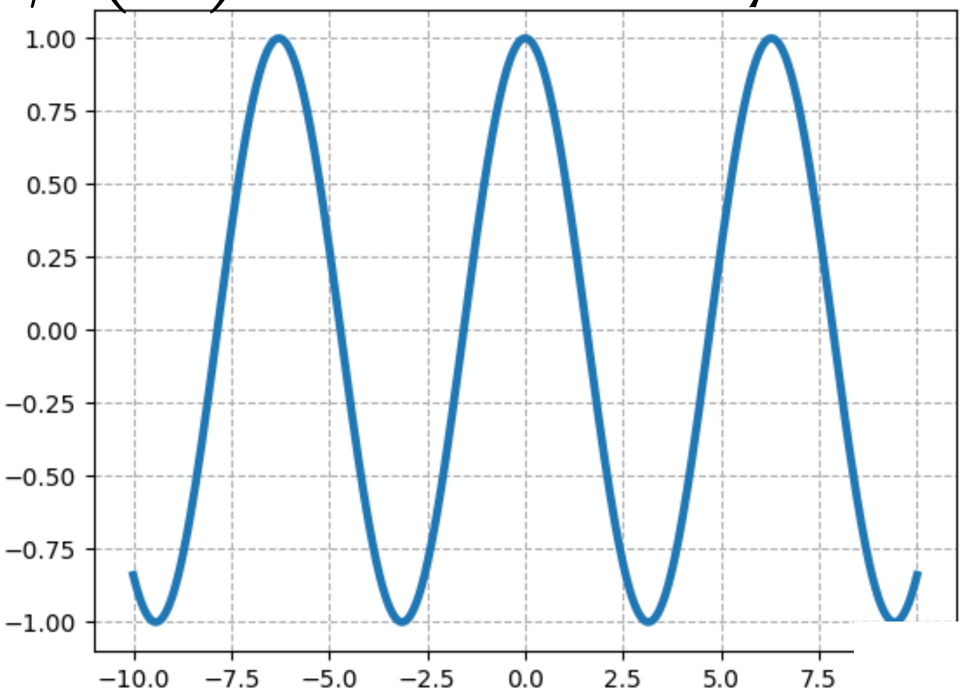
WaveFunctions

Postulate 1: The state of a particle/system is completely specified by the wave function.

Is it possible to observe a wave function?

Is any function a valid wave function?

$\psi(x)$



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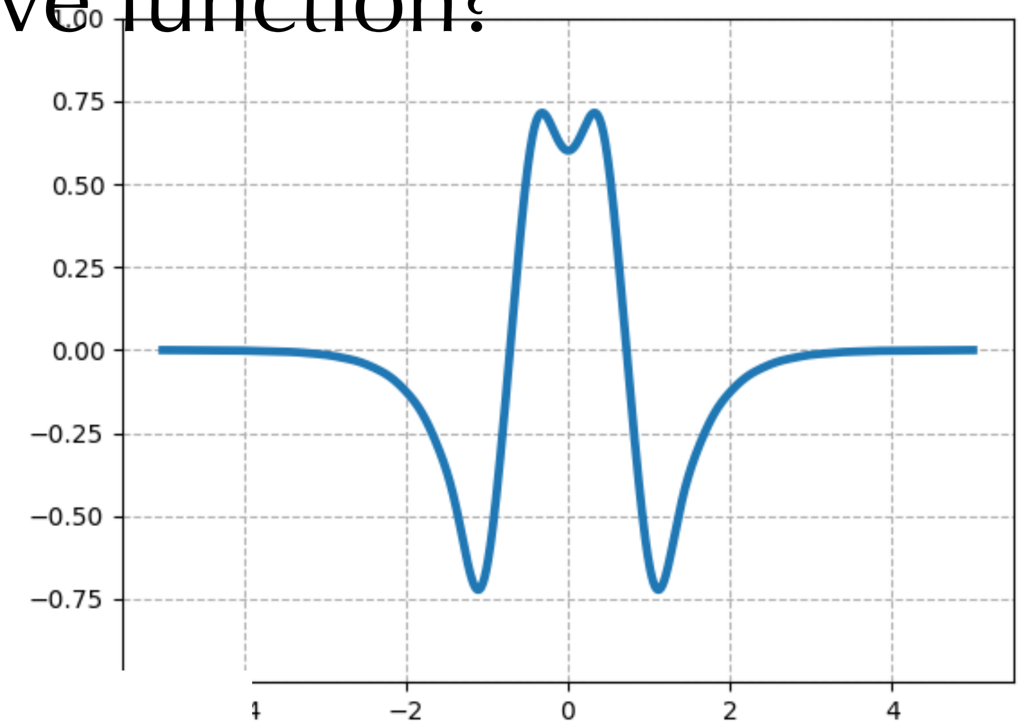

