



PH 220

Lance Nelson

Answer the following questions with your neighbor.

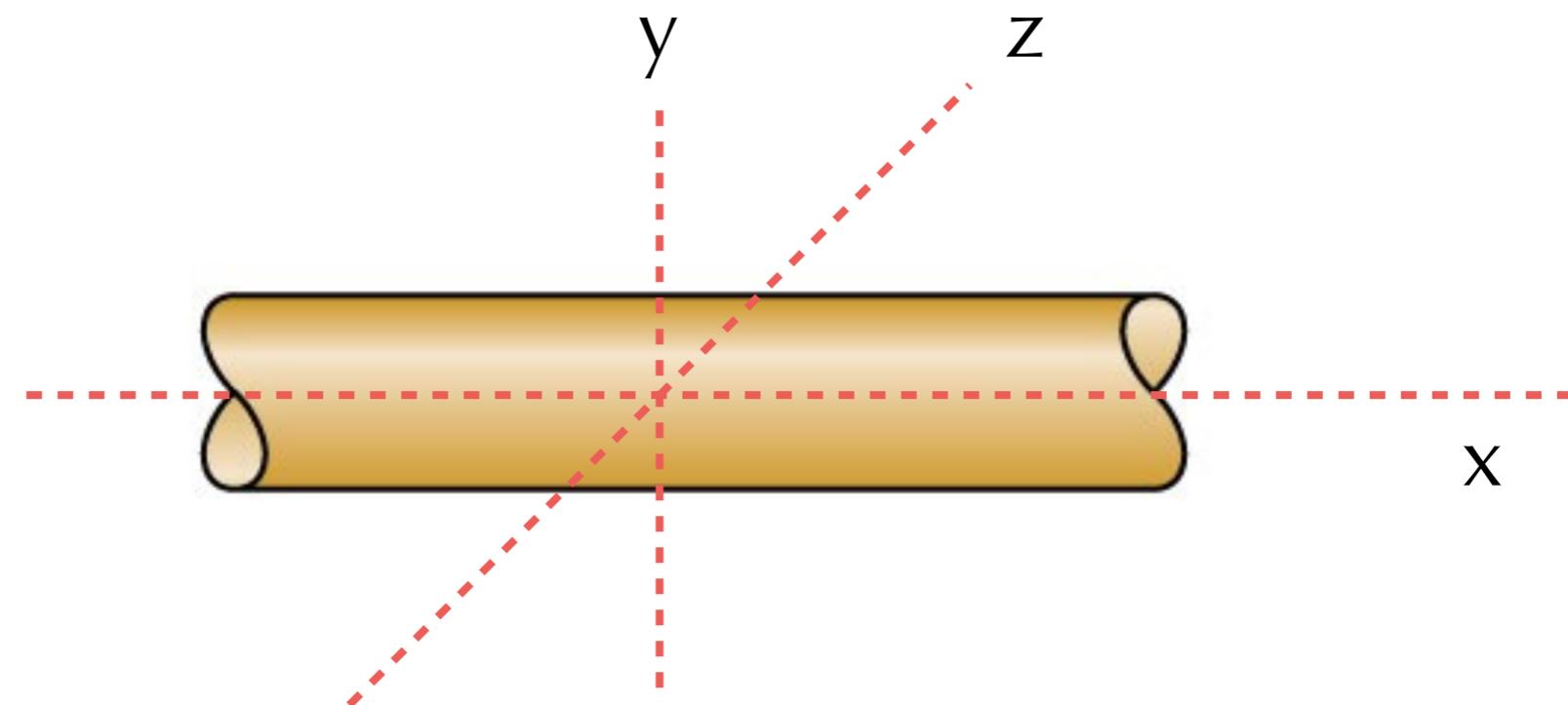
What is the electric field?

How do you calculate the electric field for
point charges?
continuous charge distributions?

How do you calculate the force on an object
placed in an electric field?

Symmetry

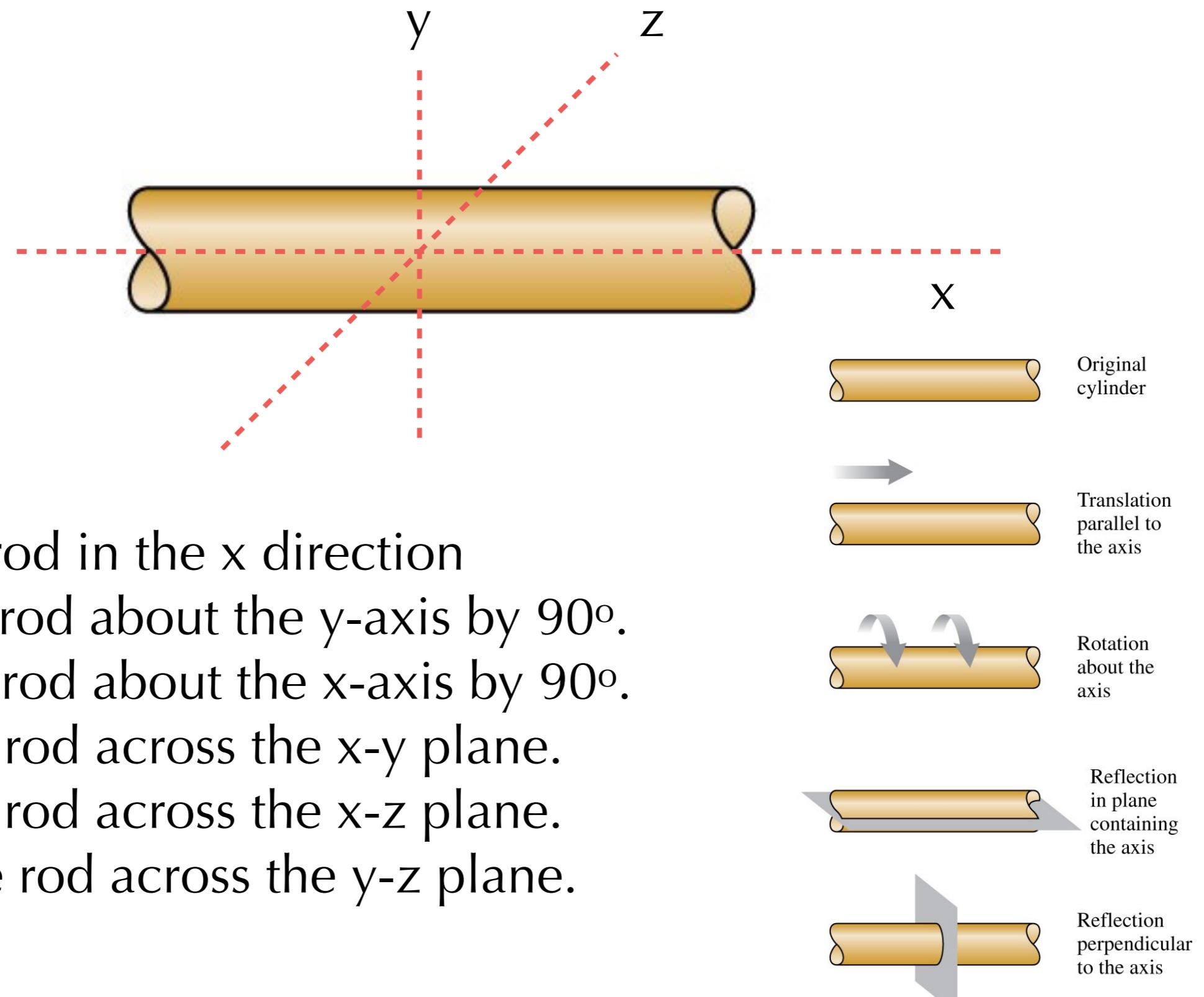
What transformation **does not** leave the rod unchanged?



- A: Move the rod in the x direction
- B: Rotate the rod about the y-axis by 90° .
- C: Rotate the rod about the x-axis by 90° .
- E: Reflect the rod across the x-y plane.
- F: Reflect the rod across the x-z plane.
- G: Reflect the rod across the y-z plane.

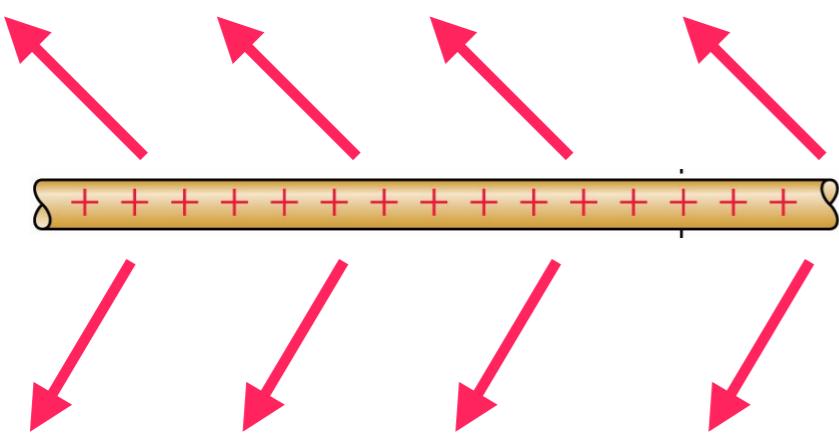
Symmetry

What transformation does not leave the rod unchanged?

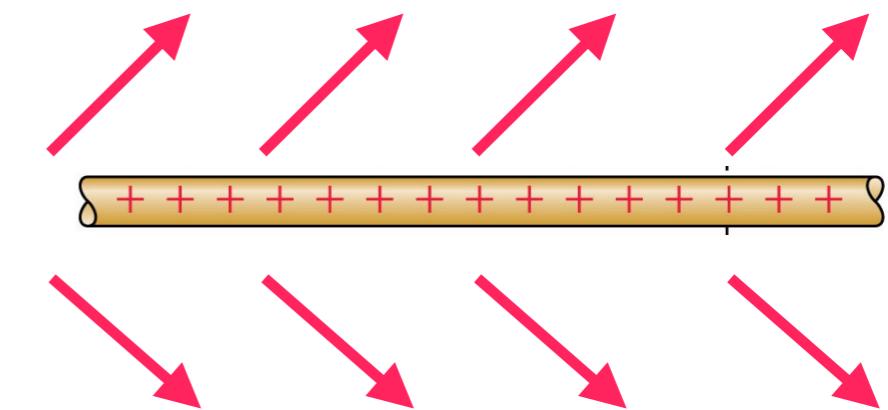


Based on the symmetry of the rod only, which electric field is possible.

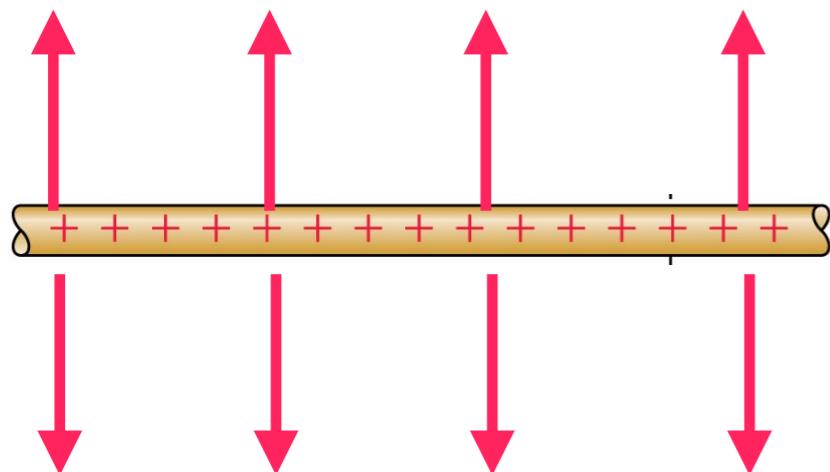
A



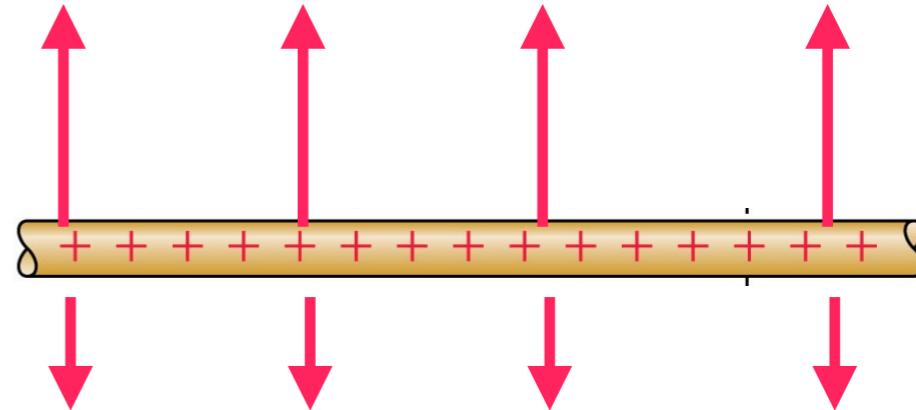
B



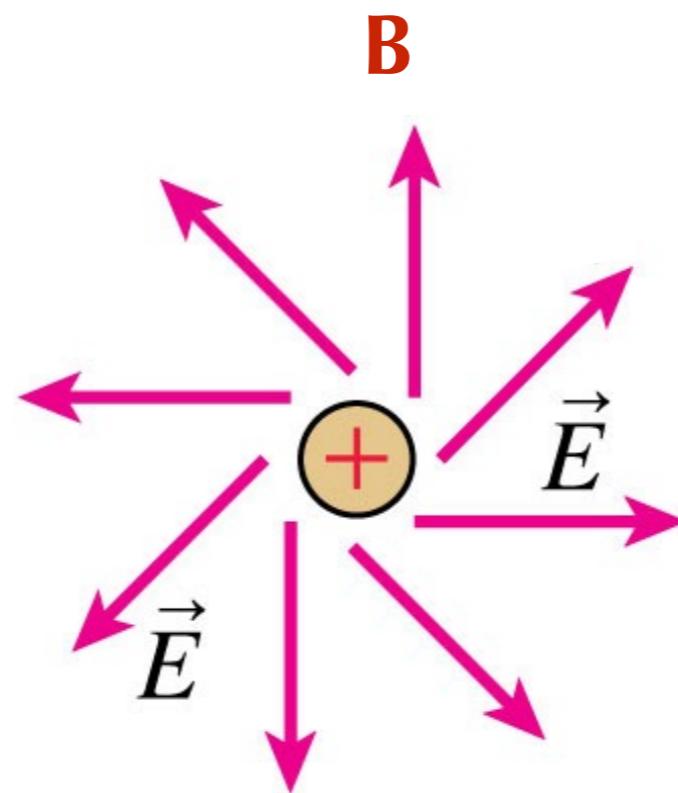
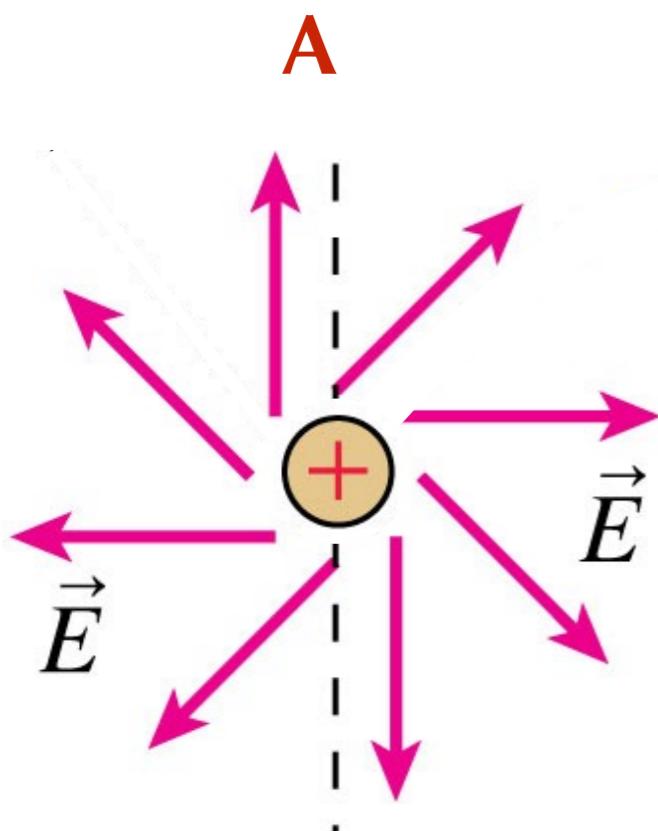
C



D



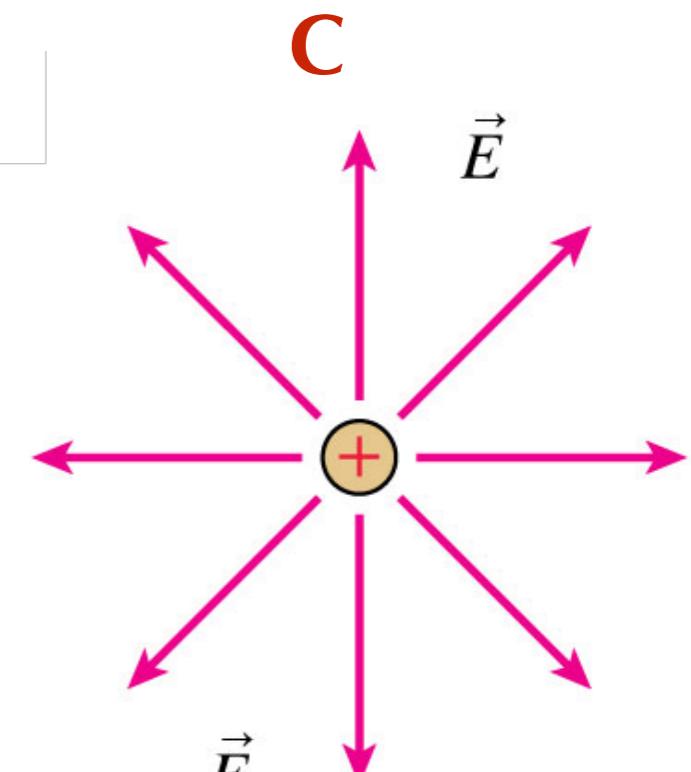
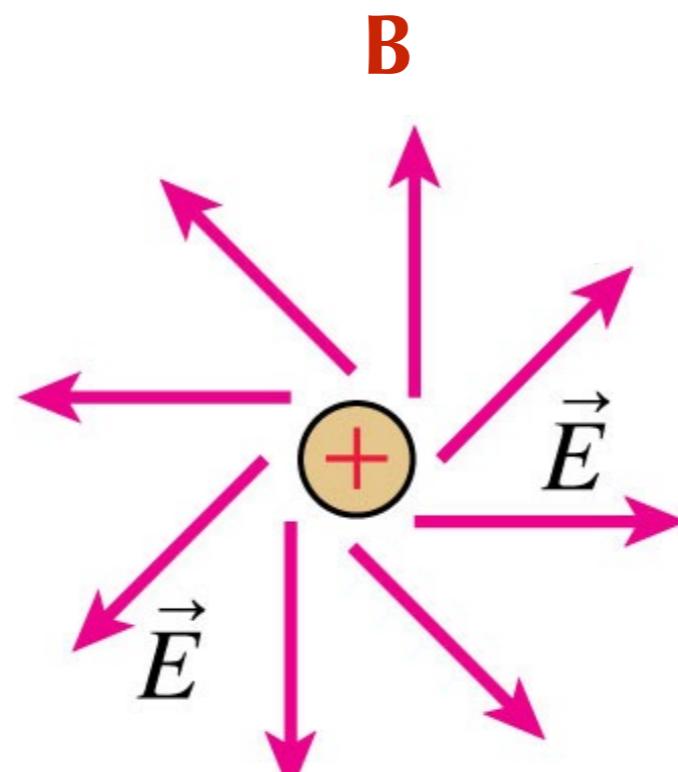
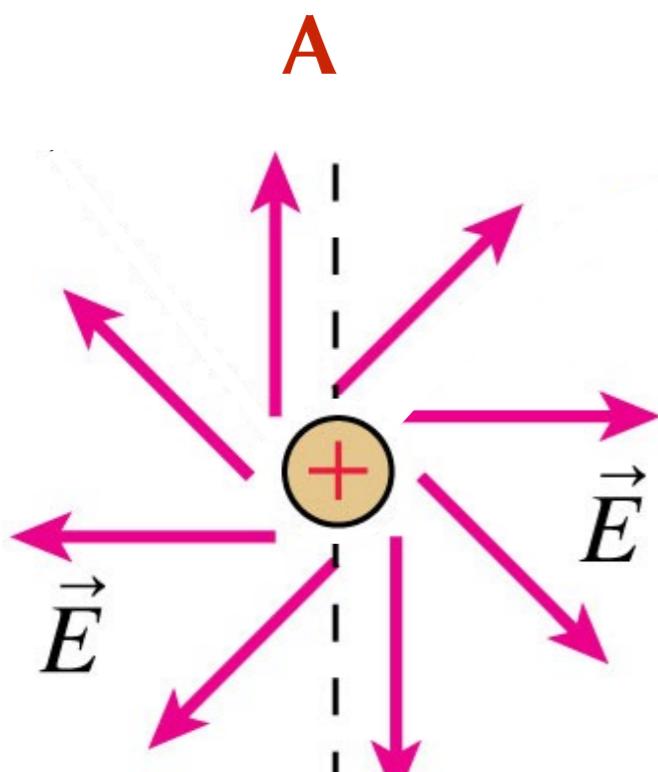
An end view of a very long rod is shown below. Which electric field(s) are not possible?



