An Adaptive Evolutionary Multi-objective Algorithm Based on R2 Indicator

Fei Li,Jianchang Liu, *Northeastern University,* and Huaitao Shi,Peiqiu Huang *Northeastern University*

***Abstract*—Multi-objective optimization problems can be solved by giving a set of preferences in some evolutionary multi-objective optimization algorithms(EMOAs). Each potential solution is ranked by considering the contribution to the preference. R2-EMOA is an EMOA which is on the foundation of decision makers preferences. However, the preference of R2-EMOA had better balance the convergence and diversity. How to build preferences and adaptively modify decision makers preference get few attention. In this paper, we propose an adaptive R2-EMOA, called CL-AR2-EMOA, which incorporates the Coulomb law to generate weight vectors and the adaptive strategy with the feature of Pareto front. The original weight vectors generation strategy, which two weight vectors can be seen as like charges in the objective space, is based on Coulomb law. The adaptive strategy is introduced to dynamically interacts information between the weight vectors and the scales of objective functions. The performance of CL-AR2-EMOA is evaluated using standard unconstraint benchmark problems, i.e. DTLZ1-DTLZ4 for 3,5,8 and 10 objectives, the scaled DTLZ problems also introduced, WFG1-WFG9 for 2 and 3 objectives. Our experimental results show that the proposed CL-AR2-EMOA performs competitively with respect to other R2-EMOAs.**

Keywords*—Multi-objective optimization, R2-indicator, Coulomb law, weight generator, adaptive strategy.

I. Introduction

MULTI-OBJECTIVE optimization problems(MOPs) refer to more than one objective function during the optimization, which lead to the notion of optimum that is different compared the optimum in single-objective optimization problems(SOPs). In general, the objective functions are conflict or contradict with each other. The primary mission of solving MOPs is to find a set of trade-off solutions among the objectives. Evolutionary algorithms(EAs) have been recognized to be well suited for MOPs since early 1980s because they can process a set of solutions in parallel, thereby can obtain an approximation of the Pareto front which consists of multiple Pareto optimal solutions in a single run [1].

Over the past three decades, various EMOAs have been proposed to solve MOPs effectively. Most of these approaches are on the basis of Pareto dominance and diversity maintaining strategy suggested by Deb,i.e. NSGA-II [2], SPEA2 [3] and MOPSO [4]. It is that Pareto dominance based EMOAs have superior performance when cope with low dimension objective problems and some real world probelms [5], [6]. However, their search capability lack of sufficient selection pressure when dealing with many-objective optimization problems(MaOPs). The diversity maintaining strategy should be properly devised for different Pareto front.

In addition to the Pareto dominance selection strategy, there has been considerable efforts invested in other types of EMOAs. One of the most promising alternatives is the decomposition based evolutionary multi-objective algorithm(DMOEA). DMOEA encompasses concept or framework that takes inspiration from the "divide and conquer" paradigm, by essentially breaking a MOPs into several subproblems for which solutions for original global problem are computed and aggregated in a cooperative manner. Most of the decomposition based methods have the following mechanisms. Firstly, a mechanism to generate a uniform distribution weight vectors. Secondly, a class of scalarizing function to evaluate solutions. Finally, a mechanism to obtain an overall ranking of the solutions derived from the evaluation of each scalarizing function.

MOEA/D [7], [8] is a recently decomposition based method. MOEA/D explicitly decomposes the MOP into a number of scalarizing optimization subproblems by means of a set of weight vectors, which solves these subproblems simultaneously by evolving a population of solutions. At each generation, the population is composed of the best solutions found so far for each subproblem. However, as discussed in some recent studies [9], [10], uniformly distributed weight vectors, used in the early MOEA/D, might not always lead to evenly Pareto front when using Tchebycheff scalarizing function.

Indicator based EMOAs are other types of EMOAs, e.g., IBEA [11], HypE [12] and SMS-EMOA [13]. It adopts performance indicator to assess the potential solutions balancing the convergence and diversity. The frequently used performance indicator is hypervolume owing to its nice theoretical properties [14]. In fact, maximizing the hypervolume can obtain solutions which will be well spread along the Pareto front. Nevertheless, the computational complexity of hypervolume is expensive with the objective increasing.

An excellent performance indicator has been reported several years: R2 [15]. R2 indicator has desirable properties (i.e., it is weakly monotonic and can be computed in a fast manner, and it produces well-distributed solutions) that makes it a viable candidate to be embedded into indicator-based EMOA. It has given rise to a new generation of EMOAs and the relationship between R2 indicator and decomposition based method will be given in the near future.

R2-EMOA [15] has become an increasing popular selection method for posteriori EMOAs. It has some advantages, such as its scalability to problems with more than three objectives

M. Shell is with the Department of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta,GA, 30332 USA e-mail: (see http://www.michaelshell.org/contact.html).

J. Doe and J. Doe are with Anonymous University.

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[16], applicability for noisy MOPs [17], easy embedded with swarm intelligence algorithm [18]. R2 indicator is composed of three main components: weight generator, scalarizing method and nadir point update. Firstly, simplex-lattice method [7], systematic sampling method [19] and uniform method [20] are commonly used to design weight vectors for R2-EMOA. But all of the weight generator can hardly set arbitrarily or too complicated. Secondly, many scalarizing methods exist, including the prominent examples of weighted sum, Tchebycheff and augmented weighted Tchebycheff method [21]. Thirdly, fixed nadir point, adaptive nadir point adjustment and record data structure nadir point adjustment [22] are applicable to update the nadir points.

Characterizing the exact optimal placements of Pareto front depending on the number and distribution of weight vectors. Designing an efficient and effective weight generation strategy is particularly crucial for R2-EMOA. R2 indicator usually requires a set of weight vectors uniformly distributed in objective space. It is generally accepted that weight vectors uniformly distributed can guide uniformly distributed along the true Pareto front.

CL-AR2-EMOAs which introduce Coulomb law to generate weight vectors and R2 indicator as a selection method to assess the individual of population is proposed in this paper. We regard the point in the quarter circle as like charges inspired from the Coulomb law in Physics. Like charges repel each other due to the effect of Coulomb force. The amplitude of Coulomb force is used as the distribution indicator of each potential weight vector in the weight vector space. Through the analyze of scalarizing function, we use R2 indicator based on Tchebycheff method to balance the convergence and diversity. CL-AR2-EMOA is expected to have superiority over other weight vectors generation strategy in the following aspects: (1) weight vectors are generated in any given positive integer. (2) adaptive weight vectors according to the nadir point and ideal point is proposed.

The remainder of this paper is organized as follows: Section II introduces some preliminaries and background that will be used in the paper. The proposed algorithm is presented in Section III, while Section IV shows a comparative study with respect to other algorithms. Section V provides our conclusions and some future directions.

II. Preliminaries and Background

In this section, we first give some basic definitions in MOPs. Since the proposed approach is on the foundation of R2 indicator and Coulomb law, we should provide some details about them before proposing our algorithms.

A. Multi-objective optimization

Without loss of generality, continuous and unconstrained minimization multi-objective optimization problems can be stated as follows:

$$\begin{aligned} min \qquad & F(x) = \{f_1(x), f_2(x), ..., f_m(x)\} \\ & \qquad subject \quad to \quad x \in \Omega \end{aligned} \tag{1}$$

where $x = (x_1, x_2, ..., x_n)$ is the decision vector, n is the dimension of solution space ,m is the number of objectives, $F(x)$ is the m dimensional objective vector, $f_i(x)$ is the ith objective to be minimized.

$x1$ is said to dominate $x2$ (denoted as $x1 \prec x2$) if and only if $f_i(x1) \leq f_i(x2)$ for every $i \in \{1, ..., m\}$,$f_j(x1) < f_j(x2)$ for at least one index $j \in \{1, ..., m\}$. A solution x^* is Pareto optimal to equation (1) if there is no other solution $x \in \Omega$ such that $x \prec x^*$. $F(x^*)$ is called a Pareto optimal front. The set of all Pareto optimal solutions is called the Pareto-optimal set.

B. R2 Indicator

The family of R indicator is on the foundation of utility functions which map the objective space to utility space for assessing the quality of two sets of individuals.

Definition 1: For a set U of general utility functions, a probability distribution p on U, and a reference set R, the R2 indicator of a solution set A is defined as the expected utility

$$R2(R, A, U, p) = \int_{u \in U} (\max_{\vec{r} \in R}\{u(\vec{r})\} - \max_{\vec{a} \in A}\{u(\vec{a})\})p(u)du \tag{2}$$

Definition 2: The primary R2 indicator can be calculated by using a discrete and finite set U and a uniform distribution p over U as follows:

$$R2(R, A, U) = \frac{1}{|U|} \sum_{u \in U} (\max_{r \in R}\{u(r)\} - \max_{\vec{a} \in A}\{u(\vec{a})\}) \tag{3}$$

Definition 3: If we assume R to be constant reference set, the R2 indicator can be defined as a unary indicator

$$R2(A, U) = (-\frac{1}{|U|}) \sum_{u \in U} \max_{\vec{a} \in A}\{u(\vec{a})\} \tag{4}$$

Note that we assume R to be constant in this paper and will only refer to $Definition$ 3 when we use the term R2 indicator.

Different utility function selection can lead to different convergence and diversity in the selection process. The commonly used utility functions contain weighted sum and weighted Tchebycheff function or combined together. As suggested by Ke Li [23], we use the modified weighted Tchebycheff function $u(\vec{z}) = u_\lambda(\vec{z}) = -\max_{j \in \{1,...,m\}} \frac{1}{\lambda_j} \left| z_j^* - z_j \right|$ for R2 indicator,where $\lambda = (\lambda_1, ..., \lambda_m) \in \Lambda$ is a given weight vectors and z^* is an ideal point.

