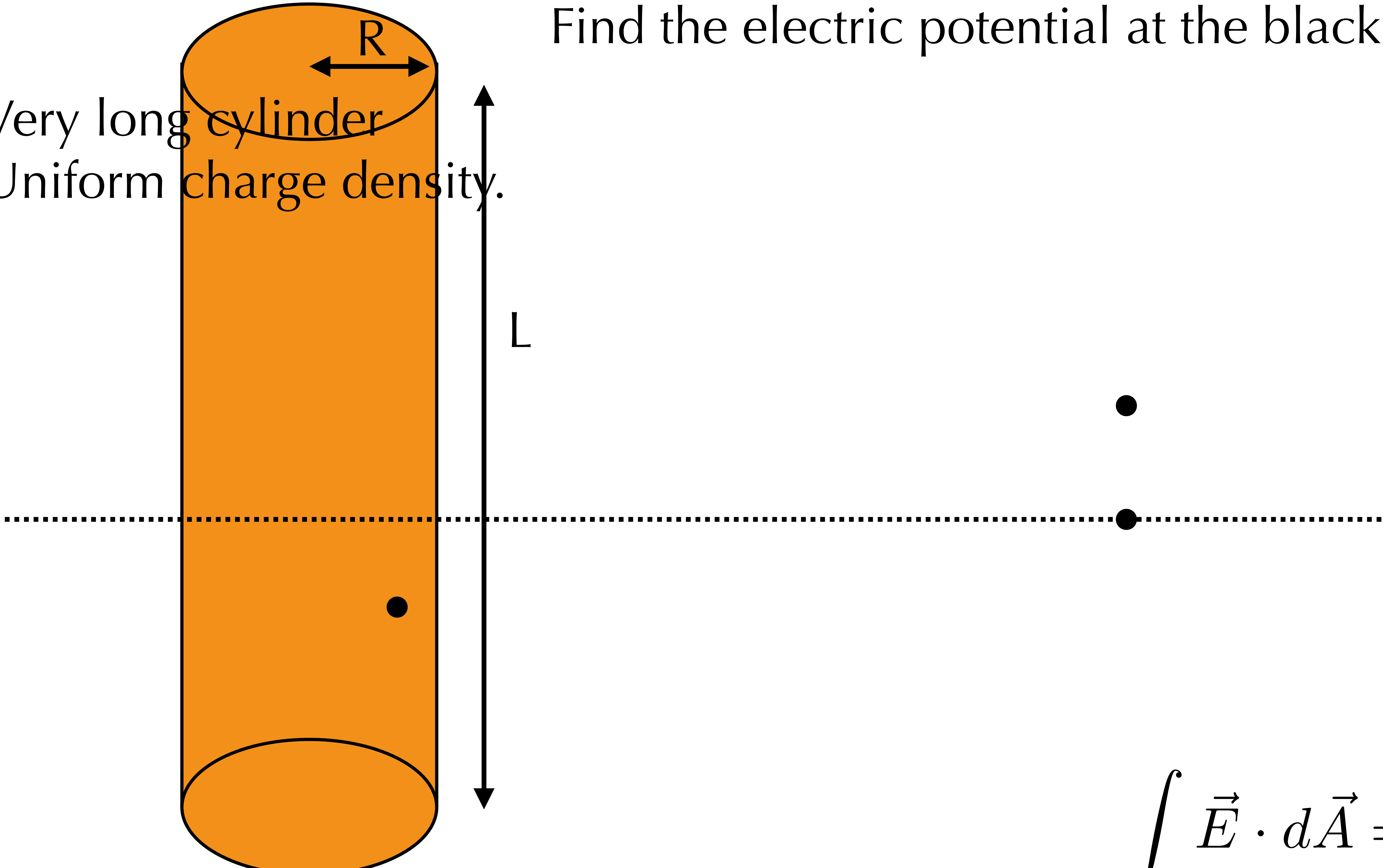


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To your neighbor:
Describe your knee-jerk reaction to the problem
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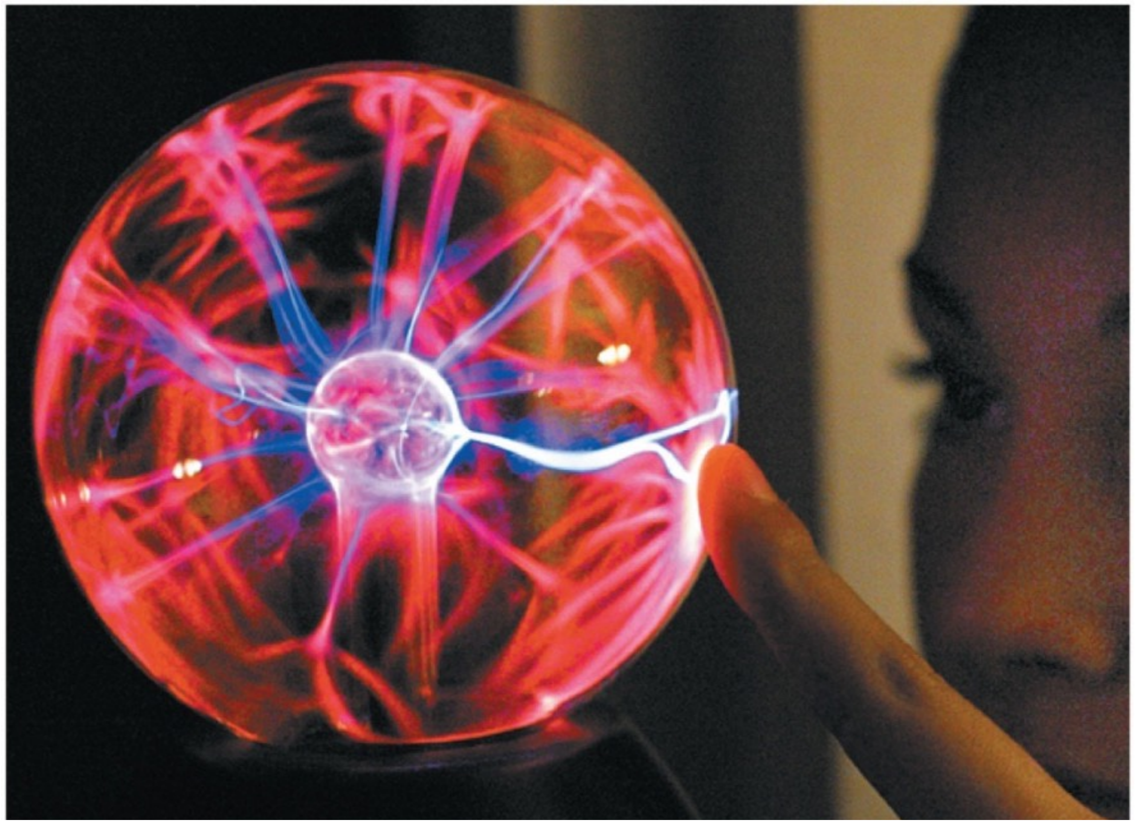
# Electric Potential of sphere of charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad \text{outside}$$



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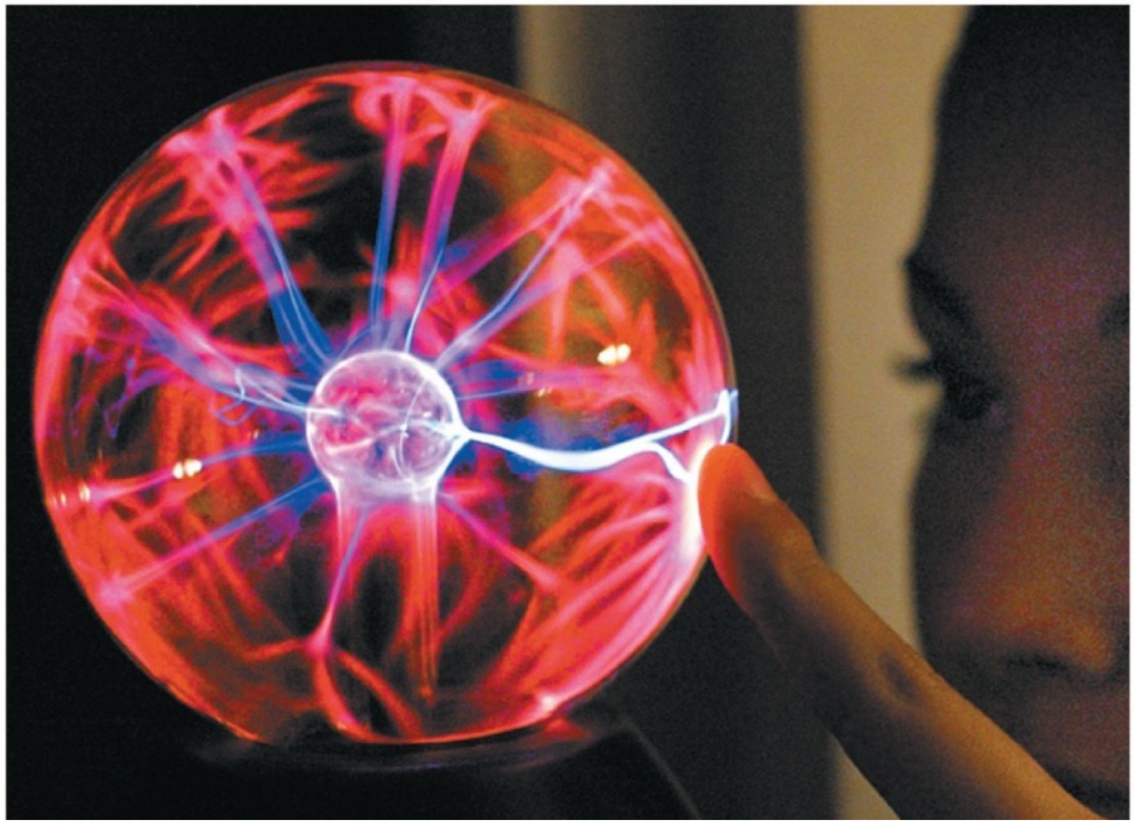
$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$



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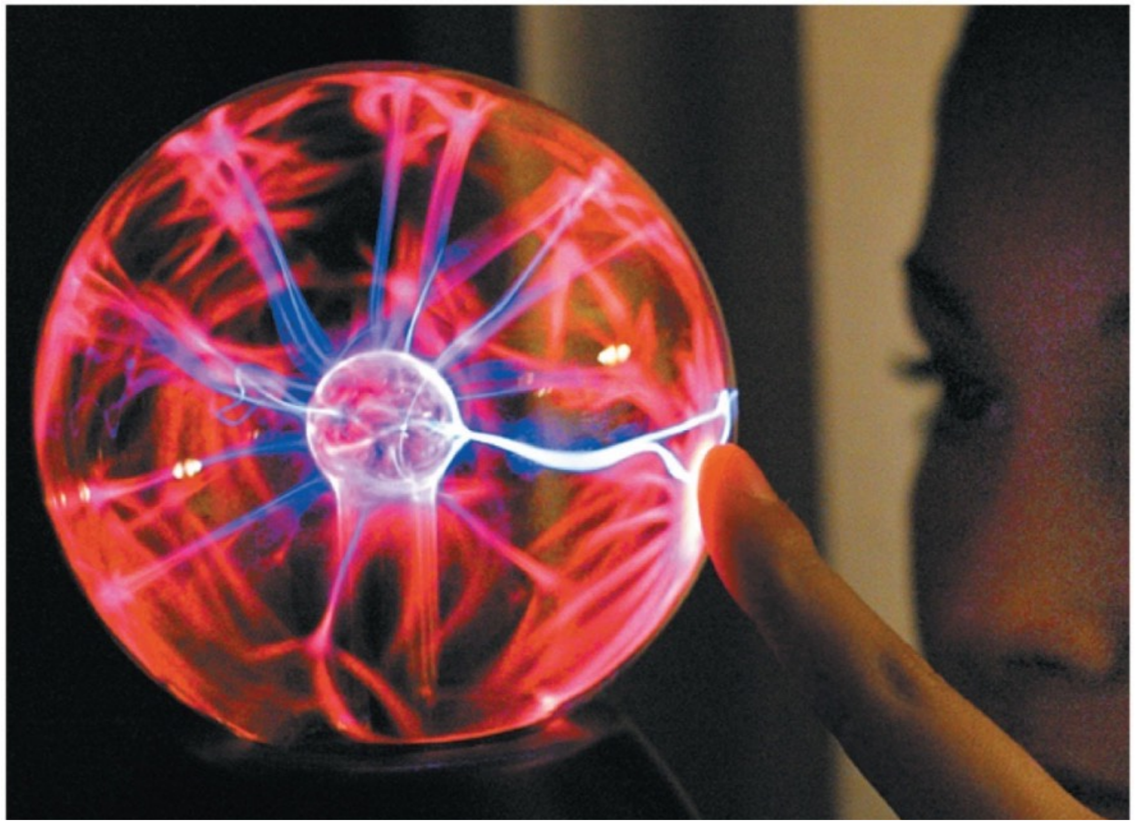

$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$Q = V_0 4\pi\epsilon_0 R$$

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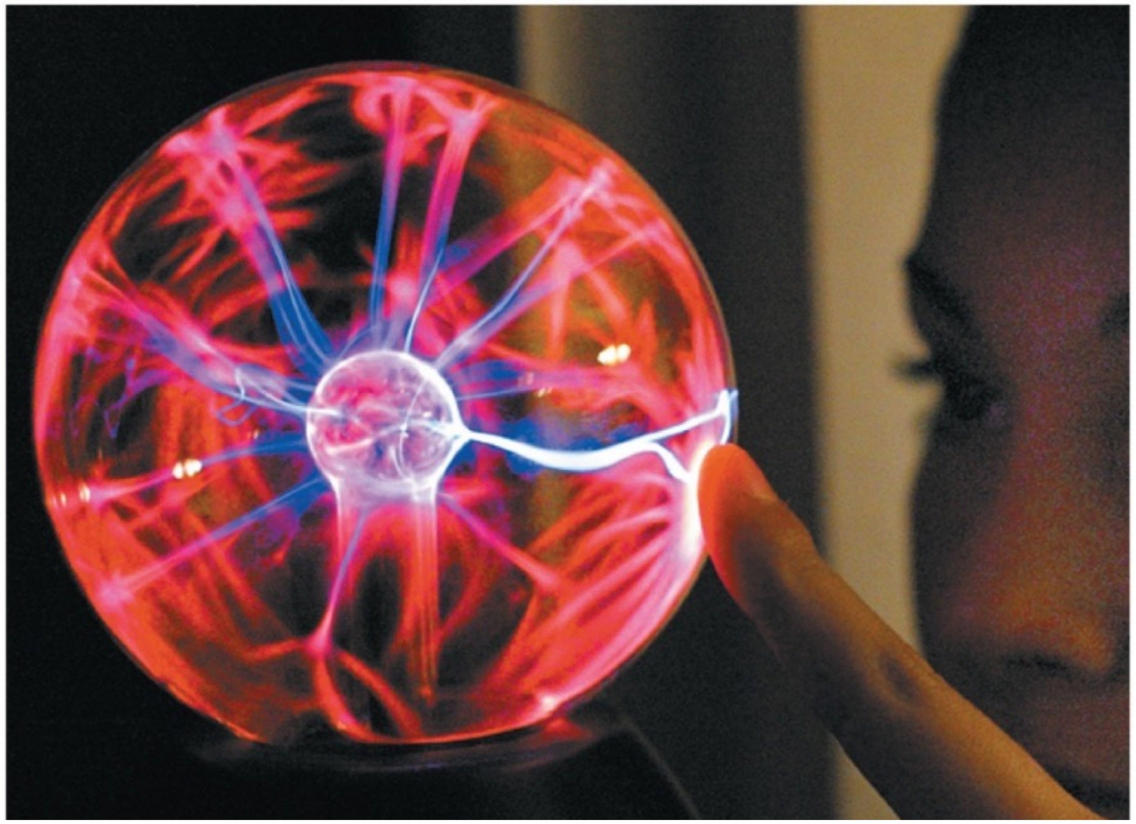
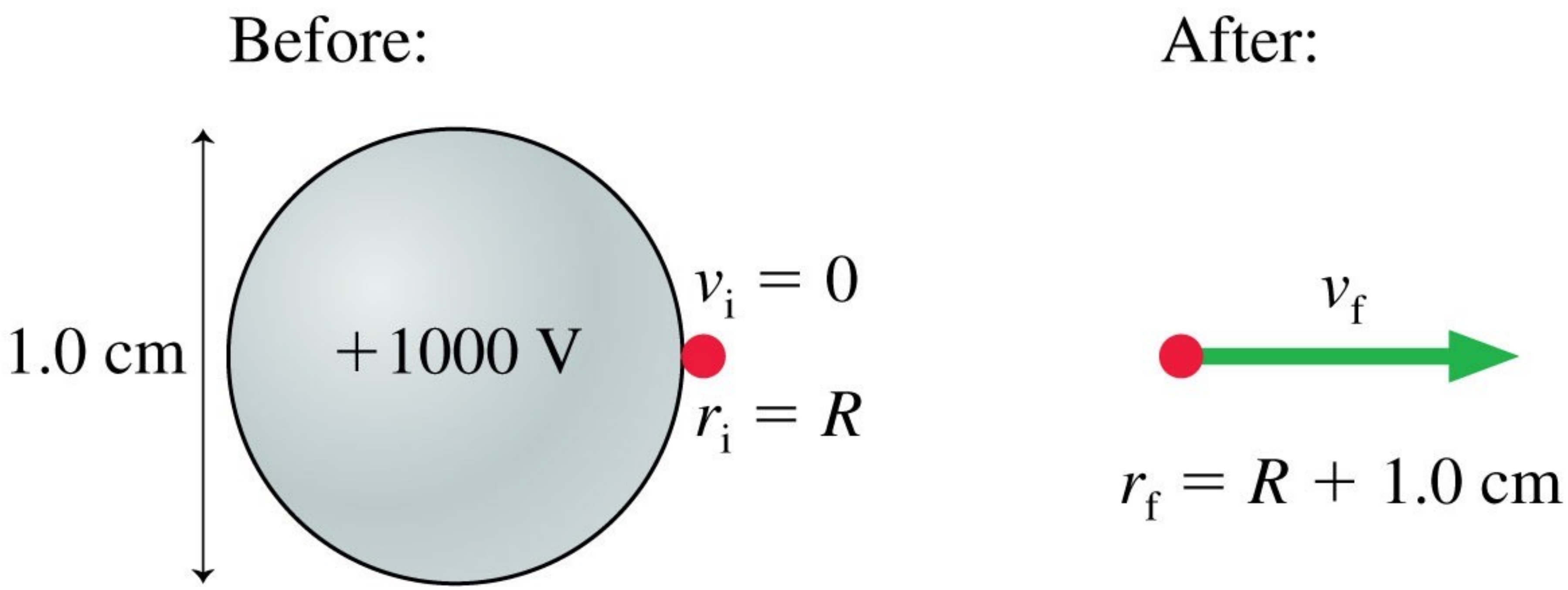