C

- D: A and D
E: A and E

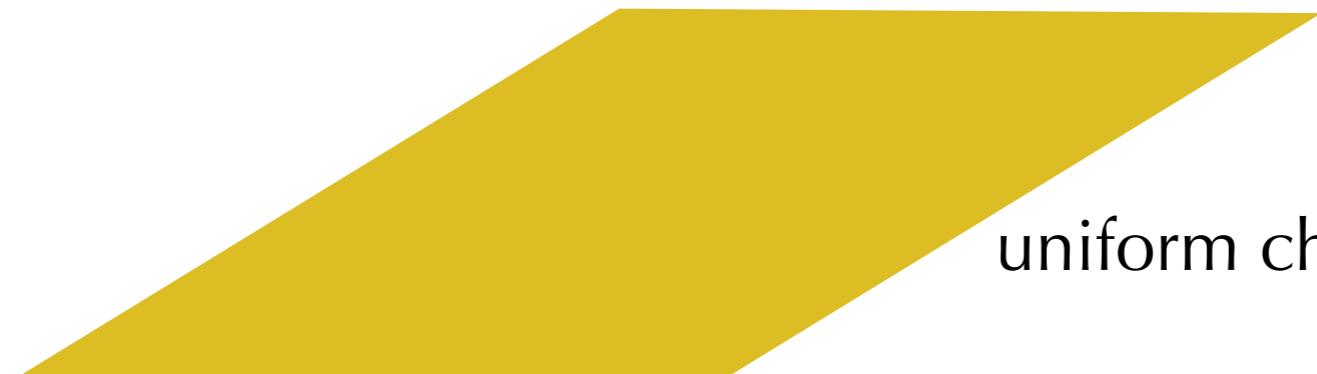
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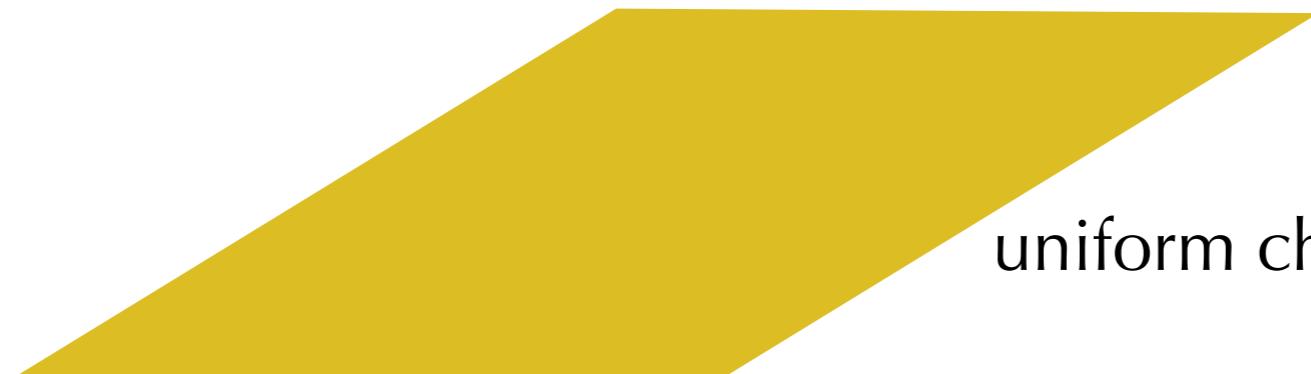
E: A and E

Based on symmetry, what must the E field look like?



uniform charge distribution

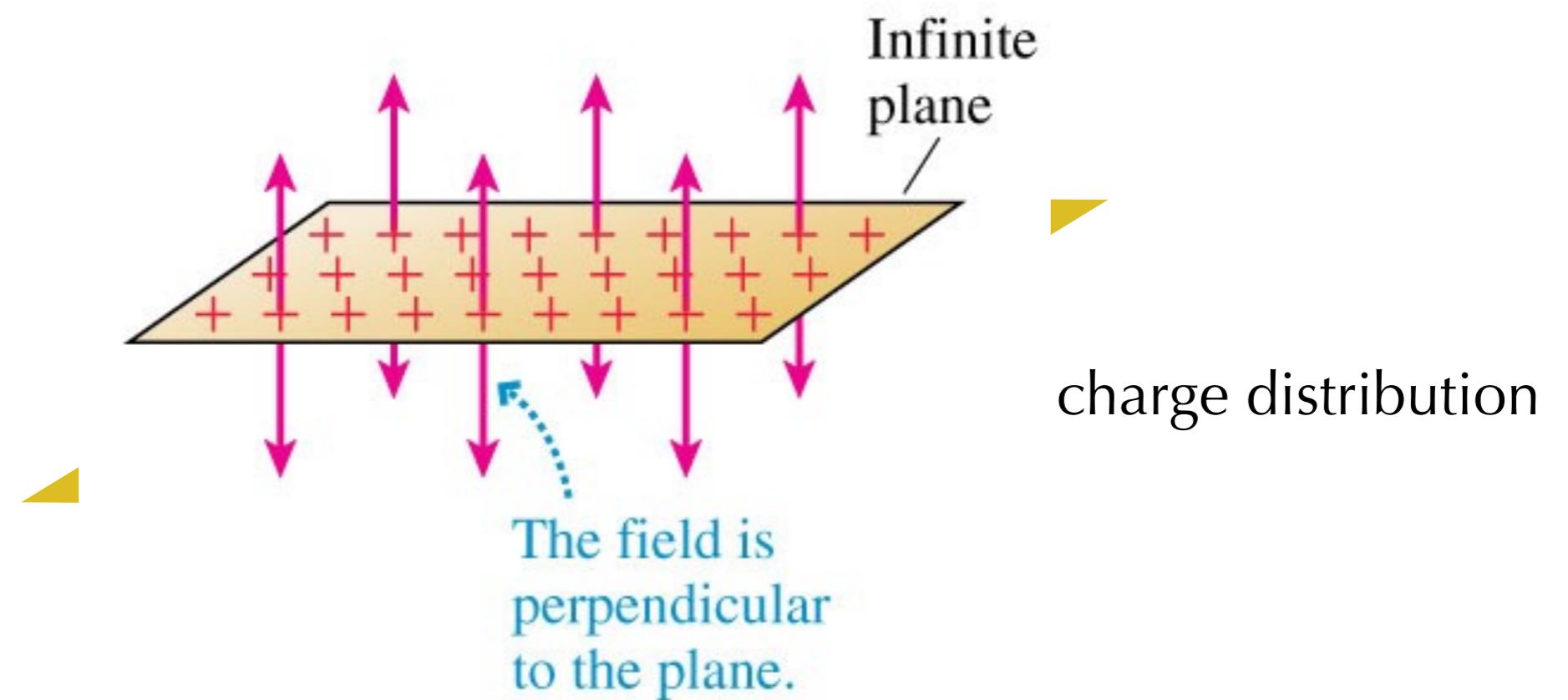
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uniform charge distribution

- Translation parallel to the plane.
- Rotation about any line perpendicular to the plane.
- Reflection in the plane.

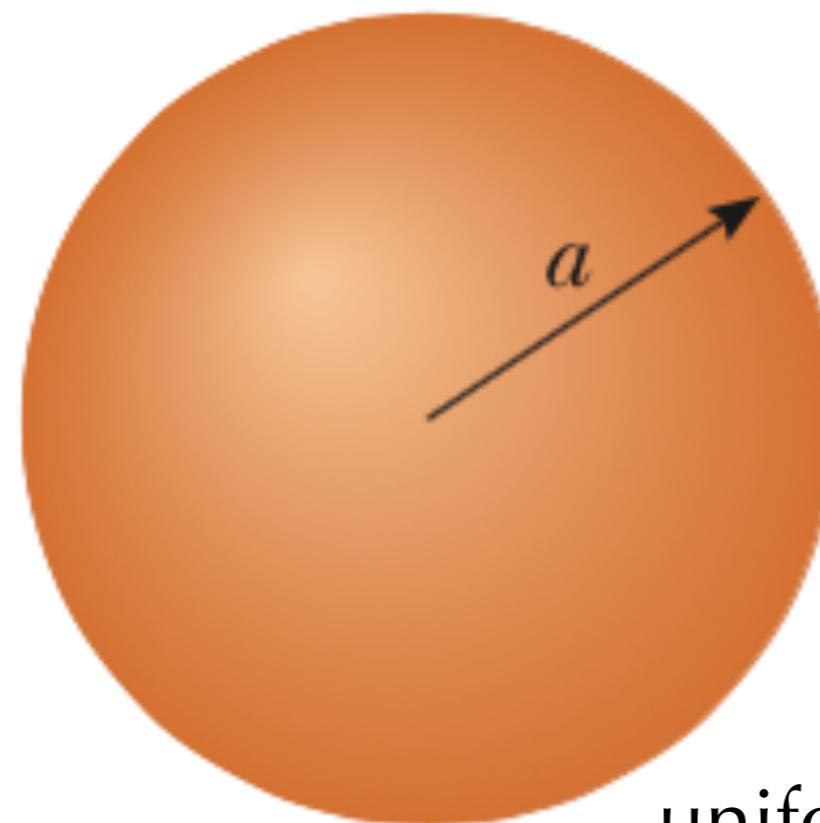
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Sphere

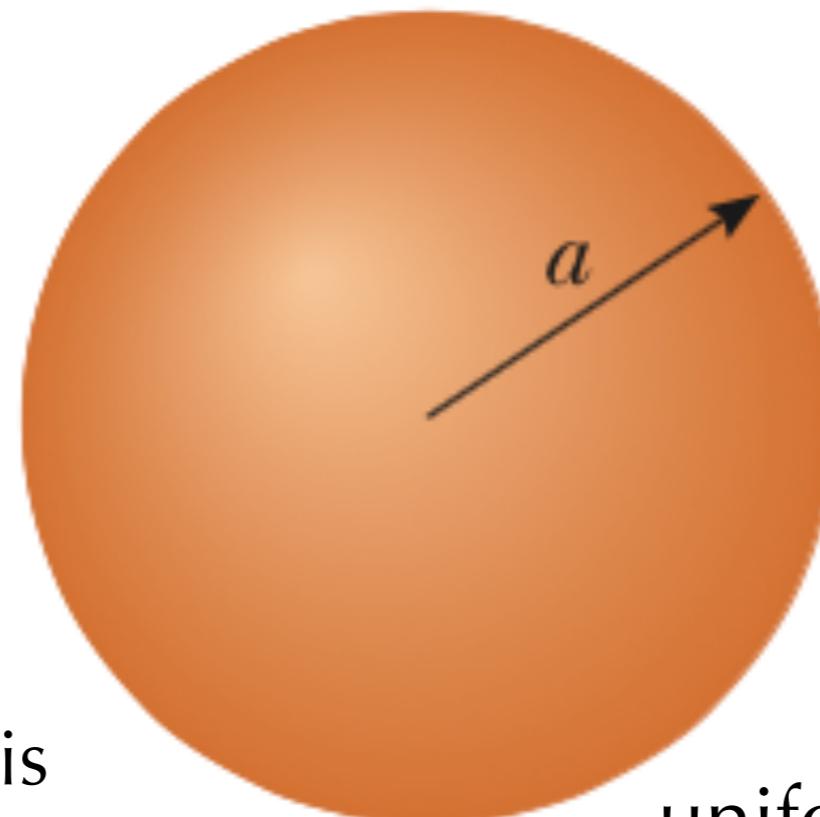
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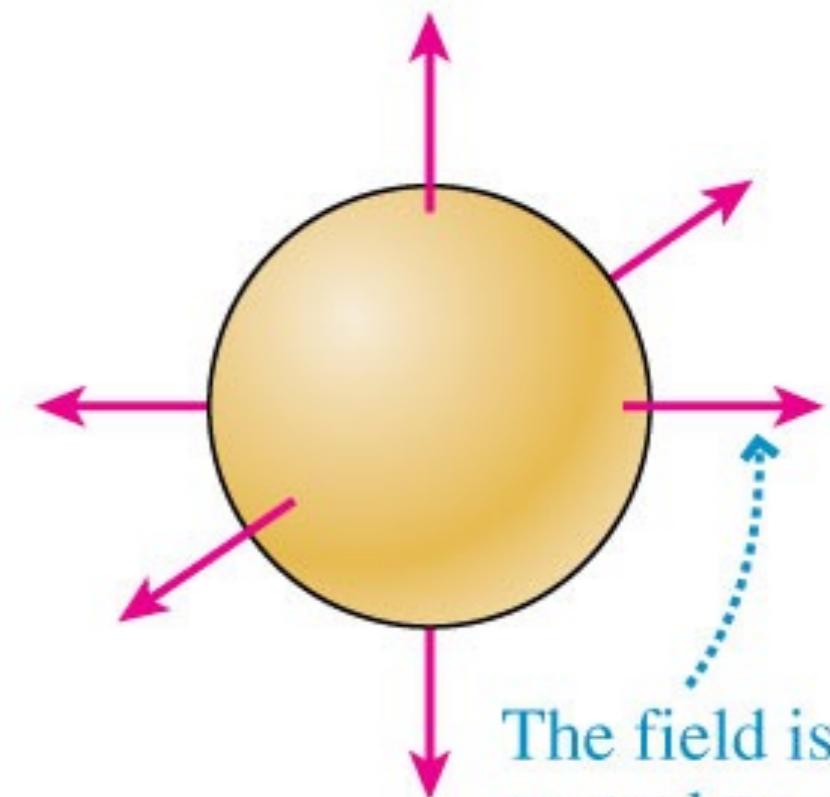
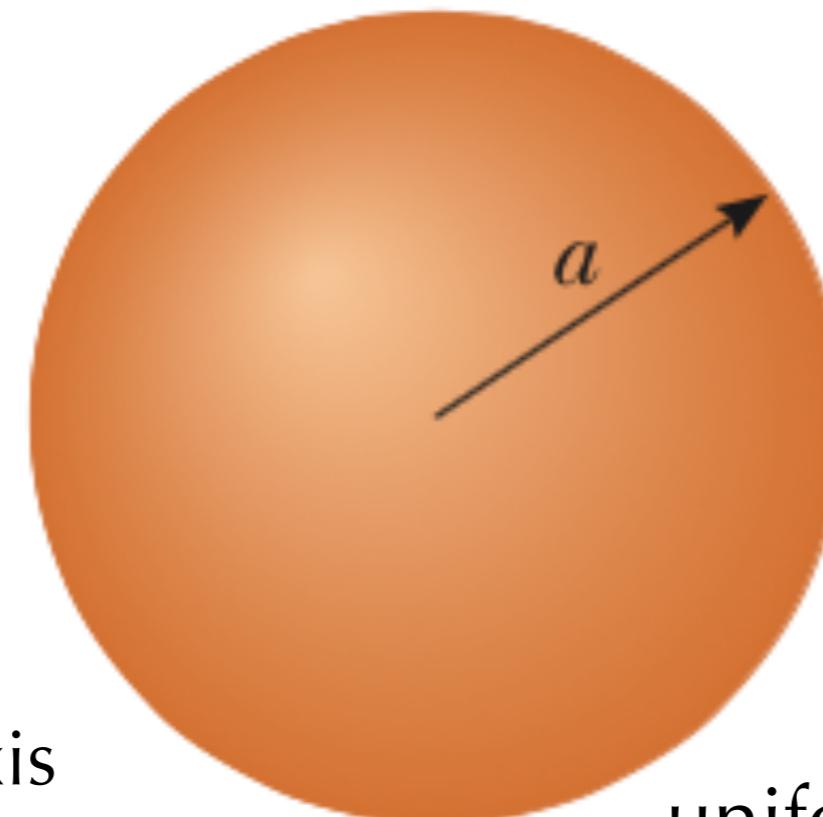


- Rotation about any axis which passes through the center point.
- Reflection in any plane containing the center point.

uniform charge distribution

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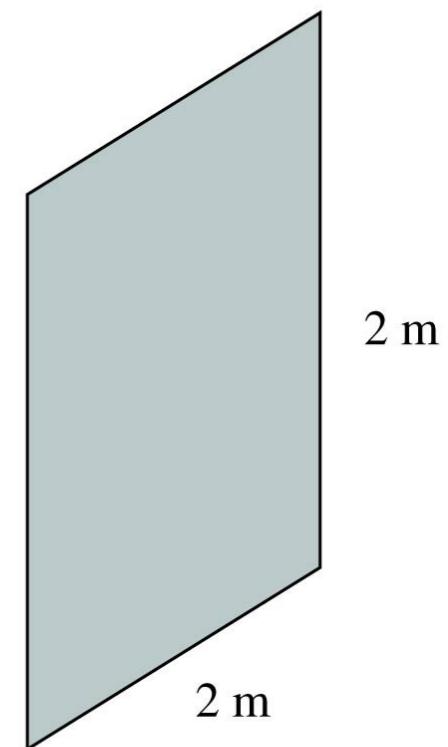
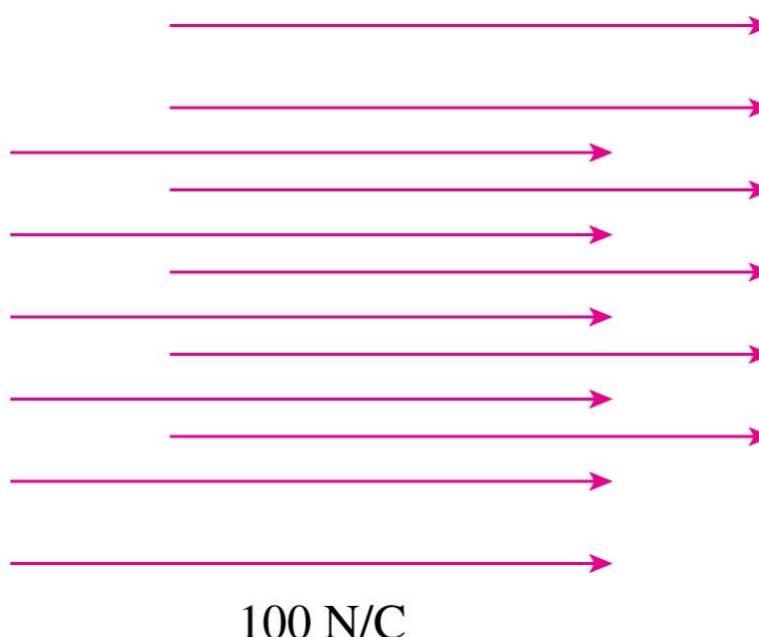
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from the cent

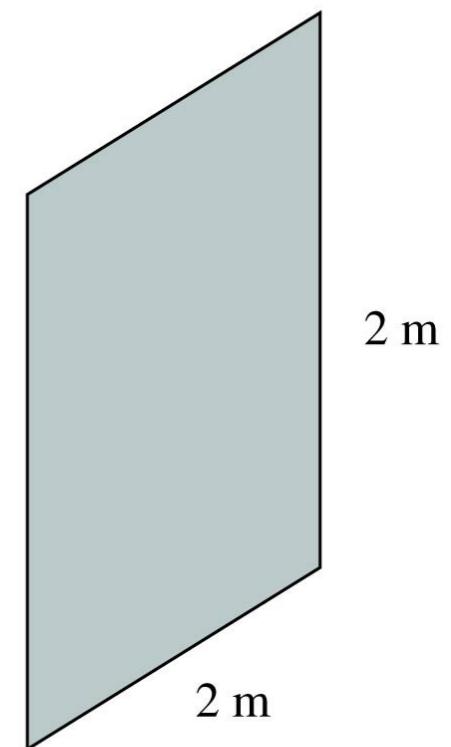
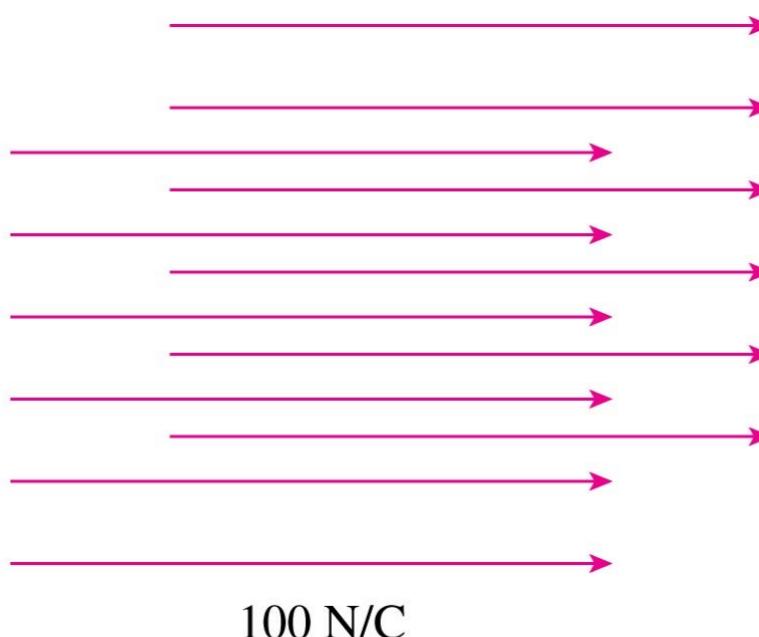
The electric flux through the shaded surface is

- A. 0.
- B. 200 N m/C .
- C. $400 \text{ N m}^2/\text{C}$.
- D. Flux isn't defined for an open surface.