Definition 4: The R2 indicator of a solution set A for a given set of weight vectors Λ and an ideal point z^* is defined as

$$R2(A, U) = \frac{1}{|U|} \sum_{\lambda \in \Lambda} \min_{\vec{a} \in A}\{ \max_{j \in \{1,...,m\}}\{\frac{1}{\lambda_j} \left| z_j^* - a_j \right|\}\} \tag{5}$$

Usually, the weight vectors are chosen uniformly distributed over the angle space. We use our weight vectors generation strategy based on Coulomb law. Weight vectors corresponding to a ray which directly crosses a point $a \in A$ is defined as λ^a.

Considering the preference of decision maker, the contribution of an individual to R2 value can be of interest other than the overall assessment of an approximation set A.

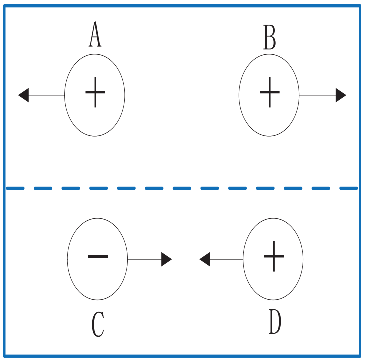

Fig. 1. The Coulomb force between charges

Definition 5: The contribution of a solution $a \in A$ to the R2 indicator is defined as

$$C_{R2}(\vec{a}, A, \Lambda, z^*) = R2(A, \Lambda, z^*) - R2(A \backslash \{\vec{a}\}, \Lambda, z^*) \quad (6)$$

C. Coulomb Law

One of the fundamental interaction in nature is the electromagnetic interaction. It is the interaction that is most directly responsible for all of the chemical and biological phenomena that you observe. Coulomb law states that the electrical force between two charged objects is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the separation distance between the two objects.

The simplest expression that demonstrates how the electrostatic force depends upon the separation between the two charged particles can be written down referring to Fig.1. The force exerted on charged particle B by charged particle A is using equation 7.

$$\overrightarrow{F_e}(on\ particle\ B) = k_e \frac{Q_A Q_B}{r_{AB}^2} \overrightarrow{r_{AB}} \quad (7)$$

Where Q_A and Q_B are the charges of particle A and B, respectively, and $\overrightarrow{r_{AB}}$ is the unit vector pointing along the radius vector from particle A to B. The constant quality k_e is called "the electric force constant". Its value is defined to be $k_e = 8.99 \times 10^9 Nm^2C^{-2}$.

D. Motivation

For R2-EMOA, the number of weight vectors is controlled by the parameter H and m using $N = C_{H+m-1}^{m-1}$ to calculate. R2-EMOA is incapable of adopting system sampling to produce an arbitrary number of evenly distributed weight vectors when the number is three or many objectives. With the change of the parameter H and m, the weight vector curve is shown in Fig. 2. For example, to maintain accurate weight vectors, we set H as a constant, e.g. $H = 18$, then the weight vector size set as 190,1330,7315,33649 respectively for problems

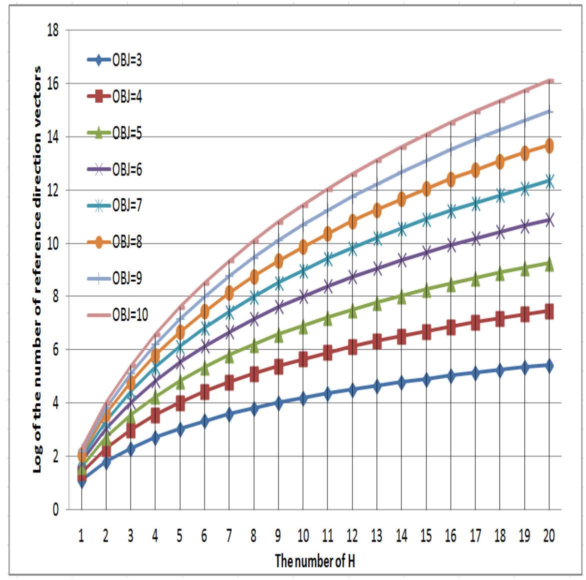

Fig. 2. The log of reference weight vectors

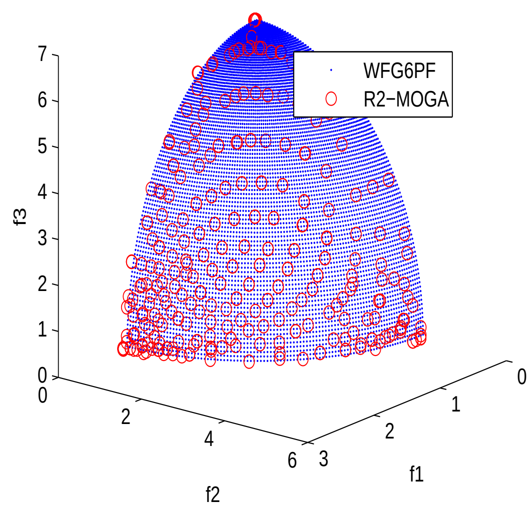

Fig. 3. R2-MOGA for WFG6

with 2-6 objectives according to the system sampling design. Hence, there is need to design efficiency and effective weight vectors. As shown in Fig. 3, the diversity of WFG6 with three objectives is not uniformly when we use R2-EMOA to deal with the scaled MOPs. The weight vectors distribution with the Pareto front should have the same geometry feature when deal with different MOPs. Then, the adaptive weight vectors generation strategy with the geometry changing should properly design.

III. Description of CL-AR2-EMOA

In this section, we propose CL-AR2-EMOA, an adaptive

Algorithm 1 General framework of CL-AR2-EMOA

Require: *Maxiter*, the maximum number of iteration;
N, the number of population;
m, the number of objective

