Conductivity and bands

$$v = \frac{d\omega}{dk}$$

$$E = \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}$$

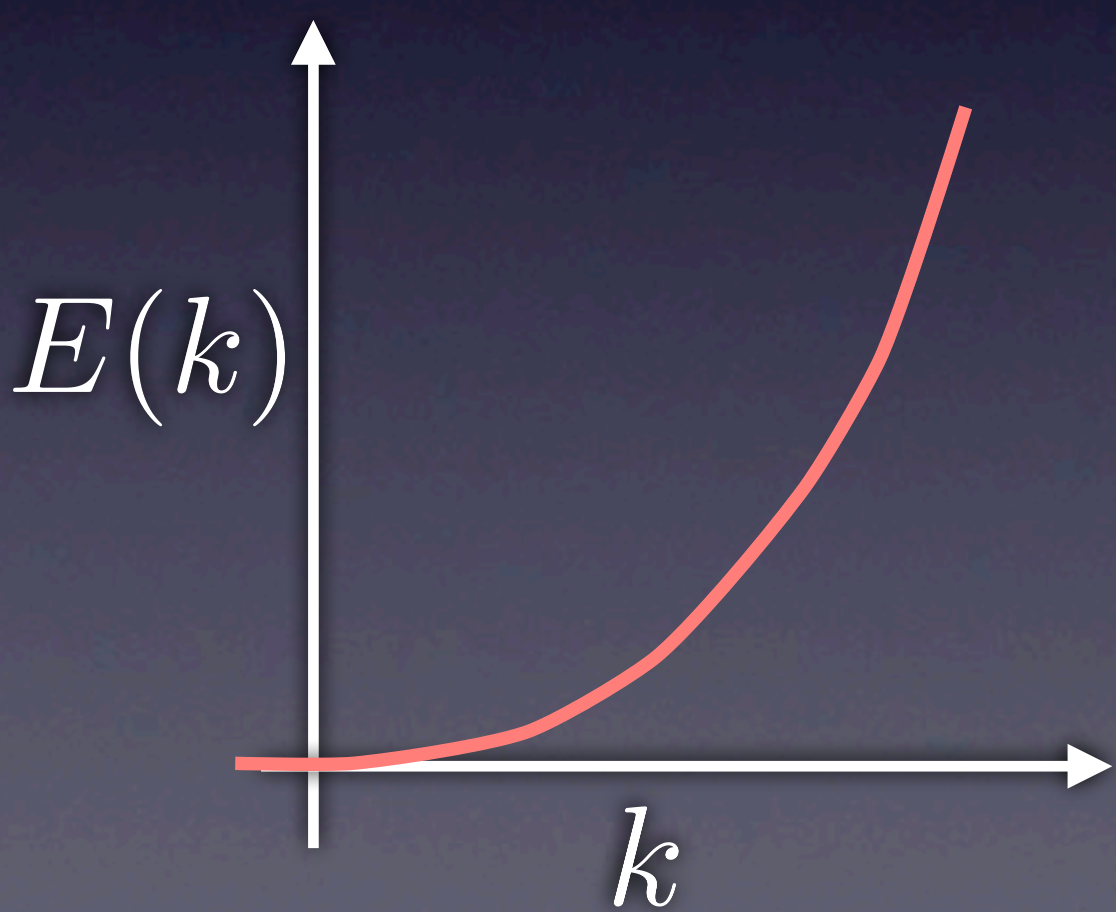
Conductivity and bands

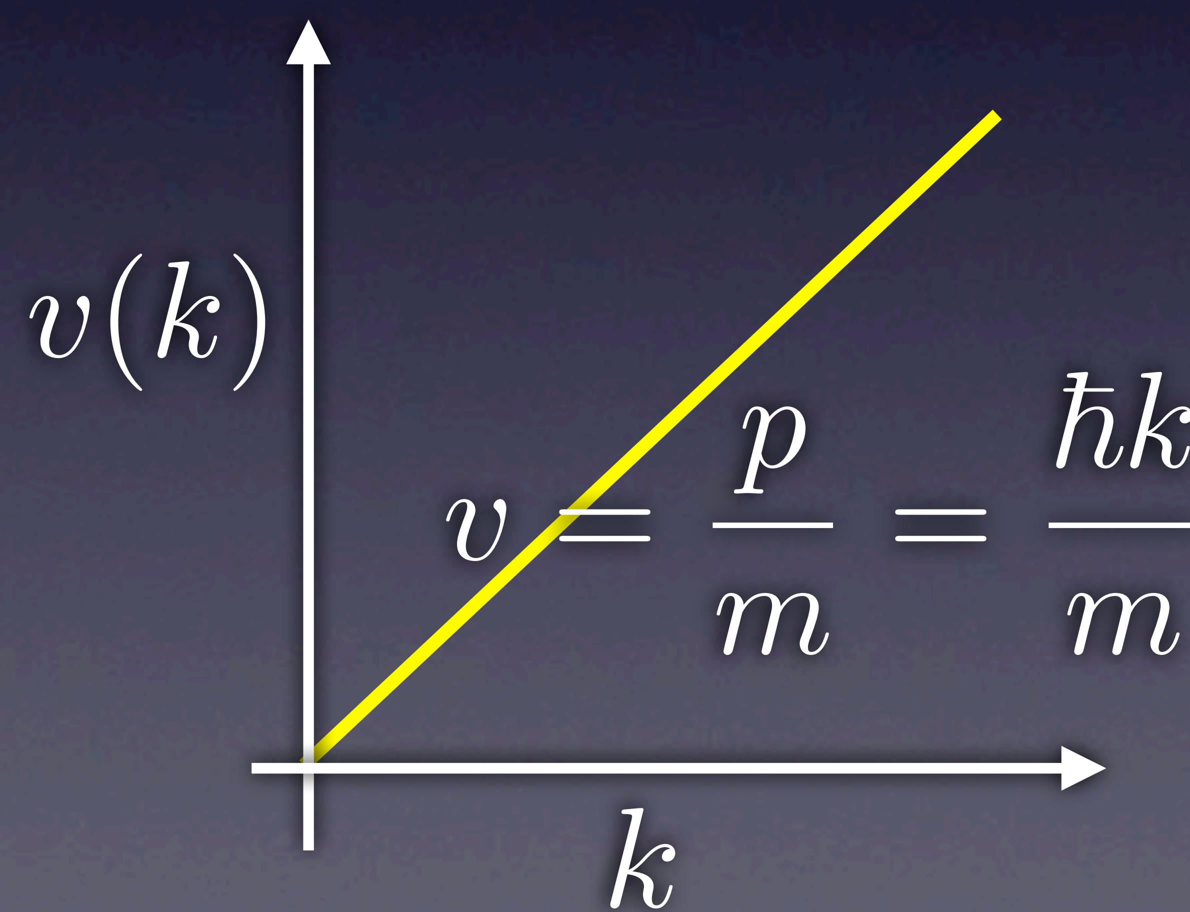
$$v = \frac{d\omega}{dk}$$

$$E = \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}$$

Free particle (no forces)

$$
\begin{aligned}
v &= \frac{d\omega}{dk} \\
E &= \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}
\end{aligned}
$$





In a crystal...

$$\begin{aligned} v &= \frac{d\omega}{dk} \\ E &= \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk} \end{aligned}$$

In a crystal...

$$v = \frac{d\omega}{dk}$$

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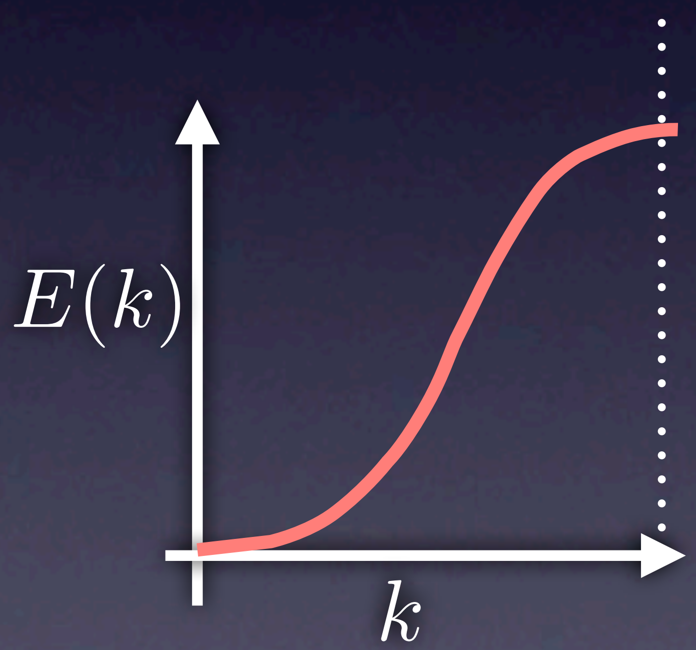
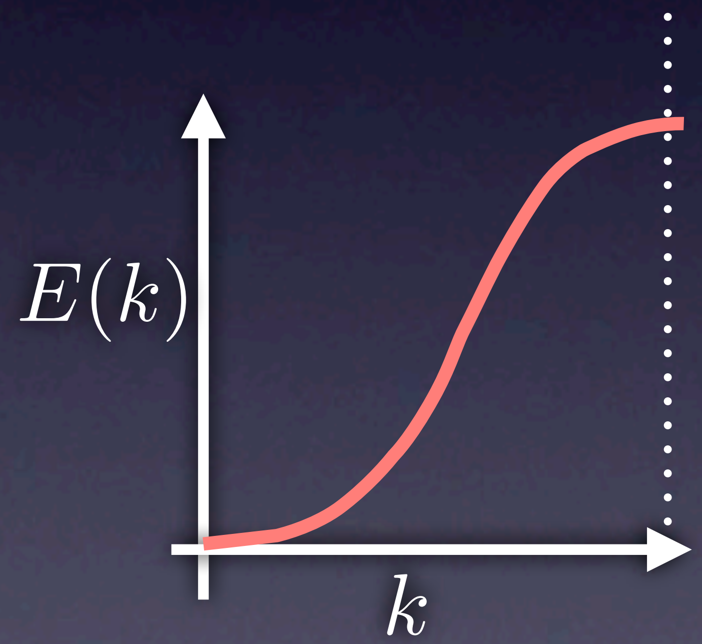


In a crystal...

$$v = \frac{d\omega}{dk}$$

$$E = \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}$$



The velocity of the electrons with the greatest energy is

(B) zero, or

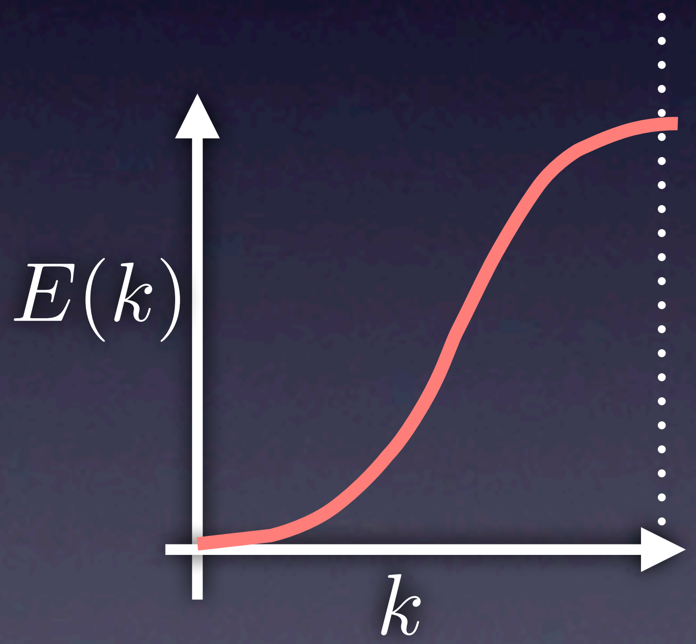
(C) not zero.

Question #10

In a crystal...

$$v = \frac{d\omega}{dk}$$

$$E = \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}$$

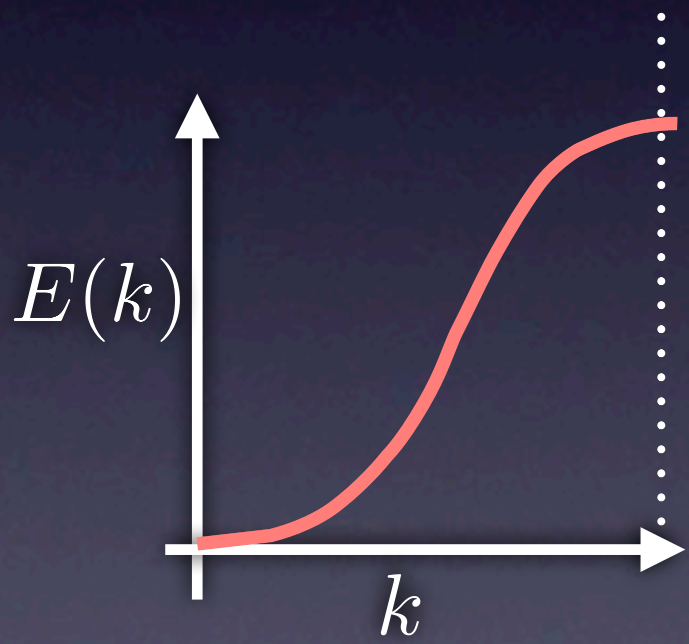


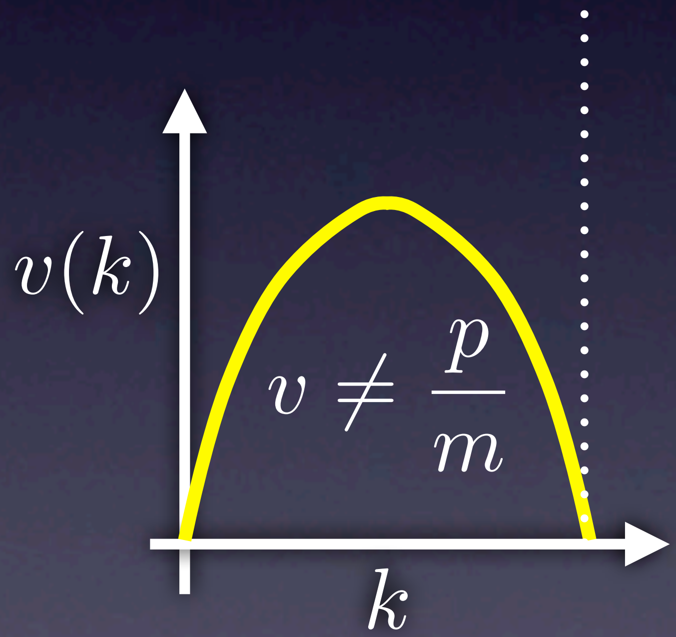
Question #10

In a crystal...

$$v = \frac{d\omega}{dk}$$

$$E = \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk}$$



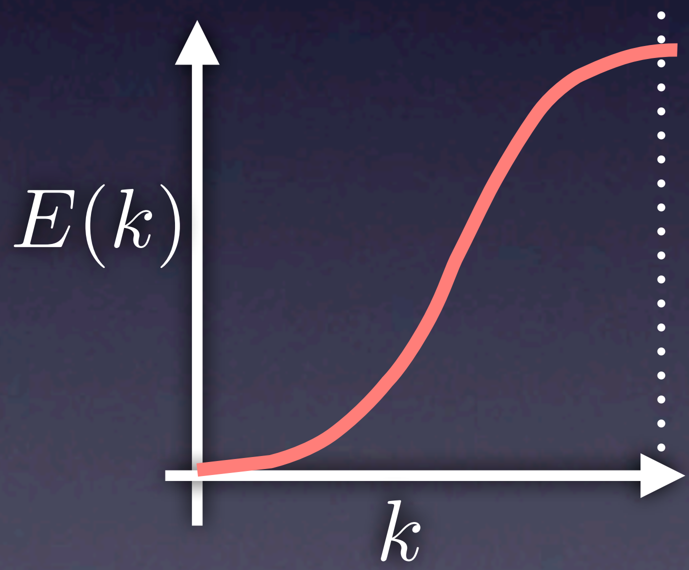


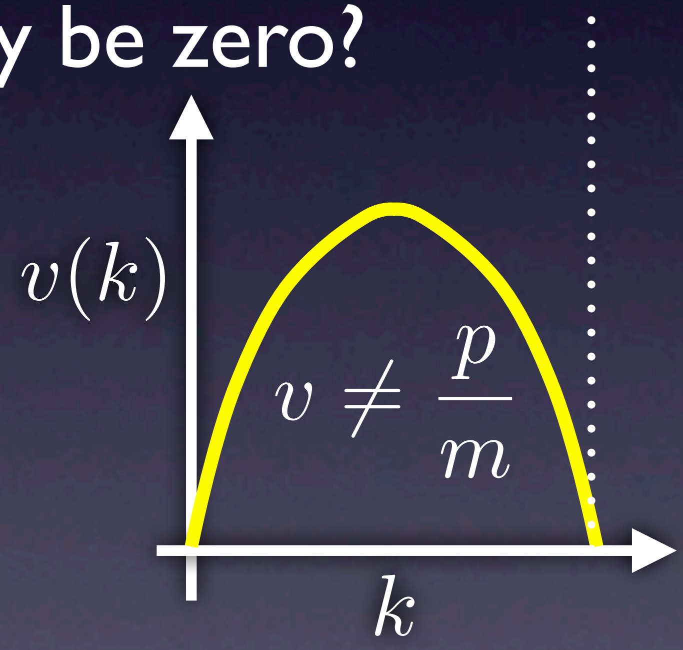
Question #10

In a crystal...

$$\begin{aligned} v &= \frac{d\omega}{dk} \\ E &= \hbar\omega \Rightarrow v = \frac{1}{\hbar}\frac{dE}{dk} \end{aligned}$$

How can the velocity be zero?





Question #10

But applying an external force causes the k-vector to change with time. So the electron is “moving in k space.”

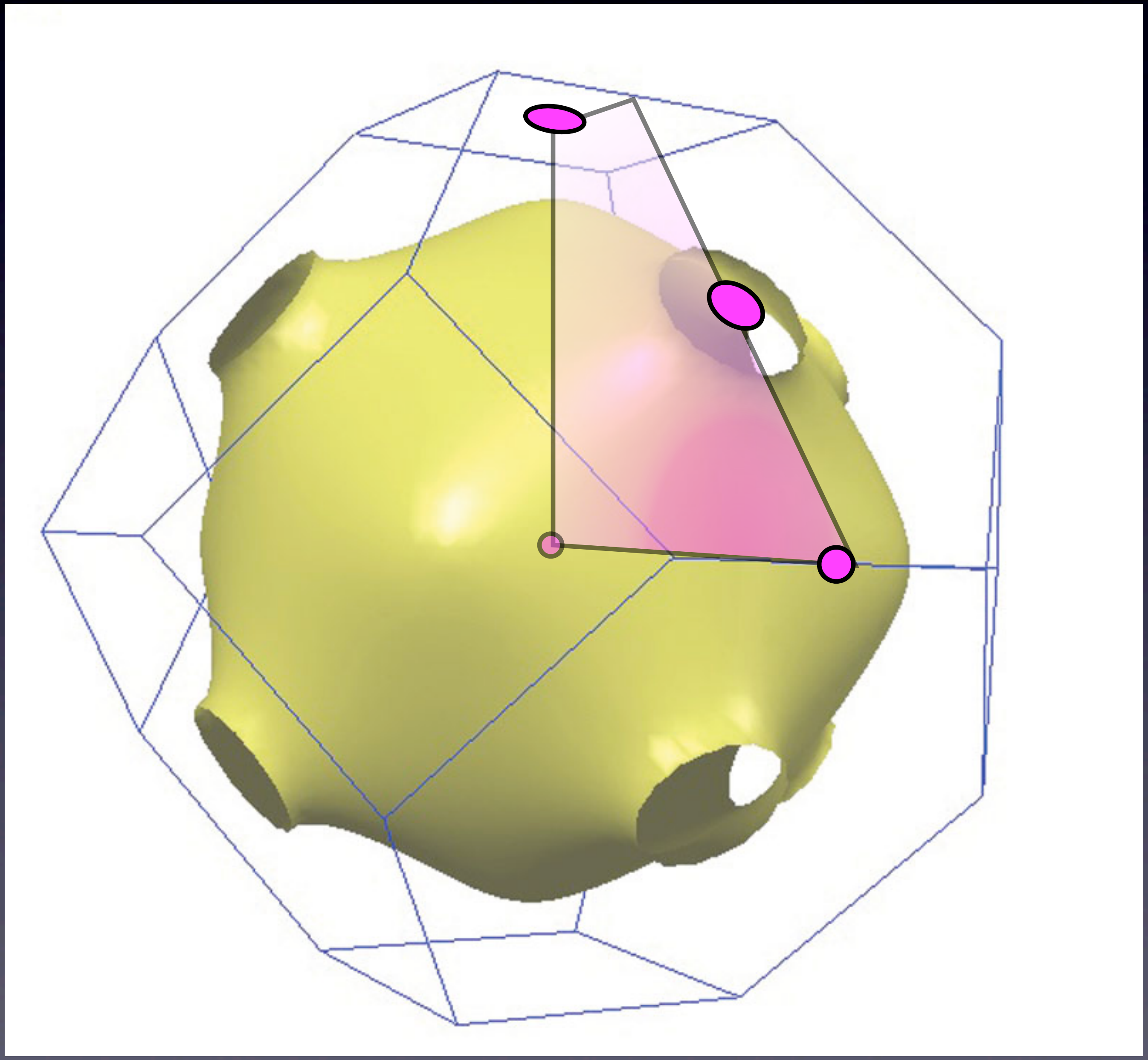
$$F_{\mathrm{ext}} = \hbar \frac{dk}{dt}$$

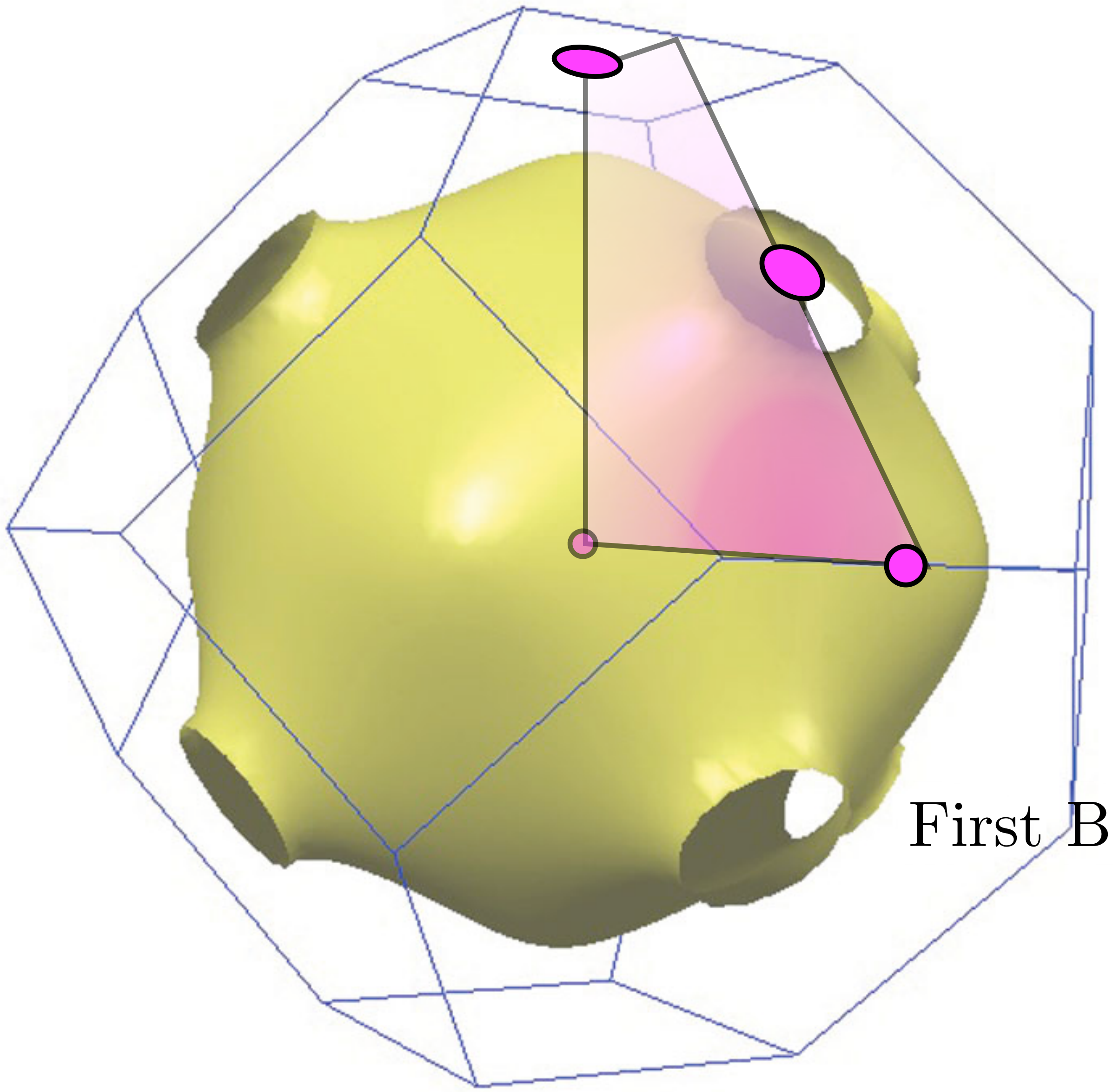
What does it mean for an electron to "move in k-space"?