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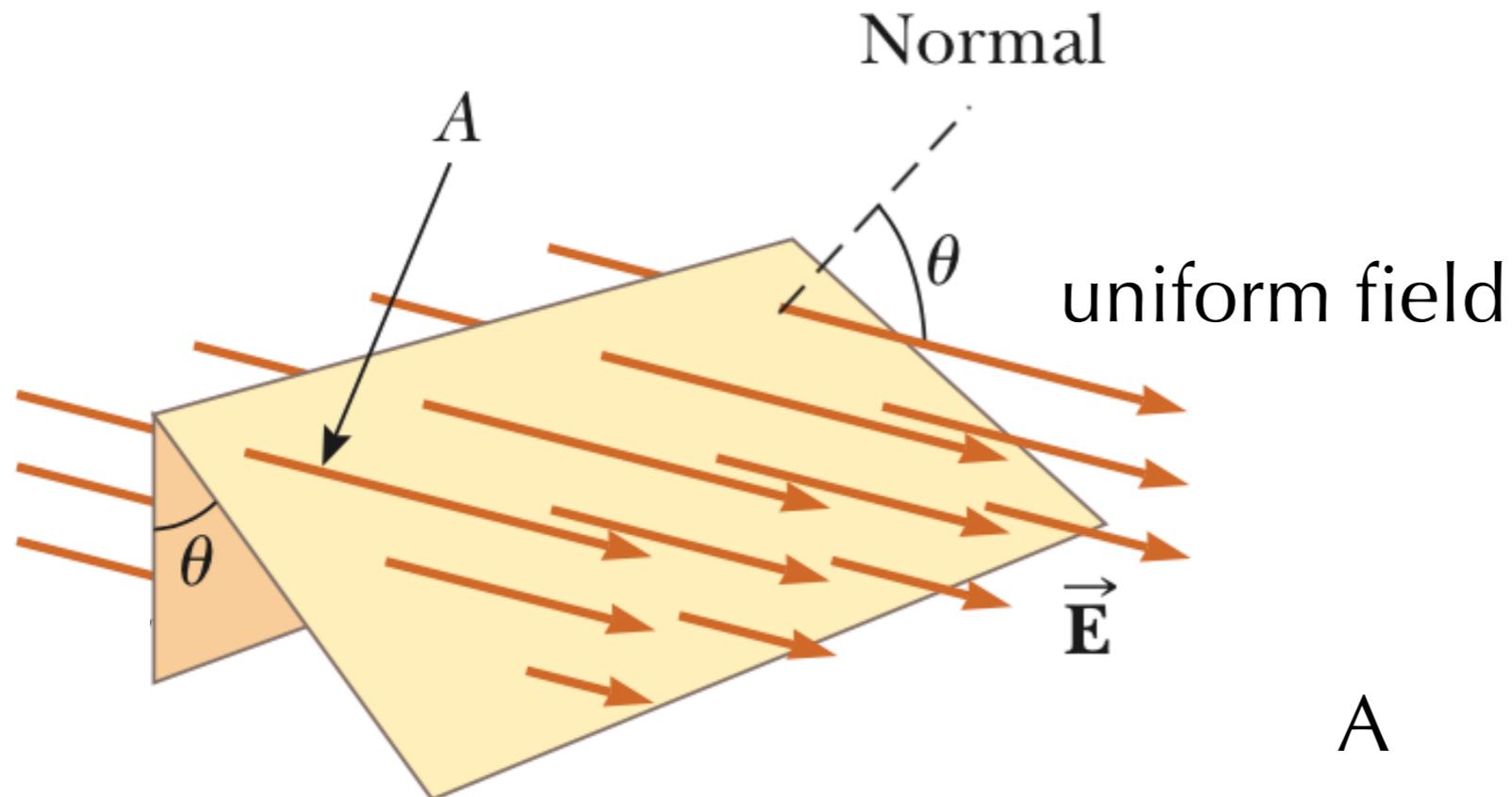
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$$\Phi = E \cdot A$$

Electric Flux

What is the flux through surface A?



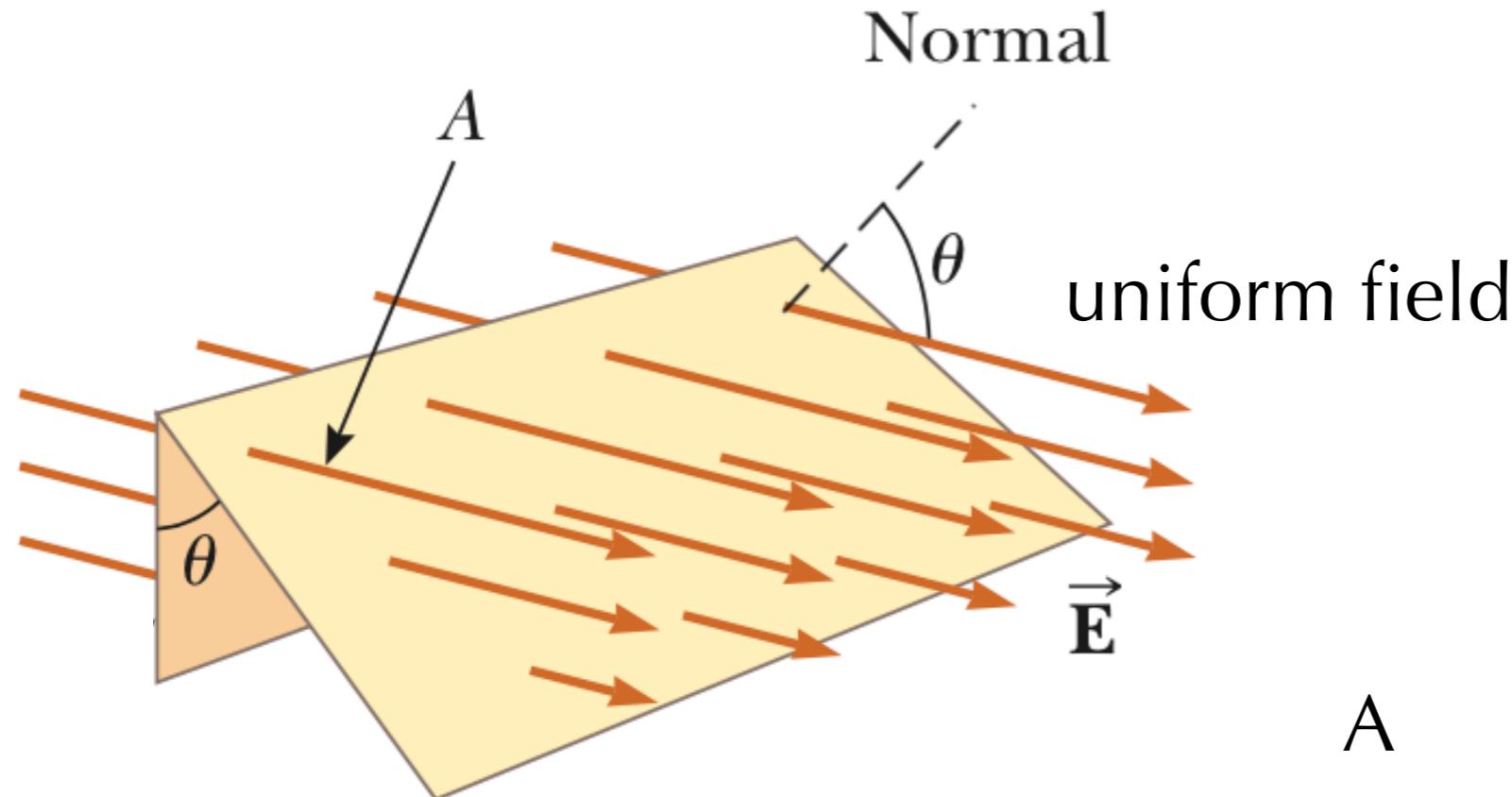
A $\Phi = E \cdot A \cdot \cos(\theta)$

B $\Phi = E \cdot A$

C $\Phi = E \cdot A \cdot \sin(\theta)$

Electric Flux

What is the flux through surface A?



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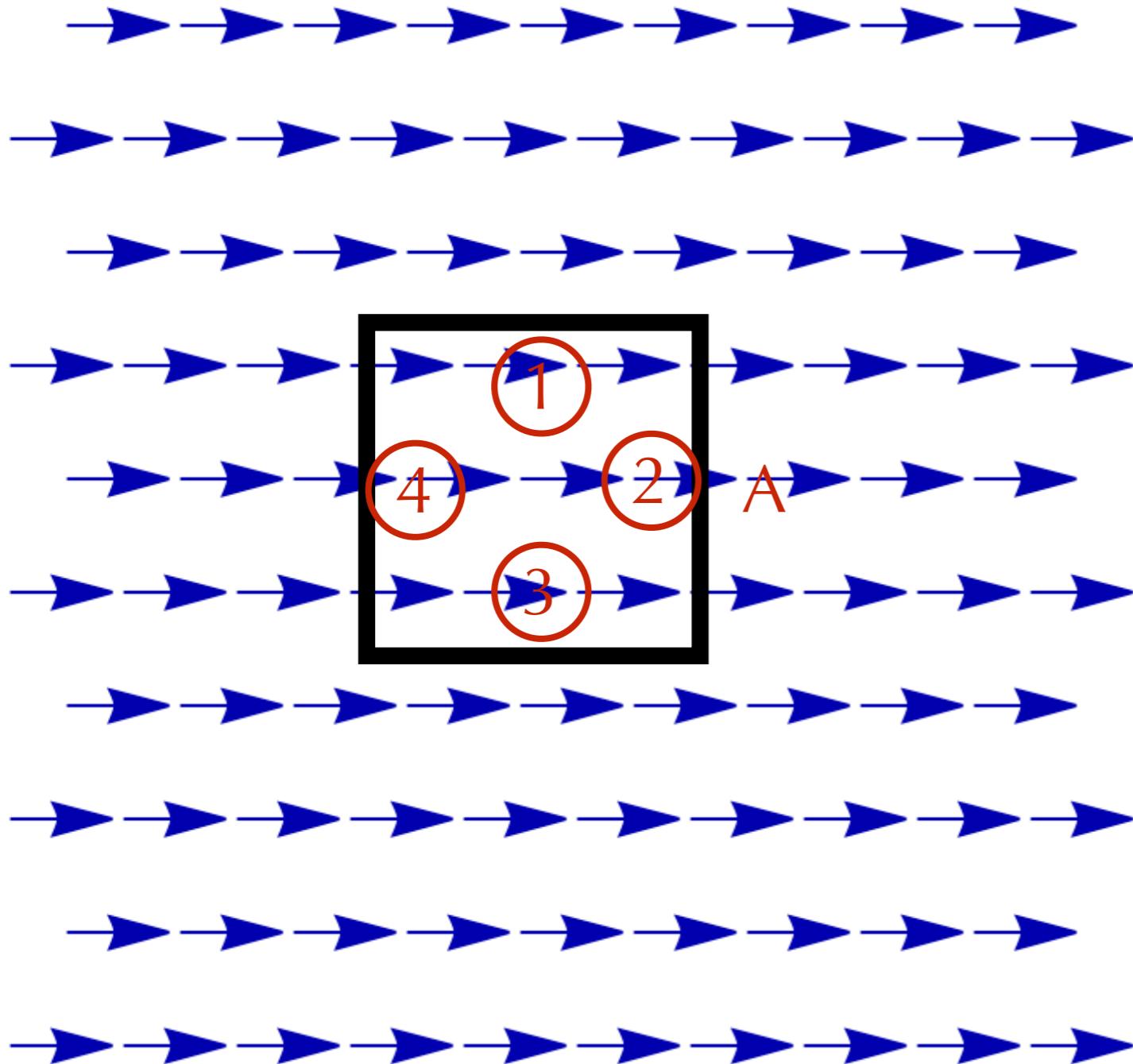
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$$\boxed{\Phi = \vec{E} \cdot \vec{A}}$$

2D Flux

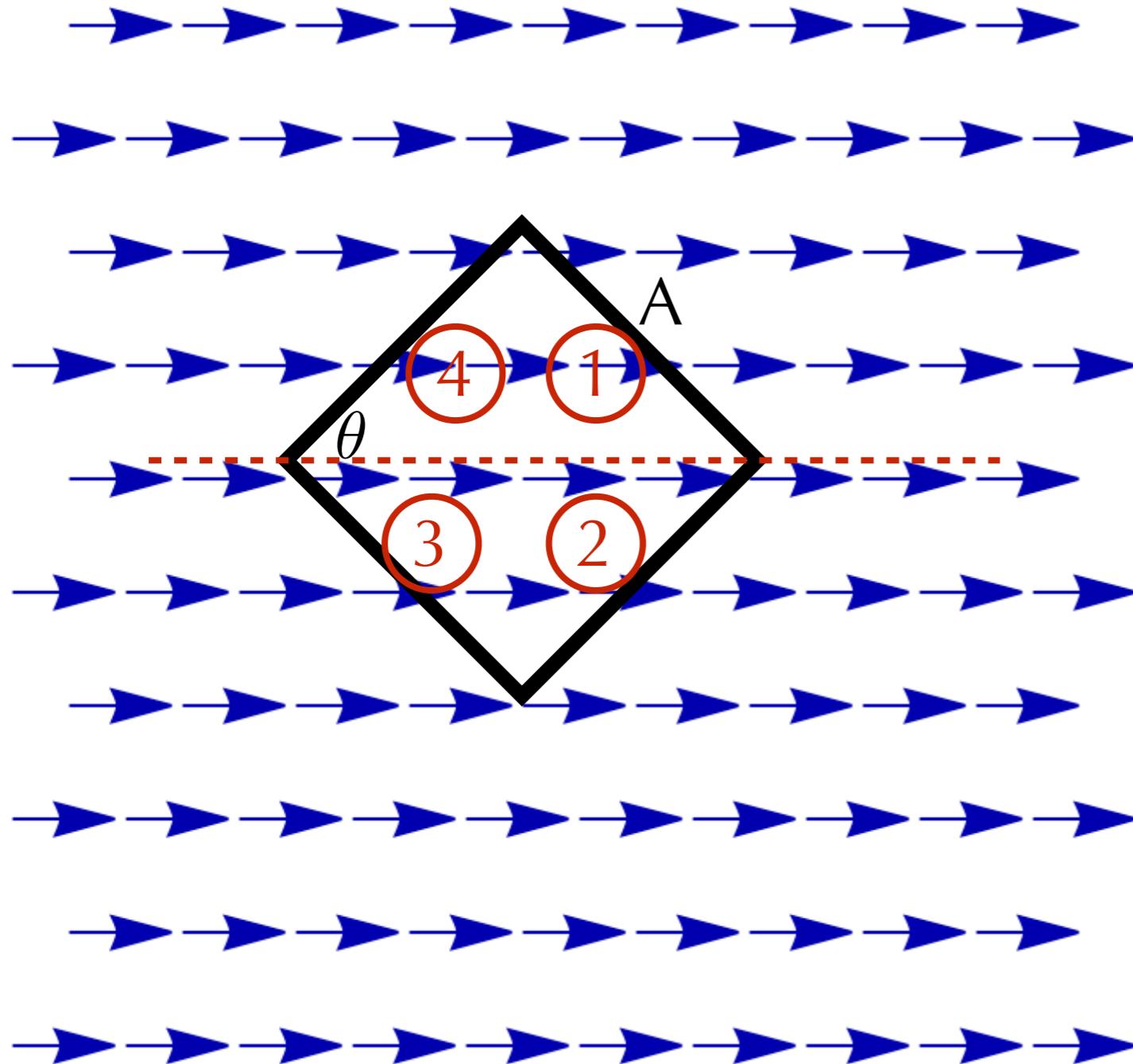
Which “surface(s)” have zero flux through them?



- a) 1 and 4
- b) 2 and 3
- c) 4
- d) 2 and 4
- e) 1 and 3

2D Flux

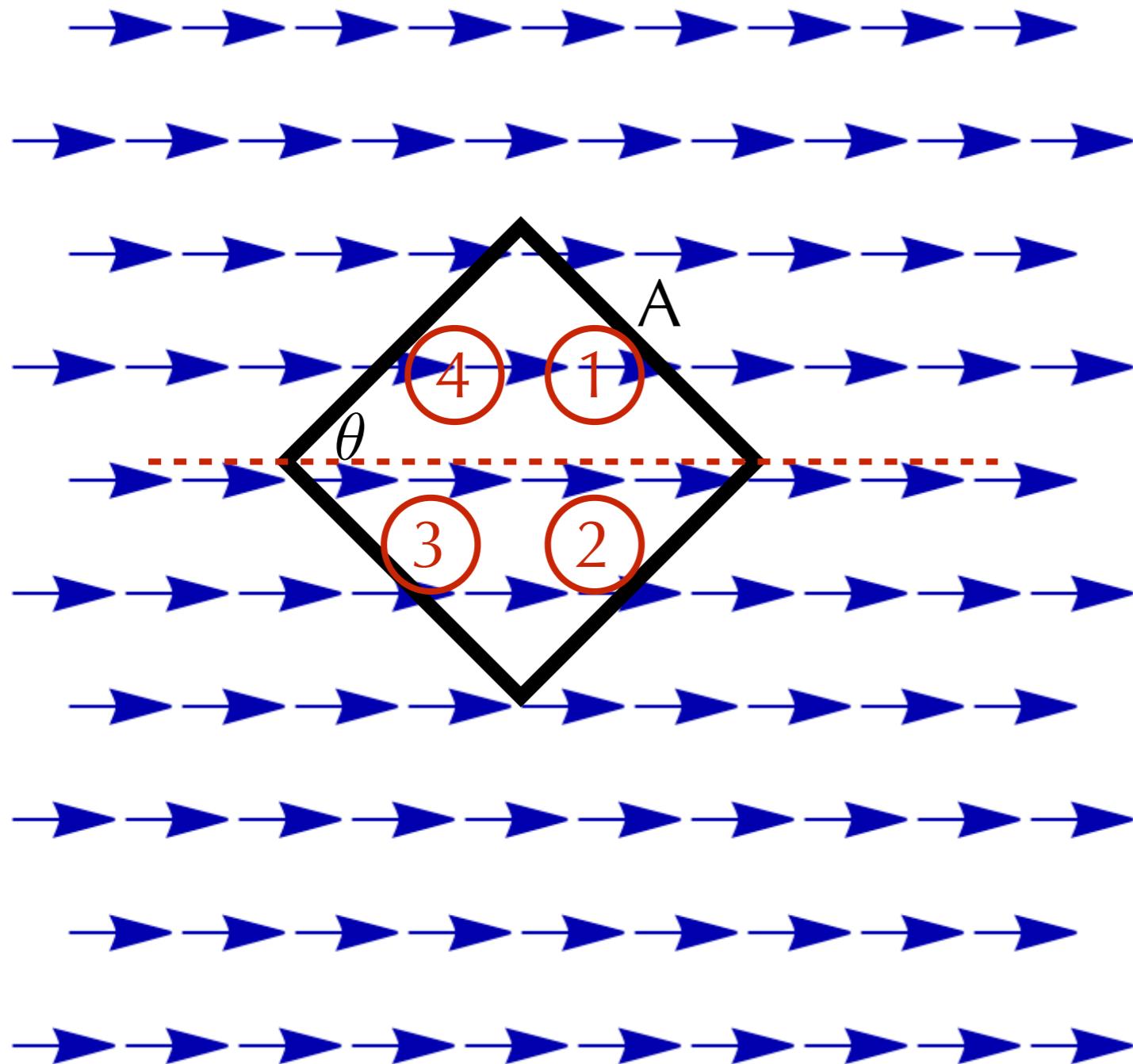
Which “surface(s)” have zero flux through them?