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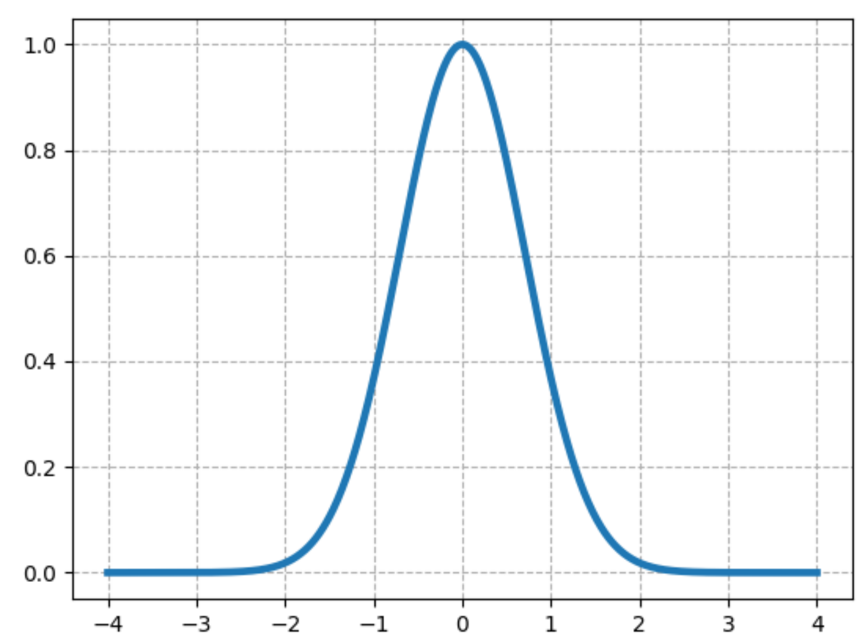
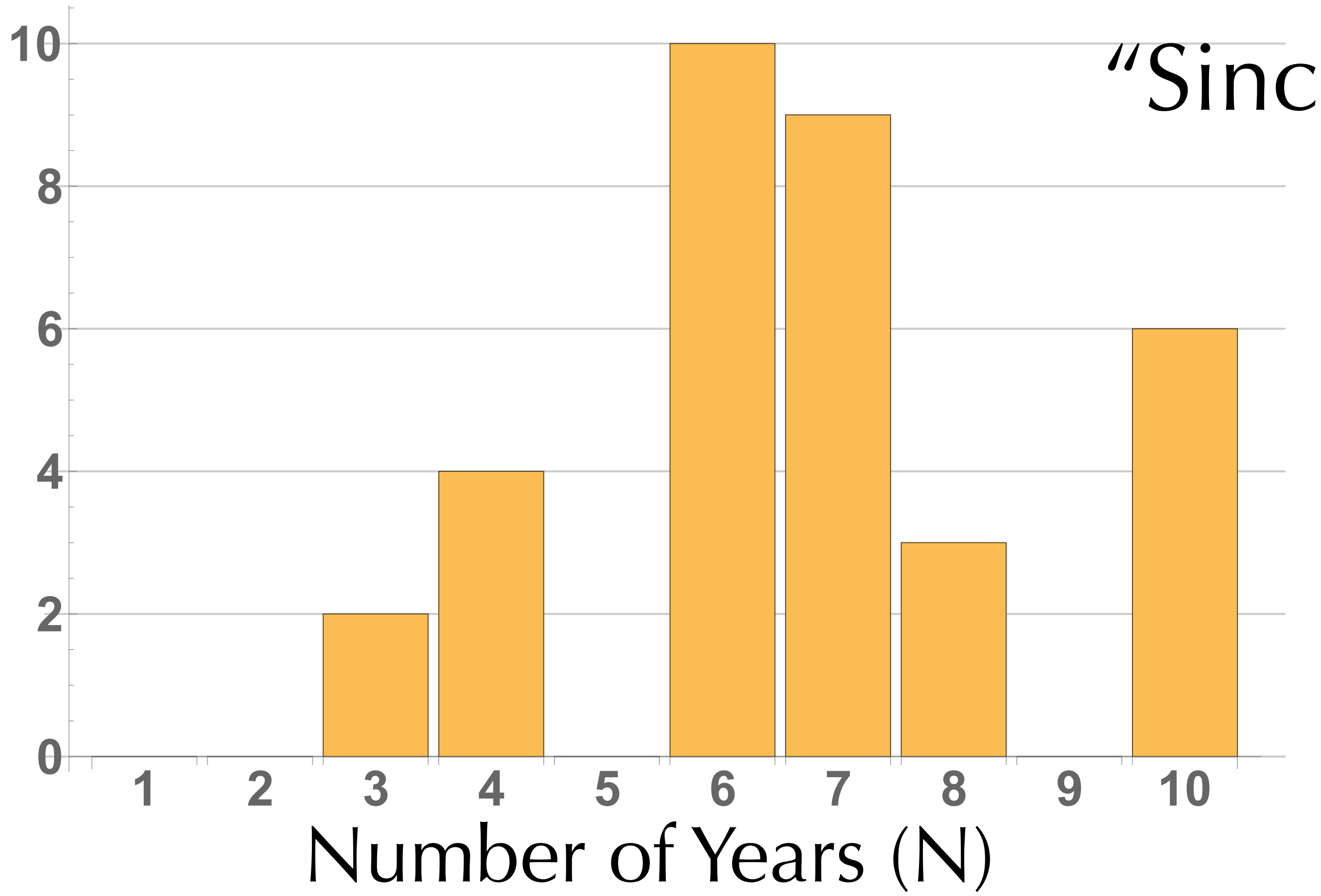


