Understanding simulated driver behavior using hysteresis loops

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Abstract:
Modeling techniques evolved a lot during the last half century. Acceleration models used for microscopic traffic simulation are heterogeneous. For instance, differential equation based models are different from agent based ones. Comparison of those models is a difficult task because studies are often conducted based on a suitable and particular theory, usually not applicable to all models. So we propose to use the hysteresis loops to understand and compare models regardless of the used modeling technique. We show how this study can reflect important properties of the models in order to compare them.

Keywords: Modeling, Control and Optimization of Transportation Systems, Hysteresis, Simulation

1. INTRODUCTION

There is a wide range of models available to simulate and study the properties of a traffic flow. Usual classification (Hoogendoorn and Bovy (2001)) recognizes two main families of models: macroscopic and microscopic. The macroscopic models allow to study the evolution of macroscopic order variables of a traffic such as the density of vehicles, the average speed and the output rate. The microscopic models study individual entities composing the traffic: the vehicles. Those models deal with microscopic variables such as individual position and speed. The interaction among individual entities results in a consistent and hopefully realistic traffic flow. Two aspects are considered in microscopic models: acceleration and lane changing. In this paper we only consider acceleration models, used to determine the acceleration of a particular vehicle according to the vehicles in front. Those models are usually referred to as "car-following" models.

During the last 50 years, modeling and simulation techniques have significantly evolved, resulting in a heterogeneous variety of acceleration models (Brackstone and McDonald (1999)). The first models in the car-following area were based on dynamical equations (see for instance Gazis et al. (1961)). Those equations have been then enriched to integrate physical values used by drivers in their decision making process (such as Gipps (1981)). Artificial intelligence (AI) and multi-agent systems (MAS) allowed to develop even more realistic models based on a wide variety of theories and techniques such as fuzzy logic theory (Kikuchi and Chakroborty (1992)), constraint satisfaction (Doniec et al. (2006)) or game theory (Champion et al. (2003)).

Even if those models are used to study the same phenomenon (the vehicle's acceleration) and its consequences at a macroscopic level (resulting flow, emergence of stop & go waves...), the variety of tools used makes the study of those models difficult. Finding the most suitable model for a given situation is then a challenging task. Benchmarking has been conducted in Brockfeld et al. (2003) to determine the most realistic resulting flow. However this approach does not give any information on how individual entities actually behave and how they can influence the macroscopic properties of the traffic.

In this paper we propose to use the hysteresis phenomenon to study and understand car-following models while being independent of the modeling techniques. The hysteresis loop can be used to understand the dynamics of a model, reflect its anticipation capabilities and study the propagation of a perturbation along a platoon of vehicles.

Section 2 presents the models we selected for this study, then Section 3 presents the hysteresis phenomenon and its application to study driver behavior. Section 4 describes the experimental protocol and Section 5 presents the results we obtained. Finally, Section 6 discusses the results and gives some further work perspectives.

2. MODELS

We selected three car-following models for this study. Each model determines at a given point of time the vehicle's acceleration. The first model is described with a simple dynamic equation, the second introduces physical values involved in the decision making process of drivers and the third is an agent based model. Figure 1 summarizes naming conventions used across this paper.

2.1 Optimal Velocity (OV)

The OV model, proposed by Bando et al. (1994), is based on the following principle : for each situation there

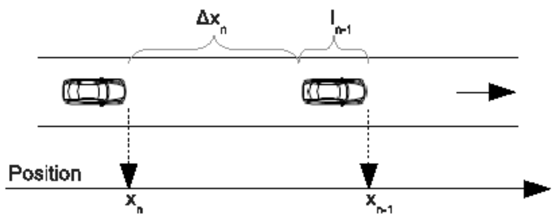


Fig. 1. Naming conventions used in model description : the n^{th} vehicle follows the $(n-1)^{\text{th}}$ vehicle. The front bumper's position of the n^{th} vehicle is called x_n, its velocity $\dot{x}_n$, and its acceleration $\ddot{x}_n$. We call l_n the length of the n^{th} vehicle. The gap available in front of the vehicle is $\Delta x_n = x_{n-1} - l_{n-1} - x_n$.

is an optimal speed to adopt. Any deviation from this optimal speed causes the vehicle to adopt an acceleration proportional to the difference between the optimum and the actual speed.

