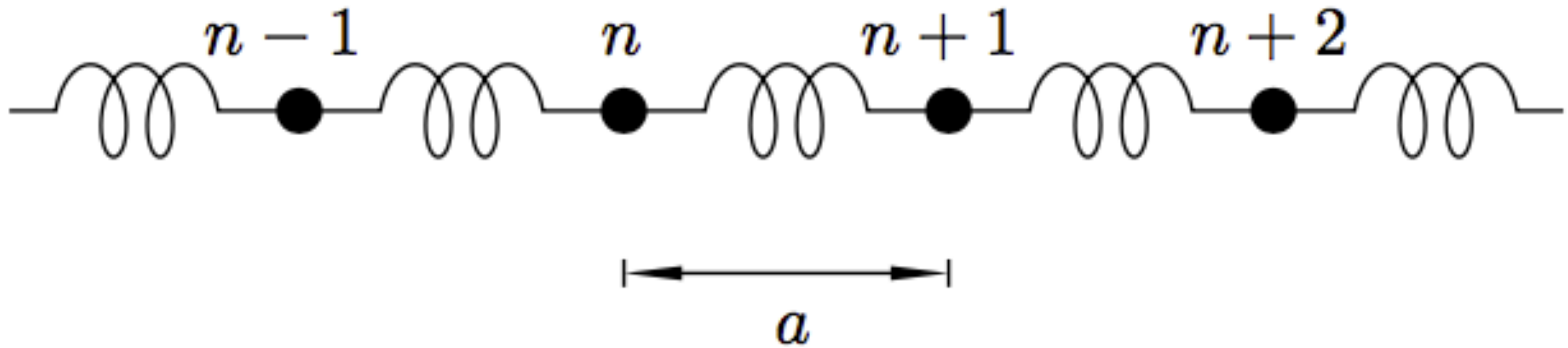


$$-\alpha(2u_n - u_{n+1} - u_{n-1}) = m\frac{d^2u_n}{dt^2}$$

$$u_n = Ae^{ikna-i\omega t}$$

Plug in the function to Newton's second law and solve for omega!

1 $$-\alpha A\left(2e^{ikna-i\omega t}-e^{ik(n+1)a-i\omega t}-e^{ik(n-1)a-i\omega t}\right)=-Am\omega^2 e^{ikna-i\omega t}$$

$$-\alpha\left(2e^{ikna}e^{-i\omega t}-e^{ik(n+1)a}e^{-i\omega t}-e^{ik(n-1)a}e^{-i\omega t}\right)=-m\omega^2 e^{ikna}e^{-i\omega t}$$

2

3 $$-\alpha\left(2e^{ikna}-e^{ik(n+1)a}-e^{ik(n-1)a}\right)=-m\omega^2 e^{ikna}$$

4 $$-\alpha\left(2e^{ikna}-e^{ikna}e^{ika}-e^{ikna}e^{-ika}\right)=-m\omega^2 e^{ikna}$$

5 $$-\alpha\left(2-e^{ika}-e^{-ika}\right)=-m\omega^2$$

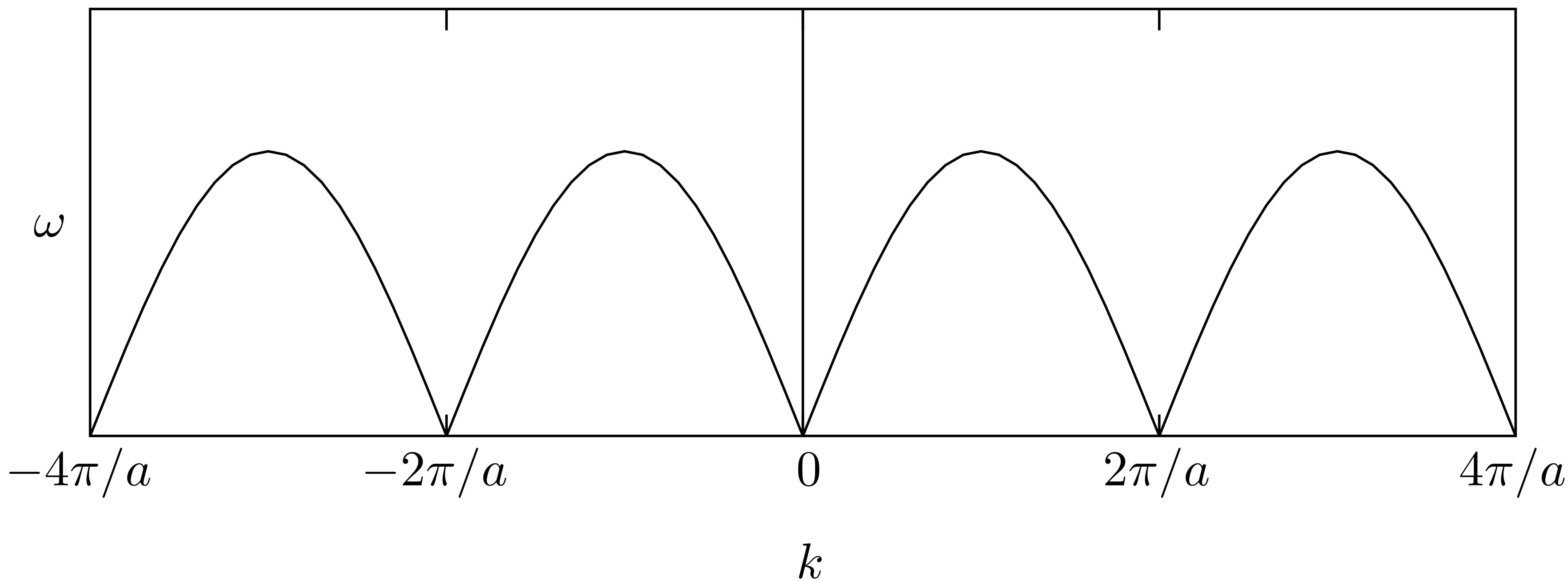
6 $$-\alpha\left(2-2\cos(ka)\right)=-m\omega^2$$