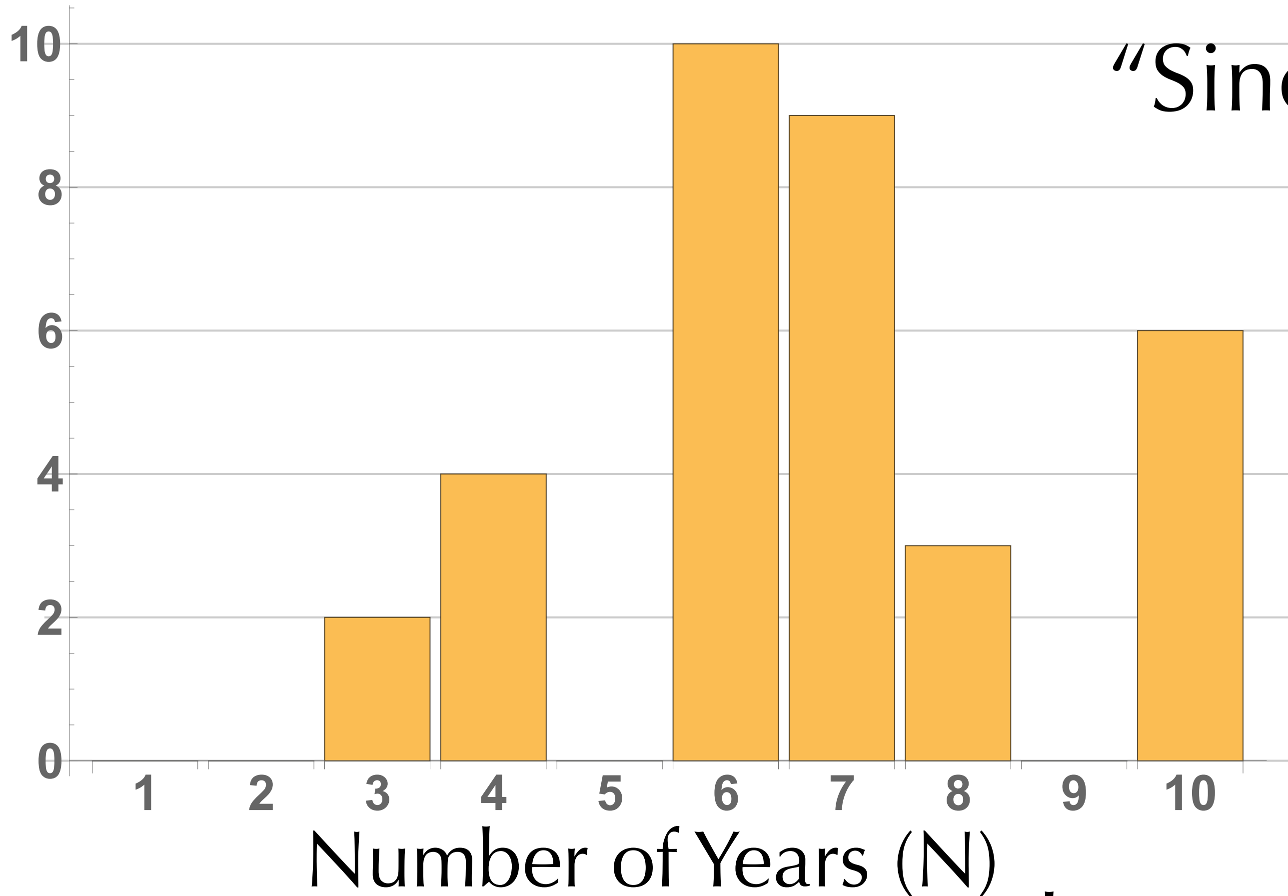
Question #21 Average (Expectation) Values



"Since high school how many years have you been in school?

34 total people

Question #21 Average (Expectation) Values



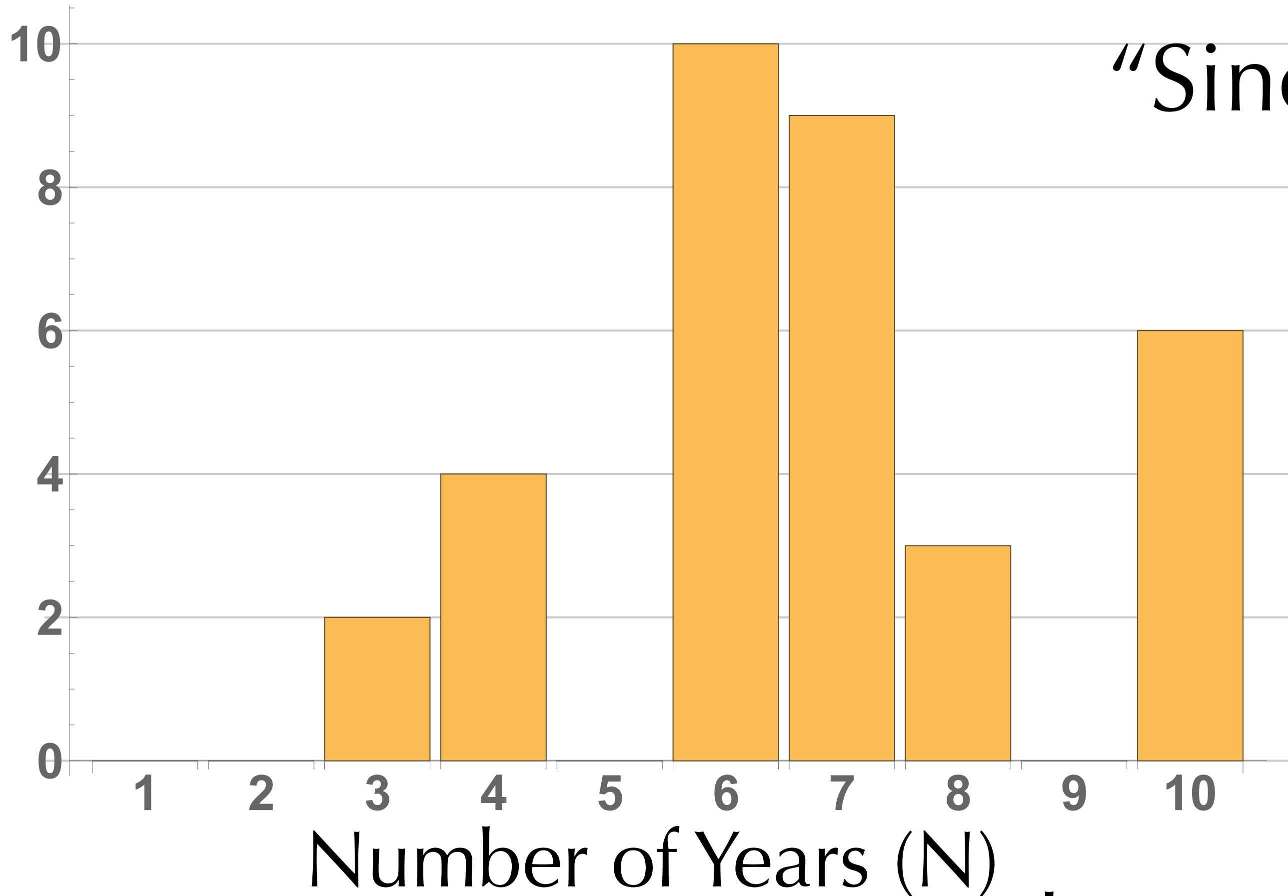
"Since high school how many years have you been in school?