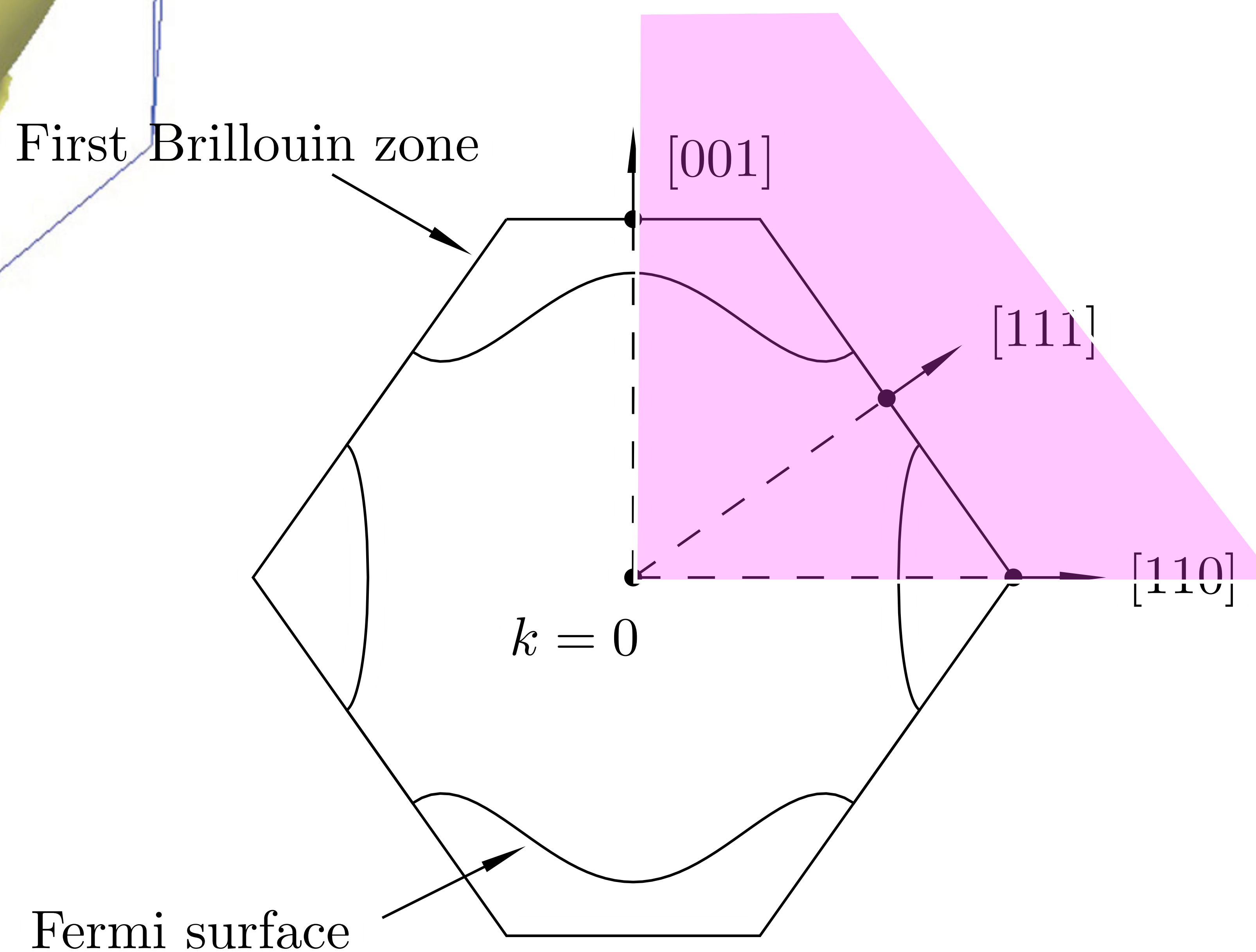
But applying an external force causes the k-vector to change with time. So the electron is "moving in k space."

