1 Day 3: Properties of Eigenfunction, Basis Sets

1. The eigenstates for the infinite square well are given by:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin(\frac{n\pi x}{a}) \tag{1}$$

Show that these eigenstates are orthonormal.

2. (Note: You should not use a computer to do this problem, it's not hard enough.) The wavefunction for a particle that is confined to an infinite square well is given by:

$$\psi(x) = \sqrt{\frac{2}{a}} \left(\frac{\sin(\frac{2\pi x}{a}) + 2\sin(\frac{\pi x}{a})}{\sqrt{5}} \right) \tag{2}$$

1. Is the wavefunction normalized?
2. What is the expectation value of energy: $\langle E \rangle$?
3. With what probability will energy E_1, be measured? (E_2, E_3?)

3. (Note: Use a computer for this one.) The wavefunction for a particle that is confined to an infinite square well ($0 < x < a$) is given by:

$$\psi(x) = \frac{\sqrt{2}}{2} \tag{3}$$

for $\frac{a}{2} - 1 < x < \frac{a}{2} + 1$
and

$$\psi(x) = 0 \tag{4}$$

elsewhere.
Let $a = 5$

1. Is the wavefunction normalized? (Ans. Yes)
2. Find the expansion coefficients c_n. (Ans: $c_n = \frac{\sqrt{5}\left(\cos(\frac{3n\pi}{10}) - \cos(\frac{7n\pi}{10})\right)}{n\pi}$
3. Reconstruct the wave function as a sum of energy eigenstates and plot to verify that it is approximating the wavefunction given.
4. What is the expectation value of energy: $\langle E \rangle$? (Ans: If I use 20 basis functions, ψ_n, to represent the wavefunction, the expectation value for energy is: $\frac{2\hbar^2}{m}$
5. With what probability will energy E_1, be measured? (E_2, E_3?) (Ans: $c_1^2 = 0.7$, $c_2^2 = 0$, $c_3^2 = 0.20$,$c_4^2 = 0$ etc.)
6. Make an animation of the wavefunction for $t > 0$