Ensure: The approximation set *P*

```
Initialize and evaluate N population P
Calculate the ideal point z* and nadir point z^nad
Generate N weight vectors based on Coulomb law
for iter = 1 to Maxiter do
    Perform tournament selection
    Generate offspring using EAs
    Evaluate the population
    Update ideal point z* and nadir point z^nad
    Update weight vectors using adaptive strategy
    Rank population using Fast R2-ranking
    Choose the best N solutions to the next generation
end for
return P
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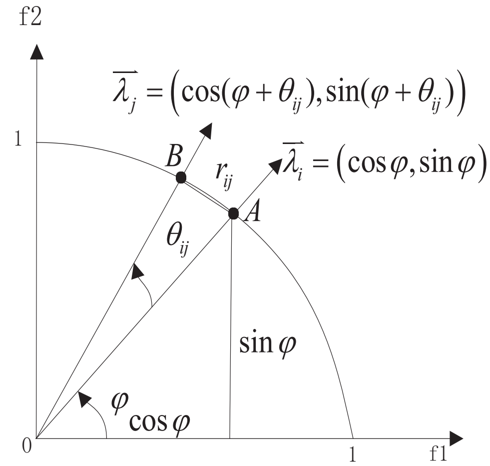

Fig. 4. Weight Vectors Generation

evolutionary algorithm that the objective function value interacts with weight vectors. As shown in CL-AR2-EMOA, the potential solutions consider the contribution to the utility functions which are defined with weight vectors and scalarizing function. On the other hand, the preference of utility function ranks each solution. The R2 indicator can combine the convergence and diversity together.

The weight vectors of utility function are adaptively modified during the search. The main features of CL-AR2-EMOA include : (1) the Coulomb law to generate the original weight vectors; (2) the maintenance of adaptive weight vectors. In this section, we present an instantiation of CL-AR2-EMOA and then the implementation details of each component will be explained.

A. Algorithm Framework

The main framework of the proposed CL-AR2-EMOA is given in Algorithm 1. First, the initialization process generates N initial solutions and calculates the ideal and nadir point. Then, the proposed weight vectors based on Coulomb law is embedded into the CL-AR2-EMOA as shown in Algorithm 2. When the loop is not met the termination, the simulated binary crossover(SBX) and polynomial mutation(PM) or differential evolution(DE) operator is used to generate offspring solutions. Then, the offspring and parent solutions are combined together to obtain the ideal and nadir point. For the scaled MOPs, we adaptively update the weight vectors considering the geometry character of PF as proposed in Algorithm 3. Finally, the combined population are pruned via using fast R2-ranking mechanism in Algorithm 4. The components in CL-AR2-EMOA will be elaborated in the following paragraphs.

B. Weight Generator Based on Coulomb Law

We use the magnitude of the Coulomb force inspired from the Coulomb law among all like charges as the performance indicator to assess the distribution of charges in the quarter circle. As shown in Fig. 4, the angle θ_{ij} between the weight vector $\overrightarrow{\lambda_i}$ and $\overrightarrow{\lambda_j}$ is called the direction angle, ϕ is the angle between the weight vector $\overrightarrow{\lambda_i}$ and the corresponding axis. Uniformly director angle can obtain the uniform weight vector and then the approximate Pareto solutions will be uniformly along the Pareto front. The modified comprehensive magnitude of the Coulomb force is defined as followed:

$$|\overrightarrow{F_i}| = \sum_{i=1, j\neq i}^{n} \frac{1}{r_{ij}^2} \tag{8}$$

$$r_{ij} = \sqrt{2 - 2cos\theta_{ij}} \tag{9}$$

$$\theta_{ij} = arccos(\overrightarrow{\lambda_i}, \overrightarrow{\lambda_j}) \tag{10}$$

Where r_{ij} is the distance between the charge i and j, θ_{ij} is the direction angle between the weight vector $\overrightarrow{\lambda_i}$ and $\overrightarrow{\lambda_j}$.

How to implement weight vectors generation via using Coulomb law is proposed in Algorithm 2. Firstly, we randomly generate N initialization points *wts* in the quarter circle. Then, the loop is begin, we have calculated the electrostatic force among the charges using Equation(8),(9),(10). We delete the point whose $|\overrightarrow{F_i}|$ is greatest, and it will generate a new charge on the hypersphere randomly. Finally, if the loop is terminated, the *wts* will be returned as weight vectors.

C. Adaptive Weight Generator Strategy

The proposed CL-AR2-EMOA is expected to obtain a set of uniformly distributed Pareto optimal solutions. However, this happens only if the function values of all objectives can be easily normalized into the same range, e.g.,[0,1]. Unfortunately, there exist many different objectives which are scaled to different ranges, e.g., the WFG and SDLTZ problems. In this

Algorithm 2 Weight Vectors Generation

Require: *Iteration*:the maximum number of iteration
n, the number of weight vectors
objDim, the number of objectives
Ensure: *wts*: Weight Vectors
Randomly generate *wts* in the quarter circle
for $it = 1 : Iteration$ **do**
  Calculate the electrostatic force among the charges using Equation(8),(9),(10);
  Delete the charge in the *wts* whose electrostatic force is greatest;
  Randomly generate a new charge on the hypersphere;
end for
Return *wts*

case, uniformly distributed weight vectors will not produce uniformly distributed solutions, as shown in Fig.3.

Then, we propose to adapt the weight vectors according to the ranges of the objective values in the following manner:

$$\lambda_i = \lambda_{oi} * (z^{nad} - z^*) \tag{11}$$

where $i = 1, ..., N$,λ_i denotes the i-th weight vectors, λ_{oi} denotes the original uniformly weight vector, which is generated in the Algorithm 2, z^{nad} and z^* denote the maximum and minimum values of each objective function, respectively.

With the use of adaptive weight generator strategy described above, the proposed CL-AR2-EMOA will be able to obtain uniformly distributed solutions when the objective function is scaled.

Algorithm 3 Adaptive Weight Generator Strategy

Require: z^{nad}, the nadir point
z^*, the ideal point
wts, the original weight vectors
iter, the generation index
Ensure: *Awts*, the adaptive weight vectors
if $\frac{iter}{10} == 0$ **then**
  Obtain z^{nad} and z^*,respectively;
  $Awts = wts * (z^{nad} - z^*)$
end if
Return *Awts*

D. Fast R2-ranking Strategy

We intend to use R2 in the selection strategy of EMOAs. We use R2-ranking to select individuals other than fast non-dominated ranking relation in NSGA-II [2]. Firstly, we combine the parent and offspring together and calculate a set of scalarizing function values, and place such solutions on top such that they get the first rank. Then, such points will then be removed and a second rank will be identified in the same manner so on and so forth. The process will continue until all the solutions had been ranked. Algorithm 4 gives the pseudo-code of fast R2-ranking strategy using in our CL-AR2-EMOA.

Algorithm 4 Fast R2-Ranking

Require: P:the combined population
W: weight vectors
z^*: the ideal point
Ensure: $p.Rank$:Ranking of the P population
$\forall p \in P$ $p.Rank = p.Utility = inf$
for all $w \in W$ **do**
  for all $p \in P$ **do**
    $p.Utility = \max_{i \in \{1,...,m\}} \frac{1}{w_i} |f_i - z_i^*|$
  end for
  Sort P using Utility and L2 value in increasing order
  $rank = 1$
  for $p \in P$ **do**
    $p.Rank = \min\{p.Rank, Rank\}$
    $Rank = Rank + 1$
  end for
end for

IV. EXPERIMENTAL RESULTS

A. Benchmark Problems