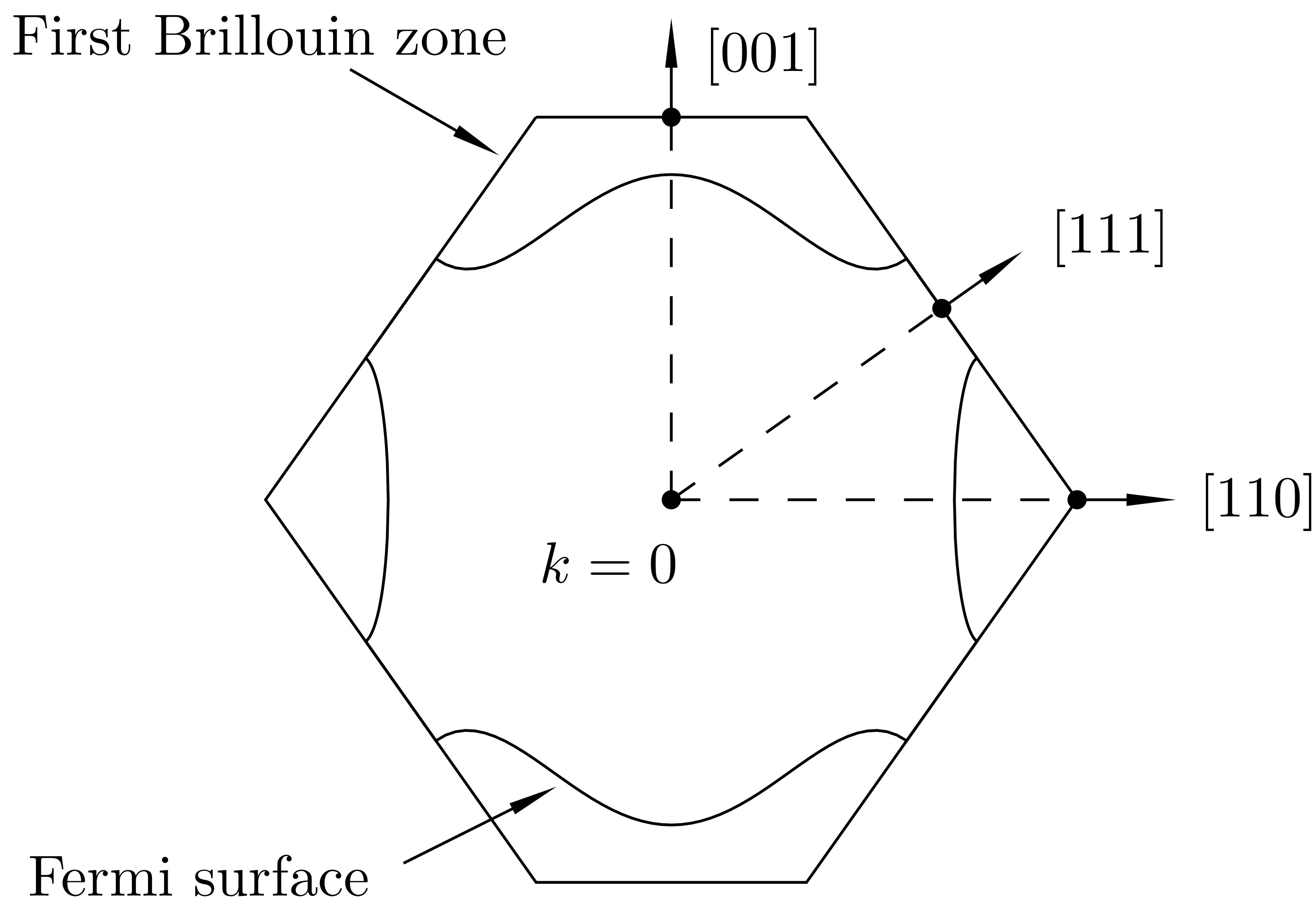
$$F_{\rm ext} = \hbar \frac{dk}{dt}$$

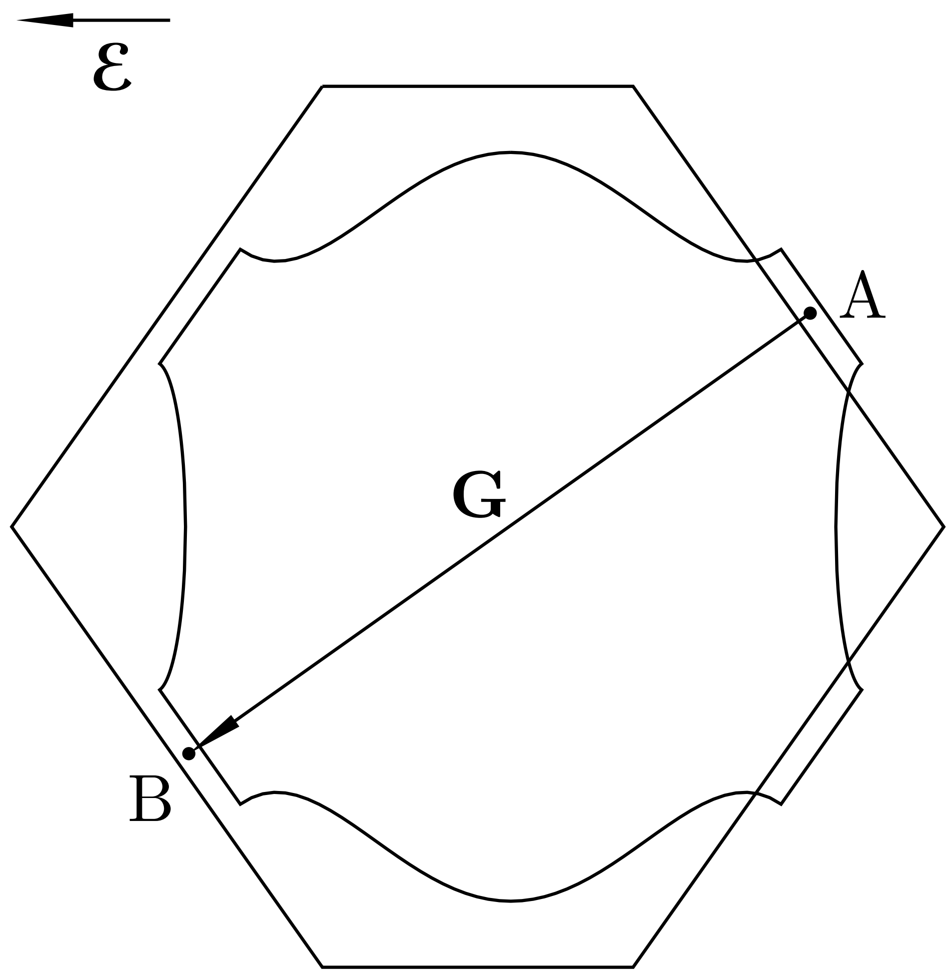
Fermi Surface of Cu





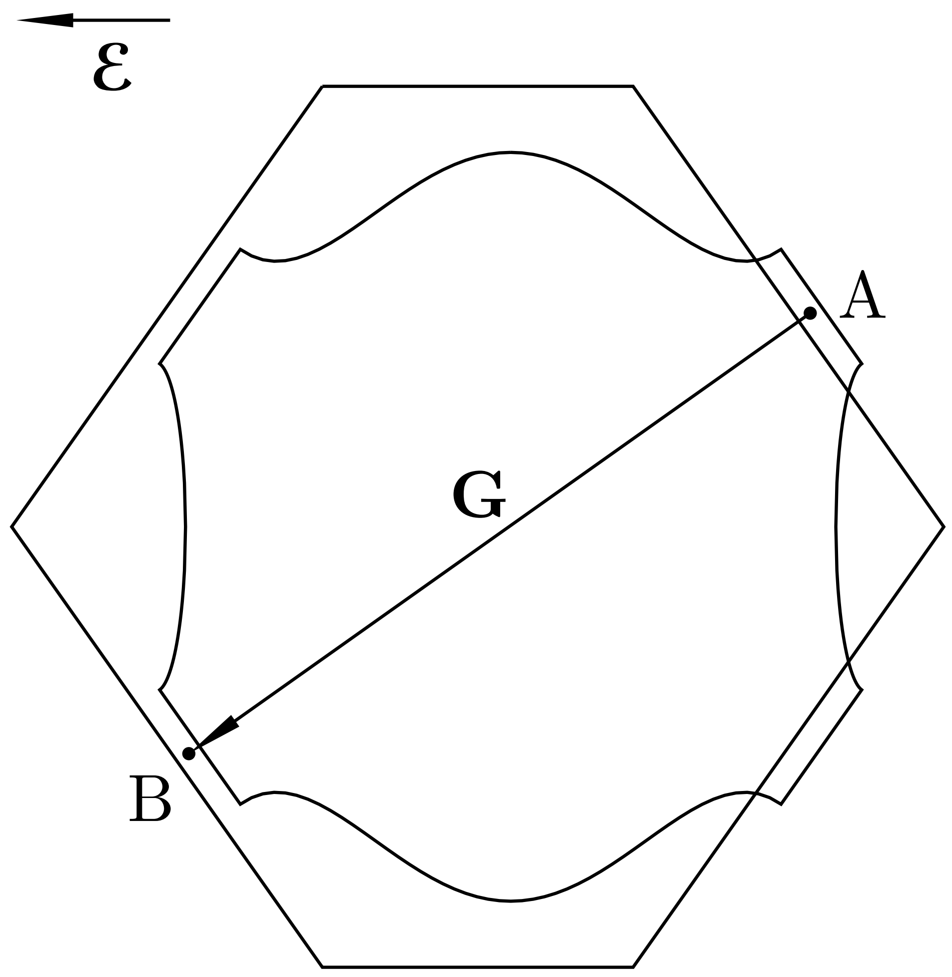

First Brillouin zone
[001]
[111]
[110]
$k = 0$
Fermi surface

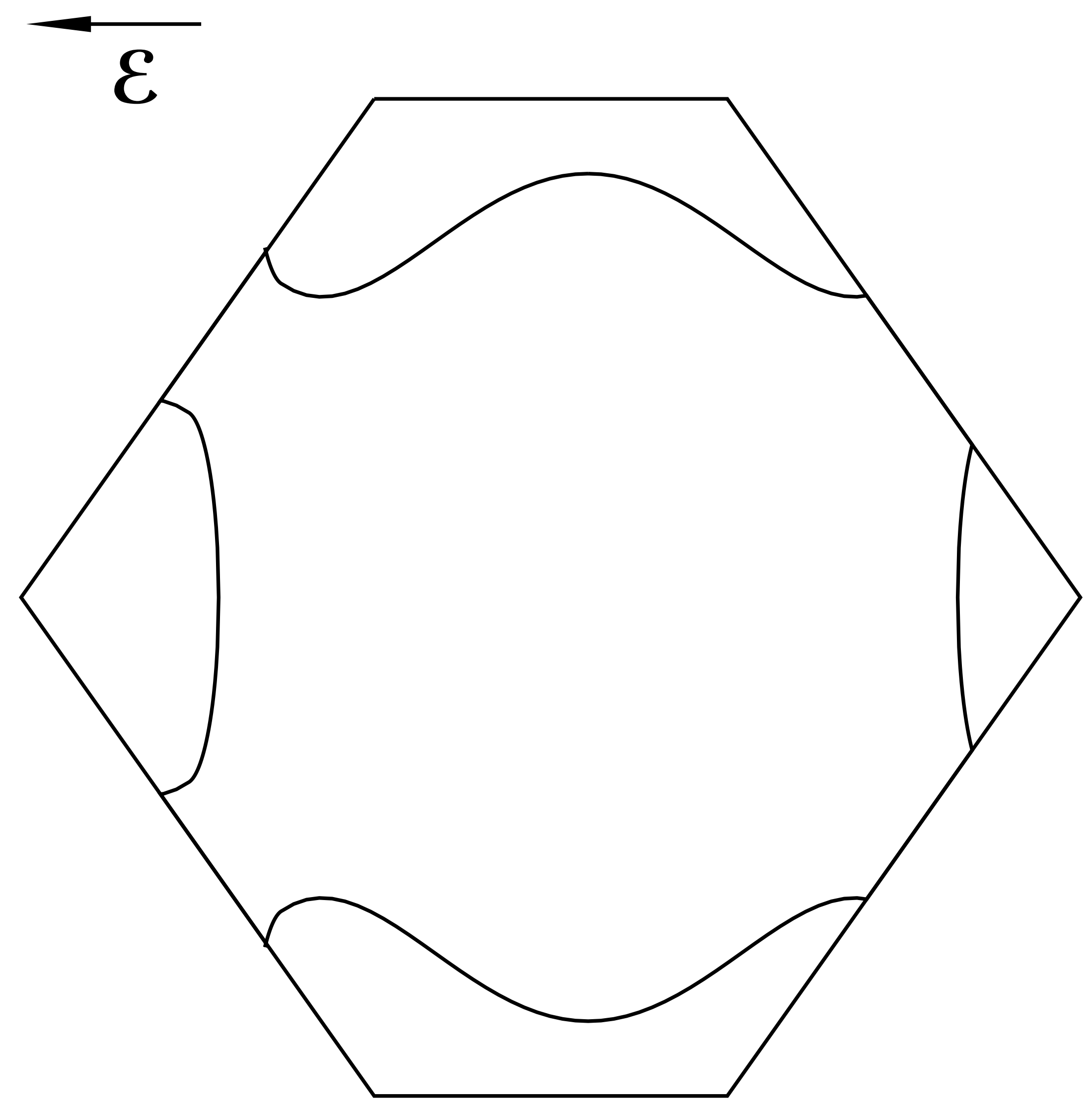

First Brillouin zone
[001]
[111]
[110]
$k = 0$
Fermi surface

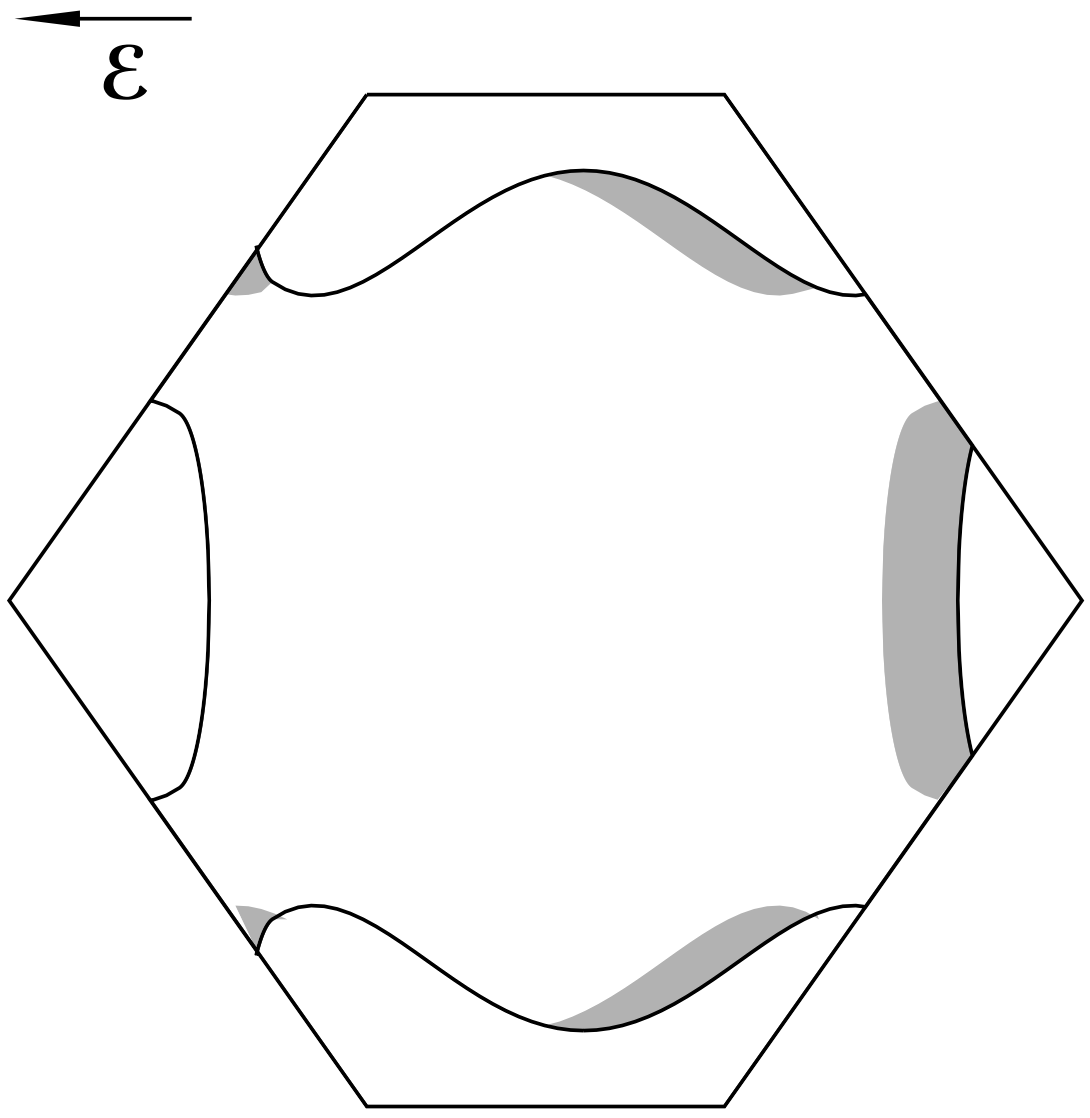


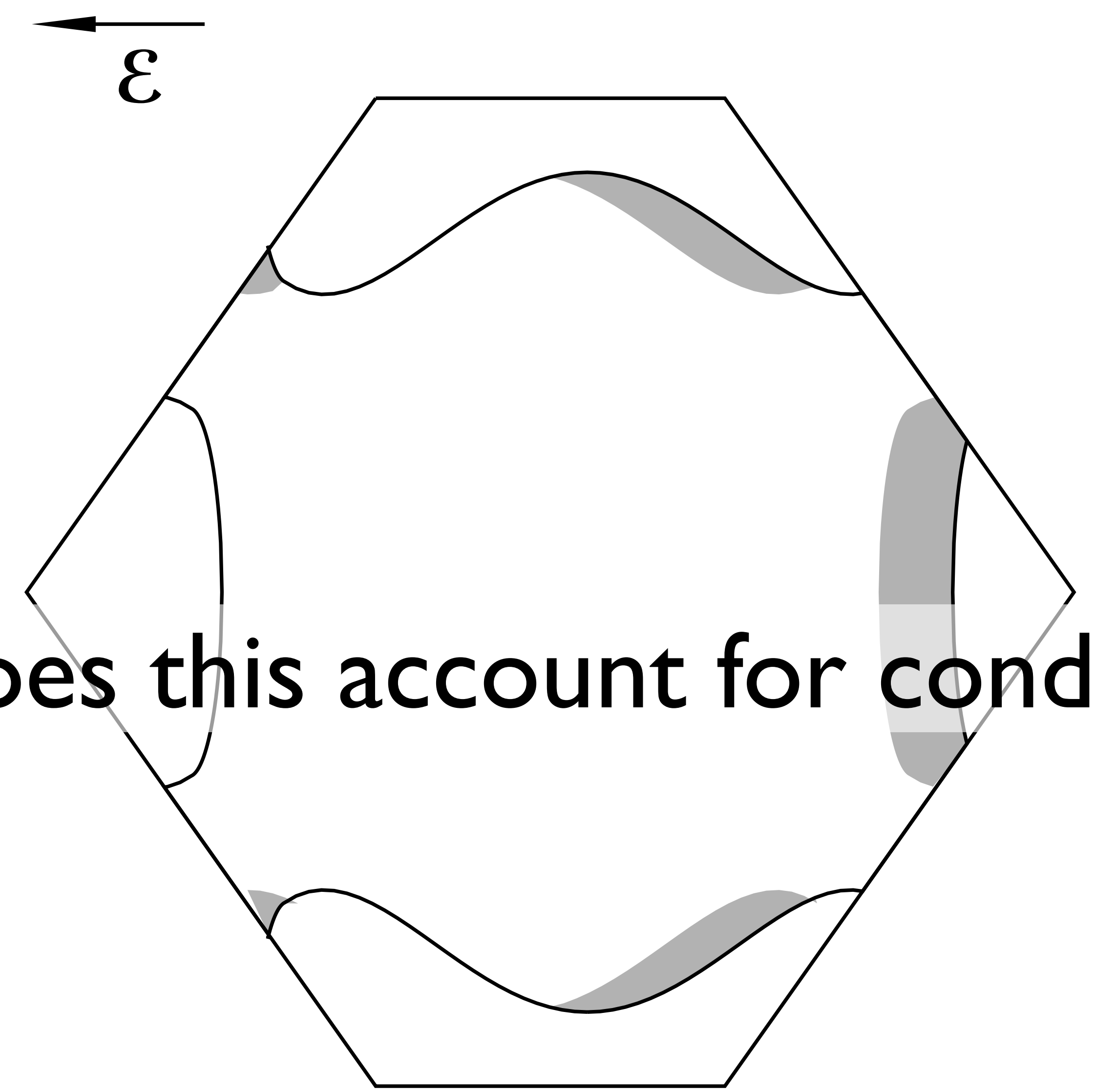
$\boldsymbol{\varepsilon}$
A
$\mathbf{G}$
B

What is happening to the electrons?




ε

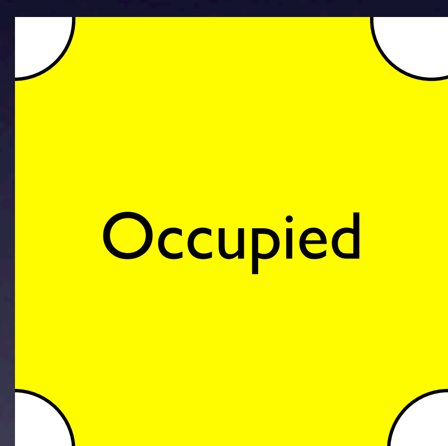

ε



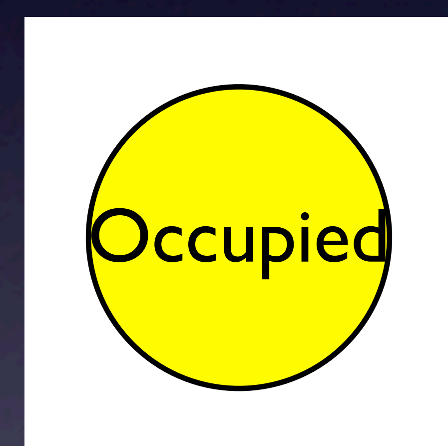
How does this account for conductivity?

Consider the conduction bands below. Each square represents the first Brillouin zone. Which band contains the most electrons?

Question #12



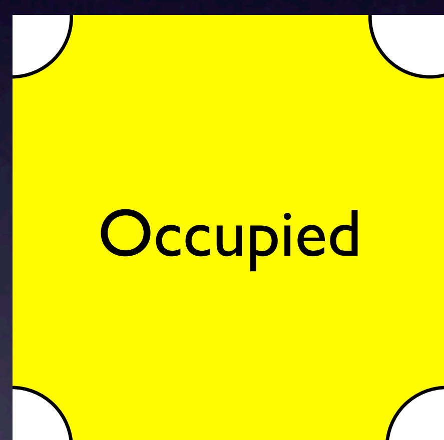
(A)



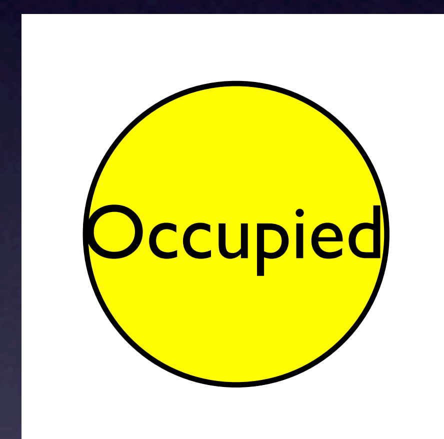
(B)

Question #13

Consider the conduction bands below. Each square represents the first Brillouin zone. If we apply the same electric field to each band, which band will produce the most current?

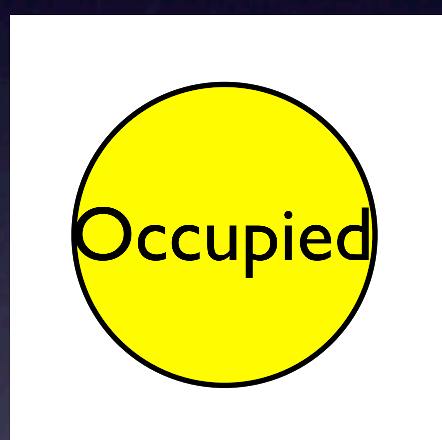


(D)

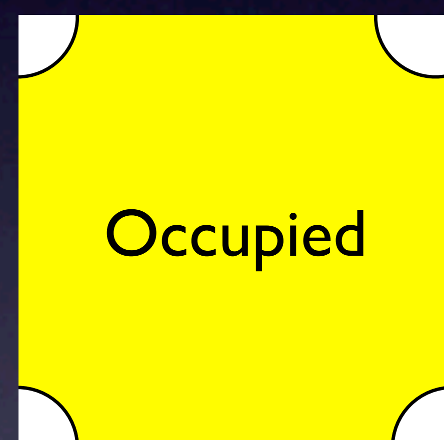


(E)

Two bands...

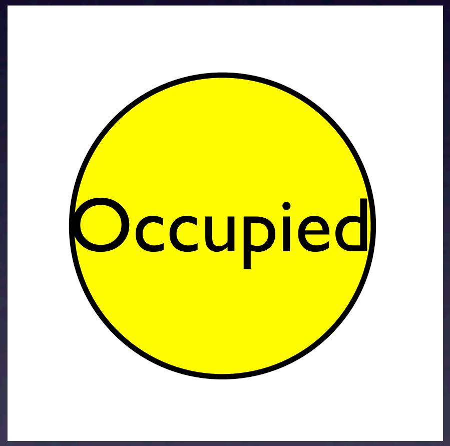


half full

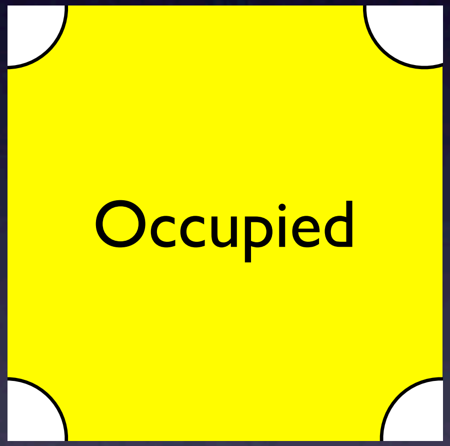


almost full

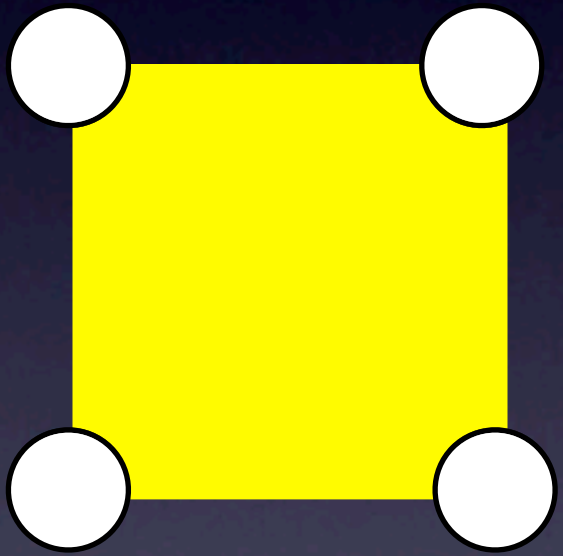
Two bands...



half full



almost full



Question #14

Which of the following has the greatest influence on how much current will flow through a metal if an electric field is applied?

(A) the number of conduction electrons
(B) number of conduction electrons with energies close to the Fermi energy
(C) the volume of states enclosed by the Fermi surface

