Time-dependent Schrodinger Equation

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

assume that.. $\psi = X(x)T(t)$

$$i\hbar\dot{T}X = -\frac{\hbar^2}{2m}\ddot{X}T + VXT$$

$$\overbrace{i\hbar\frac{\dot{T}}{T}}^{\text{function of time}} = \underbrace{-\frac{\hbar^2}{2m}\frac{\ddot{X}}{X} + V\frac{X}{X}}_{\text{function of position}}$$

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$$-\frac{\hbar^2}{2m}\frac{\ddot{X}}{X} + V\frac{X}{X} = \overbrace{E}^{\text{constant}}$$ (harder... maybe)

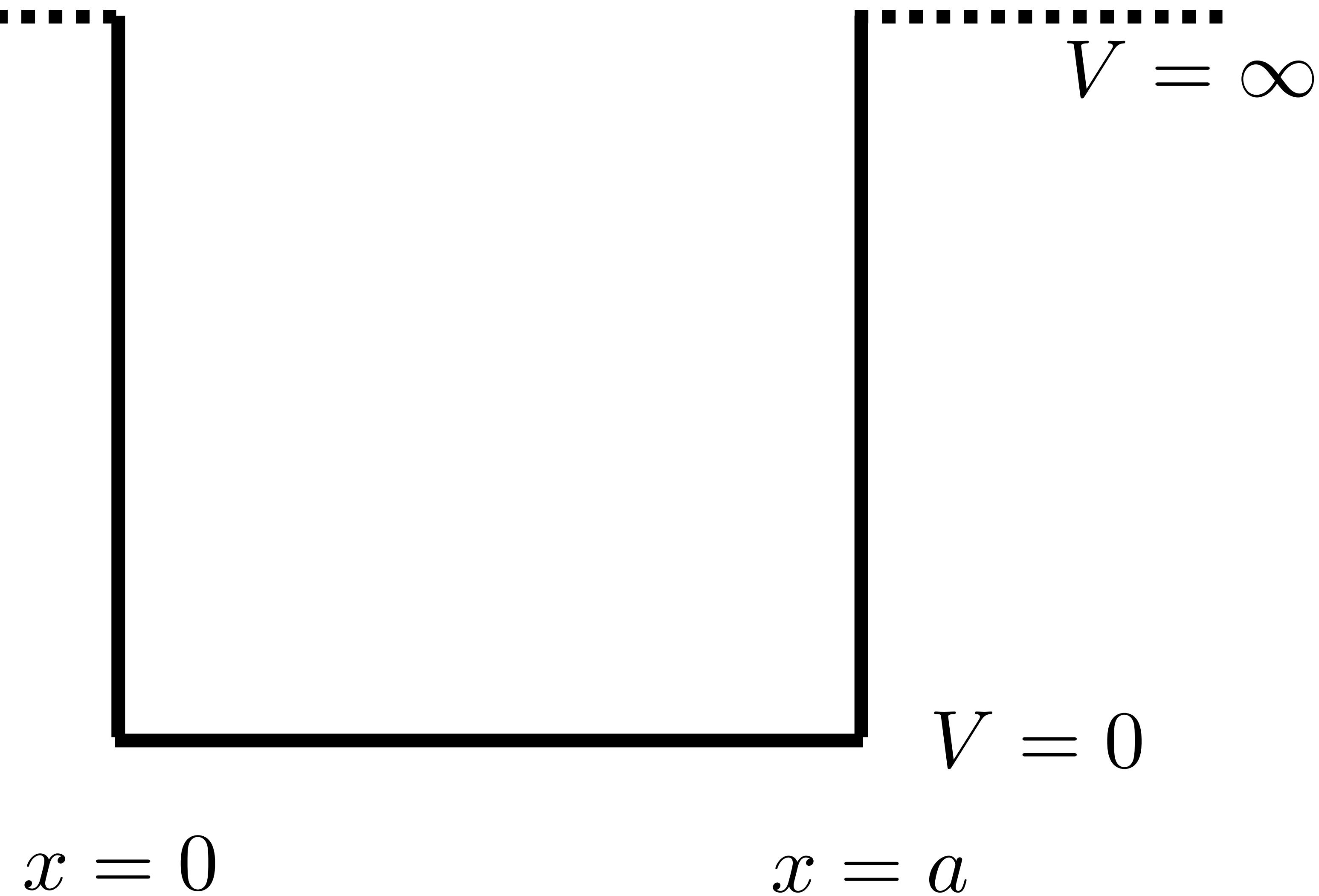
$$i\hbar\frac{\dot{T}}{T} = E$$ (easy)

$$T = e^{-i\frac{E}{\hbar}t}$$

Let's try it… on the infinite square well.