9 $$\sqrt{\frac{4\alpha}{m}}\left|\sin\frac{ka}{2}\right|=\omega$$

7 $$-\alpha\left[2-2\left(1-2\sin^2\frac{ka}{2}\right)\right]=-m\omega^2$$

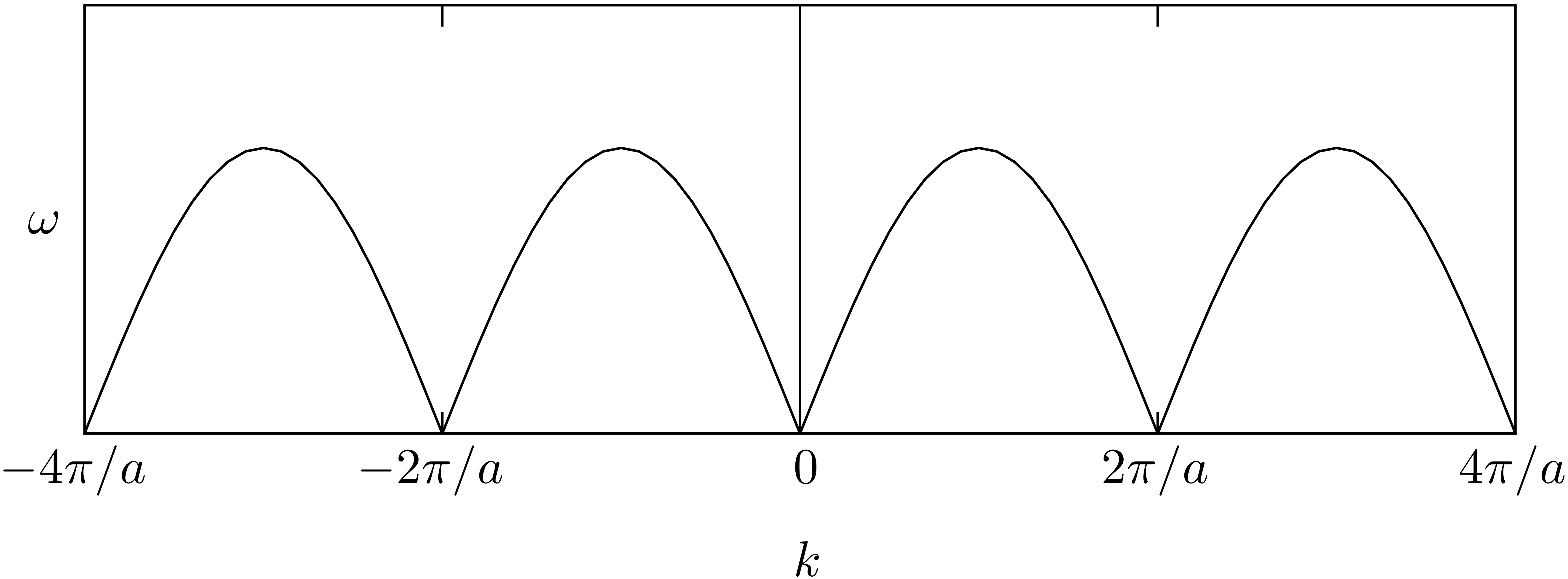
8 $$-\alpha\left[4\sin^2\frac{ka}{2}\right]=-m\omega^2$$