Classical vs. Quantum Model

Classical vs. Quantum Model

$$\sigma = \frac{ne^2\tau}{m}$$

Classical vs. Quantum Model

$$\sigma = \frac{ne^2\tau}{m}$$

$$\sigma \propto n$$

Classical vs. Quantum Model

$$\sigma = \frac{ne^2\tau}{m}$$

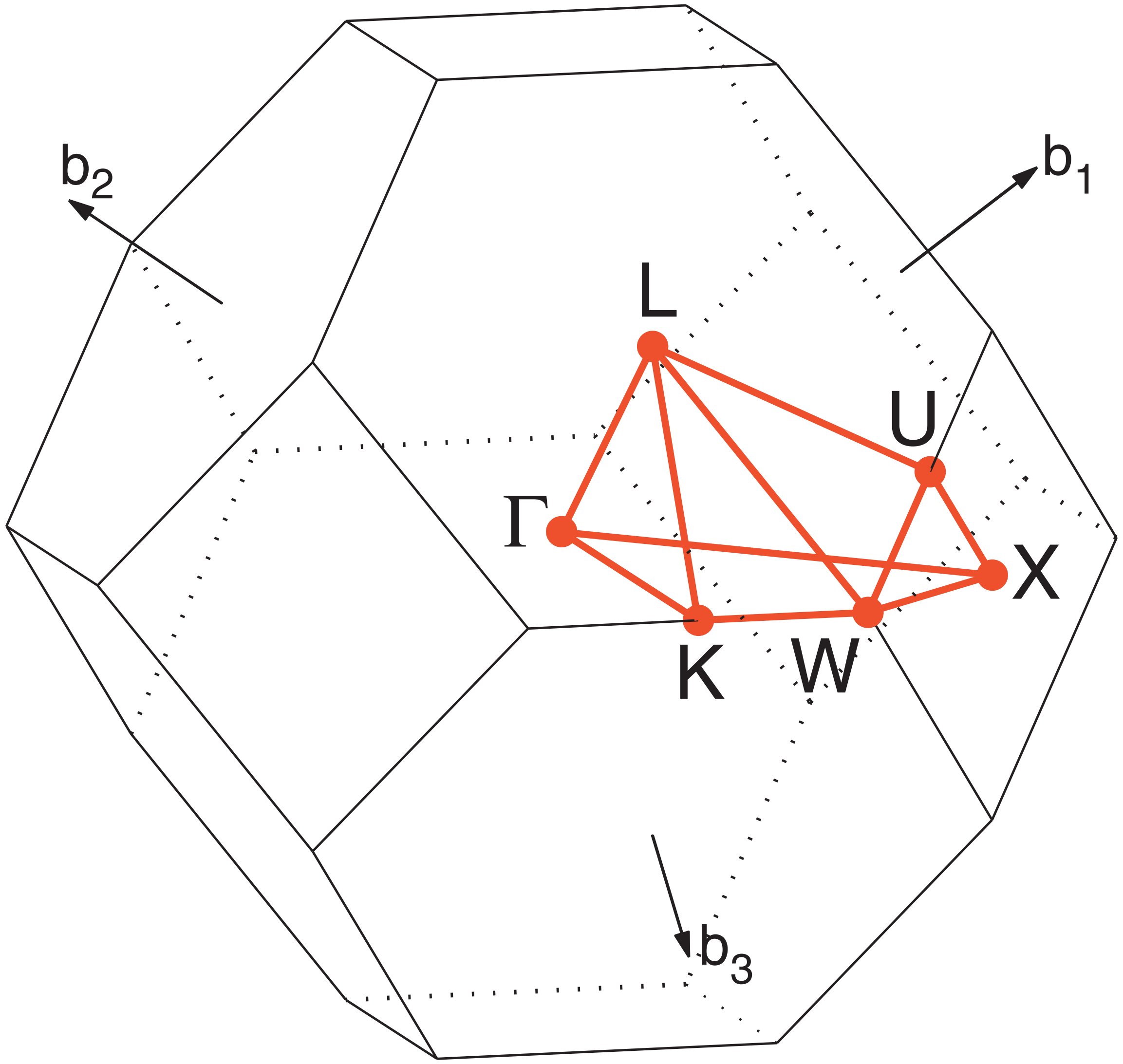
$$\sigma \not\propto n$$

Classical vs. Quantum Model

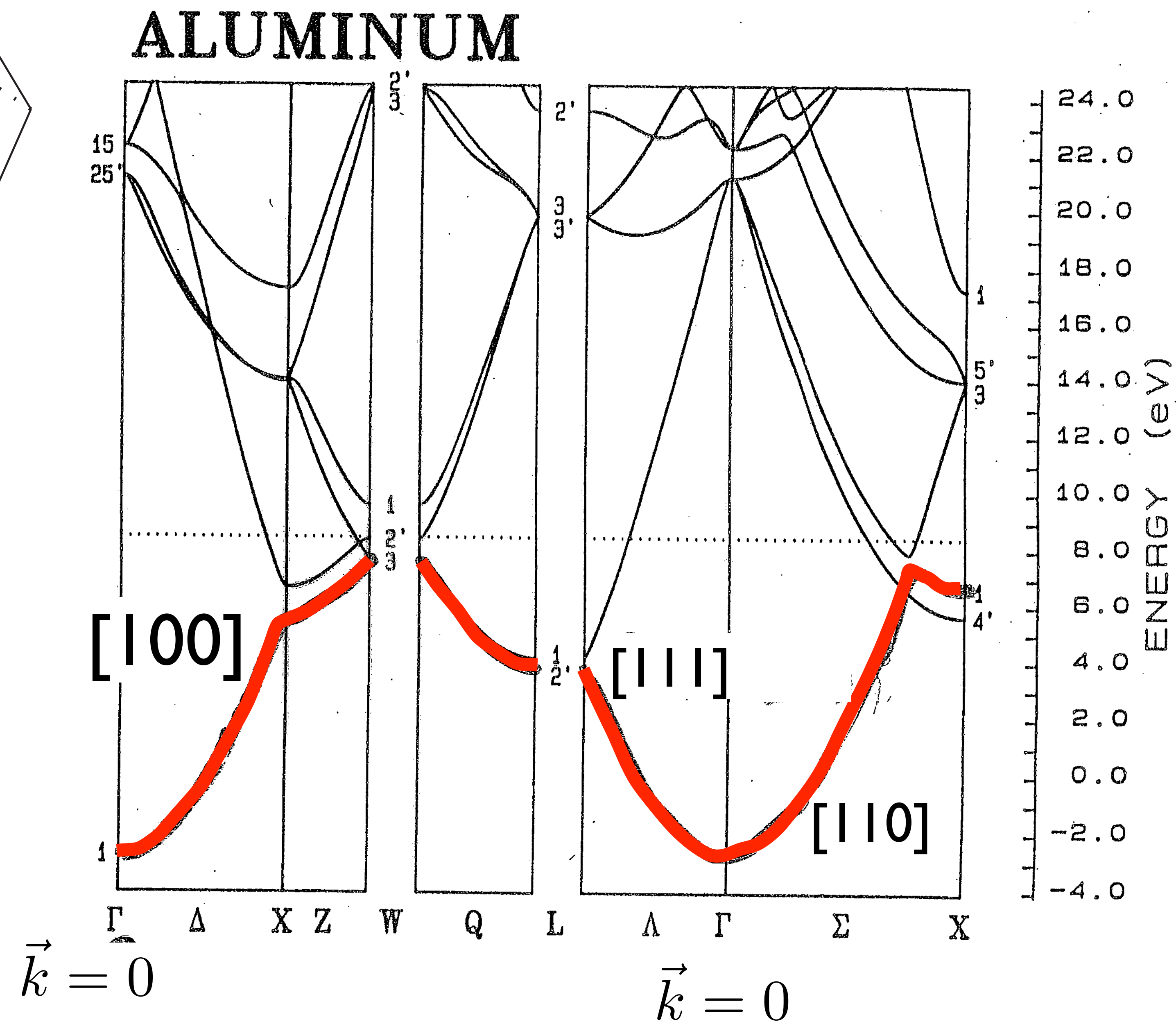
$$\sigma = \frac{ne^2\tau}{m}$$

$$\sigma \not\propto n$$

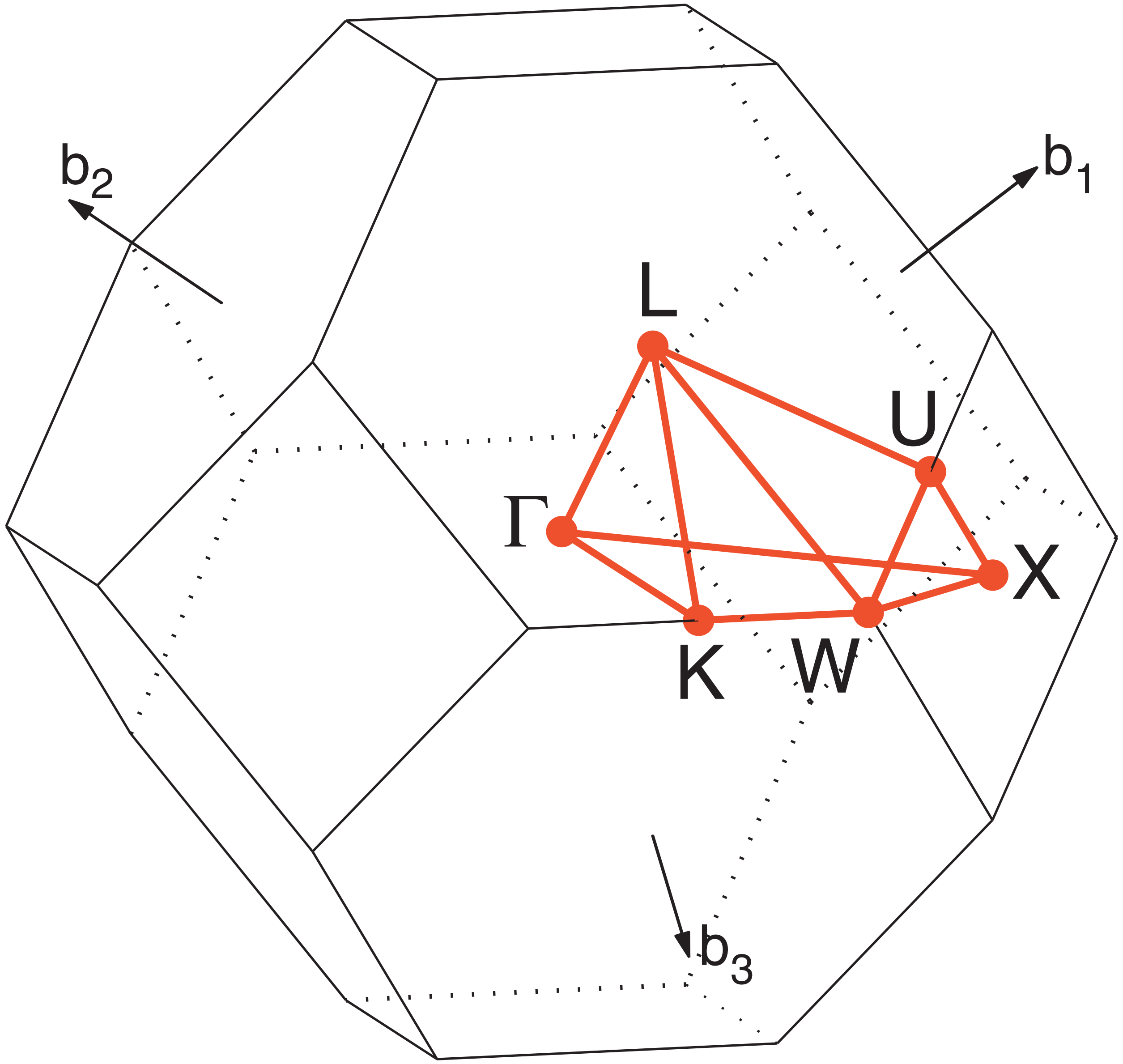
$$\sigma \propto A_{\mathrm{FS}}$$



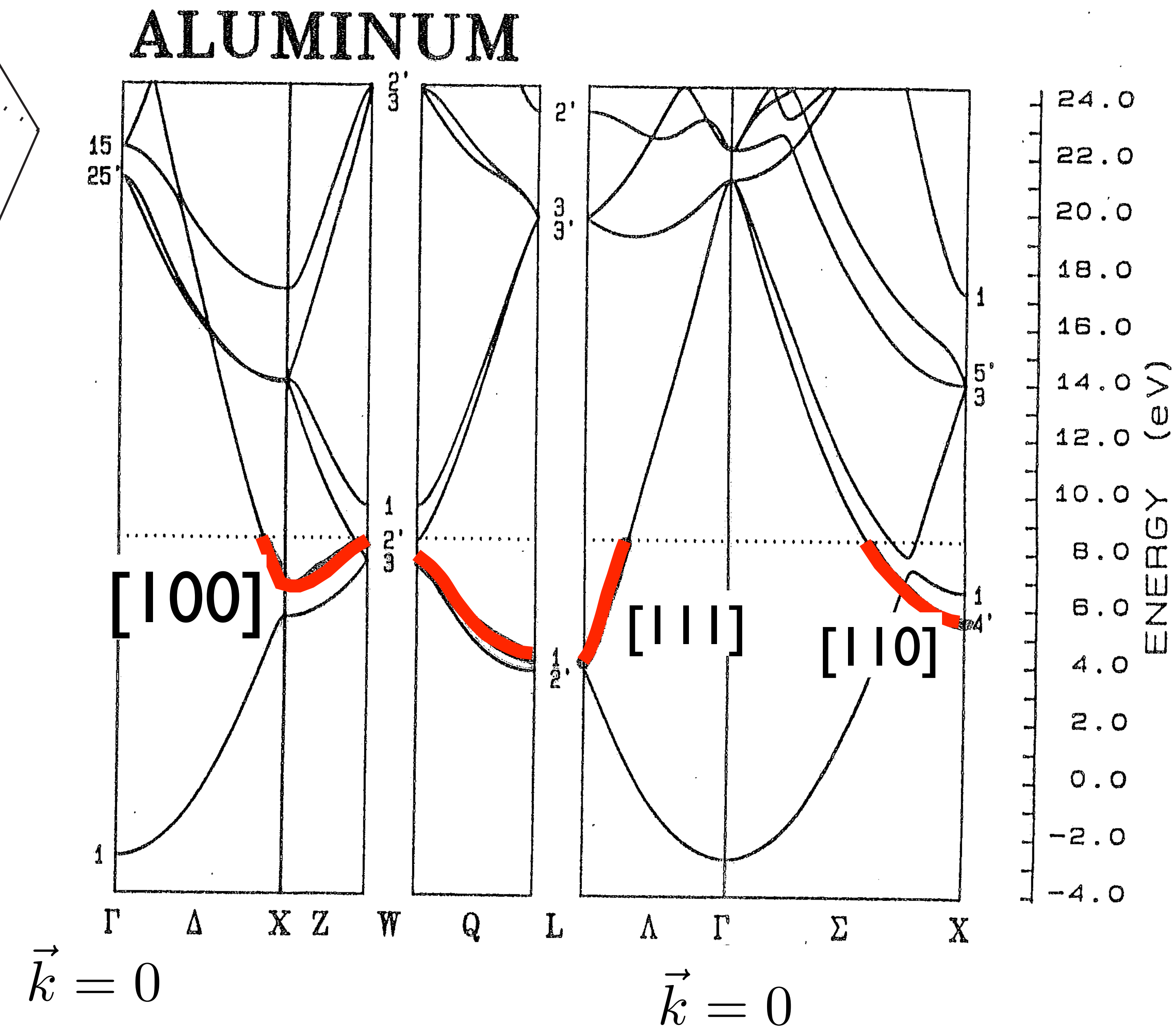
b2
b1
L
U
Γ
X
K
W
b3



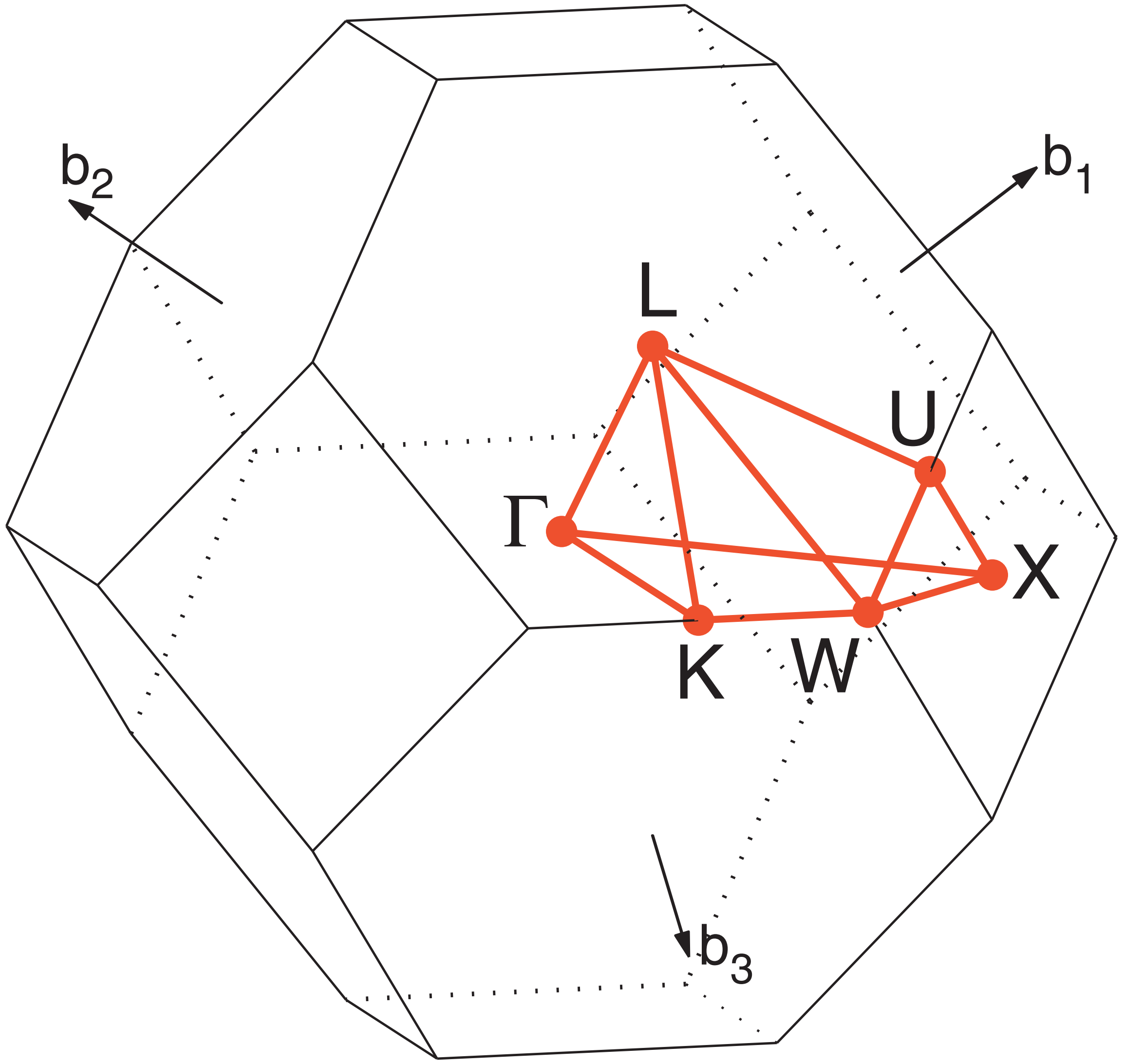
ALUMINUM
[100]
[111]
[110]
Γ Δ X Z W Q L Λ Γ Σ X
ENERGY (eV)
24.0
22.0
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
$\vec{k} = 0$
$\vec{k} = 0$



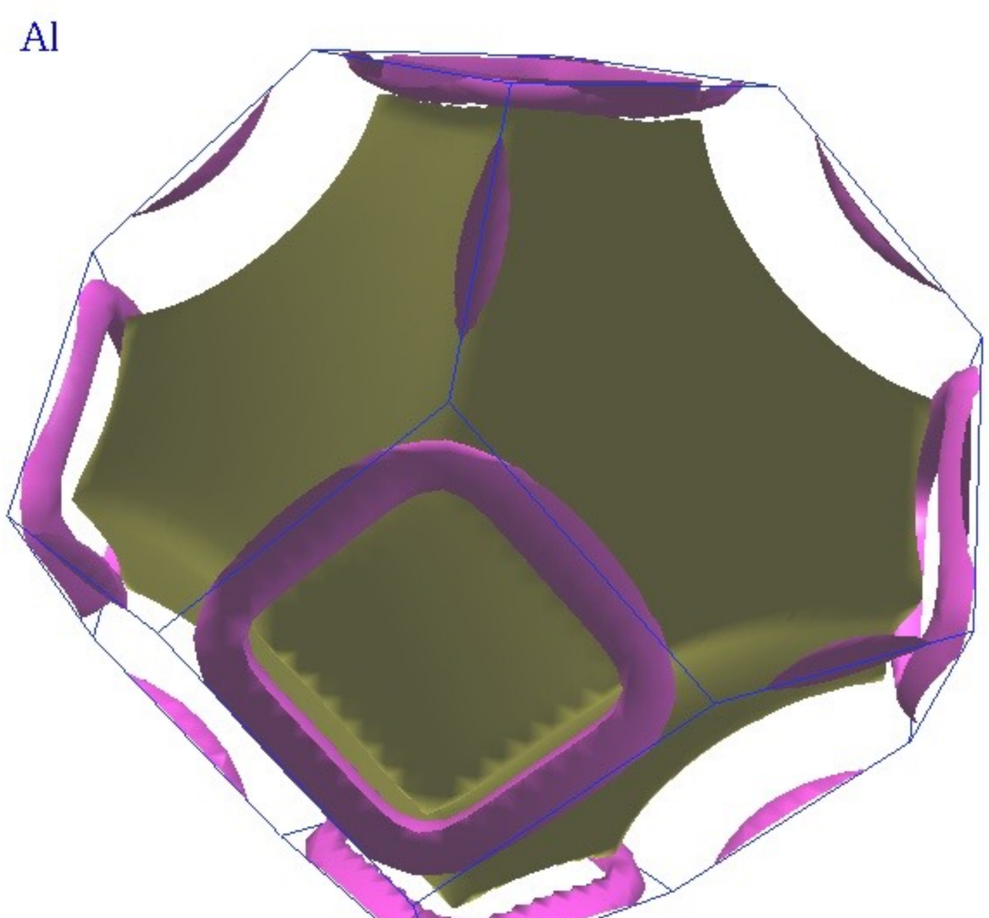
b2
b1
L
U
Γ
X
K
W
b3



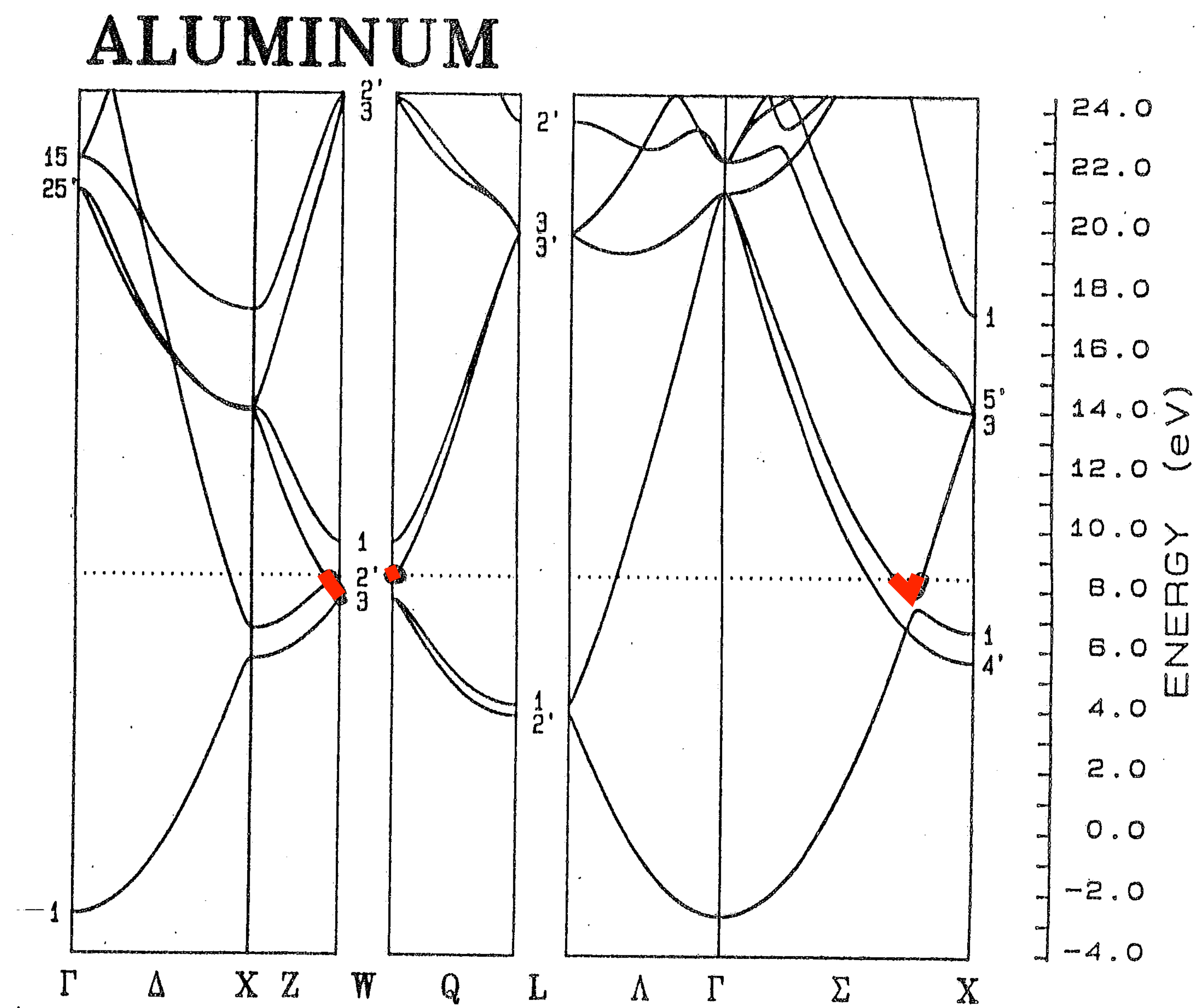
ALUMINUM
[100]
[111]
[110]
Γ Δ X Z W Q L Λ Γ Σ X
ENERGY (eV)
24.0
22.0
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
$\vec{k} = 0$
$\vec{k} = 0$



b2
b1
L
U
Γ
X
K
W
b3



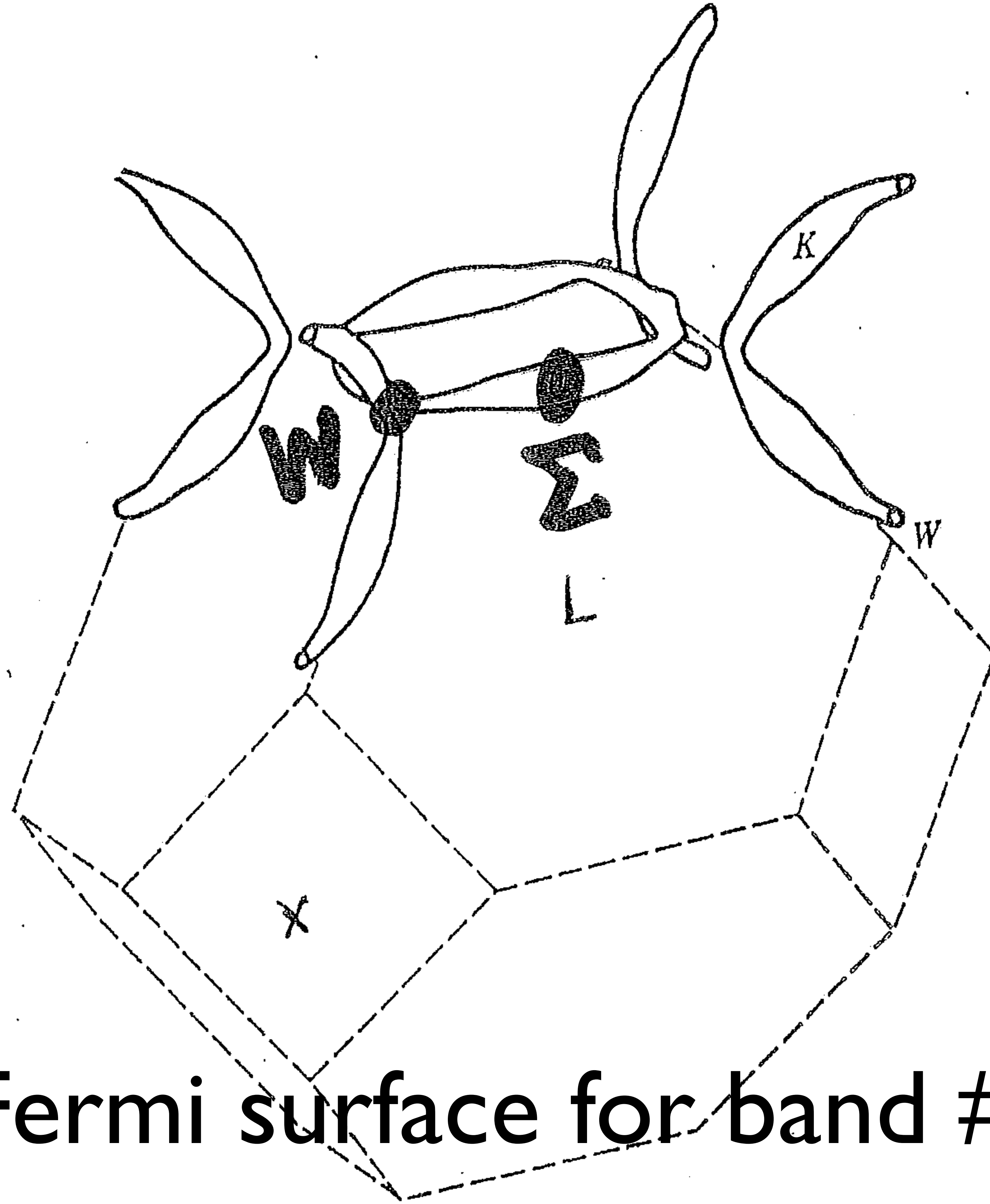
Al



ALUMINUM
ENERGY (eV)
24.0
22.0
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
Γ
Δ
X
Z
W
Q
L
Λ
Γ
Σ
X