- a) 1 and 4
- b) 2 and 3
- c) 1 and 3
- d) 2 and 4
- e) none

2D Flux

How would you calculate the flux through surface 4?



A $E \cos \theta$

B $-Ea \sin \theta$

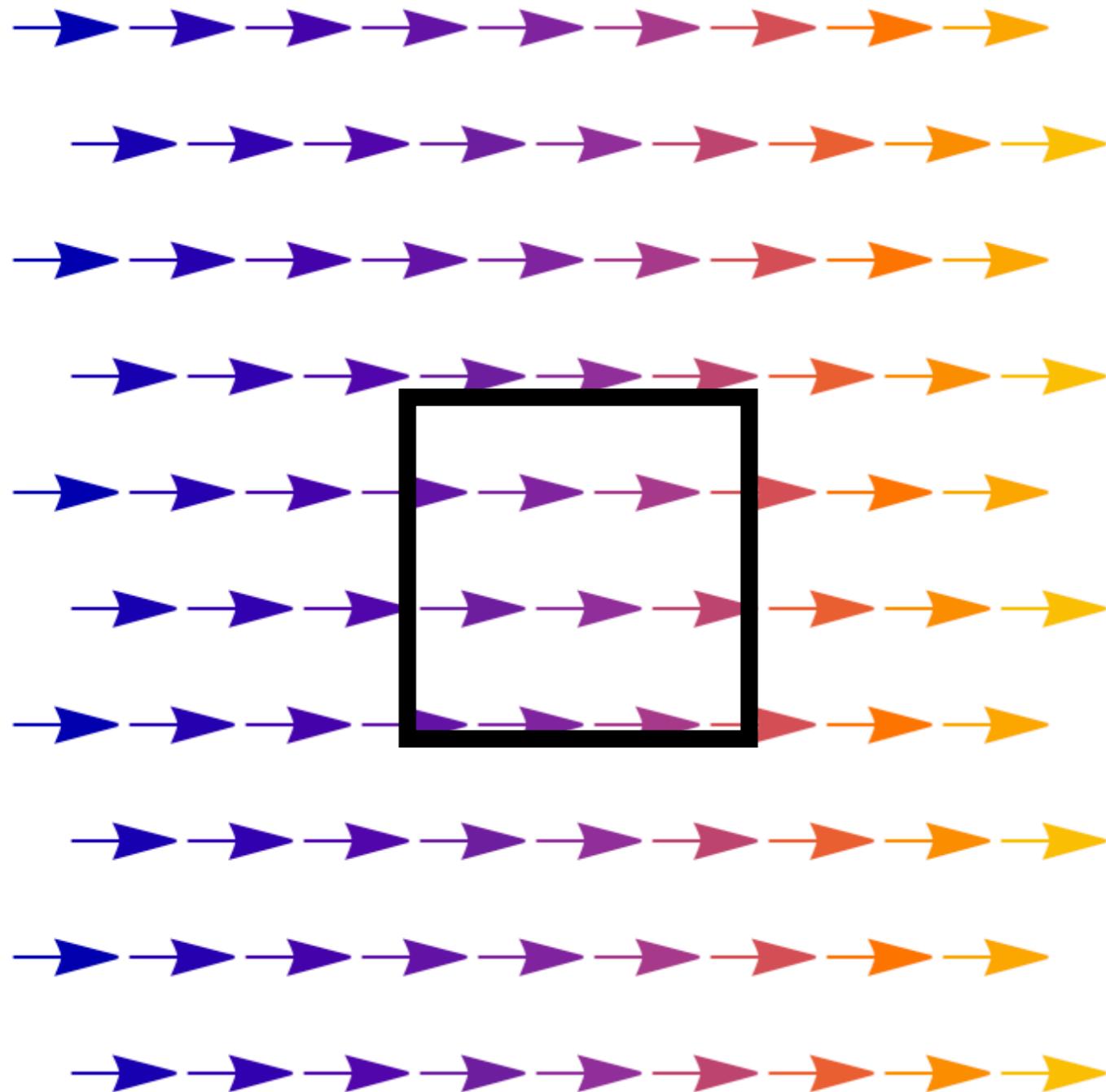
C $Ea \sin \theta$

D $Ea \cos \theta$

E Ea

2D Flux

The net flux through this cube is....



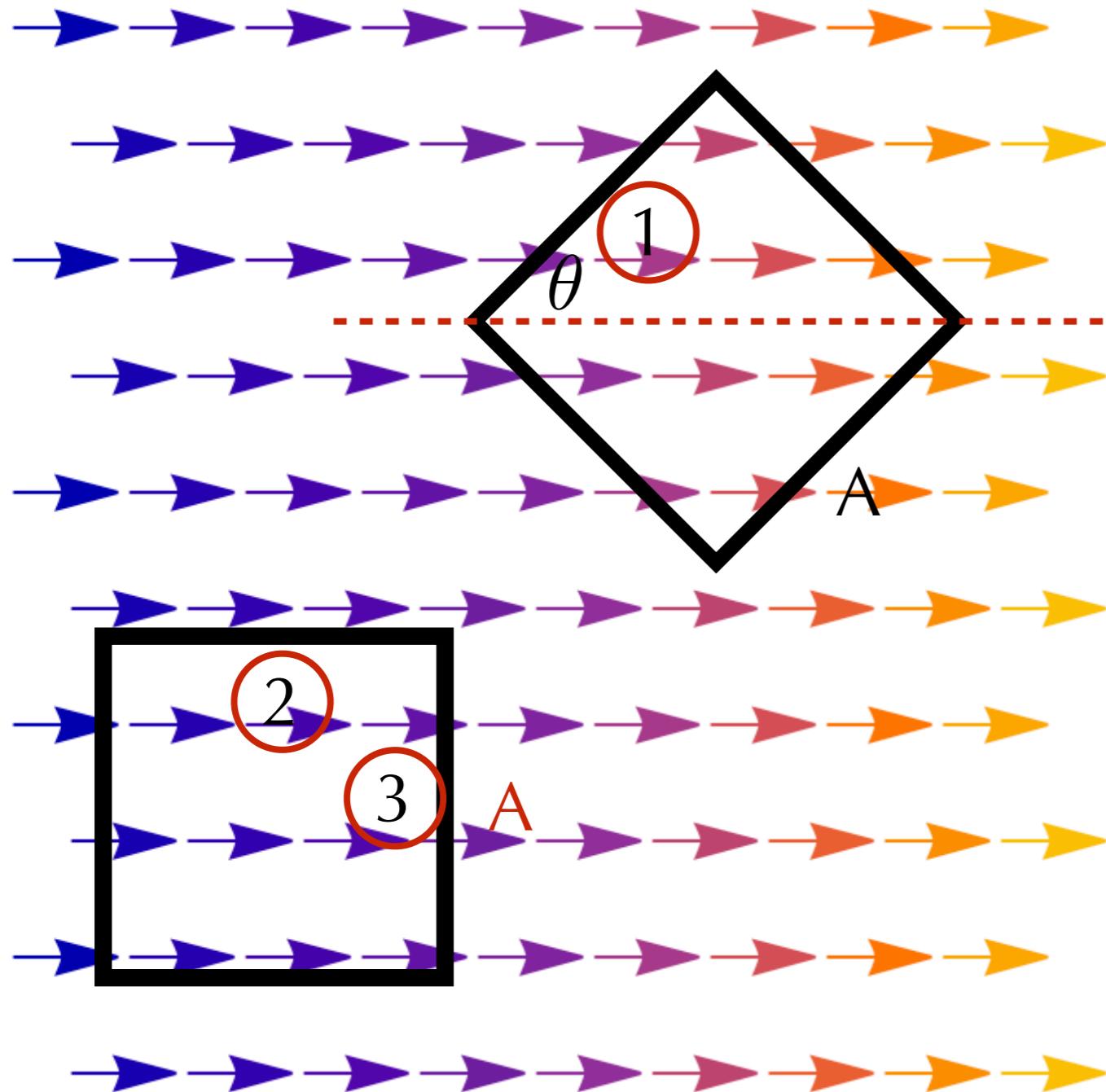
- a) Positive
- b) Negative
- c) Zero

Weaker field

Stronger field

2D Flux

Which flux calculation will be most difficult?



- a) Surface 3
- b) Surface 2
- c) Surface 1
- d) They're all equally hard

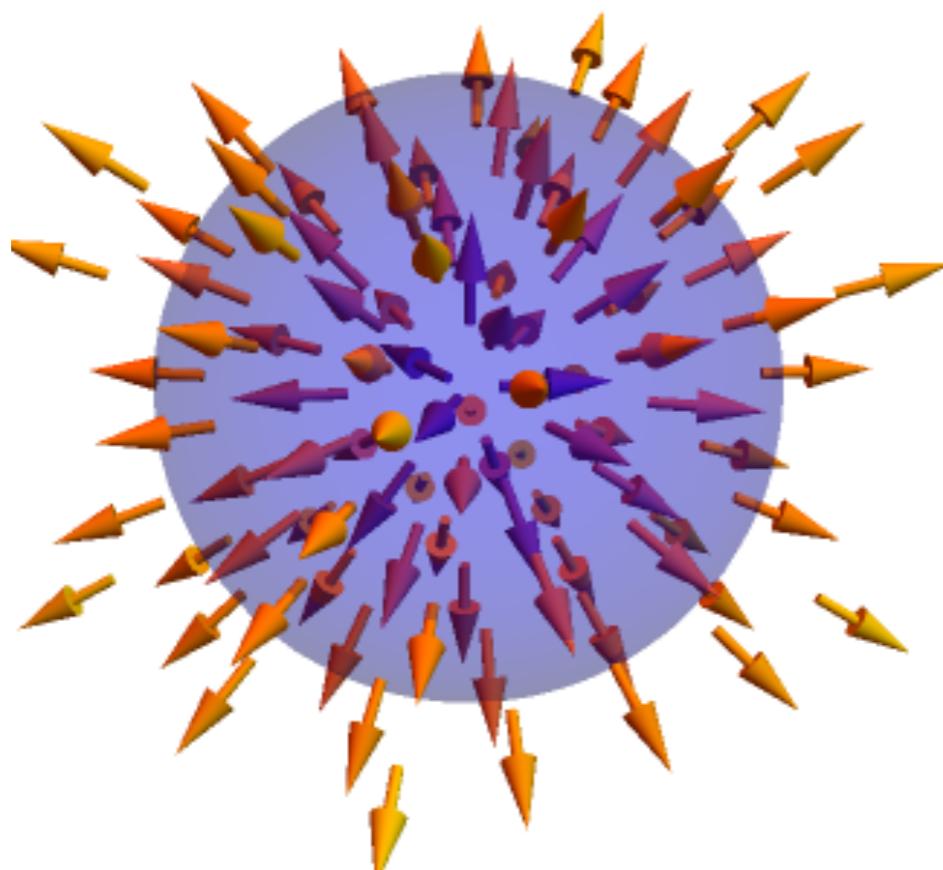
3D Flux

Shown is the electric field due to a point charge. What is the flux through the blue sphere ($r = 0.5$ m).

For ease of calculation, let:

$$q = 5 \text{ C}$$

$$k = 1$$



A $\frac{20}{3}\pi$

B 20π

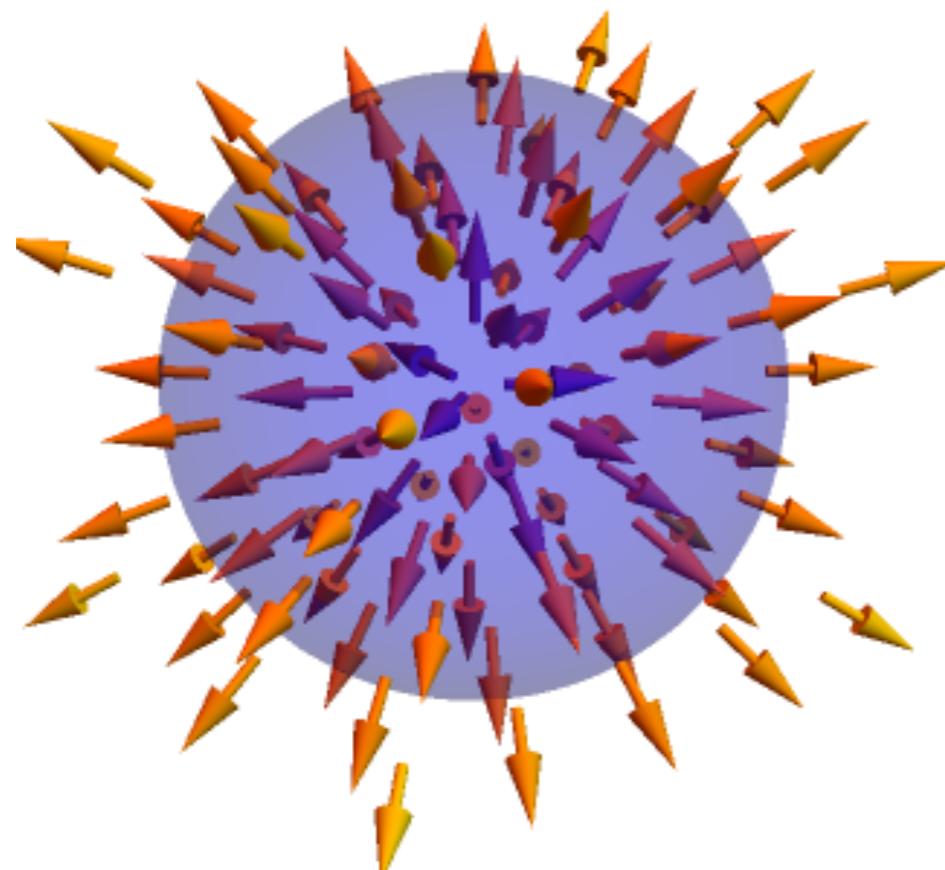
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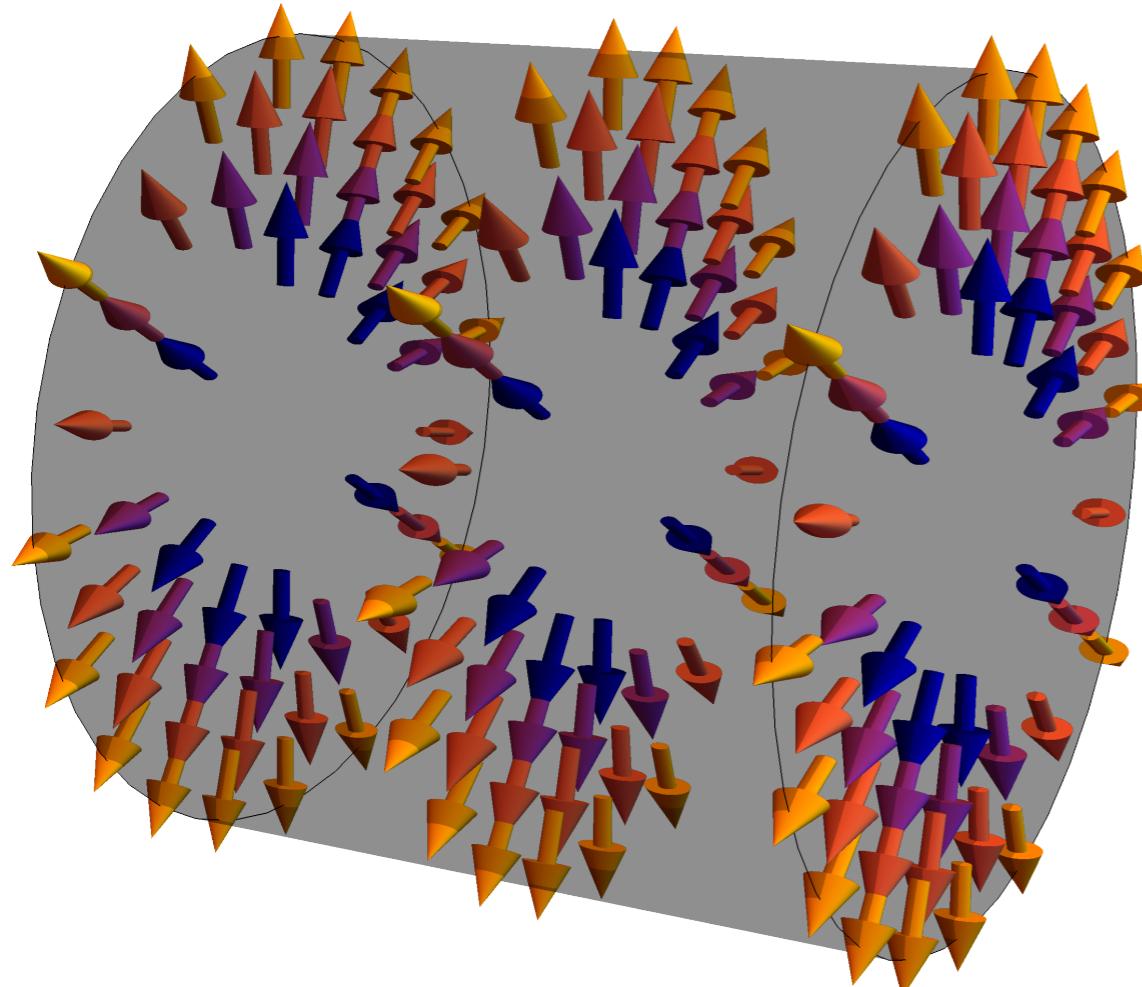
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$$A_{\text{sphere}} = 4\pi r^2$$

3D Flux

Shown is the electric field due to a very long line charge. What is the flux through the gray cylinder ($r = 0.5$ m).