$$\cos(2\theta)=1-2\sin^2\theta$$



$$\omega = \omega_m |\sin(ka/2)|$$

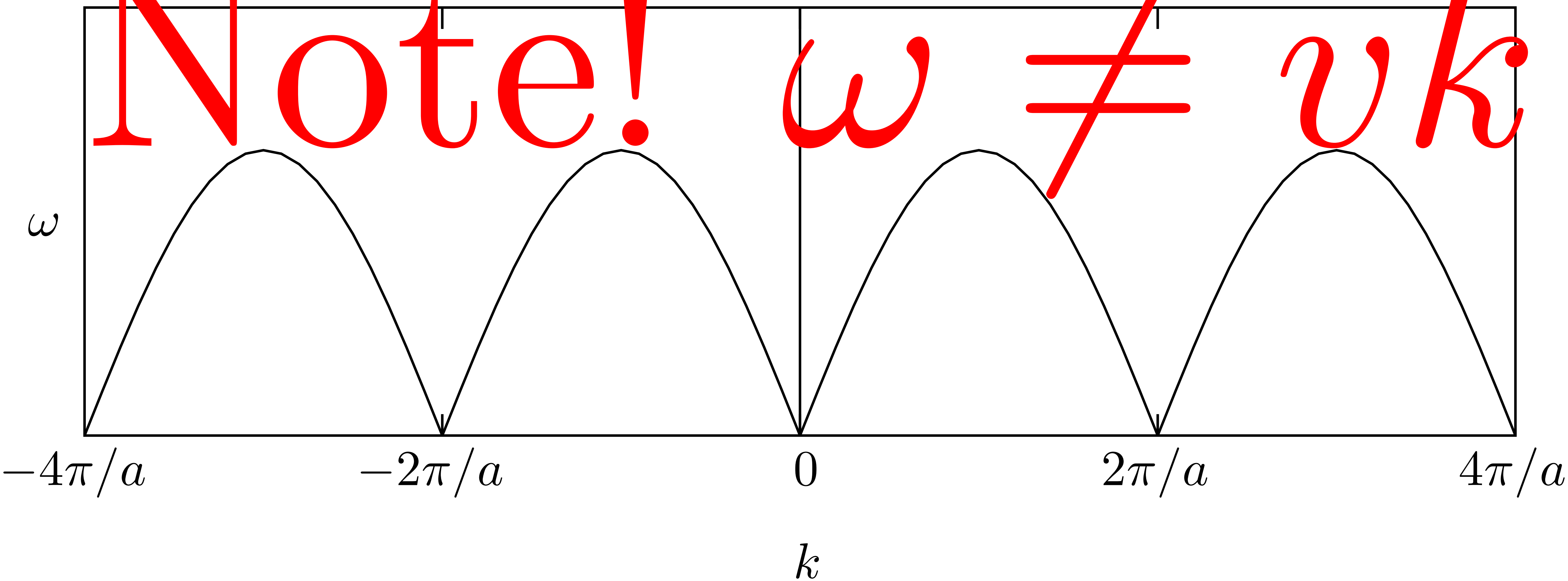
Dispersion relation



$$\omega = \omega_m |\sin(ka/2)|$$

Dispersion relation

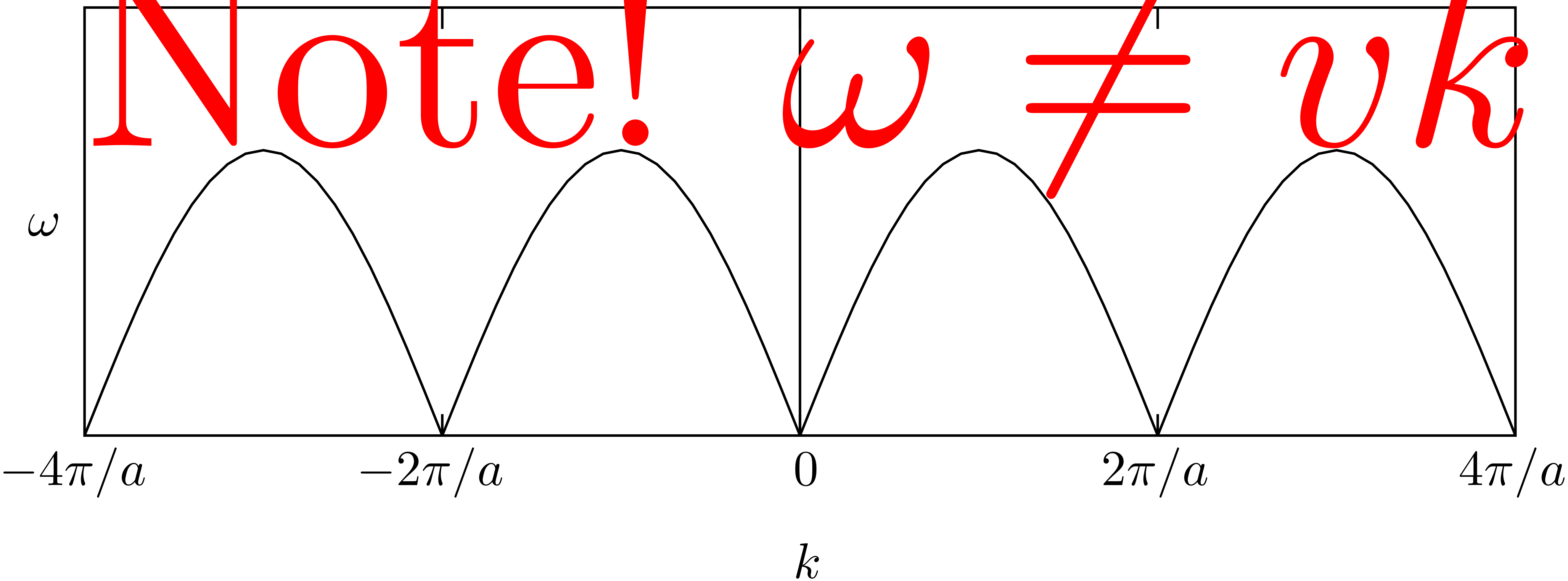
Note! $\omega \neq vk$



$$\omega = \omega_m |\sin(ka/2)|$$

Dispersion relation

Note! $\omega \neq vk$



$$\omega = \omega_m |\sin(ka/2)|$$

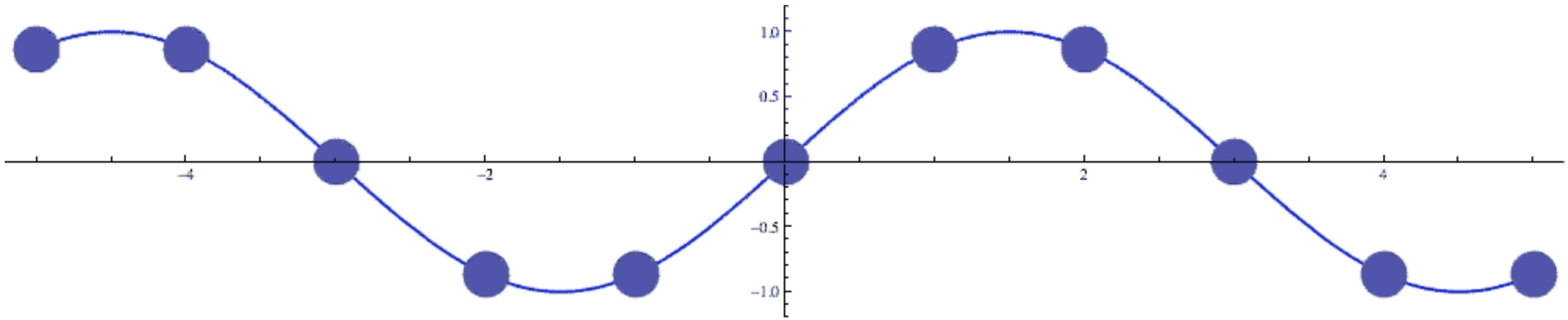
No Rainbows without dispersion!

Question #7

What is the wave vector for the lattice vibration shown?

$\frac{\pi}{4a}$	$\frac{\pi}{2a}$	$\frac{\pi}{8a}$	$\frac{\pi}{10a}$
C	B	A	D

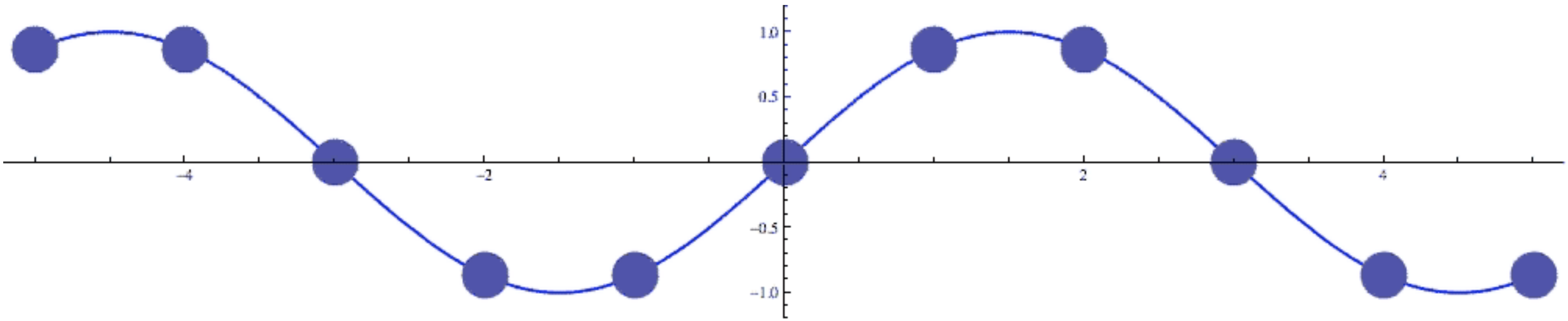
Question #8



What is k for this lattice wave?