$$-\frac{\hbar^2}{2m}\frac{\ddot{X}}{X} + V\frac{X}{X} = \overbrace{E}^{\text{constant}}$$

$$-\frac{\hbar^2}{2m}\ddot{\psi} + V\psi = E\psi$$



$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$$ (Eigenfunctions)

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$ (Eigenvalues)

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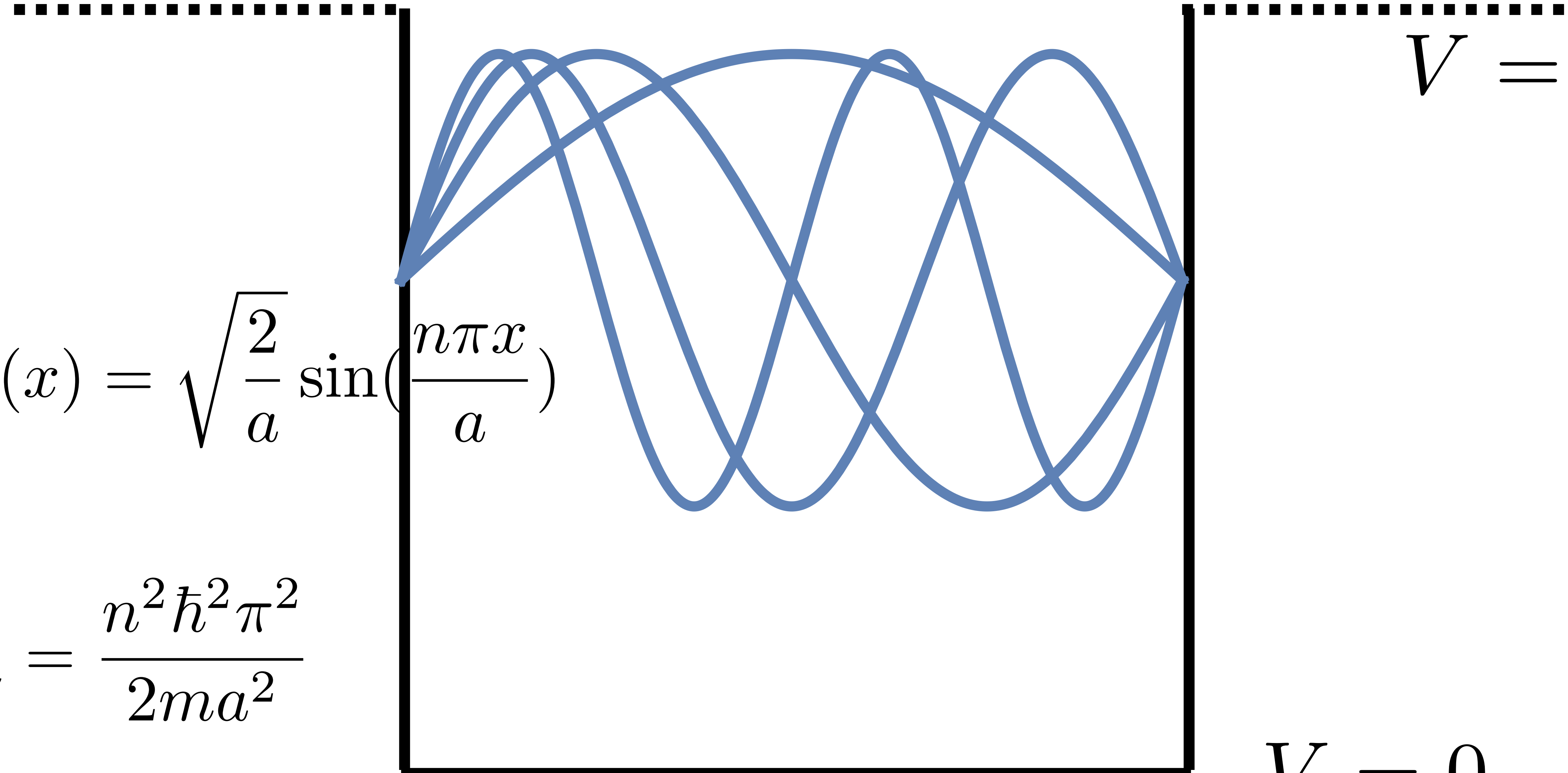

