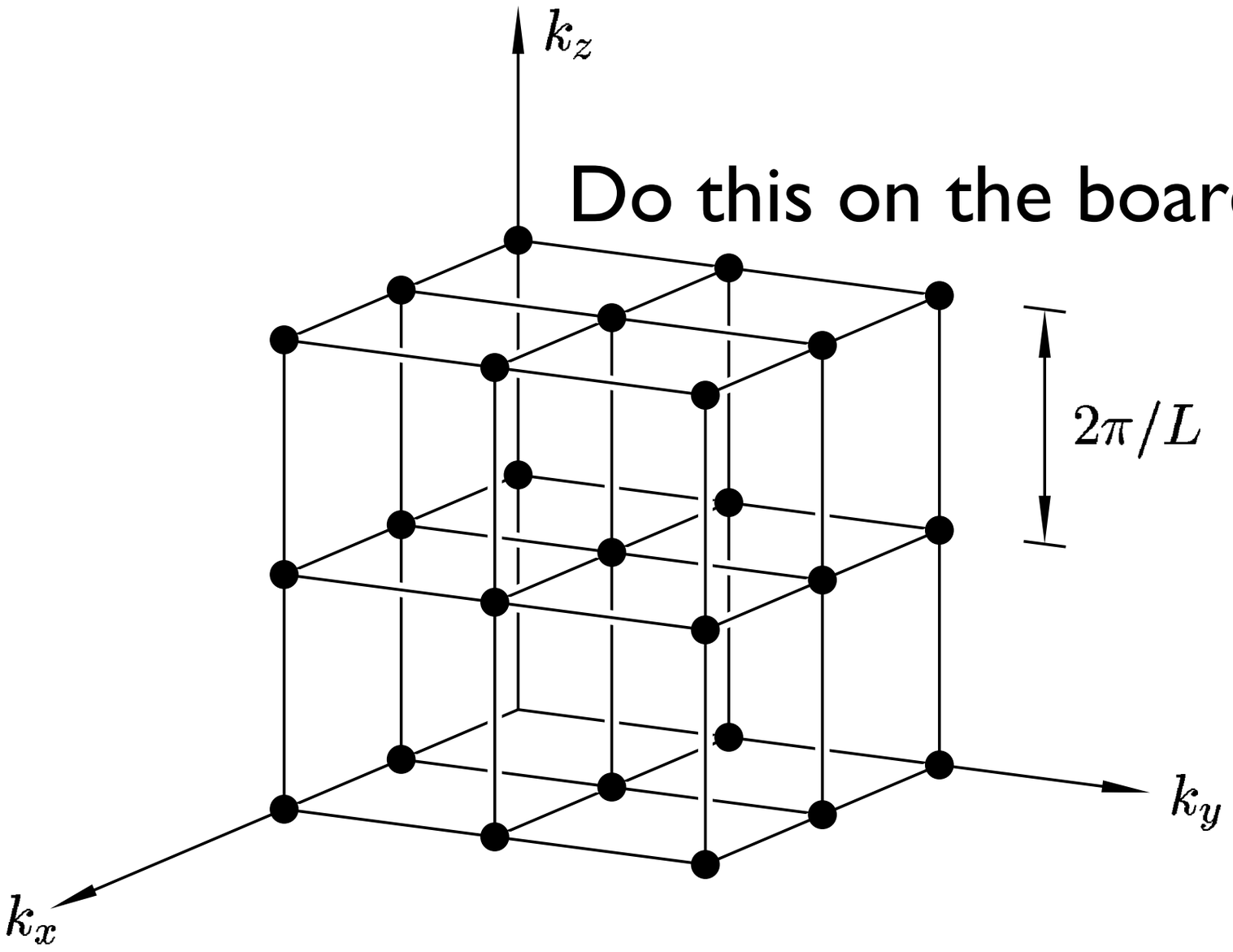
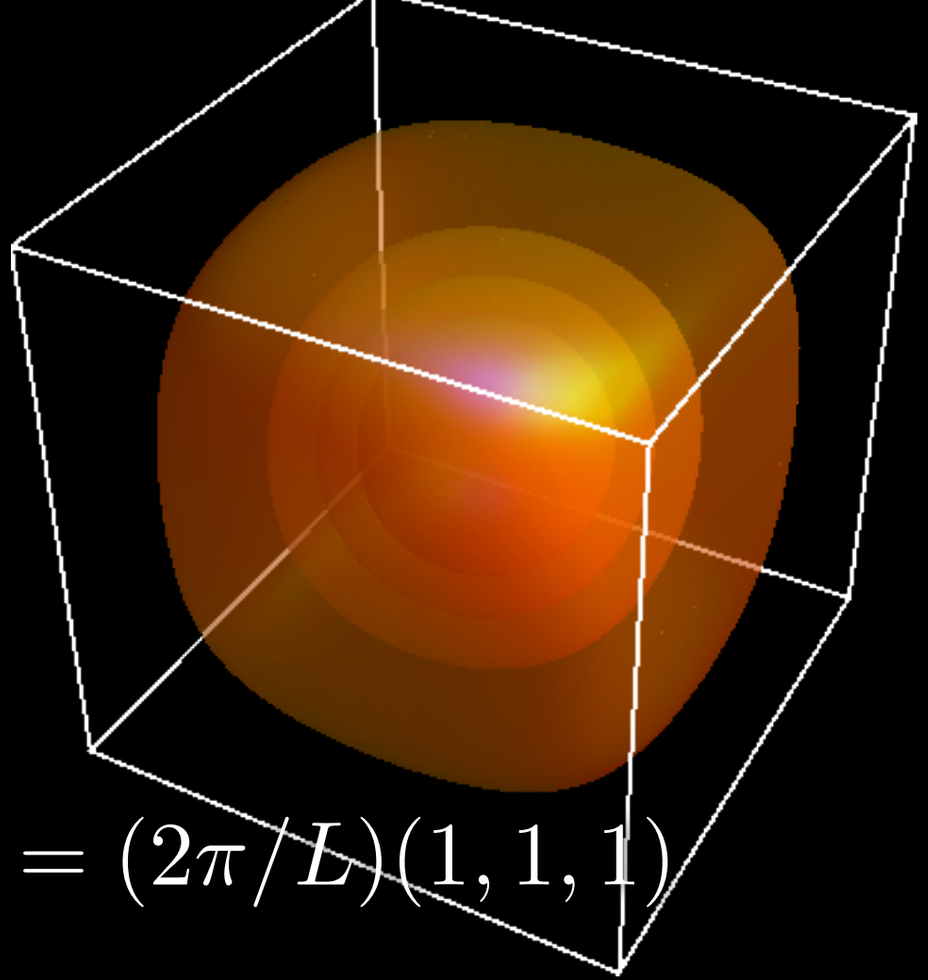
Particle in a 3D box

