1 Quantum Review - A summary

1. (Can use a computer) A particle in an **infinite square well** has the initial wave function:

$$\psi(x) = \psi(x,0) = Ax^2(a-x) \tag{1}$$

1. Normalize the wavefunction. (Ans: $A = \sqrt{\frac{105}{a^7}}$)
2. Plot the wavefuntion. If you were guessing, which energy eigenstates would you say are most dominant in this wavefunction. (i.e. which c_n are biggest?)
3. Find the coefficients c_n. (Ans: $c_n = -\frac{\sqrt{210}}{n^4\pi^4}(2n\pi + 4n\pi\cos(n\pi))$)
4. Find the expectation value of energy $\langle E\rangle$ by truncating the expansion to have 1000 terms. (The full summation diverges and therefore the expectation value is infinite.) (Ans: $\approx 7\frac{\hbar^2}{ma^2}$)
5. Find $\psi(x,t)$ and use Mathematica's `Manipulate` function to make a movie of it.

2. (Can be done by hand) A particle in an **infinte square well** has the initial wavefunction

$$\psi(x) = \sqrt{\frac{2}{a}}\frac{3\sin(\frac{2\pi x}{a}) + 5\sin(\frac{4\pi x}{a}) + 4\sin(\frac{5\pi x}{a})}{\sqrt{45}} \tag{2}$$

1. Is the wavefunction normalized? If not, how would you modify the function so it is normalized?
2. Plot the wavefuntion.
3. Find the coefficients c_n.
4. Find the expectation value of energy $\langle E\rangle$.
5. Find $\psi(x,t)$ and use Mathematica's `Manipulate` function to make a movie of it.

3. Use the energy eigenfunctions from the infinite square well as a basis to solve the problem of a **finite** square well with a square "bump" in the well that is off-center. In other words, solve Schrodinger's equations for the following potential:

$$V = \begin{cases} -V_0 & 0 < x < \frac{a}{3} \\ -V_0 + V_1 & \frac{a}{3} < x < \frac{a}{2} \\ -V_0 & \frac{a}{2} < x < a \\ 0 & \text{otherwise} \end{cases} \tag{3}$$