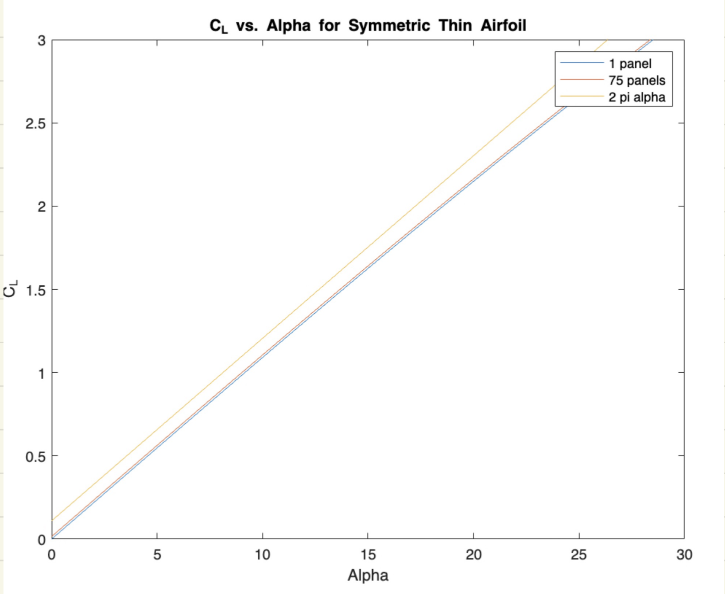
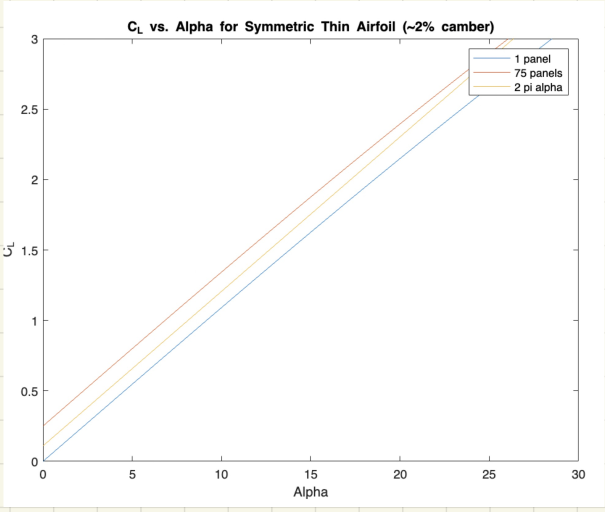
Lance Tan
Project 1

(1)



The computed plots seem to have the same linear trend that coincides with the C_L = 2*pi*alpha relationship. We can see closely that there seems to be a small discrepancy in the plots between 1 and 75 panels. This might be due to the fact that 75 panels shows a higher accuracy in describing the C_L across the panels, whereas 1 panel generalizes the relationship and loses that level of detail.

(2)



We can see a similar relationship between all 3 panels, with 75 panels having the most closest match to the expected properties of C_L = 2*pi*alpha. This is because we see a y-intercept in addition to seeing the linear relationship, which may be captured by having more panels, which add more detail to the plot.

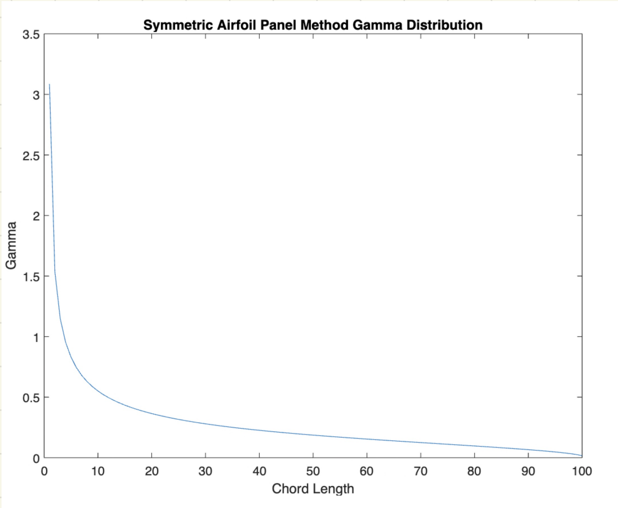
(3) (as seen from the MATLAB code attached below)