Do this on the board

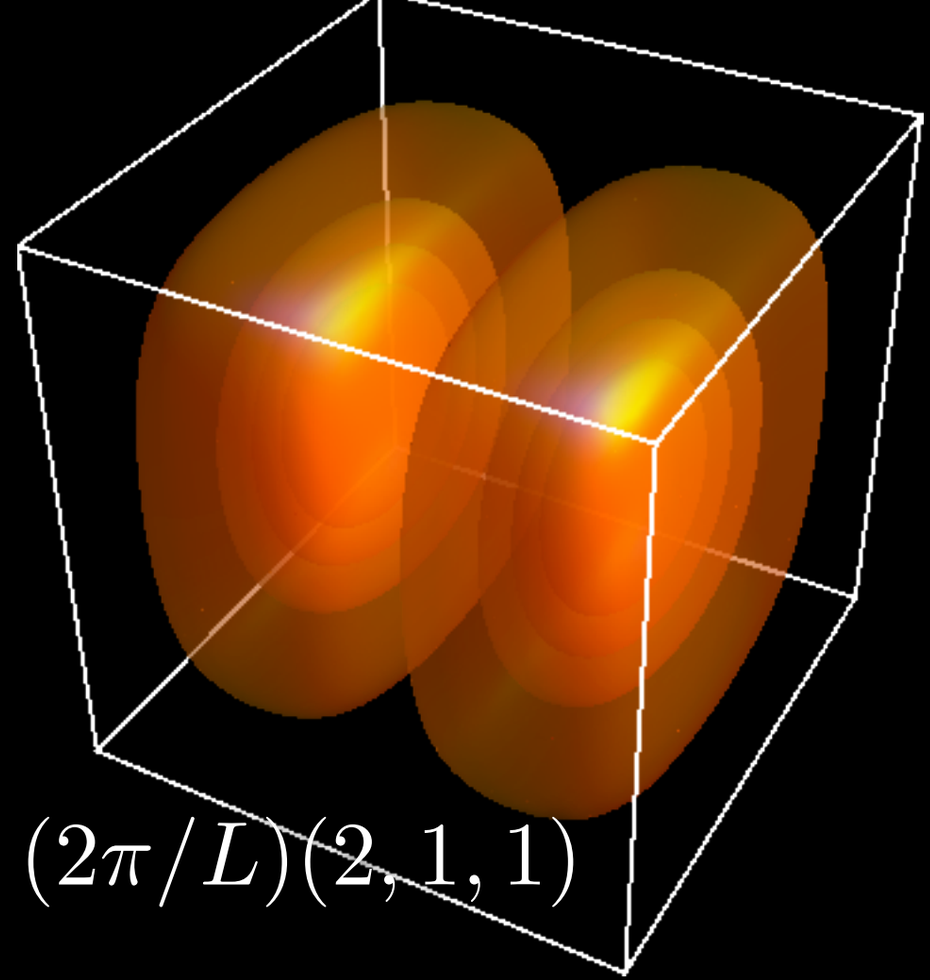
Particle in a 3D box



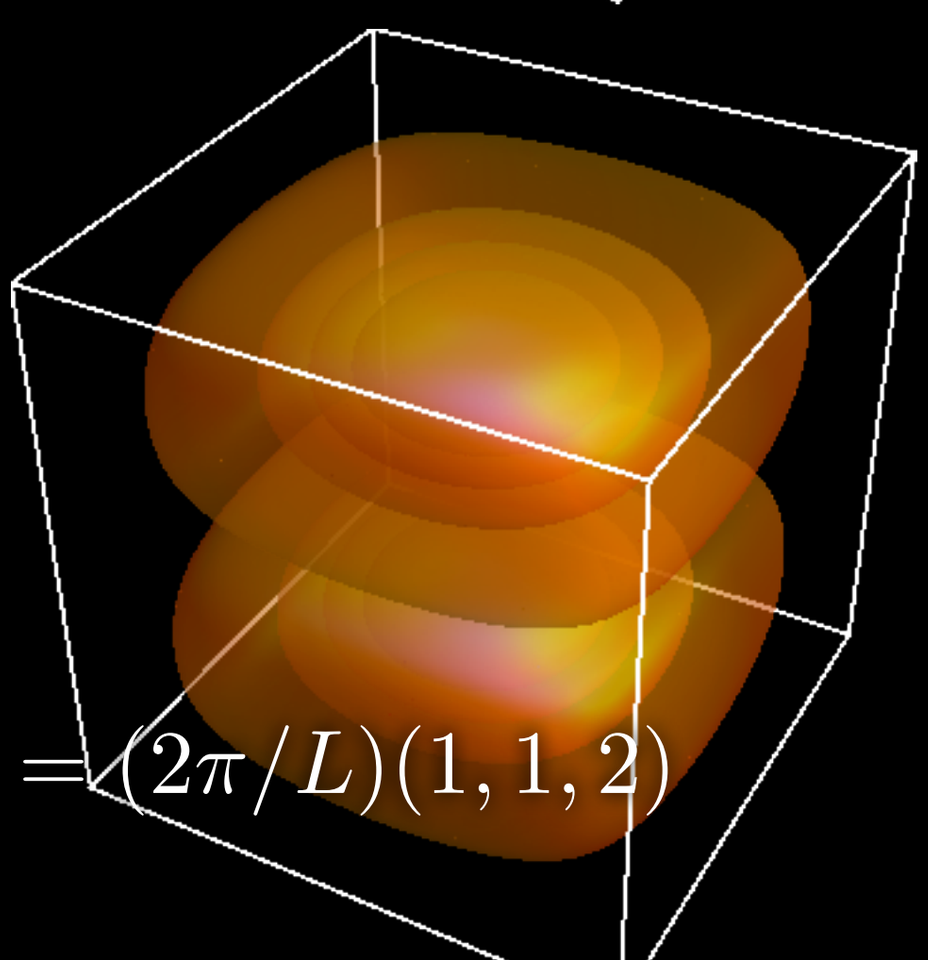
Do this on the board



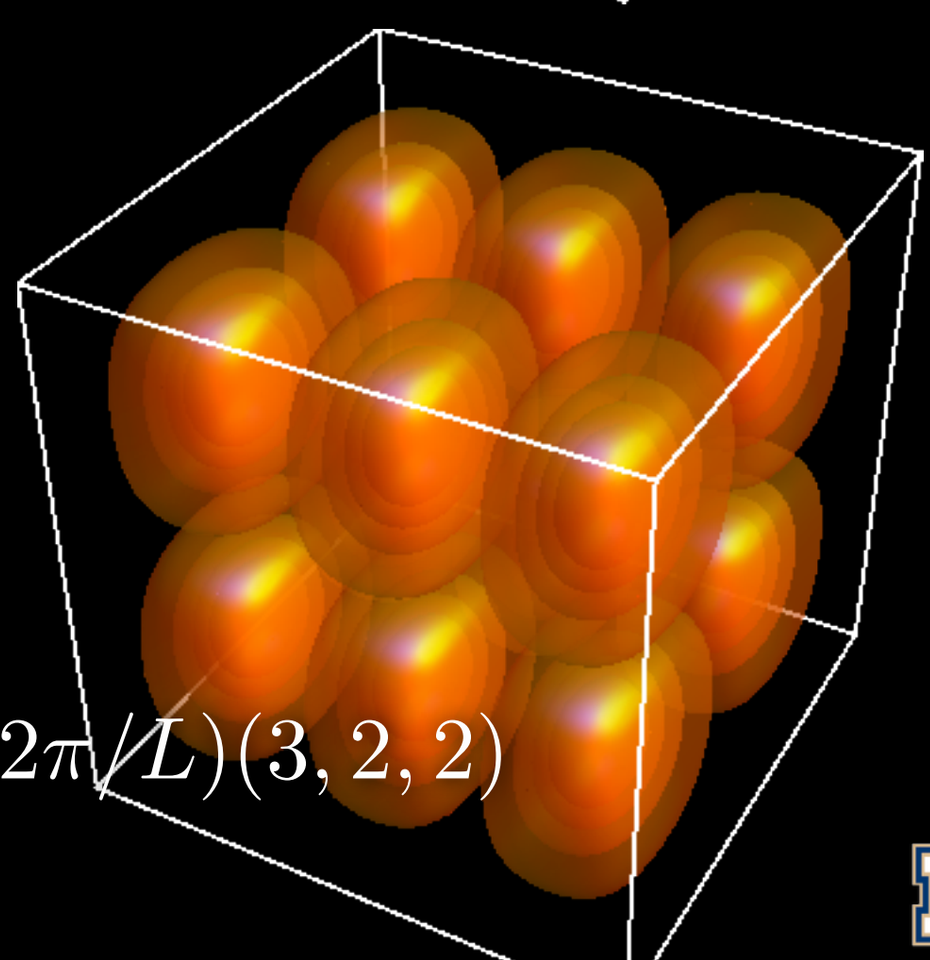
$\mathbf{k} = (2\pi/L)(1,1,1)$



$\mathbf{k} = (2\pi/L)(2,1,1)$

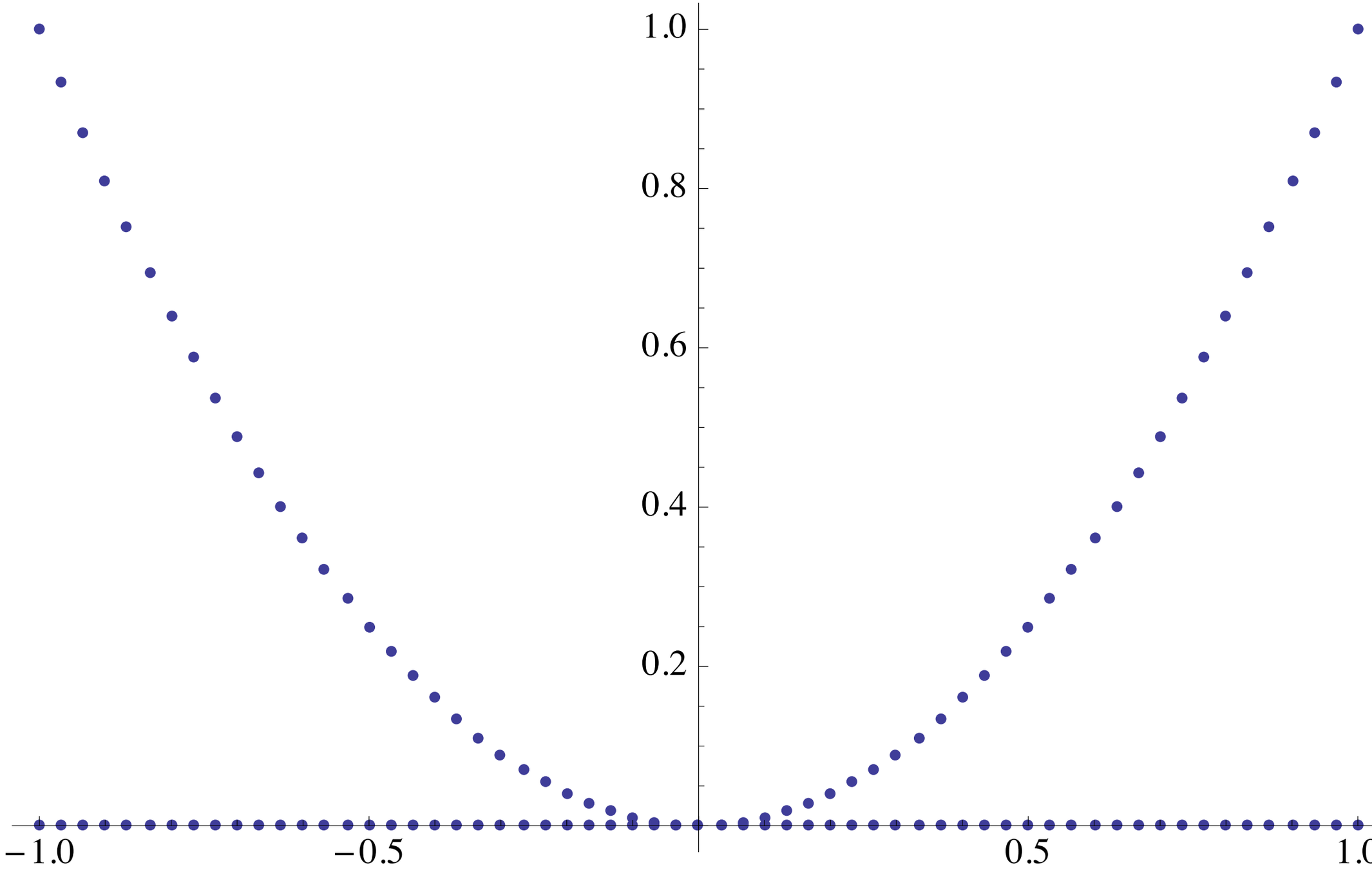


$\mathbf{k} = (2\pi/L)(1,1,2)$

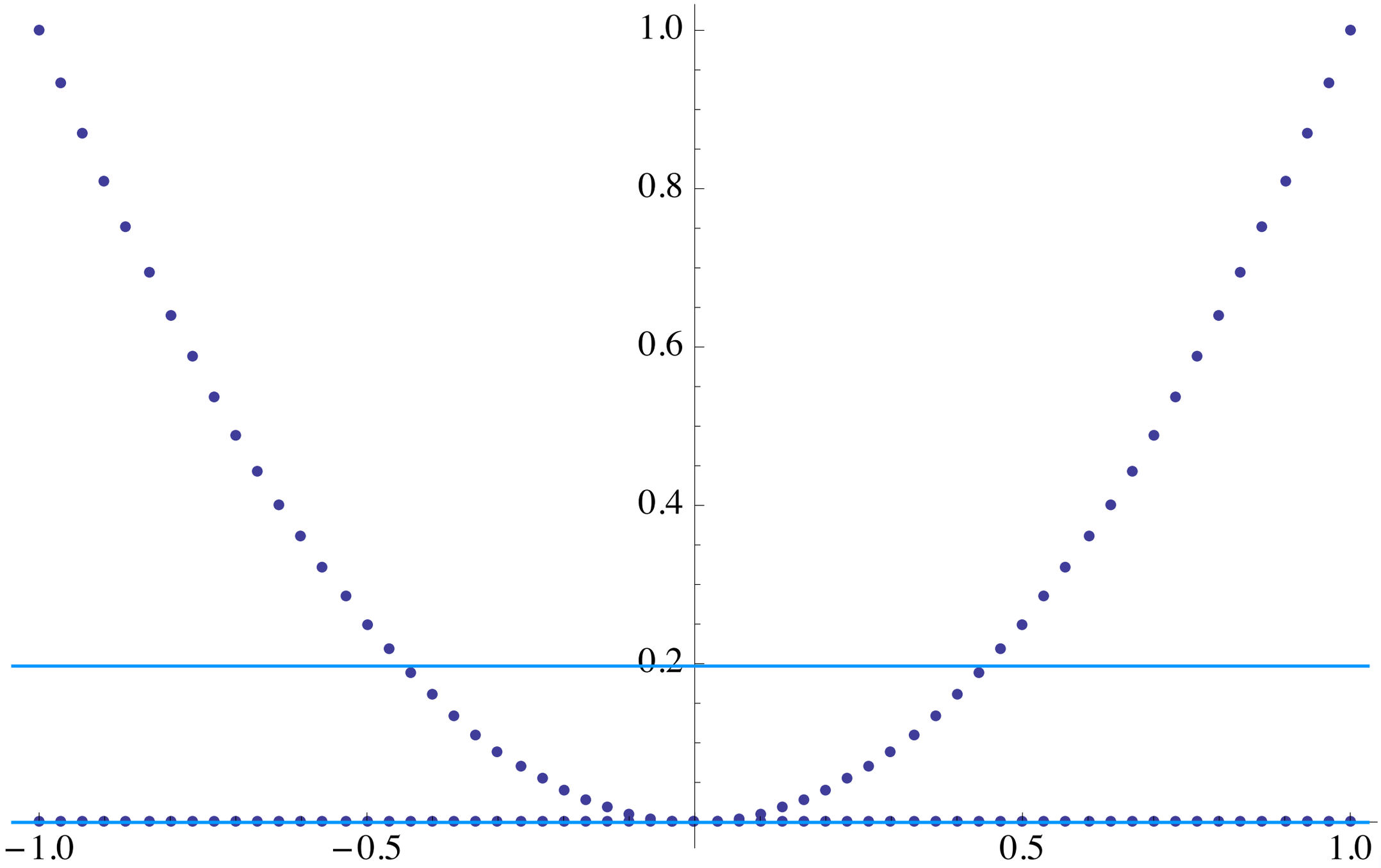


$\mathbf{k} = (2\pi/L)(3,2,2)$

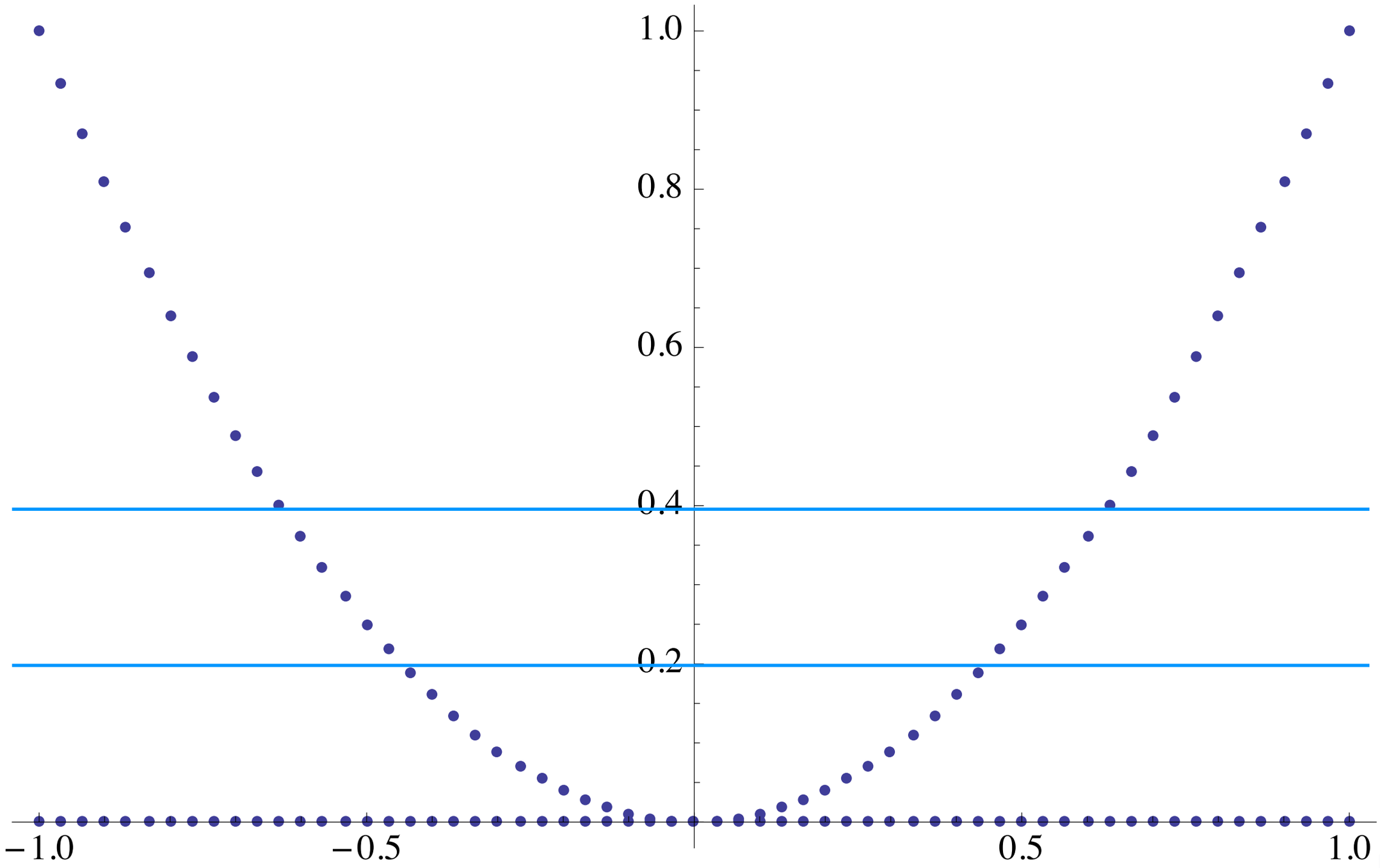
$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$