$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

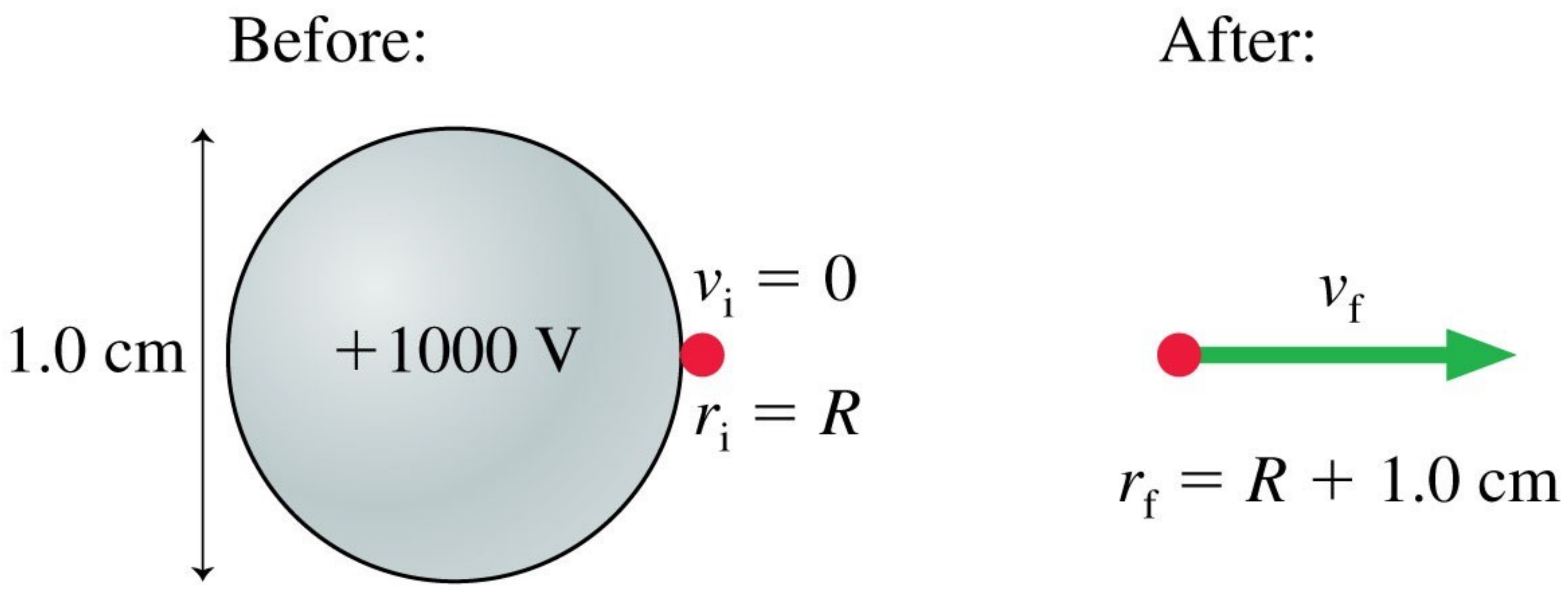
$$Q = V_0 4\pi\epsilon_0 R$$

$$V = \frac{R}{r}V_0$$ potential of a charged sphere

How much charge is on this sphere?

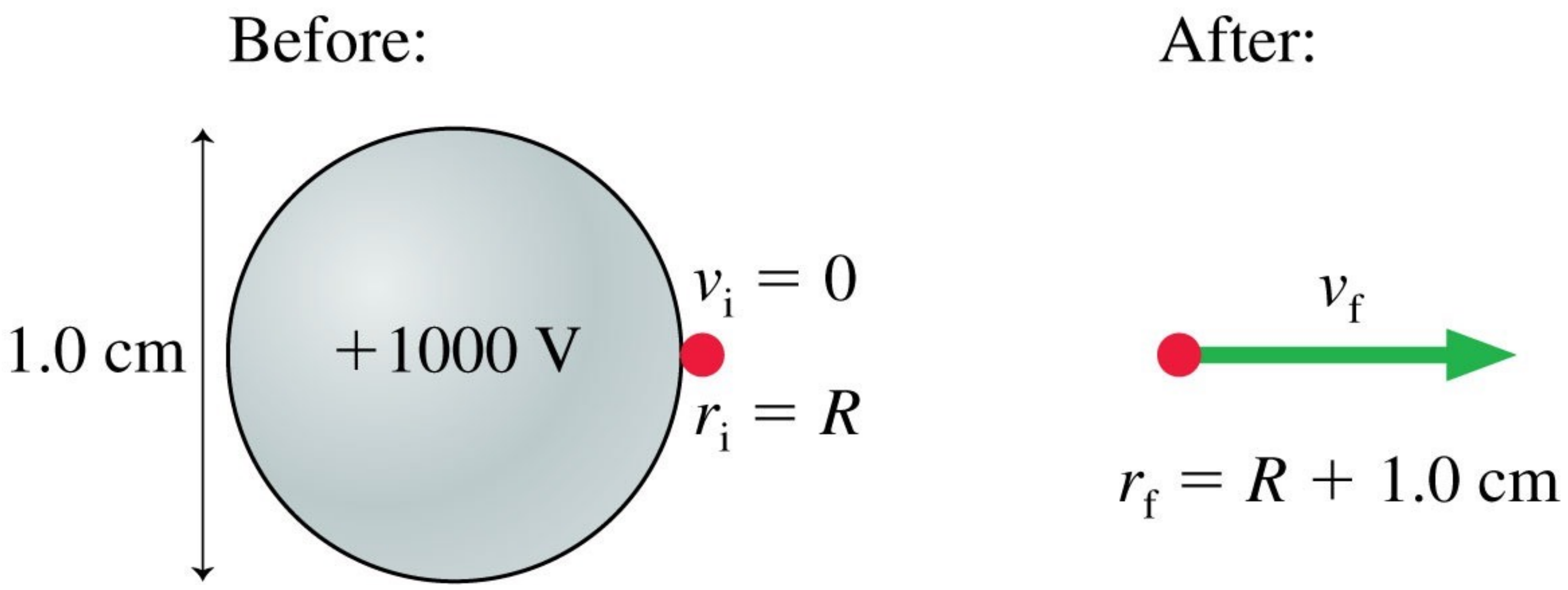


How much charge is on this sphere? $0.56\ \mathrm{nC}$



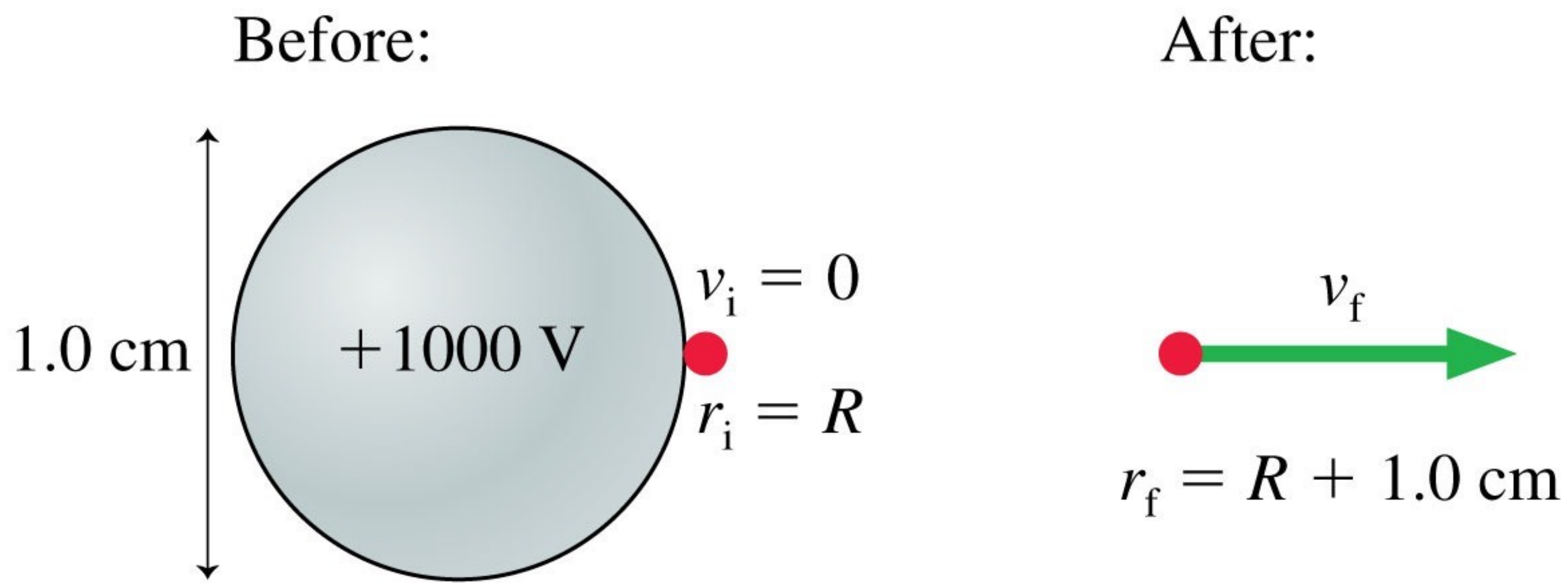
How much charge is on this sphere? $0.56\ \mathrm{nC}$

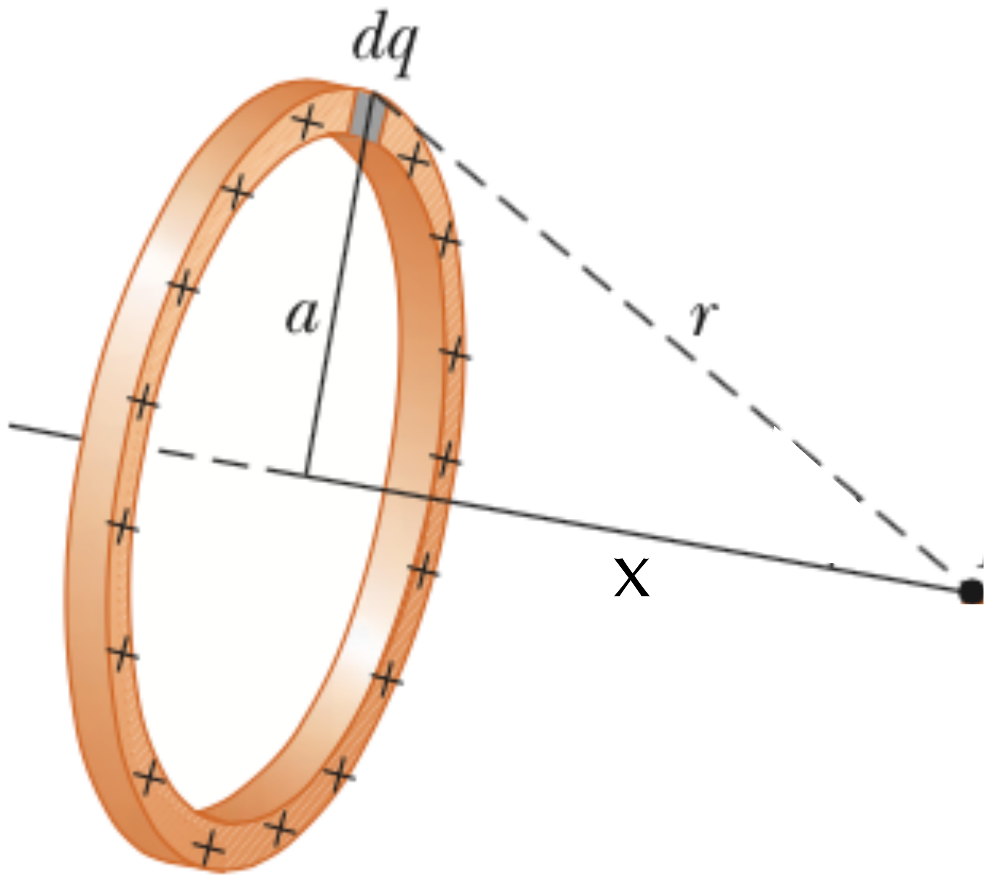
If a proton is released from rest at the surface of the sphere, how fast will it be going when it is 1.0 cm away from the surface?