$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2\pi}{4}$
C	B	A	D

Question #8



What is k for this lattice wave?

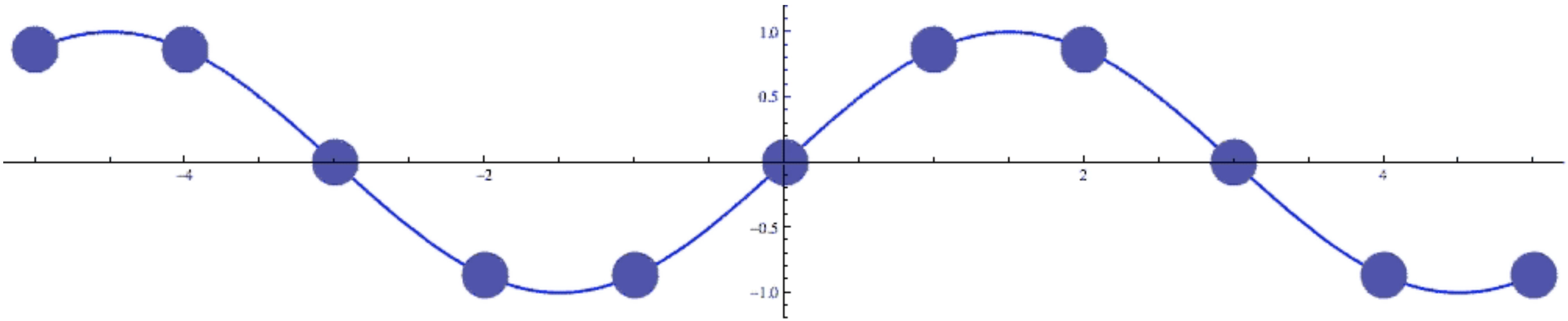
$\frac{\pi}{6}$ C

$\frac{\pi}{3}$ B

$\frac{2\pi}{3}$ A

$\frac{2\pi}{4}$ D

Question #8



What is k for this lattice wave?

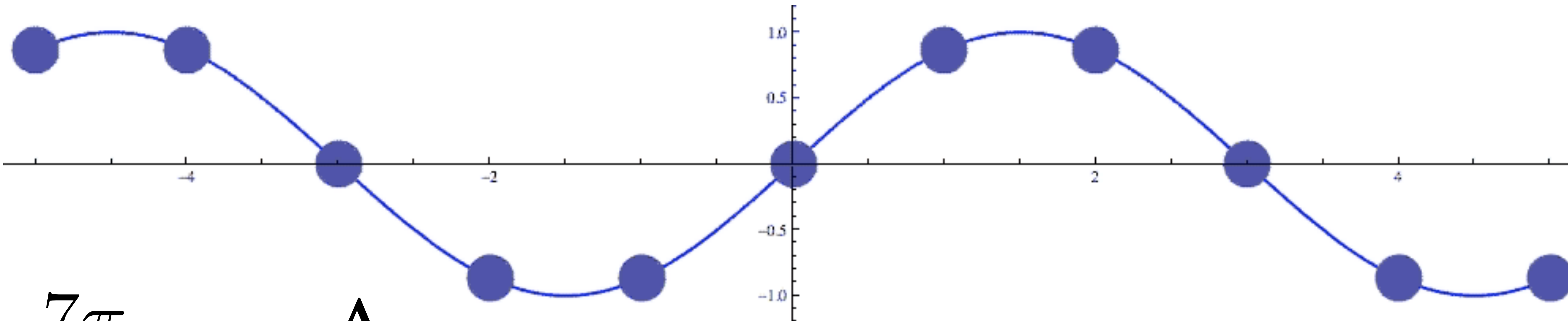
$\dfrac{\pi}{6}$ C

$\dfrac{\pi}{3}$ B

$\dfrac{2\pi}{3}$ A

$\dfrac{2\pi}{4}$ D

Question #9



What is $k' = k + \frac{2\pi}{a}$?

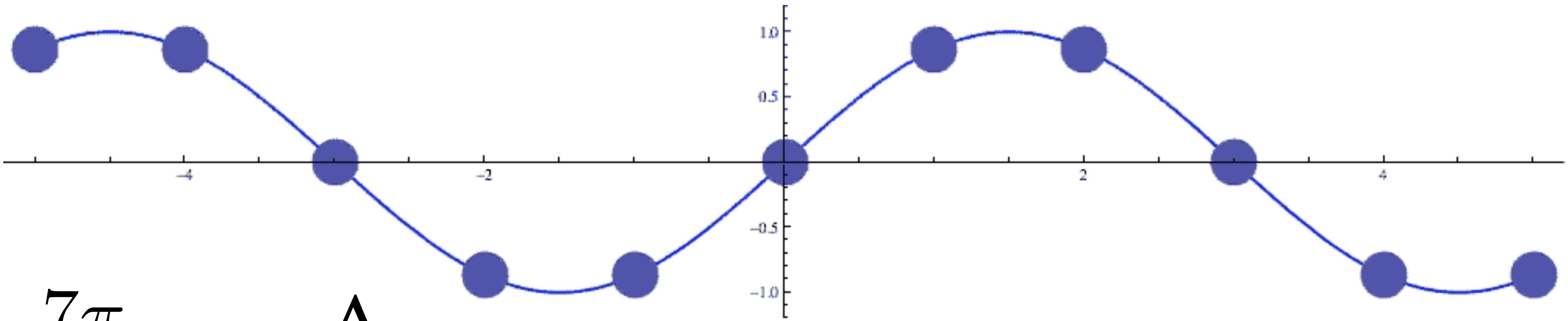
C: $\frac{7\pi}{3}$

A: $\frac{7\pi}{6}$

B: 3π

D: $\frac{9\pi}{4}$

Question #9



What is $k' = k + \frac{2\pi}{a}$?

