Physics 220 Equations

Chapter 22: Electric Forces

$F = \frac{kq_1q_2}{r^2}$	Coulomb's law.

Chapter 23: The Electric Field

$\vec{E} = \frac{kq}{r^2}\hat{r}$	Electric Field of a point charge.
$\vec{F} = q\vec{E}$	Force on a point charge in presence of electric field.
$\vec{p} = q\vec{s}$	Dipole moment (vector points from negative to positive
$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$	Electric field created by a dipole on a line going through both point charges.
$\vec{E} = \frac{kQ}{r\sqrt{r^2+(\frac{L}{2})^2}}$	Electric field created by a finite line of uniformly distributed charge on a line that bisects the line charge.
$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$	Electric Field of dipole (on line perpendicular to dipole passing through its center)
$\vec{\tau} = \vec{p} \times \vec{E}$	torque on dipole in uniform field
$\lambda = \frac{Q}{L}$	linear charge density (one dimensional)
$\eta = \frac{Q}{A}$	Surface charge density (two dimensional)
$\rho = \frac{Q}{V}$	Volume charge density (three dimensional)
$\frac{2k\lambda}{r}$	E-field created by an infinite line of charge
$E = \frac{\eta}{2\epsilon_0}$	E-field created by an infinite plane of charge
$E = \frac{\eta}{\epsilon_0}$	E-field inside of a parallel plate capacitor
$E = \frac{kzQ}{(z^2+R^2)^{3/2}}$	E-field created by a uniformly charge ring (on axis perpendicular to ring and passing through the center of the ring)
$E = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$	E-field created by a uniformly charged disk (on axis perpendicular to ring and passing through the center of the ring)

Chapter 24: Gauss's Law

$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$	Definition of Flux

$\int_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$	Gauss's Law
$\vec{E} = (\frac{\eta}{\epsilon_0}$, perpendicular to surface)	Electric field at surface of conductor

Chapter 25: The Electric Potential

$K = \frac{1}{2}mv^2$	Kinetic Energy
$\Delta U = Eq\Delta s$	Difference in electric potential energy of a point charge in a uniform field.
$U = \frac{kq_1q_2}{r}$	electric potential energy of two point charges
$U = -\vec{p} \cdot \vec{E}$	electric potential energy of a dipole in uniform field:
$V = \frac{U}{q}$	Electric potential (not potential energy)
$V = Es$	electric potential inside parallel plate capacitor
$V = \frac{kq}{r}$	electric potential of a point charge
$V = \frac{kq}{r} = \frac{R}{r}V_0$	electric potential of a charged solid sphere
$V = \frac{kQ}{\sqrt{x^2+R^2}}$	electric potential of a uniformly charged ring(on symmetry axis)
$V = \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{R^2+x^2} - x\right)$	electric potential of a uniformly charged disk(on symmetry axis)

Chapter 26: Potential and Field

$\Delta V = -\int E_s ds$	Relationship between electric potential and the electric field
$E_s = -\frac{dV}{ds}$	Inverse relationship between the electric field and the electric potential in <u>one dimension</u>
$E_s = -\nabla V$	Electric field from the electric potential in <u>three dimension</u>
$\Delta V_{\text{loop}} = 0$	Kirchoff's loop law
$C = \frac{Q}{\Delta V_c}$	Definition of capacitance
$C = \frac{\epsilon_0 A}{d}$	Capacitance of a parallel-plate capacitor
$C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots$	Equivalent capacitance of capacitors in <u>parallel</u>
$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$	Equivalent capacitance of capacitors in <u>series</u>.
$U = \frac{1}{2}C(\Delta V_C)^2 = \frac{Q^2}{2C}$	Energy stored in a capacitor.

$\kappa \equiv \frac{E_0}{E}$	Definition of the dielectric constant κ.
$C = \kappa C_0$	How dielectric affects the capacitance.

