

## 1 Day 3: Properties of Eigenfunction, Basis Sets

1. The eigenstates for the infinite square well are given by:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad (1)$$

Show that these eigenstates are orthonormal.

2. (Note: You should not use a computer to do this problem, it's not hard enough.) The wavefunction for a particle that is confined to an infinite square well is given by:

$$\psi(x) = \sqrt{\frac{2}{a}} \left( \frac{\sin(\frac{2\pi x}{a}) + 2 \sin(\frac{\pi x}{a})}{\sqrt{5}} \right) \quad (2)$$

1. Is the wavefunction normalized?
  2. What is the expectation value of energy:  $\langle E \rangle$ ?
  3. With what probability will energy  $E_1$ , be measured? ( $E_2$ ,  $E_3$ ?)
3. (Note: Use a computer for this one.) The wavefunction for a particle that is confined to an infinite square well ( $0 < x < a$ ) is given by:

$$\psi(x) = \frac{\sqrt{2}}{2} \quad (3)$$

for  $\frac{a}{2} - 1 < x < \frac{a}{2} + 1$   
and

$$\psi(x) = 0 \quad (4)$$

elsewhere.

1. Is the wavefunction normalized?
2. Find the expansion coefficients  $c_n$ .
3. Reconstruct the wave function as a sum of energy eigenstates and plot to verify that it is correct.
4. What is the expectation value of energy:  $\langle E \rangle$ ?
5. With what probability will energy  $E_1$ , be measured? ( $E_2$ ,  $E_3$ ?)
6. Make an animation of the wavefunction for  $t > 0$