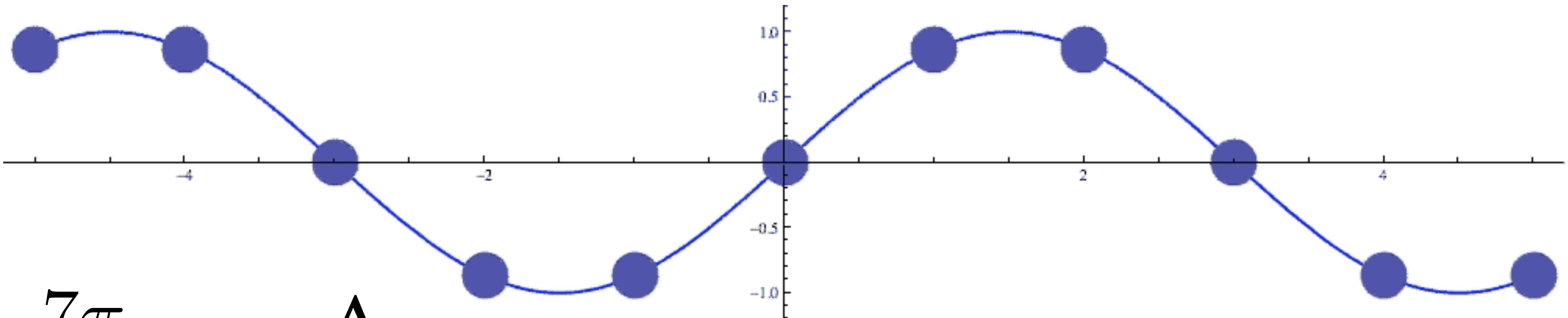
A $\frac{7\pi}{3}$

B 3π

C $\frac{7\pi}{6}$

D $\frac{9\pi}{4}$

Question #9



What is $k' = k + \frac{2\pi}{a}$?

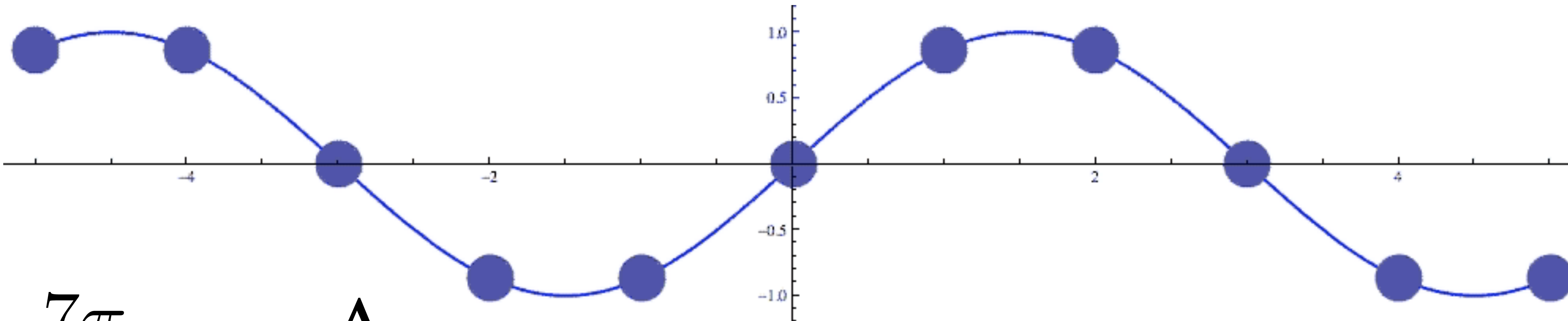
A $\frac{7\pi}{3}$

B $\frac{7\pi}{6}$

C 3π

D $\frac{9\pi}{4}$

Question #9



$\frac{7\pi}{3}$

A

What is $k' = k + \frac{2\pi}{a}$?

$\frac{7\pi}{6}$

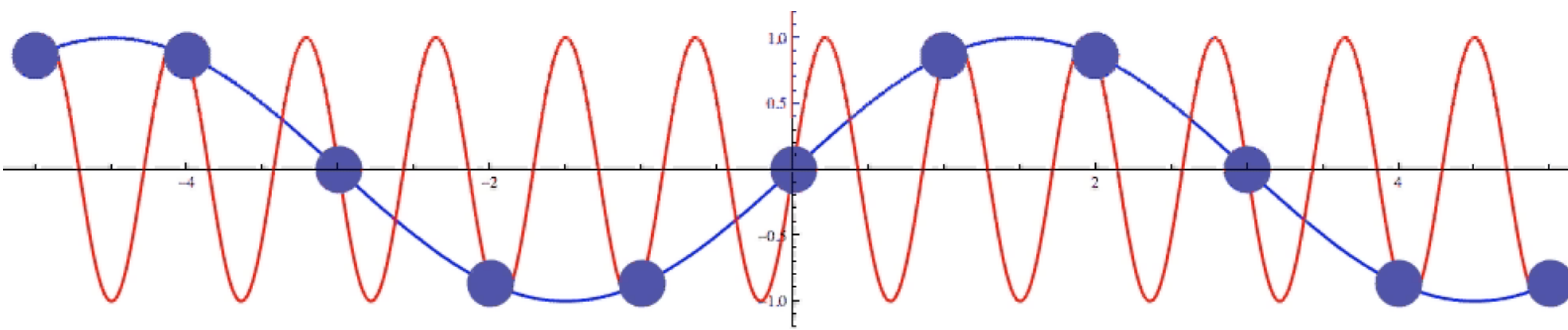
3π

$\frac{9\pi}{4}$

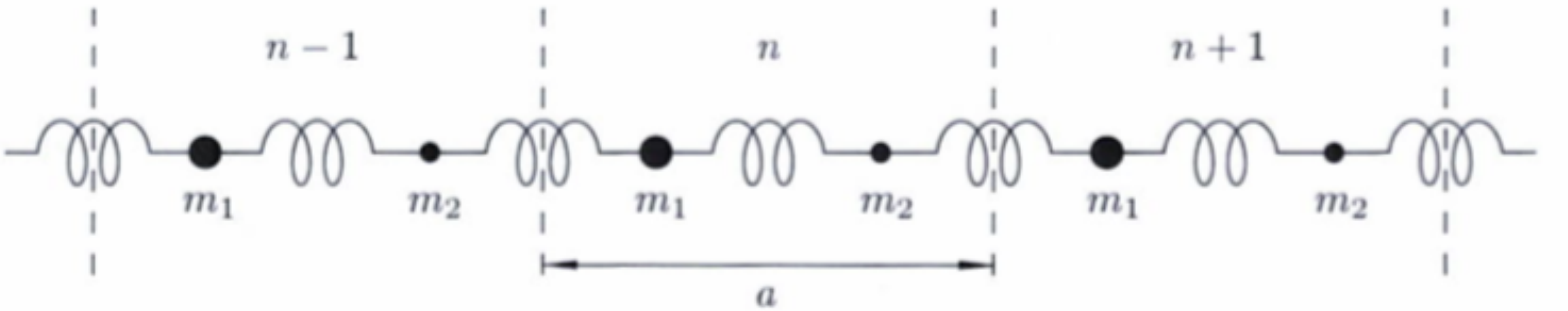
C

D

B



Question #10



What is the force on m_1 in cell n?