$\vec{k} = 0$

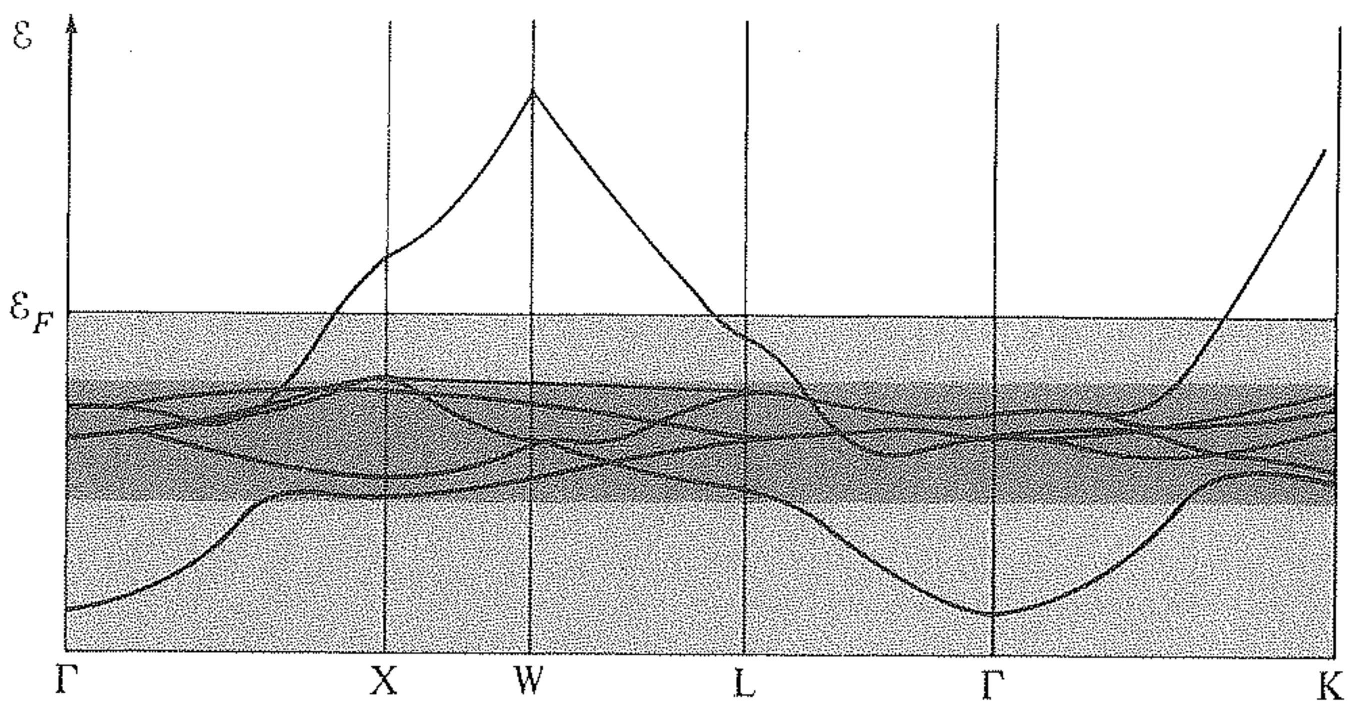
$\vec{k} = 0$



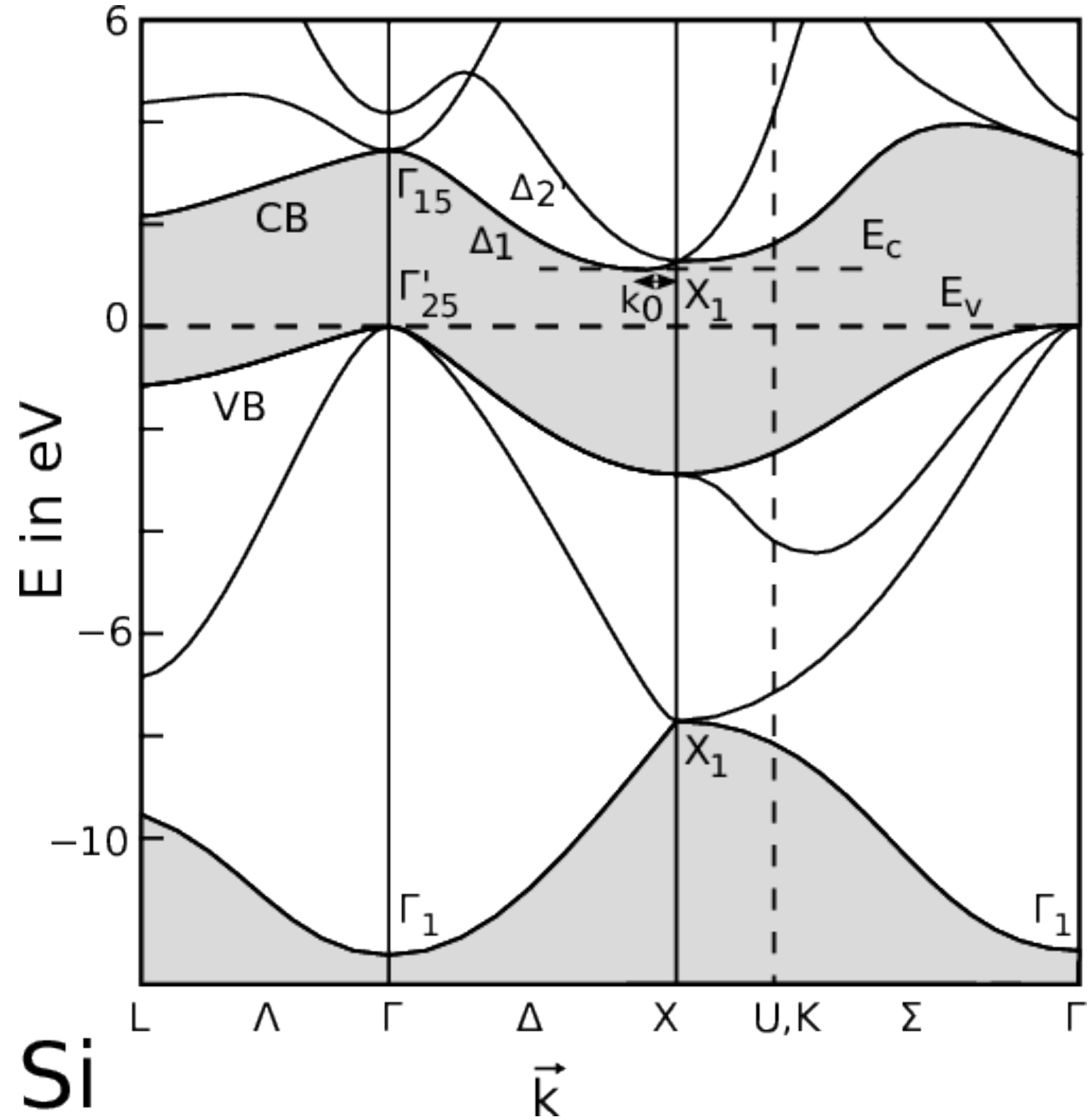
Fermi surface for band #3

Question #12

Which is the insulator?



D



E

Resistitivity

Question #13

The electrons in solids collide with

a) all atoms in the crystal
b) Only imperfections in the crystal
c) About half of the atoms in the crystal.

Resistitivity

Question #13

5. It can be shown quite generally (Appendix E) that an electron in a level specified by band index n and wave vector $\mathbf{k}$ has a nonvanishing mean velocity, given by

$$\boxed{\mathbf{v}_n(\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}}\,\mathcal{E}_n(\mathbf{k}).} \tag{8.51}$$

This is a most remarkable fact. It asserts that there are stationary (i.e., time-independent) levels for an electron in a periodic potential in which, in spite of the interaction of the electron with the fixed lattice of ions, it moves forever without any degradation of its mean velocity. This is in striking contrast to the idea of Drude that collisions were simply encounters between the electron and a static ion. Its implications are of fundamental importance, and will be explored in Chapters 12 and 13.

Resistivity

For which type of collision is the number of collisions independent of temperature?

a) Collisions with phonons
b) .
c) Both are independent of temperature.
d) Collisions with crystal defects.

Question #14

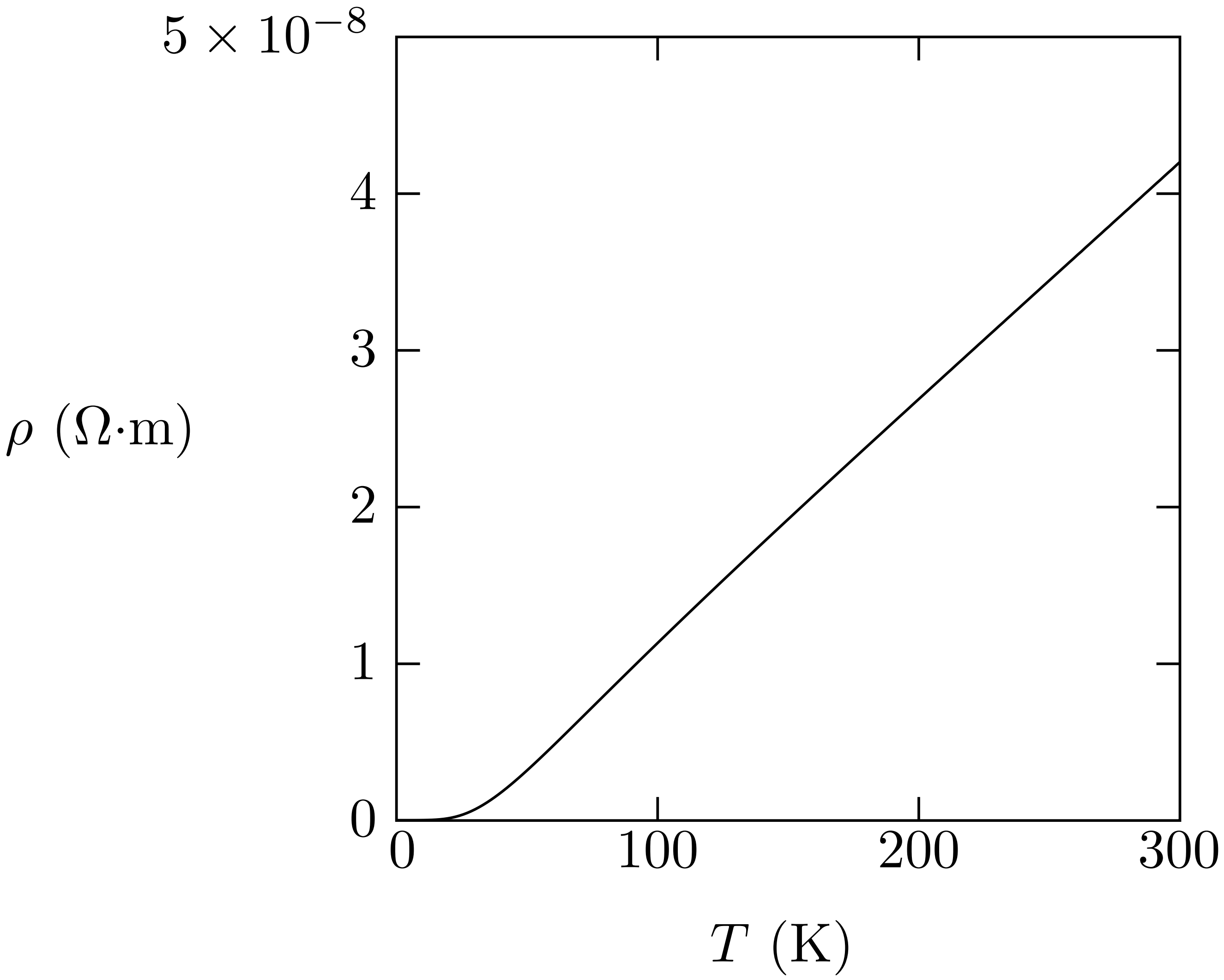
Resistivity

Would you expect the resistivity of a metal to increase or decrease with temperature?

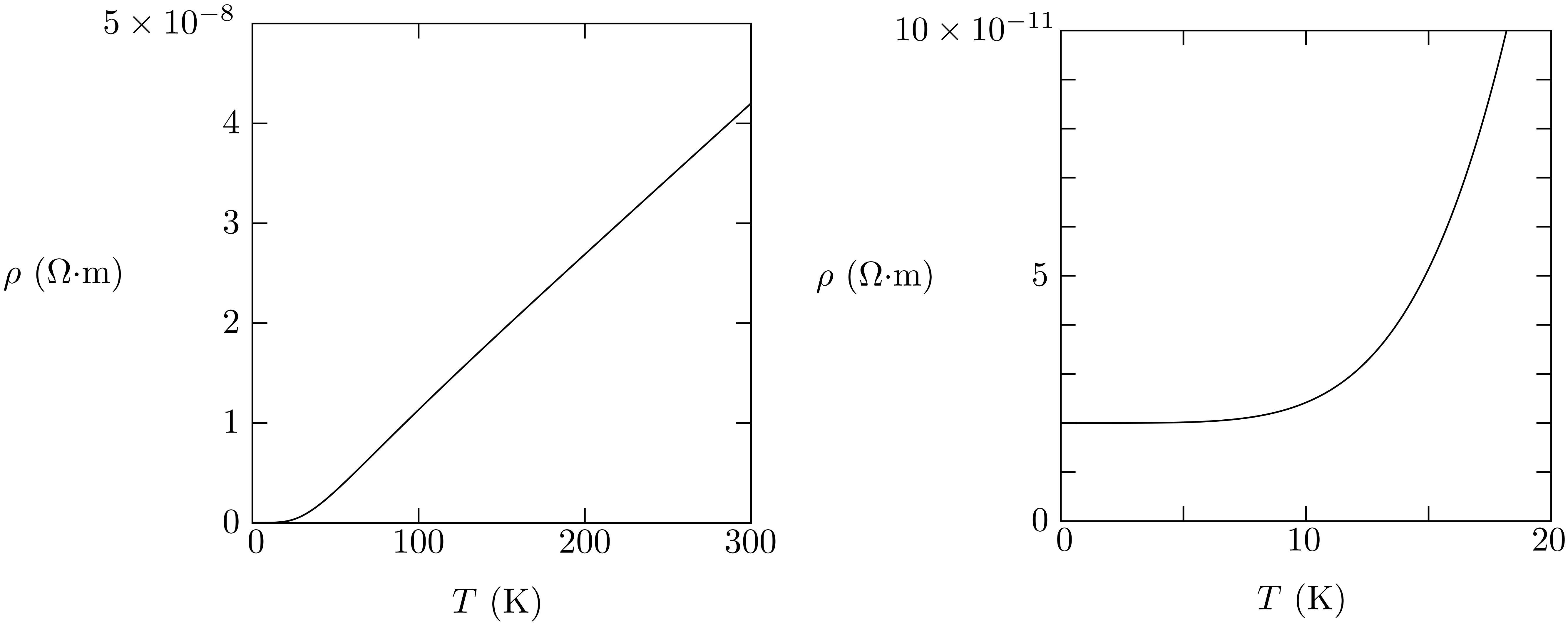
a) .
b) Decrease
c) Stay Constant.
d) Increase.

Question #15

Resistivity and Temperature



Resistivity and Temperature

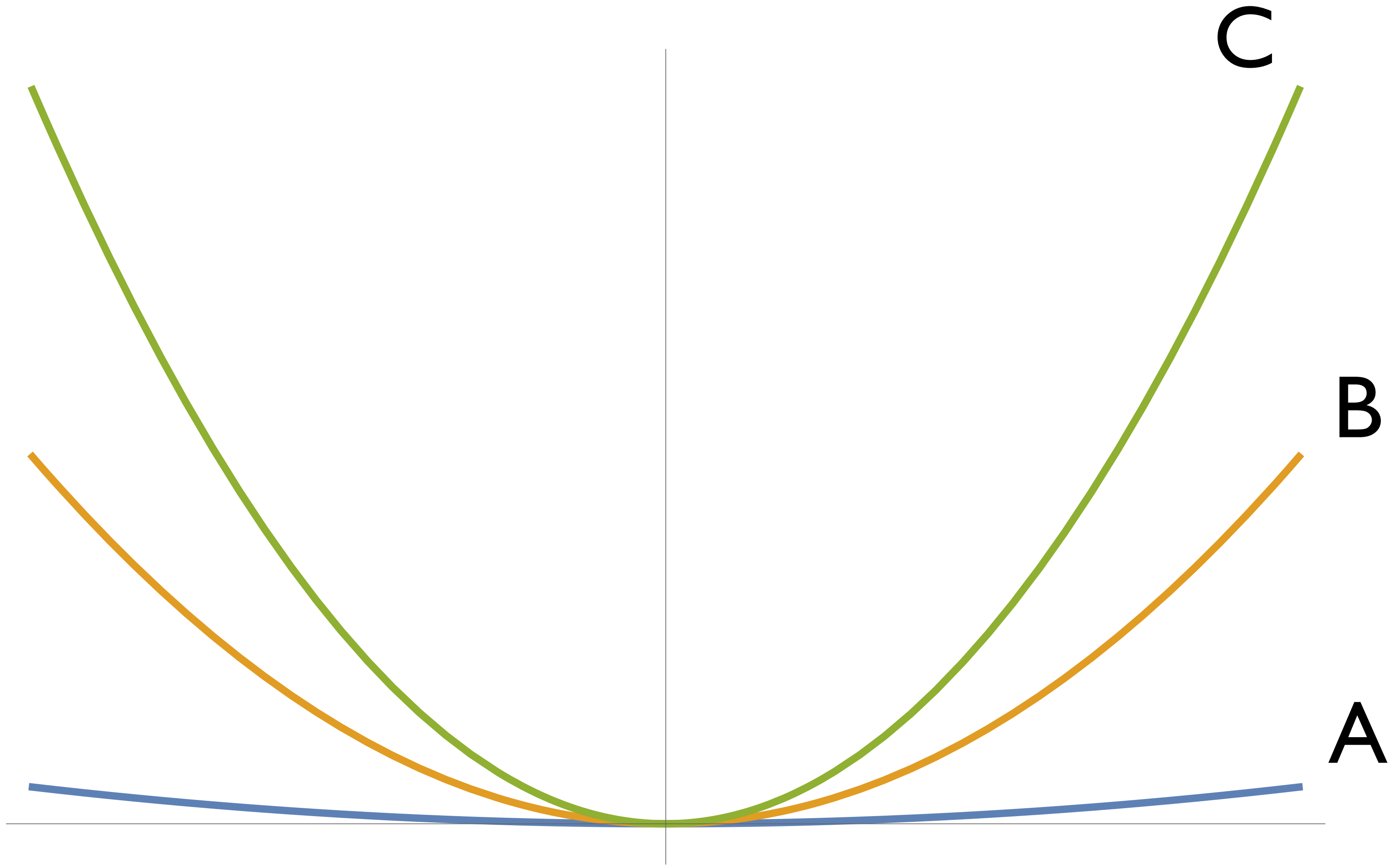


Applying an External Force

$$
\begin{aligned}
a &= \frac{dv}{dt} = \frac{dv}{dk}\frac{dk}{dt} \\
a &= \left(\frac{1}{\hbar}\frac{d^2E}{dk^2}\right)\left(\frac{1}{\hbar}F_{\text{ext}}\right) \\
\Rightarrow m^* &= \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1} \\
a &= \frac{F_{\text{ext}}}{m^*}
\end{aligned}
$$

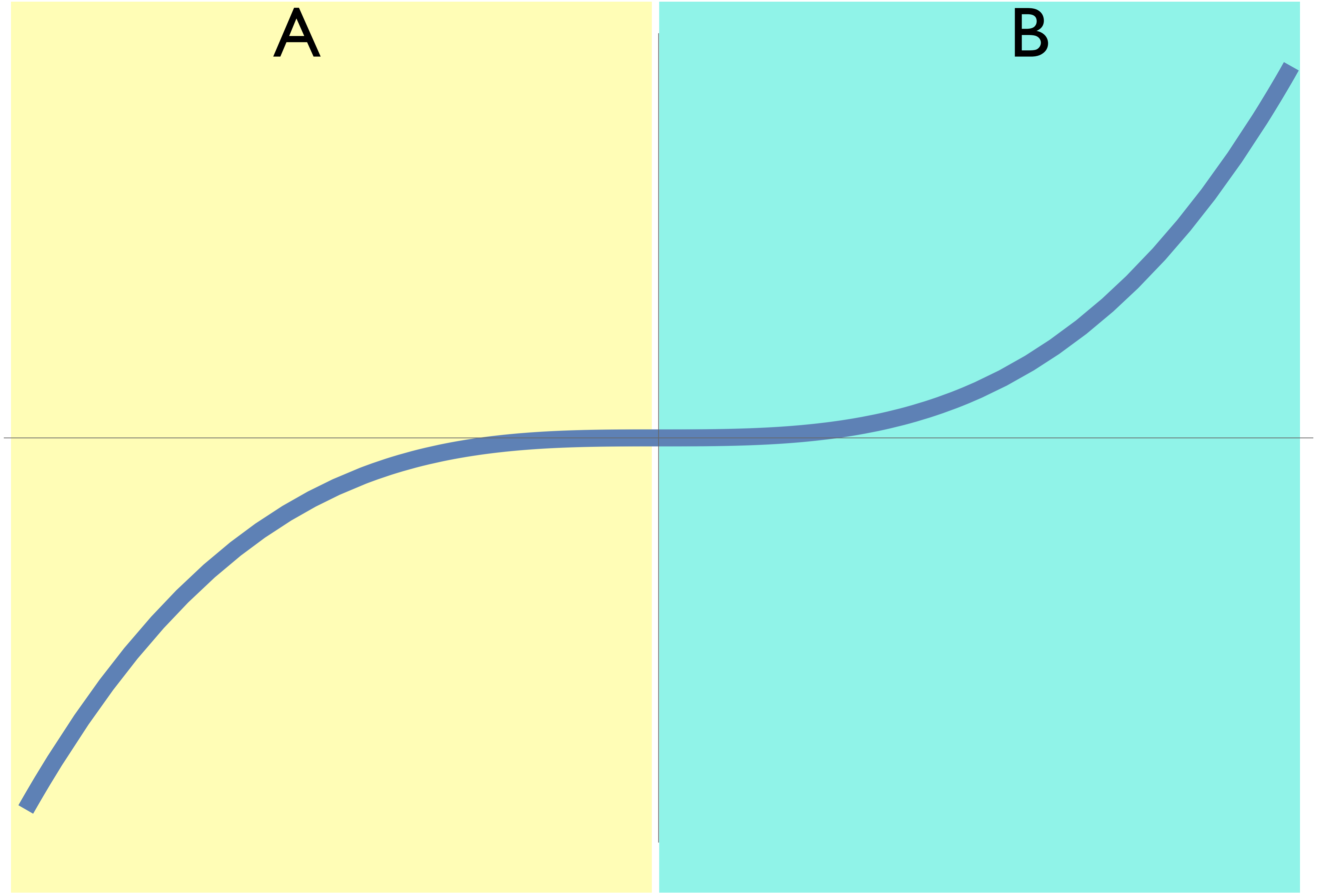
Question #16

For which function is the second derivative largest?

