1 Day 1: Wavefunctions, Probabilities, and Normalization

1. Consider the following wavefunctions:

$$\psi_1(x) = Ae^{\frac{-y^2}{4}} \tag{1}$$

$$\psi_2(x) = Aye^{\frac{-y^2}{8}} \tag{2}$$

and

$$\psi_3(x) = A\left(e^{\frac{-y^2}{4}} + ye^{\frac{-y^2}{8}}\right) \tag{3}$$

   1. Normalize all three states over the interval $-\infty < y < \infty$
   2. What is the probability of finding the particle in the region $0 < y < 1$ for all three states.
   3. Is the probability of finding the particle in the region $-1 < y < 1$ when it is in state ψ_3 the same as the sum of the probabilities when the particle is in states ψ_1 and ψ_2

2. Consider the following wavefunction:

$$\psi(x) = e^{\frac{(x-a)^2}{4b^2}} e^{icx} \tag{4}$$

where $a = 0$, $b = 5$ and $c = 0.3$

   1. Normalize the wavefunction.
   2. What is the expectation value of position $\langle x \rangle$
   3. What is the expectation value of the square of the position $\langle x^2 \rangle$
   4. What is the variance of the wavefunction $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

3. Consider the following wavefunction defined over the domain $0 < x < a$:

$$\psi(x) = \frac{1}{\sqrt{6.84592}} \cos(\frac{x}{10}) \left(\sin(\frac{2\pi x}{a}) + \sin(\frac{6\pi x}{a}) + \sin(\frac{8\pi x}{a})\right) \tag{5}$$

with $a = 5$

   1. Plot the wavefunction and the square of the wavefunction.
   2. Verify that the wavefunction is normalized.
   3. What is the expectation value of position $\langle x \rangle$
   4. What is the expectation value of the square of the position $\langle x^2 \rangle$
   5. What is the variance of the wavefunction $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$