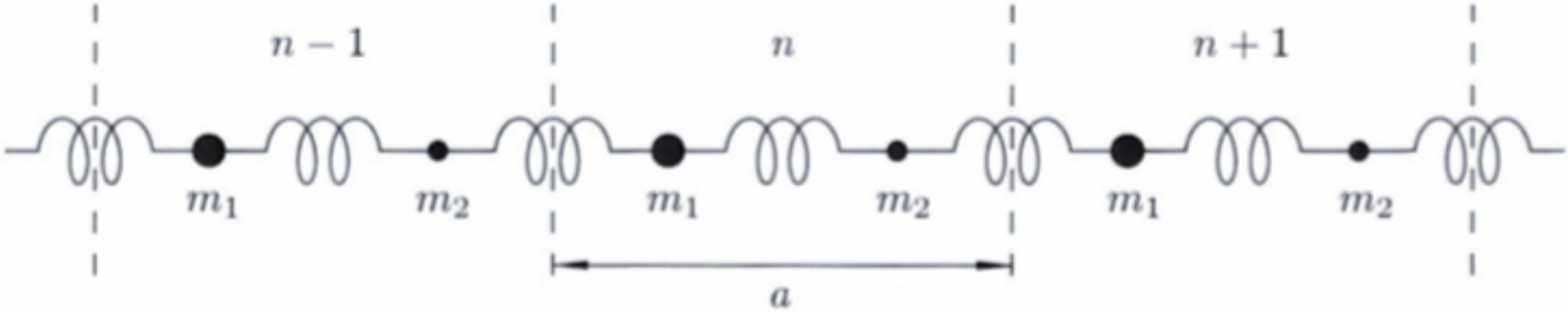
$$-\alpha\left(2u_{1,n} - u_{2,n} - u_{2,n-1}\right)$$ E

$$-\alpha\left(2u_{2,n} - u_{1,n} - u_{1,n-1}\right)$$ D

$$\alpha\left(2u_{1,n} - u_{2,n} - u_{2,n-1}\right)$$ B

$$\alpha\left(2u_{2,n} - u_{1,n} - u_{1,n-1}\right)$$ C

Question #11



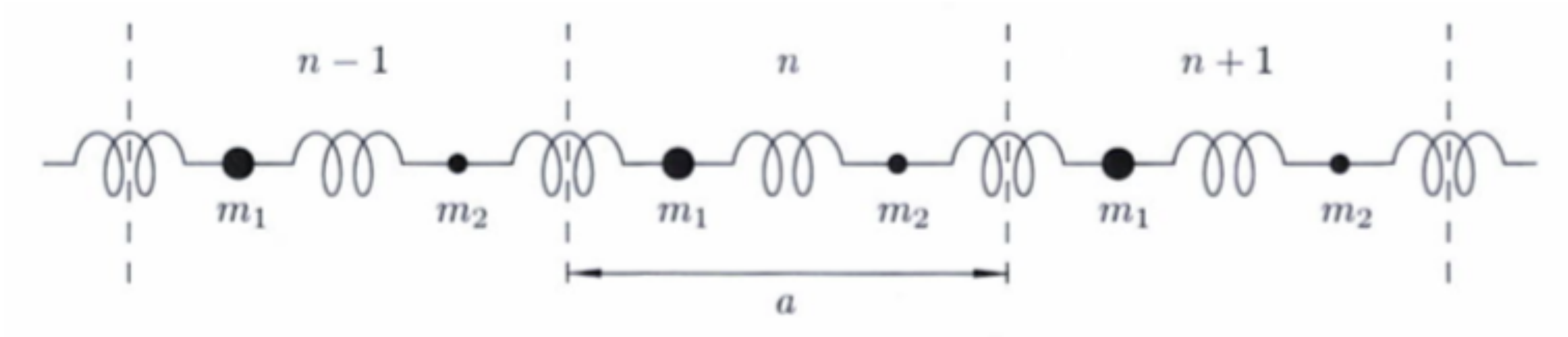
What is the force on m_2 in cell n?

$$-\alpha\left(2u_{2,n+1}-u_{1,n}-u_{1,n-1}\right)$$ B

$$-\alpha\left(2u_{2,n-1}-u_{1,n}-u_{1,n+1}\right)$$ E

$$-\alpha\left(2u_{2,n}-u_{1,n+1}-u_{1,n}\right)$$ D

$$-\alpha\left(2u_{1,n}-u_{2,n+1}-u_{2,n}\right)$$ C

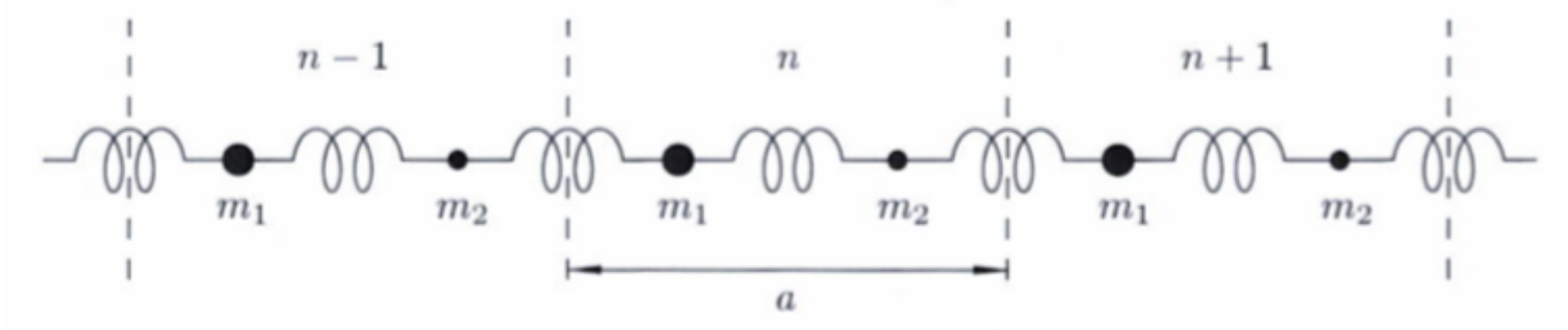


$$-\alpha\left(2u_{2,n} - u_{1,n+1} - u_{1,n}\right) = m_2 \frac{d^2 u_{2,n}}{dt^2}$$