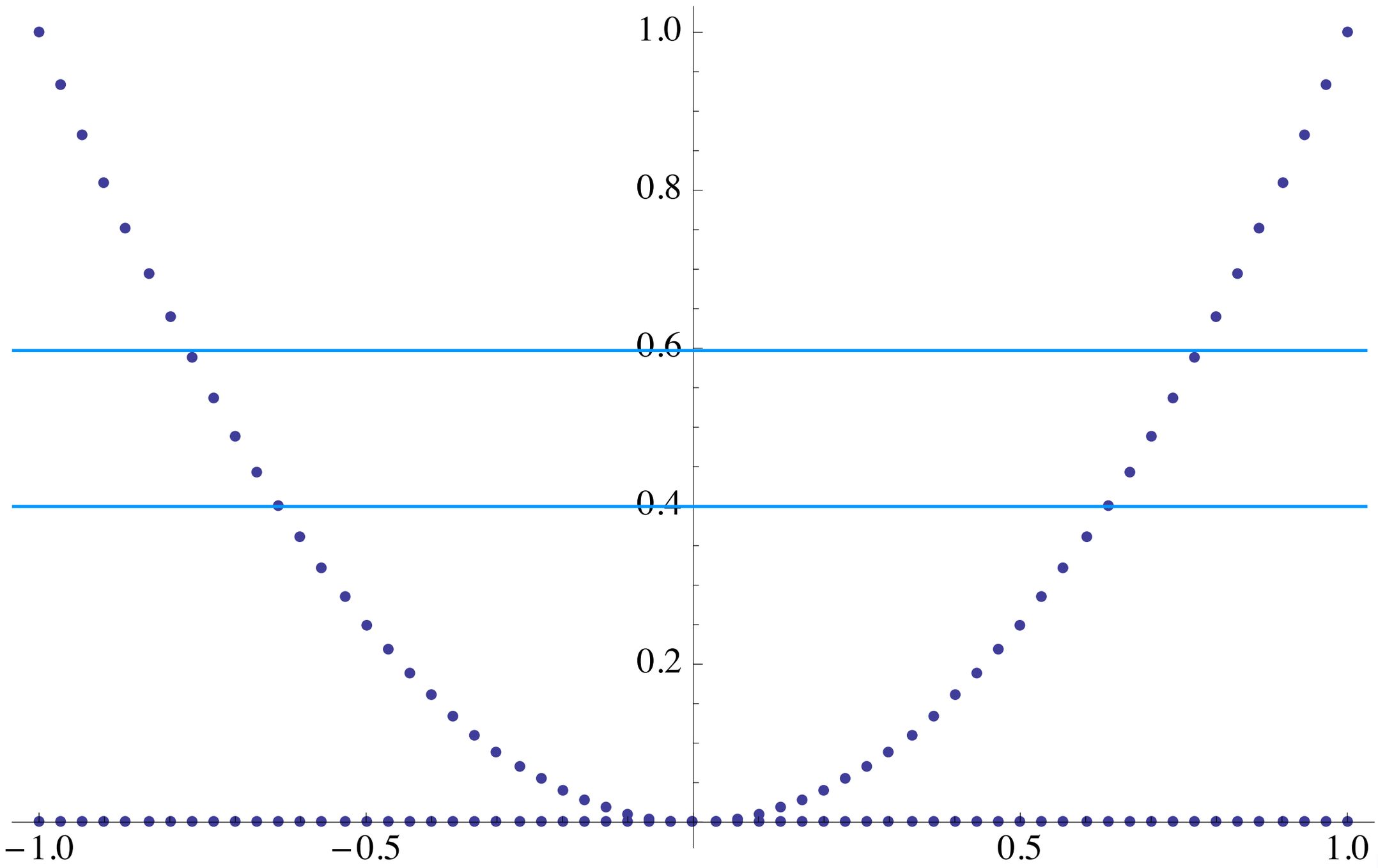
$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$



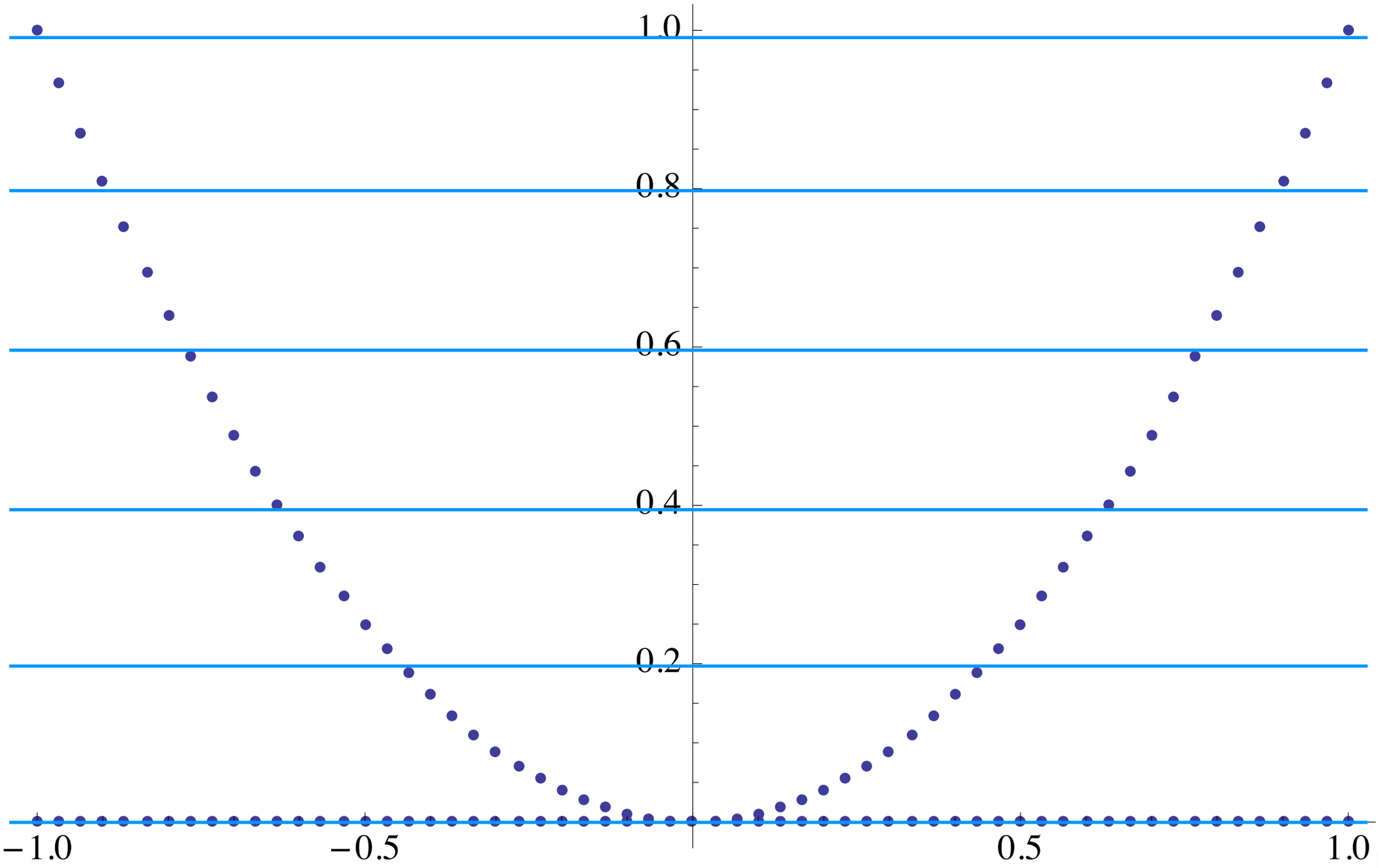
$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

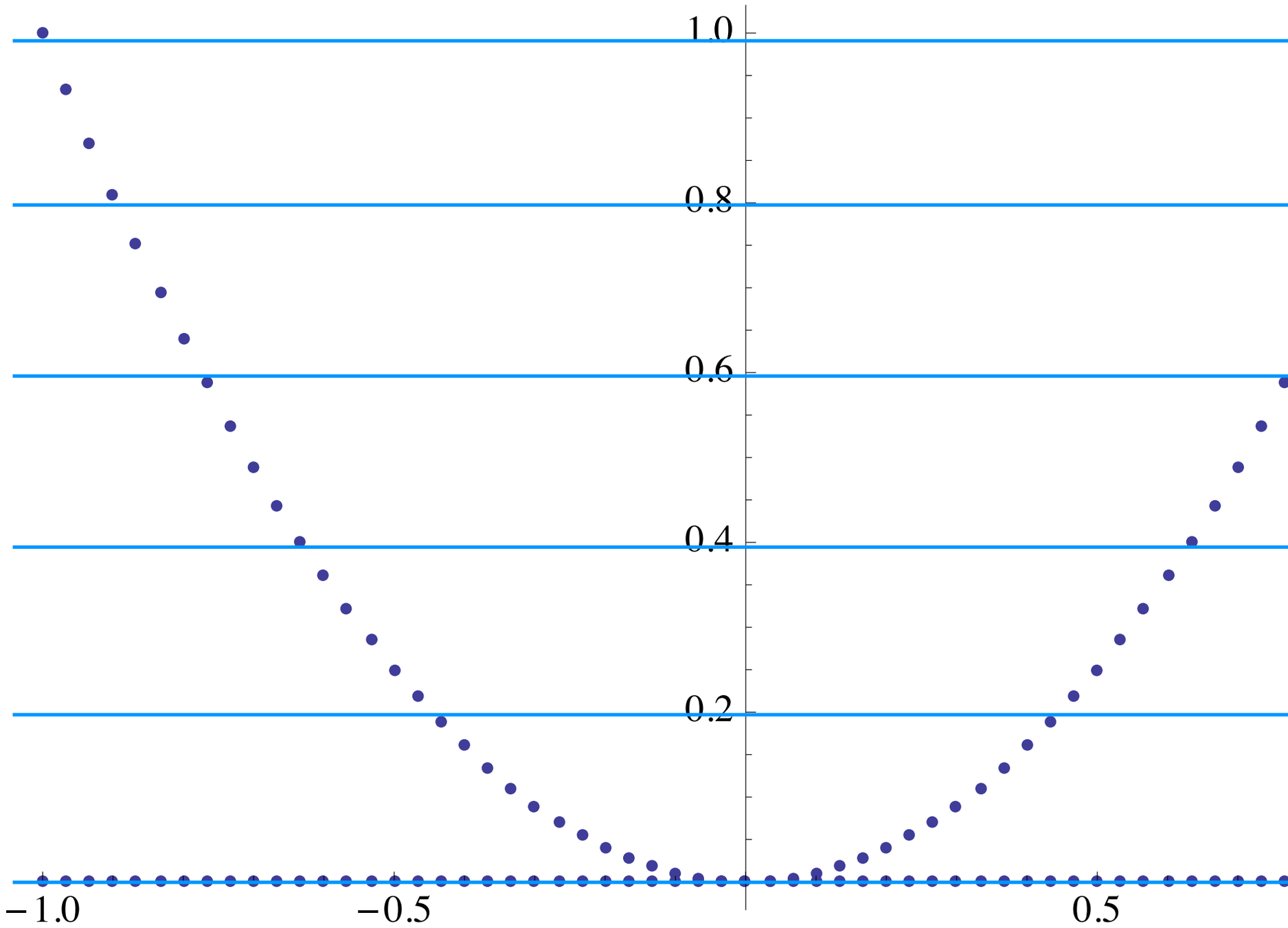


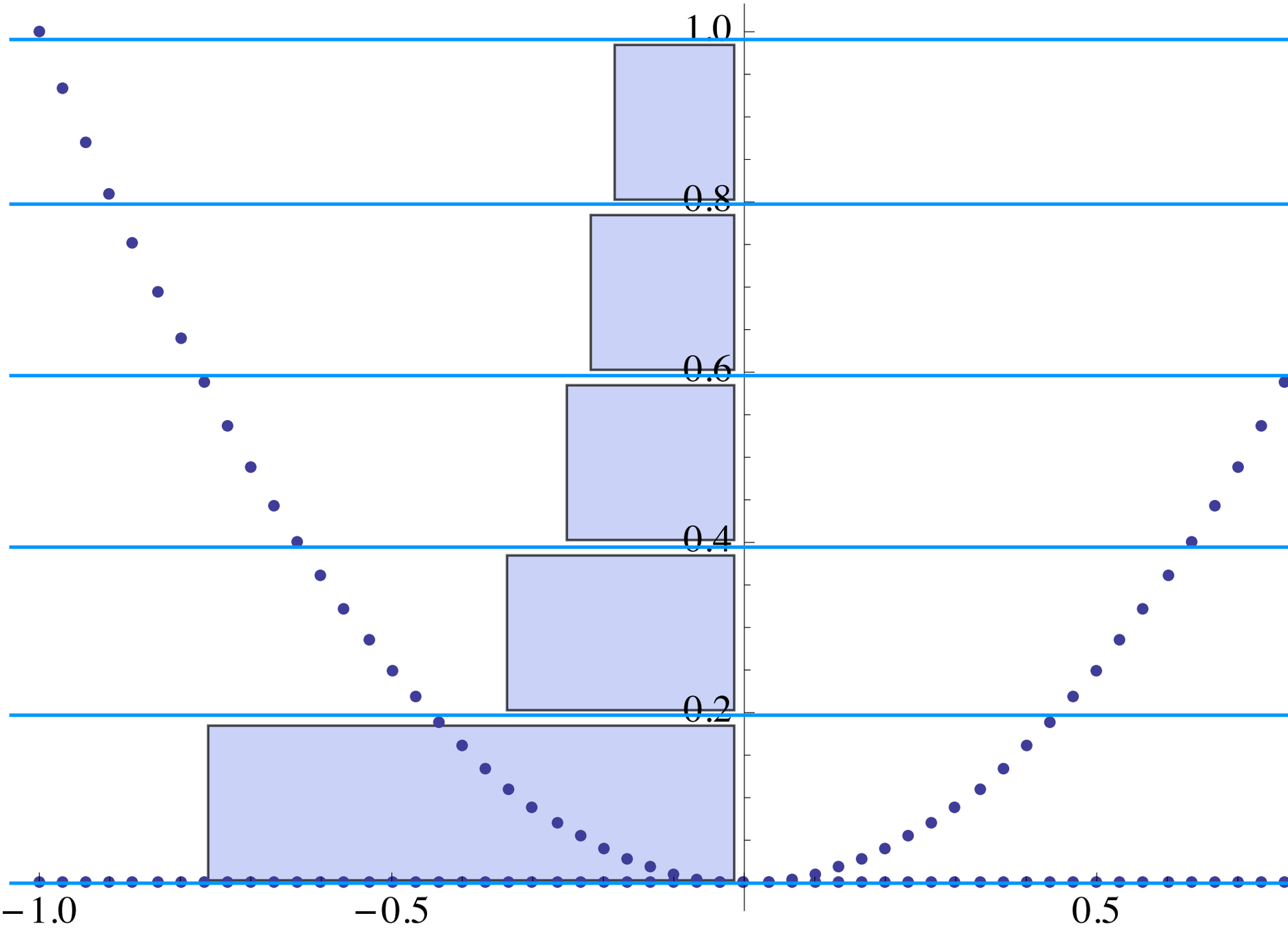
$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