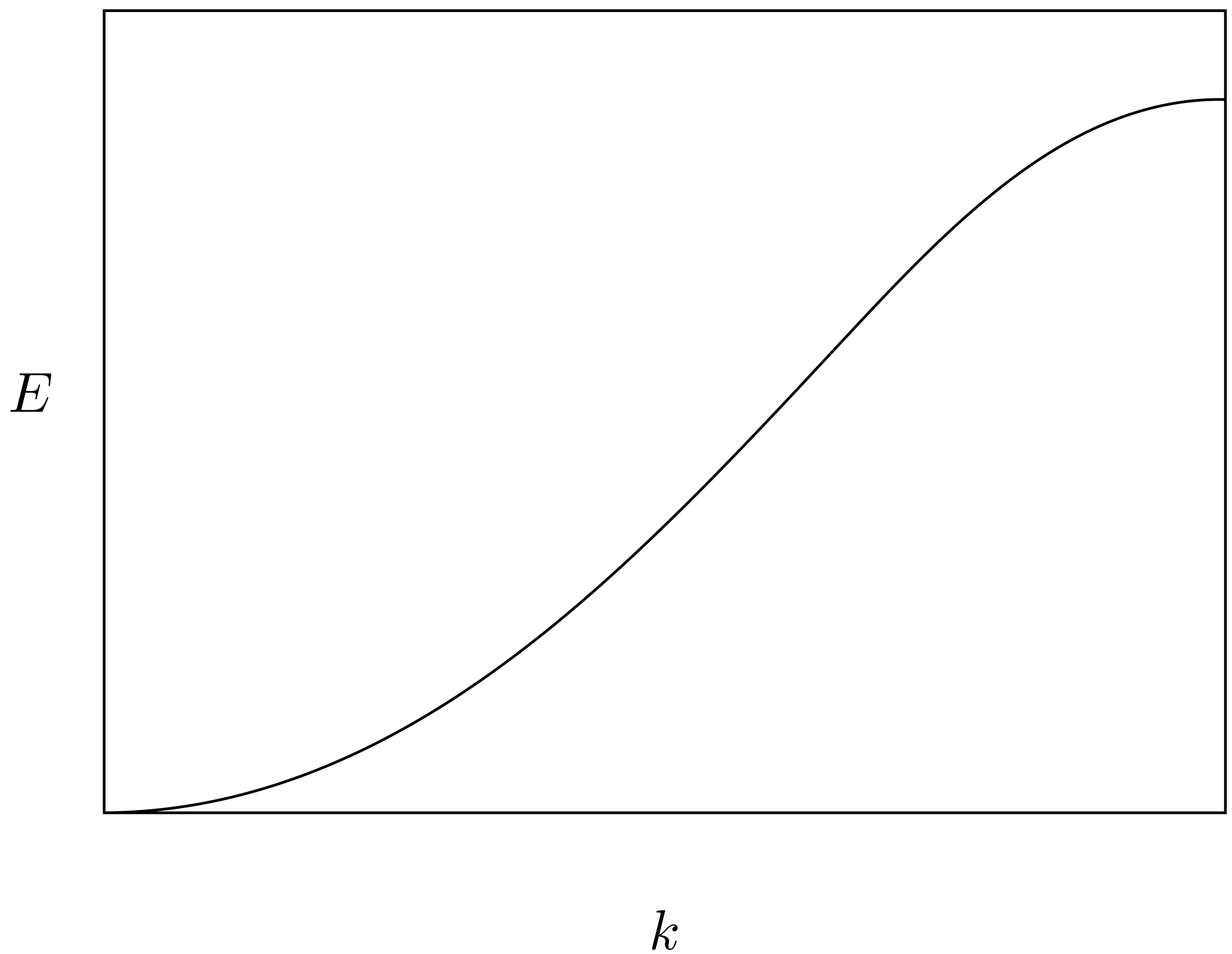


Where is the second derivative of this function negative

Question #17

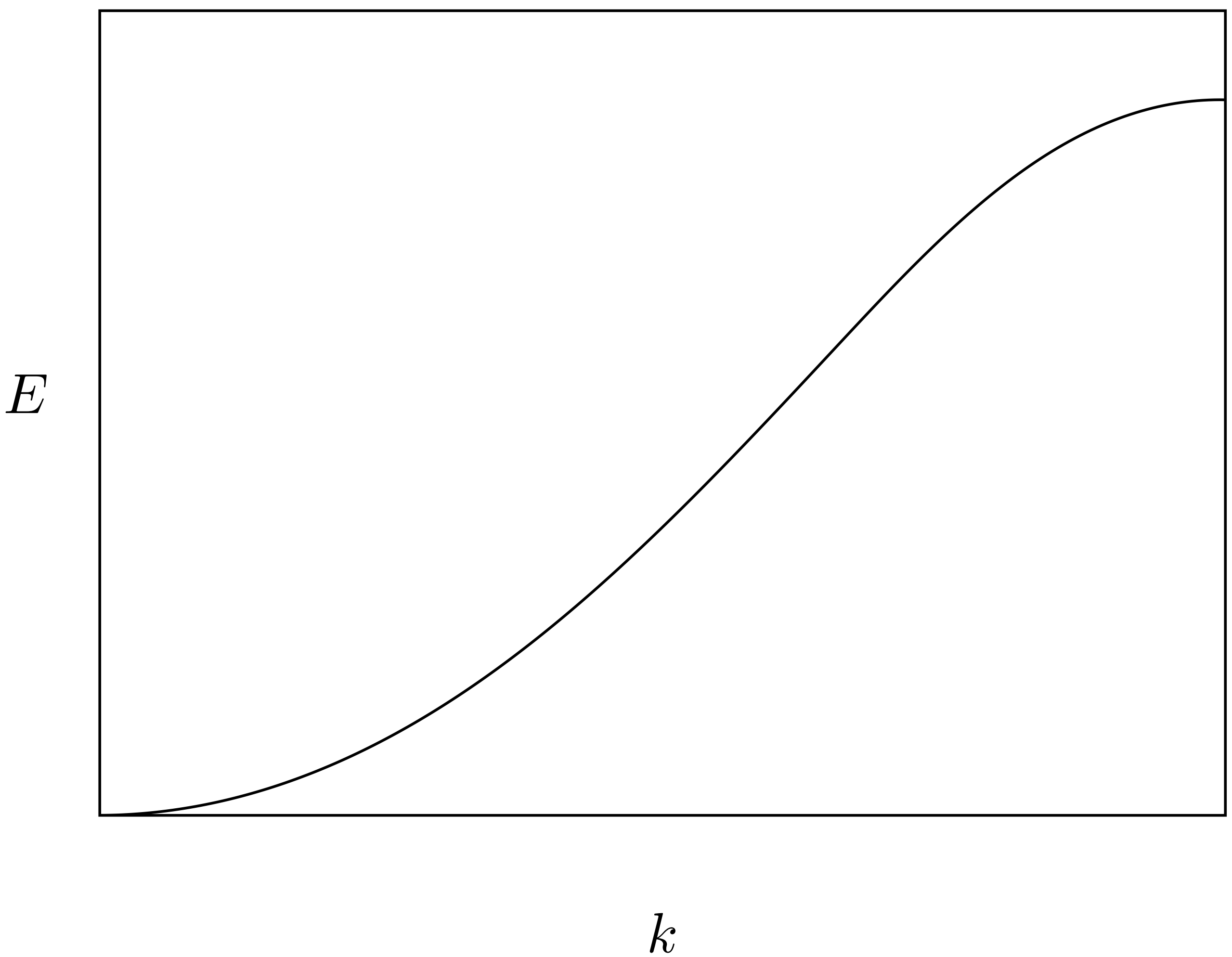


Effective Mass



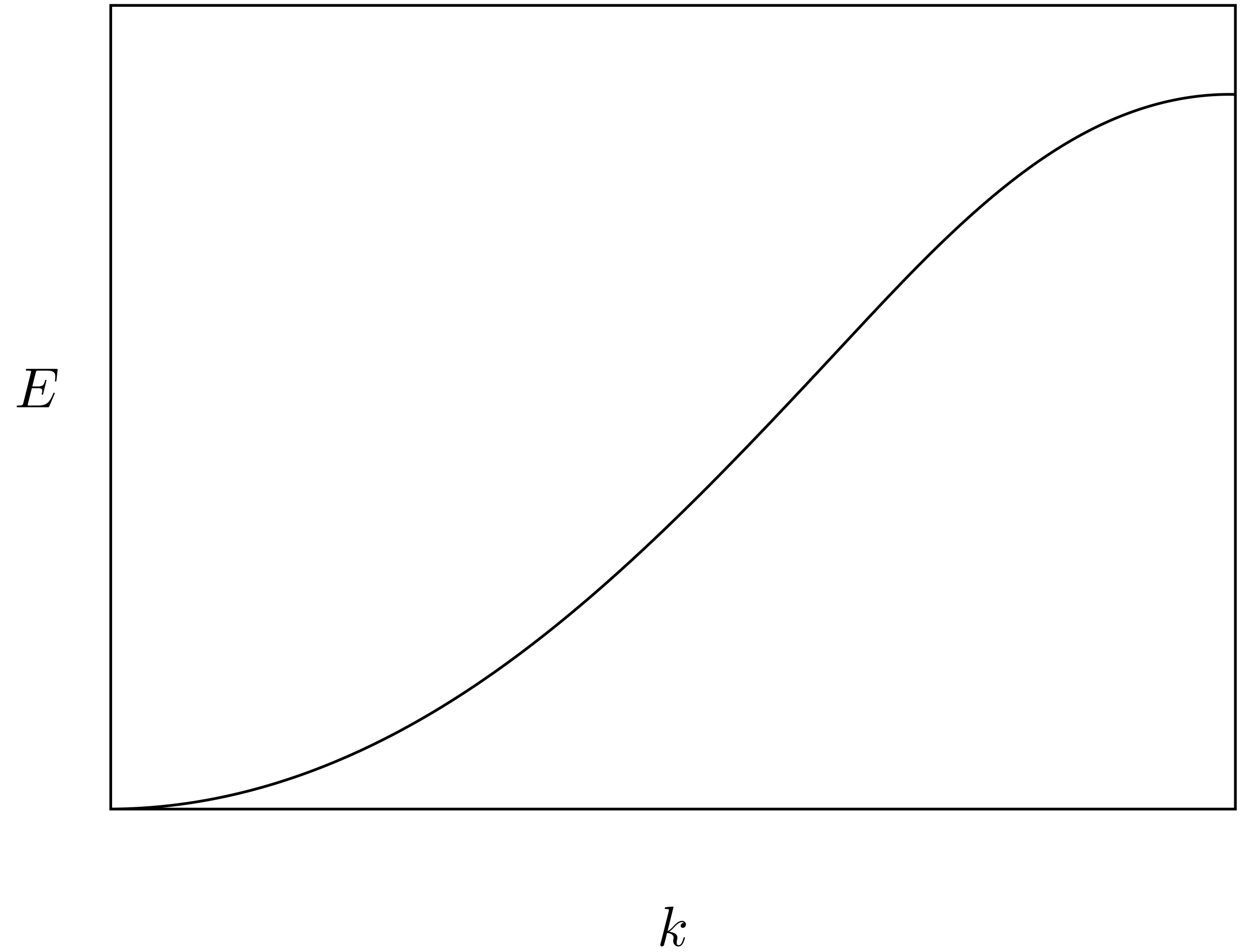
Effective Mass

$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$



Effective Mass

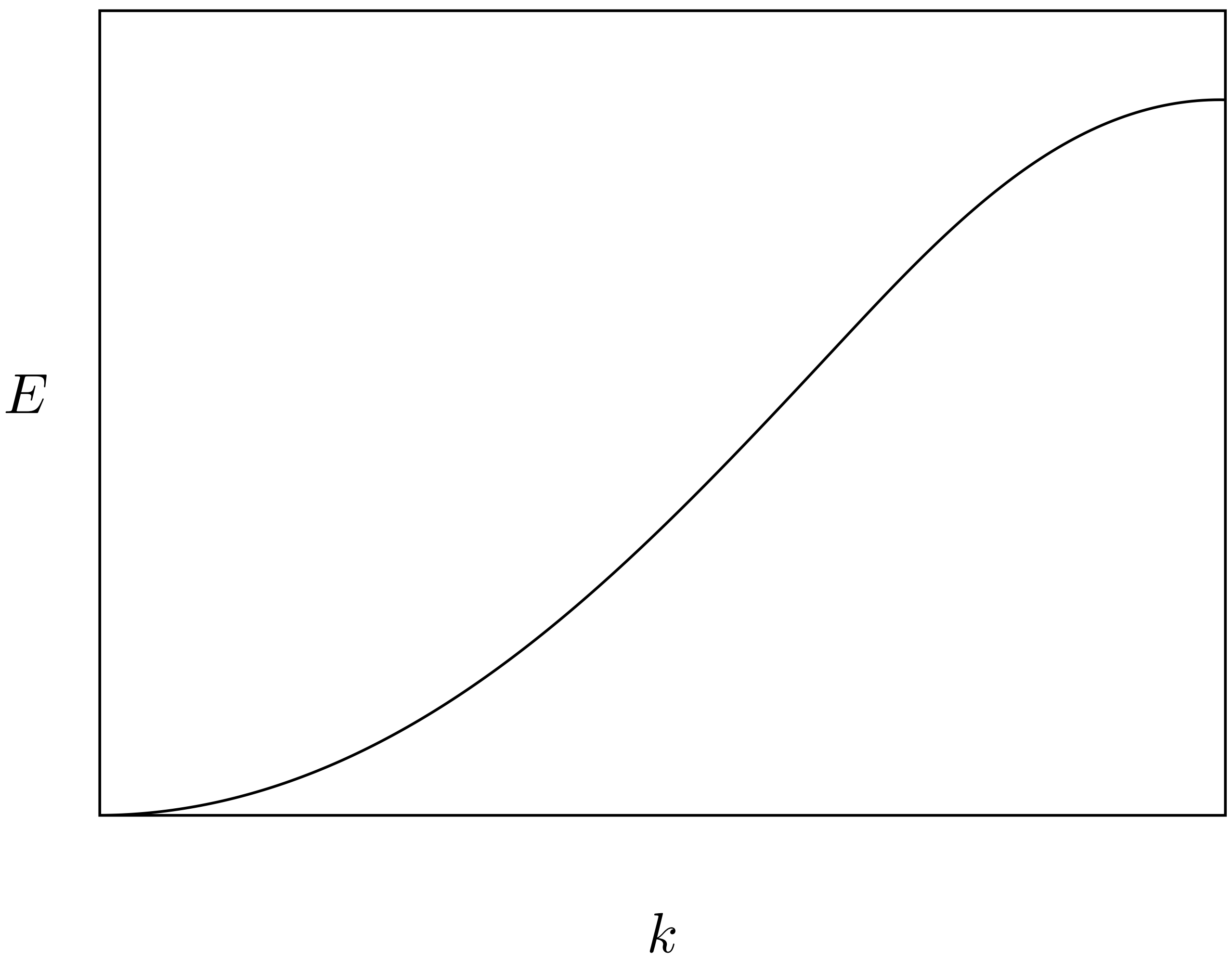
$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$

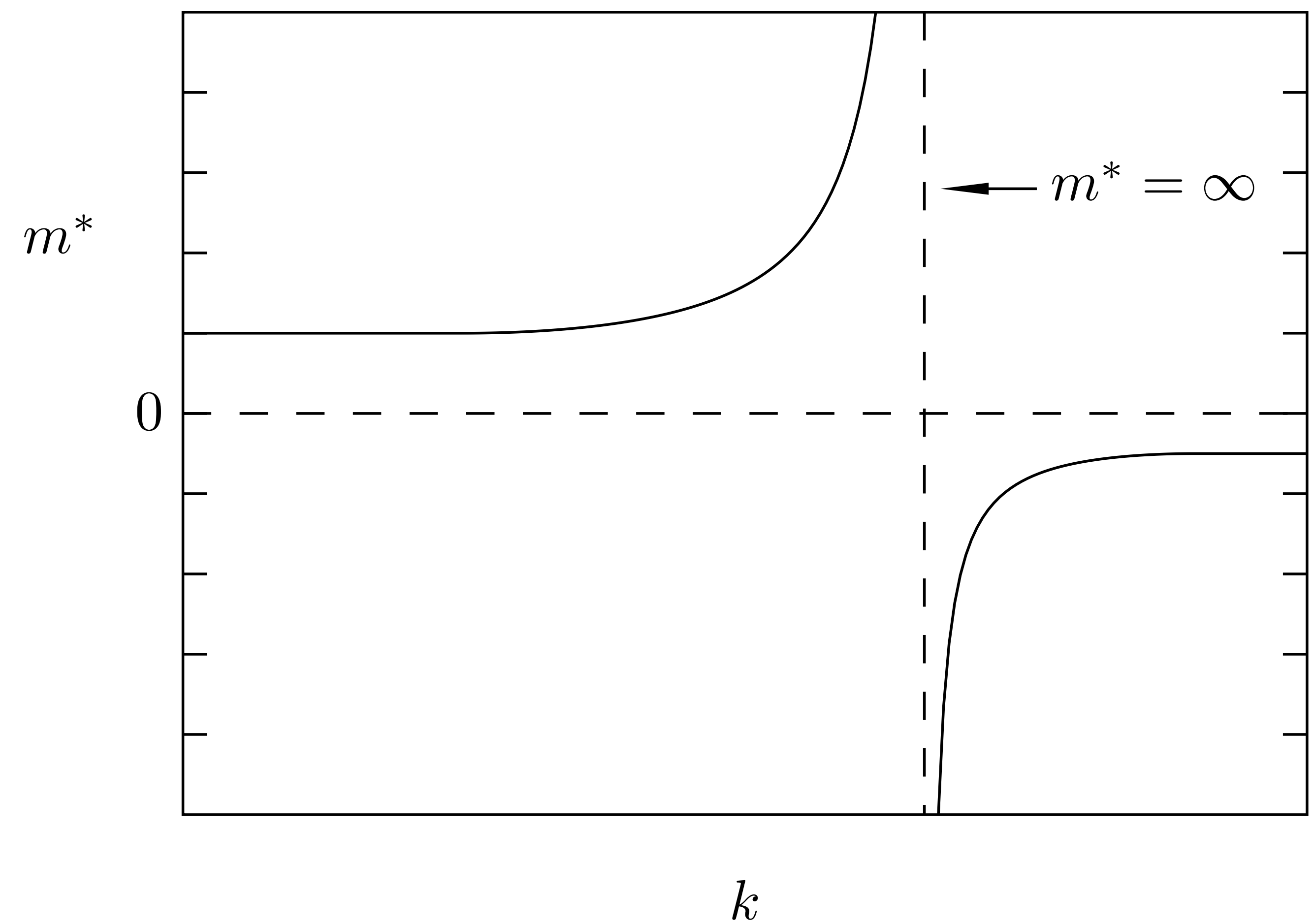


Draw the effective mass as a function of *k* for this band. When you are finished, push (A) on your clicker.

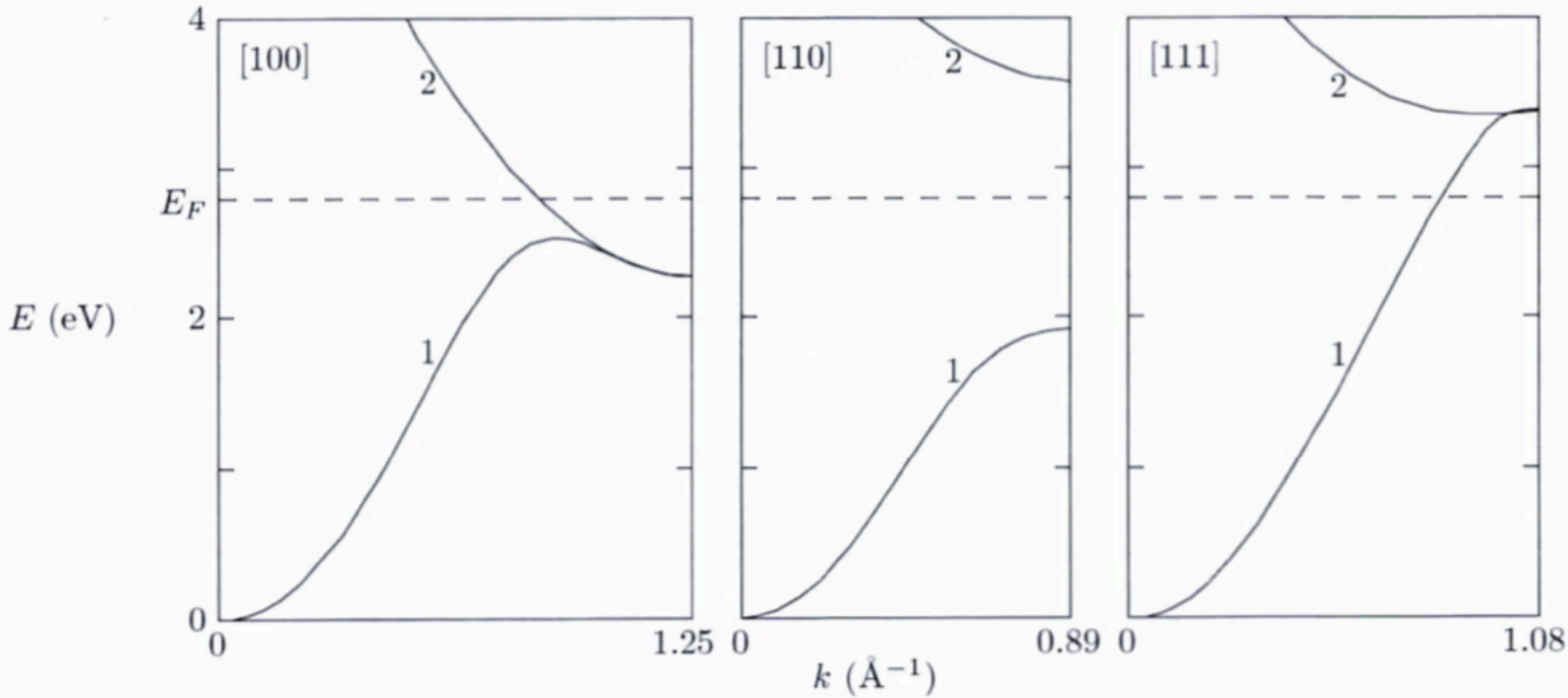
Effective Mass

$$m^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$$



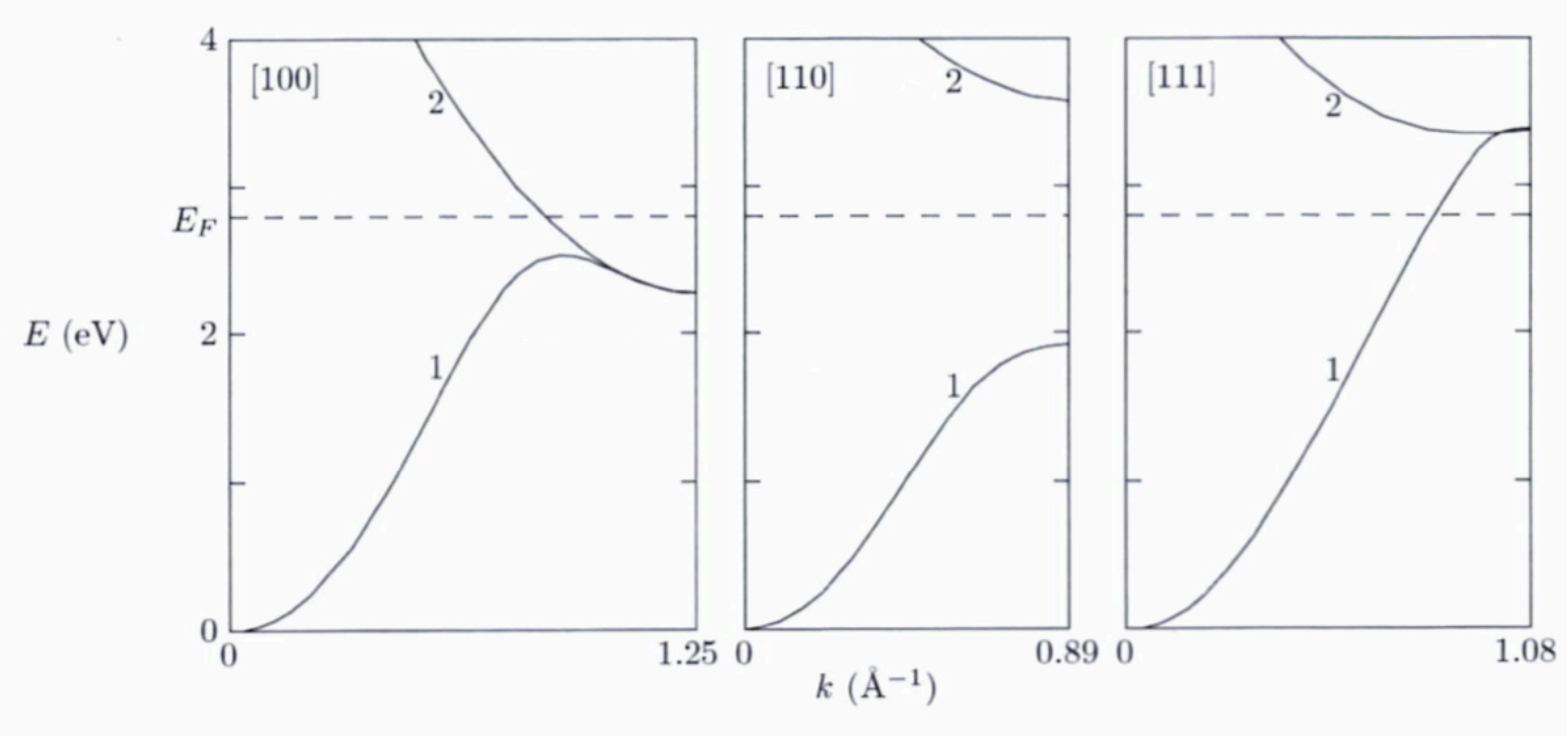


Which electrons have negative effective mass?