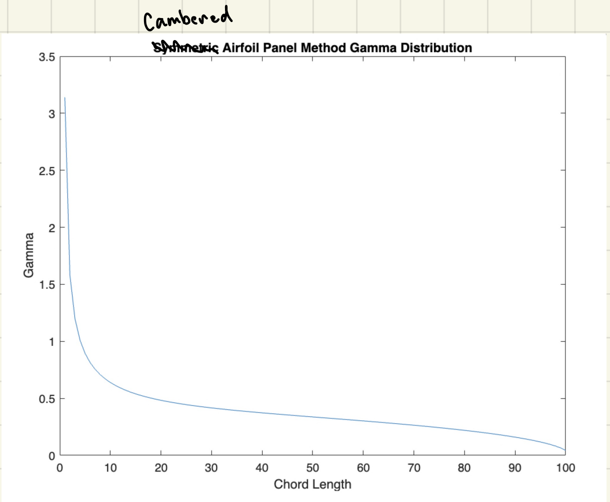
circulation = 28.16

circulation due to the analytical solution = 27.4156

As seen from the circulation values, there is a small discrepancy in circulation values.

(4)



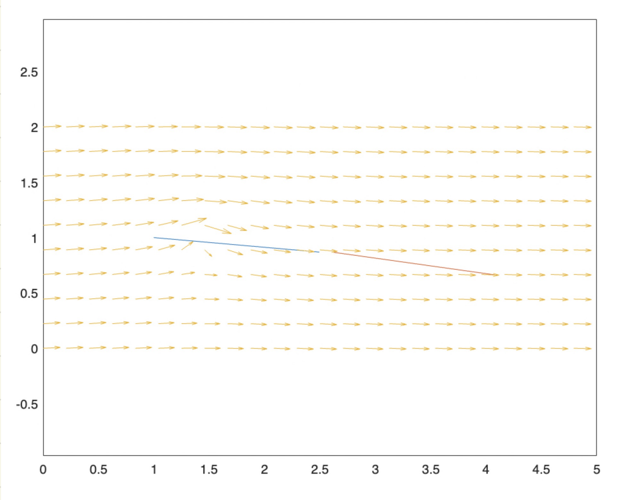


As seen from both plots, there seems to be a general drop in gamma as the chord length. However, for the cambered airfoil, we see higher gamma values, which lines up with the concept that cambered airfoils generally produce more lift.