34 total people

What is the **average** number of post-high-school years in school?

a) 7.26
b) 6.74
c) 6.23
d) 6.52
e) 6.85

Question #21 Average (Expectation) Values



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What is the **average** number of post-high-school years in school?

34 total people

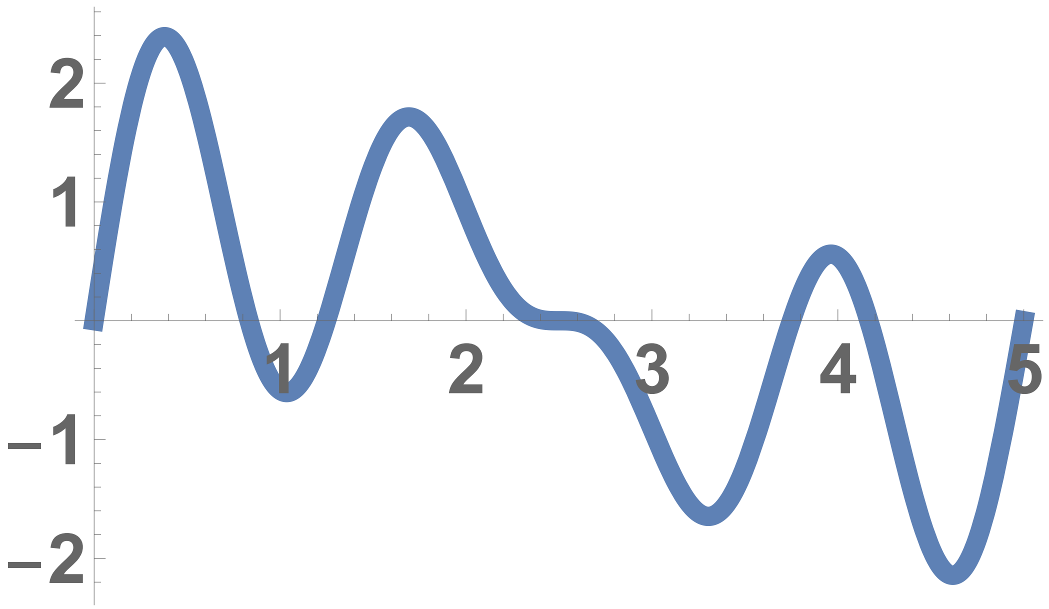
$$\langle N \rangle = \sum_i N_i P(N_i)$$

a) 7.26
b) 6.74
c) 6.23
d) 6.52
e) 6.85

Question #22

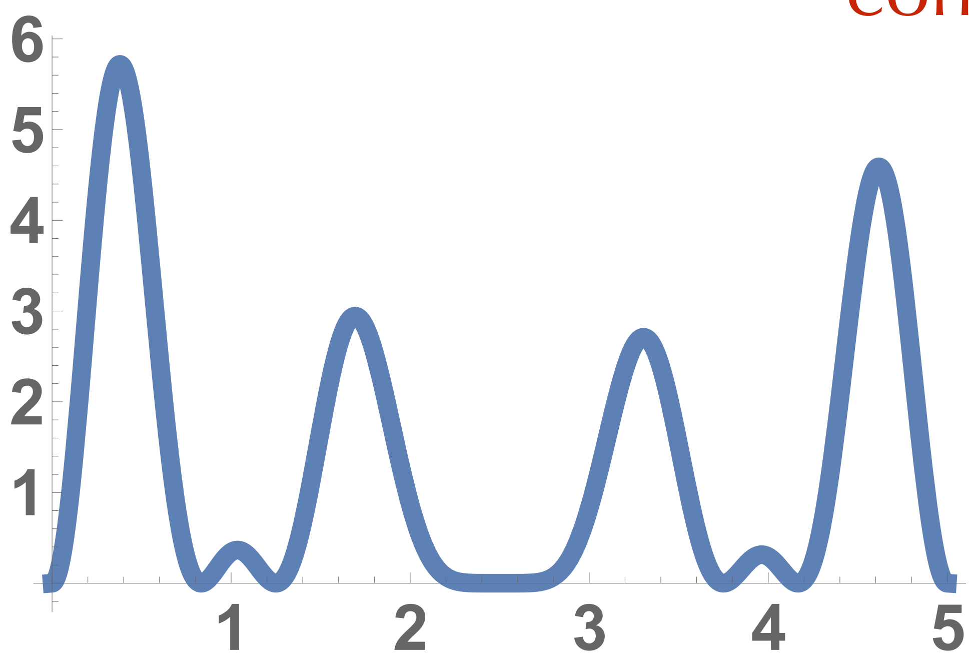
Continuous Probability Distributions

$$\psi(x) = \frac{1}{\sqrt{6.84592}} \cos(\frac{x}{10}) \left(\sin(\frac{2\pi x}{a}) + \sin(\frac{6\pi x}{a}) + \sin(\frac{8\pi x}{a}) \right)$$