To assess the convergence and diversity of the proposed algorithms CL-AR2-EMOA, the well-known test suites are involved for experiments. WFG1 to WFG9 are taken from WFG test suit [24] and DTLZ1 to DTLZ4 are taken from DTLZ test suit [25]. For each WFG instance, the number of objectives is set as $m = \{2, 3\}$ and the number of decision variables is set to $n = k + l$,where $k = 2 * (m - 1)$ and $l = 20$. For each DTLZ instance, the number of objectives is set as $m = \{3, 5, 8, 10\}$ and the number of decision variables is set to $n = m + k - 1$, where $k = 5$ for DTLZ1 and $k = 10$ for DTLZ2, DTLZ3 and DTLZ4. Moreover, we also introduce four scaled test problems, i.e., scaled DTLZ1 to DTLZ4 problems, which is the modification of DTLZ test suites. The scaling method is that each objective is multiplied by a coefficient p^{i-1}, where p is a parameter that controls the scaling size and i is the objective index. We use $p = 10$ for all of the SDTLZ test suites. We compare their performance with respect to other R2-MOEAs that include R2-MOGA, R2-MODE and MOMBI. The features of all test instances are summarized in Table I.

TABLE I. FEATURES OF BENCHMARK PROBLEMS

TEST PROBLEMS	FEATURES
DTLZ1/SDTLZ1	linear, multimodal
DTLZ2/SDTLZ2	concave
DTLZ3/SDTLZ3	concave, multimodal
DTLZ4/SDTLZ4	concave, biased
WFG1	mixed,biased
WFG2	convex, disconnected,multimodal,nonsepareble
WFG3	linear,degenerate,nonseparable
WFG4	concave,multimodal
WFG5	concave,deceptive
WFG6	concave,nonseparable
WFG7	concave,biased
WFG8	concave,biased,nonseparable
WFG9	concave,biased,multimodal,deceptive,nonseparable

B. Performance Metrics

In our experimental studies, three performance metrics, the generational distance (GD) [2] ,spacing metric(SP) [2] and the inverse generational distance (IGD) [26] are used.

(1)GD reports how far the approximate Pareto front is from the true Pareto front on average, which is considered for convergence aspect of performance.

$$GD = \frac{\sum_{i=1}^{N} d_i}{N} \tag{12}$$

where d_i is the Euclidean distance from solution i to the nearest solution in Pareto front, and N is the size of P. This measure is considered for convergence aspect of performance. Therefore, it could happen that the set of solutions is very close to Pareto front, but it does not cover the entire Pareto front.

(2)SP describes the spread of the objective vectors in the approximate Pareto front, which takes into account diversity aspect of performance.

$$SP = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\bar{d} - d_i)^2} \tag{13}$$

where d_i is the Euclidean distance from solution i to the nearest solution in P, $\bar{d}$ is the average of d_i, and N is the size of P.

(3)IGD reports how far the true Pareto front is from the approximate Pareto front, which is considered for both convergence and diversity.

$$IGD = \frac{\sum_{i=1}^{\overline{N}} \bar{d}_i}{\overline{N}} \tag{14}$$

where $\bar{d}_i$ is the Euclidean distance from solution i in true Pareto front to the nearest solution in P, $\overline{N}$ is the size of true Pareto front.

In the comparison tables in the following section, the best experimental results are denoted by the **bold** font.

C. Parameter Settings

For each MOP, the empirical results are the average values of 30 independent runs. SBX+PM ($p_c = 0.9$, $p_m = 1/n$, $\eta_c = 15$, $\eta_m = 20$) are used in R2-MOGA, CL-AR2-MOGA and MOMBI. In R2-MODE and CL-AR2-MODE, scaling factor is set to $F = 0.5$, crossover constant is set to $CR = 0.5$. The number of population is set to 91 for m=2, 210 for m=5, 156 for m=8 and 275 for m=10,the maximal number of generations for DTLZ1, DTLZ2 and DTLZ4 is set to 400 for m=2, 600 for m=5, 750 for m=8 and 1000 for m=10, while the maximal number of generations for DTLZ3 is set to 1000 for m=2, 5, 8 and 1500 for m=10. When we take WFG problems which have different features as shown in Table I into considering. The maximal number of generations for WFG is set to 250, and the number of population is set to 100 for bi-objective and 300 when the number of objective is 3.

D. Experimental Results on DTLZ Problems

Experimental results of our proposed CL-AR2-EMOAs and other three EMOAs in terms of GD,SP,IGD values are given in Table II,III and IV, respectively. The best values are highlighted by the **bold** font. From these comparison results, it is clear that CL-AR2-EMOAs are the best optimizer of almost all results. In the following discussion, we analyze all of the benchmark test suit instance by instance.

The PF of DTLZ1 is a line hyper-plane, where is many local optima in search space. CL-AR2-MOGA is able to find a well converged set of solution with 3-, 8- and 10-objective, and CL-AR2-MOGA has the best convergence performance with 5-objective from the experimental results. The distribution performance of CL-AR2-EMOA is significantly better than the other three R2-EMOAs as SP table is given. For the whole performance, the solution set obtained by CL-AR2-MODE performs better than CL-AR2-MOGA, R2-MOGA and R2-MODE with 5-, 8- and 10-objective, and CL-AR2-MOGA is best in 3-objective.

DTLZ2 is a relatively simple test problem, which PF shape is a quarter of the unit hypersphere. R2-MODE has achieved the best converged performance among algorithms with 8- and 10-objective, while CL-AR2-MODE and MOMBI outperform R2-MODE with 3- and 5-objective. MOMBI has better distribution performance than the other algorithm with 5- and 8-objective, while CL-AR2-MODE performs best with 3-objective and R2-MOGA perform best with 8 objectives. For both convergence and diversity, CL-AR2-MODE is best in the 5- ,8- and 10-objective case, while CL-AR2-MOGA is best in three objectives.

For DTLZ3, which is a highly multimodal problem, CL-AR2-MOGA obtains an approximate PF of high quality on 3-, 8-, and 10-objective, while R2-MODE performs better than the other algorithms on 5-objective. As can be observed from Table II, each MOP cannot converge to the global PF in every instance. When considering the diversity of DTLZ3, the distribution performance of CL-AR2-MOGA and CL-AR2-MODE is of high quality on 3- 8- and 10-objective while the distribution of R2-MODE is best on 5-objective. When comprehensively consider the convergence and diversity, our proposed algorithms CL-AR2-EMOAs have an excellent IGD performance metric when deal with this kind MOPs.

For DTLZ4, which is is a strong biased problem, CL-AR2-MODE shows the best overall performance when take into account convergence and distribution performance. The new adaptive weight vector generation based on Coulomb law and the modification scalarizing method not only improve the convergence and enhance the diversity of these kind MOPs. It is very interesting that R2-MODE obtains the excellent convergence when we use GD as an performance indicator. The reason is that the obtained PF of R2-MODE converge to the same point in the true PF. When we take SP into account, the CL-AR2-MOGA, R2-MOGA, R2-MODE and MOMBI get the best diversity when deal with DTLZ4 problems have different objectives.

From the Table II,III and IV. It can be seen that CL-AR2-EMOAs (CL-AR2-MOGA and CL-AR2-MODE) shows better overall performance than R2-EMOAs (R2-MOGA,R2-MODE and MOMBI)(9 out of 16 for GD, 9 out of 16 for SP, 16 out of 16 for IGD) for DTLZ instance.

