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APPLICATION OF THE ENERGY-DISSIPATION MODEL OF TURBULENCE TO THE CALCULATION OF FLOW NEAR A SPINNING DISC

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Introduction

In the last few years a number of models of turbulent heat and momentum transport have been developed in which the effective transport coefficients are related to local values of certain turbulent correlations; these correlations are computed simultaneously with the mean field variables. Models of this kind achieve significantly greater breadth of applicability than do simpler approaches based on mean-flow quantities alone. One of the more successful of these newer approaches is the energy-dissipation model developed by Jones and Launder (1,2). Its originators applied it to the calculation of numerous boundary layer flows with severe streamwise pressure gradient or surface mass transfer. No applications have been reported, however, of its use to predict swirling flows, an omission that the present note remedies. The flow considered (that generated by a rotating disc in a quiescent atmosphere) produces very high gradients of swirl velocity in the vicinity of the disc which in turn brings to prominence terms in the kinetic energy and dissipation equations that have formerly been absent or of only small importance. This application thus provides a test of the generality of the model for an important class of fluid flows.

The Turbulence Model

The turbulent transport coefficients μ_T and Γ_T are obtained from the following system of differential and auxiliary equations; explanation of the origin of the equations is provided in refs (1,2).

Turbulent viscosity: $\mu_T = c_\mu \rho k^2/\varepsilon$ (1)

Turbulent thermal or mass diffusivity : $\Gamma_T = \mu_T/0.9\rho$ (2)

(i.e. Turbulent Prandtl/Schmidt number = 0.9)

Turbulence kinetic energy equation:

$$\rho U \frac{\partial k}{\partial r} + \rho V \frac{\partial k}{\partial y} = \frac{1}{r}\frac{\partial}{\partial y}\left[r \quad \mu\left(+ \frac{\mu_T}{\sigma_k}\right)\frac{\partial k}{\partial y}\right]$$

$$+ \mu_T\left[\left(\frac{\partial U}{\partial y}\right)^2 + \left(r\,\frac{\partial V_\theta/r}{\partial y}\right)^2\right] - \rho\varepsilon - 2\mu\left(\frac{\partial k^{\frac{1}{2}}}{\partial y}\right)^2 \qquad (3)$$

Turbulence energy dissipation equation:

$$\rho U \frac{\partial \varepsilon}{\partial r} + \rho V \frac{\partial \varepsilon}{\partial y} = \frac{1}{r}\frac{\partial}{\partial y}\left[r\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right]$$

$$+ c_1\frac{\varepsilon}{k}\,\mu_T\left[\left(\frac{\partial U}{\partial y}\right)^2 + \left(r\,\frac{\partial V_\theta/r}{\partial y}\right)^2\right] - c_2\rho\varepsilon^2/k$$

$$+ 2.0\,\frac{\mu\mu_T}{\rho}\left\{\frac{\partial}{\partial y}\left[\left(\frac{\partial U}{\partial y}\right)^2 + \left(r\,\frac{\partial V_\theta/r}{\partial y}\right)^2\right]^{\frac{1}{2}}\right\}^2 \qquad (4)$$

where $c_\mu \equiv 0.09 \exp\left[-3.4/(1 + R_T/50)^2\right]$ (5)

$c_2 \equiv 1.92\left[1.0 - 0.3 \exp(-R^2{}_T)\right]$ (6)

and $R_T \equiv \rho k^2/\mu\varepsilon$, the turbulent Reynolds number.

The other empirical coefficients take the following uniform values:

$c_1 = 1.44; \quad \sigma_k = 1.0; \quad \sigma_\varepsilon = 1.3.$

The independent variables r and y are respectively the radial distance from the disc axis and the normal distance from the disc surface. The corresponding velocities are U and V; V_θ denotes the circumferential velocity. All other notation is the same as in ref (1,2).

The above system of equations differ from that in (1) and (2) in two respects:

(i) Extra source terms involving gradients of (V_θ/r) appear in the equations for k and ε. Their appearance is due to the conversion of the Cartesian-tensor from of these equations to the present coordinate frame. They are not ad hoc terms.