discrete $\langle N \rangle = \sum_i N_i P(N_i)$

continuous



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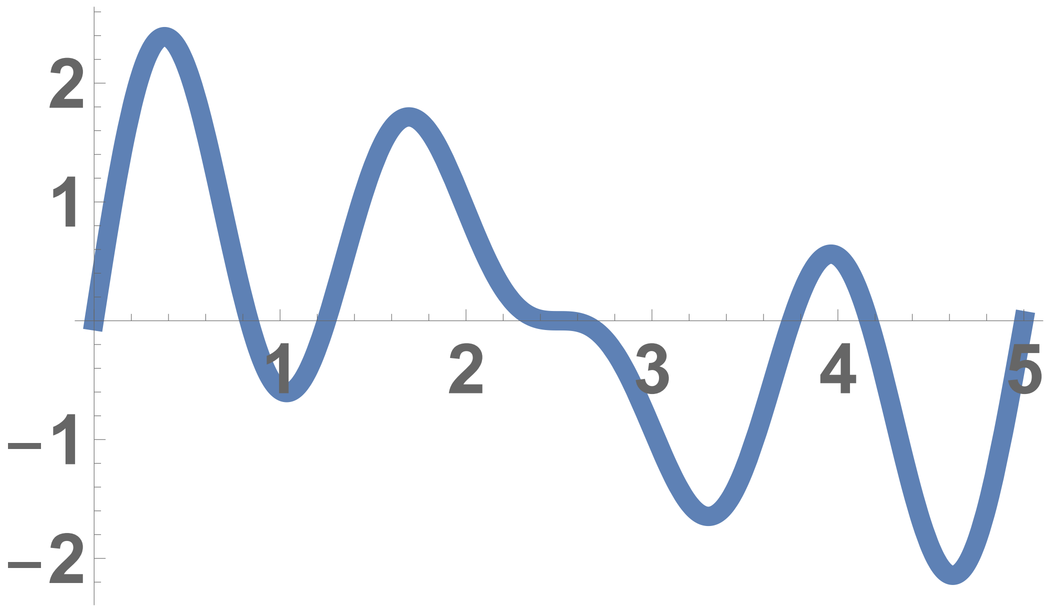

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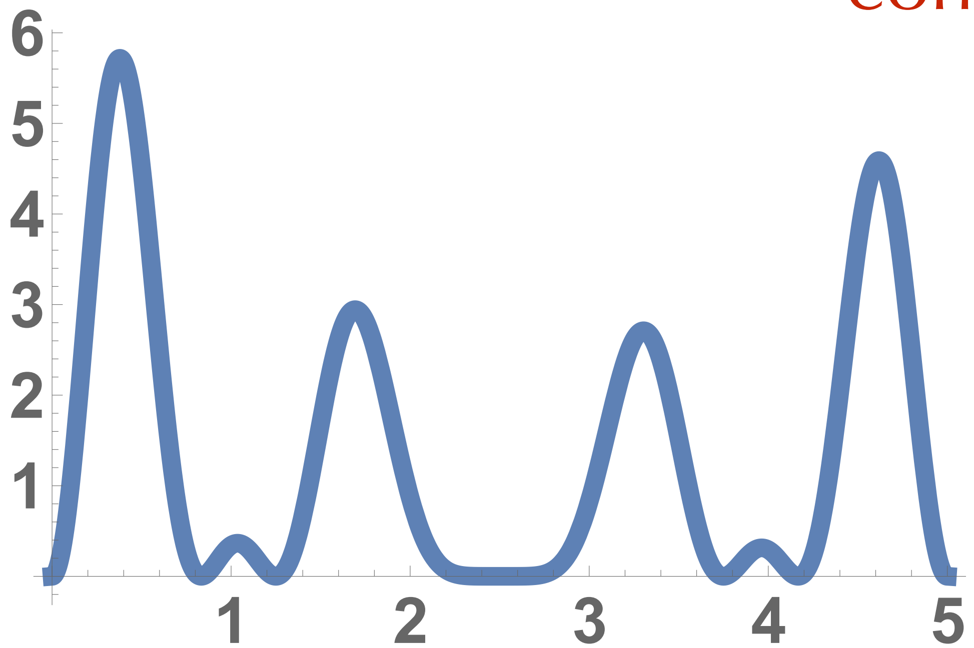