This behavior is expressed in the following equation:

$$\ddot{x}_n = a_n \left(V(\Delta x_n) - \dot{x}_n \right) \tag{1}$$

Where $V(\Delta x)$ is the optimal velocity for a given situation represented by the available gap, a_n is a parameter of the model influencing how strong the reaction is.

The optimal velocity function used in this paper is inspired from the one proposed in Bando et al. (1994). The function maintains the main properties of the one previously used but introduces a parameter v_0^n to specify the desired speed of the vehicle :

$$V(\Delta x) = \left(tanh\left(\frac{2\Delta x}{v_0^n} - 2 \right) + tanh(2) \right) * \frac{v_0^n}{2} \tag{2}$$

2.2 Intelligent Driver Model (IDM)

The IDM model, proposed by Treiber et al. (2000), is designed as the aggregation of two behaviors : the first one leads the vehicle to accelerate until it reaches its desired speed, the second one forces it to decelerate to maintain a safe gap ahead.

The model is described as follow :

$$\ddot{x}_n = a_n \left[1 - \left(\frac{\dot{x}_n}{v_0^n} \right)^{\delta} - \left(\frac{s_n^*(\dot{x}_n, \Delta v_n)}{\Delta x} \right)^2 \right] \tag{3}$$

Where a_n is the maximum acceleration of the n^{th} vehicle, v_0^n its desired speed, δ the acceleration exponent and Δv_n is the approaching rate of the leading vehicle given by $\Delta v_n = \dot{x}_n - \dot{x}_{n-1}$. s_n^* is a function that calculates the desired gap to maintain for a particular (speed / approaching rate) couple :

$$s_n^*(\dot{x}_n, \Delta v_n) = s_0^n + s_1^n \sqrt{\frac{\dot{x}_n}{v_0^n}} + T_n \dot{x}_n + \frac{\dot{x}_n \Delta v_n}{2\sqrt{a_n b_n}} \tag{4}$$

Where T_n is the safe time headway, b_n represents the conformable deceleration, and s_0^n and s_1^n are jam distance parameters.

Equation 3 reflects the two sub-behaviors of IDM : $a_n \left(1 - \left(\dot{x}_n / v_0^n \right)^{\delta} \right)$ is the acceleration part on a free road and $-a_n \left(s_n^* \left(\dot{x}_n, \Delta v_n \right) / \Delta x \right)^2$ represents the part influenced by other vehicles applying a constraint and forcing the vehicle to decelerate to maintain the situation safe.

2.3 Archisim

The Archisim model, presented in Espié et al. (1994), is based on in-depth studies conducted by driving psychologists and developed with MAS techniques. The behavior of an Archisim vehicle is influenced not only by the leading vehicle, but by potentially all the surrounding vehicles. During each time step, the vehicle receives a list of its acquaintances (vehicles in perception range). From this list, it determines which one of the vehicles down the road causes the biggest constraint (causes the biggest deceleration). The constraint is then classified as short term or long term depending on the context and a corresponding strategy is adopted as proposed in Espié et al. (2007). A short term constraint corresponds to a situation where the constraint is known to reduce with no particular action of the driver.

The short term strategy consists in maintaining short minimal bumper to bumper time given by the S_{bbt} parameter. The bumper to bumper time is defined as the time necessary for the front bumper of the follower vehicle to reach the position of the rear bumper of the leader vehicle. Thus, we can deduce a minimal gap to maintain given by the expression $\Delta x_{min} = \dot{x}_n * S_{bbt}$.

If the vehicle chooses the long term strategy, the minimal safe bumper to bumper time is given by $E_{bbt} = \alpha_n S_{bbt} + (1-\alpha_n) L_{bbt}$ with E_{bbt} the effective bumper to bumper time targeted, L_{bbt} the longest bumper to bumper time and α_n a vehicle specific parameter.