$$-\alpha\left(2u_{1,n} - u_{2,n} - u_{2,n-1}\right) = m_1 \frac{d^2 u_{1,n}}{dt^2}$$

$$u_{1,n} = A_1 e^{ikna - i\omega t} \qquad u_{2,n} = A_2 e^{ikna - i\omega t}$$

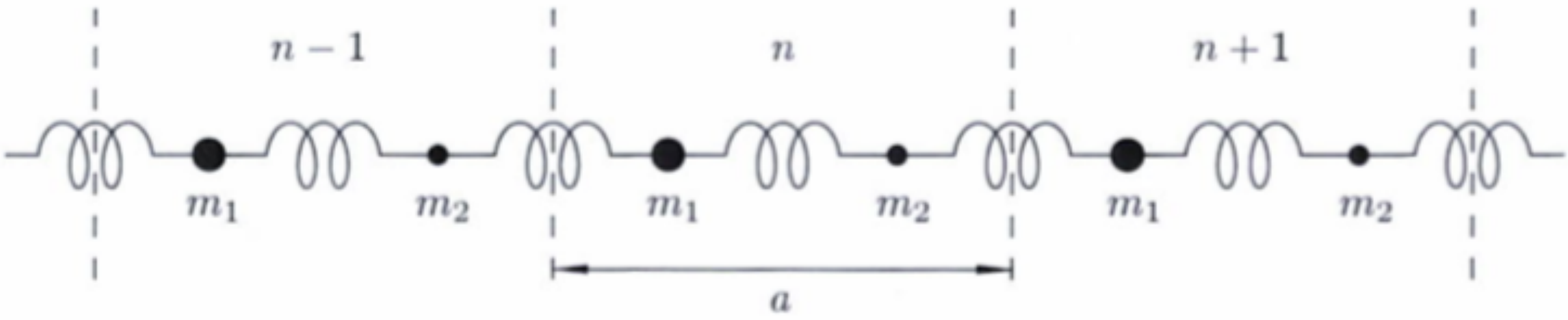
Plug these functions in and arrive at the eigenvalue problem.



$$\left(m_1\omega^2 - 2\alpha\right) A_1 + \alpha\left(1 + e^{-ika}\right) A_2 = 0$$

$$\alpha\left(1 + e^{ika}\right) A_1 + \left(m_2\omega^2 - 2\alpha\right) A_2 = 0$$

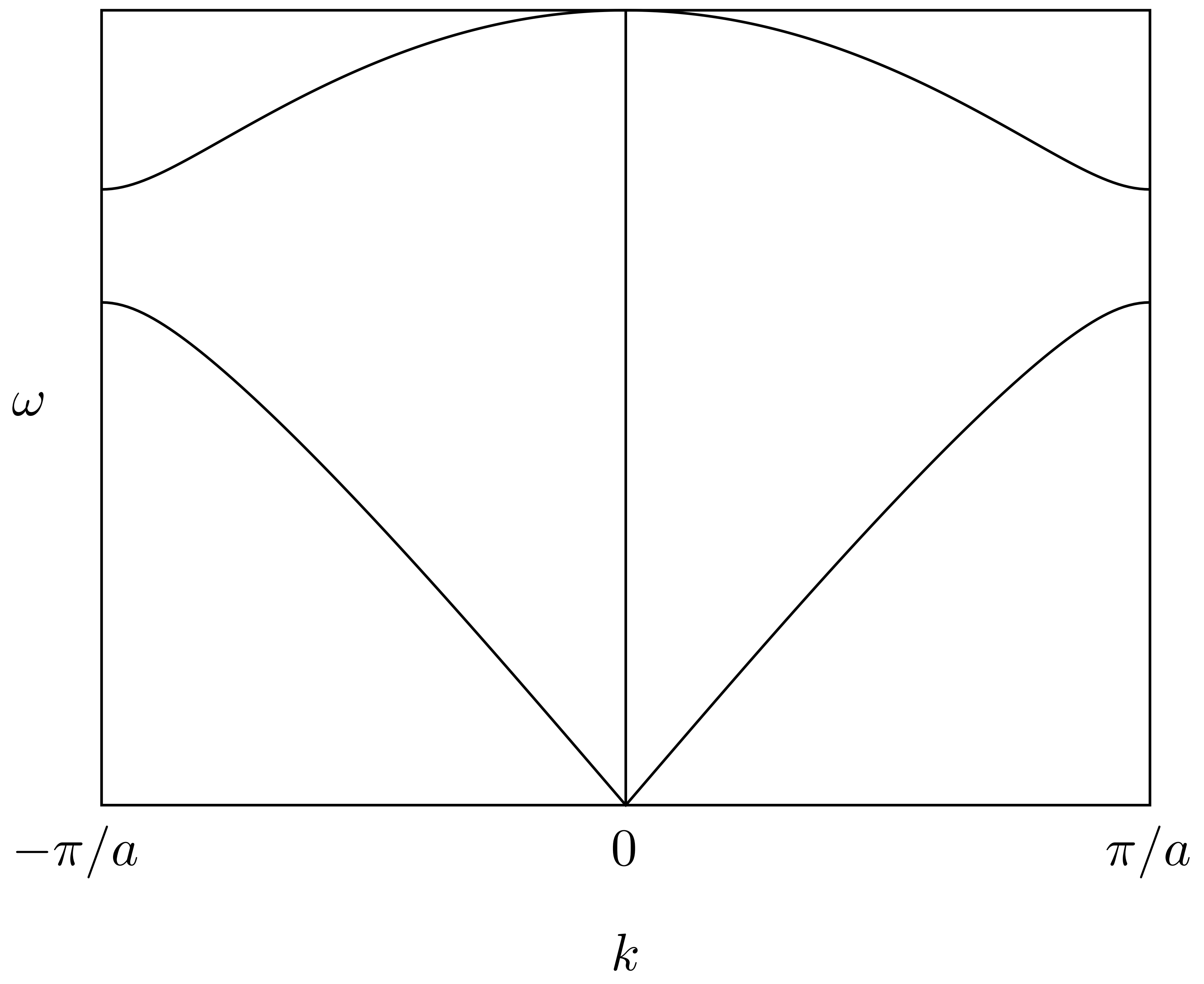
$$\omega^2 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \pm \alpha\sqrt{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 - \frac{4\sin^2\frac{ka}{2}}{m_1 m_2}}$$