Fig.5 and Fig.6 show the final Pareto front obtained by R2-EMOAs when deal with DTLZ2 with 3 and 10 objectives problems. Notice that some algorithms have similar character such as R2-MOGA,R2-MODE and MOMBI. From Fig.5, our proposed CL-AR2-EMOAs have an excellent convergence and diversity when deal with these problems in low-dimentional space. Fig.6 compares the Pareto front of DTLZ2 with 10 objectives when we use R2-MOGA, R2-MODE, MOMBI, CL-AR2-MOGA and CL-AR2-MODE. All of the R2-EMOAs have achieved an excellent convergence when we consider the GD performance indicator. However, CL-AR2-EMOAs obtain the best IGD value for DTLZ2 with 10 objectives. Fig. 10 compares the evolution of the median IGD metric value versus the number of function evaluations of five algorithms on DTLZ2 with 3 objectives. The property of these five algorithms is CL-AR2-MOGA, CL-AR2-MODE, MOMBI, R2-MOGA and R2-MODE. The figure of IGD plot is consistent with the experimental results we obtained. Clearly, these figures further show that the utilization of the proposed adaptive weight vectors generation strategy based on Coulomb law is effective when dealing with DTLZ test suit.

E. Experimental Results on WFG Problems

To observe the convergence and diversity of CL-AR2-EMOAs on problems having differently scaled objective values. WFG1-WFG9 are selected as the test problems to compare R2-MOGA, R2-MODE, MOMBI and CL-AR2-EMOAs for bi-objective WFG test problems. Table V, VI and VII illustrate that our proposed methods can well solve these scaled problems no matter the convergence and diversity. Considering the GD performance metrics, CL-AR2-MOGA and CL-AR2-MODE obtains 3 optimum problems compared with R2-MODE. Meanwhile, diversity is another important performance metric when compared with other algorithms. CL-AR2-MOGA and CL-AR2-MODE obtains 6 optimum problems compared with MOMBI, R2-MOGA and R2-MODE. The distribution of WFG1,WFG2 and WFG8 are better than our proposed CL-AR2-EMOAs. IGD can simultaneously measure the convergence and diversity of obtained solutions. The lower IGD value, the better quality of solutions for approximating the whole PF. The combination of CL-AR2-MOGA and CL-AR2-MODE gain 7 optimum problems while R2-MODE only work well for WFG6 and WFG8 problems.

Table VIII, IX and X show our algorithms better than other algorithm when the objective is three. This is similar to the experimental results in the bi-objective problems. When we considering the convergence of our proposed algorithms, GD performance metric can be used to compared with other algorithms. CL-AR2-MOGA obtains the best convergence when deal with WFG1,WFG4,WFG6,WFG9. MOMBI get the better GD when the problems are WFG2,WFG3,WFG7,WFG8 and R2-MODE can well solve the WFG5 problem. CL-AR2-MOGA obtains the best diversity when the geometry of Pareto front is concave which is similar to ellipse. Other algorithms obtain well diversity when deal with WFG1,WFG8,WFG2 and WFG3 problems. Still more interesting is that our proposed CL-AR2-MOGA achieve all best IGD when the objective is three of all the WFG test suites. From the empirical studies on WFG test suit, we conclude that the promising results obtain by CL-AR2-EMOAs should well solve MOPs with complicated PF which has 3 objectives. Fig.6 and 8 have already validated our conclusion when handle the WFG4 with bi-objective and WFG6 with tri-objectives. Fig. 10 compares the convergence characteristics of the IGD on WFG4-2 and WFG6-3 during the R2-MOGA, R2-MODE, MOMBI, CL-AR2-MOGA and CL-AR2-MODE search processes. The figures further show that the utilization of the adaptive weight vectors generation strategy based on Coulomb law improve the convergence and diversity for the algorithm to approximate the PF.

F. Experimental Results on SDTLZ Problems

The scaled test problems,i.e., scaled DTLZ1, DTLZ2,DLTZ3 and DTLZ4 problems, which are the modification of DTLZ test suites. All of the SDTLZ problems scaling factor is 10^{i-1}. In our experiments, we call scaled DTLZ1 to DTLZ4 problems are SDTLZ1 to SDTLZ4 for short. Table XI, XII and XIII have given the result of these benchmark problems. When considering GD and SP, CL-AR2-MOGA have obtain the best convergence and diversity when solve SDTLZ1 with three objectives. CL-AR2-MODE and R2-MODE obtains the best GD and SP for high dimensional SDTLZ1 problems. When the IGD is used, CL-AR2-MODE gets an excellent performance. SDTLZ2 is easy to cope. R2-MODE obtains the best GD when the objective is larger than 5. MOMBI have the best diversity no matter the dimension is low or high. It is interesting that when we use IGD to examine the convergence and diversity together, our proposed CL-AR2-MODE is the best of all compared algorithms. Fig.9 and 10 provide the Pareto front of SDTLZ2 with 3 objectives and the evolution of the median IGD values versus the number of generations. The graphical conclusion is fully consistent with the table experimental results. For SDTLZ3 test problems, the reproduction operator based on $SBX + PM$ can well deal with these problems. Our proposed CL-AR2-MOGA has achieved satisfactory results for most of SDLTZ3 problems. For SDTLZ4 problem, R2-MODE and MOMBI are better than our proposed CL-AR2-EMOAs when the GD and SP is introduced for comparison. However, when we used GD and SP, these performance indicator only analyze one aspect of the EMOAs. Then, IGD is the most important when we make comparison. Our proposed CL-AR2-MODE achieves the best effect of all test suites except SDTLZ3 with 5 objectives and SDTLZ4 with 3 objectives. The total optimal is 14 compared with MOMBI and R2-MODE only obtains one respectively. Experiment results show that the proposed CL-AR2-EMOAs have an excellent convergence and diversity.

V. Conclusion

In this paper, we have suggested an adaptive weight vectors generation strategy based on Coulomb law. Combining

TABLE II. Mean And Standard Deviation GD Value On DTLZ Instances

GD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
DTLZ1_3	Mean	0.117748984	0.169382583	0.163707415	**0.00688418**	0.069462086
	Std	0.584140799	0.204911579	0.36874659	**0.0039359**	0.143802538
DTLZ1_5	Mean	1.591030605	0.095720877	0.632397243	0.044955274	**0.04422191**
	Std	2.123876093	0.10503773	0.919629605	0.001115439	**0.00088183**
DTLZ1_8	Mean	1.475120925	0.119316396	0.270396753	**0.08779475**	0.098133751
	Std	1.982931026	0.073599426	0.372396205	**0.0037519**	0.002322448
DTLZ1_10	Mean	0.901461048	0.120878883	0.543382294	**0.09944517**	0.109219349
	Std	0.864469268	0.014479599	0.599754535	**0.00331079**	0.002956918
DTLZ2_3	Mean	0.007264369	0.076897644	0.00630646	0.005966596	**0.00595026**
	Std	0.000450587	0.0032716	5.59E-04	0.000215565	**0.00020996**
DTLZ2_5	Mean	0.029020691	0.028085915	**0.02082606**	0.128668035	0.127074745
	Std	0.001307008	0.003531959	**0.00240212**	0.002750649	0.002430453
DTLZ2_8	Mean	0.028087229	**0.011411148**	0.023076928	0.248648623	0.264378264
	Std	0.005415042	**0.002527377**	0.006319946	0.008712022	0.008262743
DTLZ2_10	Mean	0.011721216	**0.002909001**	0.014396517	0.293358407	0.318153753
	Std	0.002839132	**0.000744203**	0.006001567	0.005976921	0.007232507
DTLZ3_3	Mean	0.085976095	10.78714328	0.024639444	**0.01290876**	14.10146222
	Std	0.294618592	5.790274961	0.035554414	**0.00291979**	7.829611753
DTLZ3_5	Mean	0.341613679	**0.027720655**	0.12391699	0.126780246	0.446320283
	Std	0.474431715	**0.003340709**	0.23279484	0.003192668	0.561712057
DTLZ3_8	Mean	8.861551037	13.47867535	4.524638906	**0.2266936**	0.369618382
	Std	5.53979643	9.963873762	5.819318584	**0.00758384**	0.28300629