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(Eigenfunctions)

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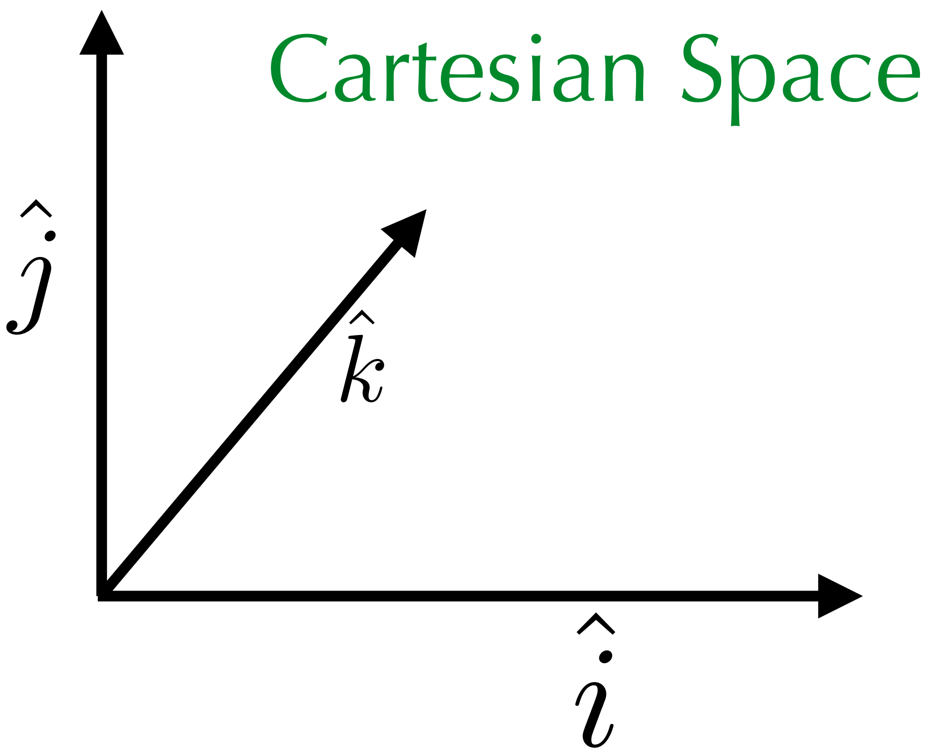
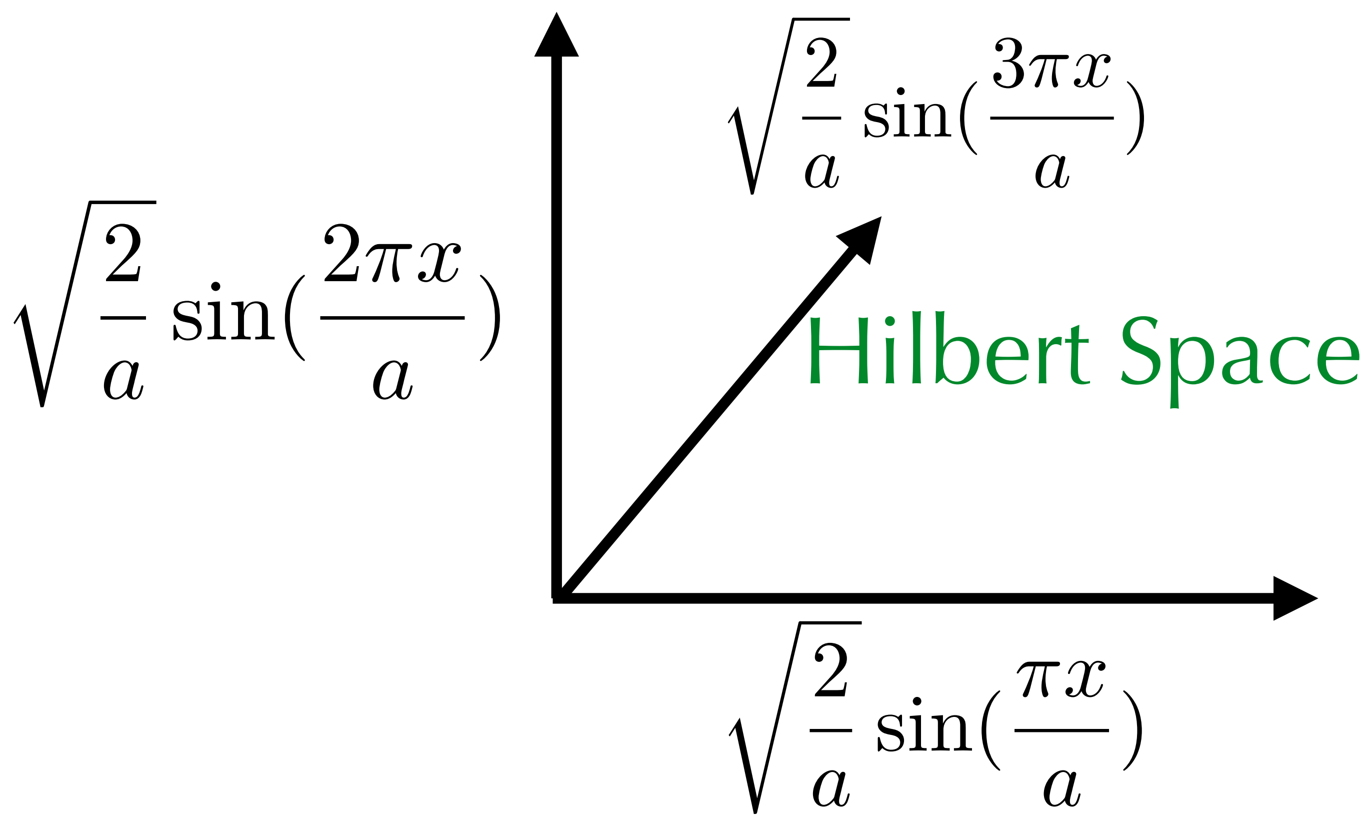
(Eigenvalues)