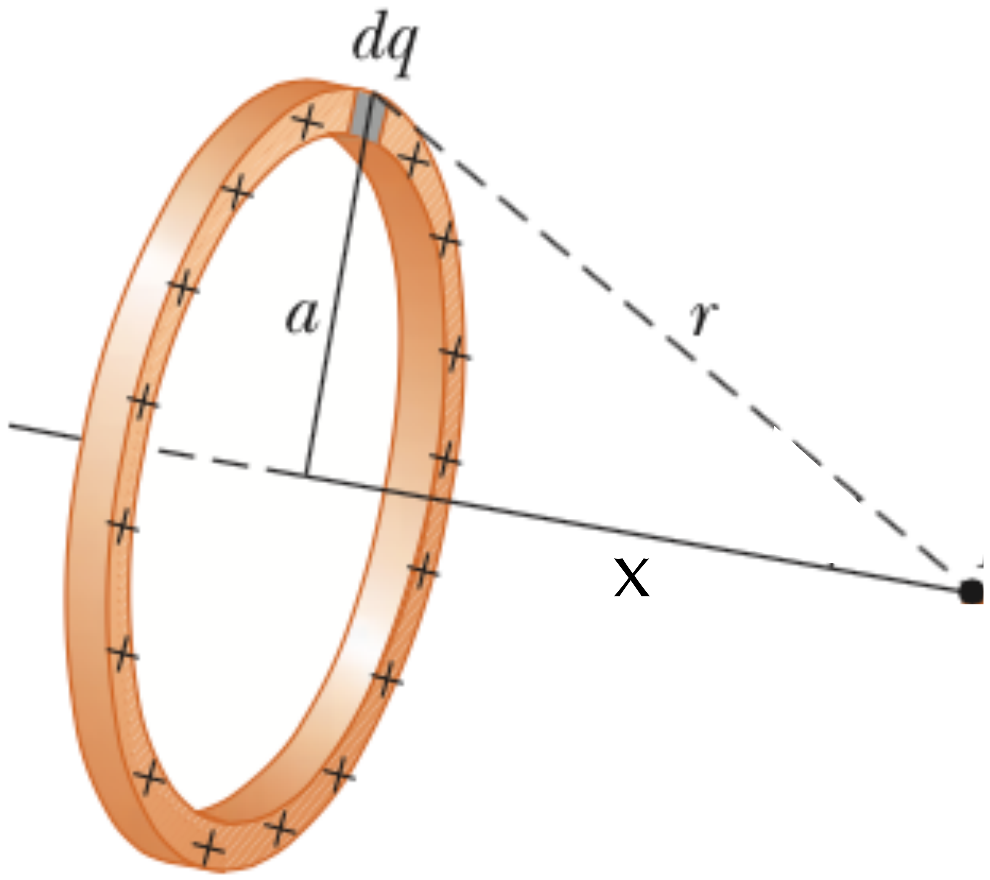
How much charge is on this sphere? $0.56\ \mathrm{nC}$

If a proton is released from rest at the surface of the sphere, how fast will it be going when it is 1.0 cm away from the surface? $3.6 \times 10^{5}\ \mathrm{m/s}$






Can you calculate the electric potential due to this ring of charge?




Can you calculate the electric potential due to this ring of charge?

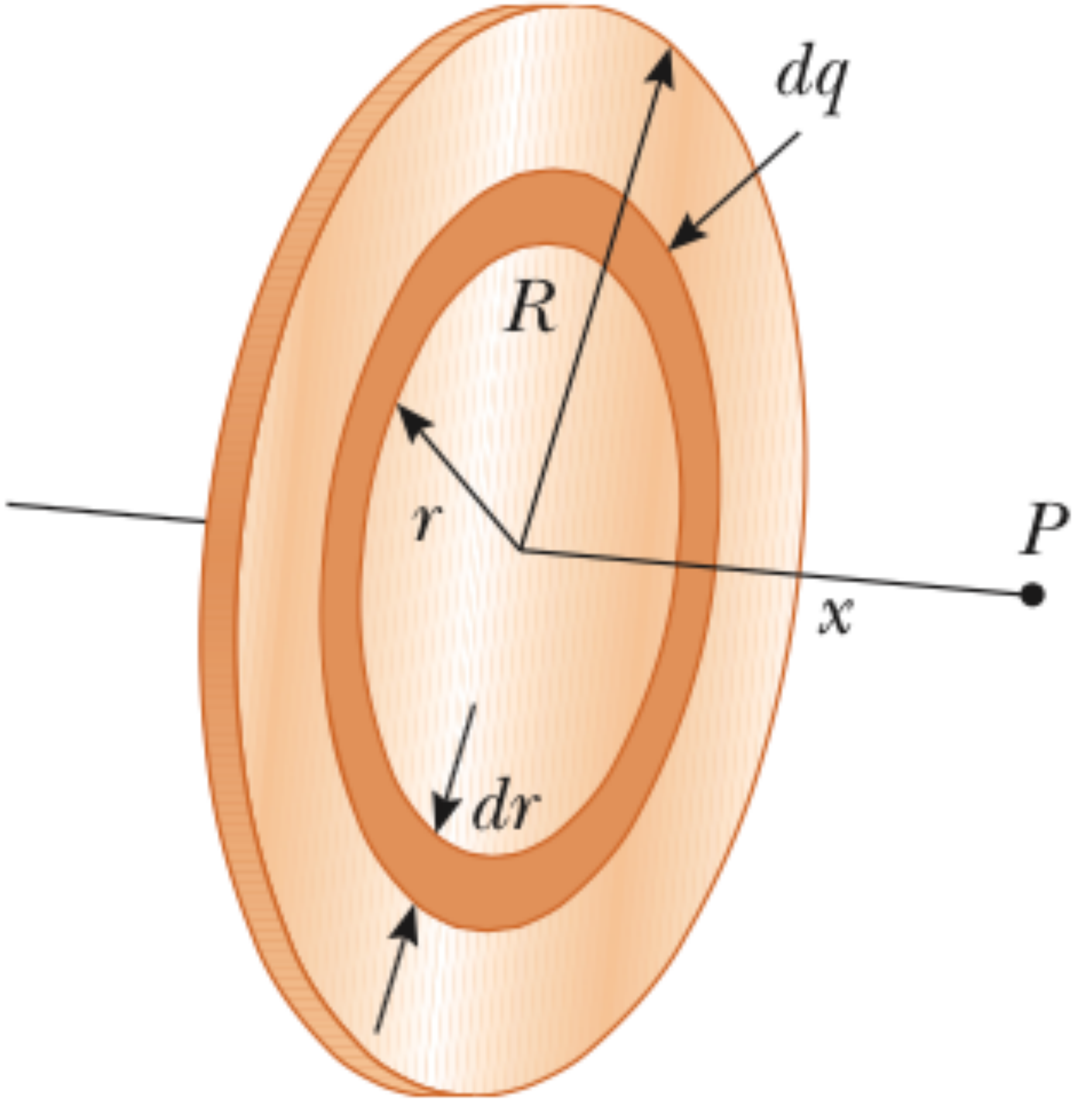
$$V = \frac{kq}{\sqrt{x^2 + a^2}}$$

$R = 17.5$ mm

$Q = +\ 5.00$ nC

Find the potential of this disk at x ?

Evaluate at x=0 and x = 1.00 cm.



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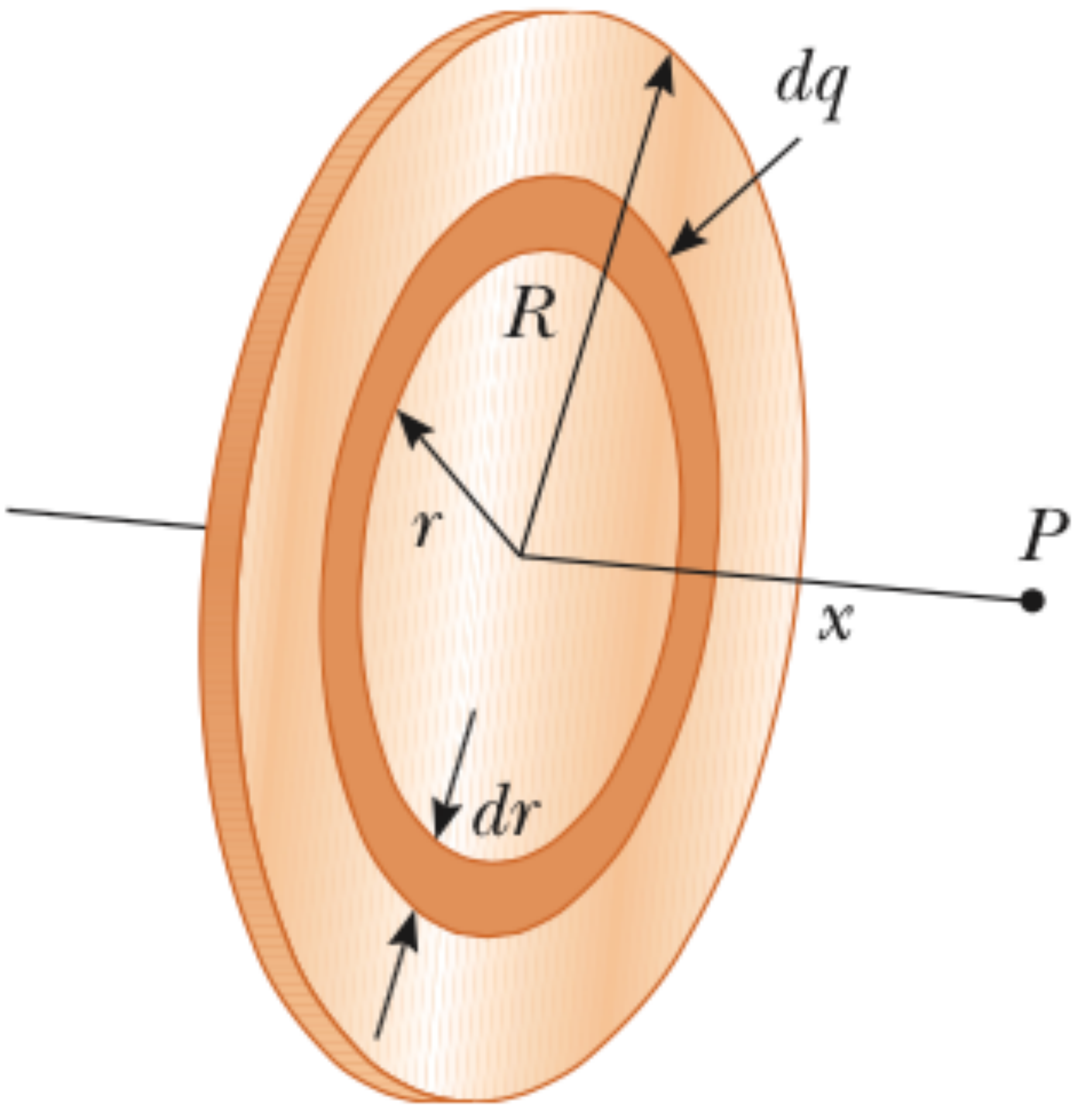
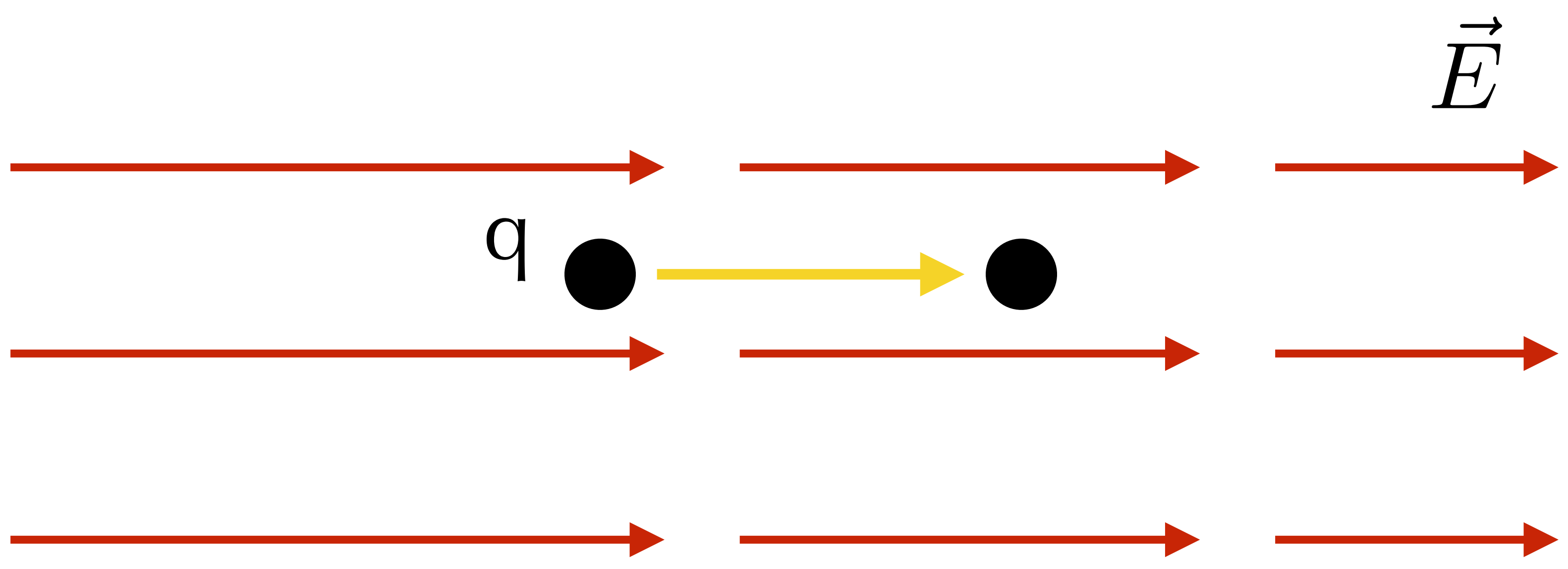



Evaluate at x=0 and x = 1.00 cm.

$$V = \frac{Q}{2\pi\epsilon_0 R^2}(\sqrt{R^2 + x^2} - x)$$

Which expression correctly gives the change in potential energy of the charge as it moves from left to right?



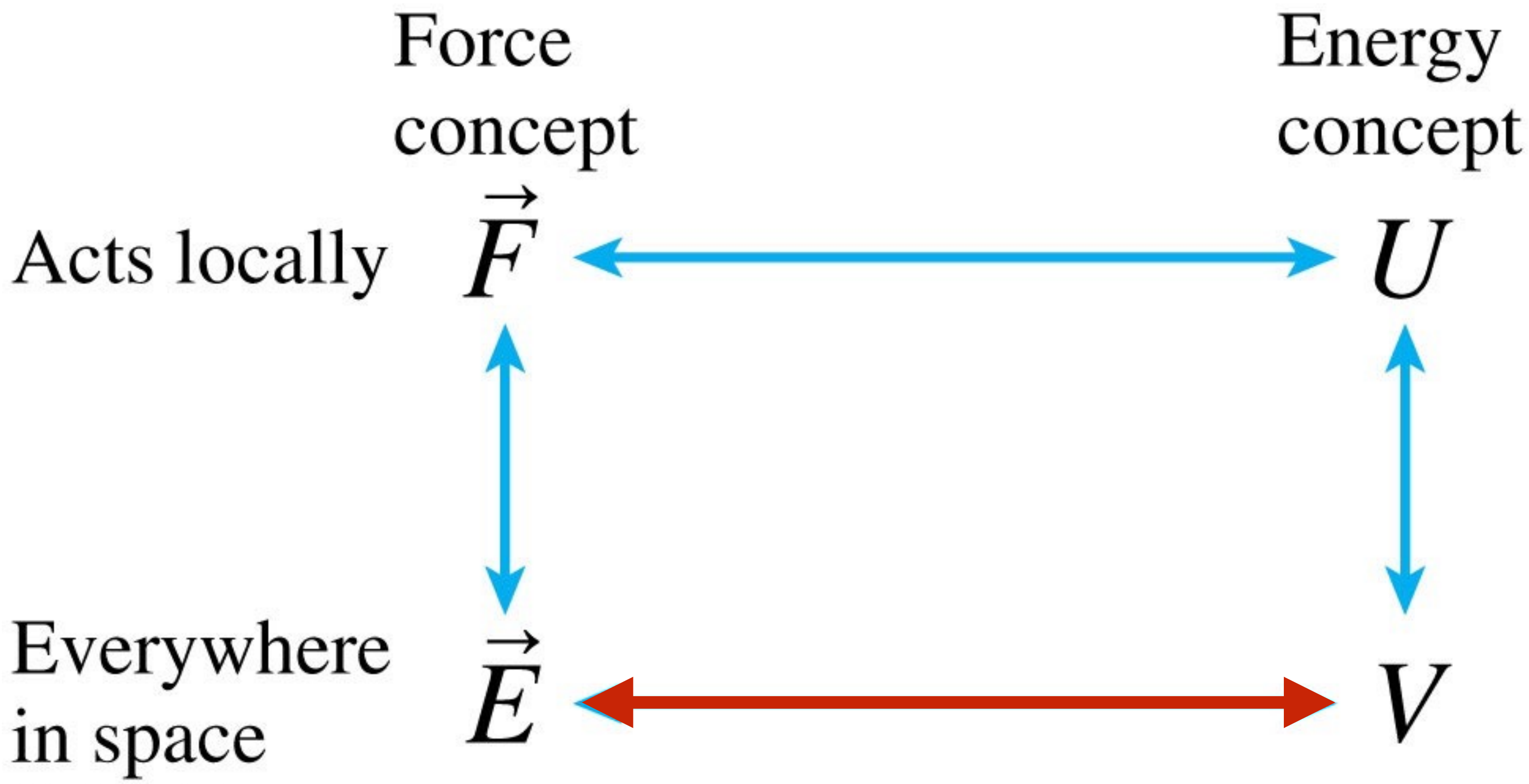

a) $\Delta U = -qE\Delta x$

b) $\Delta U = q\Delta x$

c) $\Delta U = -q\int_i^f E(x)dx$

d) $\Delta U = q\int_i^f E(x)dx$

# Big Picture: Where are we headed



$$\Delta U = -q \int_i^f E(x) dx$$

What if I divide both sides by q?

$$\Delta U = -q \int_i^f E(x)dx$$

What if I divide both sides by q?

$$\boxed{\Delta V = -\int_i^f E(x)dx}$$

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What if I take the derivative of both sides?