Which electrons have negative effective mass?

Hey, they're always the states at the top of a band.



Question #18

Consider an electron with negative effective mass. If we apply an external force on that electron in the $+x$ direction, in which direction will the electron move in k space?

(A) $+x$
(B) $-x$
(C) It depends on the initial velocity
(D) I don't know

How do you know?

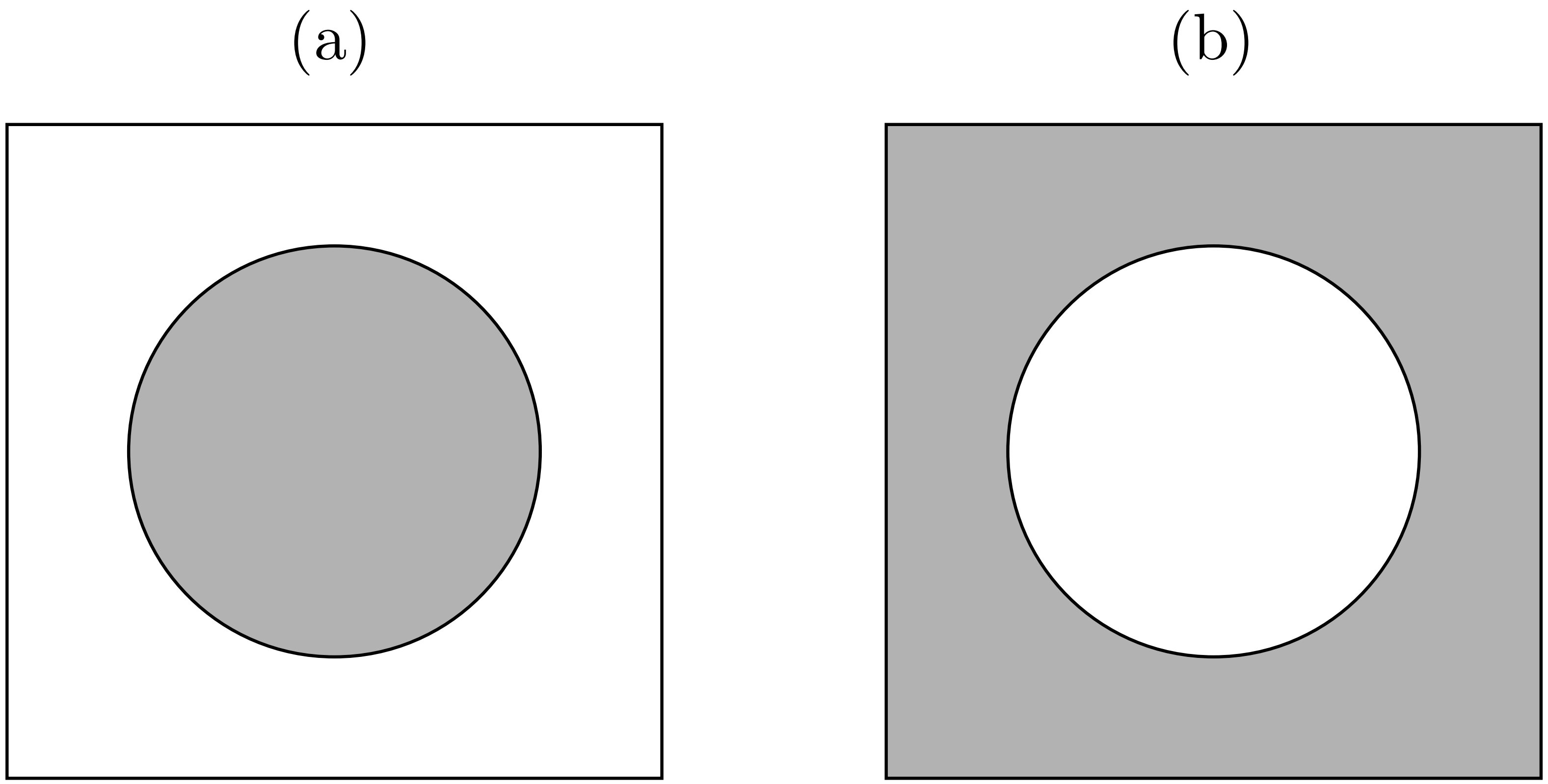
Effective Mass Tensor

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

$$E(\mathbf{k}) = a_1(k_x + k_y)^2 + a_2(k_x - k_y)^2 + a_3 k_z^2$$

Go to the board and build the tensor

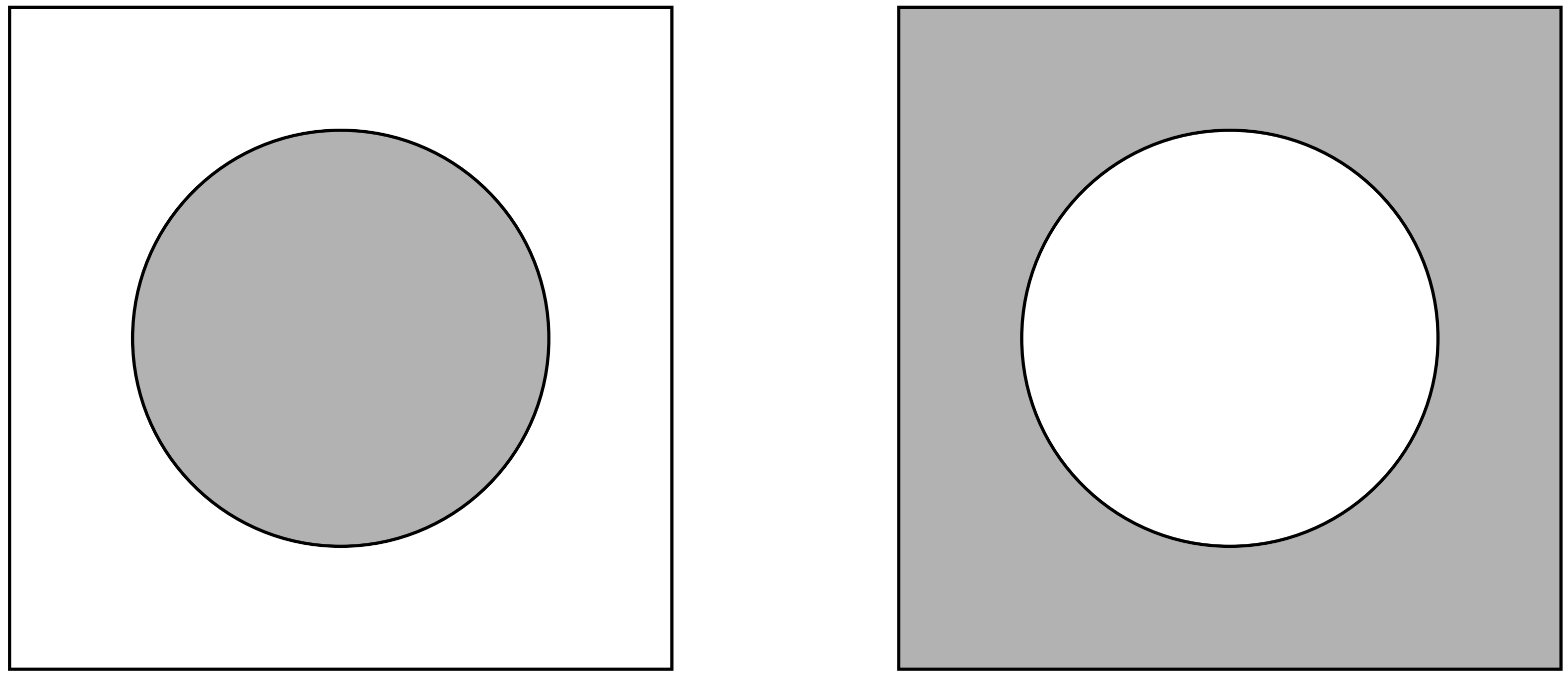
Holes



If you take an electron out of a band, what happens?

Holes

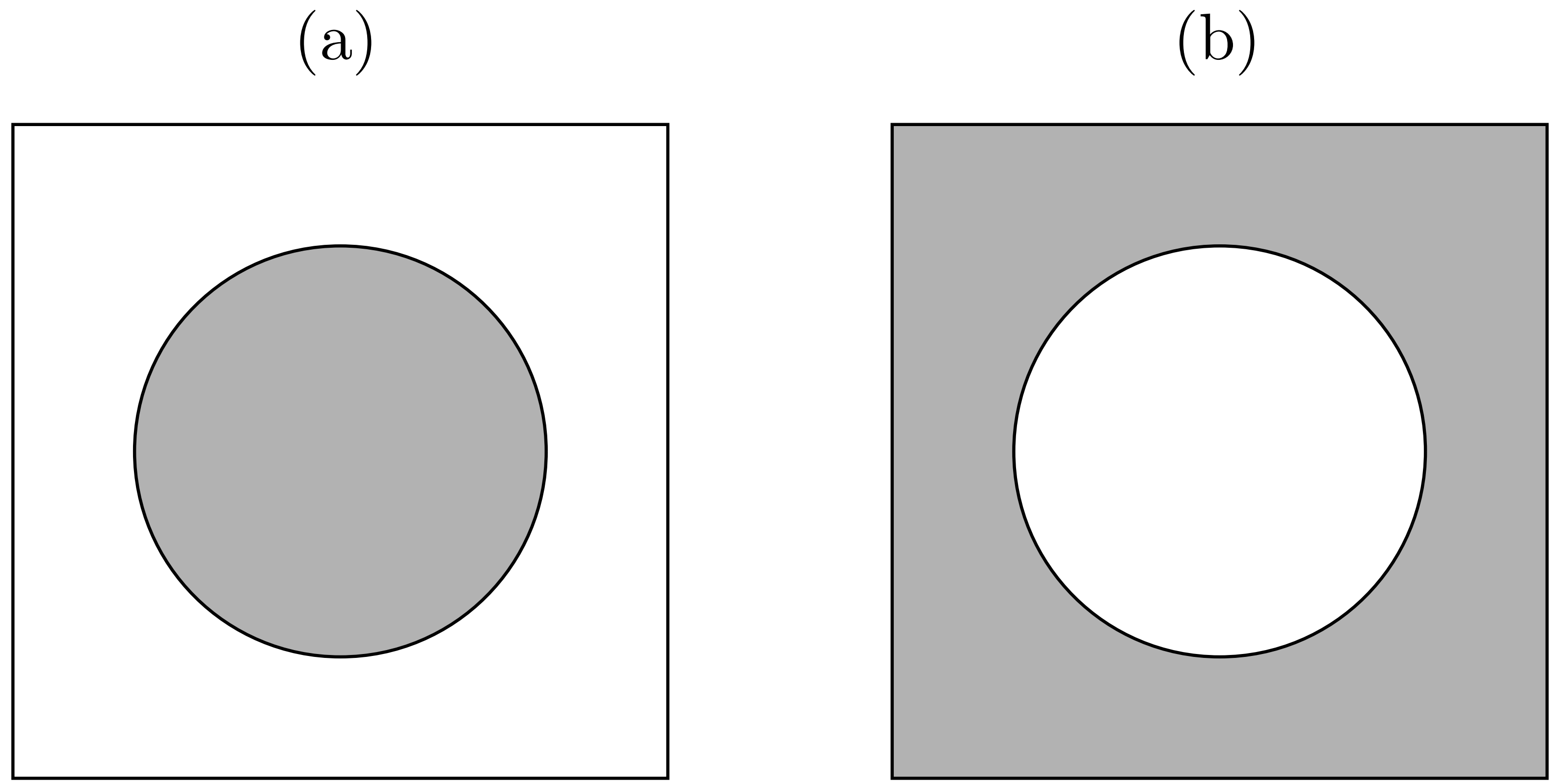
(a) (b)



If you take an electron out of a band, what happens?

$$E_p = -E_n$$

Holes

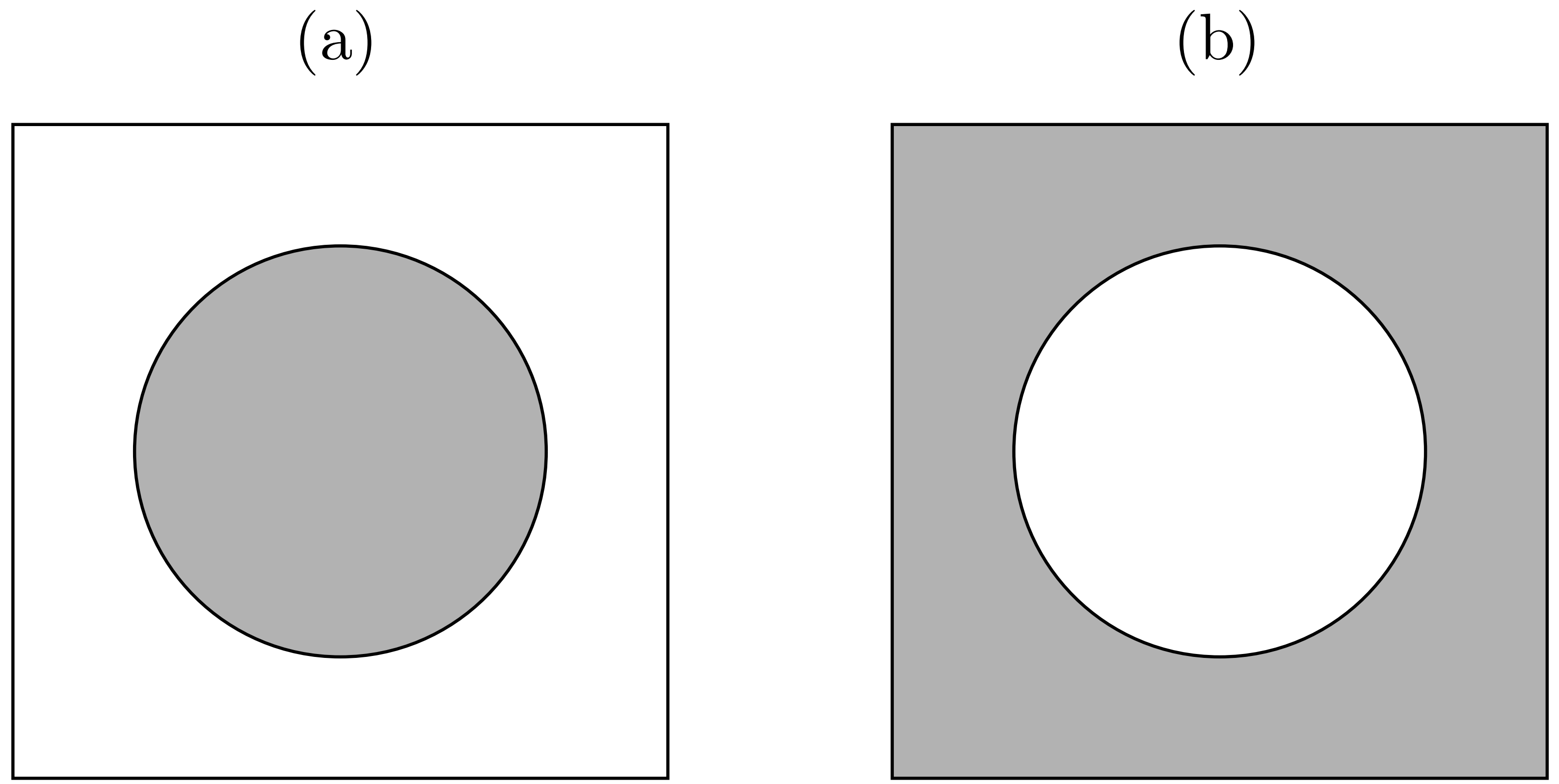


If you take an electron out of a band, what happens?

$$E_p = -E_n$$

The “missing” energy is the energy of the hole

Holes



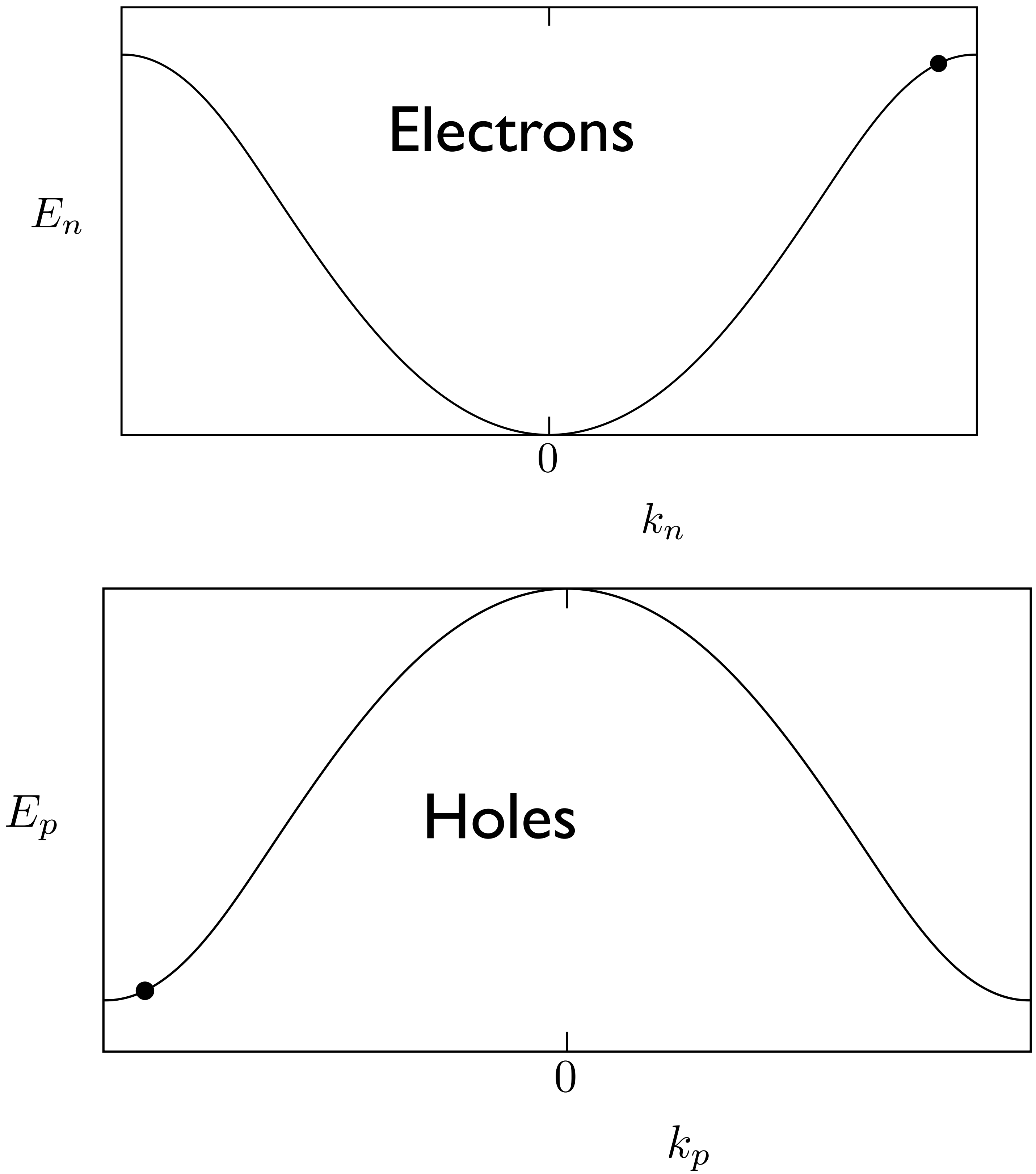
If you take an electron out of a band, what happens?