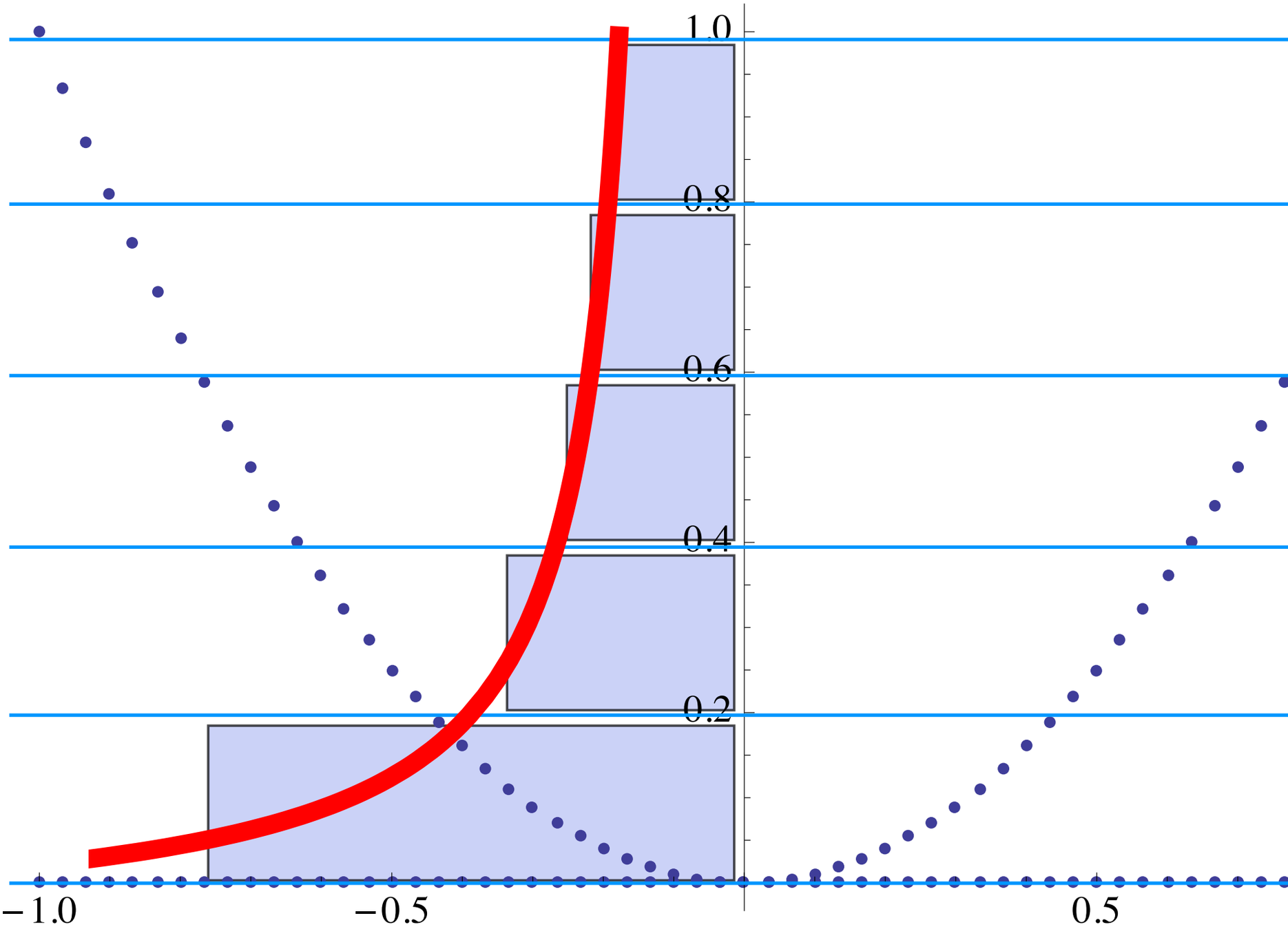


$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

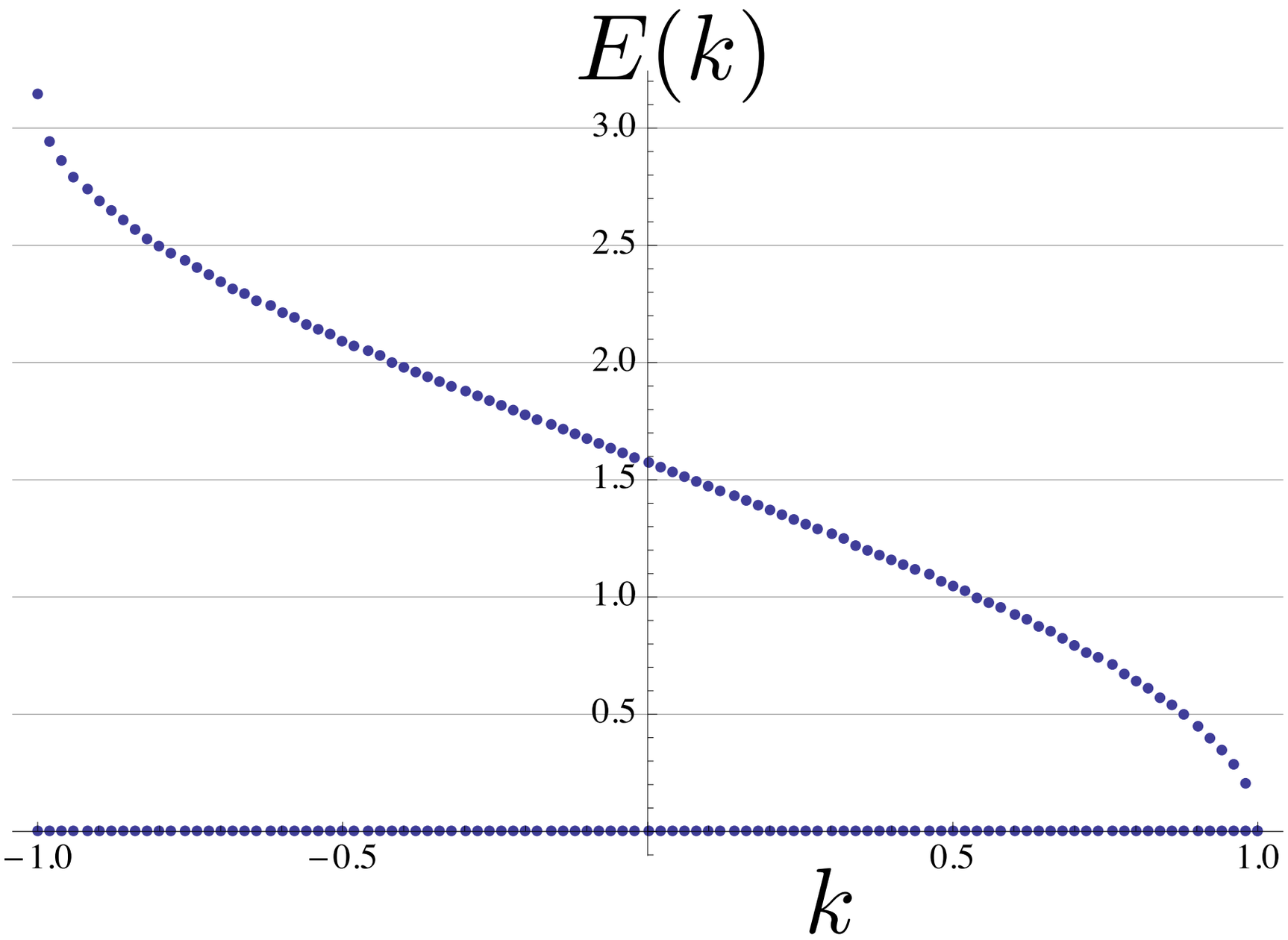



1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5

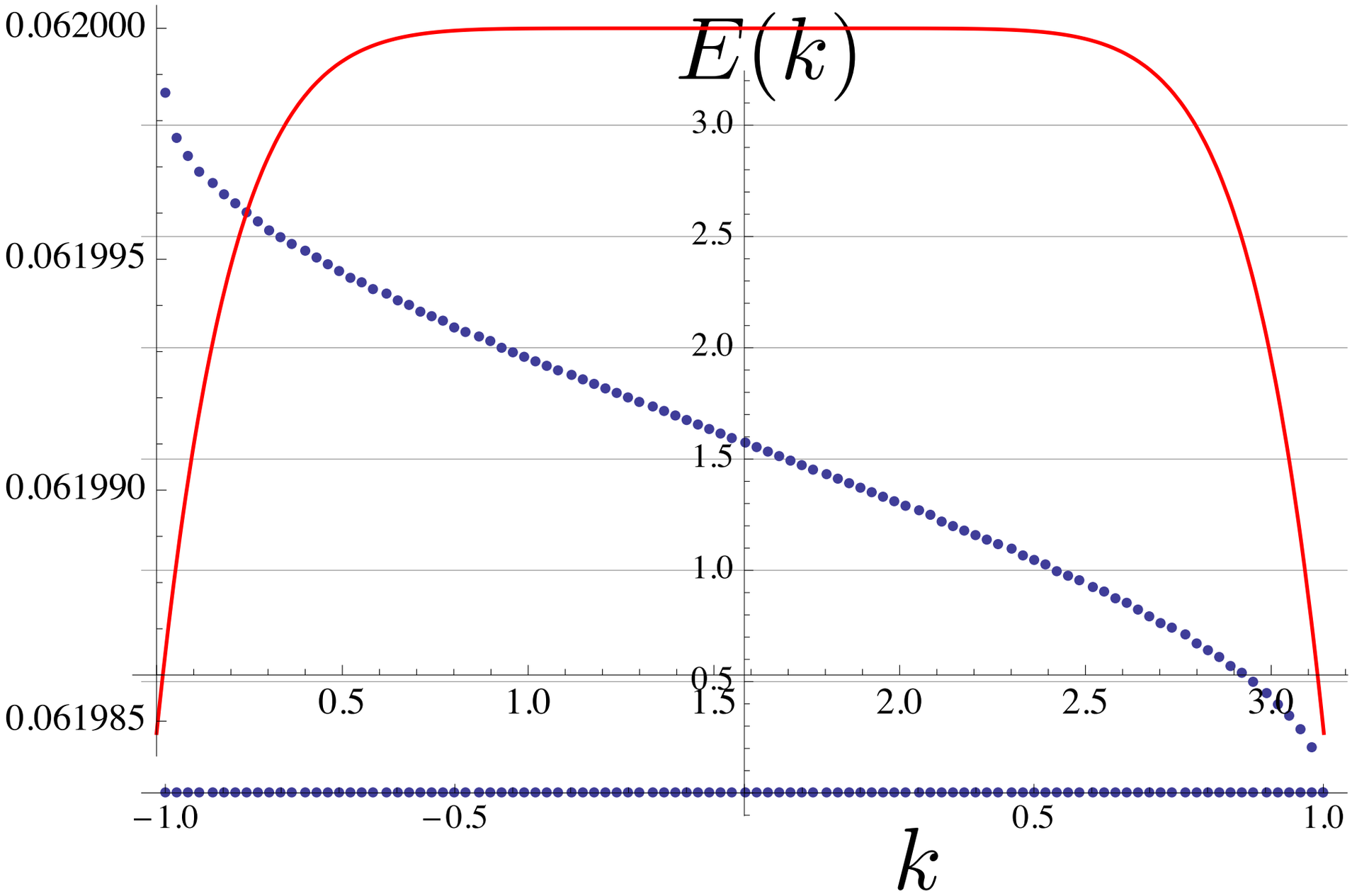

1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5


1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5

Sketch the graph for $g(E)$



Sketch the graph for $g(E)$



$$\psi(x,t) = A\sin(kx)\exp(-i\omega t) \qquad k = \frac{2\pi n}{L}$$

What is the density of these points?

Question #9

C

$$g(k) = \frac{L}{2\pi}$$

B

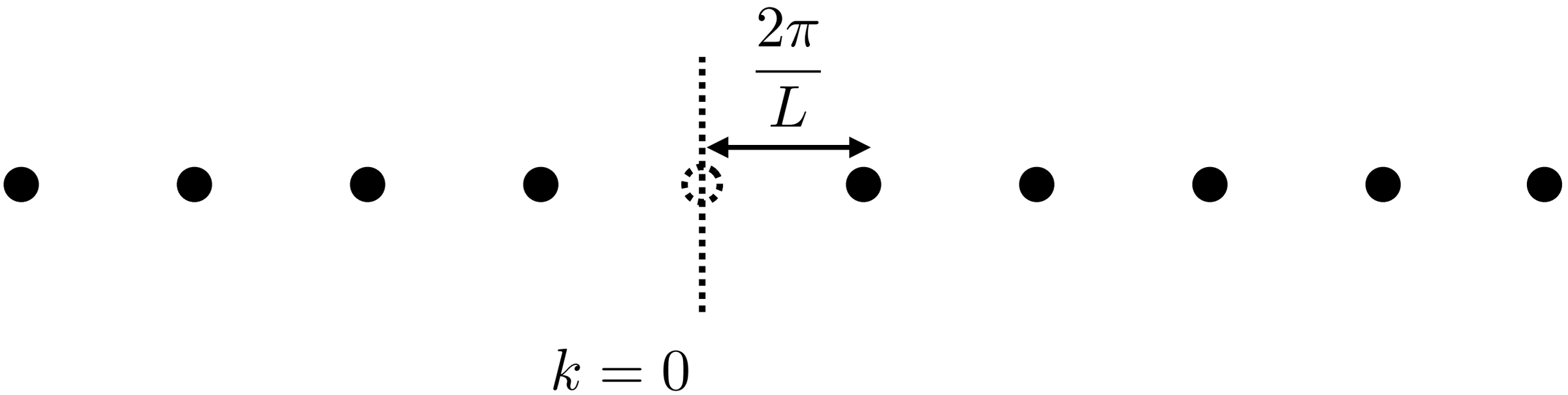
$$g(k) = \frac{2\pi}{L}$$

A

$$g(k) = \frac{2\pi}{L^2}$$

$$g(k) = \frac{L^2}{2\pi}$$

D



$$\psi(x,t) = A\sin(kx)\exp(-i\omega t) \qquad k = \frac{2\pi n}{L}$$

How many states are there in a “circle” of radius k?