Chapter 27: Current and Resistance

$i_e = n_e A v_d = \frac{n_e e \tau A}{m} E$	Electron current given the physical properties/dimensions of the material.
$v_d = \frac{e\tau}{m} E$	Drift velocity of electrons moving through a conductor.
$I = \frac{dq}{dt}$	Definition of current.
$J = \frac{I}{A}$	Definition of current density.
$\sum I_{\text{in}} = \sum I_{\text{out}}$	Conservation of current/ Kirchoff's junction rule.
$\vec{J} = \sigma \vec{E}$	Relationship between current density (J) and the applied E field (E)
$\rho = \frac{1}{\sigma}$	Relationship between conductivity (σ) and resistivity (ρ).
$I = \frac{\Delta V}{R}$	Ohm's law stating how much current will flow when a given potential difference is applied.
$R = \frac{\rho l}{A}$	Resistance of a conductor.

Chapter 28: DC circuits

$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$	Power delivered by a source or power dissipated by a resistor.
$R_{\text{eq}} = R_1 + R_2 + R_3 + \ldots$	Equivalent resistance of resistors in series.
$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$	Equivalent resistance of resistors in parallel.
$Q = Q_0 e^{-t/RC}$	Charge on a capacitor, or network of capacitors, as a function of time as it discharges through a resistor, or network of resistors.
$Q = Q_0(1 - e^{-t/RC})$	Charge on a capacitor in an RC circuit as a function of time as it is charged by a battery.
$\tau = RC$	Time constant found in charge/discharge of a capacitor for RC circuits.

Chapter 29: Magnetic Fields

$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$	Magnetic field created by a moving point charge.
$B = \frac{\mu_0 I}{2\pi d}$	Magnetic field created by a long, straight wire.

$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$	Magnetic field created by a steady state current.
$B = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}$	Magnetic field created by a circular loop of wire of radius R.
$\vec{\mu} = AI$, south to the north pole.	Magnetic dipole moment.
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}$	Ampere's Law.
$B_{\text{solenoid}} = \frac{\mu_0 NI}{l}$	Magnetic field created by a solenoid(inductor if you like).
$\vec{F} = q\vec{v}\times\vec{B}$	Magnetic force on a moving charge.
$f = \frac{qB}{2\pi m}$	Cyclotron motion frequency.
$\vec{F} = I\vec{l}\times\vec{B}$	Magnetic force on a current carry wire.
$\vec{\tau} = \vec{\mu}\times\vec{B}$	Torque on magnetic dipole.

Chapter 30: Induced fields

$\Phi_M = \vec{A}\cdot\vec{B}$	Magnetic flux.
$\oint \vec{E}\cdot d\vec{s} = \mathcal{E} = \lvert\frac{d\Phi_M}{dt}\rvert$	Faraday's law.
$L = \frac{\mu_0 N^2 A}{l}$	Inductance.
$\Delta V_L = -L\frac{dI}{dt}$	Potential difference across an inductor.
$U = \frac{1}{2}LI^2$	Energy stored in an inductor.
$I(t) = I_{\text{max}}\sin\omega t$	Current as a function of time in an LC circuit.
$Q(t) = Q_{\text{max}}\cos\omega t$	Charge as a function of time in an LC circuit.
$\omega = \sqrt{\frac{1}{LC}}$	Frequency for an LC circuit.
$I(t) = I_0 e^{-tR/L}$	Current as a function of time in an LR circuit.

Chapter 31: EM Waves

$\oint \vec{B}\cdot d\vec{A} = 0$	Gauss's law for magnetism.
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0\mu_0\frac{d\Phi_{\text{E}}}{dt}$	Ampere-Maxwell law

$\vec{E}_{\mathrm{B}} = \vec{E}_{\mathrm{A}} + \vec{v}_{\mathrm{BA}} \times \vec{B}_{\mathrm{A}}$	Field transformation equation
$\vec{B}_{\mathrm{B}} = \vec{B}_{\mathrm{A}} - \frac{1}{c^2}\vec{v}_{\mathrm{BA}} \times \vec{E}_{\mathrm{A}}$	Field transformation equation
$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$	Poynting vector
$c^2 = \frac{1}{\epsilon_0\mu_0}$	speed of EM wave
$I = \frac{P}{A} = \frac{E_0^2}{2c\mu_0} = \frac{E_0^2 c\epsilon_0}{2}$	Definition of intensity in general (first equation) and specifically for EM waves (last two equations)
$E = cB$	Relationship between E and B fields
$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$	Lorentz Force law