(ii) The Reynolds number functions c_μ and c_2 and the coefficient c_1 are slightly different. This is the result of an overall reoptimisation of coefficients reported in (3). We repeated the computation of some of the

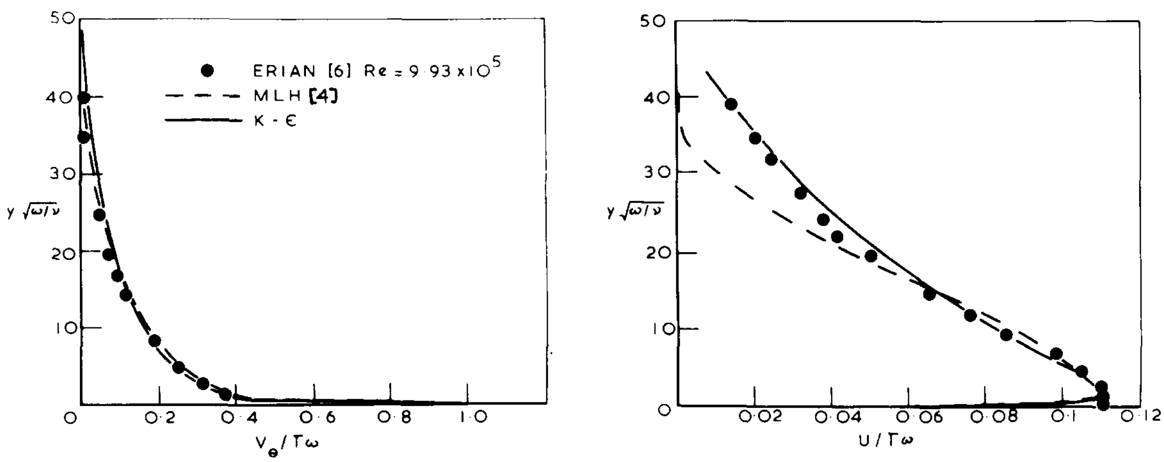

Fig. 1
Turbulent flow velocity profiles near a spinning disc

flows studied in (1,2) with the present coefficients: there were no significant differences in the results from those originally reported.

Solution of Equations

The above system of equations have been solved simultaneously with that governing the mean flow: i.e. the radial and angular momentum equations, the continuity equation and (for problems of heat or mass diffusion) the enthalpy or chemical species equation. The mean-flow equations are given in the Appendix together with the boundary conditions. The precise form is given in (4). The numerical solving scheme used in basically the Patankar-Spalding (5) procedure modified for the inclusion of swirl as outlined in (4). Seventy nodes were used to span the boundary layer with a substantial concentration very near the wall (this is about 60% more than are needed to obtain grid-independent results when the mixing-length model is used). The forward step used was typically 15% of the boundary layer thickness leading to computer times per run of about 50 s on a CDC 6600 computer.

Discussion of Results

Some predicted characteristics of the rotating-disc flow are compared with experimental data in Figs. 1-3. The calculated radial and circumferential velocity profiles are seen, from Fig. 1 to be closely in agreement with Erian's (6) experimental data. The mixing-length results from (4) are also shown: although here agreement with experiment is also reasonably satisfactory, the radial velocity falls to zero faster than experiment suggests due to the turbulent viscosity falling to zero too quickly near the outer edge.