DTLZ3_10	Mean	2.218713883	88.7	0.835698058	**0.27921404**	0.33253472
	Std	2.489833956	12.1	1.137891881	**0.01008199**	0.005813696
DTLZ4_3	Mean	0.035181101	0.047346943	0.02830356	**0.02798448**	0.037717599
	Std	0.015164868	0.001549981	0.01476531	**0.01333184**	0.001286297
DTLZ4_5	Mean	0.036031349	**0.026064508**	0.12391699	0.123236654	0.123951662
	Std	0.015350315	**0.003246278**	0.015784318	0.005994838	0.002828698
DTLZ4_8	Mean	0.030692961	**0.009673229**	0.038162854	0.25604791	0.221468339
	Std	0.006621436	**0.002396631**	0.012386278	0.010353082	0.006864355
DTLZ4_10	Mean	6.549549069	**0.002333549**	0.016430909	0.328101129	0.260731698
	Std	6.665387176	**0.001337705**	0.004562748	0.007892374	0.007115335

TABLE III. Mean And Standard Deviation SP Value On DTLZ Instances

SP		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
DTLZ1_3	Mean	0.165236155	0.027222111	1.505309456	0.024814929	**0.01087138**
	Std	0.545113474	0.010856924	3.499205825	0.030363285	**0.00449513**
DTLZ1_5	Mean	2.858469939	0.030618254	2.477242994	0.02077216	**0.01476081**
	Std	2.820867976	0.012183079	2.850776253	0.008013325	**0.00109427**
DTLZ1_8	Mean	3.314548399	0.053570597	0.831401784	**0.03192399**	0.045319023
	Std	3.799429414	0.014457787	1.732859153	**0.00398025**	0.004085544
DTLZ1_10	Mean	1.738421222	0.05519875	1.868614594	**0.02789948**	0.050187102
	Std	1.785348716	0.008406557	2.109070807	**0.00380214**	0.002663325
DTLZ2_3	Mean	0.050346033	0.058666557	0.051972588	0.025216476	**0.02349976**
	Std	0.002192641	0.003168179	0.003449871	0.002332973	**0.0024261**
DTLZ2_5	Mean	0.010087657	0.033858297	**0.00910073**	0.058169816	0.049805339
	Std	0.014602191	0.029891252	**0.01283704**	0.003272688	0.002965487
DTLZ2_8	Mean	**0.009130083**	0.047322984	0.009244117	0.116917215	0.098602735
	Std	**0.009114358**	0.035800485	0.01405777	0.009673822	0.00852909
DTLZ2_10	Mean	0.008670117	0.067312587	**0.00393317**	0.123800067	0.122512039
	Std	0.015794362	0.037038566	**0.00298585**	0.009395878	0.007205863
DTLZ3_3	Mean	0.72005521	0.633635876	0.155880699	**0.03026769**	0.335932774
	Std	2.805800974	0.314207472	0.329345127	**0.00486192**	0.178608833
DTLZ3_5	Mean	1.574044146	**0.025305879**	0.789175084	0.064506725	0.064334455
	Std	2.834303481	**0.028964603**	2.171319543	0.005569089	0.027461973
DTLZ3_8	Mean	7.329940895	22.28004236	4.011226373	0.108583245	**0.10120069**
	Std	6.564901094	23.40404748	6.620718739	0.015026345	**0.03192502**
DTLZ3_10	Mean	5.406344642	52.71226514	3.613974048	**0.09195803**	0.102653528
	Std	5.218279457	18.17158127	7.212852726	**0.0101726**	0.006662965
DTLZ4_3	Mean	0.043076455	0.05363115	0.040145171	**0.02428887**	0.026391251
	Std	0.020304607	0.001857851	0.027456911	**0.01779901**	0.00330029
DTLZ4_5	Mean	**0.018048665**	0.032768419	0.019019633	0.058612044	0.055783806
	Std	**0.020323177**	0.020538709	0.017373042	0.006873763	0.003512905
DTLZ4_8	Mean	0.049893781	**0.028579833**	0.030390619	0.101525982	0.10117667
	Std	0.030324851	**0.028058139**	0.027405691	0.010266026	0.008052423
DTLZ4_10	Mean	4.085914583	0.048290739	**0.04543766**	0.105636407	0.123580859
	Std	4.781041521	0.040165943	**0.02759686**	0.007561329	0.006479323

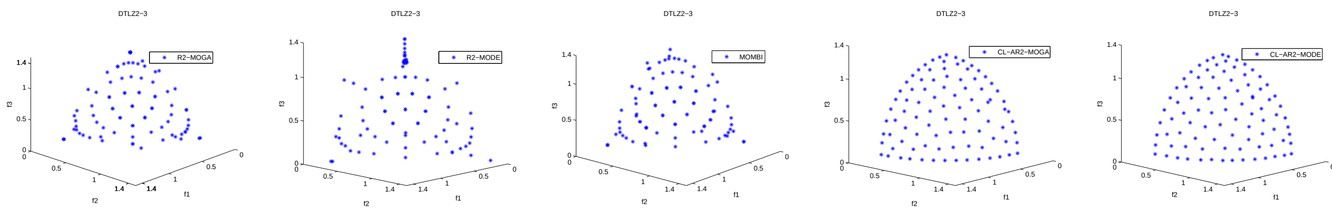

Fig. 5. Final nondominated solutions of the DTLZ2-3 problem using five algorithms

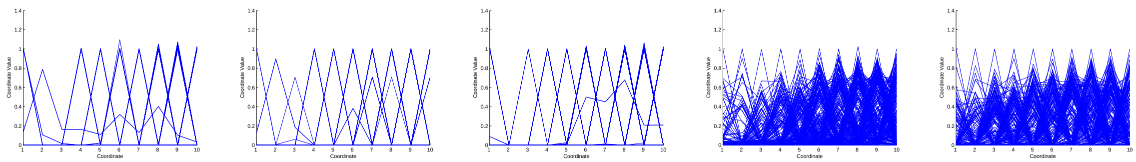

Fig. 6. Final nondominated solutions of the DTLZ2-10 problem using five algorithms

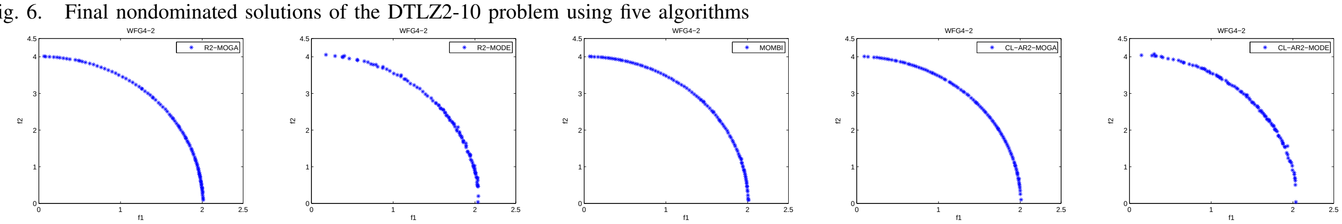

Fig. 7. Final nondominated solutions of the WFG4-2 problem using five algorithms

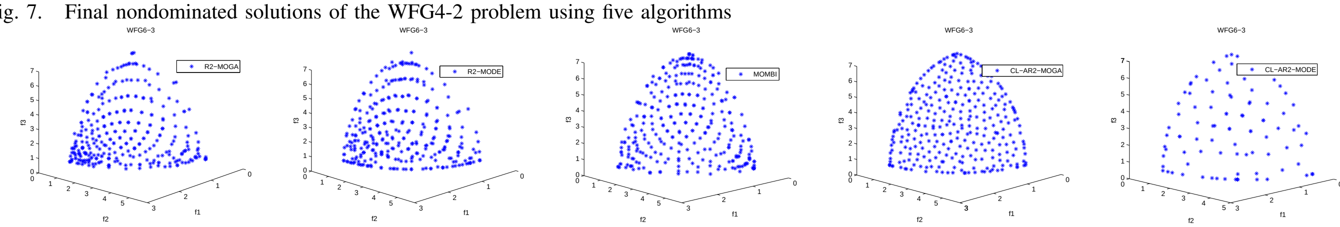

Fig. 8. Final nondominated solutions of the WFG6-3 problem using five algorithms

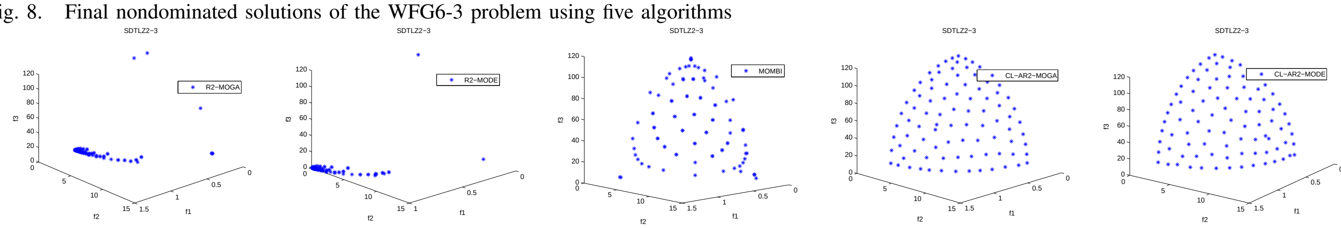

Fig. 9. Final nondominated solutions of the SDTLZ2-3 problem using five algorithms

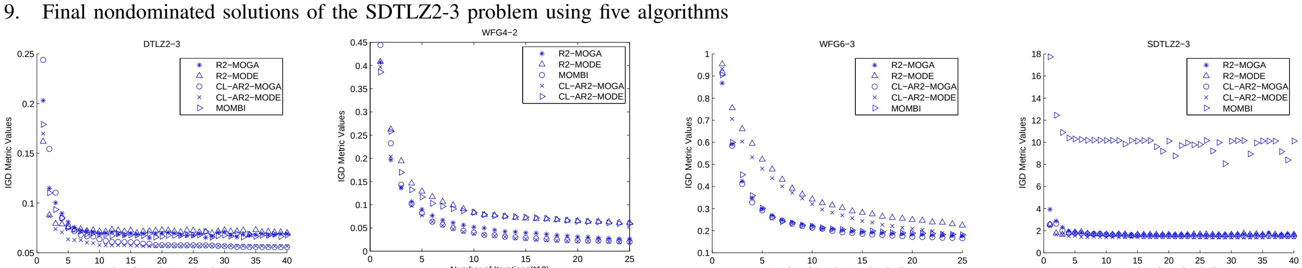

Fig. 10. Evolution of the median IGD metric values versus the number of generations

TABLE IV. MEAN AND STANDARD DEVIATION IGD VALUE ON DTLZ INSTANCES

IGD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
DTLZ1_3	Mean	0.029674424	0.171347419	0.028852928	**0.02671076**	0.076819591
	Std	0.001565434	0.178469554	0.002123327	**0.02215691**	0.118794384
DTLZ1_5	Mean	0.157267799	0.120010789	1.67E-01	0.052807353	**0.05013222**
	Std	0.007373341	0.0697386	0.012991375	0.001181535	**0.00091909**
DTLZ1_8	Mean	0.205526332	0.160008122	0.262498786	0.176570542	**0.11449306**
	Std	0.011132027	0.038410173	0.015355479	0.026986451	**0.00532135**
DTLZ1_10	Mean	0.242364298	0.187335721	0.269399382	0.21312211	**0.15048694**
	Std	0.013900064	0.010664707	0.009106698	0.017507831	**0.00592531**
DTLZ2_3	Mean	0.0691518	0.108735215	0.068955542	**0.0560455**	0.056376669
	Std	0.001293052	0.001097245	2.26E-03	**0.0005998**	0.000733221
DTLZ2_5	Mean	0.451525776	0.437140599	0.439466647	0.14975778	**0.14357035**
	Std	0.007161714	0.030021685	0.006855547	0.00359089	**0.00309357**
DTLZ2_8	Mean	0.574487427	0.572692718	0.594917324	0.289859566	**0.26190908**
	Std	0.006011978	0.011597668	0.021294374	0.034949979	**0.00759561**
DTLZ2_10	Mean	0.678665074	0.669635557	0.697441617	0.395064081	**0.35380906**
	Std	0.015146002	0.02136682	0.009880522	0.032536046	**0.00858173**
DTLZ3_3	Mean	0.071644007	10.79025575	0.071348325	**0.05892939**	14.10344973
	Std	0.001433866	5.790251908	2.65E-03	**0.00141443**	7.82950003
DTLZ3_5	Mean	0.450742527	0.441680482	0.434365588	**0.1529606**	0.458823921
	Std	0.009500372	0.025596545	0.010048141	**0.00397354**	0.557805584
DTLZ3_8	Mean	2.164725722	2.774332181	1.259180094	0.378380916	**0.36378663**
	Std	1.65405199	1.636219735	1.034349029	0.103483803	**0.28594383**
DTLZ3_10	Mean	0.700906472	9.800962548	0.733821334	0.564631373	**0.33907**
	Std	0.043277198	2.248704242	0.028762742	0.062174032	**0.00632827**
DTLZ4_3	Mean	0.128709775	0.076416625	1.64E-01	0.135719195	**0.03399317**
	Std	0.114543242	0.003706098	1.34E-01	0.149026651	**0.00174891**
DTLZ4_5	Mean	0.465614955	0.439168322	0.465023798	0.166875891	**0.14587478**
	Std	0.06218976	0.018303173	0.04244007	0.059791902	**0.00320052**
DTLZ4_8	Mean	0.600910258	0.578051937	0.628262904	0.281275421	**0.22243925**
	Std	0.032349937	0.007350519	0.038389934	0.040164888	**0.00618373**
DTLZ4_10	Mean	1.80323082	0.67306405	6.89E-01	0.341950287	**0.28613219**
	Std	1.790758411	0.019202061	0.032036967	0.009597567	**0.00581614**

TABLE V. MEAN AND STANDARD DEVIATION GD VALUE ON WFG*_2 INSTANCES

GD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_2	Mean	1.034620351	1.139417811	**1.02842168**	1.035708374	1.123507014
	Std	0.020401337	0.010602944	**0.01950996**	0.013459478	0.007903104
WFG2_2	Mean	0.034067505	0.031333936	0.036600256	**0.02960377**	0.032953138
	Std	0.013258349	0.003127071	0.012977413	**0.00894172**	0.003880068
WFG3_2	Mean	0.028690343	0.012439219	0.029403405	0.031543835	**0.01227946**
	Std	0.009974511	7.11E-04	0.010326361	1.10E-02	**6.95E-04**
WFG4_2	Mean	0.019072588	0.061082193	**0.01585814**	0.016627046	0.059782135
	Std	0.00107769	0.002604051	**0.00211257**	0.001996209	0.00236514
WFG5_2	Mean	0.06289056	**0.061922081**	0.066782358	0.065775776	0.064284734
	Std	6.15E-04	**8.23E-04**	5.31E-04	5.98E-04	5.70E-04
WFG6_2	Mean	0.062673922	**0.060946731**	0.069234933	0.062901861	0.062819373
	Std	0.008718675	**0.022173107**	0.00921498	0.011243771	0.02175298
WFG7_2	Mean	0.008776649	**0.007353002**	0.008303488	0.008905835	0.007667758