Chapter 32: AC circuits

$\omega = \frac{1}{RC}$	Crossover frequency (RC AC circuit).
$X_L = \omega L$	Inductive reactance.
$X_C = \frac{1}{\omega C}$	Capacitive reactance.
$I = \frac{\mathcal{E}_0}{\sqrt{R^2+(X_L-X_C)^2}}$	Ohm's law for AC circuits.
$\tan\phi = \frac{X_L-X_C}{R}$	Phase angle.
$\omega_0 = \frac{1}{\sqrt{LC}}$	Resonant frequency.
$P_{\mathrm{avg}} = \frac{1}{2}I_R^2 R = (I_{\mathrm{rms}})^2 R = \frac{(V_{\mathrm{rms}})^2}{R} = I_{\mathrm{rms}}V_{\mathrm{rms}}$	Power dissipated by resistor.
$V_{\mathrm{rms}}(I_{\mathrm{rms}}) = \frac{V_R(I_R)}{\sqrt{2}}$	Relationship between peak value and root mean square value.
$P = I_{\mathrm{rms}}\mathcal{E}_{\mathrm{rms}}\cos\phi$	Power delivered by a source in AC circuit.

Useful Math equations

quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric functions:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Dot Product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = |A||B| \cos\alpha$ (α is the angle between the two vectors)

Cross Product: $\vec{A} \times \vec{B} = |A||B| \sin\alpha$ (α is the angle between $\vec{A}$ and $\vec{B}$. Direction given by the right-hand rule)

surface area of sphere: $4\pi r^2$

volume of a sphere: $\frac{4}{3}\pi r^3$

differential volume in spherical coordinates: $dV = r^2 \sin\phi dr d\phi d\theta$

differential area in polar coordinates: $dA = r dr d\theta$

differential area in spherical coordinates: $dA = R^2 \sin\phi d\theta d\phi$

Logarithmic properties

$\log_{\text{b}}(mn) = \log_{\text{b}}(m) + \log_{\text{b}}(n)$

$\log_{\text{b}}(m/n) = \log_{\text{b}}(m) - \log_{\text{b}}(n)$

$\log_{\text{b}}(m^n) = n \cdot \log_{\text{b}}(m)$

Helpful Integrals

$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{a^2 + x^2}}$

$\int \frac{x\ dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$

$\int \frac{dx}{a+x} = \ln(a + x)$

$\int \frac{x\ dx}{a+x} = x - a\ln(a + x)$

$\int \frac{1\ dx}{((x-b)^2 + a^2)^{3/2}} = \frac{x-b}{a^2\sqrt{a^2 + (b-x)^2}}$

$\int \frac{x\ dx}{((x-b)^2 + a^2)^{3/2}} = \frac{x-a^2}{a^2\sqrt{a^2 + (b-y)^2}}$

$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$

$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$

$\int x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4}$

$\int \frac{1}{x} dx = \ln(x)$

$\int \frac{dx}{\sqrt{x^2 + b^2}} = \ln\left[x + \sqrt{x^2 + b^2}\right]$

Constants

Mass of Earth: 5.98×10^{24} kg
Radius of Earth: 6.37×10^{6} m
Free fall acceleration: $g = 9.80$ m/s^2
Gravitational constant: $G = 6.67 \times 10^{-11}$ N m^2/kg^2
Fundamental charge: $e = 1.60 \times 10^{-19}$ C
Coulomb constant: $k = 1/4\pi\epsilon_0 = 8.99 \times 10^{9}$ N $\cdot$ m^2/C
Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N$\cdot$ m^2
mass of proton: $m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
mass of electron: $m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
permeability of free space: $\mu_0 = 1.26 \times 10^{-6}$ N / A^2

Conversions
1 in = 2.54 cm
1 mile = 1.609 km
1 meter = 39.37 in
1 mph = 0.447 m/s
1 rad = $180^\circ/\pi = 57.3^\circ$
1 rev = $360^\circ = 2\pi$ rad