$$E = \frac{2k|\lambda|}{r}$$

For ease of calculation, let:

$$L = 1 \text{ m}$$

$$\lambda = 5 \text{ C/m}$$

$$k = 1$$

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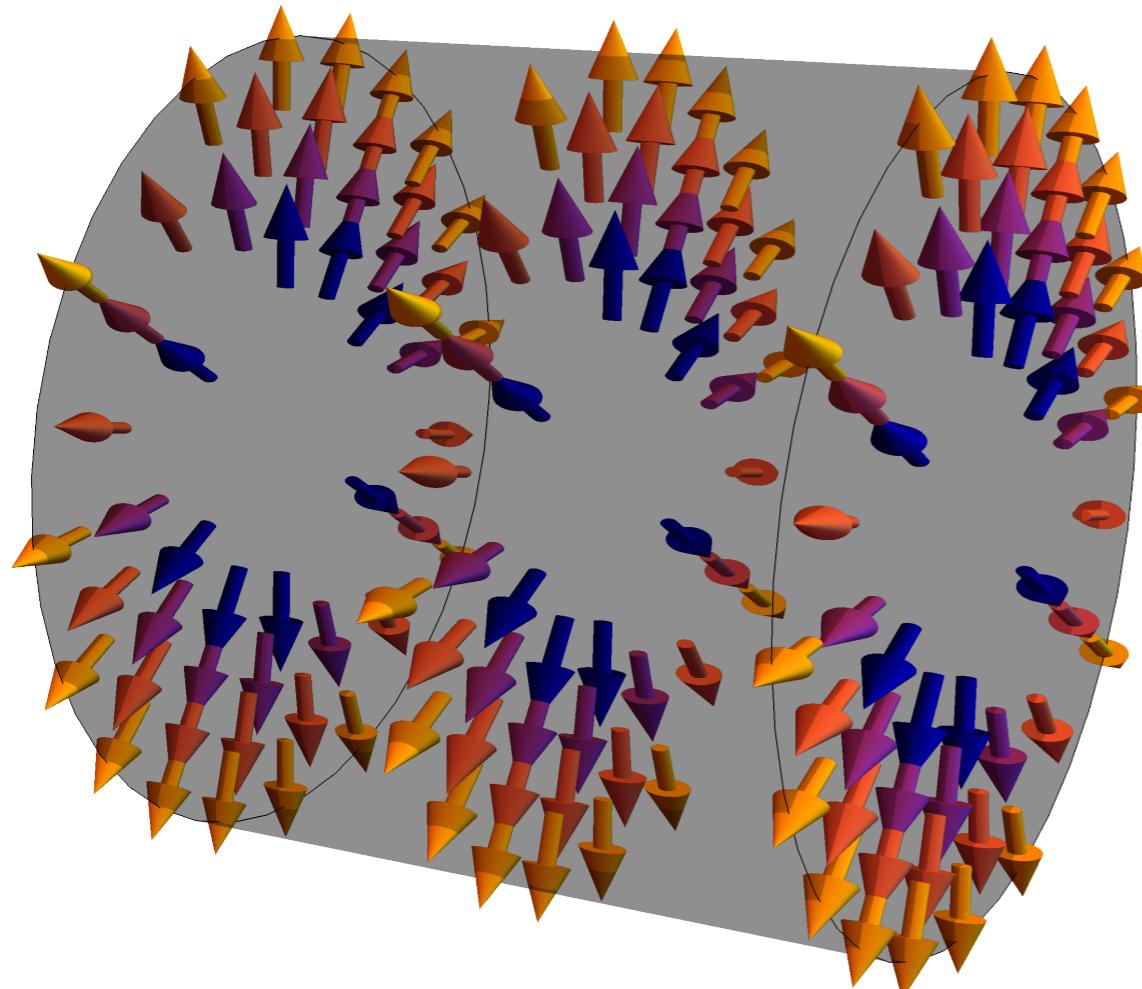
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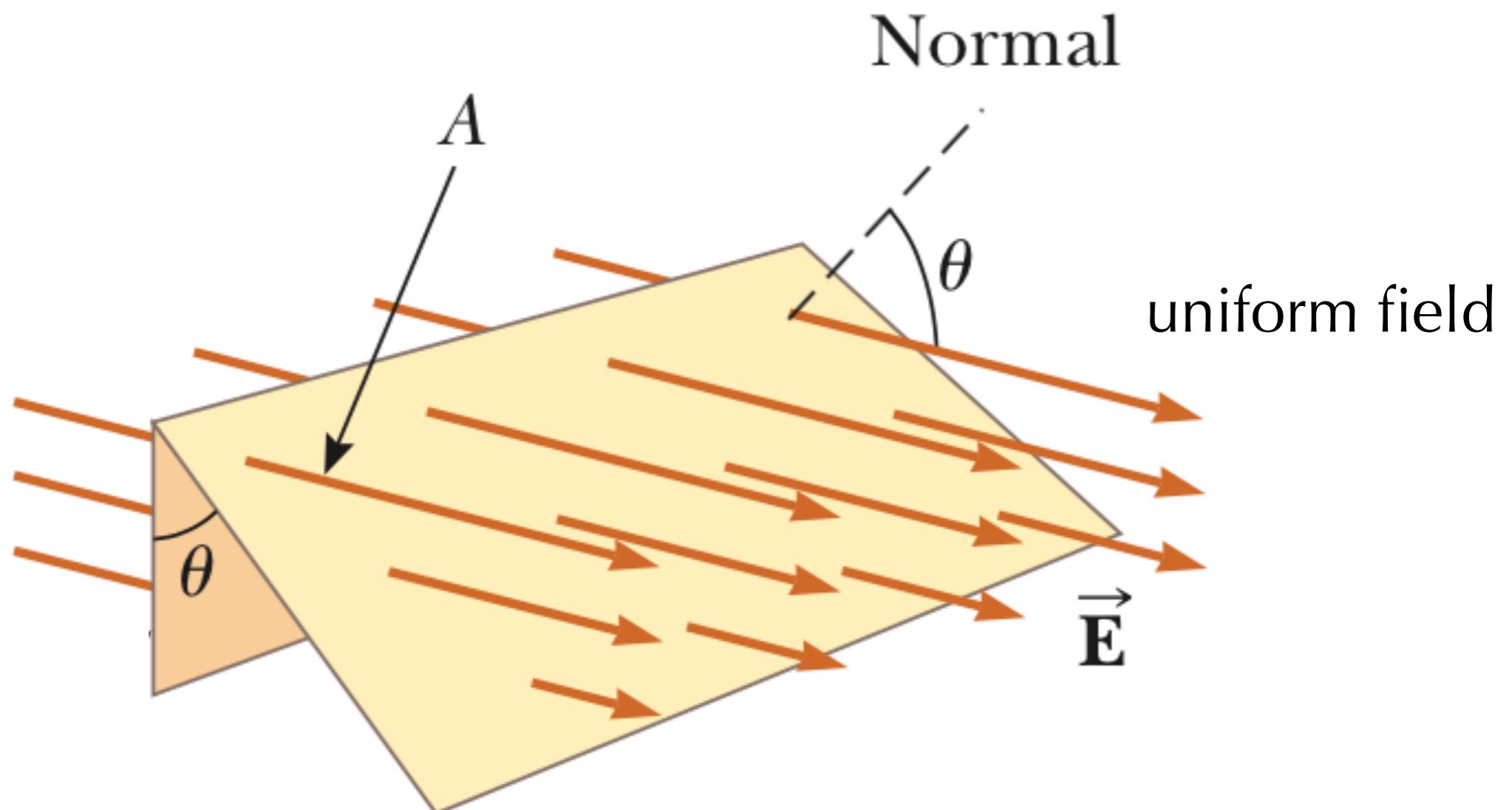
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$$E = \frac{2k|\lambda|}{r} \quad A_{\text{cylinder}} = 2\pi r L$$

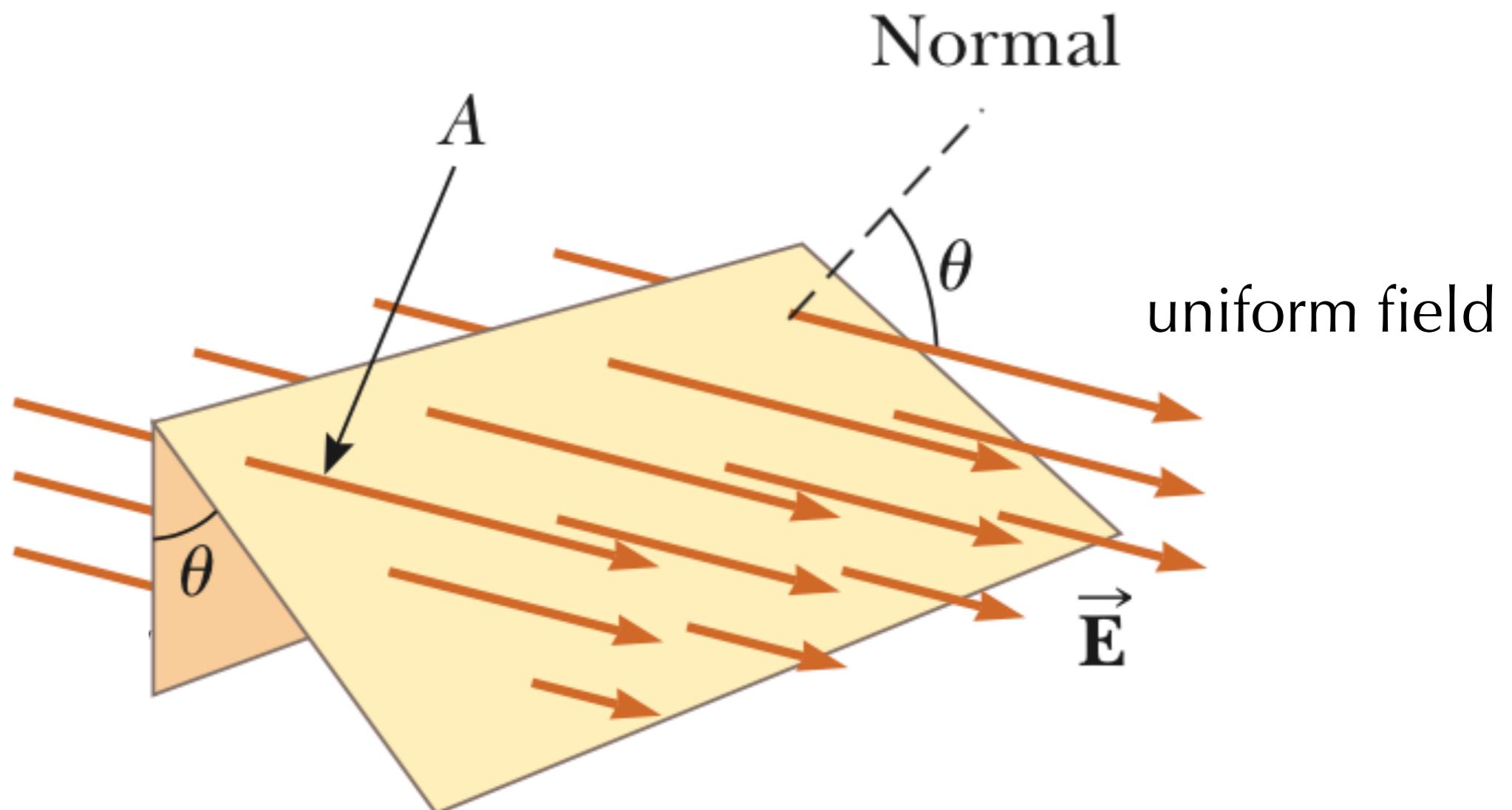
Uniform Field

What is the flux through surface A?



Uniform Field

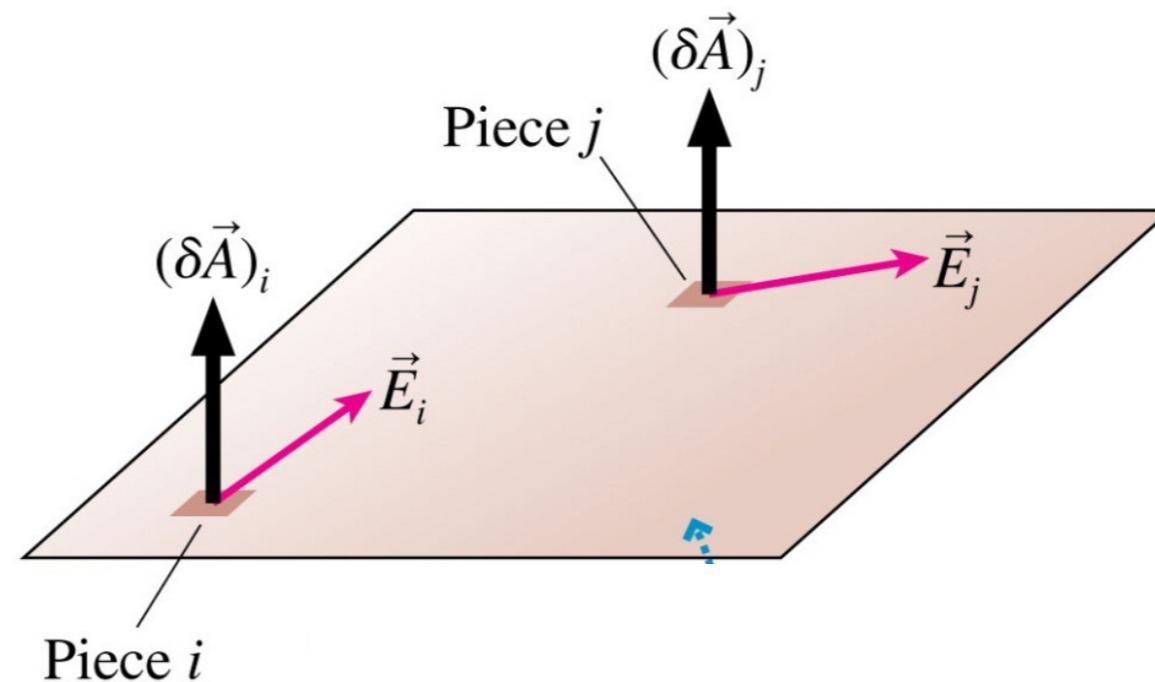
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$$\Phi = \vec{E} \cdot \vec{A}$$

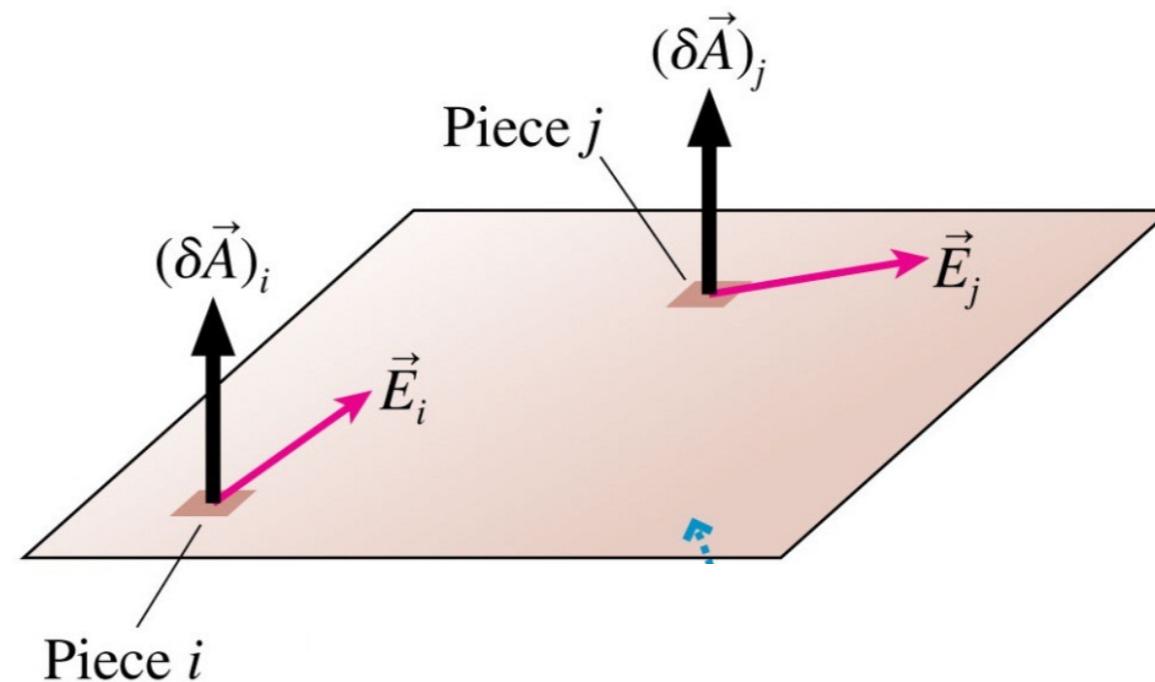
Non uniform field

What if the Electric field is not uniform??



Non uniform field

What if the Electric field is not uniform??

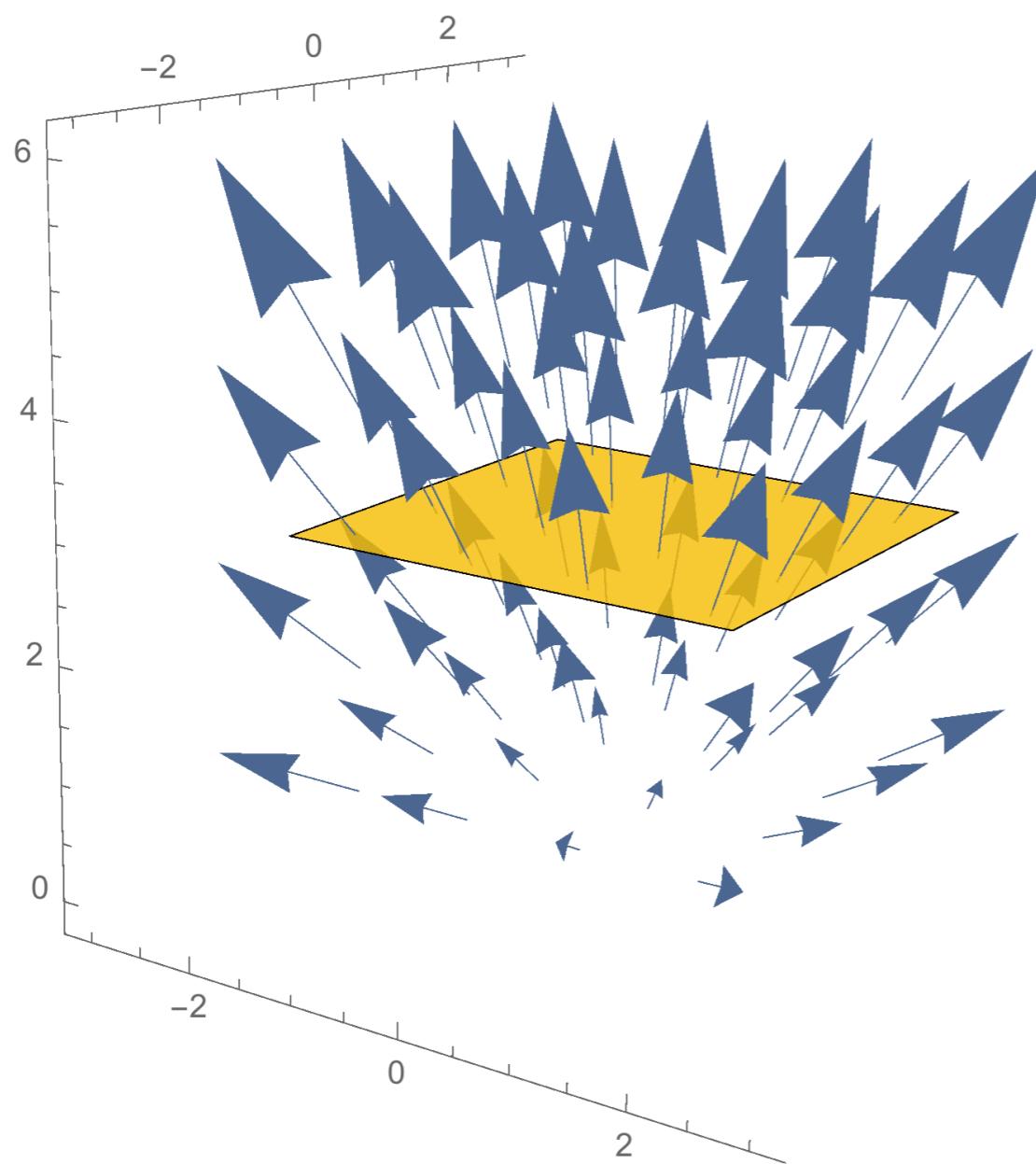


$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$$= \int_{\text{surface}} \vec{E} \cdot \hat{N} dS$$

Example

$$\vec{E} = x\hat{i} + y\hat{j} + z\hat{k}$$

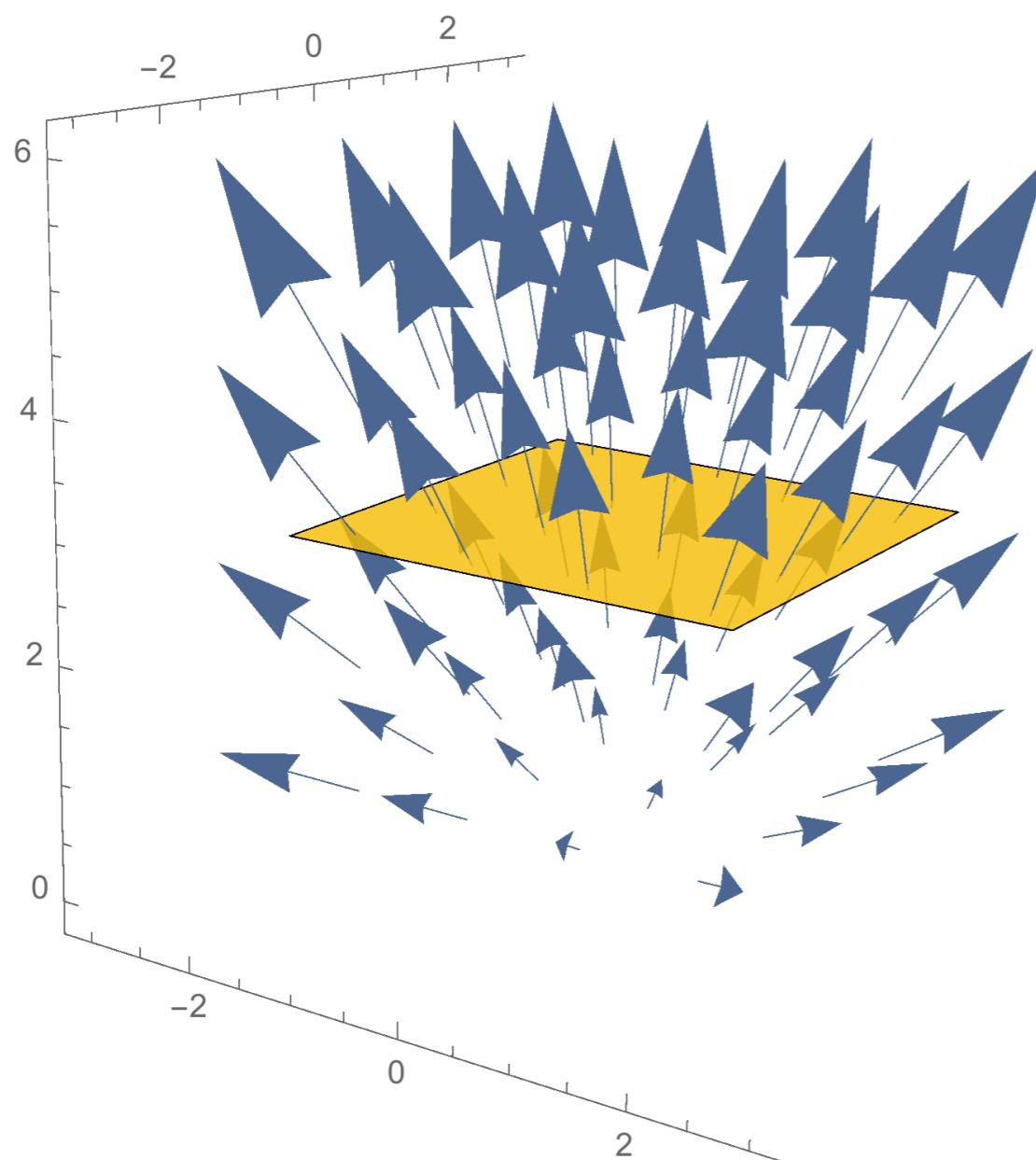


The surface is an origin-centered 4×4 square at a height $z = 3$.