3. THE HYSTERESIS LOOP

The hysteresis, from Greek *husteros* meaning "coming after", is the lag of a reaction compared to its cause. It was first introduced by the physicist James Alfred Ewing as follows :

> When there are two quantities $\mathcal{M}$ and $\mathcal{N}$, such that cyclic variations of $\mathcal{N}$ cause cyclic variation of $\mathcal{M}$, then if the changes of $\mathcal{M}$ lag behind those of $\mathcal{N}$, we may say that there is hysteresis in the relation of $\mathcal{M}$ and $\mathcal{N}$.

Figure 2 shows the graphical representation of the hysteresis phenomenon. The loop shows the values taken by the reaction as a function of the effect. The reaction shows different values for the same effect when this effect changes.

In this paper, we suppose that for each stable value of the effect there is one stable value of the reaction and it is unique. When the system is in a situation where both effect and reaction are stable, the system is told to be in an *equilibrium state*. Figure 2 shows what could be the equilibrium states on the dashed line.

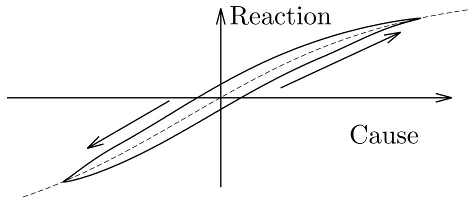


Fig. 2. Hysteresis phenomenon showing the lag of the reaction compared to the effect.

In traffic theory, Bando et al. (1994) observed that when stop and go waves appear, it is possible to plot on a graph the speed of individual vehicles as a function of the gap available in front of them and obtain an hysteresis loop. Other authors did notice this property in Jiang et al. (2001); Helbing and Tilch (1998); Bando et al. (1998); Davis (2003) and discussed the influence of model parameters on the loop's size and shape.

We present here an interpretation of the hysteresis loop where the cause is a constraint applied to a vehicle — a vehicle present in front of it, given by the gap available — and the reaction is the behavior of the vehicle given by its speed. In this interpretation, the equilibrium states correspond to desired states the models will eventually converge to. The equilibrium states are also considered as optimal because they represent the state where a vehicle can, given a particular gap, maximize its speed while staying safe. Any point below the equilibrium state is considered as under constrained and any point above is over constrained.

4. SIMULATION PROTOCOL

The three selected models are tested in the same environment. We selected the Archisim platform (Espié et al., 1994) to conduct this experiment because it allows us to:

- create a repeatable scenario,
- record simulation traces (individual vehicle trajectories),
- select or define the model used to guide the vehicle in the simulation.

This platform also provides the reference implementation for the Archisim model so we only had to add the two remaining models through two new kind of vehicle agents.

The scenario developed in this study involves a platoon of vehicles we want to study led by a controlled vehicle on a straight one lane free road. The speed and acceleration of the leader vehicle are controlled by the modeler. This allows to apply a known and controlled constraint on the platoon to study its reactions. For each studied model, the scenario is instantiated with the same leader vehicle, assuring a comparable constraint.

During a simulation run, the leader's speed varies to study the reaction of the platoon following it. The leader's speed changes from 4 to 22 $m.s^{-1}$ and vice-versa. Each speed variation, of an intensity of 3 $m.s^{-2}$, occurs every 2 minutes. The follower can thus adapt to the new constraint and eventually join a new equilibrium state before the leader changes its speed again.

Table 1. Parameters used during this study

Model	**Parameter**	**Value**
OV	a	1.0
IDM	T	$1.2s$
IDM	a	$0.8m.s^{-2}$
IDM	b	$1.25m.s^{-2}$
IDM	s_0	$1m$
IDM	s_1	$10m$
IDM	δ	3
Archisim	S_{bbt}	$0.7s$
Archisim	L_{bbt}	$2.5s$
Archisim	α	0.5
*	v_0	$25m.s^{-1}$

Inside the platoon, all the studied vehicles are given the same set of parameters. The desired speed is the same for all the three models: 25 $m.s^{-1}$. The model-specific parameters are set to the values originally proposed by the authors. The parameters for IDM are from Treiber et al. (2000), those for OV are from Bando et al. (1994) and those for Archisim are the default values provided by the authors. Table 1 summarizes the parameters used.