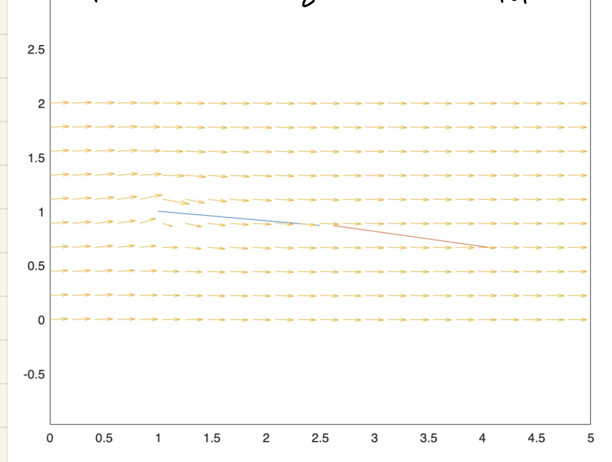
(5) N = 1

N = 75

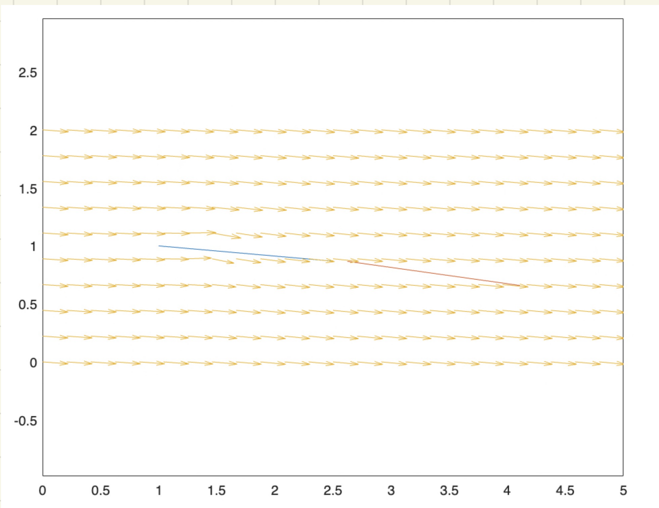
(a)



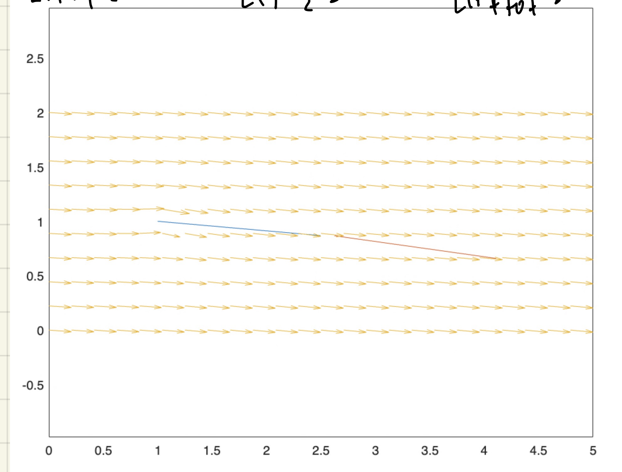
$Lift_1 = 17.4$ $Lift_2 =$ $Lift_{tot} =$



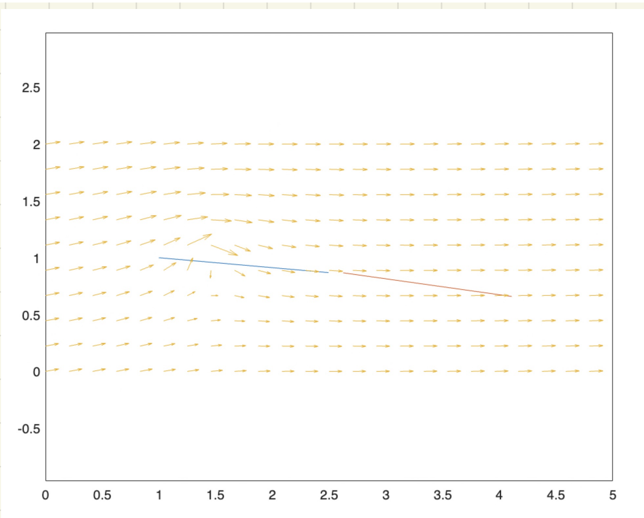
(b)