What is the flux?

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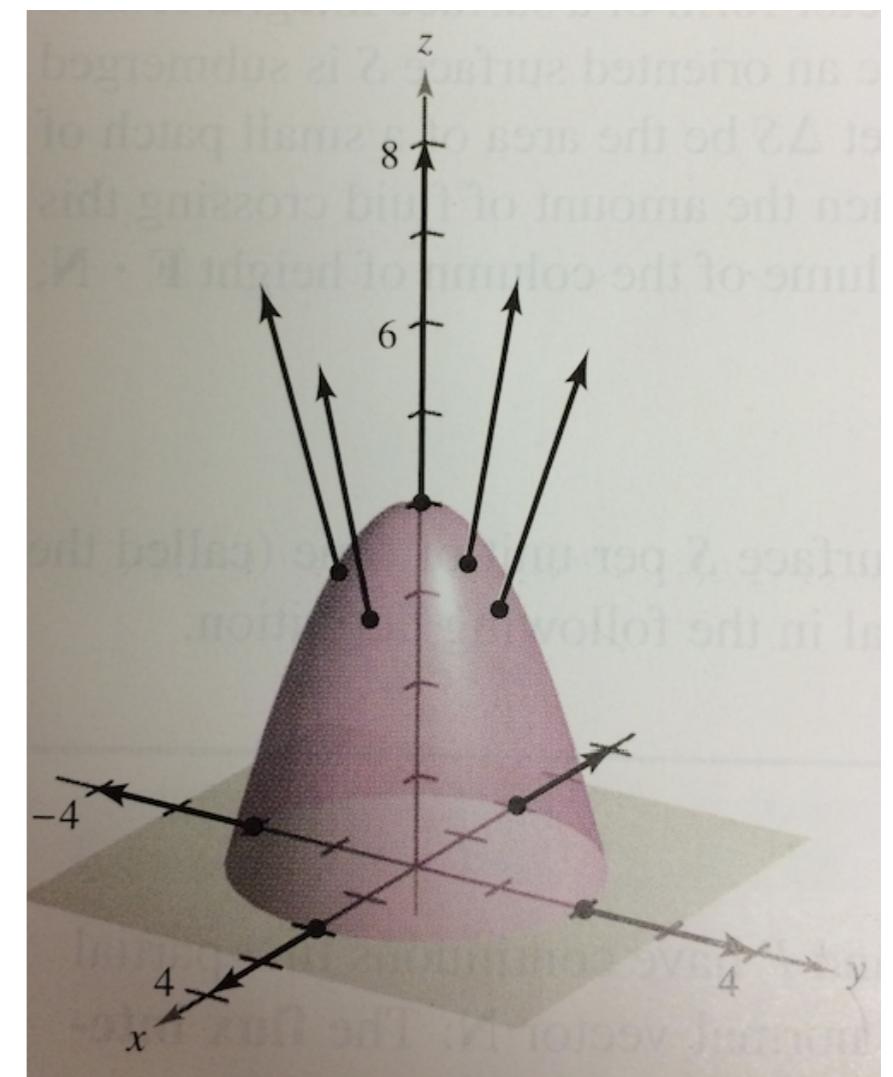
Let S be the portion of the paraboloid

$$z = 4 - x^2 - y^2$$

lying above the xy -plane, oriented by an upward unit normal vector, as shown in the figure. The Electric field in the region is given by:

$$\vec{E}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$$

Find the electric flux through surface S



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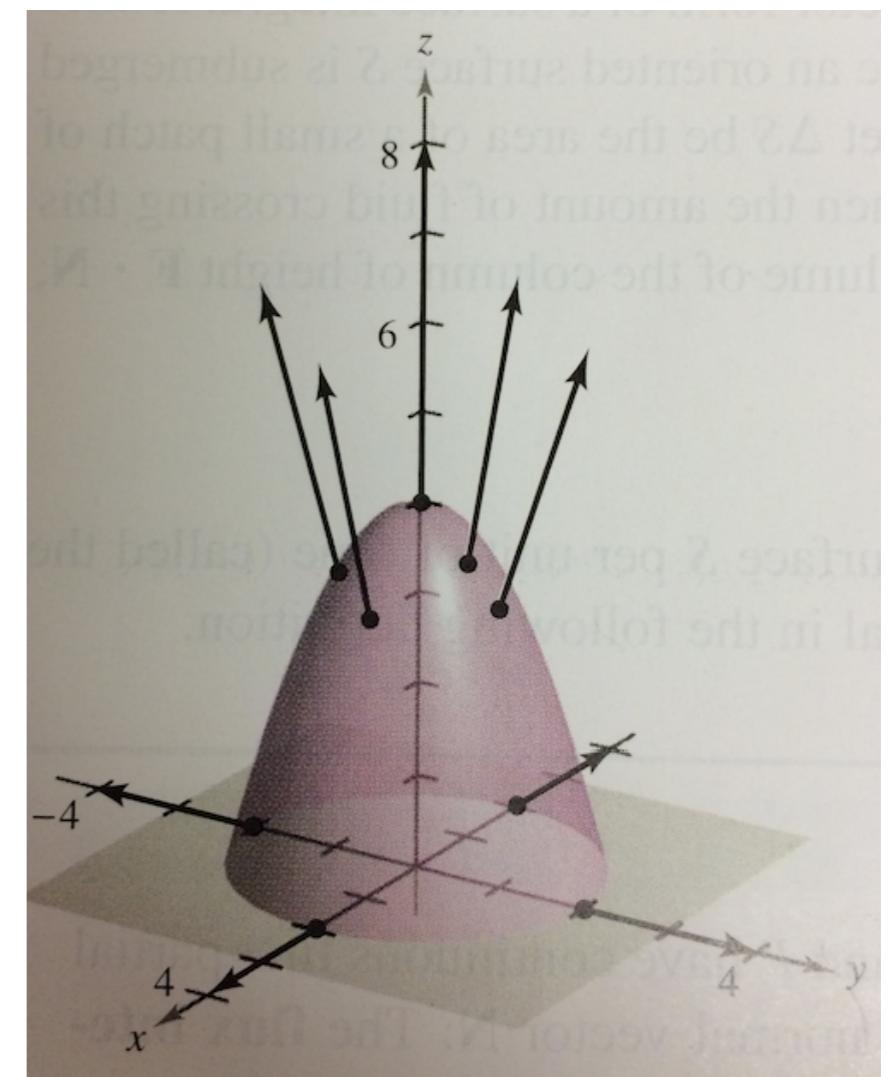
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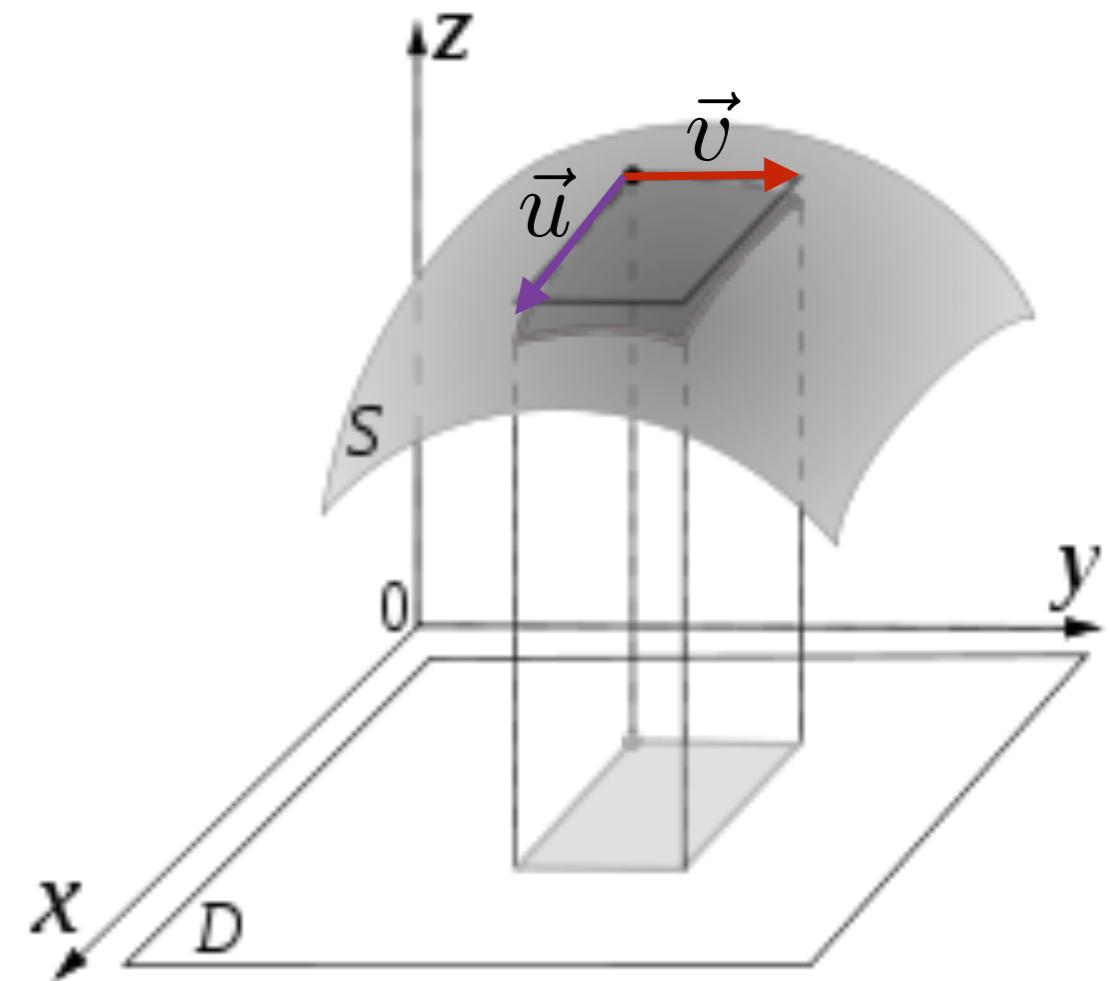
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Find the electric flux through surface S

$$\int \vec{E} \cdot \hat{N} dS = \int \vec{E} \cdot \left(-\frac{df}{dx}\hat{i} - \frac{df}{dy}\hat{j} + \hat{k} \right) dA$$

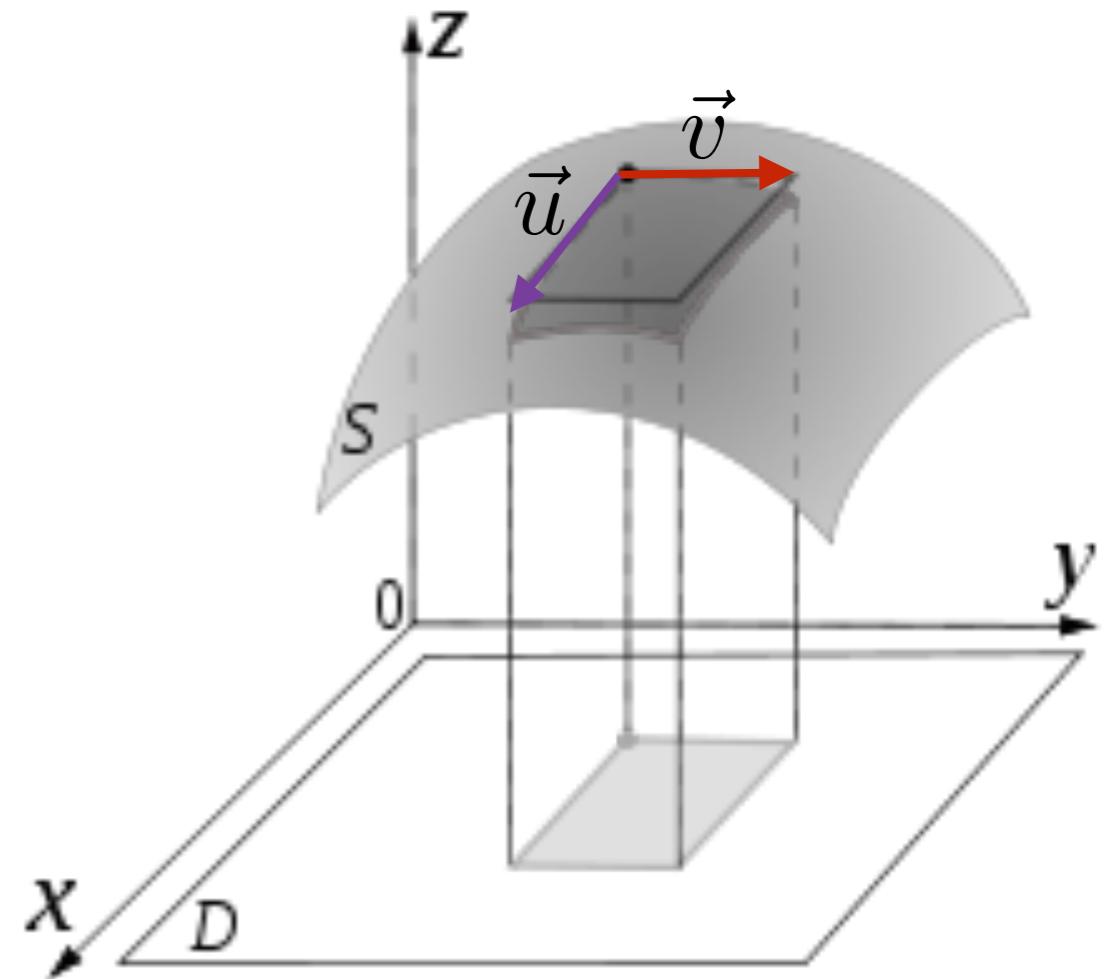


Harder Flux Integrals (Calculus Review)



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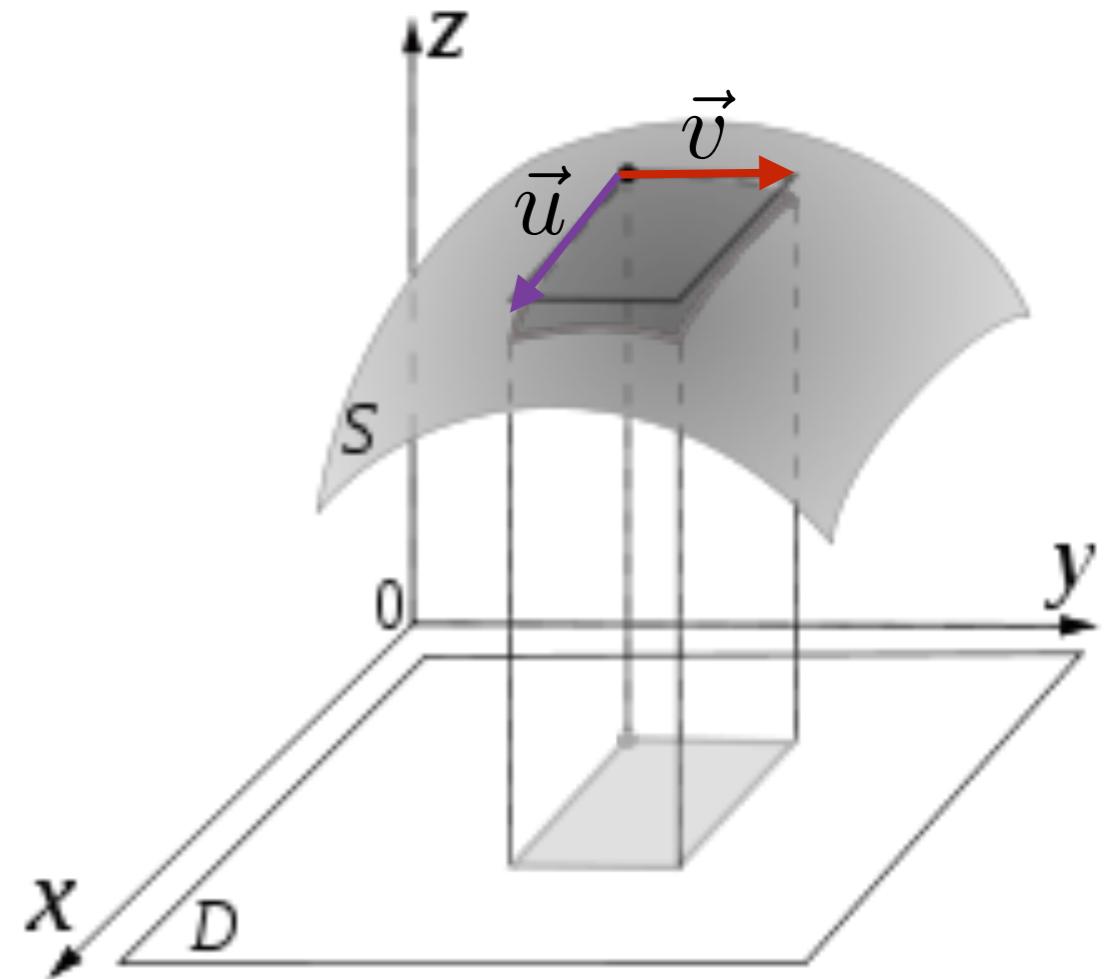
$$\vec{u} = \Delta x \hat{i} + \frac{df}{dx} \Delta x \hat{k}$$



Harder Flux Integrals (Calculus Review)

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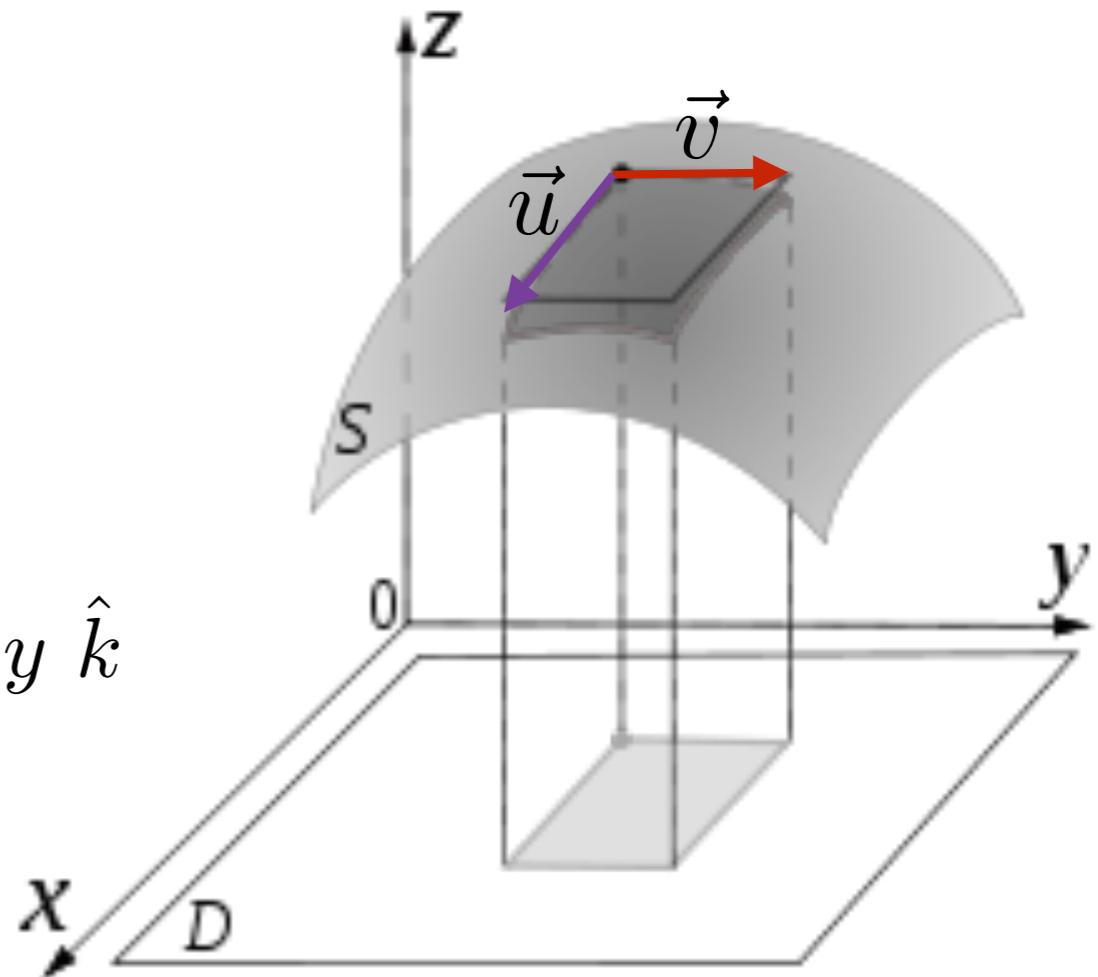