continuous $$\langle x \rangle = \int x\psi^*(x)\psi(x)dx$$

a) 4.35
b) 2.65
c) 0.56
d) 2.35
e) 1.5

Question #23

Continuous Probability Distributions

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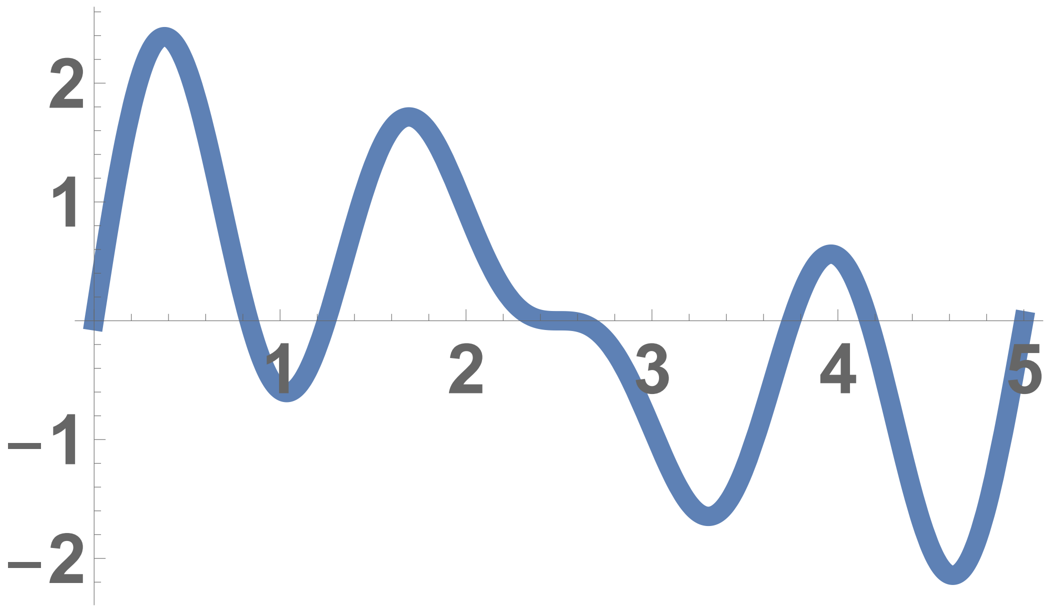
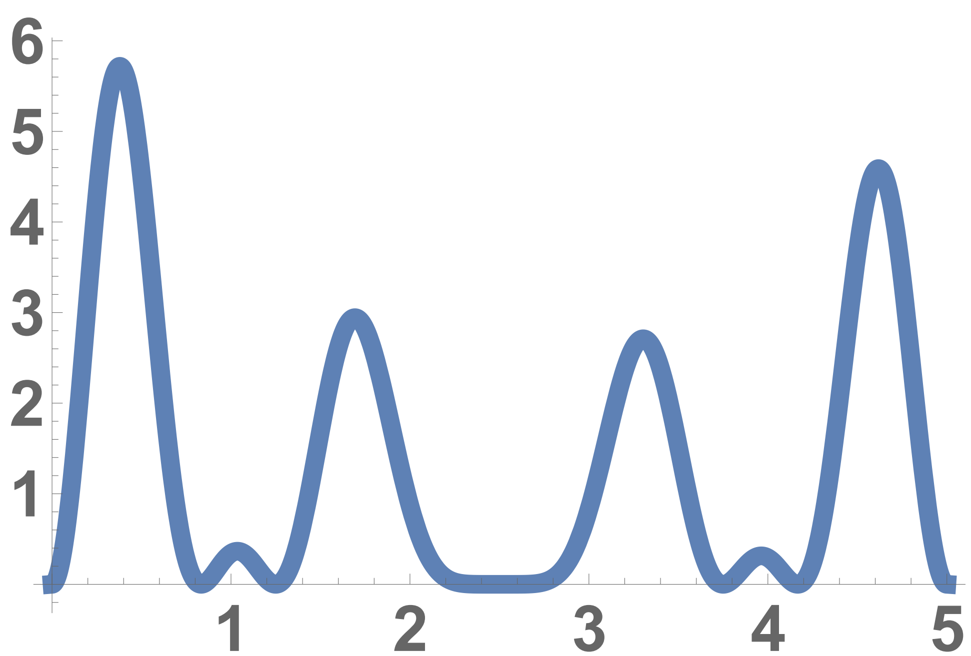


$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$



What is the uncertainty (σ^2) in x?

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Continuous Probability Distributions

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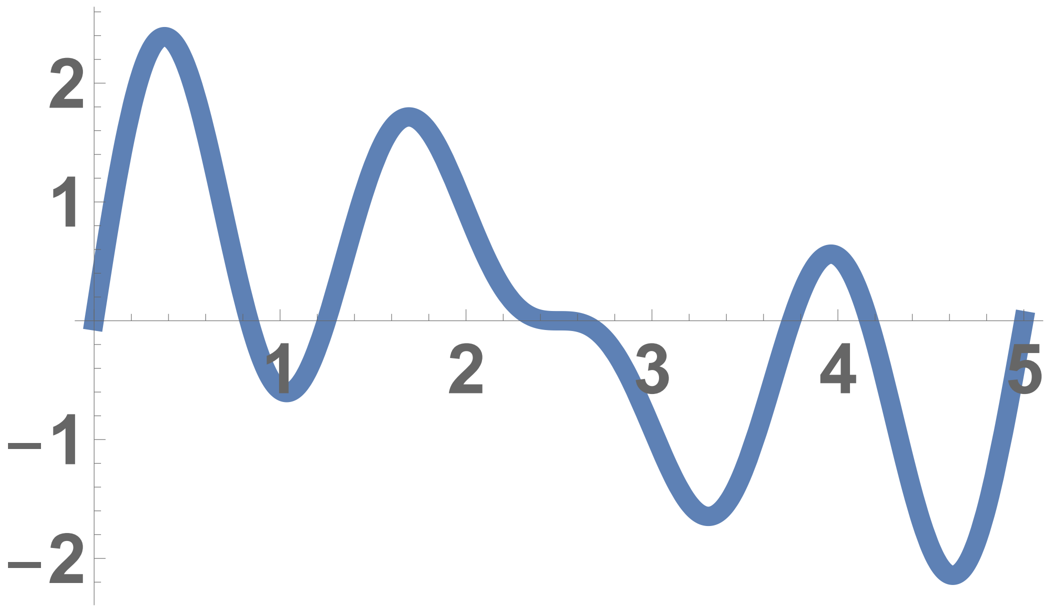

