1 Day 4: Dirac Notation

1. We have seen that the eigenstates for the infinite square well are given by:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a}) \tag{1}$$

and learned that you can write **any** function (over the interval $0 < x < a$) in terms of them. Now let's see if we can add a very small modification to the infinite square well potential and see how we would solve it. Consider the following modification to the infinite square well potential:

$$V = \begin{cases} 5 & \frac{a}{3} < x < \frac{a}{2} \\ 0 & 0 \geq x < \frac{a}{3} \\ 0 & \frac{a}{2} \geq x < a \\ \infty & \text{otherwise} \end{cases} \tag{2}$$

1. Using the first 10 energy eigenfunctions of the ideal infinite square well, construct the Hamiltonian matrix ($\langle m|\hat{H}|n\rangle$) for the modified infinite square well. (Note: $|n\rangle$ are not eigenstates of the $\hat{H}$ for this modified potential. Therefore you'll need to use a computer to build this matrix.)

2. You should begin to see an eigenvalue problem emerging (in the linear algebra sense, not the differential equation sense). Use a computer to solve the eigenvalue problem.

3. Make a plot of a few of the solutions and verify that they seem correct for the modified potential square well potential that we started with.