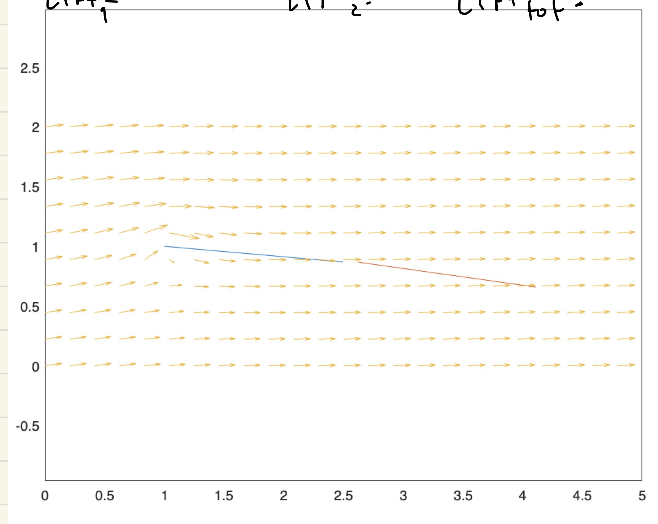
$Lift_1 =$ $Lift_2 =$ $Lift_{tot} =$



(c)



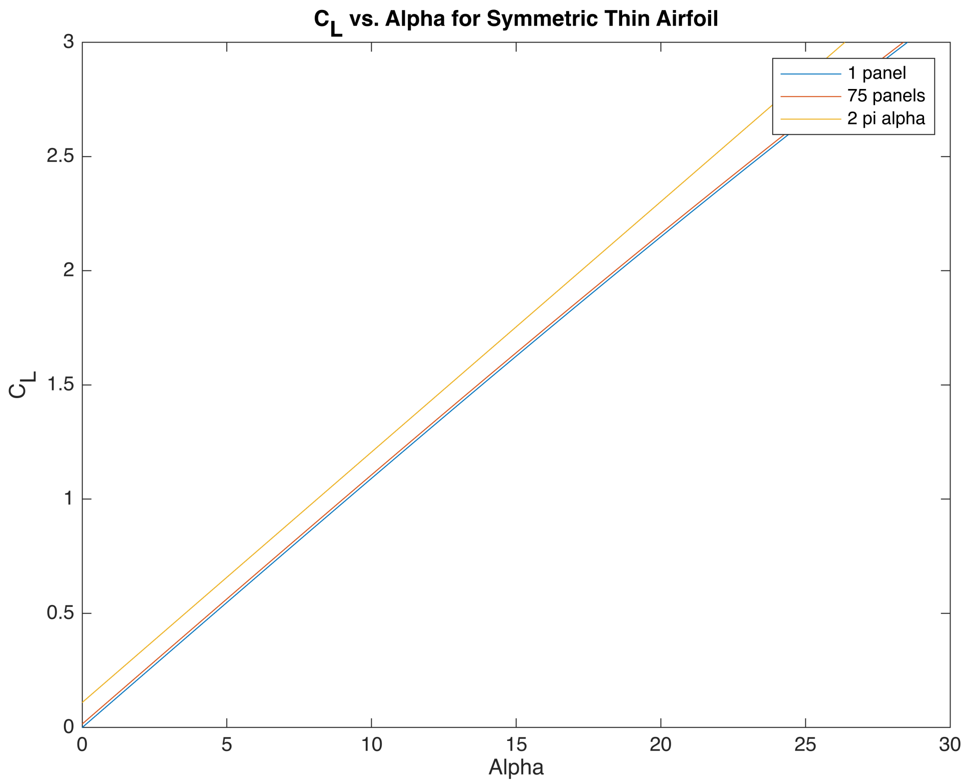
$Lift_1 =$ $Lift_2 =$ $Lift_{tot} =$



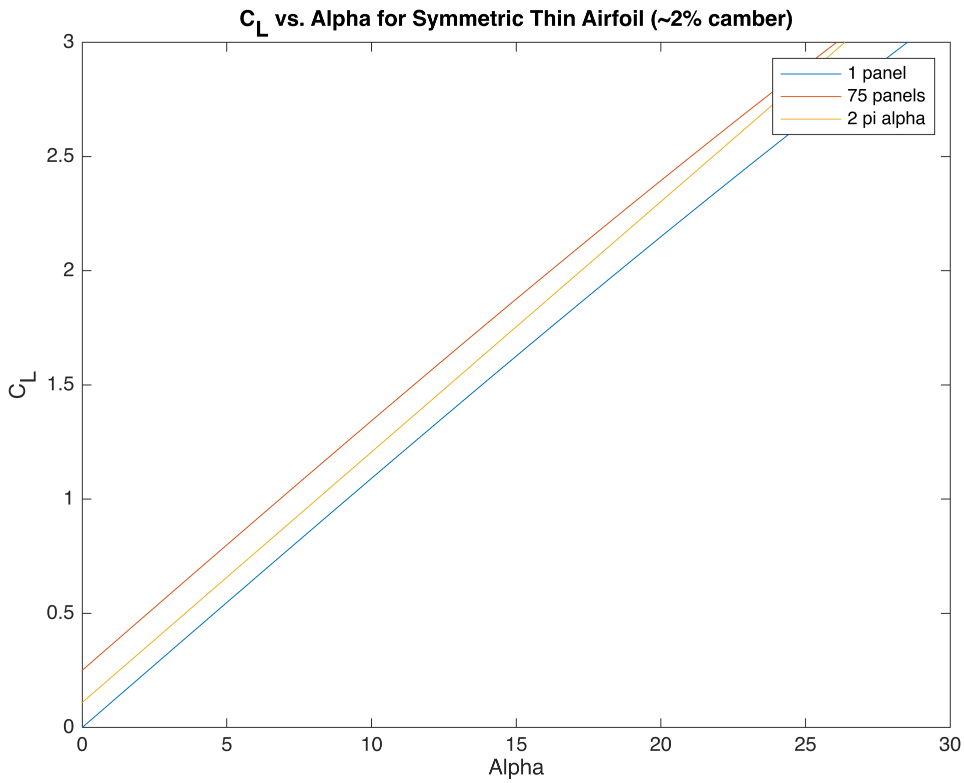
As seen from the plots, there is a strong similarity between the plots and lift values for both numbers of panels. However, as we change the conditions for climbing and descending, we see a change in the vortex shape.

```
clear all
close all
clc
```

```
% Problem 1
for alpha=1:31

    [~,C_l,~,~,~,~,~,~] = computePanelData(10,-1000,alpha-1,1,1);
    Cl_1_panel(alpha)=C_l;

    [~,C_l,~,~,~,~,~,~] = computePanelData(10,-1000,alpha-1,75,1);
    Cl_75_panel(alpha)=C_l;

    Cl_eqn(alpha)= 2*pi*(alpha*(pi/180));

end

alpha = linspace(0,30,31);
Cl_combined = [Cl_1_panel;Cl_75_panel;Cl_eqn];
plot(alpha,Cl_combined);
title('C_L vs. Alpha for Symmetric Thin Airfoil')
xlabel('Alpha')
ylabel('C_L')
legend('1 panel', '75 panels', '2 pi alpha')
ylim([0,3])
xlim([0,30])
```




```
% Problem 2

for alpha=1:31

    [~,C_l,~,~,~,~,~,~] = computePanelData(10,-62,alpha-1,1,1);
    Cl_1_panel(alpha)=C_l;

    [~,C_l,~,~,~,~,~,~] = computePanelData(10,-62,alpha-1,75,1);
    Cl_75_panel(alpha)=C_l;

    Cl_eqn(alpha)= 2*pi*(alpha*(pi/180));

end

alpha = linspace(0,30,31);

Cl_combined = [Cl_1_panel; Cl_75_panel; Cl_eqn];

plot(alpha,Cl_combined);
title('C_L vs. Alpha for Symmetric Thin Airfoil (~2% camber)')
xlabel('Alpha')
ylabel('C_L')
legend('1 panel', '75 panels', '2 pi alpha')
ylim([0,3])
xlim([0,30])
```



