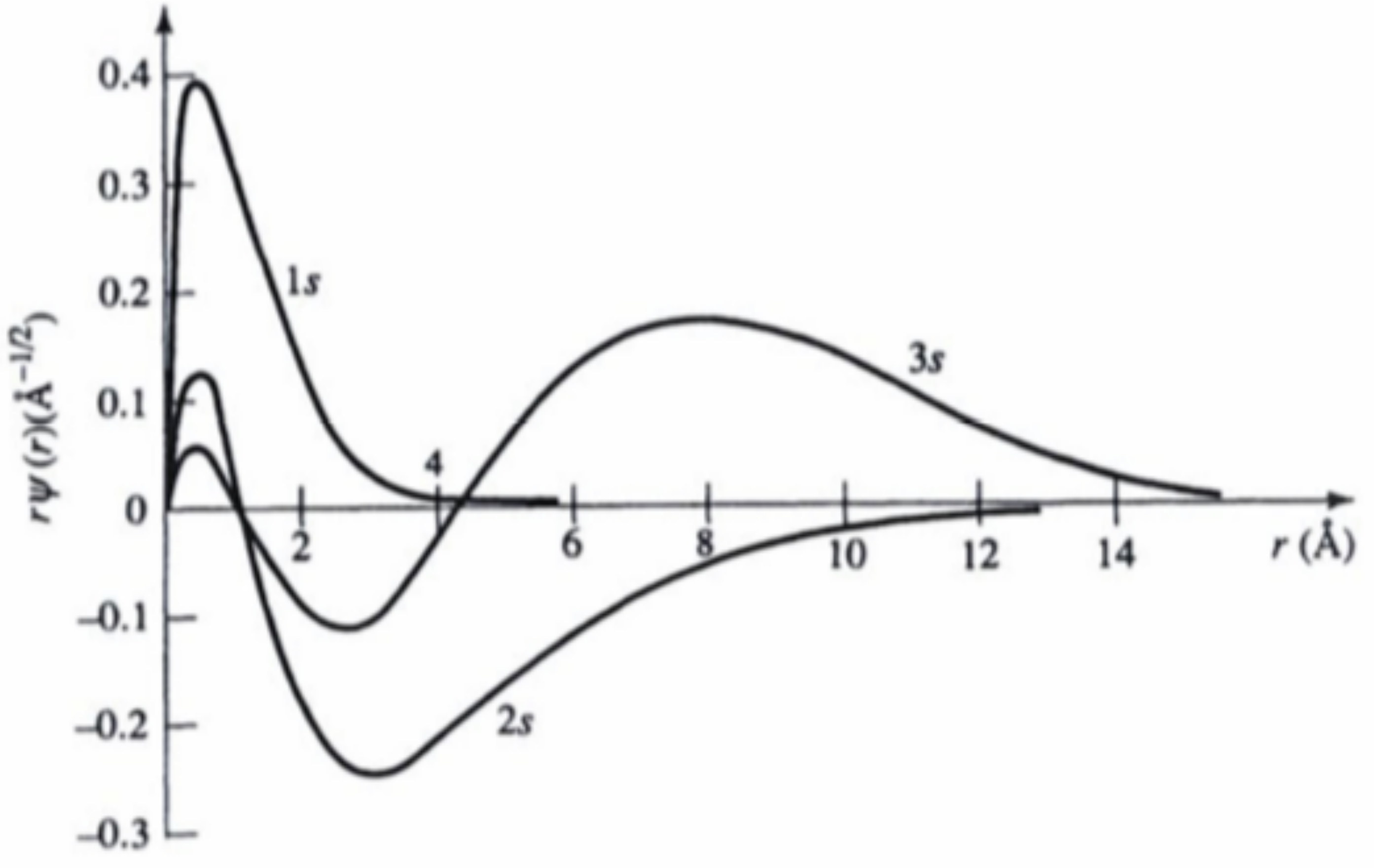
$$H_1|1\rangle = E_f|1\rangle \qquad H_2|2\rangle = E_f|2\rangle$$