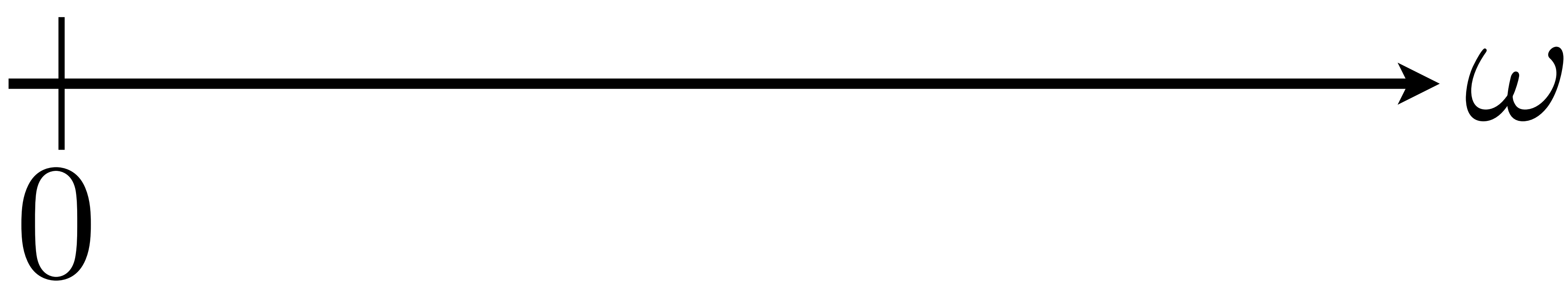
$$\omega^2 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \pm \alpha \sqrt{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 - \frac{4\sin^2\frac{ka}{2}}{m_1 m_2}}$$

What does this expression become when k is very small?

Phonon dispersion for diatomic crystal

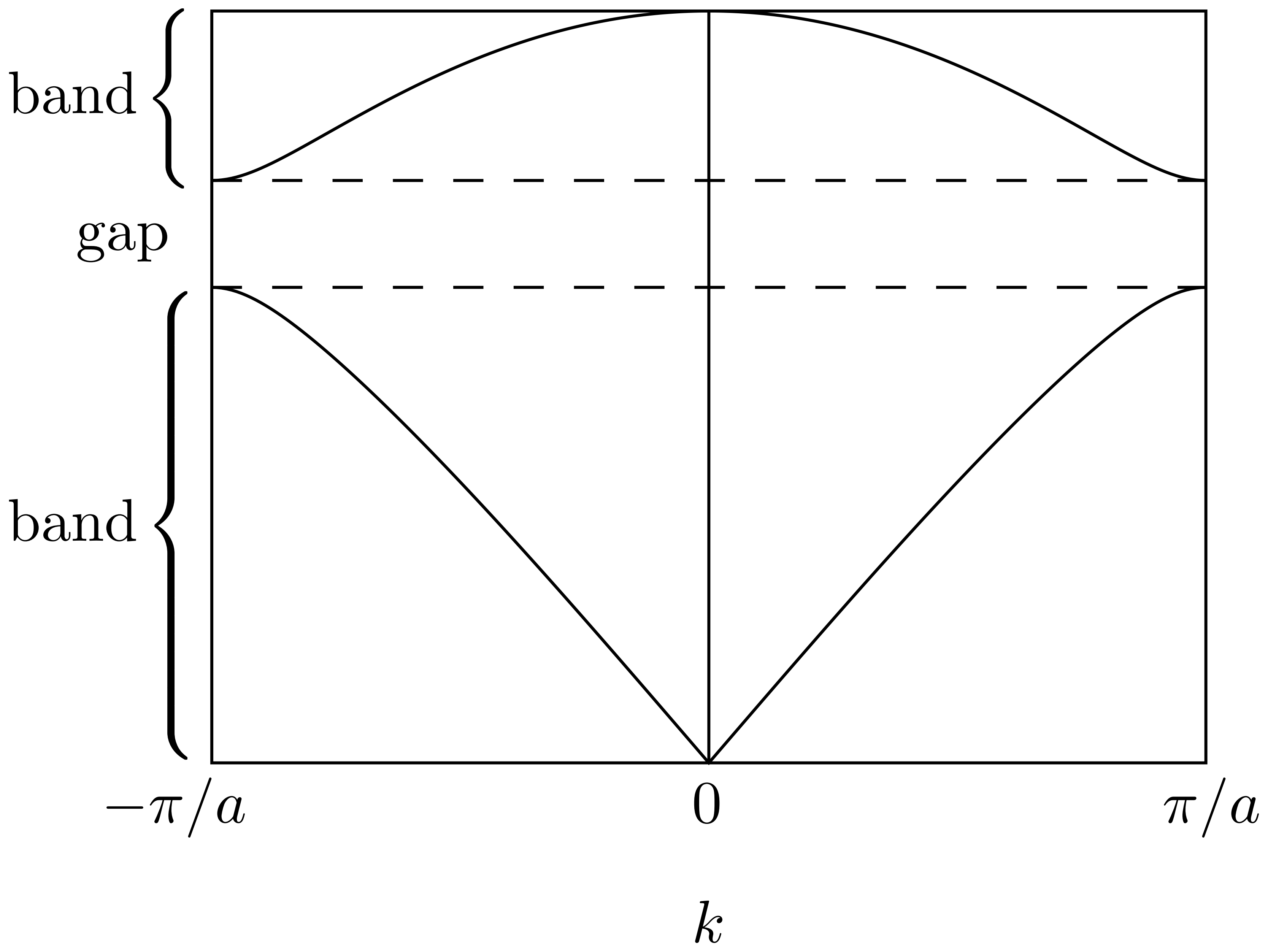


Gaps



Gaps

Gaps



Question #12

Consider lattice waves in a one-dimensional diatomic crystal. Inside the first Brillouin zone, the waves with the lowest frequencies have

(C) the shortest or

(E) the longest wavelengths.

Question #13

Consider lattice waves in a one-dimensional diatomic crystal. Inside the first Brillouin zone, the waves with the highest frequencies have

(A) the shortest or

(B) the longest wavelengths.