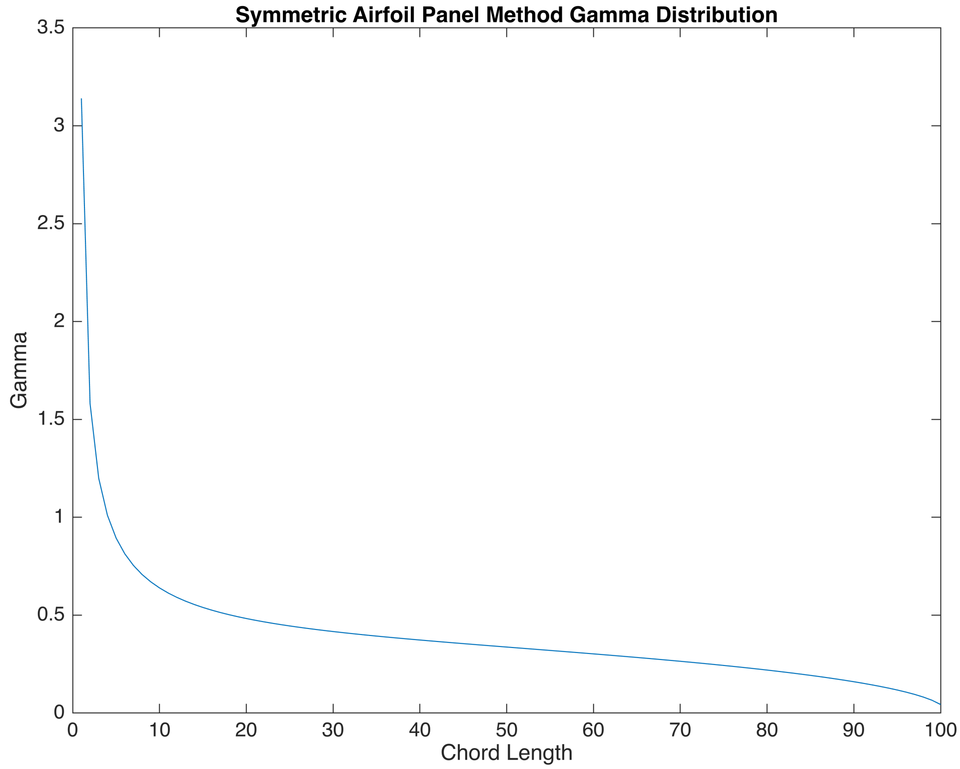

```
% Problem 3

[gamma,~,~,~,~,~,~,~] = computePanelData(10,-1000,5,75,1);

circulation = sum(gamma)
```

```
circulation = 28.1533
```

```
circulation_eqn = 10*10*pi*5*(pi/180)
```

```
circulation_eqn = 27.4156
```

```
% Problem 4

[gamma,~,camber,~,~,~,~,~] = computePanelData(10,-62,5,100,1);
l = linspace(1,length(gamma),length(gamma));
plot(l,gamma)
title('Symmetric Airfoil Panel Method Gamma Distribution');
xlabel('Chord Length');
ylabel('Gamma');
```




```
% Problem 5

N = 75;
% v_inf = 203.2;
v_inf = 202.5;
% alpha_d = 4.68;
alpha_d = 0;
alpha_r = alpha_d*(pi/180);
grid_res = 5;
v_inf_x = v_inf*cos(alpha_r);
v_inf_y = v_inf*sin(alpha_r);
rho = 0.00238;

% create the upstream airfoil
x_i_u = 1;
y_i_u = 1;
x_f_u = 2.494;
y_f_u = 0.869;

% create the downstream airfoil
x_i_d = 2.624;
y_i_d = 0.867;
x_f_d = 4.109;
y_f_d = 0.659;
```

```
% compute points for the upstream airfoil
x_u = linspace(x_i_u,x_f_u,N+1);
y_u = linspace(y_i_u,y_f_u,N+1);
plot(x_u,y_u);

hold on

% compute points for the downstream airfoil
x_d = linspace(x_i_d,x_f_d,N+1);
y_d = linspace(y_i_d,y_f_d,N+1);
plot(x_d,y_d);

axis equal
xlim([0,5])
ylim([0,2])

% creating the panels

panel_coords_g = zeros(N,4);
panel_coords_g_u = zeros(N,4);
panel_coords_g_d = zeros(N,4);

for i=1:N
    panel_coords_g_u(i, 1) = x_u(i);
    panel_coords_g_u(i, 2) = y_u(i);
    panel_coords_g_u(i, 3) = x_u(i+1);
    panel_coords_g_u(i, 4) = y_u(i+1);
end

for i=1:N
    panel_coords_g_d(i, 1) = x_d(i);
    panel_coords_g_d(i, 2) = y_d(i);
    panel_coords_g_d(i, 3) = x_d(i+1);
    panel_coords_g_d(i, 4) = y_d(i+1);
end

panel_coords_g =[panel_coords_g_u; panel_coords_g_d];

% declaring panel length

L = 1.5/N;

% calculating the normal and tangential vectors

x_g = zeros(2*N,2);
for i=1:2*N
    x_g(i,1) = (panel_coords_g(i,3)-panel_coords_g(i,1))/L;
    x_g(i,2) = (panel_coords_g(i,4)-panel_coords_g(i,2))/L;
end
```

