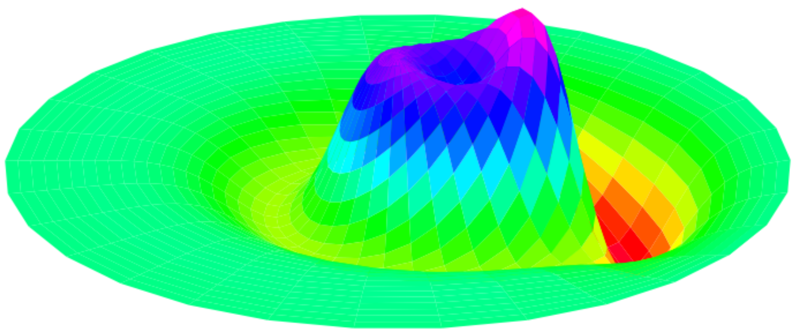
Machine Learning for Physicists



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Last Revised: April 23, 2020

Preface

This is a lab notebook intended to give you experience solving ordinary and partial differential equations numerically. The objectives of this course are

- to help you learn how to solve a differential equation numerically; in situations when a paper-and-pencil solution is impossible or impractical.
- to help you gain greater skills programming a computer and using loops, logic, functions, and classes.
- that your ability to produce a high-quality, professional scientific document will increase.

Text with a bold P designation (**P1.1** for example) indicate tasks that will be done together in class. Text with a bold H designation are homework problems and should be completed out of class. (working in groups is encouraged.)

Python is the programming language that we will be using You can obtain a free copy of Python here. Any computer code that you create should be uploaded to the Google Drive folder provided.

There is a companion book to this one entitled, "Introduction to Python". It is intended to help you learn to use Python to do the tasks contained herein.

Contents

Part I

Simple Probability

Chapter 1

Introduction to Probability

Simple Probability

For most people, the idea of simple probability is pretty intuitive. The probability of rolling a 5 on a 6-sided die is $\frac{1}{6}$ and the probability of drawing the queen of hearts in a 52-card deck is $\frac{1}{52}$. Generalizing this idea yields the equation:

$$P(\text{event A}) = \frac{\text{No. of ways A can happen}}{\text{Total no. of things that could happen}} \tag{1.1}$$

Probability of multiple events

Things get a little harder when you consider multiple events because you have to consider whether the outcome of one event will affect the outcome of the next. For example, suppose you roll a 6-sided die twice and you want to know the probability that both rolls come up 5. Having a 5 come up on the first roll doesn't affect the probability that a 5 comes up on the second. These two events are independent of one another. The outcome of one event does not affect the outcome of the other.

In contrast, consider randomly selecting marbles out of a bag which initially contains 3 red marbles and 8 green marbles. If you reach into the bag and grab two marbles, what is the probability that they will both be red? Notice that selecting a red marble on my first reach into the bag modifies the number or red marbles available for selection on my second reach into the bag. Hence the two probabilities are dependent on each other. This is an example of dependent events.

1-1	1-2	1-3	1-4	1-5	1-6
2-1	2-2	2-3	2-4	2-5	2-6
3-1	3-2	3-3	3-4	3-5	3-6
4-1	4-2	4-3	4-4	4-5	4-6
5-1	5-2	5-3	5-4	5-5	5-6
6-1	6-2	6-3	6-4	6-5	6-6

Figure 1.1 All 36 outcomes from rolling two fair dice. Out of 36 possible outcomes, only one yields a 5 on both rolls. Hence the probability of rolling two 5's is $\frac{1}{36}$.

Intersection of two events (event A and event B)

We'll start with the case where two events, A and B, both occur. (Also called the intersection of two events. See figure 1.2) The math for this sort of situation is:

$$P(A \cap B) = P(A) \times P(B)$$

but let's see some examples so it makes more sense.

Independent Events

Consider rolling a fair 6-sided dice. What is the probability that 5 comes up on the first two rolls? Your intuition is probably telling you that rolling two 5's is less likely than rolling just one, so you expect the probability to be less than $\frac{1}{6}$. In figure 1.1 you will see all 36 possible outcomes when rolling a fair dice two times. Of those 36 possible outcomes how many of occurrences of two 5's being rolled were there? Just one. Hence, the probability is $\frac{1}{36}$. You may have noticed that $\frac{1}{36} = \frac{1}{6} \times \frac{1}{6}$. In other words, the probability of event A happening **and** event B happening is just the product of the two individual probabilities:

$$P(A \text{ and } B) = P(A) \times P(B) \tag{1.2}$$

sometimes $P(A \text{ and } B)$ is written using the symbol for intersection ($\cap$):

$$P(A \cap B) = P(A) \times P(B) \tag{1.3}$$

Dependent Events

When the outcome of one event is affected by the outcome of a previous event, we say they events are dependent. For example, consider a bag containing 3 red marbles and 8 green marbles. If I reach into the bag and select two marbles, what is the probability that they are both red. The probability that my first selection is red is:

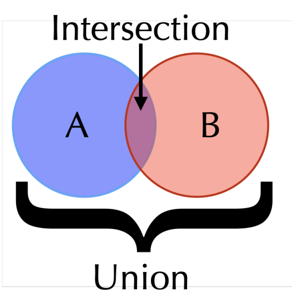

Figure 1.2 If the blue circle represents the probability of A occuring and the red circle represents the probability of B occuring, then the overlap of these two circles is the probability of both events occuring. By sliding one cirle left or right we can adjust the size of the overlap (aka $P(A|B)$)

$$P(A) = \frac{3}{11}$$

because 3 out of theh 11 choices are red. What about the second selection. After the first selection, I am left with 10 marbles, 2 of them being red. Hence, the probability that I draw a red marble on my second selection is:

$$P(B) = \frac{2}{10}$$

The probability that event A (first selection is a red marble) **and** event B is

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= \frac{3}{11}\frac{2}{10} \\ &= \frac{6}{110} \\ &= 0.055 \\ &= 5.5\% \end{aligned}$$

In general, the probability of two events occuring is given by:

$$P(A \cap B) = P(A) \times P(B|A) \tag{1.4}$$

where $P(A|B)$ is called "conditional probability" and means the probability of B happening **given that** A already happened. It's important to recognize that equation (1.3) is the most general equation for two events and that equation This equation reduces to equation (1.3) when the events are independent.

We can do a little algebra on equation (1.4) to get:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \tag{1.5}$$

Furthermore, if A and B are independent events, then $P(A \cap B) = P(B \cap A) = P(B) \times P(A|B)$. Hence, the above equation becomes:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \tag{1.6}$$

$$= \boxed{\frac{P(A|B)P(B)}{P(A)}} \tag{1.7}$$

1-1	1-2	1-3	1-4	1-5	1-6
2-1	2-2	2-3	2-4	2-5	2-6
3-1	3-2	3-3	3-4	3-5	3-6
4-1	4-2	4-3	4-4	4-5	4-6
5-1	5-2	5-3	5-4	5-5	5-6
6-1	6-2	6-3	6-4	6-5	6-6

Figure 1.3 All 36 outcomes from rolling two fair dice. <first roll> - <second roll>

This is known as Bayes' Theorem or Bayes' Rule and is of profound importance going forward.

Union of two events (event A or event B)

Part II

Bayesian Statistics

Part III

Regression

Part IV

Optimization Algorithms

Commented out by Michael Ware. Code below inserts index

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