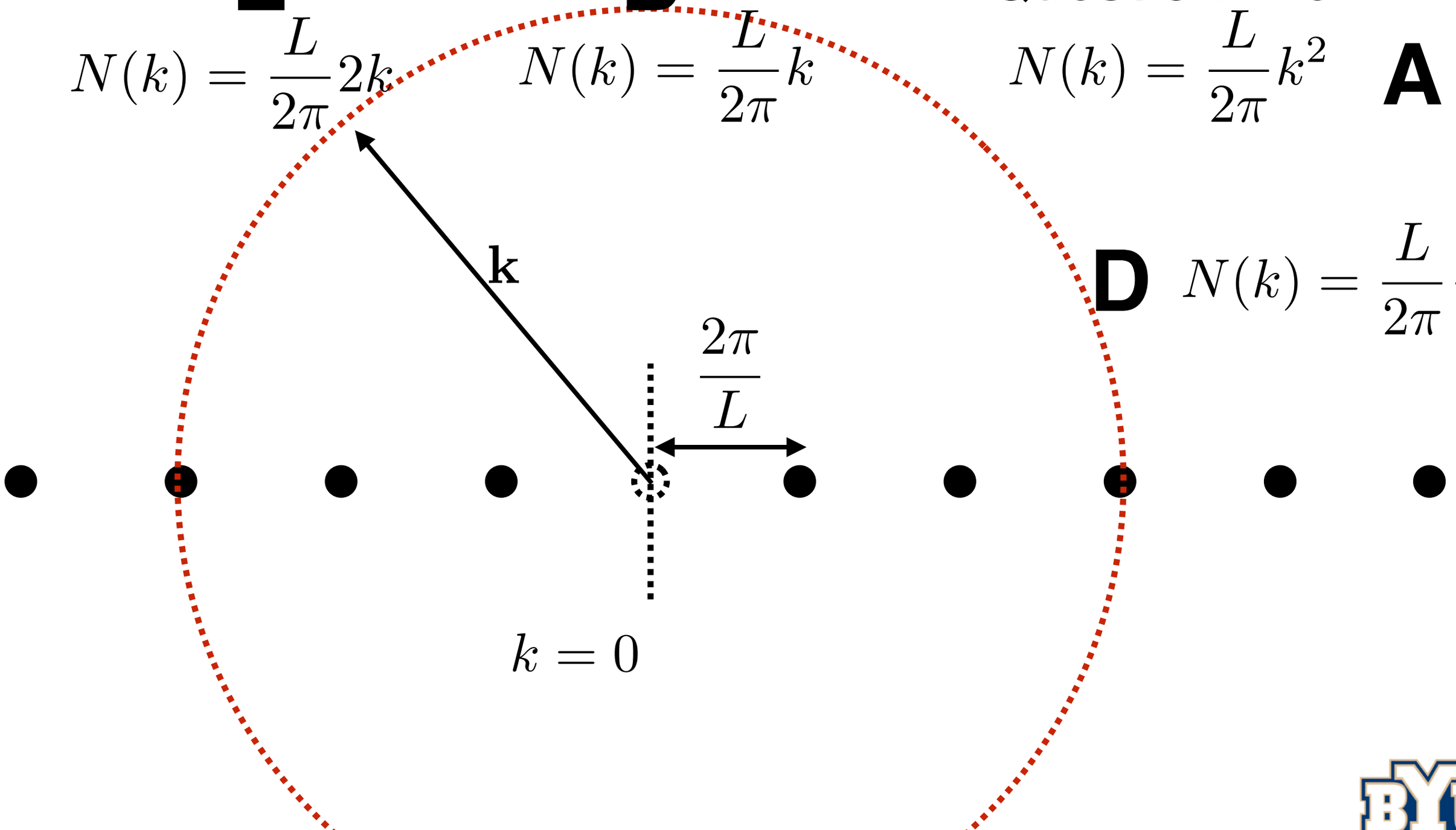
Question #10

E $N(k) = \frac{L}{2\pi} 2k$

B $N(k) = \frac{L}{2\pi} k$

A $N(k) = \frac{L}{2\pi} k^2$

D $N(k) = \frac{L}{2\pi} \frac{1}{k}$



Question #11

What is the density of states $g(E)$

$$N(k) = \frac{L}{2\pi} 2k$$

$$E = \frac{\hbar^2 k^2}{2m}$$ **1**

$$N(E) = \frac{L}{\pi\hbar}\sqrt{2mE}$$ **2**

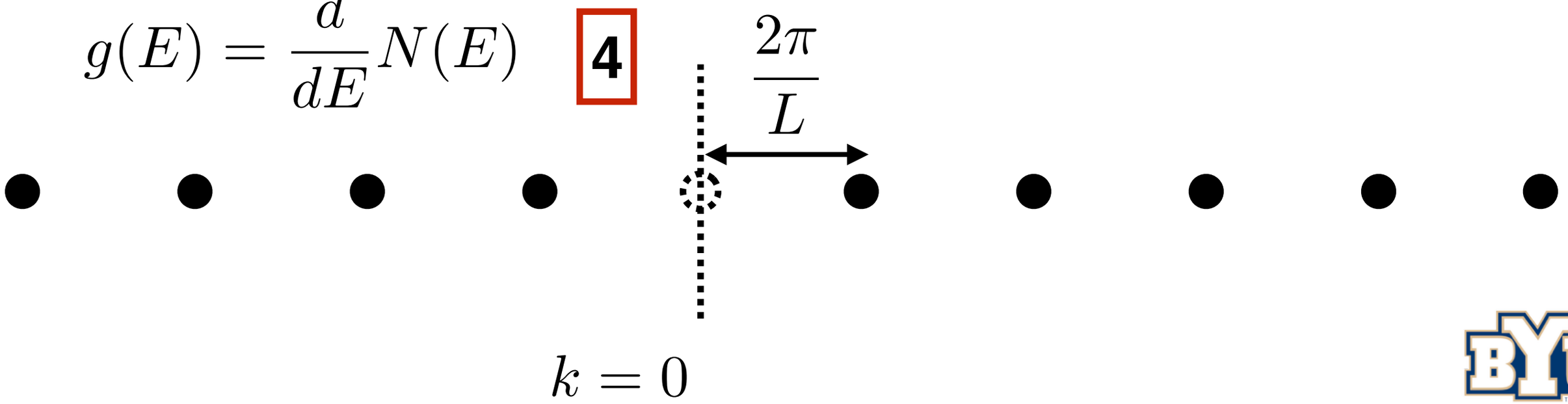
$$N(E) = \int g(E) dE$$ **3**

$$g(E) = \frac{d}{dE} N(E)$$ **4**

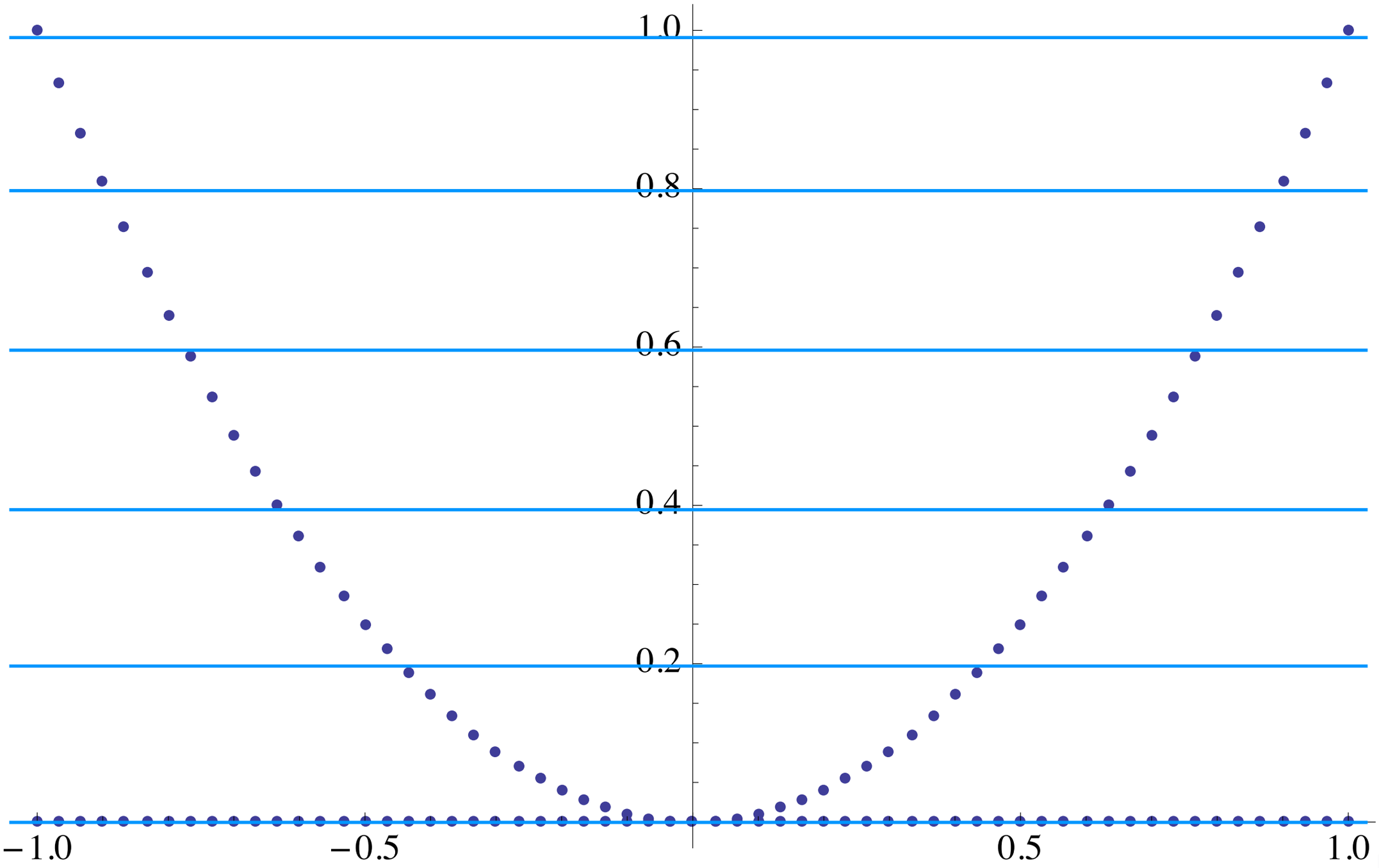
D $$N(E) = \frac{L}{2\pi\hbar}\sqrt{2m}\frac{1}{\sqrt{E}}$$

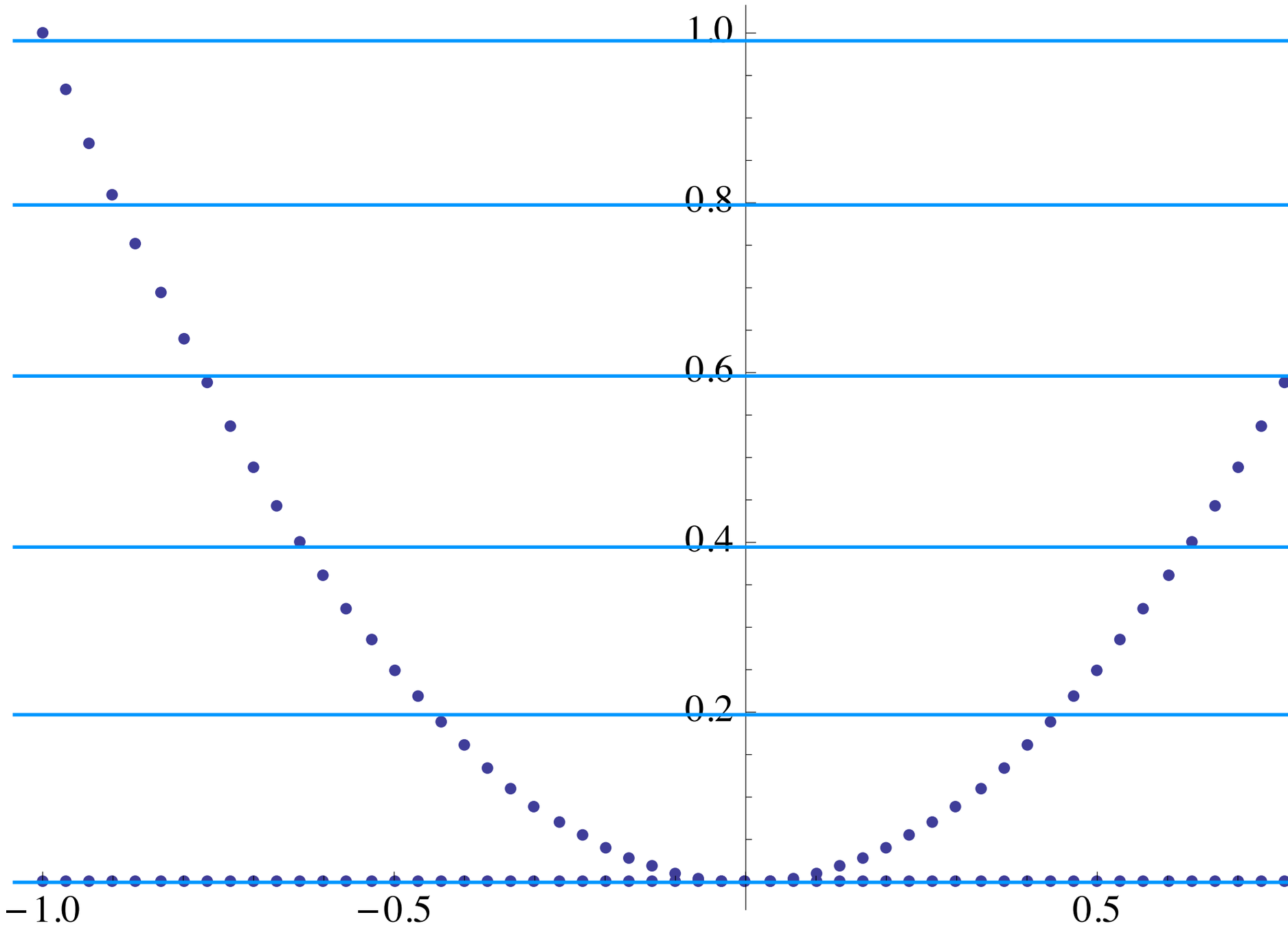
B $$g(E) = \frac{L\hbar\sqrt{2m}\sqrt{E}}{2\pi}$$