	Std	0.001003617	**5.51E-04**	0.001131756	1.04E-03	2.28E-03
WFG8_2	Mean	0.629167788	0.525686003	0.535902444	0.571427491	**0.49930806**
	Std	0.044818199	0.011053819	0.074340719	0.059236384	**0.01418378**
WFG9_2	Mean	**0.01653277**	0.021107908	0.021075816	0.018018843	0.022075112
	Std	**0.01653277**	9.85E-04	0.004065914	3.90E-03	8.84E-04

it with the fast R2-ranking strategy, CL-AR2-EMOA(CL-AR2-MOGA and CL-AR2-MODE) are proposed for solving multi-objective optimization problems. The proposed CL-AR2-EMOAs has been used to solve 2-10 objectives benchmark problems. The Pareto front of these problems include convex,concave,scaled objective values,biased density of solutions over the front. In all such problems, the proposed CL-AR2-EMOA approach has been able to successfully find a set of well-converged and distributed solutions. The performance of CL-AR2-EMOA has also been compared with R2-MOGA, R2-MODE and MOMBI of a recently proposed algorithm based on R2 indicator. The experimental results show that CL-AR2-EMOA is better than the compared algorithms. In the future, firstly, in order to obtain evenly distributed solutions based on different problem geometry , we want to dynamically generate weight vectors for MaOPs with the evolving population. The related work can be found in Wang, Purshouse and Fleming [27], Cheng and Jin [28]. Secondly, another direct use of our proposed CL-AR2-EMOA would be for robust multi-objective optimization problems [29] , where the problem can be formulated as MaOPs. These directions are currently being explored by the authors.

TABLE VI. MEAN AND STANDARD DEVIATION SP VALUE ON WFG*_2 INSTANCES

SP		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_2	Mean	**0.005961419**	0.051262772	0.006716611	0.006720891	0.052332308
	Std	**0.001312682**	0.011656362	0.001161388	1.42E-03	6.39E-03
WFG2_2	Mean	0.030460534	0.043830053	**0.02132502**	0.038184748	0.030925198
	Std	0.010720902	0.011961506	**0.00480403**	1.07E-02	7.93E-03
WFG3_2	Mean	0.018200776	0.01883913	0.016282692	0.013503705	**0.01208763**
	Std	0.002664829	0.001623144	0.002636585	2.10E-03	**1.24E-03**
WFG4_2	Mean	0.019957496	0.022058363	0.023028234	**0.01642867**	0.028032167
	Std	0.001830116	0.002638281	0.002913427	**3.21E-03**	8.71E-03
WFG5_2	Mean	0.019173631	0.019206922	0.027702721	**0.011652**	0.013250569
	Std	0.00117183	0.001003289	0.002231588	**2.52E-03**	3.70E-03
WFG6_2	Mean	0.020177961	0.023038456	0.027015926	**0.01381084**	0.020394047
	Std	0.001264307	0.002981957	0.002499934	**3.48E-03**	5.35E-03
WFG7_2	Mean	0.020177961	0.021385333	0.014599708	**0.01348276**	0.01624338
	Std	0.001264307	0.003556475	0.005888779	**4.70E-03**	3.36E-03
WFG8_2	Mean	0.023929291	**0.022737886**	0.02675811	0.024822643	0.029015187
	Std	0.00254671	**0.005473044**	0.00397702	3.39E-03	7.81E-03
WFG9_2	Mean	0.017833605	0.018873386	0.017790397	0.01178769	**0.01168668**
	Std	0.002148996	9.68E-04	0.002472452	2.39E-03	**1.66E-03**

TABLE VII. MEAN AND STANDARD DEVIATION IGD VALUE ON WFG*_2 INSTANCES

IGD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_2	Mean	1.899107475	1.118926597	1.919885544	1.862585482	**1.11101205**
	Std	0.060455965	0.00943023	0.055350062	0.071524275	**0.00861711**
WFG2_2	Mean	0.191791061	0.055083008	0.102872204	0.098034389	**0.04561158**
	Std	0.059156763	0.01159327	0.040487388	0.010618909	**0.00829643**
WFG3_2	Mean	0.16990757	0.144554178	0.190475345	0.166980555	**0.14306047**
	Std	0.01024185	0.001098152	0.019956464	0.011353849	**0.00061732**
WFG4_2	Mean	0.021444008	0.062380219	0.019029169	**0.01845234**	0.059940071
	Std	0.001167465	0.002487461	0.001751957	**0.00164434**	0.002289468
WFG5_2	Mean	0.06963307	0.067520312	0.070171861	0.068979856	**0.06701195**
	Std	6.51E-04	6.55E-04	6.98E-04	0.00058664	**0.00023459**
WFG6_2	Mean	0.068350245	**0.052358107**	0.074209665	0.065470924	0.063329831
	Std	0.009392647	**0.021125585**	0.009369938	0.010759292	0.020092247
WFG7_2	Mean	0.05296884	0.017263901	0.147374681	0.071146906	**0.01609698**
	Std	0.08086147	0.009382565	0.273193291	0.118677105	**0.00919356**
WFG8_2	Mean	0.536599248	**0.418977082**	0.485557535	0.482171961	0.423096393
	Std	0.066373727	**0.011040482**	0.07646948	0.060279543	0.010038631
WFG9_2	Mean	0.026310907	0.024255132	0.021984093	**0.01941635**	0.022739891
	Std	0.02791704	5.08E-04	0.00383975	**0.0033781**	0.00062494

TABLE VIII. MEAN AND STANDARD DEVIATION GD VALUE ON WFG*_3 INSTANCES

GD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_3	Mean	1.141070639	1.342174899	1.074863077	**1.00772137**	1.302023236
	Std	0.011285194	0.00533708	0.022915994	**0.00992851**	0.010807162
WFG2_3	Mean	0.102075667	0.110274156	**0.08932277**	0.098124989	0.108005261
	Std	0.0083828	0.004700628	**0.00790675**	0.007766876	0.012656169
WFG3_3	Mean	0.472002507	0.604662351	**0.10512527**	0.19645918	0.296510085
	Std	0.022608692	1.76E-02	**0.01436739**	0.017424314	0.044757787
WFG4_3	Mean	0.101284705	0.13138814	0.073162868	**0.07169388**	0.124287254
	Std	0.003732986	0.002762732	0.002922843	**0.00275075**	0.003636678
WFG5_3	Mean	0.085448586	**0.082990174**	0.094114154	0.085060976	0.085491927
	Std	1.38E-03	**1.35E-03**	0.00279641	0.000943791	0.003434668
WFG6_3	Mean	0.090951808	0.144421038	0.100747856	**0.08271663**	0.147208248
	Std	0.007657654	0.021049482	0.011129792	**0.00665639**	0.036908098
WFG7_3	Mean	0.064578038	0.072829054	**0.03777235**	0.042706978	0.065908748
	Std	0.019240696	3.07E-03	**0.00682917**	0.005676601	0.004058642
WFG8_3	Mean	0.678208409	7.13E-01	**0.43393596**	0.520177894	0.628413949
	Std	0.030631878	0.014694769	**0.02478536**	0.021045251	0.023011578
WFG9_3	Mean	0.046043596	0.048867476	0.038057038	**0.03699672**	0.051280823
	Std	0.003300374	1.07E-03	0.004399616	**0.00210408**	0.002252921

TABLE IX. Mean And Standard Deviation SP Value On WFG*_3 Instances

SP		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_3	Mean	0.041379839	0.121818074	**0.03037807**	0.049813749	0.228188921
	Std	0.00616461	0.057307703	**0.00532107**	0.009324437	0.032603651
WFG2_3	Mean	**0.066385888**	0.115292635	0.070257058	0.099844278	0.189886694
	Std	**0.009263076**	0.046415796	0.015746037	0.00810212	0.038084191
WFG3_3	Mean	0.07036109	**0.058451077**	0.06726395	0.069966457	0.108263373
	Std	0.005850054	**6.68E-03**	0.005991686	0.004792366	0.011104932
WFG4_3	Mean	0.109158879	0.093485857	0.116027341	**0.08184991**	0.204800864
	Std	0.003005408	0.004284555	0.005252862	**0.00423952**	0.014115825
WFG5_3	Mean	0.112282221	0.0988621	0.119182064	**0.08053648**	0.187073762
	Std	3.69E-03	3.85E-03	0.005224613	**0.00649017**	0.011456222
WFG6_3	Mean	0.109753096	0.092854891	0.115734828	**0.08219058**	0.195832506
	Std	0.003516131	0.004803291	0.004074536	**0.00486017**	0.013734876
WFG7_3	Mean	0.10883269	0.101449904	0.095499082	**0.07897691**	0.167842491
	Std	0.005853561	3.59E-03	0.017607445	**0.00326404**	0.013444921
WFG8_3	Mean	0.067013983	0.083152698	**0.04337006**	0.113488373	0.216313154
	Std	0.016022763	0.005129586	**0.01768868**	0.007565138	0.012280167
WFG9_3	Mean	0.106715791	0.100771812	0.119210753	**0.08965194**	0.18945569
	Std	0.003819207	2.73E-03	0.00976328	**0.00379339**	0.012131678

TABLE X. Mean And Standard Deviation IGD Value On WFG*_3 Instances

IGD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
WFG1_3	Mean	1.324603164	1.41728964	1.489249118	**1.17331121**	1.366129379
	Std	0.087577802	0.009052665	0.068743322	**0.08592241**	0.013100389