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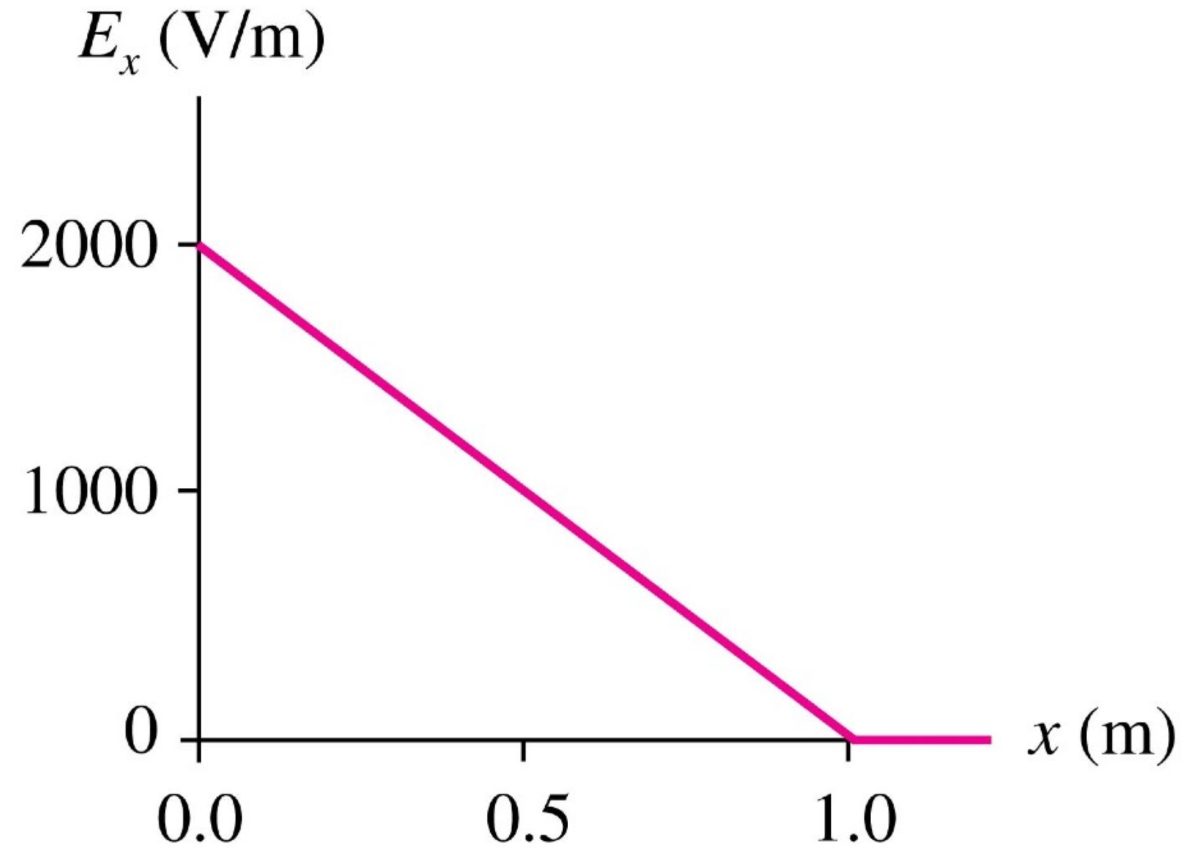
What if I take the derivative of both sides?

$$\boxed{E = -\frac{dV}{dx}}$$

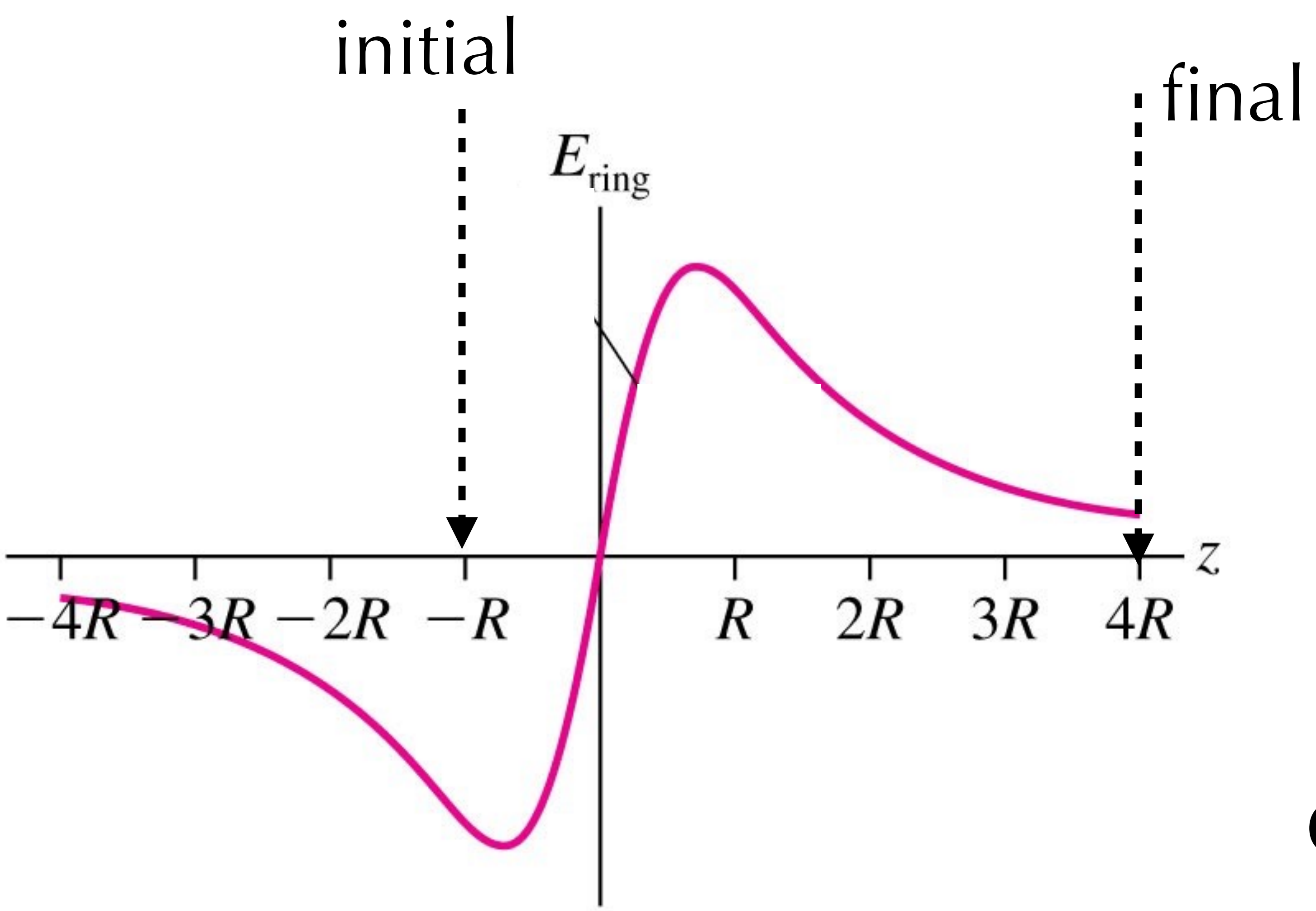
This is a graph of the $x$-component of the electric field along the $x$-axis. The potential is zero at the origin. What is the potential at $x = 1$m?




a) 2000 V
b) 1000 V
c) 0 V
d) -1000 V
e) -2000 V

The potential difference between these two points is?

b) positive e) negative d) zero



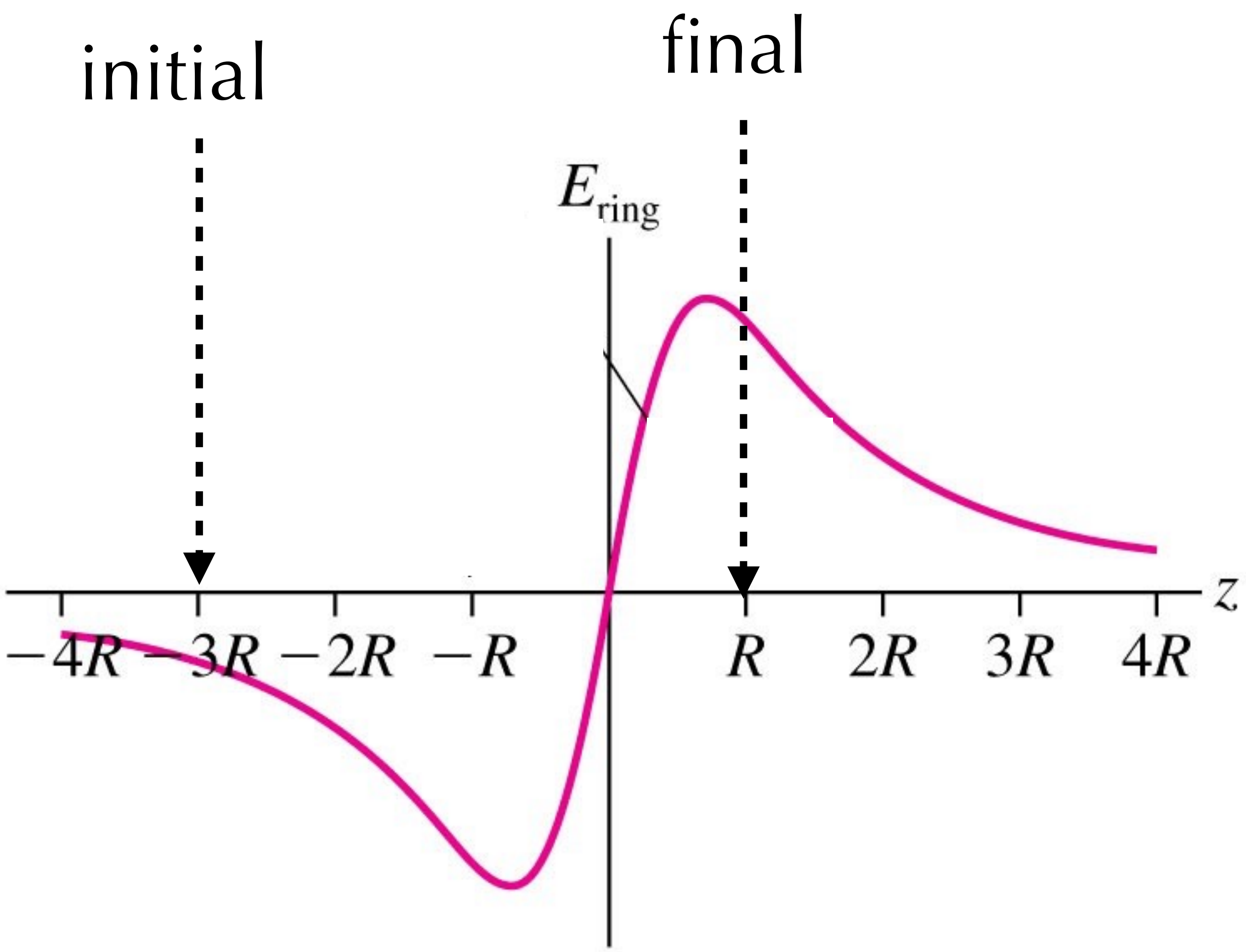

Question #30

The potential energy of an electron

a) decreases b) increases c) remains unchanged

The potential difference between these two points is?

b) positive e) negative d) zero




Question #30

The potential energy of an electron

a) decreases b) increases c) remains unchanged

# 2D Electric Field

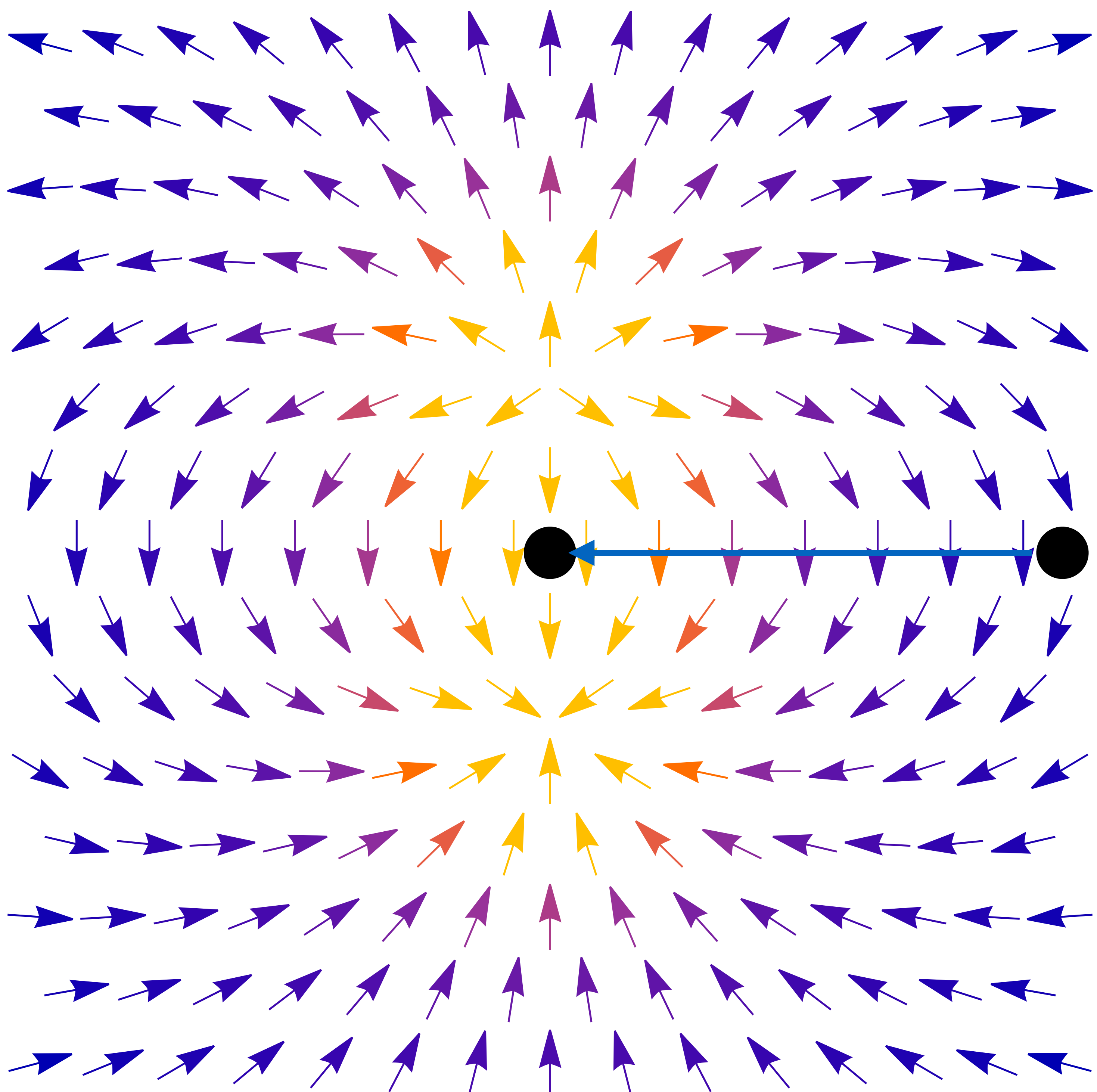


What is the potential difference between these two points?