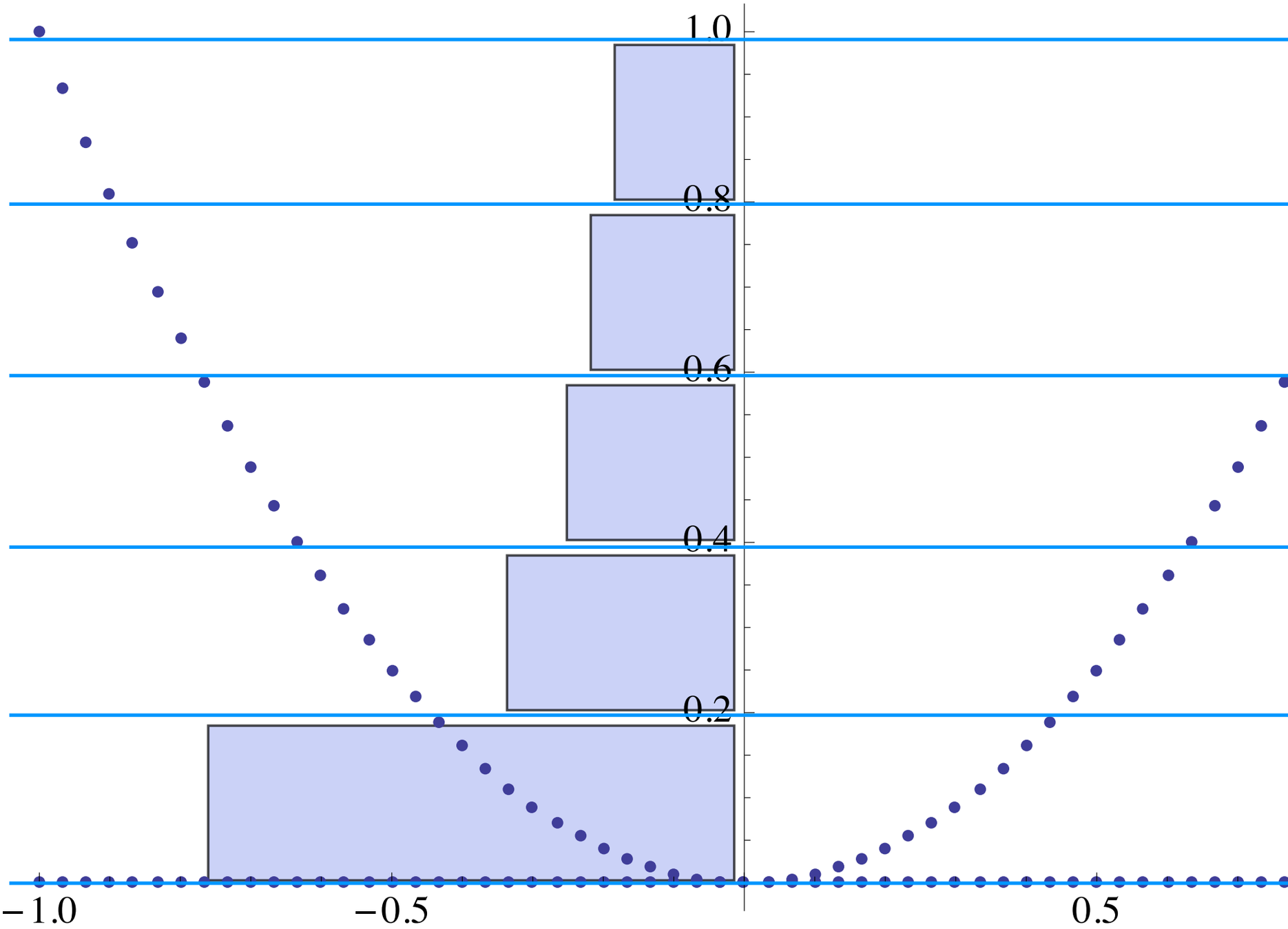
C $$g(E) = \frac{L\hbar\sqrt{2m}}{\pi\sqrt{E}}$$

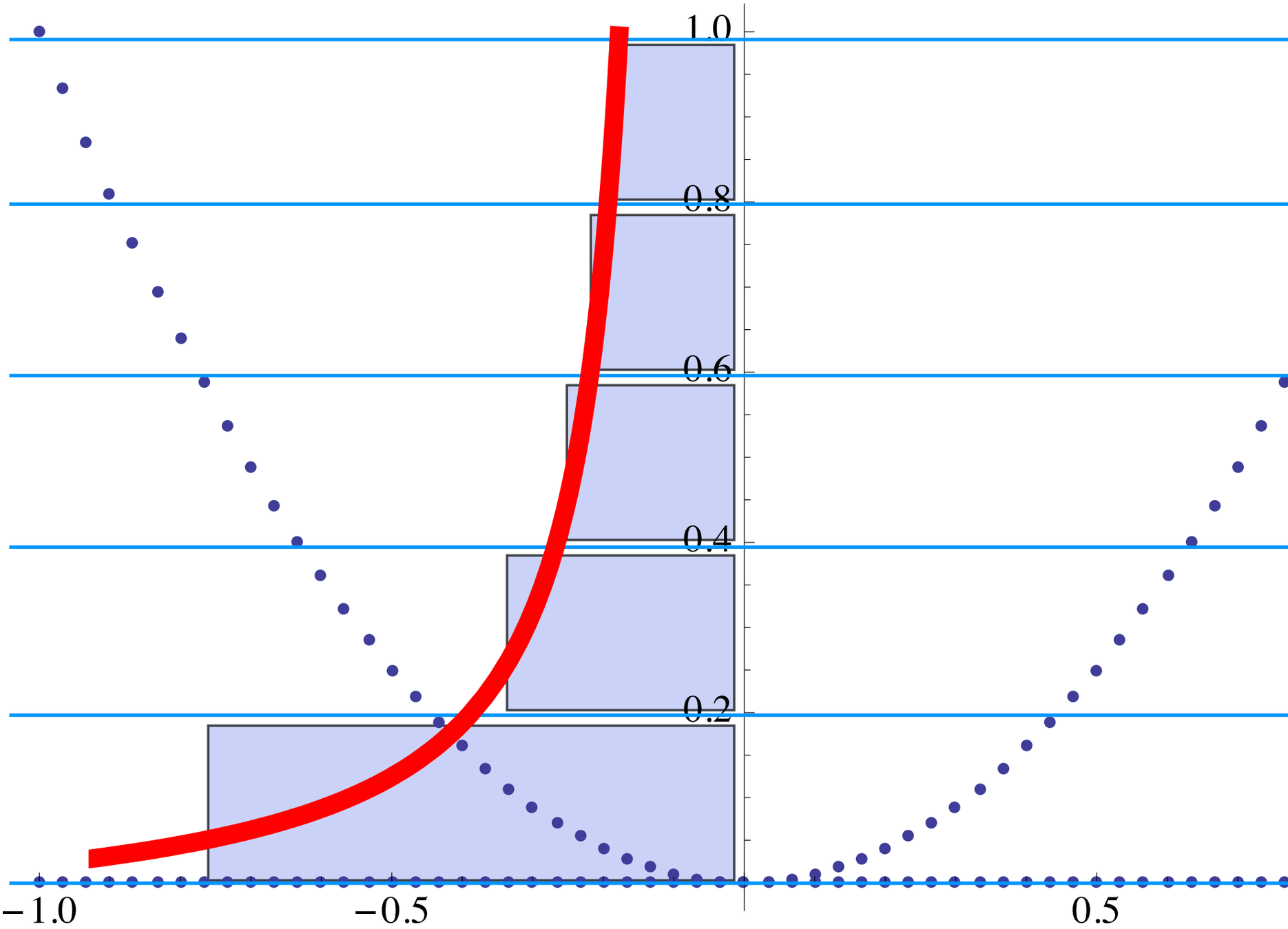


$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$




1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5


1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5


1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5

$$\psi(x,y,t) = A\sin(k_x x)\sin(k_y y)\exp(-i\omega t) \qquad k_x = \frac{2\pi n}{L}$$

Question #12

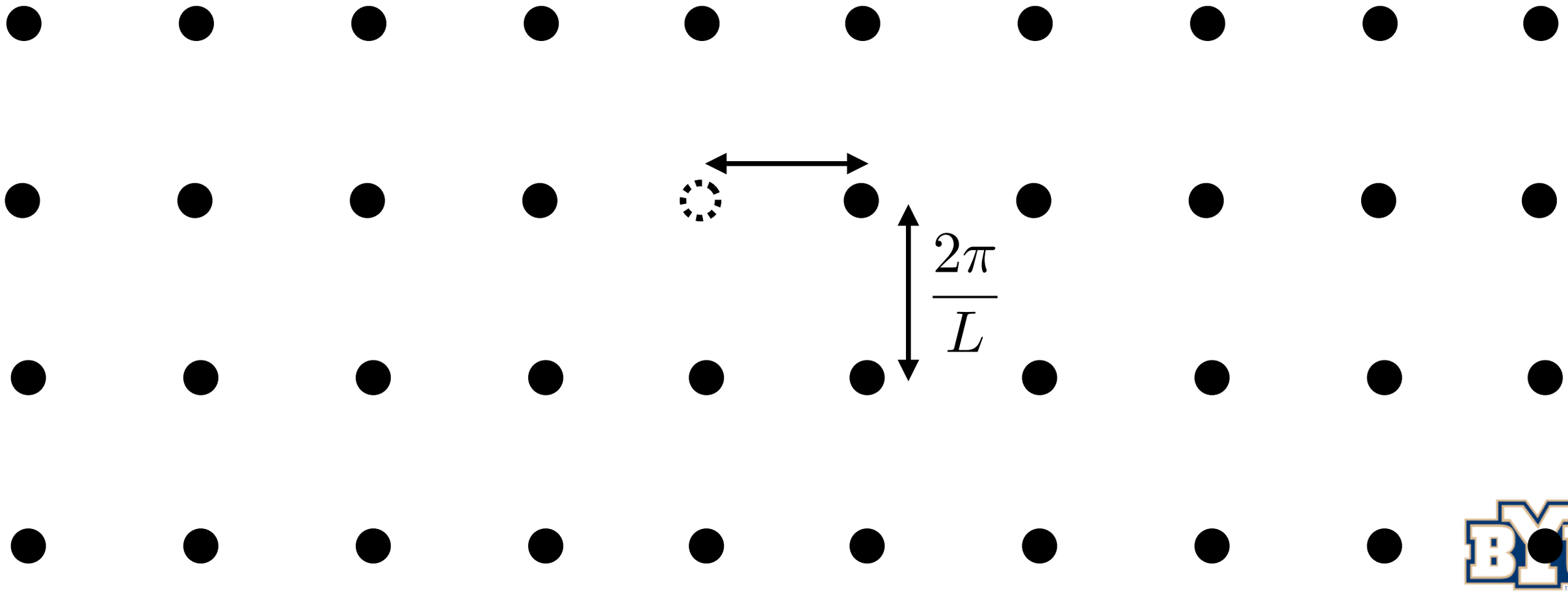
What is the density of the k-points below? $k_y = \frac{2\pi m}{L}$