5. RESULTS AND HYSTERESIS LOOP'S INTERPRETATION

In this section we present the equilibrium states and simulation results of each model.

5.1 Equilibriums

To analytically identify equilibriums, we have to find points of the (gap / speed) plan where the acceleration adopted by the vehicle is null. For each model we have to solve the equation $\mathcal{A}_{\mathcal{M}}(\Delta x, \dot{x}) = 0$ where $\mathcal{A}_{\mathcal{M}}$ represents the acceleration given by a particular model $\mathcal{M}$.

The calculation for OV is straightforward. Considering the optimal velocity function proposed in Equation 2, the equilibrium line is given by :

$$V(\Delta x_n) = \frac{v_0 \left(tahn\left(\frac{2\Delta x_n}{v_0^n} - 2\right) + tanh(2)\right)}{2} \tag{5}$$

For the IDM model, we have to consider that the approaching rate is null : $\Delta \dot{x}_n = 0$. This hypothesis was also used by Treiber in Treiber et al. (2000) to determine stable flow. The equilibrium line is then given by :

$$\Delta x_n(\dot{x}_n) = \frac{s_0^n + s_1^n \sqrt{\frac{\dot{x}_n}{v_0^n}} + T_n \dot{x}_n}{\sqrt{1 - \left(\frac{\dot{x}_n}{v_0^n}\right)^{\delta}}} \tag{6}$$

For Archisim, we consider that an equilibrium state is reached when the bumper to bumper time is the one given by the long term strategy. Indeed the short term strategy cannot be, given its definition, stable over time. Also, once the vehicle reaches its desired speed, it will not continue to accelerate. As a consequence, the equilibrium line for the Archisim model can be expressed as :

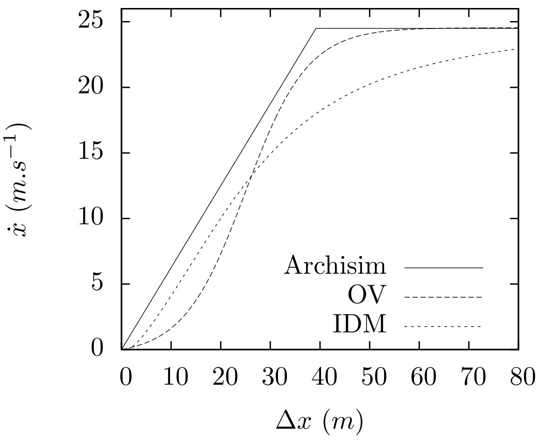


Fig. 3. Equilibrium states for the three studied models.

$$\dot{x}_n\left(\Delta x_n\right) = Min\left(\frac{\Delta x_n}{\alpha S_{bbt} + (1-\alpha)\, L_{bbt}}, v_0^n\right) \qquad (7)$$

The same results for equilibrium states have been approached experimentally by setting stable flows at different speeds and recording the behavior adopted by vehicles.

Figure 3 summarizes the three theoretical equilibriums. It can provides important elements of comparison because gaps and speeds have immediate implication on macroscopic variables such as flow density and speed. If all vehicles are in stable state, the flow is stable with the same speed as individual vehicles. Also, with the approximation of vehicles length, the flow density is the reverse of individual gap head. Some authors discussed the impact of different microscopic parameters on macroscopic variable (Treiber et al. (2000)).

Considering the parameters we used in this study, for every speed adopted by the leader, the Archisim model heads for smaller gaps than the other models, implying higher density. For small speeds (less than about $12m.s^{-1}$), the OV model goes for bigger gaps than IDM. On the opposite, for high speeds, the IDM model adopts very large gaps compared to the two others models, implying lower concentration. Finally, we can deduce from those results that a IDM platoon can not travel at desired speed because the equilibrium line never reaches it.