The Eigenfunctions

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$$

1. ***Orthonormal***
2. ***Complete***





$\hat{i} \cdot \hat{j}$ Inner Product $\int \psi_n^* \psi_m dx$

The Eigenfunctions

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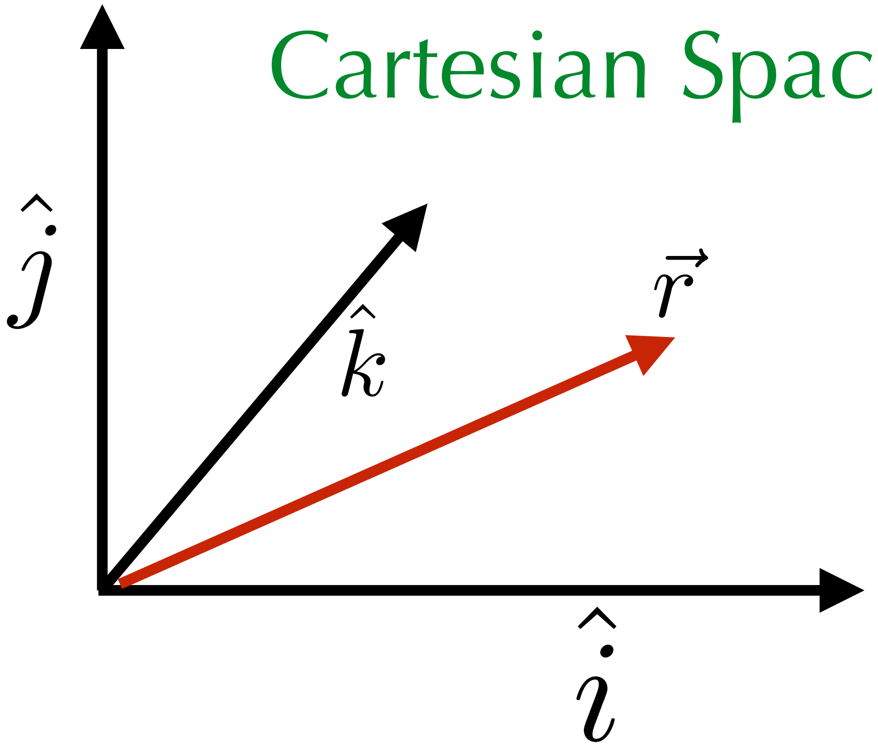
1. ***Orthonormal***
2. ***Complete***