C $g(k) = \frac{L}{2\pi}$

B $g(k) = \frac{2\pi}{L}$

D $g(\mathbf{k}) = \frac{4\pi^2}{L^2}$

$g(\mathbf{k}) = \frac{L^2}{4\pi^2}$ **E**

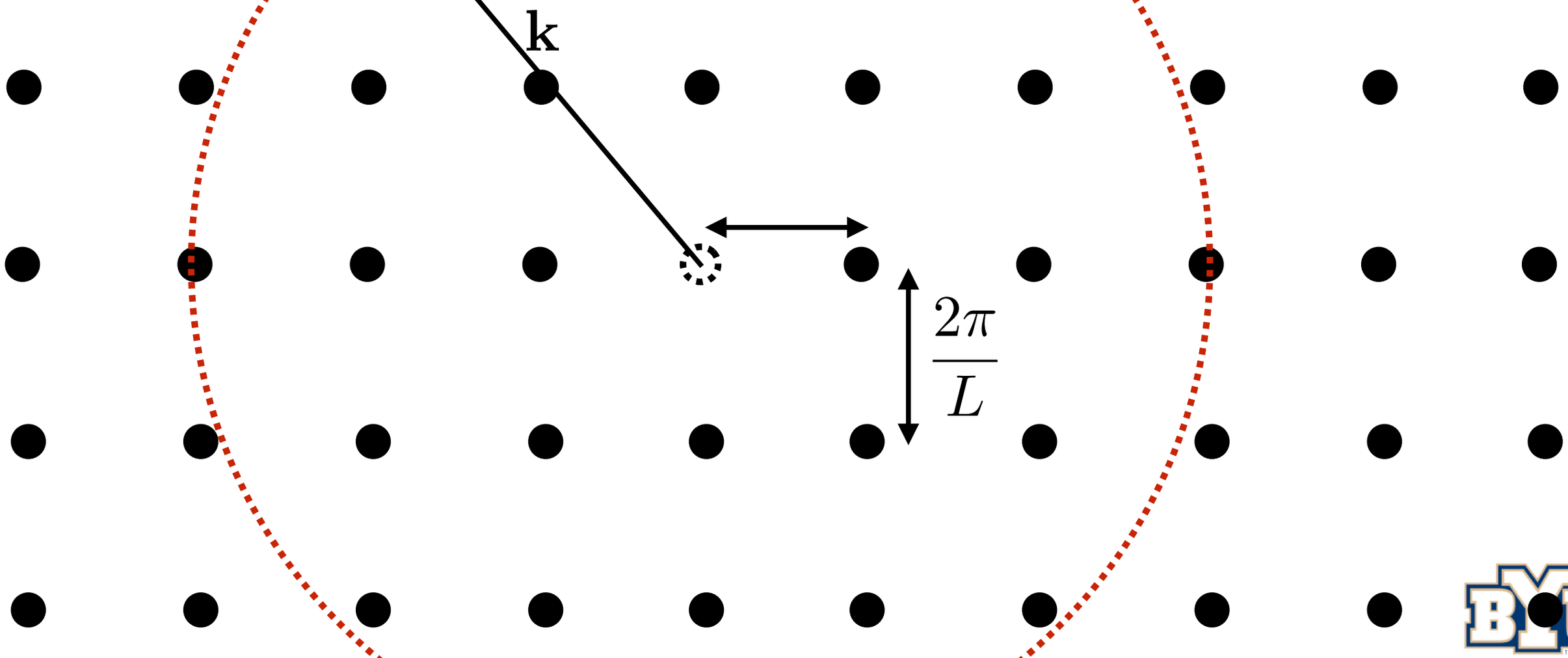


$$\psi(x,y,t) = A\sin(k_x x)\sin(k_y y)\exp(-i\omega t) \qquad k_x = \frac{2\pi n}{L}$$

Question #13

How many states are there in a circle of radius k? $\qquad k_y = \frac{2\pi m}{L}$

C $N(\mathbf{k}) = \frac{L^2}{4\pi^2} 2\pi k^2$ **A** $N(\mathbf{k}) = \frac{L^2}{4\pi^2} 2\pi k$ **B** $N(\mathbf{k}) = \frac{L^2}{4\pi^2} \pi k^2$