Those results do not provide any information about the dynamic behavior of the models and how they actually adapt to a constraint variation.

5.2 Delay and anticipation

We focus on the results obtained after using the scenario described in Section 4. The objective is to be able to study how a model operates the transition from one equilibrium state to another when the constraint applied to the vehicle changes. In this section, we only consider the hysteresis loop of the vehicle immediately following the leader. For each observed loop, we first propose an interpretation and then link it to the knowledge we have about the models to validate that the hysteresis loop reflects the behavior.

Figure 4 — representing the reaction of an OV vehicle — has one half of the hysteresis loop staying below the

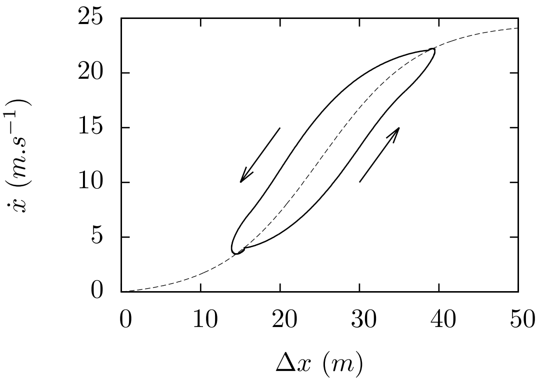


Fig. 4. Reaction of an OV vehicle to a constraint varying between $4m.s^{-1}$ and $22m.s^{-1}$

equilibrium line, and the other half above. This shape is similar to usual hysteresis loops seen in general literature. The part below corresponds to the growth of the available gap during the acceleration phase. During this phase, for all the given gaps, the associated velocity is below the optimal one. The constraint applied to the vehicle is therefore lower than the optimal one, letting the vehicle free to accelerate in order to join the equilibrium line. The deceleration phase (above the equilibrium) has analogous properties and shows that the vehicle only decelerates when it is in an over constrained situation.

This information perfectly matches the description made of the model. It is purely reactive and only engages reaction when a variation from optimal state is detected.

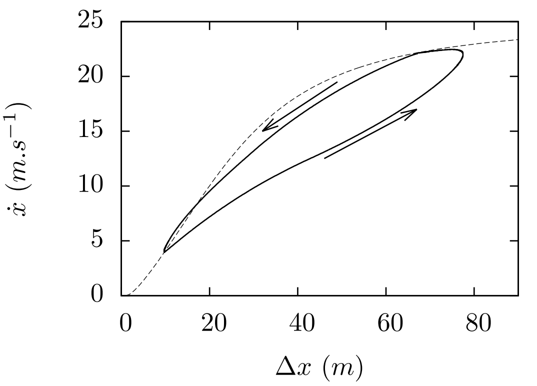


Fig. 5. Reaction of a IDM vehicle to a constraint varying between $4m.s^{-1}$ and $22m.s^{-1}$

Figure 5 represents the adaptation of the IDM model on the same constraint's variation. The overall shape of the hysteresis phenomenon drawn is still a loop. Moreover, when the follower's speed matches the leader's one (at 4 and 22 $m.s^{-1}$), the vehicle is in an equilibrium state. However, there is a clear distinction to make with the OV model : the loop is nearly always below the equilibrium line. This position of the loop shows that during almost all the speed transitions, the IDM model maintains the vehicle in an under constrained situation. This reveals two properties of the model. First during the acceleration phase, the IDM vehicle's reaction lags compared to the constraint variation, exactly like a OV vehicle does. Second, during the deceleration phase, the vehicle manages to anticipate

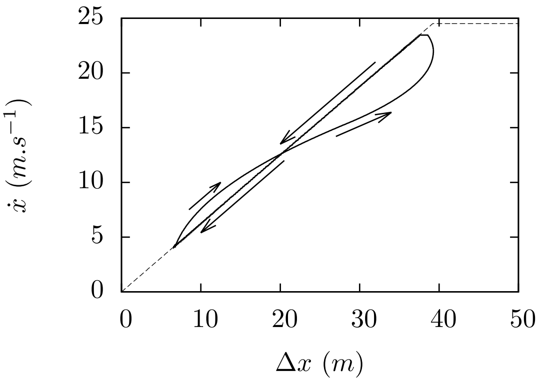


Fig. 6. Reaction of an Archisim vehicle to a constraint varying between $4m.s^{-1}$ and $22m.s^{-1}$

the reduction of the gap before it happens and reduces its speed early. This clearly shows the ability of the model to anticipate the constraint's variation.

