

1 Day 1: Wavefunctions, Probabilities, and Normalization

1. Consider the following wavefunctions:

$$\psi_1(x) = Ae^{\frac{-y^2}{4}} \quad (1)$$

$$\psi_2(x) = Aye^{\frac{-y^2}{8}} \quad (2)$$

and

$$\psi_3(x) = A \left(e^{\frac{-y^2}{4}} + ye^{\frac{-y^2}{8}} \right) \quad (3)$$

1. Normalize all three states over the interval $-\infty < y < \infty$
 2. What is the probability of finding the particle in the region $0 < y < 1$ for all three states.
 3. Is the probability of finding the particle in the region $-1 < y < 1$ when it is in state ψ_3 the same as the sum of the probabilities when the particle is in states ψ_1 and ψ_2
2. Consider the following wavefunction:

$$\psi(x) = e^{\frac{(x-a)^2}{4b^2}} e^{icx} \quad (4)$$

where $a = 0$, $b = 5$ and $c = 0.3$

1. Normalize the wavefunction.
 2. What is the expectation value of position $\langle x \rangle$
 3. What is the expectation value of the square of the position $\langle x^2 \rangle$
 4. What is the variance of the wavefunction $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$
3. Consider the following wavefunction defined over the domain $0 < x < a$:

$$\psi(x) = \cos\left(\frac{x}{10}\right) \left(\sin\left(\frac{2\pi x}{a}\right) + \sin\left(\frac{6\pi x}{a}\right) + \sin\left(\frac{8\pi x}{a}\right) \right) \quad (5)$$

with $a = 5$

1. Plot the wavefunction and the square of the wavefunction.
2. Normalize the wavefunction over the domain $0 < x < 5$.
3. What is the expectation value of position $\langle x \rangle$
4. What is the expectation value of the square of the position $\langle x^2 \rangle$
5. What is the variance of the wavefunction $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$