$$E_p = -E_n$$

$$\mathbf{k}_p = -\mathbf{k}_n$$

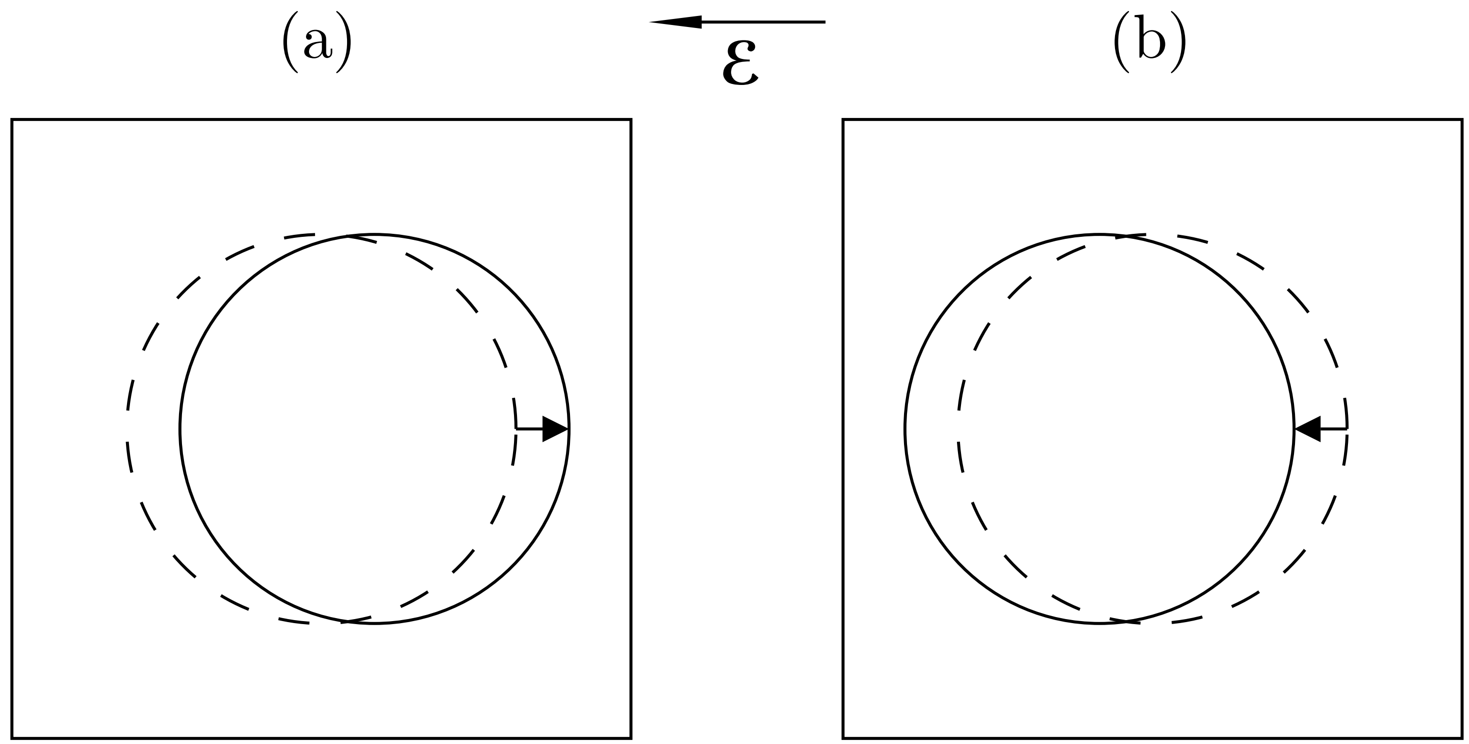
$$q_p = -q_n = +e$$

Holes in a band



Current

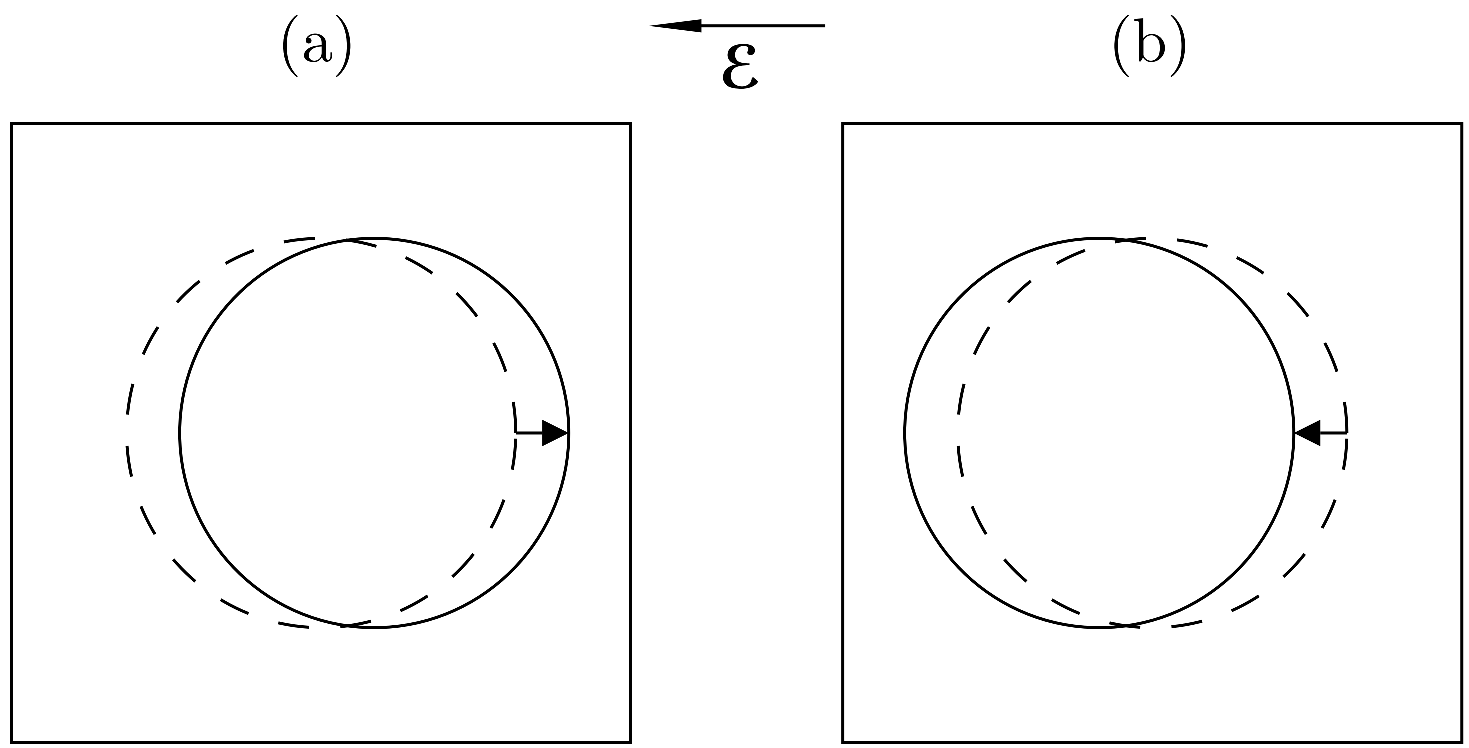
Electrons



Holes

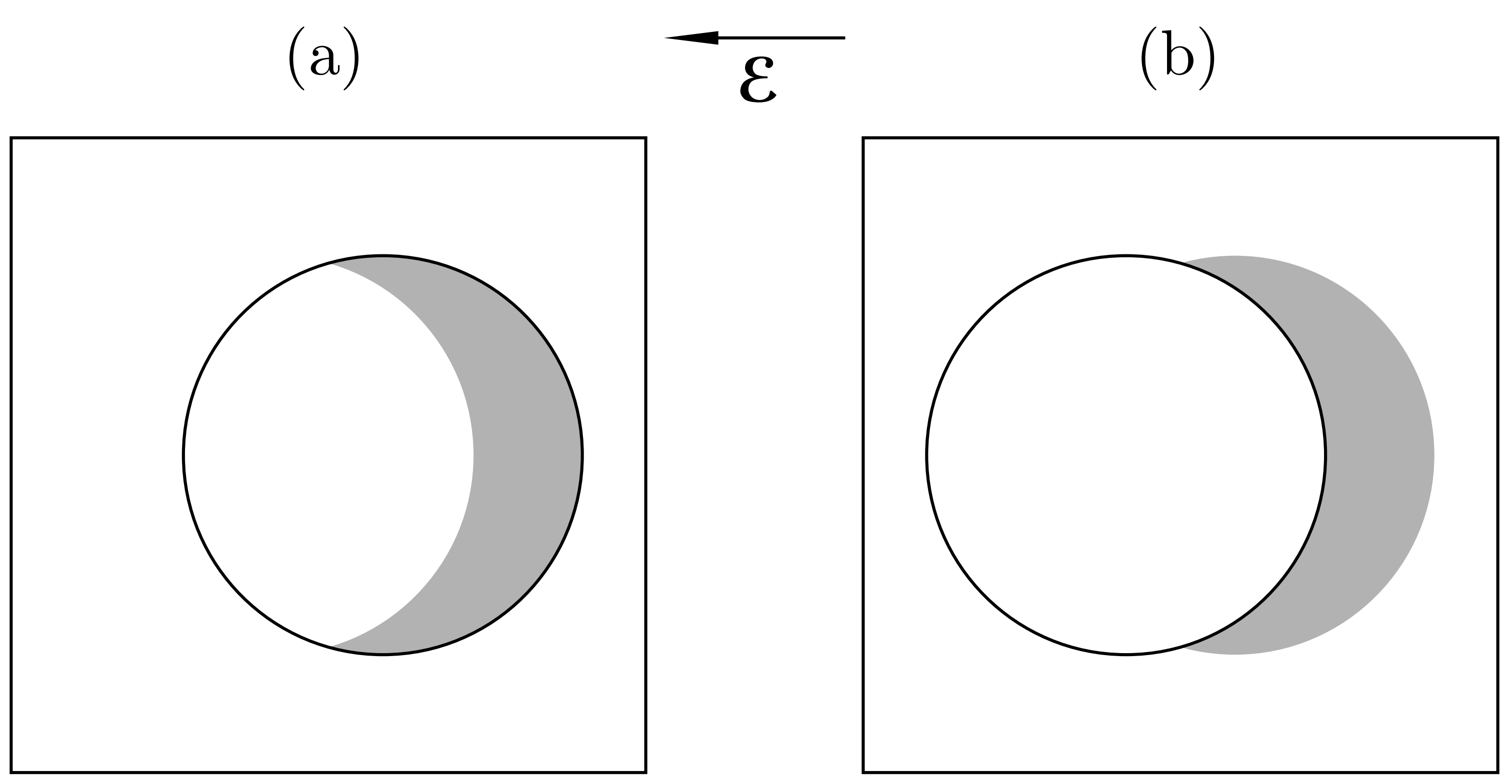
Current

Electrons



Holes

Electrons



Holes

The Hall effect

$$\mathbf{F} = -e\mathbf{v} \times \mathbf{B} \qquad 1$$

$$\boxed{F_{\text{ext}} = \hbar \frac{dk}{dt}} \qquad 2$$

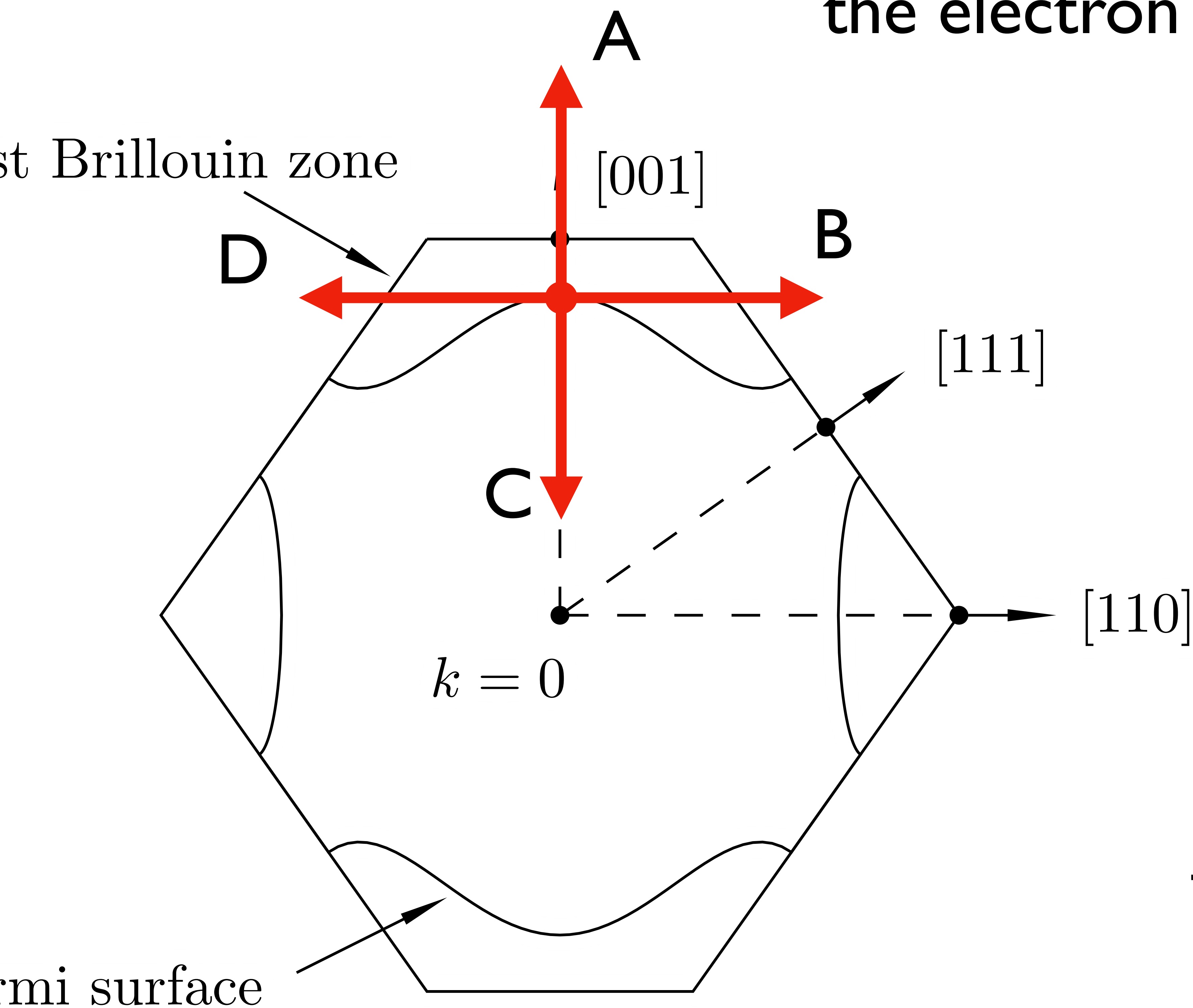
$$\mathbf{v} = \frac{1}{\hbar} \nabla_{\mathbf{k}} E \qquad 3$$

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar^2} \nabla_{\mathbf{k}} E \times \mathbf{B} \qquad 4$$

The Hall effect

Question #19

What is the group velocity vector for the electron in the state below?

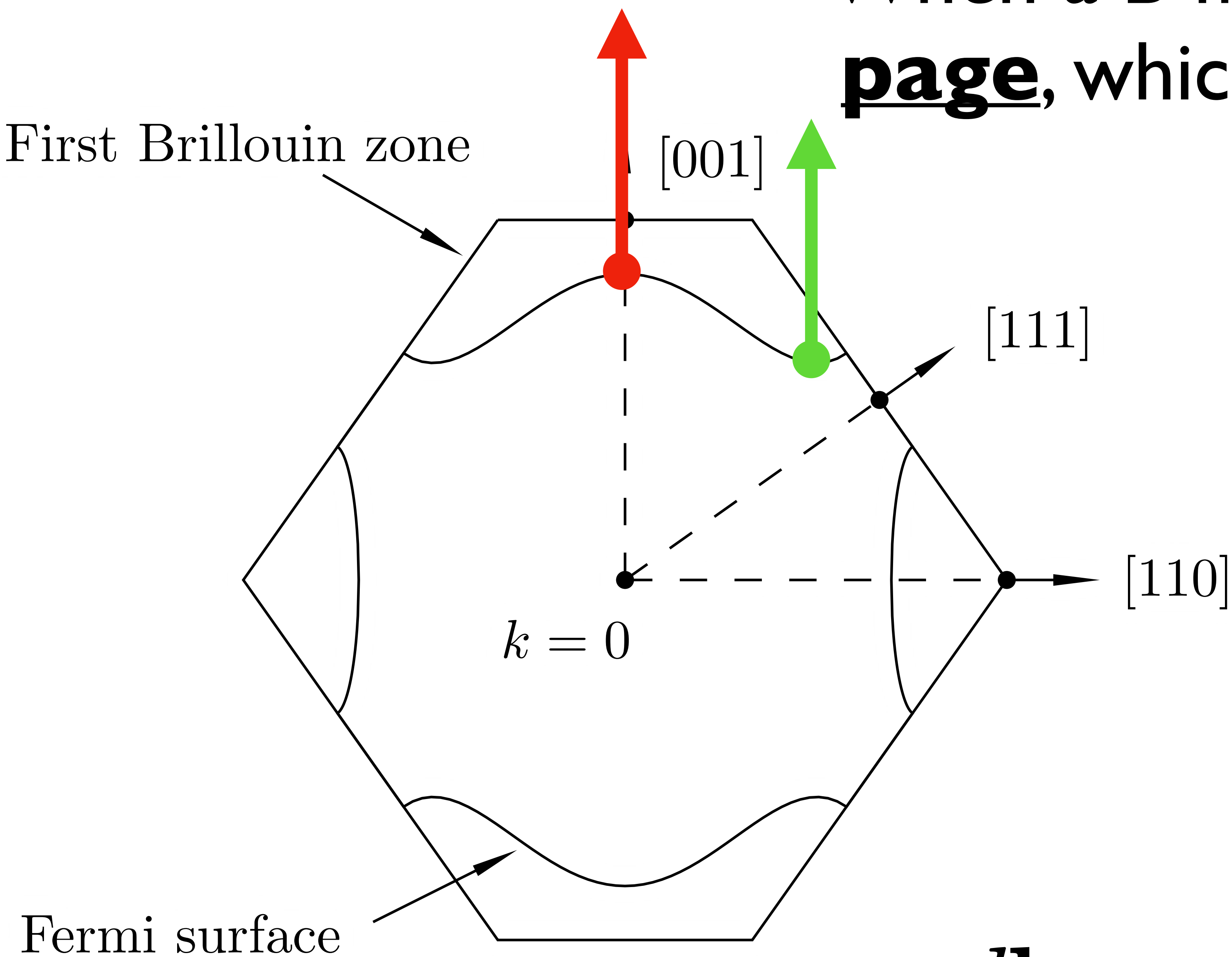


$$\mathbf{v} = \frac{1}{\hbar}\nabla_{\mathbf{k}}E$$

The Hall effect

Question #18

When a B field is applied **<u>out of the page</u>**, which electron drifts leftward? (in real space)



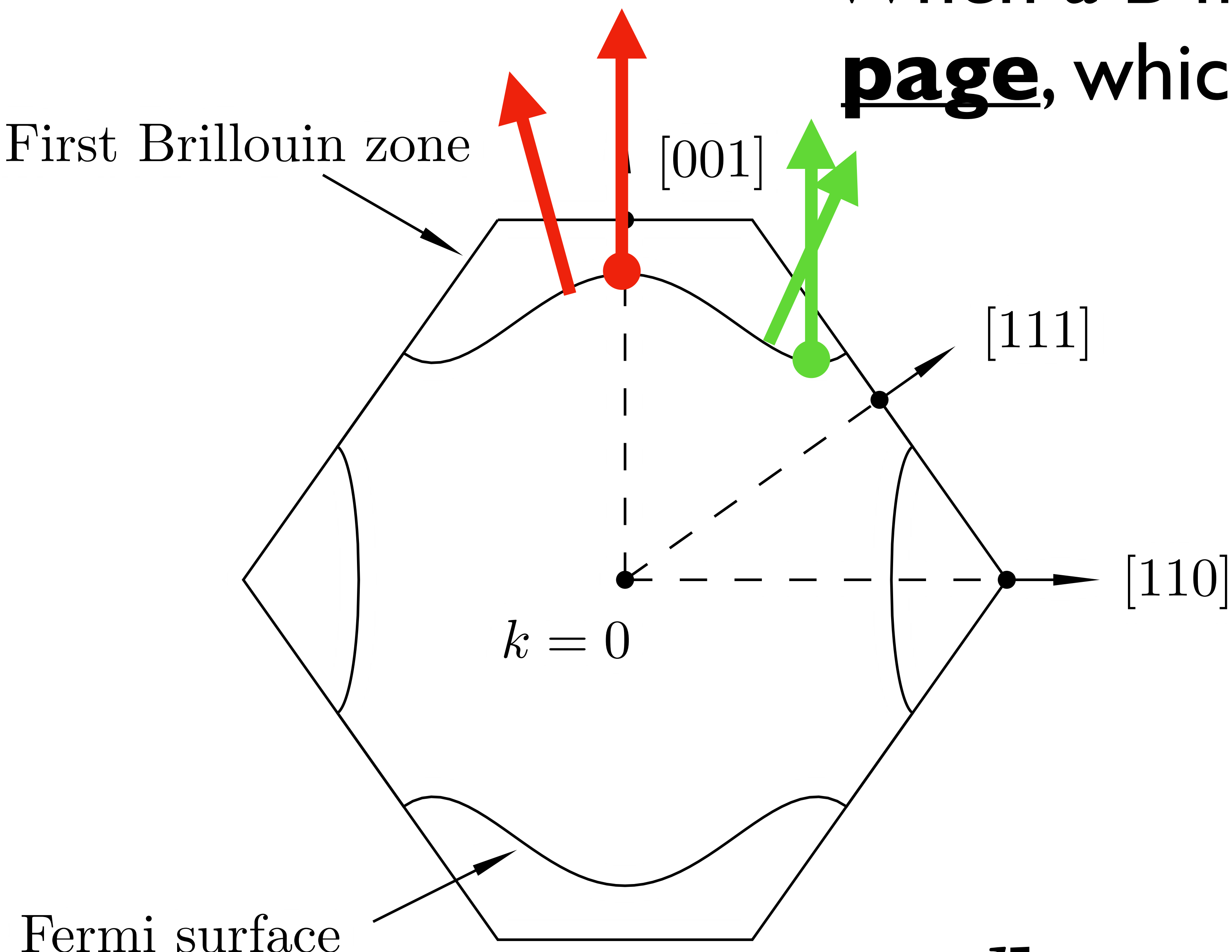
A) they both do
B) Green state
C) Red state
D) Neither do.

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar^2}\nabla_{\mathbf{k}}E \times \mathbf{B}$$

The Hall effect

Question #18

When a B field is applied **out of the page**, which electron drifts leftward? (in real space)



A) they both do
B) Green state
C) Red state
D) Neither do.

$$\frac{d\mathbf{k}}{dt} = -\frac{e}{\hbar^2}\nabla_{\mathbf{k}}E \times \mathbf{B}$$