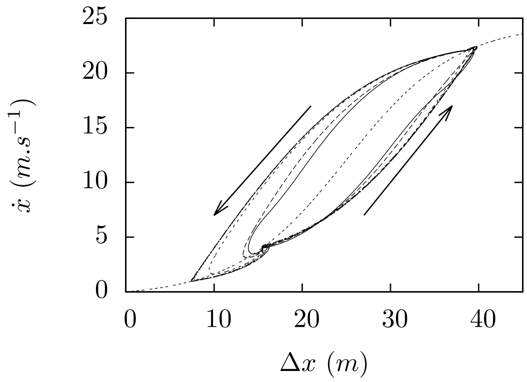
Those results reflect the initial description of the model given in Section 2. We mentioned the fact that IDM is an aggregation of two sub-behaviors. One manages the acceleration, the other manages the deceleration and takes into account the approaching rate of the leading vehicle. Knowledge of the approaching rate allows the model to anticipate the future reduction of the available gap and explains the anticipation capabilities exhibited by this model.

Finally, we study the hysteresis loop for the Archisim model (see Figure 6). This loop has distinctive properties compared to the two previous ones. It has an "8" shape : at some point the trace of the acceleration phase is above the deceleration phase and at some point it is below. First, we consider the deceleration phase given by the straight line on the loop. It exactly matches the equilibrium line meaning that there is no hysteresis when decelerating. The model manages to maintain an optimal reaction during this phase. The acceleration phase is slightly more complex because the vehicle is, at the beginning of the acceleration phase, above the equilibrium line and, at the end, under the equilibrium line. This loop shows that, at least for the beginning of the acceleration, the follower chooses to be in an over constrained situation.

Once again, those observations match the original description of the model. During the acceleration phase, the leader vehicle is considered as a short term constraint because it goes faster and is accelerating. Those two properties imply that the available gap will continue to grow, making the constraint weak. The position of the acceleration phase above the equilibrium line is considered as an expression of the anticipation capabilities of the model, exactly like the fact that the deceleration phase of IDM being below the equilibrium reflects anticipation.

5.3 Propagation of a perturbation

The previous part was about the interpretation of the hysteresis phenomenon on a single vehicle. We now consider a platoon of 30 vehicles and study the propagation of the hysteresis phenomenon in this context.



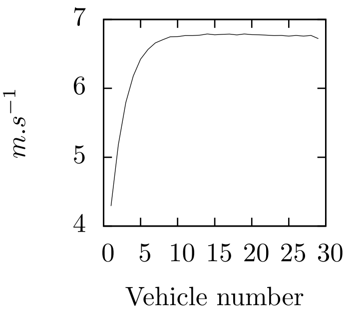


Fig. 7. Study of a platoon of 30 vehicles using the Optimal Velocity model. Top figure shows hysteresis for the vehicle number 1, 2, 6, 10, 14, 18, 22 and 26 in the platoon and bottom figure shows hysteresis to equilibrium pseudo distance evolution along the platoon

For this study we introduce a notion of pseudo distance between the hysteresis loop and the equilibrium line. This distance is defined as the maximum difference between the equilibrium speed and the velocity used by the vehicle during the simulation.