a) positive

b) zero

c) negative



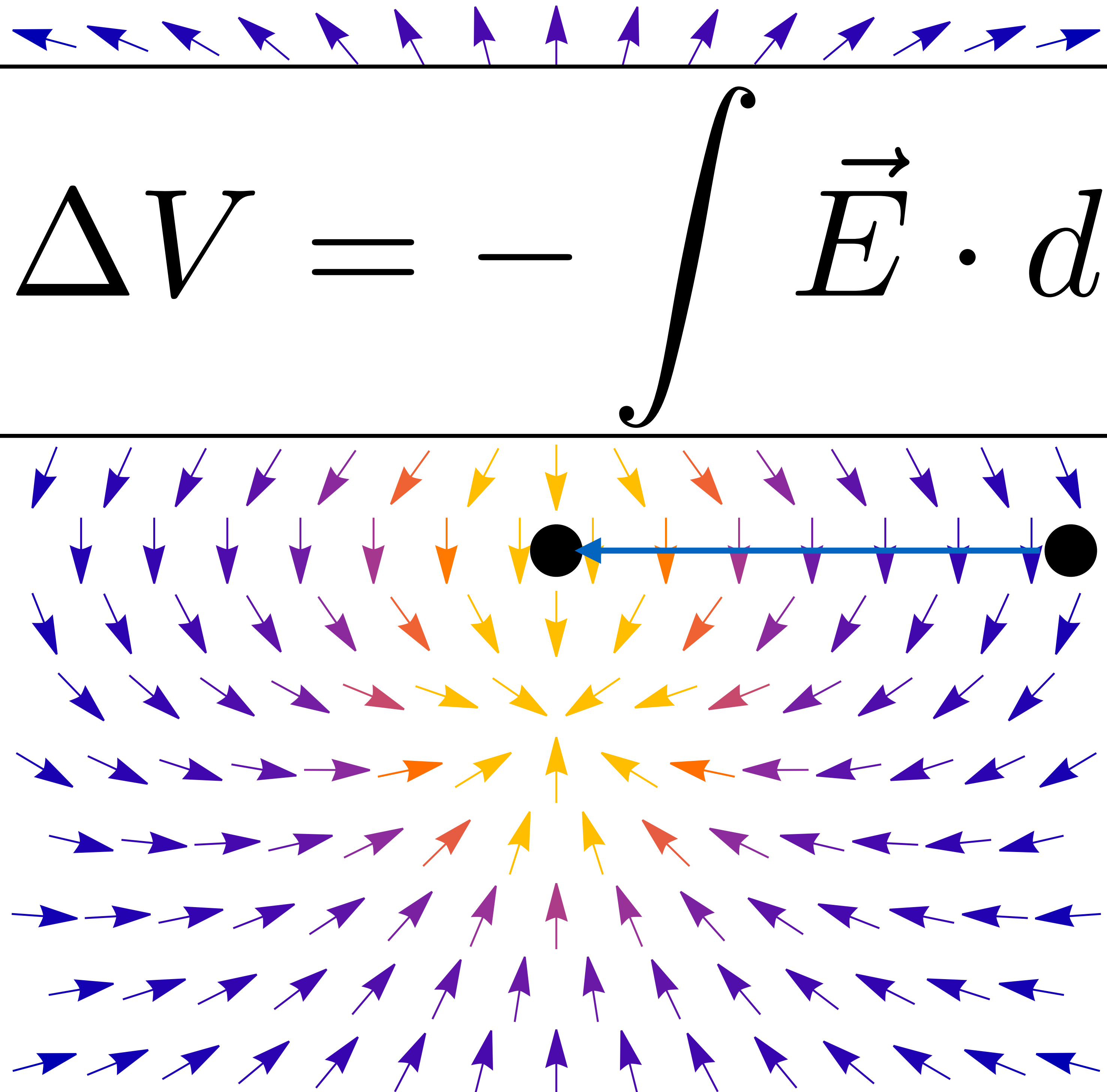
What is the potential difference between these two points?

a) positive

b) zero

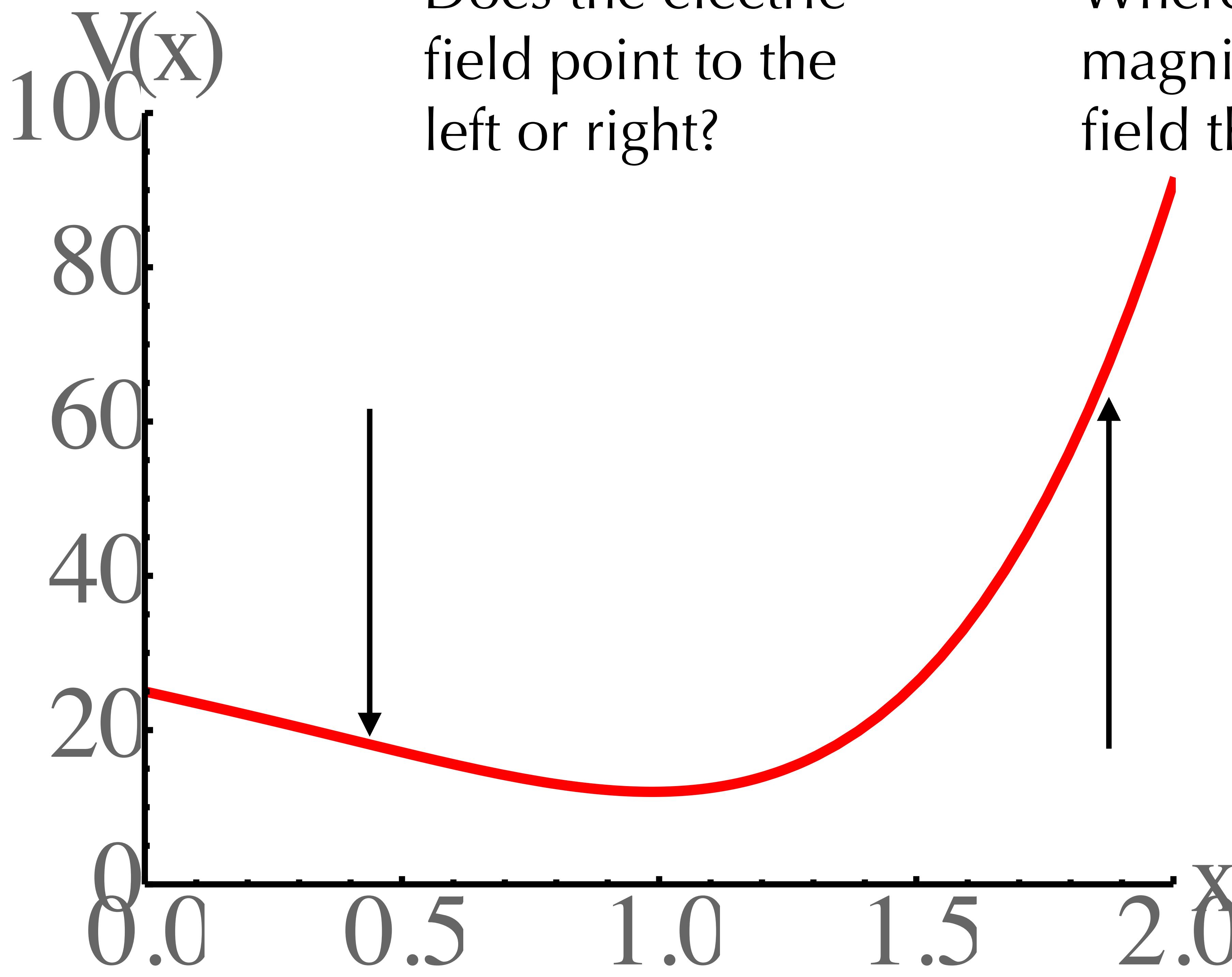
c) negative

$$\boxed{\Delta V = -\int \vec{E} \cdot d\vec{r}}$$



Does the electric field point to the left or right?

Where is the magnitude of the field the strongest?



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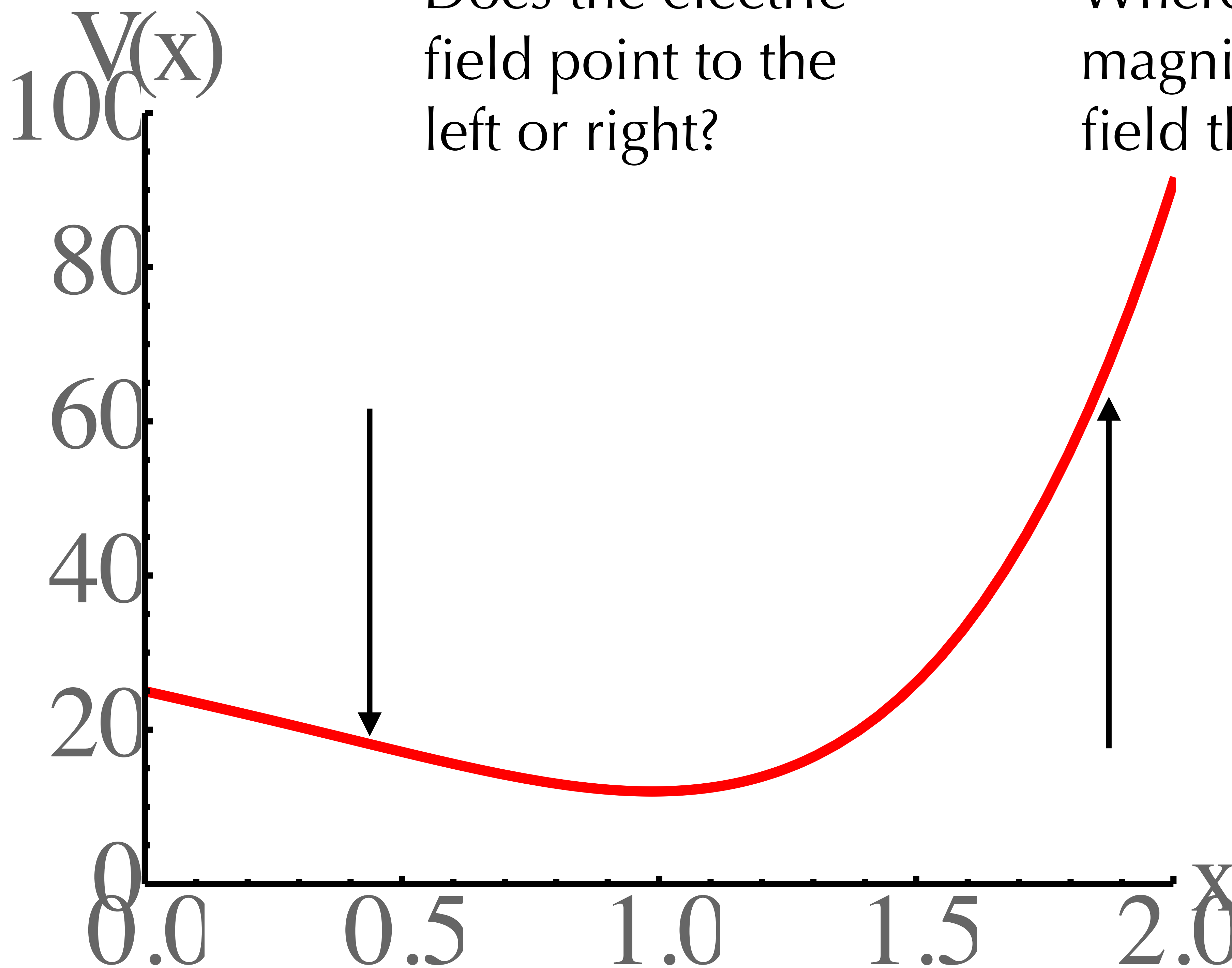


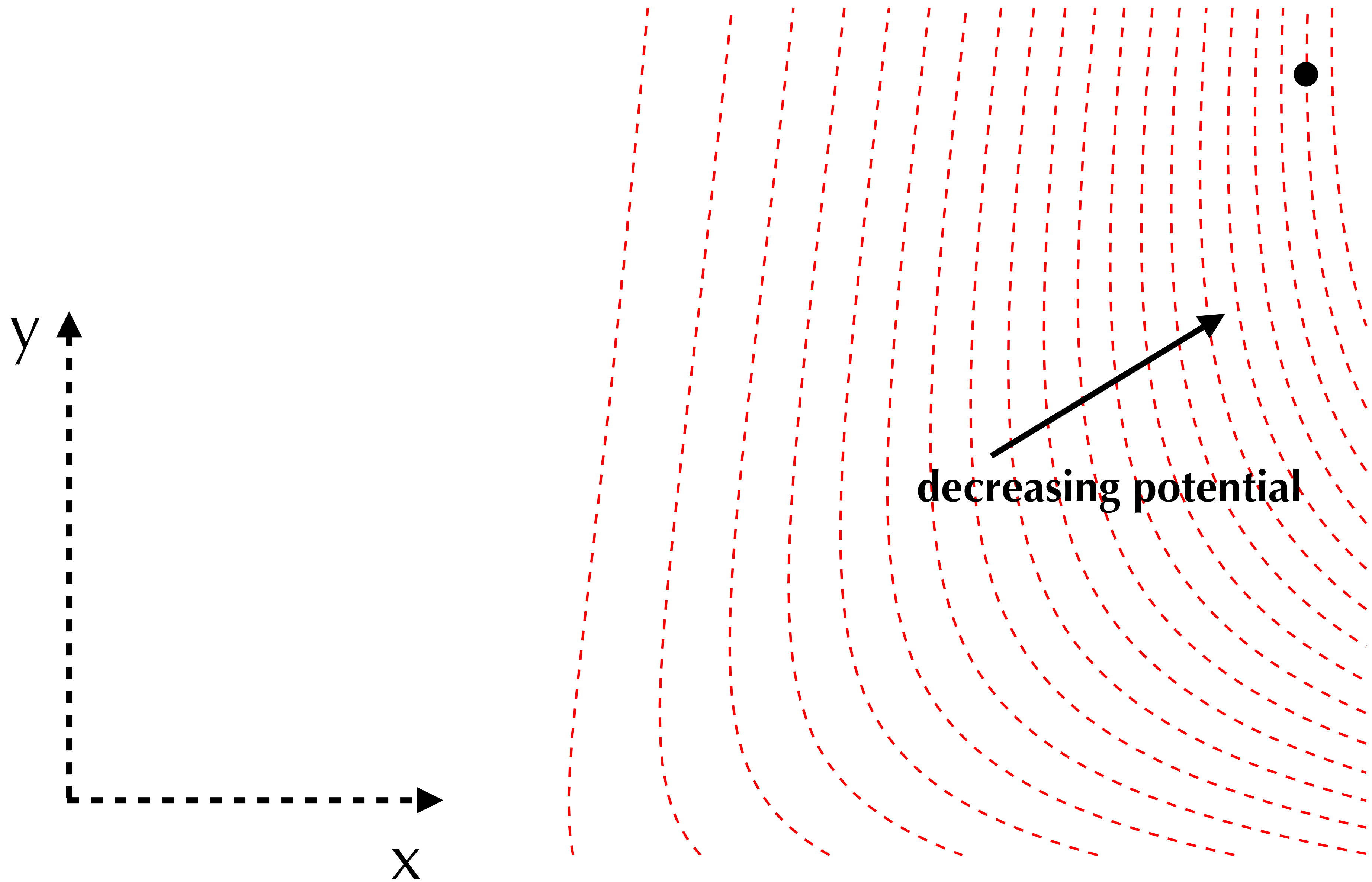

$$\boxed{E = -\frac{dV}{dx}}$$

# What about in higher dimensions

Question #32

The x and y components of the field at the dot are:

a) + and + b) 0 and + c) + and 0 d) - and + e) + and -




contour plot of potential

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

# What about in higher dimensions

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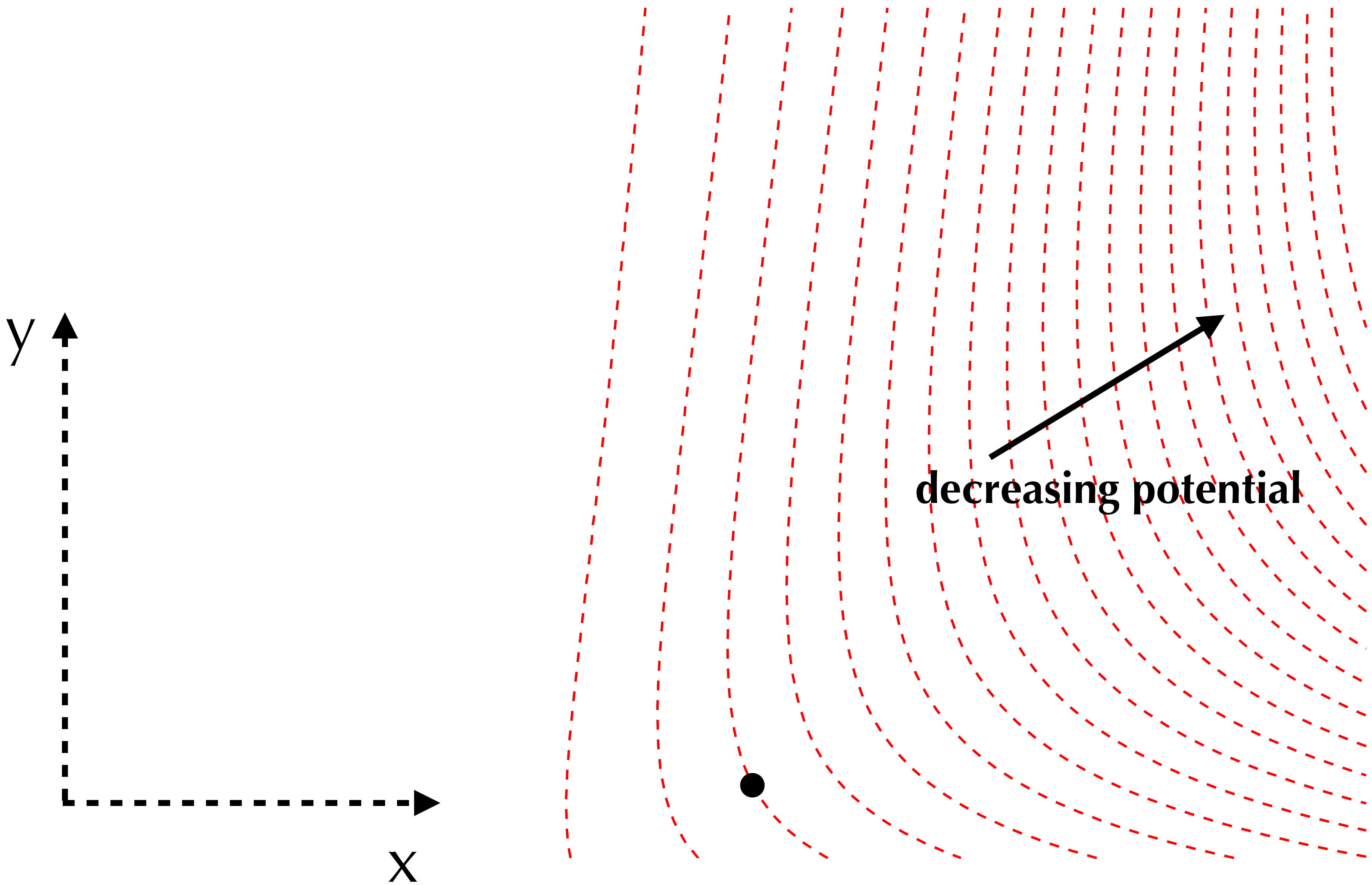


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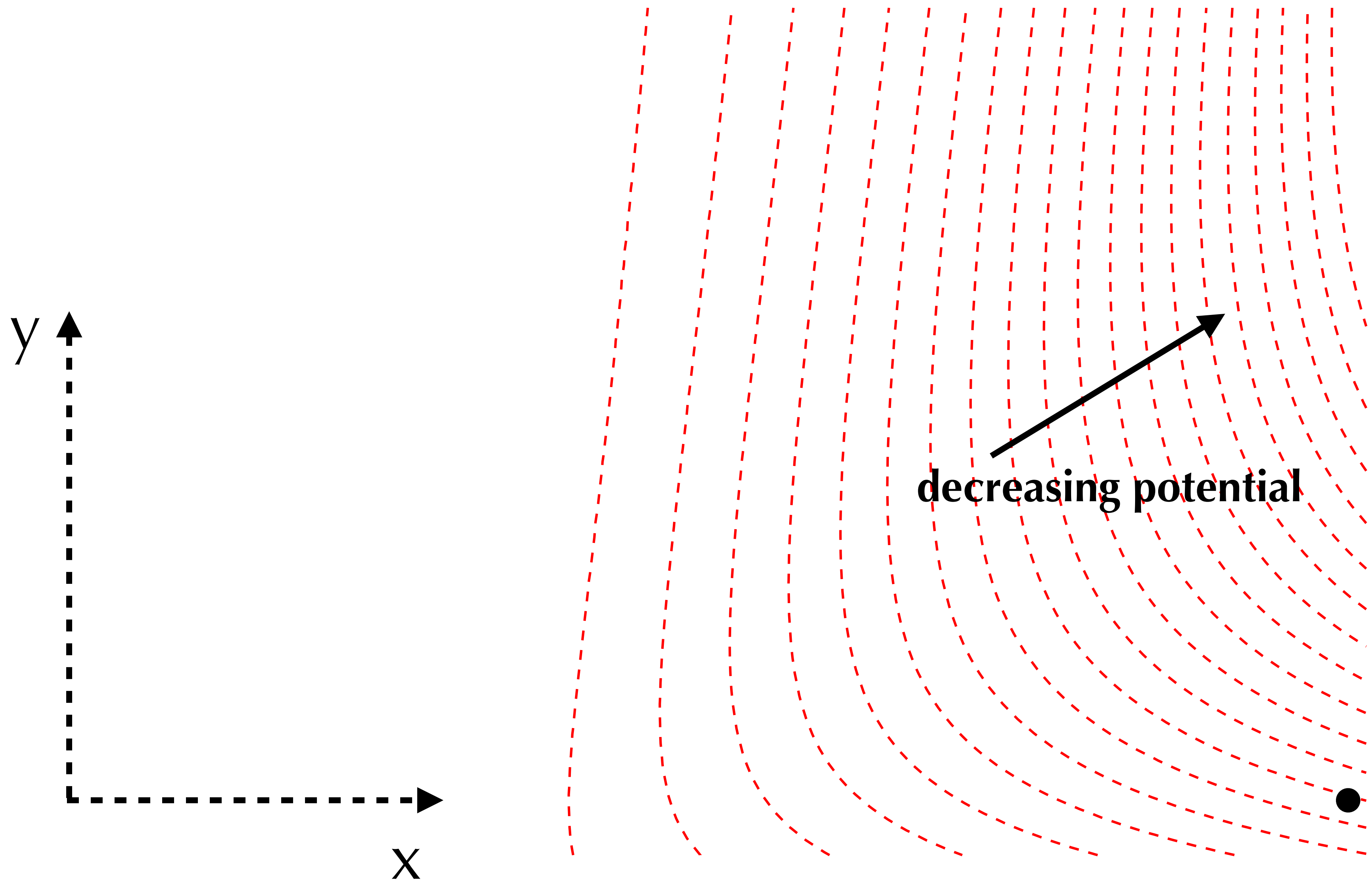
$$\boxed{\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)}$$

# What about in higher dimensions



The x and y components of the field at the dot are:

a) + and +  b) 0 and +  c) + and 0  d) - and +  e) + and -




contour plot of potential

$$\vec{E} = -\nabla V = -(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k})$$

# What about in higher dimensions



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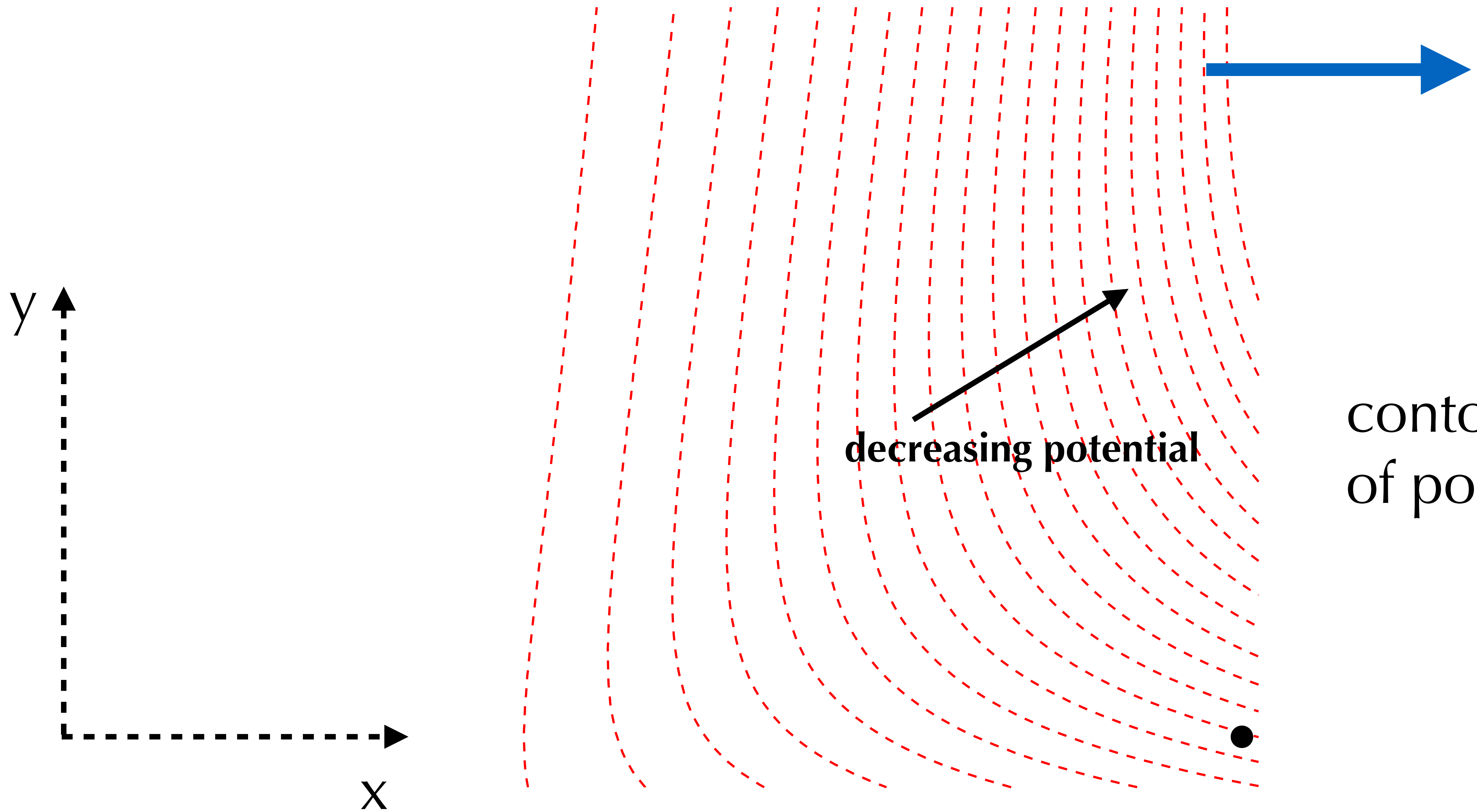
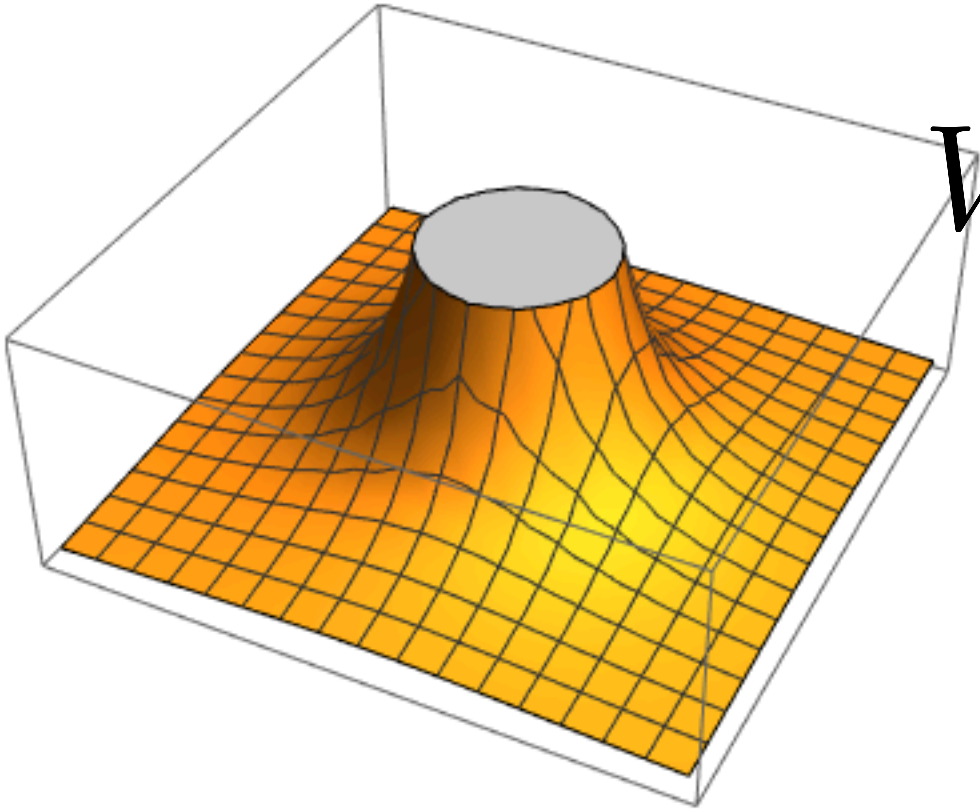


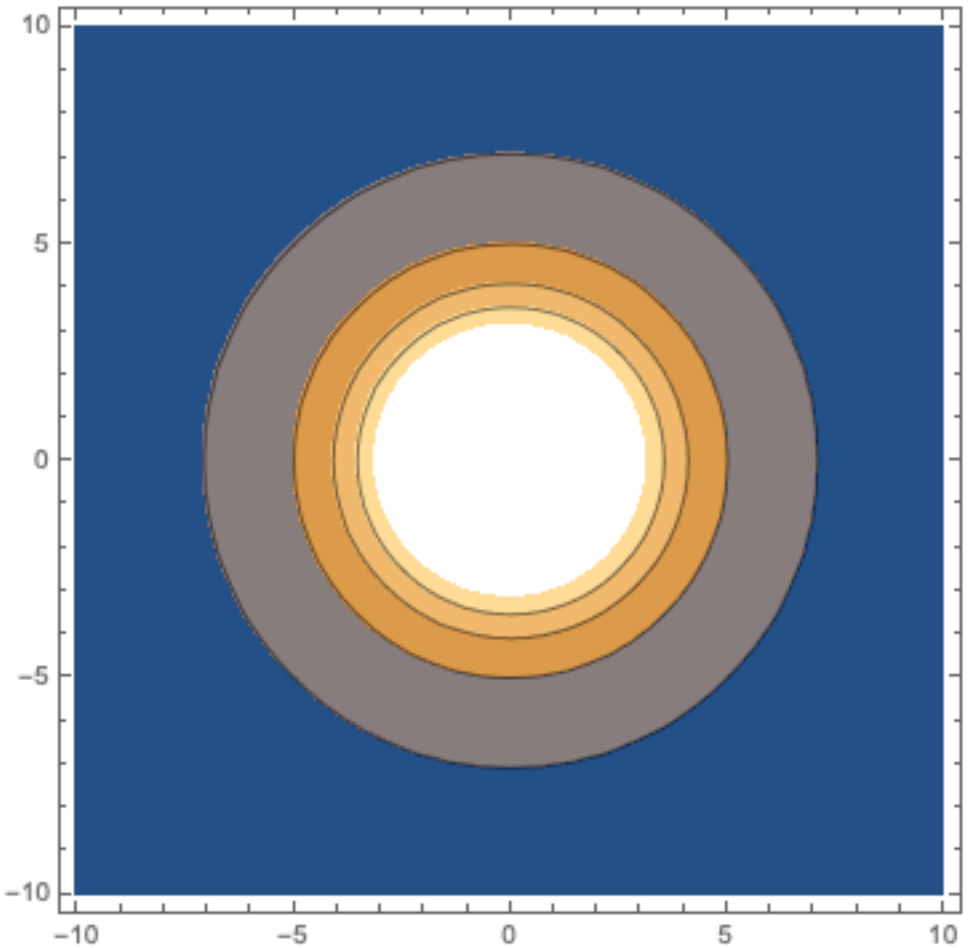

contour plot of potential

$$\vec{E} = -\nabla V = -(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k})$$

