Physics 323, Section 1, Fall Semester 2019
Instructor: Lance Nelson
Exam #1
Oct 11 – Oct 17

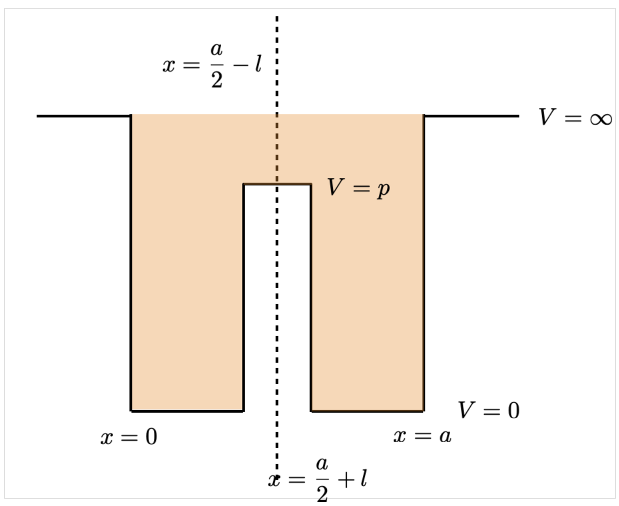
This is an open book(s), untimed exam. Your are prohibited from i) receiving help from another person, and ii) using any code that existed before you began taking the exam. Please be thorough and complete in your work. If a problem requires hand-written work, include a neat, well-organized copy of your work. If you use a computer for any calculations, please include a commented version of that code with your work.
Good Luck... and have fun.

1. [25 pts] The Chevyshev polynomials of the first kind are solutions to the Chebyshev differential equation. The first few Chebyshev polynomials are given by:

$$T_0(x) = 1 \tag{1}$$
$$T_1(x) = x \tag{2}$$
$$T_2(x) = 2x^2 - 1 \tag{3}$$
$$T_3(x) = 4x^3 - 3x \tag{4}$$
$$T_4(x) = 8x^4 - 8x^2 + 1 \tag{5}$$
$$T_5(x) = 16x^5 - 20x^3 + 5x \tag{6}$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1 \tag{7}$$
$$\tag{8}$$

   (a) Plot these functions over the interval $-1 < x < 1$
   (b) Is this set of functions orthogonal on the interval $-1 < x < 1$?
   (c) Is this set of functions normalized on the interval $-1 < x < 1$?

2. [30 pts] Consider the modified infinite square well shown in the figure (not to scale). Using



the eigenfunctions of the **ideal infinite square well** as a basis($\sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$), solve the time-independent Schrodinger equation for this potential.

   (a) Make sure to show all of the paper-and-pencil work needed to form the eigensystem.
   (b) You are free to choose values for p,l, a, and the number of basis functions you use in your expansion but please report those numbers in your solution.
   (c) If you use a computer to perform integrations, include a copy of the code.
   (d) Plot the first few solutions and verify that they seem appropriate.

3. [25 pts] (This problem must be done by hand).The wavefunction for a particle confined to an infinite square well is given by:

$$\psi(x) = \sqrt{\frac{2}{a}}\left(\frac{3\sin(\frac{2\pi x}{a}) + 6\sin(\frac{5\pi x}{a}) + \sin(\frac{6\pi x}{a})}{\sqrt{46}}\right) \tag{9}$$

(a) Write the wavefunction in Dirac notation?

(b) Is the wavefunction normalized? (Show all work. A simple yes/no is not sufficient.)

(c) What is the expectation (average) value of the energy $\langle E \rangle$? (Show your work!!)

(d) With what probability will energy E_1, E_2, and E_5 be measured.

4. [10 pts] The wavefunction for electrons in solids always takes the form of a ____________ part multiplied by a ____________ part. These functions are know as ____________ functions.

5. [10 pts] The region in k-space where all unique electronic states are found is called the ____________ ____________ ____________.