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$$\vec{u} \times \vec{v} = -\frac{df}{dx} \Delta x \Delta y \hat{i} - \frac{df}{dy} \Delta x \Delta y \hat{j} + \Delta x \Delta y \hat{k}$$



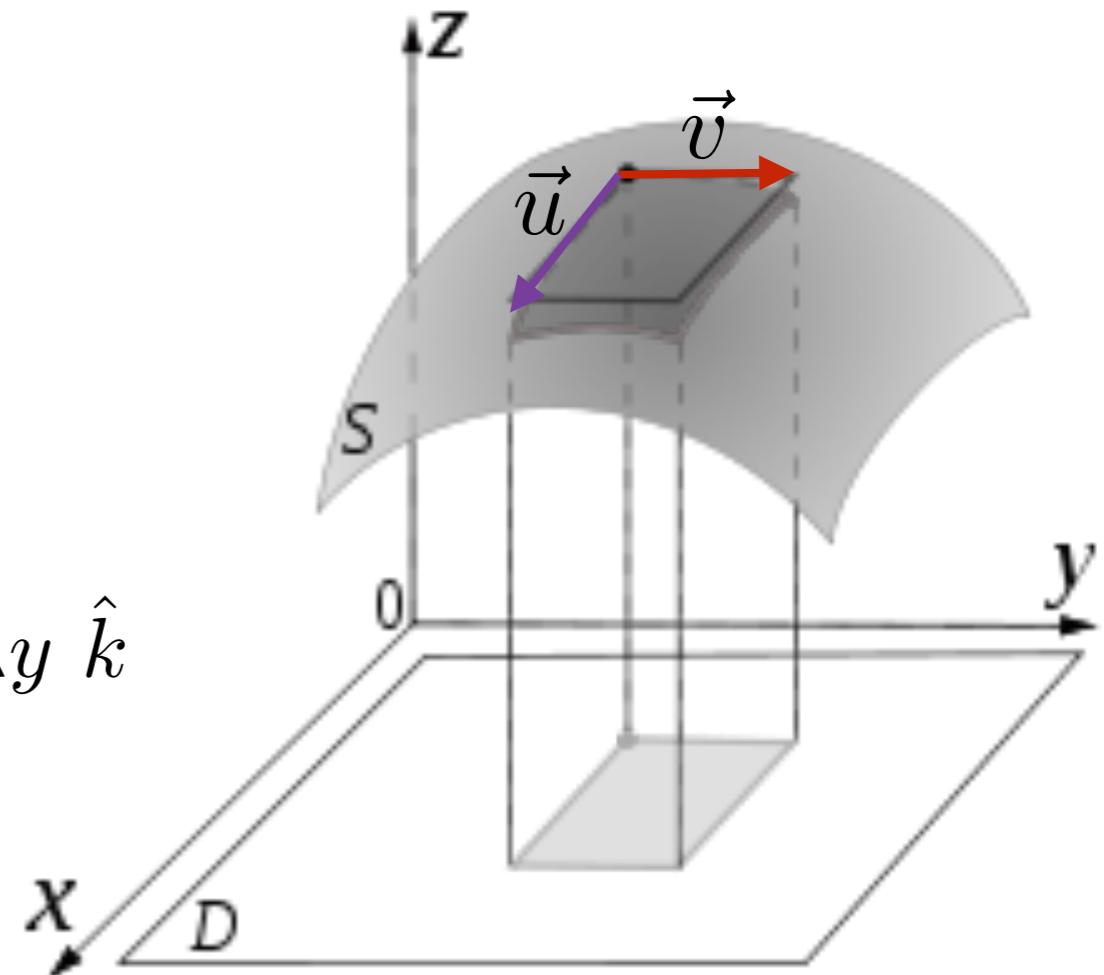
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$$= \left(-\frac{df}{dx} \hat{i} - \frac{df}{dy} \hat{j} + 1 \hat{k} \right) \Delta A$$



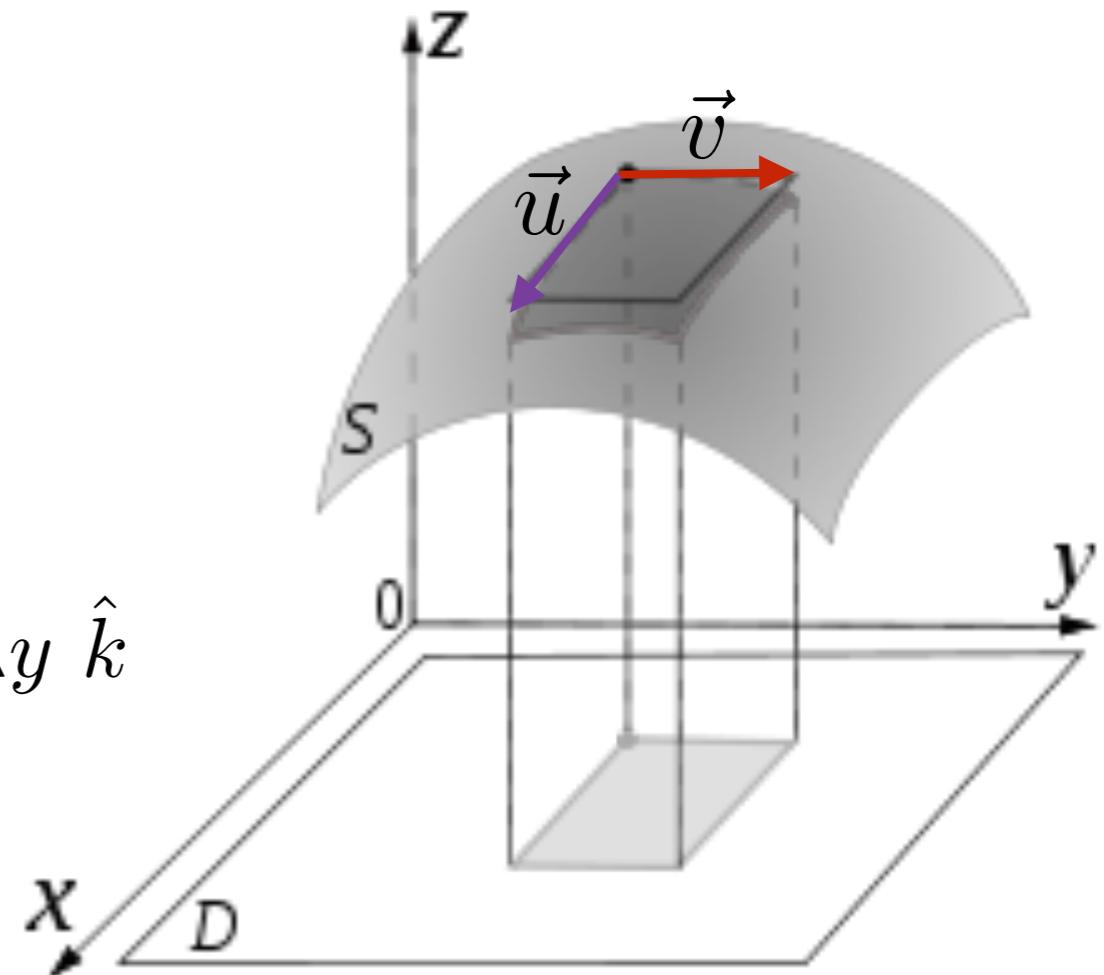
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$$|\vec{u} \times \vec{v}| = \Delta A \sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2 + 1}$$

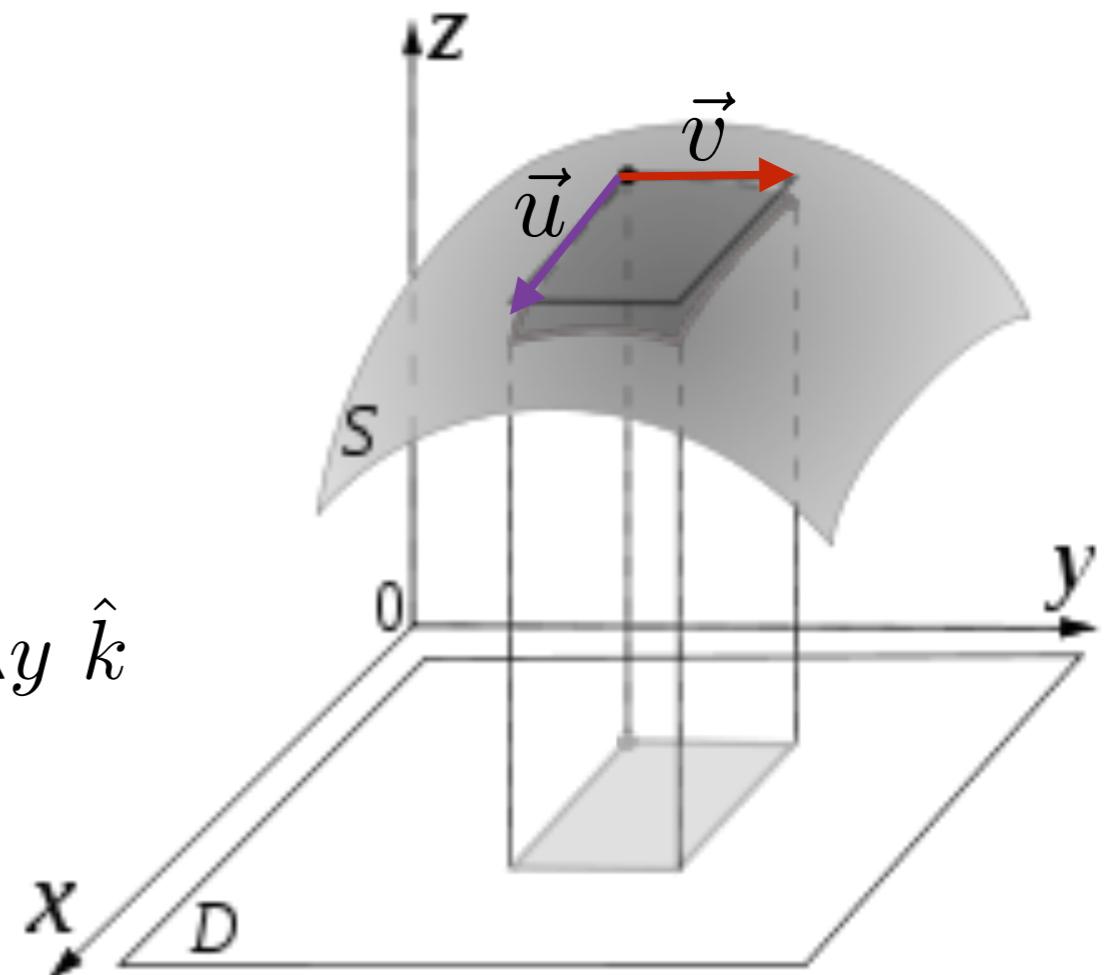
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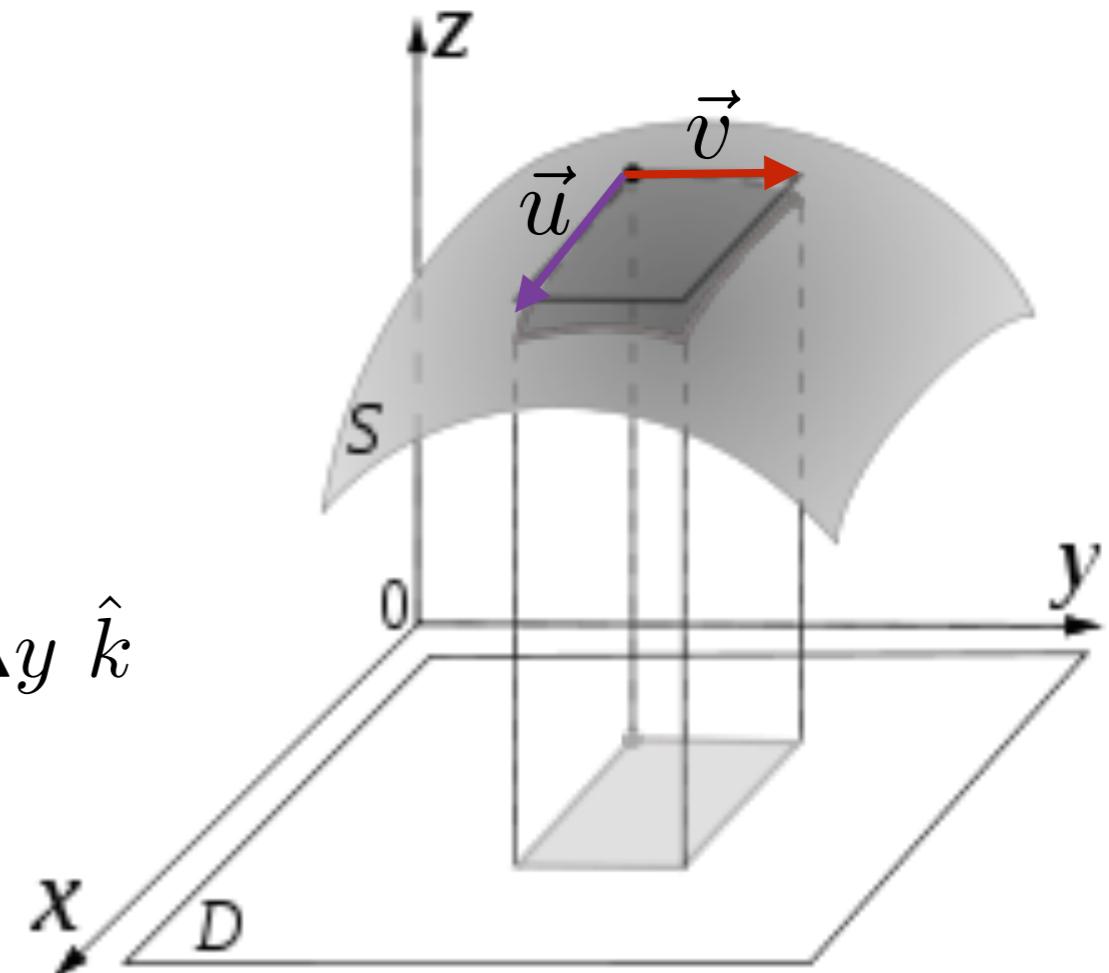
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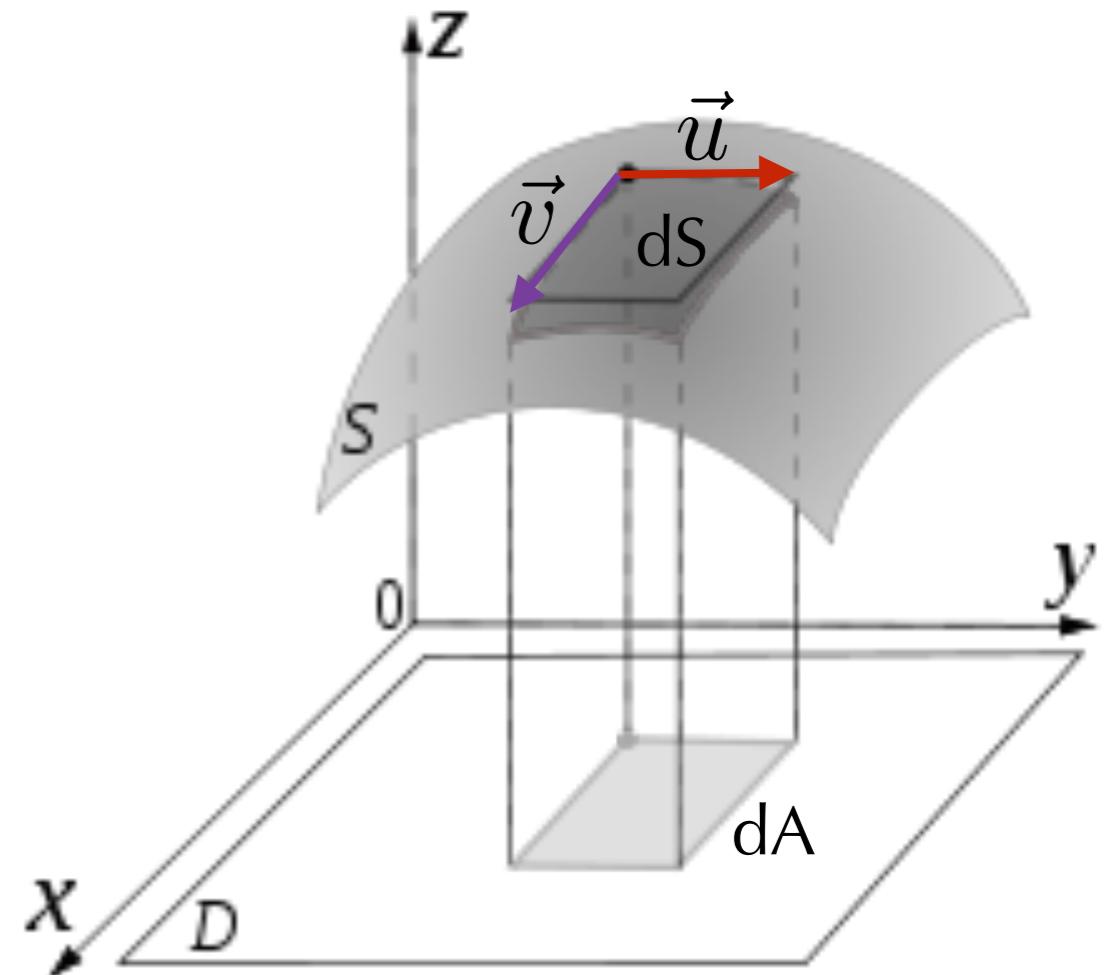
$$\Phi = \int_{\text{surface}} \vec{E} \cdot \hat{N} dS$$

$$\int \vec{E} \cdot \hat{\vec{N}} dS$$

$$\hat{\vec{N}} = \frac{\nabla f(x, y, z)}{|\nabla f(x, y, z)|}$$

$$= \frac{-\frac{df}{dx}\hat{i} - \frac{df}{dy}\hat{j} + 1\hat{k}}{\sqrt{\left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2 + 1}}$$

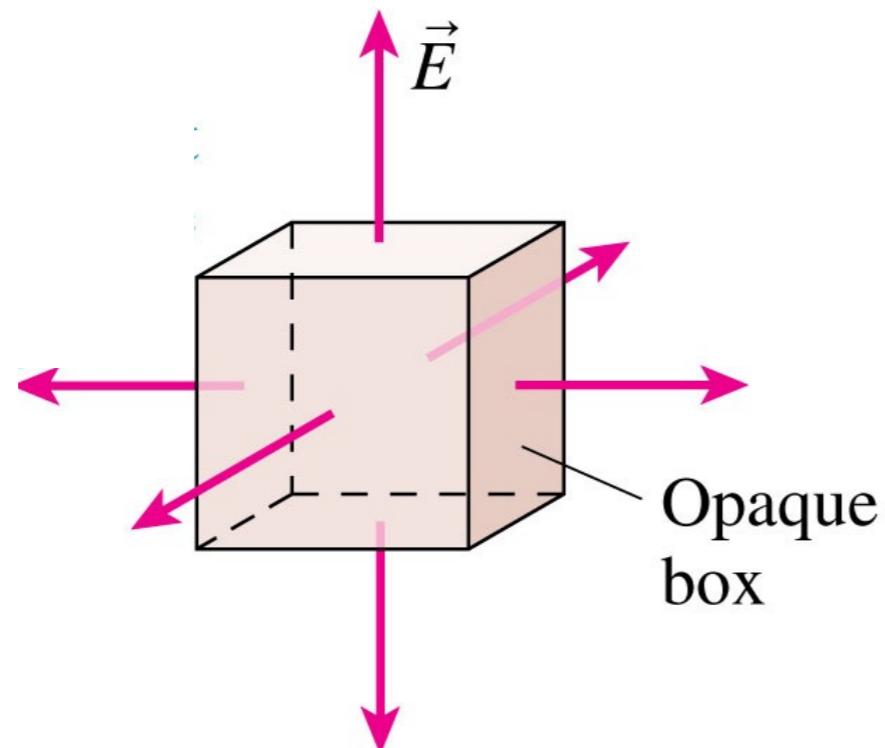
$$\int \vec{E} \cdot \hat{\vec{N}} dS = \int \vec{E} \cdot \left(-\frac{df}{dx}\hat{i} - \frac{df}{dy}\hat{j} + 1\hat{k}\right) dA$$



Why Flux? What does it even tell us?