# Find the Electric Field



$$V = \frac{5}{x^2 + y^2}$$

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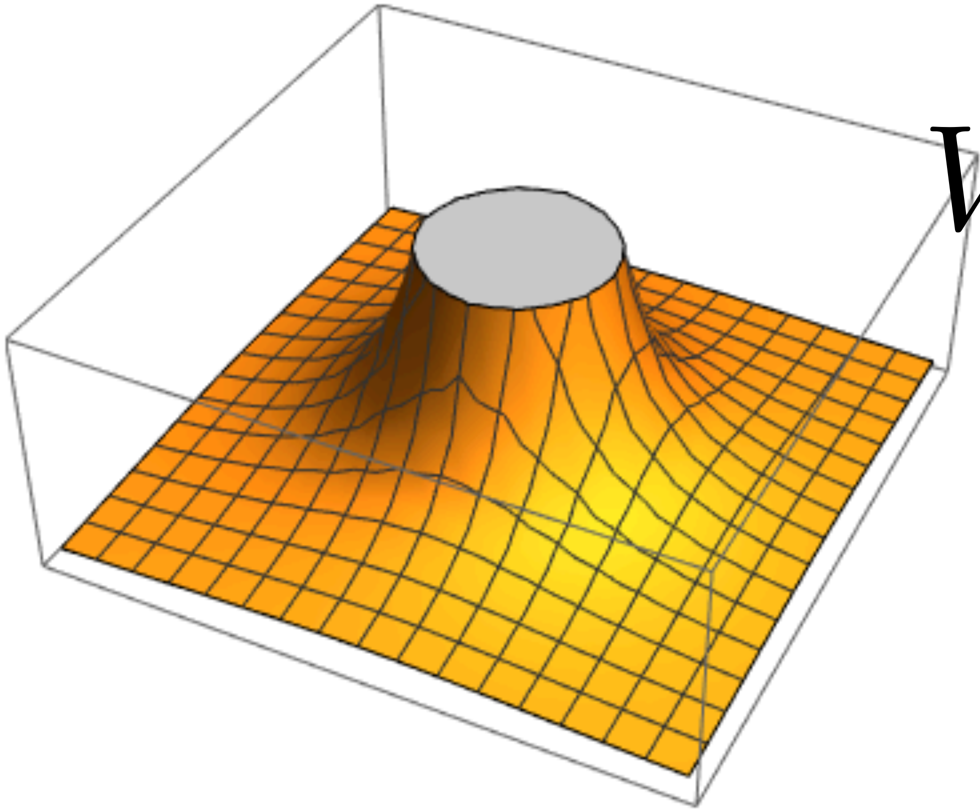
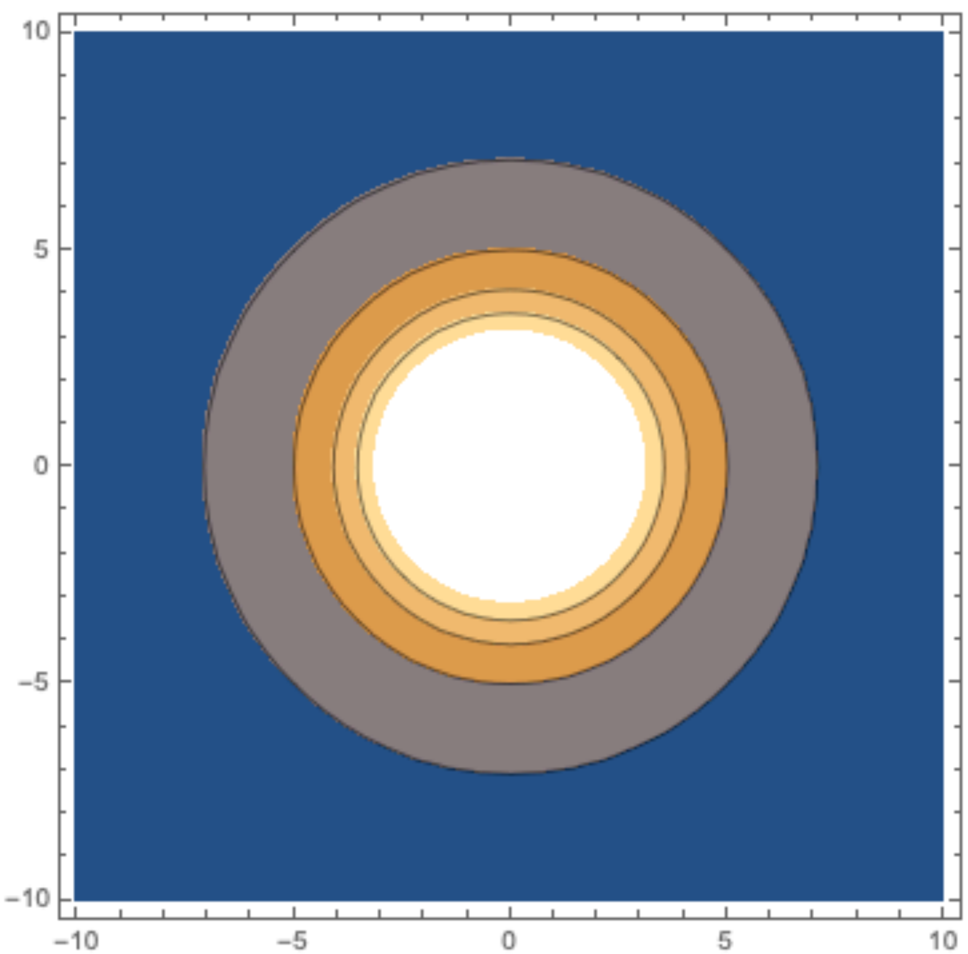


$$V = \frac{5}{x^2 + y^2}$$

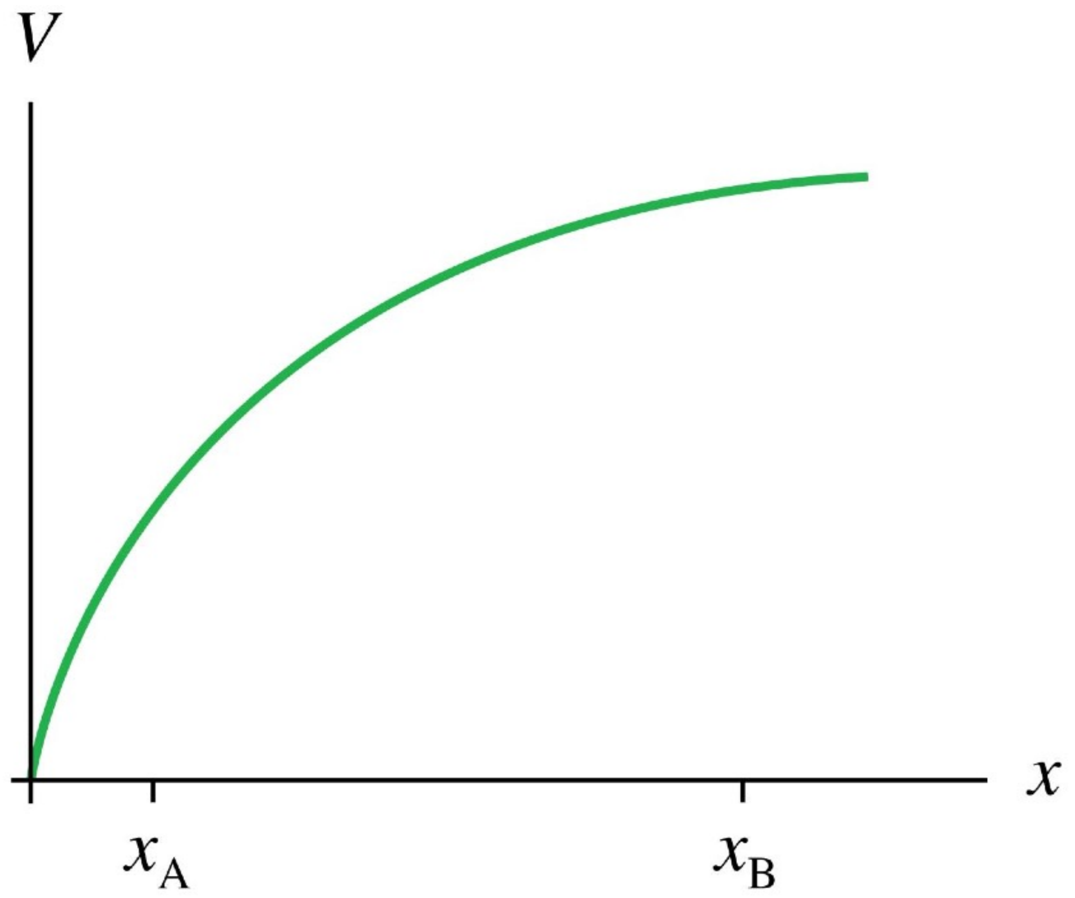



$$\vec{E} = \frac{-10}{(x^2 + y^2)^2}\left(x\hat{i} + y\hat{j}\right)$$

$$\boxed{\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)}$$

At which point is the electric field stronger?

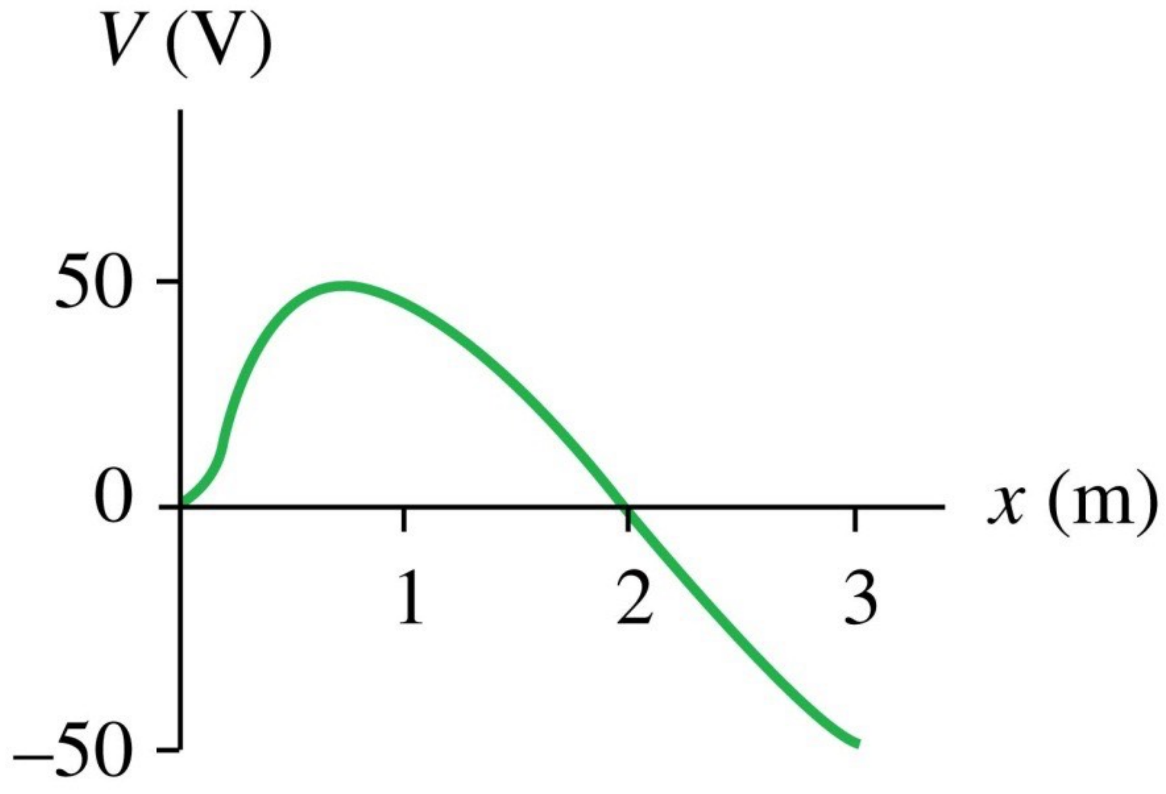
A. At $x_A$.
B. At $x_B$.
C. The field is the same strength at both.
D. There's not enough information to tell.



# Question #34

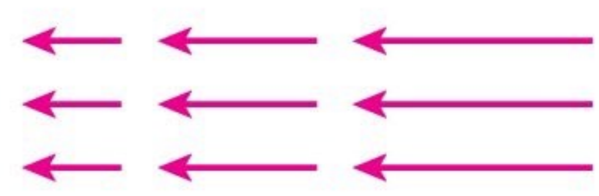
An electron is released from rest at $x = 2$ m in the potential shown. What does the electron do right after being released?

A. Stay at $x = 2$ m.
B. Move to the right ($+x$) at steady speed.
C. Move to the right with increasing speed.
D. Move to the left with increasing speed.
E. Move to the left ($-x$) at steady speed.



# Question #35

Which set of equipotential surfaces matches this electric field?



A


D


C


B


E


F

# Question #35

$$E = -\frac{dV}{dx}$$

Which set of equipotential surfaces matches this electric field?