$$\overbrace{\psi(x)}^{\text{any function}} = \sum_n \boxed{c_n}\psi_n(x)$$

How do we find these?

Cartesian Space



The Eigenfunctions

$$\psi(x) = \sqrt{\frac{2}{a}}\left(\frac{\sin(\frac{2\pi x}{a}) + 2\sin(\frac{\pi x}{a})}{\sqrt{5}}\right)$$

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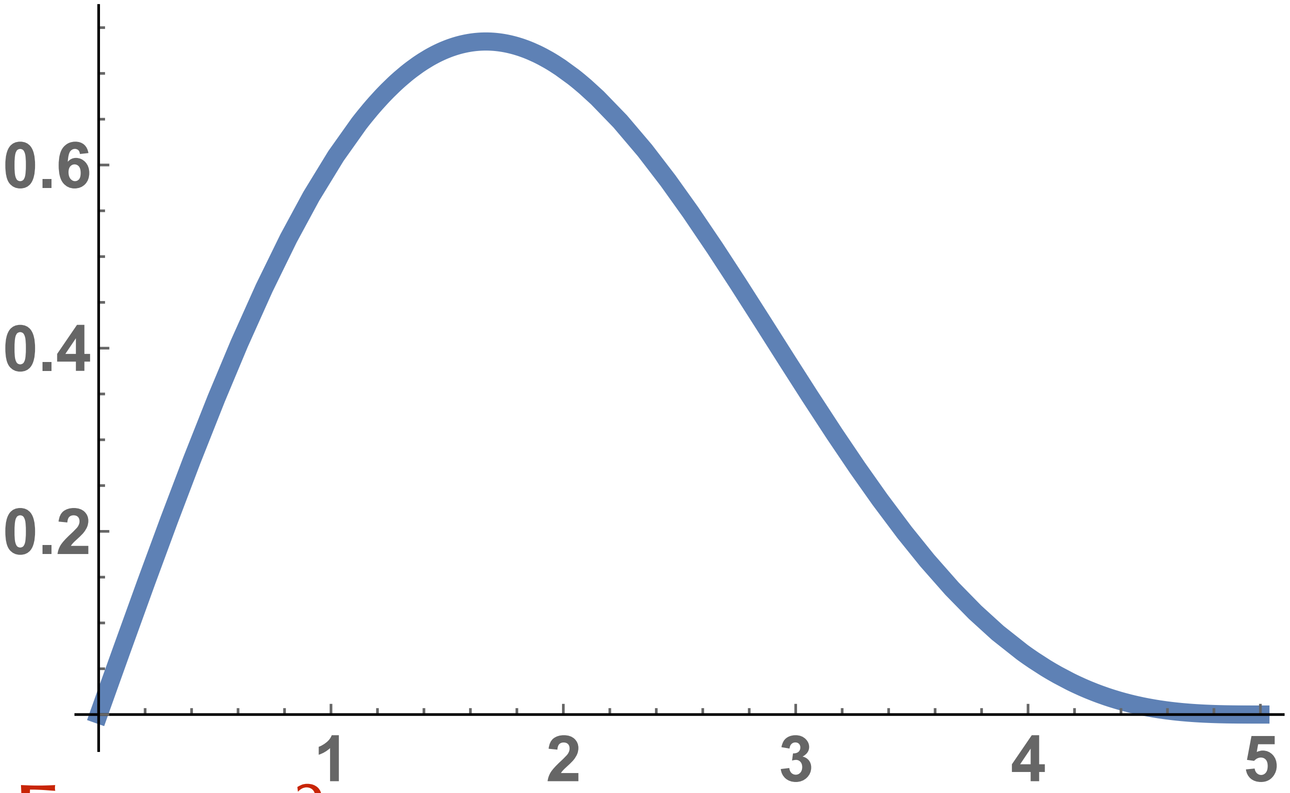

What are the c_n?