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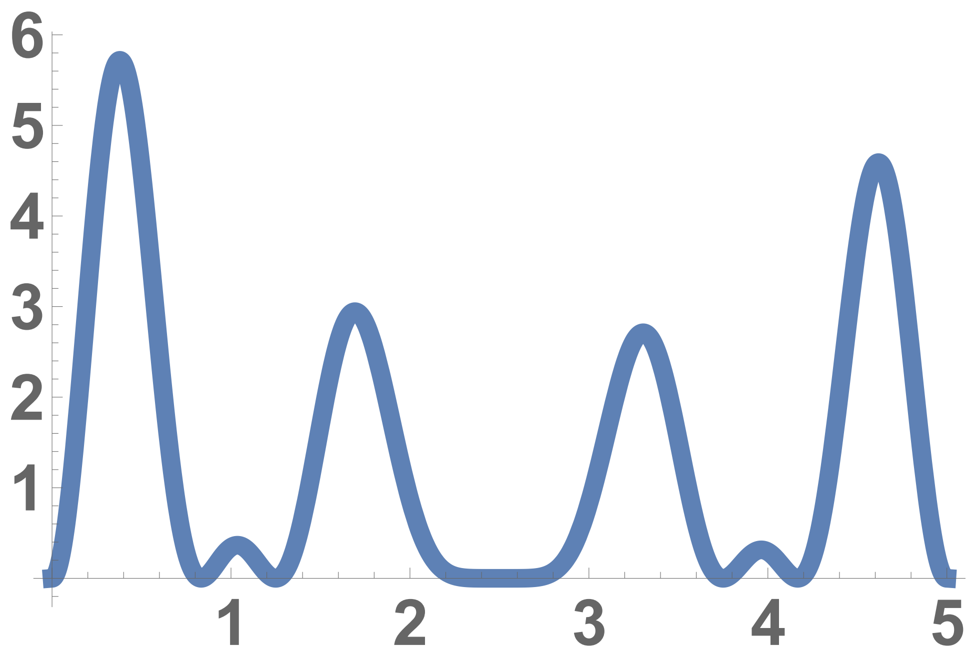

What is the uncertainty (σ^2) in x?

a) 5.5
b) 3.54
c) 1.98
d) 6.11
e) 2.95

Question #24

Operators

Postulate 2: For every physical observable there is a corresponding operator in Quantum Mechanics.

$$\hat{p} = -i\hbar\frac{d}{dx}$$

If you were guessing, what would you say the kinetic energy operator is?

Hint: Can you write kinetic energy in terms of momentum?

$$\hat{K} =$$

a) $\hat{K} = \frac{\hbar^2}{2m}\frac{d}{dx}$

b) $\hat{K} = \frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

c) $\hat{K} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

Operators

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Hint: Can you write kinetic energy in terms of momentum?

$$\hat{K} =$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

a) $\hat{K} = \frac{\hbar^2}{2m}\frac{d}{dx}$

b) $\hat{K} = \frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

c) $\hat{K} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

Question #25

Operators

$$\hat{p} = -i\hbar\frac{d}{dx}$$

$$-i\hbar\frac{d}{dx}\psi(x) = p\psi(x)$$

Eigenvalue problem

a) $\psi(x) = e^{ikx}$

b) $\psi(x) = e^{kx}$

c) $\psi(x) = e^{-kx}$

Hint: The eigenvalue must be real!

Back to Expectation Values

$$\langle x \rangle = \int \boxed{x}\, \psi^*(x)\psi(x)dx$$

actually...