A


D


C

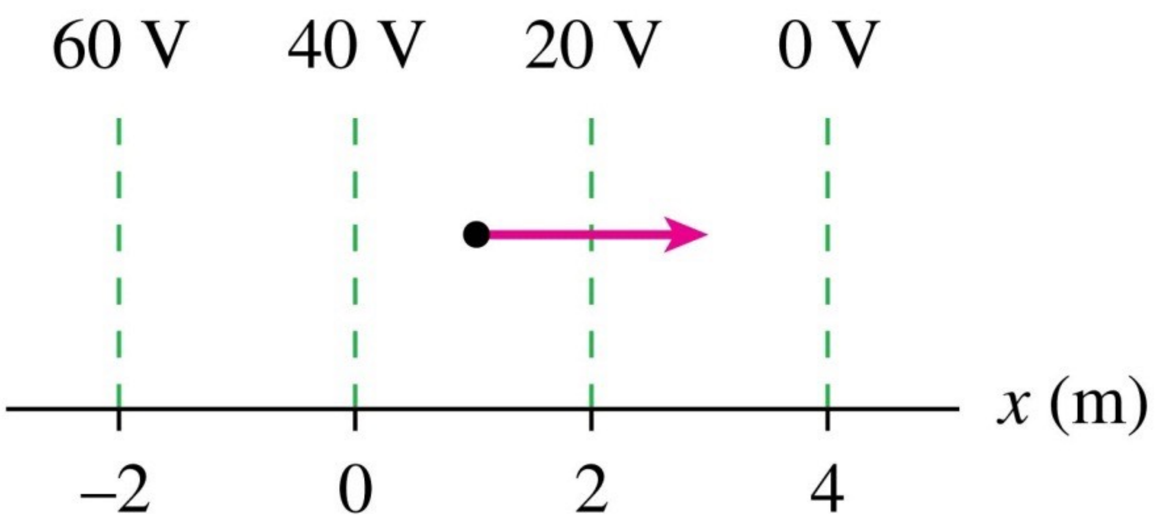

B


E


F

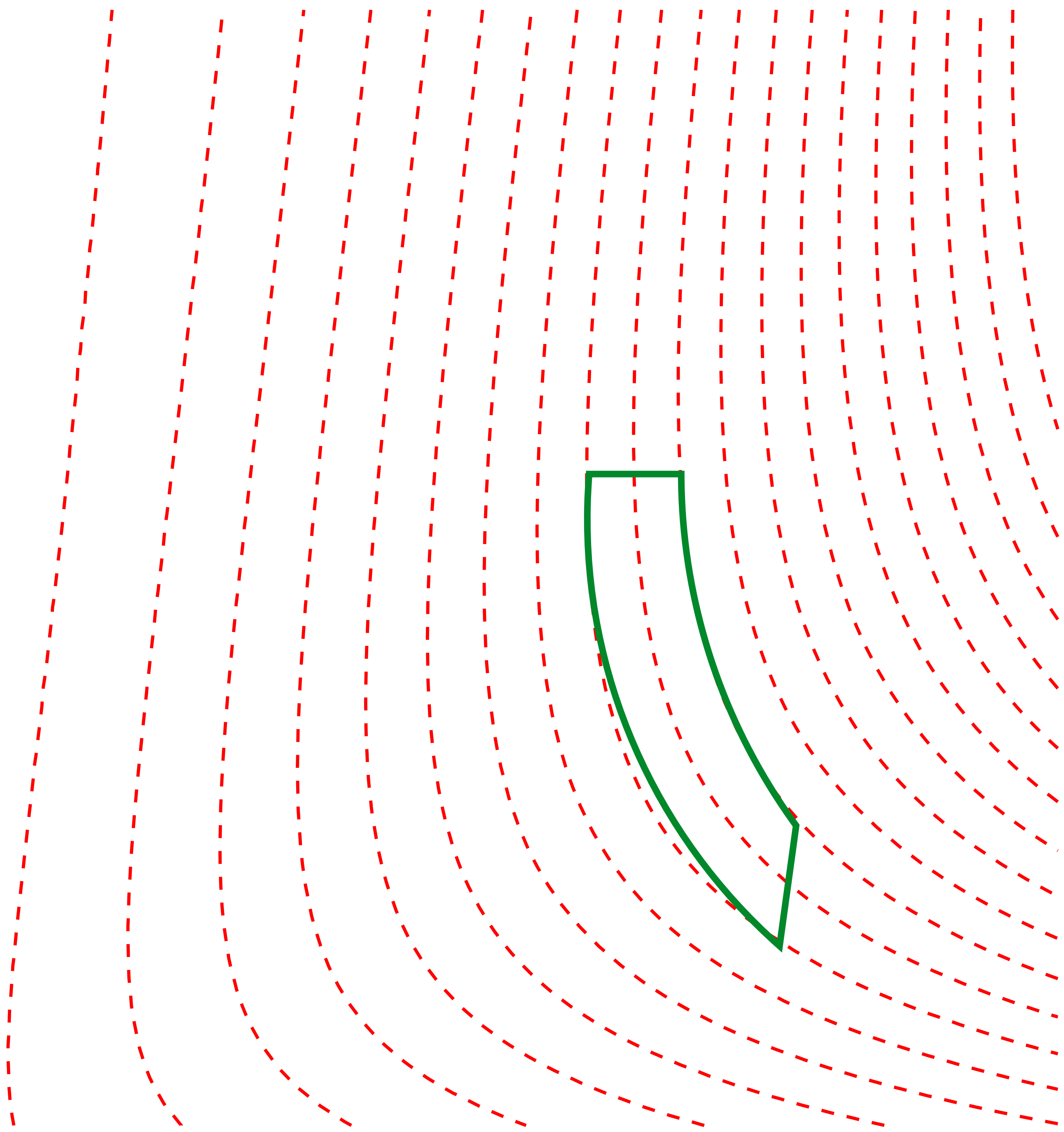
The electric field at the dot is

A. $20\hat{\imath}$ V/m.
B. $-10\hat{\imath}$ V/m.
C. $10\hat{\imath}$ V/m.
D. $30\hat{\imath}$ V/m.
E. $-30\hat{\imath}$ V/m.



# Kirchoff's Loop

What is the potential difference for this closed path?



# Kirchoff's Loop

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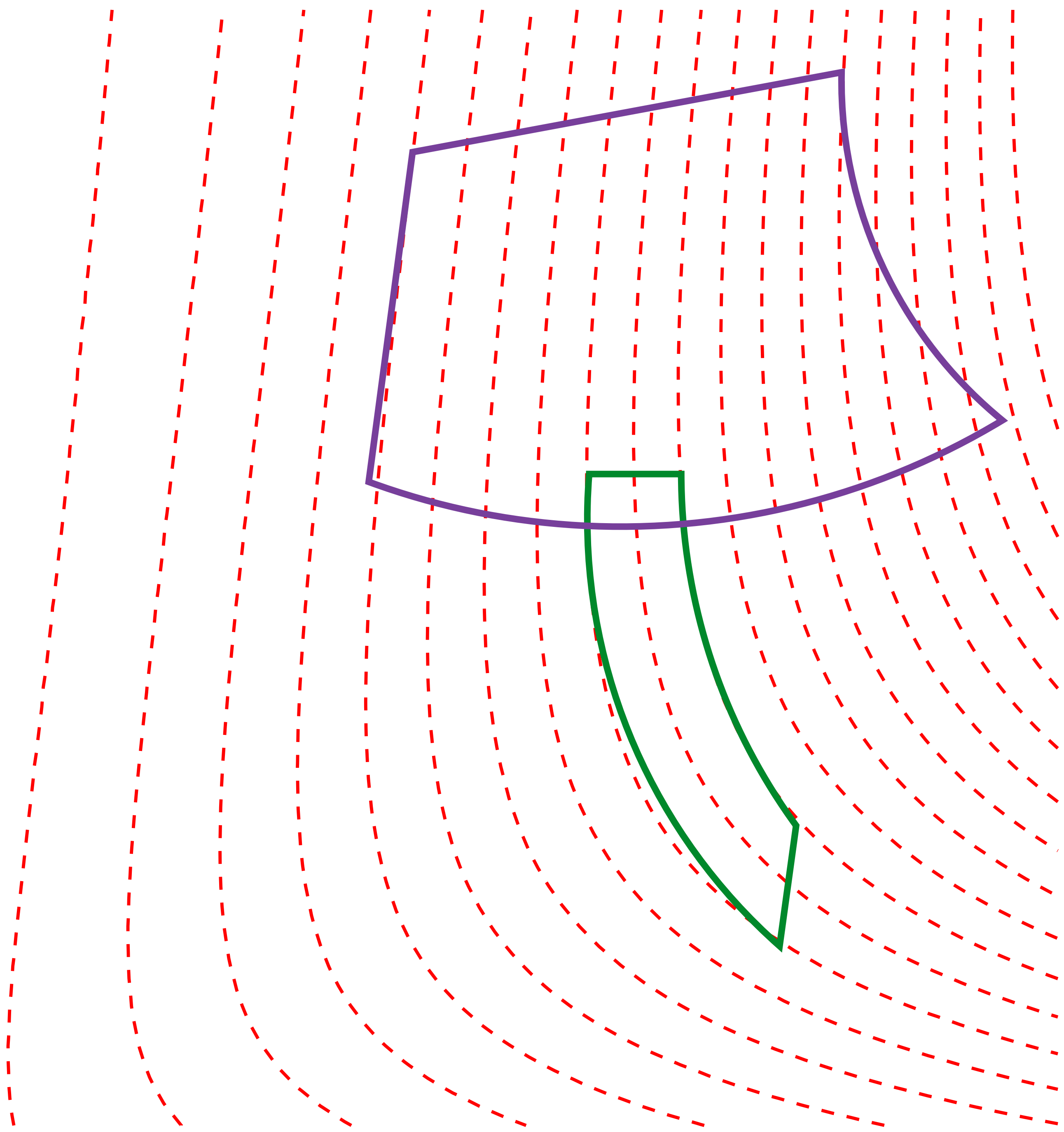
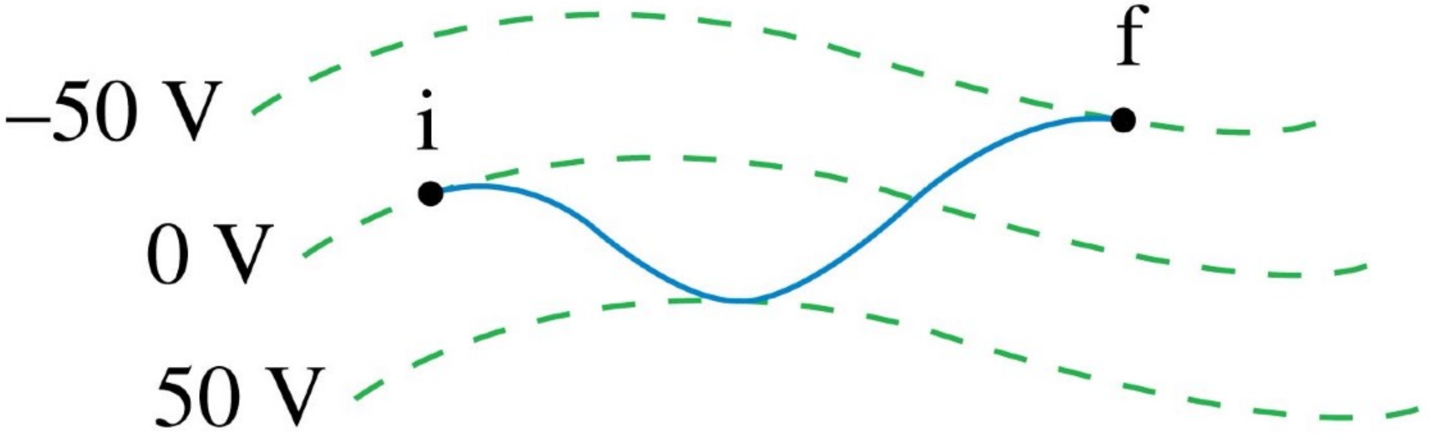
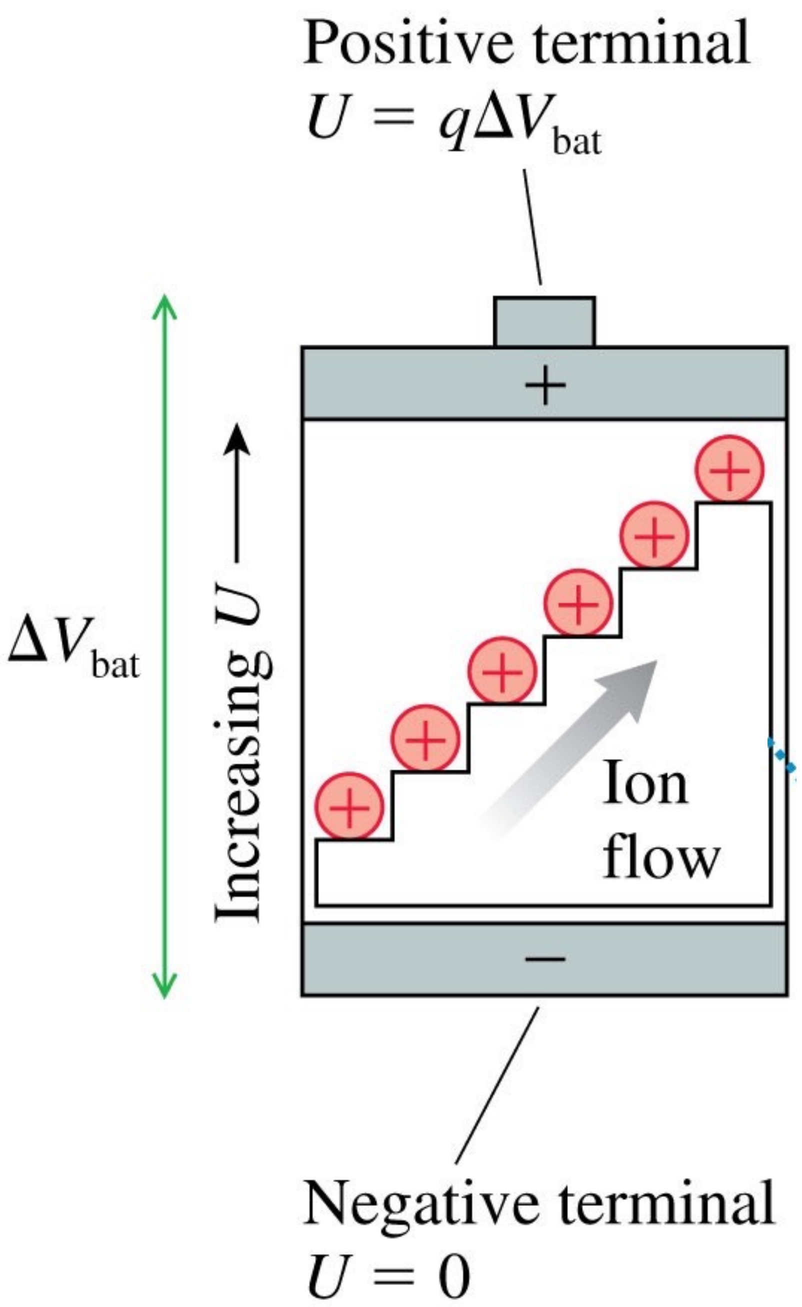


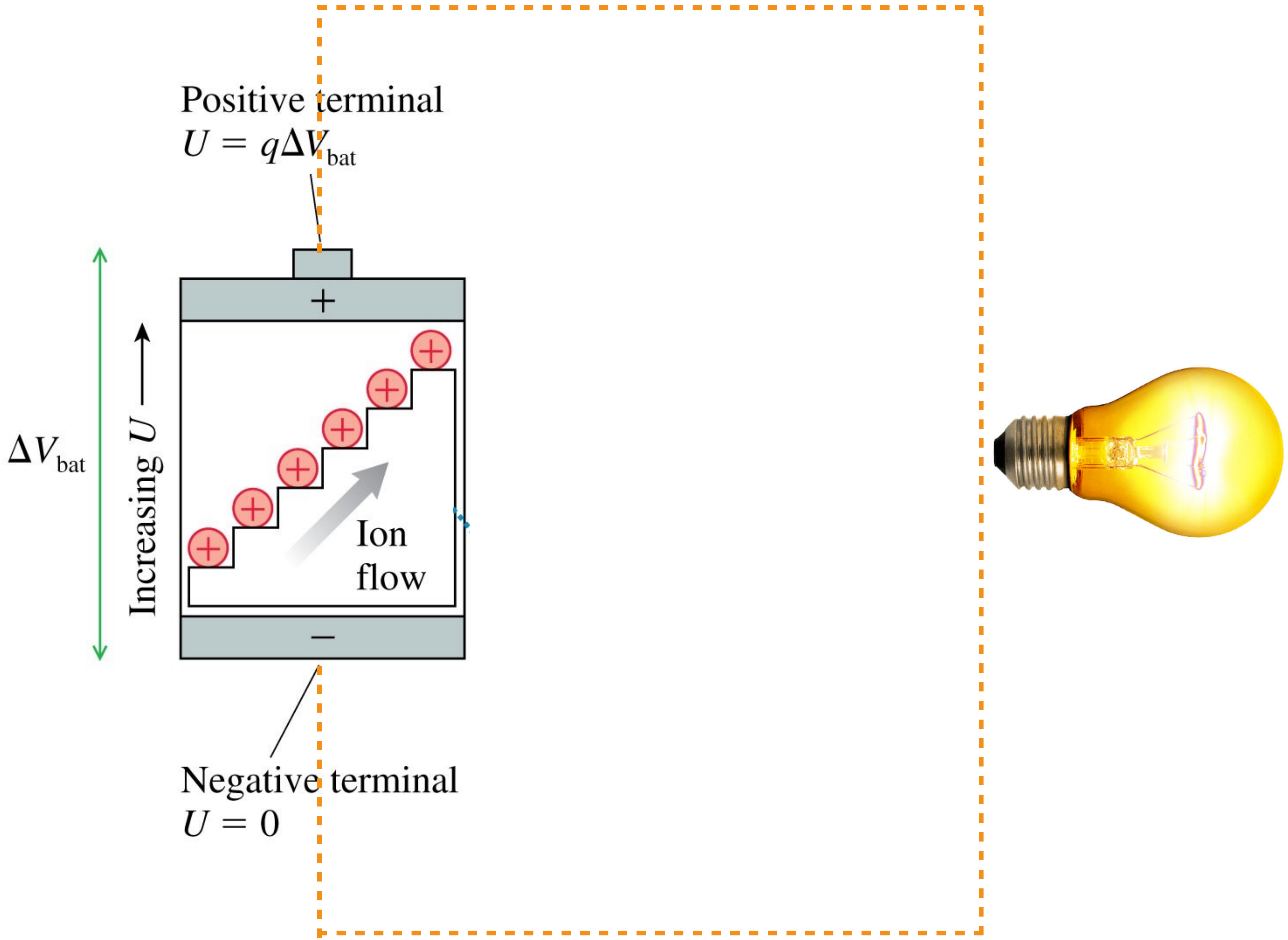
A particle follows the trajectory shown from initial position i to final position f. The potential difference $\Delta V$ *is*

A. 100 V.
B. 50 V.
C. 0 V.
D. –50 V.
E. –100 V.





Positive terminal
$U = q\Delta V_{bat}$
$\Delta V_{bat}$
Increasing $U$
+
Ion flow
−
Negative terminal
$U = 0$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