$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle \quad \boxed{1}$$



$$H|\Psi\rangle = E|\Psi\rangle \quad \boxed{2}$$

$$H\left(c_1|1\rangle + c_2|2\rangle\right) = E\left(c_1|1\rangle + c_2|2\rangle\right) \quad \boxed{3}$$

$$c_1\langle 1|H1\rangle + c_2\langle 1|H2\rangle = E\left(c_1\langle 1|1\rangle + c_2\langle 1|2\rangle\right) \quad \boxed{4}$$

$$c_1\langle 1|H1\rangle + c_2\langle 1|H2\rangle = E\left(c_1\langle 1|1\rangle + c_2\langle 1|2\rangle\right)$$

$$c_1\langle 2|H1\rangle + c_2\langle 2|H2\rangle = E\left(c_1\langle 2|1\rangle + c_2\langle 2|2\rangle\right) \quad \boxed{5}$$

$$\begin{array}{c} \begin{array}{cc} E_0 & \beta \end{array} \\ \begin{bmatrix} \langle 1|\mathbf{H}|1\rangle & \langle 1|\mathbf{H}|2\rangle \\ \langle 2|\mathbf{H}|1\rangle & \langle 2|\mathbf{H}|2\rangle \end{bmatrix} \\ \begin{array}{cc} \beta & E_0 \end{array} \end{array} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = E \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \boxed{6}$$

$$\begin{vmatrix} E_0 - E & \beta \\ \beta & E_0 - E \end{vmatrix} = 0 \quad \boxed{7}$$

$$E = E_0 \pm \beta \quad \boxed{11}$$

$$(E_0 - E)(E_0 - E) - \beta^2 = 0 \quad \boxed{8}$$

$$E_0^2 + E^2 - 2E_0E - \beta^2 = 0 \quad \boxed{10}$$

$$i\hbar\frac{\partial}{\partial t}\psi = \mathbf{H}\psi \tag{1}$$

$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle \tag{2}$$

$$i\hbar\frac{\partial c_1}{\partial t} = (c_1 H_{11} + c_2 H_{12}) \tag{6}$$

$$i\hbar\frac{\partial c_2}{\partial t} = (c_1 H_{21} + c_2 H_{22}) \tag{7}$$

$$i\hbar\frac{\partial}{\partial t}(c_1|1\rangle + c_2|2\rangle) = \mathbf{H}(c_1|1\rangle + c_2|2\rangle) \tag{3}$$

$$i\hbar(\frac{\partial c_1}{\partial t}|1\rangle + \frac{\partial c_2}{\partial t}|2\rangle) = \mathbf{H}(c_1|1\rangle + c_2|2\rangle) \tag{4}$$

$$i\hbar(\frac{\partial c_1}{\partial t}\langle 1|1\rangle + \frac{\partial c_2}{\partial t}\langle 1|2\rangle) = (c_1\langle 1|\mathbf{H}|1\rangle + c_2\langle 1|\mathbf{H}|2\rangle) \tag{5}$$

$$i\hbar\frac{\partial c_1}{\partial t} = (c_1 H_{11} + c_2 H_{12})$$

$$i\hbar\frac{\partial c_2}{\partial t} = (c_1 H_{21} + c_2 H_{22})$$

$$c_1(t) = A_1 e^{-i\omega t} \quad \boxed{8}$$

$$c_2(t) = A_2 e^{-i\omega t} \quad \boxed{9}$$

$$\hbar A_1 \omega = (A_1 H_{11} + A_2 H_{12}) \quad \boxed{10}$$

$$\hbar A_2 \omega = (A_1 H_{21} + A_2 H_{22}) \quad \boxed{11}$$

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \hbar\omega \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \quad \boxed{12}$$

$$\begin{vmatrix} H_{11} - \hbar\omega & H_{12} \\ H_{21} & H_{22} - \hbar\omega \end{vmatrix} = 0 \quad \boxed{13}$$

$$(H_{11} - \hbar\omega)(H_{22} - \hbar\omega) - H_{12}H_{21} = 0 \quad \boxed{14}$$

$$H_{11}H_{22} - \hbar\omega\,(H_{11} + H_{22}) + \hbar^2\omega^2 - H_{12}H_{21} = 0 \quad \boxed{15}$$

$$\omega = \frac{E_0 \pm \beta}{\hbar} \quad \boxed{16}$$