Figure 7 represents the evolution of the hysteresis loop along a platoon of OV vehicles. In this figure, the first vehicle's loop is in the center and as we get closer to the tail of the platoon, the loop area gets bigger. At the end of the platoon, the loops seem to reach a final state comparable to an attractor. Those readings are confirmed by the second part of Figure 7 that shows an augmentation of the distance between equilibrium and the hysteresis loops on the 10 first vehicles. After this first phase, the distance gets stable around $6.8m/s$. Figure 7 also shows that the first vehicle has a speed variation comparable to the one applied by the modeler from 4 to $22m/s$. At the end of the platoon, the speed variation has changed to become a variation from 0.9 to $22.4m/s$ which is a clear augmentation. Finally, even if the perturbation is tending to something different from what we modeled, we can still observe singular points at speed 4 and $22m/s$. This corresponds to the stable flow maintained by the platoon for a long time compared to the phase transitions.

Those results are coherent with the ones already exposed in literature. First, the emergence of a stable hysteresis loop after some time, independent of initial perturbation, was originally observed by Bando et al. (1994). This is a limit behavior observable during stop and go waves. This property was used in Nakanishi et al. (1997) to

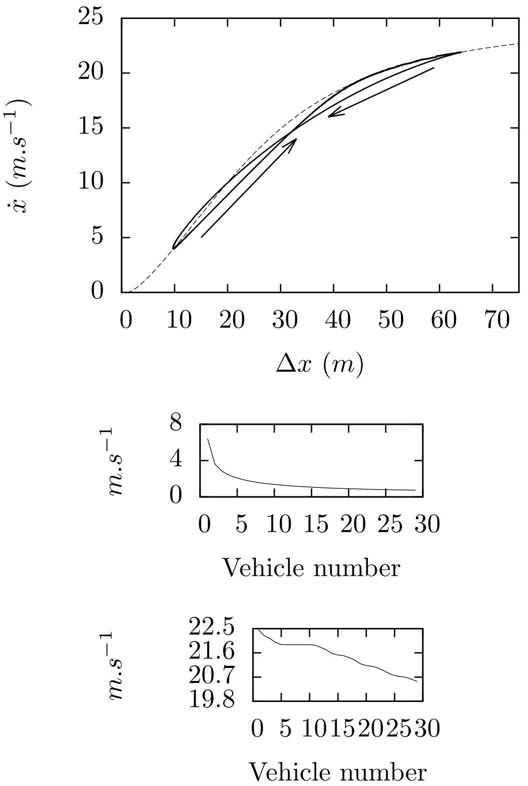


Fig. 8. Evolution of the hysteresis loop across a platoon of 30 IDM vehicles

simplify the calculation. The author supposed that the behavior of a given vehicle at time t is the same as the one of its predecessor shortly before. Analogous properties are used in other models such as Newell (1993) and present interesting results. Even if the hysteresis loop validates this approach (only after few vehicles to let the perturbation get stable), it cannot give any information about temporality: it does not give information on the time elapsed between the reactions of the two vehicles.

Figure 8 represents the propagation of a perturbation across a platoon of IDM vehicles. To reduce overlaps on the graphs, top figure only represents the hysteresis loop for vehicle 10, given the fact that all loops starting from vehicle number 2 share similar properties. It shows a “8” shape, but has to be distinguished from the one obtained in Figure 6 because the rotations are in opposite directions. It shows the deceleration phase above the acceleration phase for small gaps, and the opposite for large gaps. We can observe that the deceleration phase is really similar to the one observed in Figure 4. However, the acceleration phase seems stronger here because the leader is also an IDM vehicle, with similar acceleration capabilities, leading to a higher grade. This shape has to be linked to the results obtained by Zhang (1999) at microscopic level, who studied similar “8” shapes on the (density / speed) plan.

Middle part of Figure 8 shows that along the platoon, the pseudo distance between the hysteresis loop of the vehicles and the equilibrium tends to reduce unlike what is observed in Figure 7. Also, we notice that considering our scenario, each vehicle in the platoon reaches a lower maximal speed compared to its predecessor as shown in the bottom part of Figure 8. This reflects a really slow convergence to the final equilibrium considering that in a 2 minutes phase, the last vehicle does not have time to reach “full” speed. This propagation pattern has opposite properties compared with OV showed in Figure 7.