What is the density of states $g(E)$

Question #14

$$N(\mathbf{k}) = \frac{L^2}{4\pi^2}\pi k^2$$

$$E = \frac{\hbar^2 k^2}{2m}$$ **1**

$$g(E) = \frac{L^2}{2\pi^2\hbar^2}mE$$ **2**

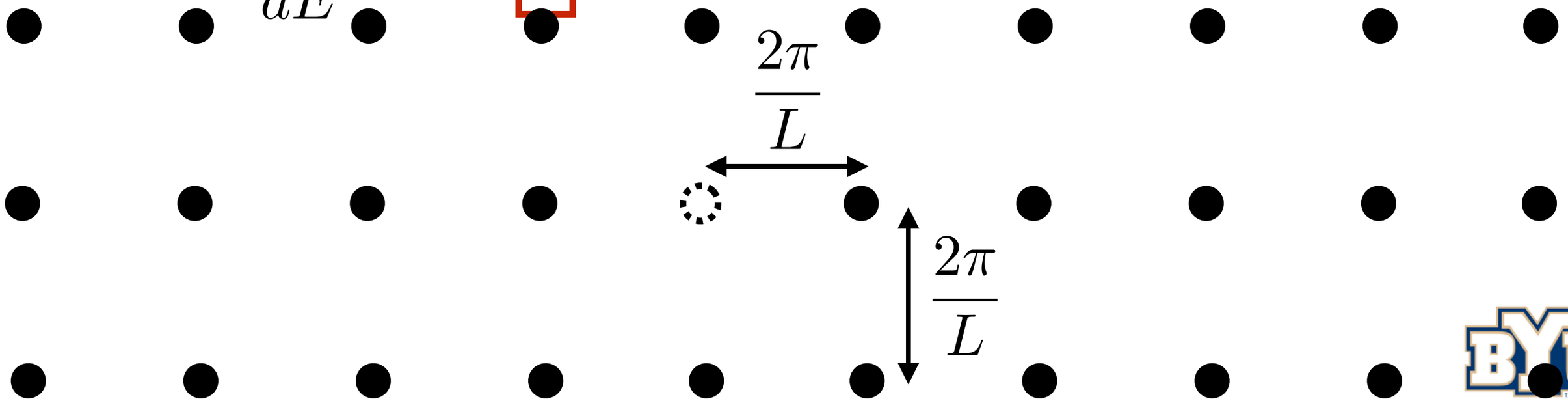
$$N(E) = \int g(E)dE$$ **3**

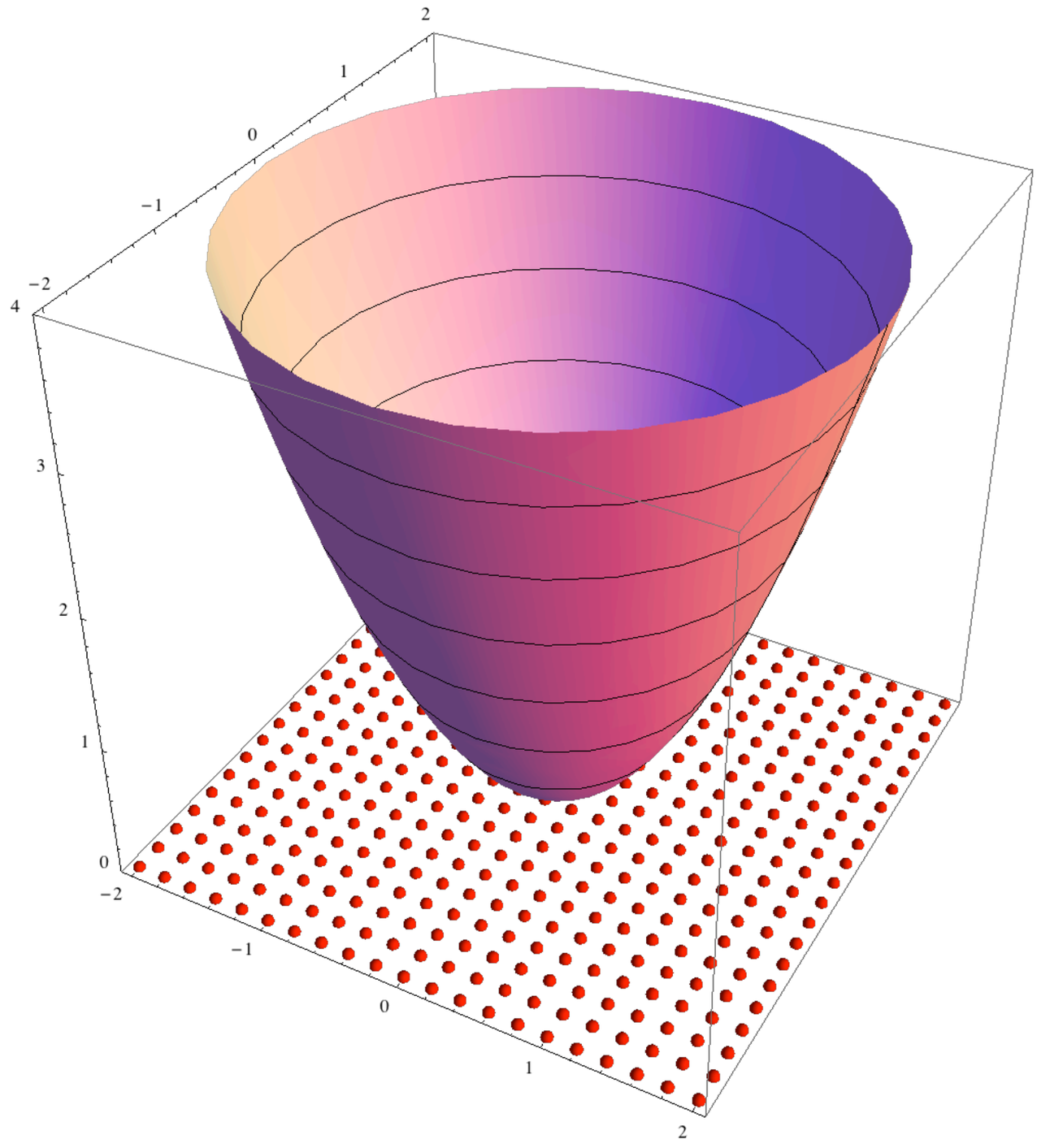
$$g(E) = \frac{d}{dE}N(E)$$ **4**

E $$g(E) = \frac{L^2}{2\pi^2}\hbar^2 m E^{3/2}$$

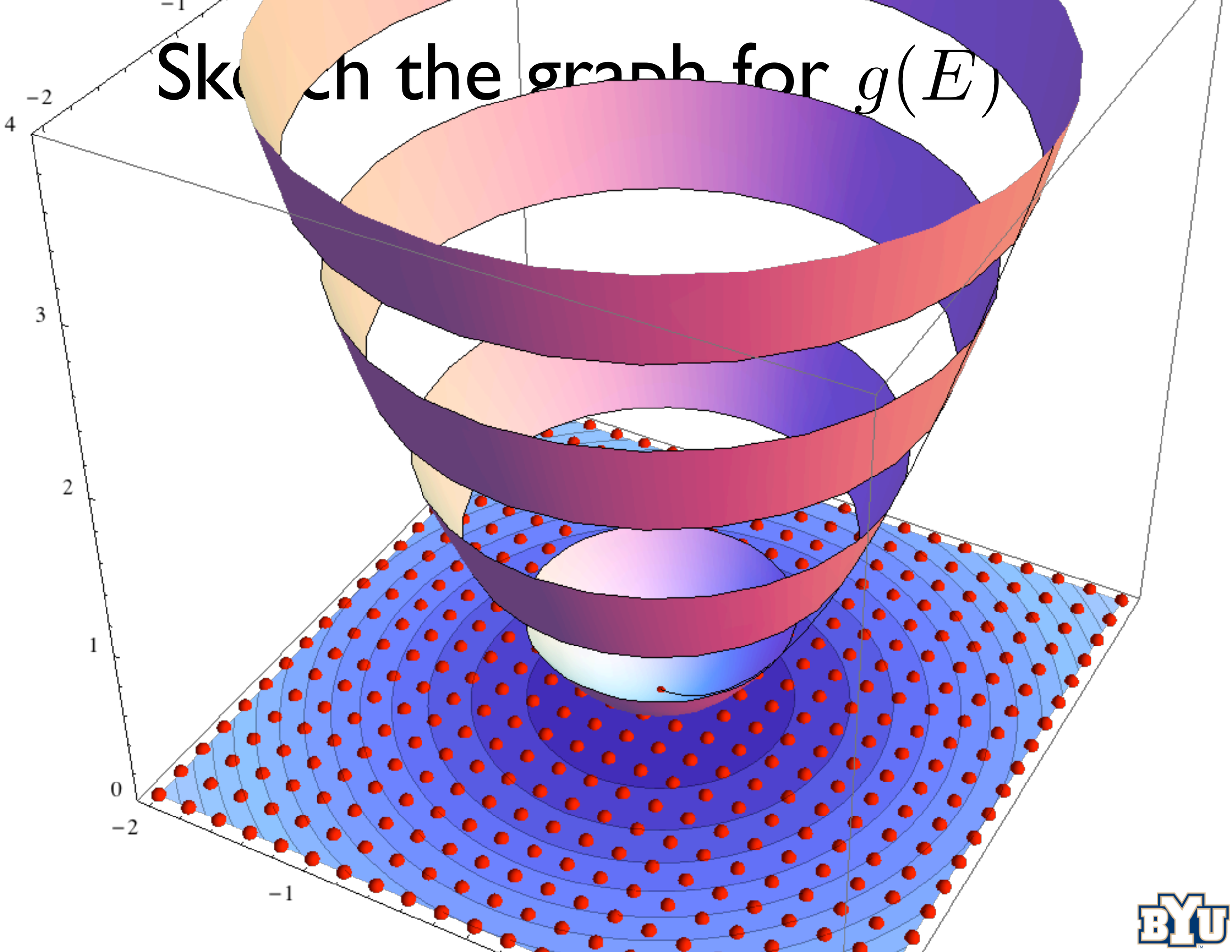
D $$g(E) = \frac{L^2}{2\pi^2\hbar^2}m$$

C $$g(E) = \frac{L^2}{4\pi^2}\hbar^2 m$$




2
1
0
−1
−2
4
3
2
1
0
−2
−1
0
1
2

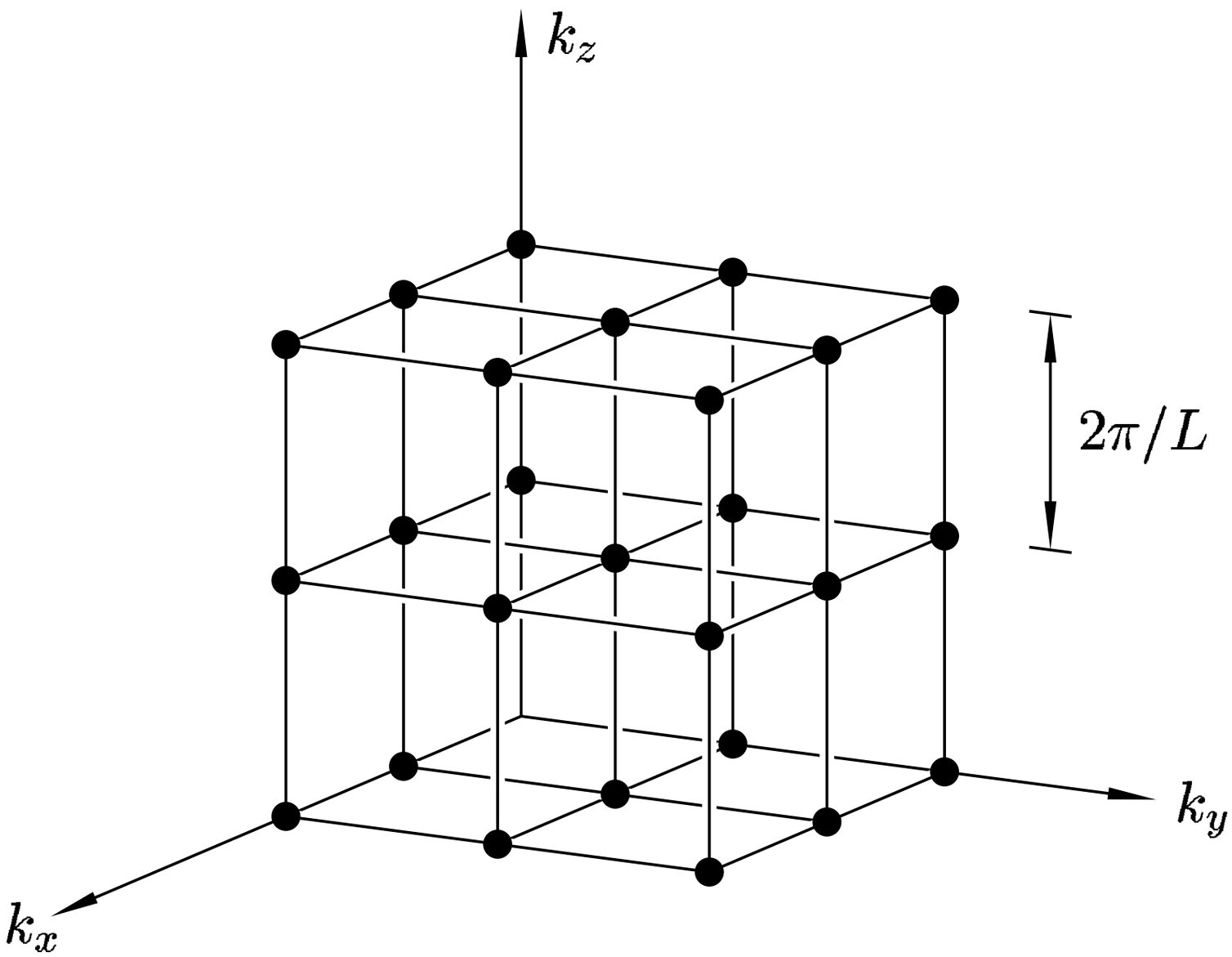
Sketch the graph for $g(E)$



$$\psi(x,y,z,t) = A\sin(k_x x)\sin(k_y y)\sin(k_z z)\exp(-i\omega t)$$

Question #15

What is the density of these points?



D $g(\mathbf{k}) = \dfrac{L^3}{8\pi^3}$

A $g(\mathbf{k}) = \dfrac{8\pi^3}{L^3}$

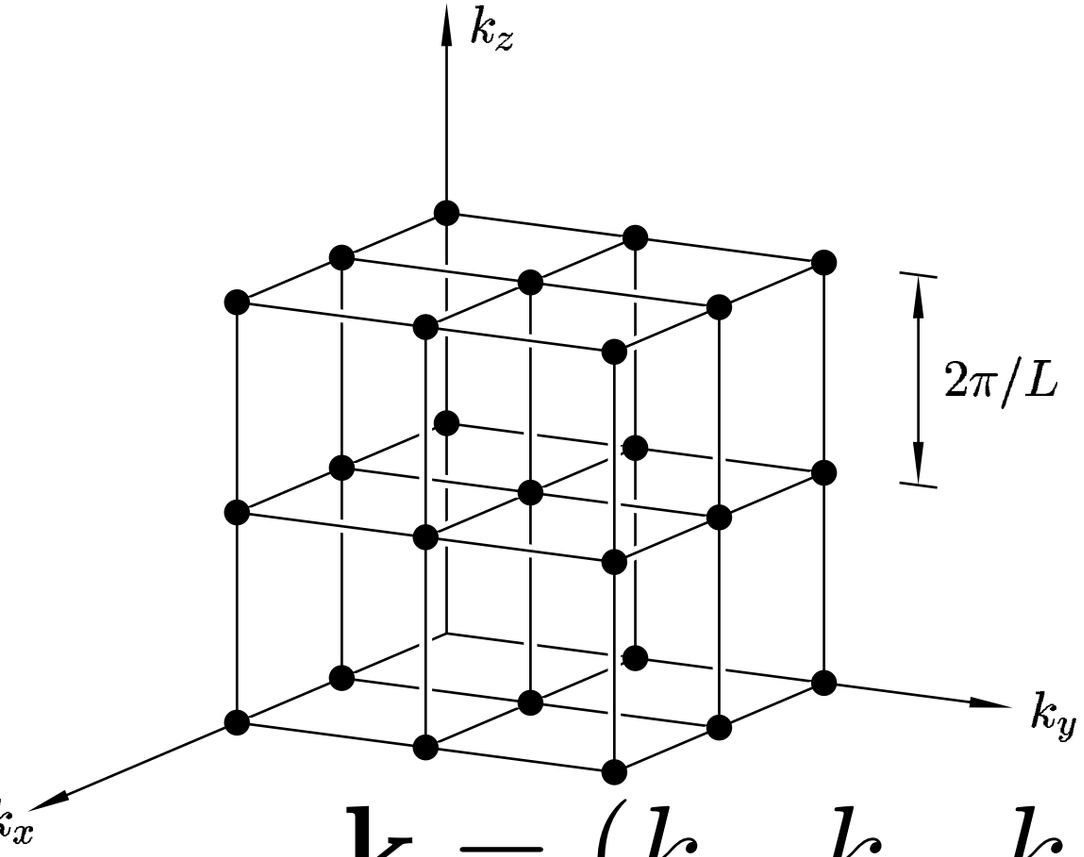
$$\mathbf{k} = (k_x, k_y, k_z) = 2\pi/L(n_x, n_y, n_z)$$

$$\psi(x,y,z,t) = A\sin(k_x x)\sin(k_y y)\sin(k_z z)\exp(-i\omega t)$$

How many states are enclosed in a sphere of radius k?

Question #16

D $N(\mathbf{k}) = \dfrac{L^3}{(4\pi)^3}\pi k^2$



C $N(\mathbf{k}) = \dfrac{L^3}{8\pi^3}\dfrac{4}{3}\pi k^3$

$$\mathbf{k} = (k_x, k_y, k_z) = 2\pi/L(n_x, n_y, n_z)$$

What is the density of states $g(E)$

Question #17

$$N(\mathbf{k}) = \frac{L^3}{8\pi^3}\frac{4}{3}\pi k^3$$

$$E = \frac{\hbar^2 k^2}{2m}$$ **1**

$$N(E) = \frac{L^3}{6\pi^2\hbar^3}(2mE)^{3/2}$$ **2**

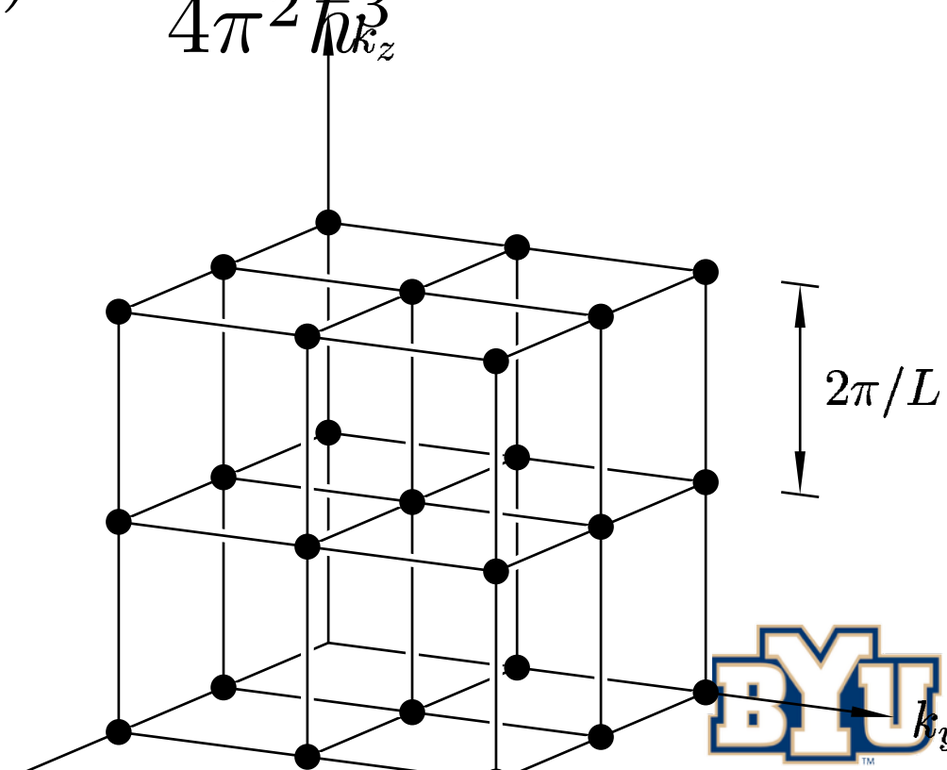
$$N(E) = \int g(E)dE$$ **3**

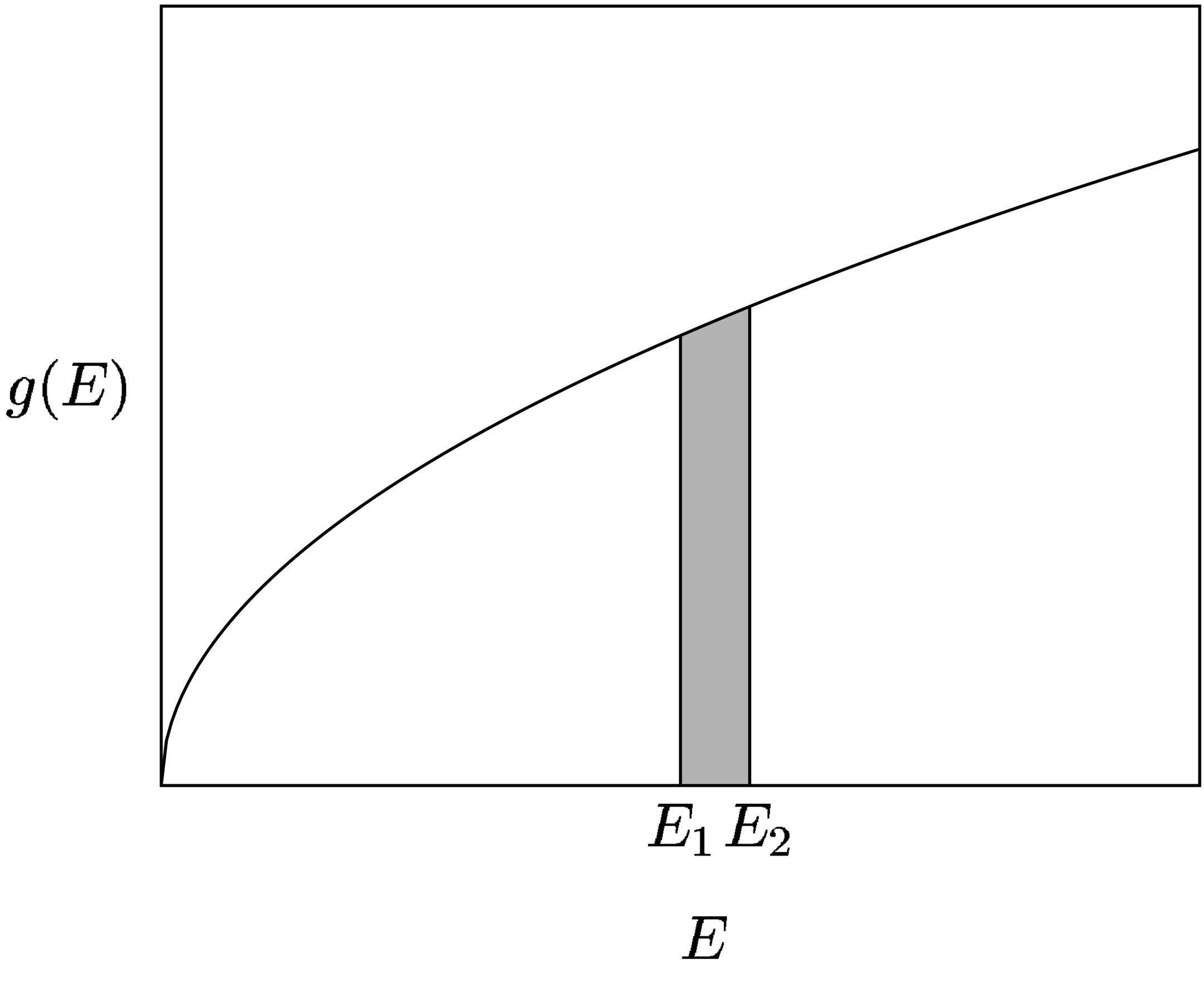
$$g(E) = \frac{d}{dE}N(E)$$ **4**

C $$g(E) = \frac{L\hbar\sqrt{2m}}{\pi\sqrt{E}}$$

B $$g(E) = \frac{1}{2}\frac{L^3}{(4\pi)^3}\frac{4}{3}\pi\hbar^3(2mE)^{5/2}$$

A $$g(E) = \frac{L^3}{4\pi^2\hbar^3}\sqrt{2mE}$$




$g(E)$
E_1 E_2
E