Is the function normalized?

What is the expectation value of Energy?

With what probability will E_1 and E_2 be measured?

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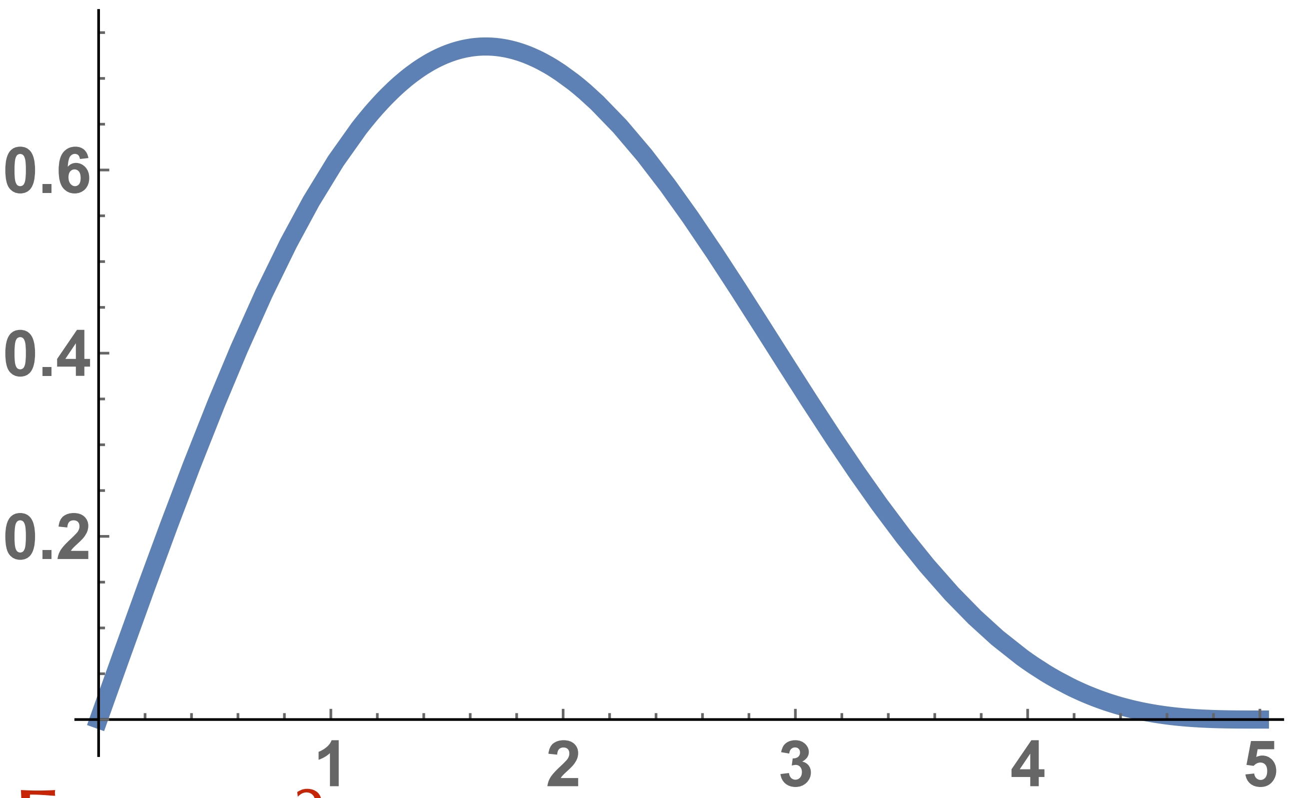

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