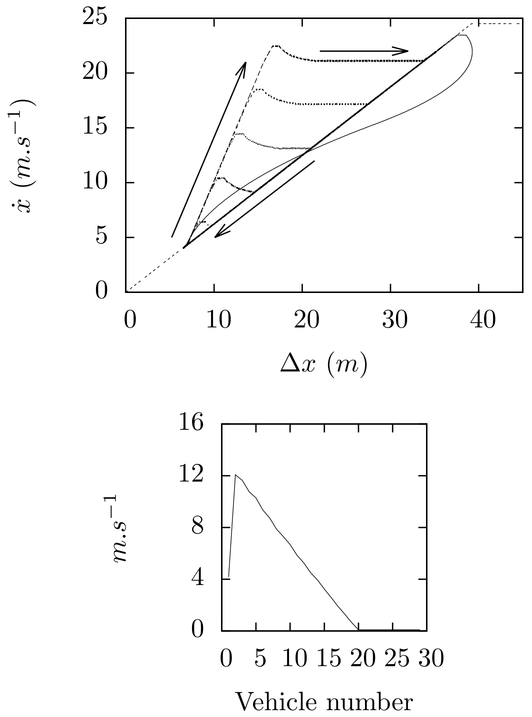


Fig. 9. Study of a platoon of 30 vehicles using the Archisim model.

Figure 9 shows the evolution of the propagation on an Archisim platoon. All the loops starting from the position number 2 present a triangular shape on the bottom left part. Even if the overall shape is not the same with the first vehicle, they share this property of exhibiting anticipation for early stages of acceleration. All the triangles share really similar shapes with a scaling factor between them. The reduction of the triangle size is interpreted as follow : it corresponds to the acceleration phase where each vehicle can perceive that the platoon is accelerating. During this phase each vehicle considers that the maximal constraint perceived is temporary and as a consequence accepts a small gap head. When the acceleration of the leader stops, each follower vehicle begins to consider it as a long term constraint and as a consequence tries to reach the equilibrium line. This transition corresponds to the horizontal lines. During the acceleration of the platoon, each vehicle is moving slightly faster than the one behind it, so when the leader stops accelerating each vehicle has its own speed, smaller than its predecessors. This explains why from one vehicle to its follower, the size of the triangle gets smaller. Bottom part of Figure 9 confirms it and shows that after 20 vehicles, the hysteresis loops match the equilibrium line, reflecting the disappearance of hysteresis in those vehicle’s behavior. Basically when the leader stops its acceleration, the 20th hasn’t started its own.

It is really important at this point to notice the time-independent nature of the hysteresis loop. During this scenario we know that all the vehicles react to the same stimuli happening at a particular point in time. However a simple reading of the evolution of hysteresis loops is not enough to reach this conclusion and we need to add additional knowledge to get a complete understanding of the situation, such as adding a temporal dimension.

6. CONCLUSION AND PERSPECTIVES

In this paper we used the hysteresis loop to understand and compare behaviors produced by different models. Studying the hysteresis phenomenon is an natural approach to evaluate a reaction as a function of the stimulus causing it. It is thus useful to study the reaction and the adaptation of an agent, in a dynamic environment. We showed that study of hysteresis loops allows to identify key properties of a behavior such as delay in reaction, anticipation capabilities and limit behavior in a platoon through comparison between equilibrium states and actual behavior. The results presented are consistent with the ones obtained by original authors of the models, but are obtained independently of the modeling technique used.

For each studied model, we determined the equilibrium state using analytical and simulation studies. Both approaches lead to the same information so the experimental approach is adequate to study those models as black boxes. However, there is no guaranty that this is sufficient for any model. As an example, stochastic models can accept surfaces to reflect states that have a high probability of being stable. Simple protocol will then not be enough to experimentally determine what are the desired states, which is a mandatory information to conduct the analysis. Determination of those equilibrium states on any “black-box” model is still an open issue that needs to be addressed to extend the validity of our approach.

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