You can't see inside the box, all you know is there an outward-pointing electric field at each face.

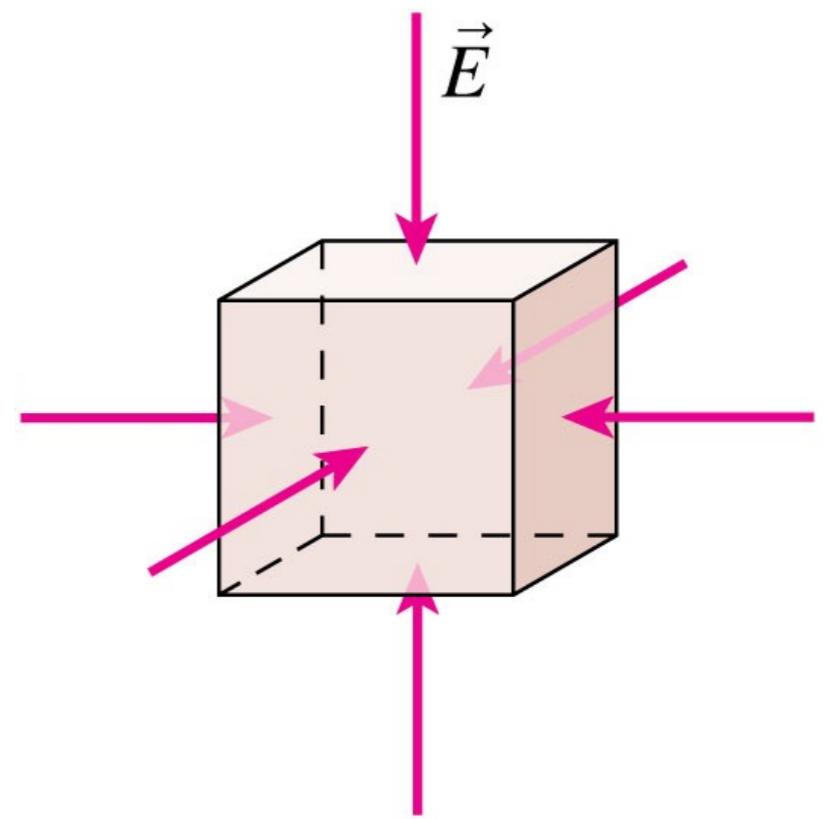
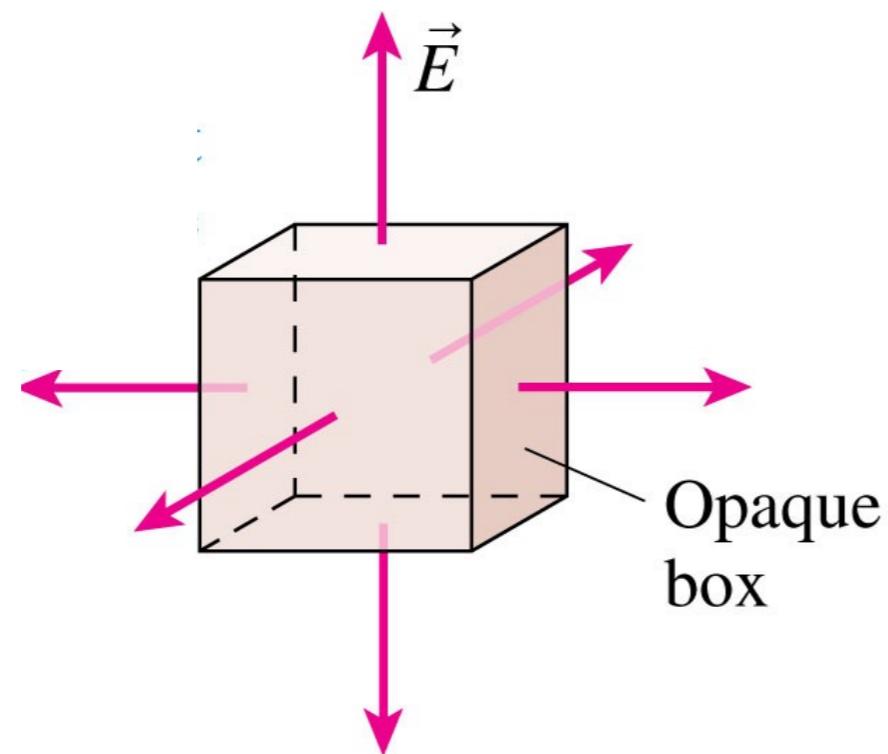
What can you tell about the charge that is inside the box?



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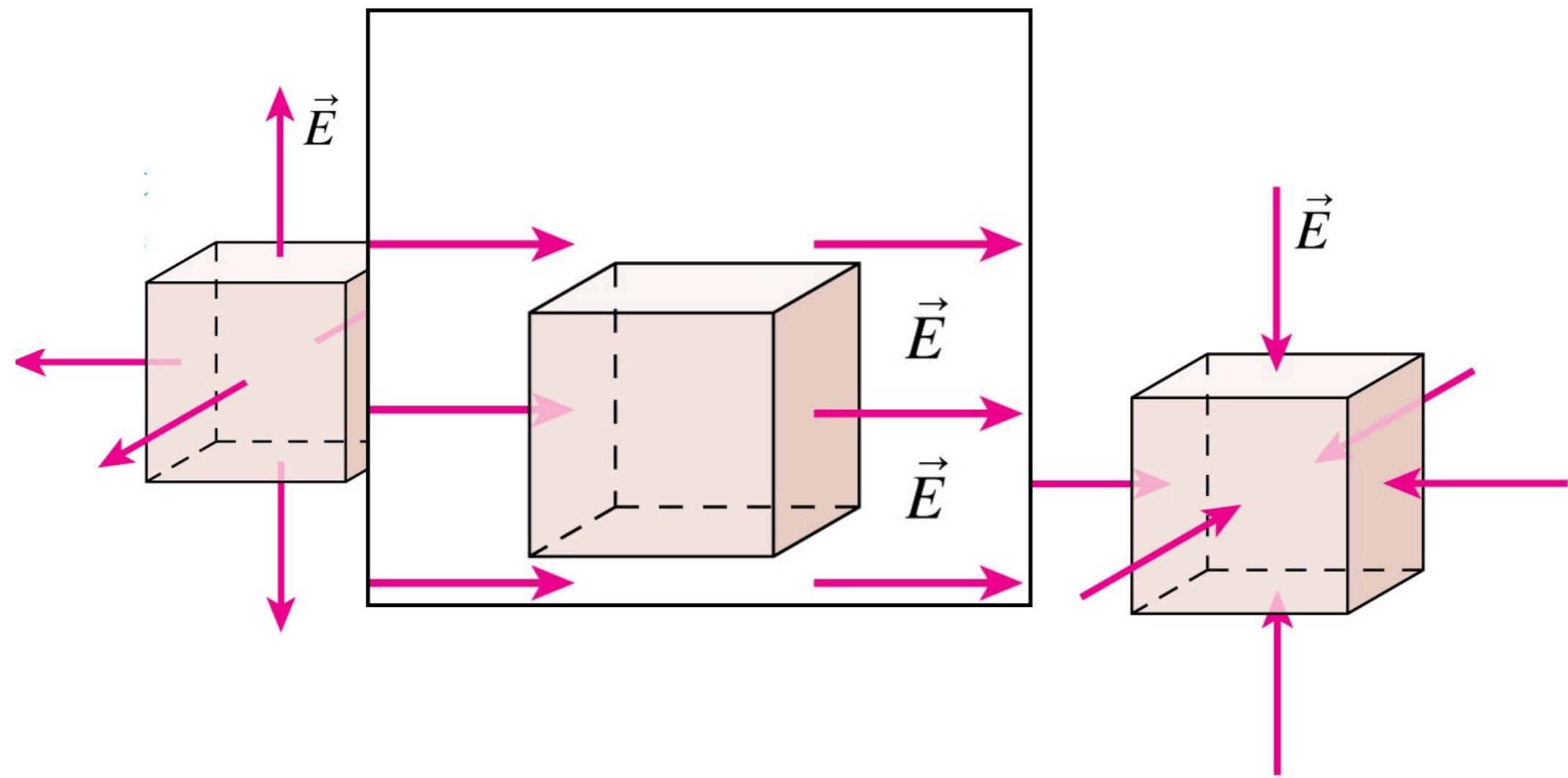
What can you tell about the charge that is inside the box?



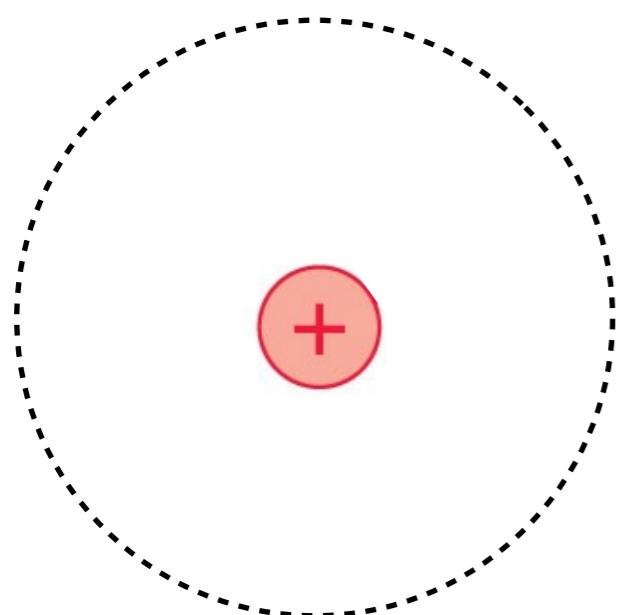
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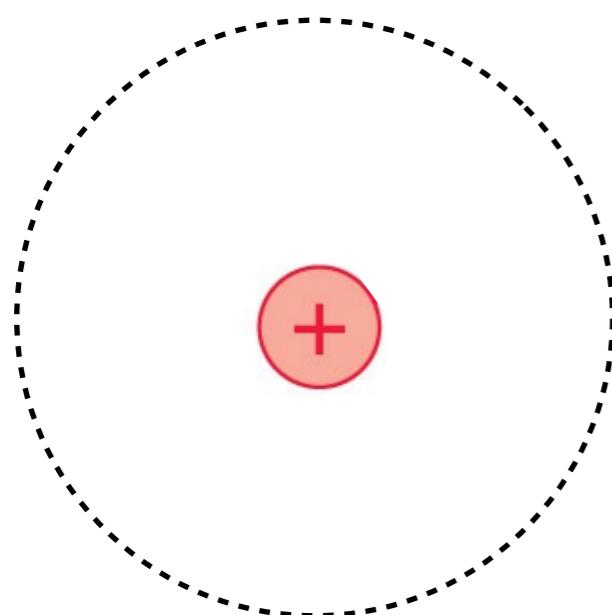


Push a button on your clicker after answering each question.



- A. Draw several electric field lines at the gaussian surface.
- B. Are the magnitudes of these electric field vectors the same or different.
- C. Calculate the electric flux through this surface.
- D. Reduce your expression as far as you can.

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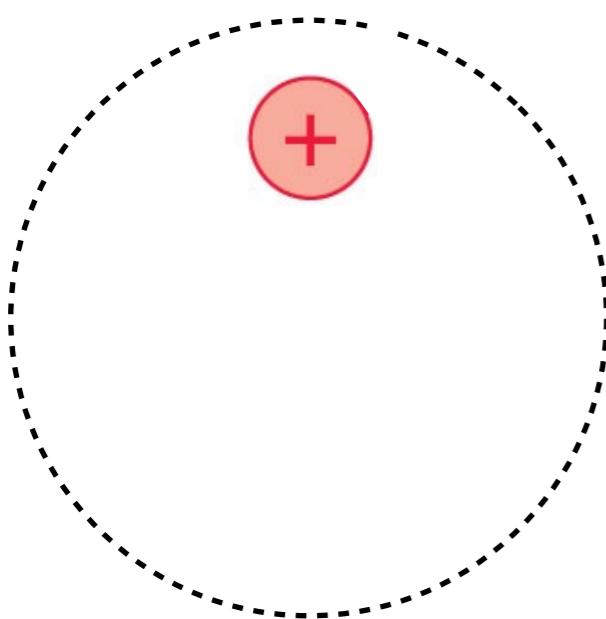
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Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

How/When is Gauss's law helpful

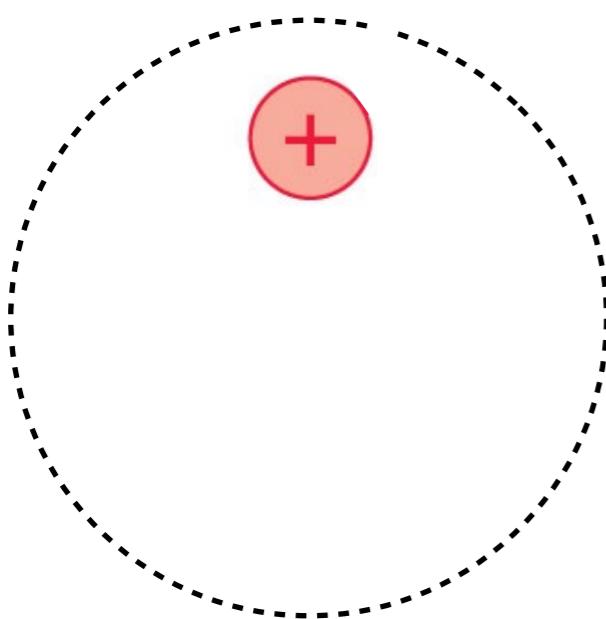
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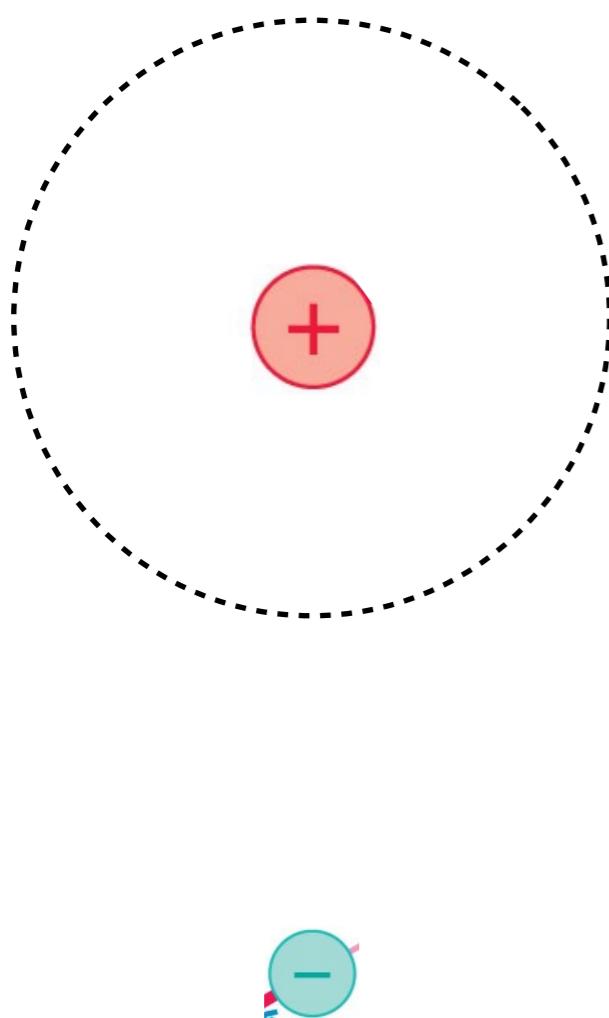
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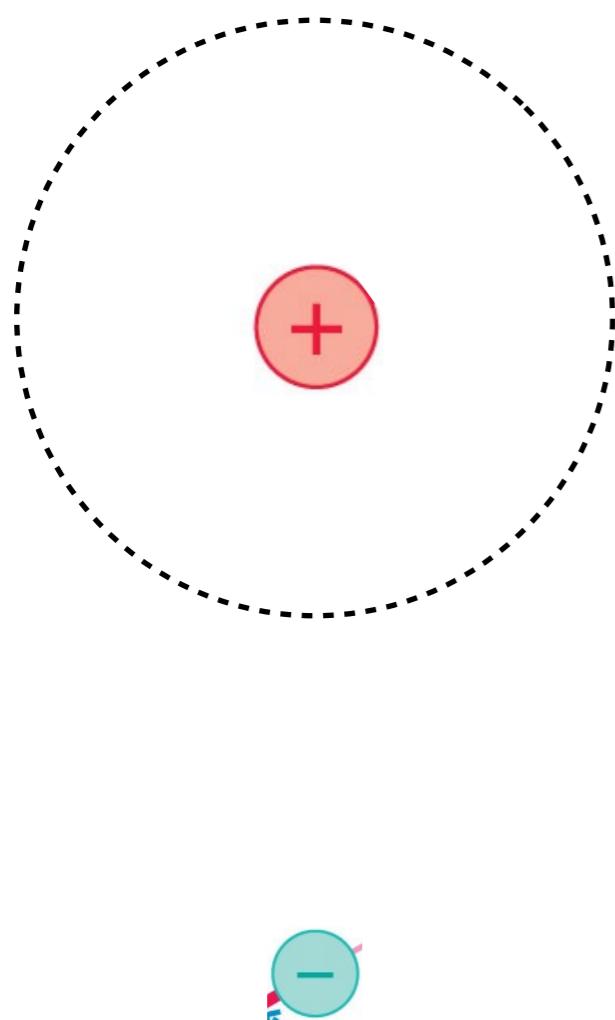


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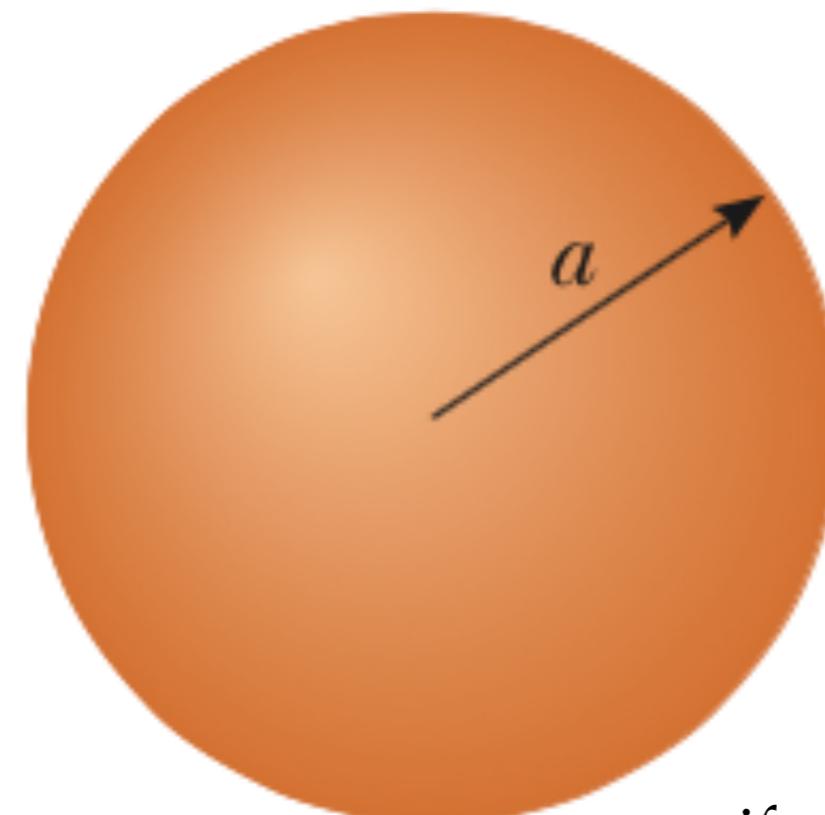


Is Gauss's Law helpful for this charge distribution?

Gauss's Law is most useful when the surface is chosen such that the integral becomes trivial (easy).

What direction must the E field point?

What is the electric field inside and outside of the sphere



uniform charge distribution