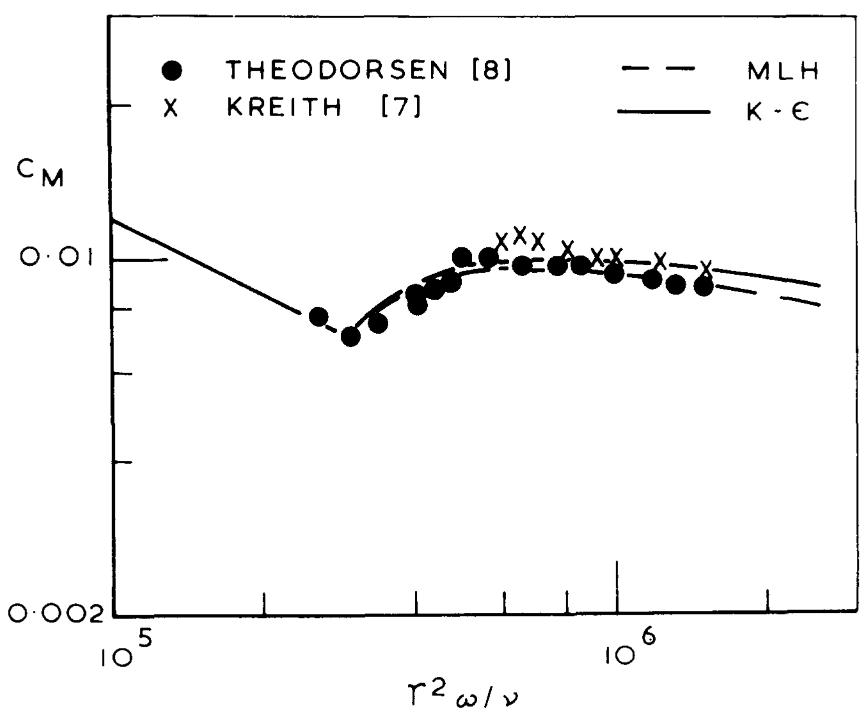

Fig. 2
Torque coefficient for spinning disc

The corresponding variation of torque coefficient with Reynolds number is shown in Fig. 2. The Reynolds number at which transition occurs must be provided (when the radial distance is so small that the spin Reynolds number is less than Re_{trans} the flow is taken as laminar and only the mean flow equations are solved). Following our practice in (4) we have adopted the value of Re_{trans} that seemed to be suggested by any particular apparatus; not surprisingly there is a significant variation in Re_{trans} from one apparatus to another due presumably to minor variations in geometry and in surface finish. The predicted variation of torque coefficient with Reynolds number is closely in line with the measured behaviour, the line representing predicted behaviour falling roughly mid-way between the data of Kreith (7) and Theodorsen and Regier (8).

Heat and mass transfer predictions are shown in Fig. 3. There is extremely close agreement with Cobb's measured mean Nusselt numbers over the full range of the experiments; the data of McComas lie about 7% below this. Agreement with the naphthalene diffusion data of Tien (11) and Kreith (12) is not quite as satisfactory, however. While there is close agreement with experiment for spin Reynolds numbers up to 4 x 10^5, beyond this the data rise progressively faster than the predictions indicate. The same kind of discrepancies were noted in (4) where the mixing length hypothesis was used. It does not seem possible to identify the cause of the disagreement in the absence of measured profiles of species concentration near the disc.

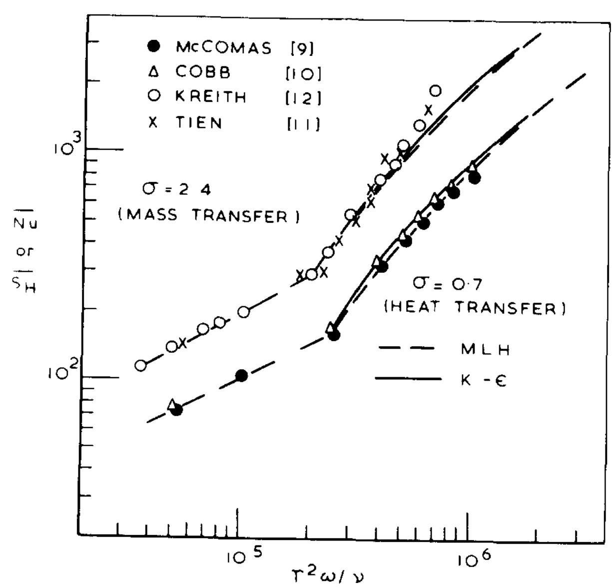

Fig. 3
Heat and mass transfer from a disc rotating in still air

Conclusion

The main conclusion is that the $k\sim\varepsilon$ model of turbulence, which had been devised (1,2) specifically to predict certain low-Reynolds-number phenomena in boundary layers and duct flows, has been found to predict accurately the flow, heat and mass transfer in the neighbourhood of a rotating disc. The result is of significance to the problem of predicting convective heat transfer rates in turbine discs.

Acknowledgement

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Appendix

The following is the system of mean-flow conservation equations solved simultaneously with equations (1-6) describing the turbulence quantities.

Streamwise momentum:

$$\rho U \frac{\partial U}{\partial r} + \rho V \frac{\partial U}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left[r(\mu + \mu_T) \frac{\partial U}{\partial y} \right] + \rho \frac{V_\theta^2}{r} \qquad \text{(A1)}$$

Angular momentum:

$$\rho U \frac{\partial rV_\theta}{\partial r} + \rho V \frac{\partial rV_\theta}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left[r^3 (\mu + \mu_T) \frac{\partial (V_\theta / r)}{\partial y} \right] \qquad \text{(A2)}$$

Enthalpy or species mass fraction (ϕ):

$$\rho U \frac{\partial \phi}{\partial r} + \rho V \frac{\partial \phi}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y} \left[r(\Gamma + \Gamma_T) \frac{\partial \phi}{\partial y} \right] \qquad \text{(A3)}$$

Boundary conditions are applied at the disc surface (y=0) and beyond the edge of the boundary layer ($y=y_\infty$) as follows:

$y = 0$: $U = k = \varepsilon = 0$; $V_\theta = \omega r$; $\phi = \phi_W$

$y = y_\infty$: $U = k = \varepsilon = V_\theta = 0$; $\phi = \phi_\infty$.