WFG2_3	Mean	0.340819037	0.375055688	1.63E-01	**0.15048918**	0.225191812
	Std	0.071142354	0.066086181	0.027897419	**0.02118668**	0.027719166
WFG3_3	Mean	0.095573351	0.18207178	0.044064549	**0.02426461**	0.124546513
	Std	0.021153781	8.93E-03	0.005798649	**0.00251633**	0.013112735
WFG4_3	Mean	0.193243533	0.225015502	0.11039501	**0.10535563**	0.240123955
	Std	0.007741646	0.004894155	0.002218169	**0.00224121**	0.011408403
WFG5_3	Mean	0.195317256	0.187273697	0.183542786	**0.14374707**	0.239857129
	Std	3.68E-03	1.96E-03	3.33E-03	**0.0010328**	0.003670117
WFG6_3	Mean	0.205626445	0.237011002	0.180156015	**0.14294617**	0.286780876
	Std	0.005848397	0.014390687	0.008549121	**0.00493807**	0.025745962
WFG7_3	Mean	0.194341403	0.15822338	0.267262086	**0.12438282**	0.209888642
	Std	0.091960522	2.69E-03	0.076546744	**0.06778238**	0.002799048
WFG8_3	Mean	0.618194866	0.613218957	0.588338308	**0.50877228**	0.625370918
	Std	0.029336556	0.014869533	0.016784923	**0.01865461**	0.012253929
WFG9_3	Mean	0.173359387	0.176470916	0.179198901	**0.11384697**	0.216430586
	Std	0.002368295	2.07E-03	0.044516627	**0.00149969**	0.003834993

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TABLE XI. Mean And Standard Deviation GD Value On SDTLZ Instances

GD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
SDTLZ1_3	Mean	1.03E+01	5.68E+00	4.621303458	**1.79E-01**	1.39E+00
	Std	2.78E+01	9.44E+00	18.28818245	**2.74E-02**	2.85E+00
SDTLZ1_5	Mean	6.12E+03	2.42E+02	3.05E+03	1.70E+02	**1.59E+02**
	Std	1.03E+04	2.23E+02	5.64E+03	8.85E+00	**2.48E+01**
SDTLZ1_8	Mean	3.74E+06	4.20E+05	8.26E+05	2.89E+05	**2.34E+05**
	Std	3.57E+06	2.31E+05	1.55E+06	2.56E+04	**1.59E+04**
SDTLZ1_10	Mean	3.17E+08	3.49E+07	1.40E+08	3.05E+07	**2.63E+07**
	Std	4.07E+08	6.57E+06	2.46E+08	1.17E+06	**1.37E+06**
SDTLZ2_3	Mean	3.05E-01	2.59E-01	0.091118611	2.73E-01	**2.53E-01**
	Std	4.57E-02	2.44E-02	0.034395982	1.93E-02	**2.01E-02**
SDTLZ2_5	Mean	2.66E+01	1.60E+01	**8.84E+00**	2.25E+02	1.91E+02
	Std	8.16E+00	5.52E+00	**3.51224212**	1.48E+01	1.12E+01
SDTLZ2_8	Mean	7.07E+04	**2.70E+04**	6.72E+04	6.09E+05	4.59E+05
	Std	1.75E+04	**2.08E+04**	1.65E+04	4.64E+04	4.09E+04
SDTLZ2_10	Mean	5.03E+06	**2.78E+05**	3.33E+06	7.16E+07	5.14E+07
	Std	1.63E+06	**8.26E+04**	9.41E+05	2.93E+06	3.91E+06
SDTLZ3_3	Mean	7.48E+00	9.23E+02	2.337388774	**3.08E-01**	4.73E+02
	Std	3.35E+01	5.61E+02	6.652527364	**2.10E-02**	3.28E+02
SDTLZ3_5	Mean	1.52E+03	8.93E+02	3.26E+03	**2.14E+02**	2.57E+03
	Std	2.38E+03	2.36E+03	6.18E+03	**1.46E+01**	4.96E+03
SDTLZ3_8	Mean	2.26E+07	6.43E+07	3.01E+06	**5.77E+05**	6.03E+05
	Std	2.21E+07	4.87E+07	6.05E+06	**5.75E+04**	6.96E+05
SDTLZ3_10	Mean	7.47E+08	4.04E+10	1.97E+08	7.04E+07	**4.97E+07**
	Std	1.27E+09	1.05E+10	4.44E+08	3.38E+06	**1.79E+07**
SDTLZ4_3	Mean	7.64E-01	9.05E-01	0.937469958	**6.63E-01**	8.93E-01
	Std	3.80E-01	1.64E-01	0.235721274	**3.64E-01**	5.45E-02
SDTLZ4_5	Mean	2.90E+01	2.05E+01	**9.74E+00**	1.65E+02	2.01E+02
	Std	1.19E+01	9.96E+00	**14.8350135**	4.17E+01	1.35E+01
SDTLZ4_8	Mean	5.03E+04	**1.56E+04**	3.01E+06	4.36E+05	4.37E+05
	Std	4.81E+04	**1.20E+04**	3.72E+04	9.42E+04	5.78E+04
SDTLZ4_10	Mean	2.26E+06	**1.33E+05**	1.55E+05	4.73E+07	3.97E+07
	Std	1.69E+06	**1.66E+05**	4.91E+05	4.65E+06	4.45E+06

TABLE XII. Mean And Standard Deviation SP Value On SDTLZ Instances

SP		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
SDTLZ1_3	Mean	6.44E+01	1.12E+00	40.39417202	**5.47E-01**	6.52E-01
	Std	2.22E+02	6.01E-01	1.75E+02	**1.79E-01**	2.33E-01
SDTLZ1_5	Mean	1.01E+04	**4.89E+01**	7.12E+03	6.10E+01	7.77E+01
	Std	1.73E+04	**3.45E+01**	1.57E+04	1.26E+01	2.00E+01
SDTLZ1_8	Mean	1.85E+07	**7.18E+04**	3.50E+06	7.67E+04	1.43E+05
	Std	2.88E+07	**4.58E+04**	1.24E+07	1.72E+04	1.81E+04
SDTLZ1_10	Mean	1.10E+09	**5.13E+06**	5.61E+08	5.49E+06	1.17E+07
	Std	1.67E+09	**2.11E+06**	1.22E+09	2.28E+06	2.06E+06
SDTLZ2_3	Mean	1.26E+00	1.36E+00	**0.7526346**	1.06E+00	1.08E+00
	Std	1.08E-01	1.29E-01	**0.51329353**	9.64E-02	9.03E-02
SDTLZ2_5	Mean	7.00E+00	3.99E+01	**4.31111196**	7.49E+01	8.23E+01
	Std	6.43E+00	5.49E+01	**4.37264771**	5.02E+00	4.46E+00
SDTLZ2_8	Mean	9.07E+03	3.18E+04	**4.31111196**	9.53E+04	1.15E+05
	Std	1.03E+04	5.49E+04	**4.37264771**	9.29E+03	1.10E+04
SDTLZ2_10	Mean	1.23E+06	5.44E+06	**8.76E+05**	6.38E+06	8.97E+06
	Std	2.02E+06	7.75E+06	**2.23E+06**	4.07E+05	9.26E+05
SDTLZ3_3	Mean	6.81E+01	2.45E+01	19.54708643	**1.11E+00**	1.12E+01
	Std	3.20E+02	1.37E+01	63.34782705	**1.04E-01**	6.37E+00
SDTLZ3_5	Mean	1.60E+04	**3.46E+01**	1.69E+04	7.64E+01	1.50E+02
	Std	3.14E+04	**4.36E+01**	3.88E+04	5.35E+00	1.31E+02
SDTLZ3_8	Mean	3.74E+07	1.38E+08	1.88E+07	**9.45E+04**	1.38E+05
	Std	4.67E+07	2.06E+08	4.67E+07	**1.04E+04**	5.08E+04
SDTLZ3_10	Mean	2.58E+09	1.63E+10	6.08E+08	**6.04E+06**	1.10E+07
	Std	5.38E+09	1.06E+10	1.35E+09	**4.82E+05**	2.11E+06
SDTLZ4_3	Mean	9.86E-01	1.41E+00	3.319111887	**7.53E-01**	1.01E+00
	Std	5.24E-01	1.68E-01	4.011486209	**4.24E-01**	7.91E-02
SDTLZ4_5	Mean	7.28E+00	**6.37E+00**	16.43704939	9.98E+01	8.33E+01
	Std	1.78E+01	**1.29E+01**	45.68959737	8.41E+01	4.45E+00
SDTLZ4_8	Mean	3.86E+04	**1.04E+04**	1.76E+04	1.22E+05	1.24E+05
	Std	5.98E+04	**2.81E+04**	5.74E+04	3.48E+04	1.61E+04
SDTLZ4_10	Mean	5.00E+06	7.02E+05	**6.06E+05**	1.27E+07	1.18E+07
	Std	6.95E+06	2.14E+06	**2.73E+06**	3.15E+06	2.17E+06

TABLE XIII. MEAN AND STANDARD DEVIATION IGD VALUE ON SDTLZ INSTANCES

IGD		R2-MOGA	R2-MODE	MOMBI	CL-AR2-MOGA	CL-AR2-MODE
SDTLZ1_3	Mean	1.07E+00	1.60E+00	4.617601805	2.28E+00	**7.18E-01**
	Std	1.71E-01	1.24E+00	0.793630308	6.05E-01	**3.94E-02**
SDTLZ1_5	Mean	3.39E+02	6.68E+01	7.16E+02	2.28E+02	**3.64E+01**
	Std	4.41E+01	2.74E+01	84.64980883	4.69E+01	**6.32E+00**
SDTLZ1_8	Mean	1.64E+05	4.75E+04	3.68E+05	3.30E+05	**2.93E+04**
	Std	5.67E+04	1.89E+04	6.13E+04	7.39E+04	**6.95E+03**
SDTLZ1_10	Mean	2.15E+07	2.81E+06	3.63E+07	3.34E+07	**1.91E+06**
	Std	8.65E+06	7.73E+05	2.86E+06	9.23E+06	**3.41E+05**
SDTLZ2_3	Mean	1.63E+00	1.62E+00	10.16314823	1.61E+00	**1.39E+00**
	Std	2.92E-02	3.82E-02	1.295253686	3.03E-02	**5.26E-02**
SDTLZ2_5	Mean	5.51E+02	6.01E+02	1.70E+03	3.79E+02	**8.98E+01**
	Std	3.89E+01	2.37E+02	17.23873984	6.28E+01	**7.90E+00**
SDTLZ2_8	Mean	3.98E+05	5.18E+05	7.21E+05	6.57E+05	**6.17E+04**
	Std	7.26E+04	1.64E+05	8.28E+04	6.55E+05	**1.36E+04**
SDTLZ2_10	Mean	7.12E+07	6.91E+07	7.76E+07	4.67E+07	**4.08E+06**
	Std	1.29E+07	2.57E+07	3.79E+06	2.23E+07	**6.99E+05**
SDTLZ3_3	Mean	1.78E+00	5.45E+01	12.47303179	6.68E+00	**1.55E+00**
	Std	6.72E-02	3.12E+01	2.38E+00	2.28E+00	**4.61E-02**
SDTLZ3_5	Mean	6.45E+02	7.16E+02	**4.37264771**	4.44E+02	9.33E+01
	Std	1.73E+02	5.08E+02	**1.02E+02**	8.81E+01	1.10E+01
SDTLZ3_8	Mean	9.17E+05	1.63E+06	7.53E+05	7.31E+05	**6.39E+04**
	Std	4.61E+05	5.79E+05	6.90E+04	3.74E+04	**1.58E+04**
SDTLZ3_10	Mean	5.74E+07	1.64E+08	7.74E+07	6.06E+07	**4.52E+06**
	Std	2.36E+07	2.38E+03	3.41E+06	2.08E+07	**9.08E+05**
SDTLZ4_3	Mean	2.69E+01	**2.09E+00**	1.10E+01	7.06E+00	7.28E+00
	Std	6.76E+00	**7.98E-01**	5.50E+00	6.58E+00	1.09E+01
SDTLZ4_5	Mean	1.02E+03	8.57E+02	2.12E+03	4.66E+02	**2.22E+02**
	Std	9.66E+02	2.45E+02	8.97E+02	5.12E+02	**5.89E+02**
SDTLZ4_8	Mean	9.21E+05	5.36E+05	1.40E+06	9.85E+05	**1.16E+05**
	Std	8.07E+05	2.64E+05	6.77E+05	1.51E+06	**3.26E+05**
SDTLZ4_10	Mean	8.26E+07	8.97E+07	1.51E+08	5.10E+07	**4.98E+06**
	Std	5.07E+07	1.67E+06	3.45E+07	4.12E+07	**2.91E+06**

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