1 Day 4: Dirac Notation

1. We have seen that the eigenstates for the infinite square well are given by:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a}) \tag{1}$$

and learned that you can write **any** function (over the interval $0 < x < a$) in terms of them. Now let's see if we can add a very small modification to the infinite square well potential and see how we would solve it. Consider the following modification to the infinite square well potential:

$$V = \begin{cases} 5 & \frac{a}{3} < x < \frac{a}{2} \\ 0 & 0 \geq x < \frac{a}{3} \\ 0 & \frac{a}{2} \geq x < a \\ \infty & \text{otherwise} \end{cases} \tag{2}$$

1. Using the first 10 energy eigenfunctions of the ideal infinite square well, construct the Hamiltonian matrix ($\langle m|\hat{H}|n\rangle$) for the modified infinite square well. (Note: $|n\rangle$ are not eigenstates of the $\hat{H}$ for this modified potential. Therefore you'll need to use a computer to build this matrix.)

2. You should begin to see an eigenvalue problem emerging (in the linear algebra sense, not the differential equation sense). Use a computer to solve the eigenvalue problem.

3. Make a plot of a few of the solutions and verify that they seem correct for the modified potential square well potential that we started with.

2. Choose some other potential (finite square well, harmonic oscillator ($\frac{1}{2}kx^2$), "Vee" potential) and solve Schrodinger's equation using infinite square well eigenfunctions as your basis.

Note: If you choose a domain that is different from the infinite square well domain, you'll need to re-solve Schrodinger's equation for the infinite square well again (It's not that big of a deal, you just have to slide the functions over to fit the domain).)