$$\langle x \rangle = \int \psi^*(x)\boxed{x}\,\psi(x)dx$$

$$\langle p \rangle =$$

Back to Expectation Values

$$\langle x \rangle = \int \boxed{x}\,\psi^*(x)\psi(x)dx$$

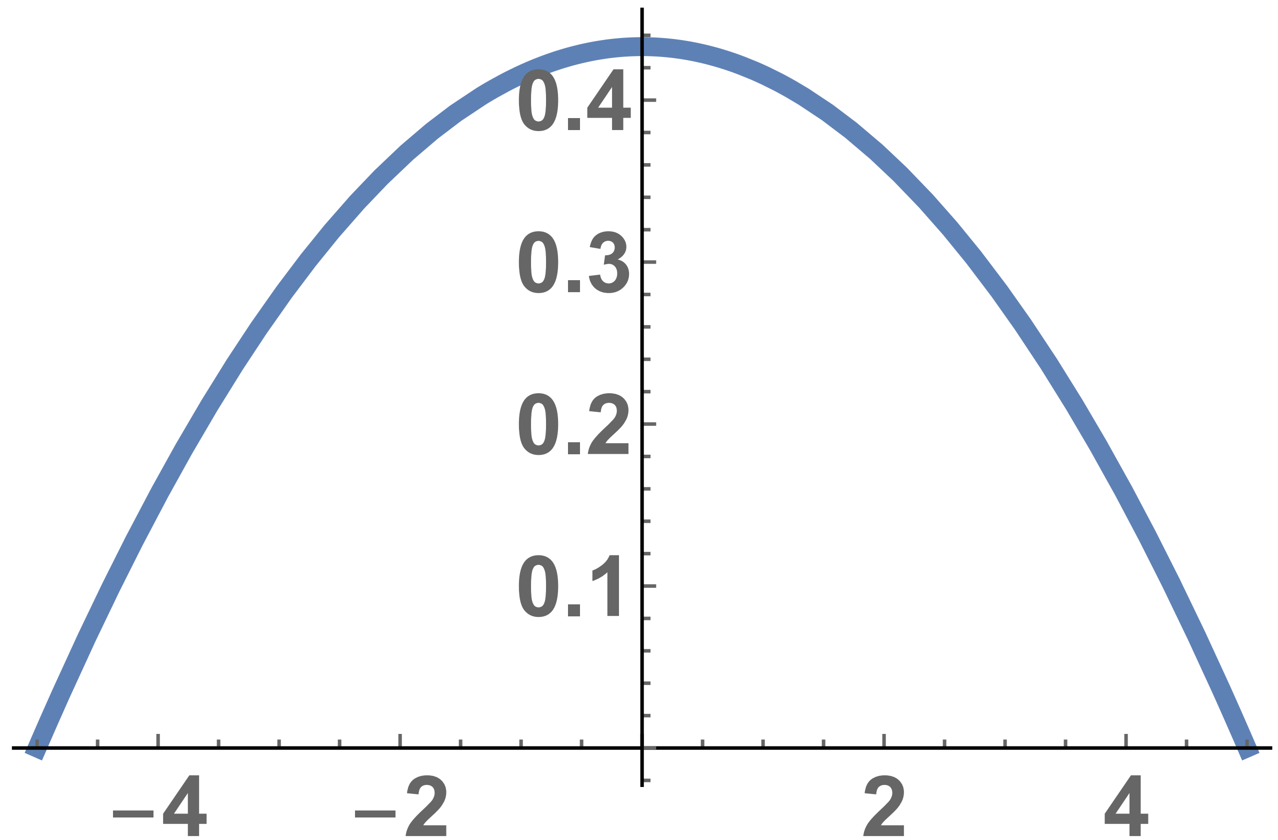
actually…

$$\langle x \rangle = \int \psi^*(x)\boxed{x}\,\psi(x)dx$$

$$\boxed{\langle p \rangle = \int \psi^*(x)\ \hat{p}\ \psi(x)dx}$$

Back to Expectation Values

A	0
$\langle x \rangle$	$\dfrac{5\hbar^2}{2a^2}$
$\langle x^2 \rangle$	$\dfrac{a}{\sqrt{7}}$
$\langle p \rangle$	$\sqrt{\dfrac{5}{2}}\dfrac{\hbar}{a}$
$\langle p^2 \rangle$	$\dfrac{a^2}{7}$
σ_x	0
σ_p	$\sqrt{\dfrac{15}{16a^5}}$

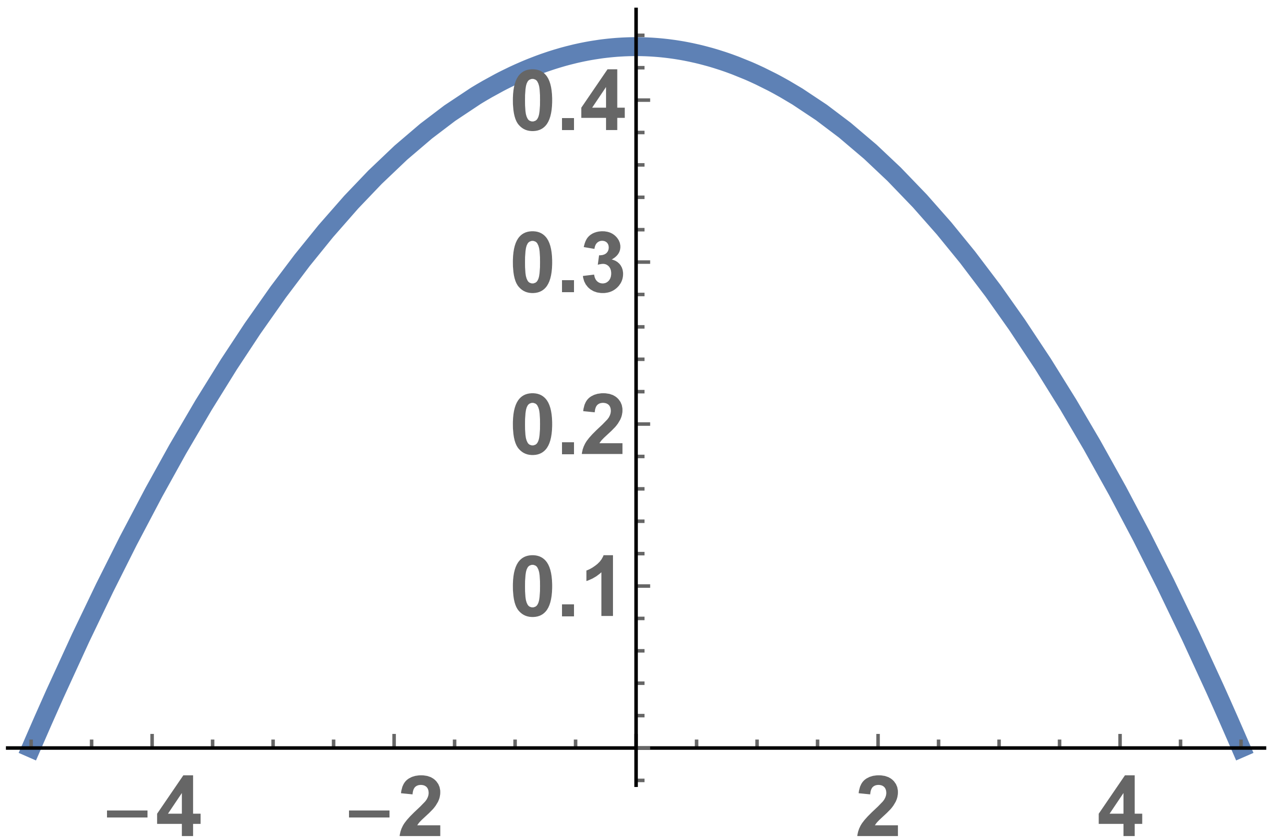


$$\psi(x) = A(a^2 - x^2)$$

$$-a \leq x \leq +a$$

Back to Expectation Values

A	$\sqrt{\frac{15}{16a^5}}$	0
$\langle x \rangle$	0	$\frac{5\hbar^2}{2a^2}$
$\langle x^2 \rangle$	$\frac{a^2}{7}$	$\frac{a}{\sqrt{7}}$
$\langle p \rangle$	0	$\sqrt{\frac{5}{2}}\frac{\hbar}{a}$
$\langle p^2 \rangle$	$\frac{5\hbar^2}{2a^2}$	$\frac{a^2}{7}$
σ_x	$\frac{a}{\sqrt{7}}$	0
σ_p	$\sqrt{\frac{5}{2}}\frac{\hbar}{a}$	$\sqrt{\frac{15}{16a^5}}$



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