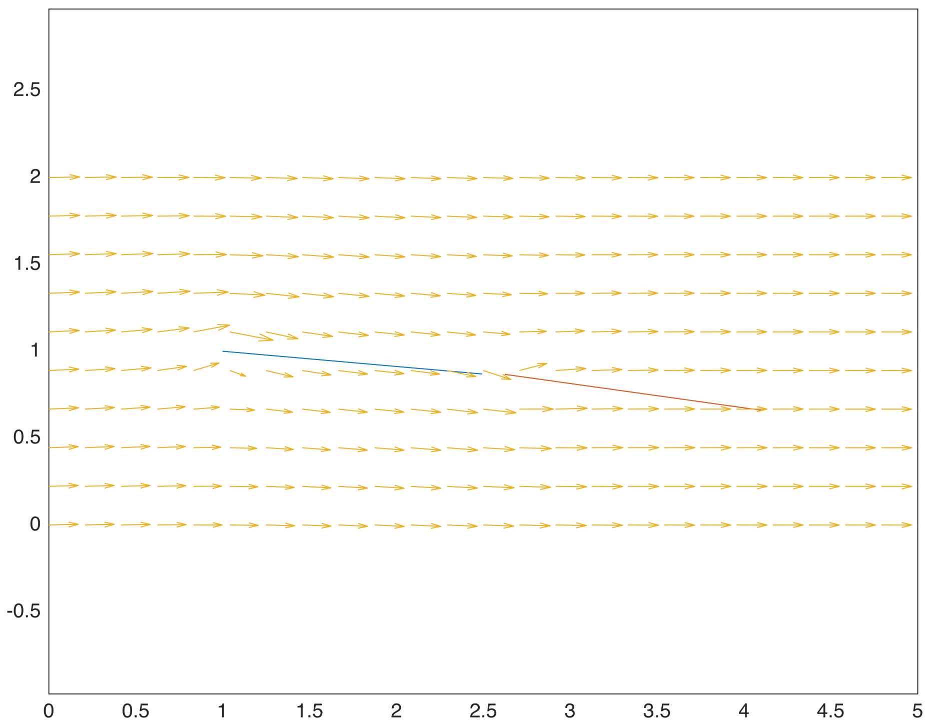
```
y_g = zeros(2*N,2);
y_g = [0 1; -1 0]*transpose(x_g);
y_g = -transpose(y_g);

% computing collocation points in global frame
collocation_g = zeros(2*N,2);
for i=1:2*N
    collocation_g(i,1) = panel_coords_g(i,1) + 0.75*L*x_g(i,1);
    collocation_g(i,2) = panel_coords_g(i,2) + 0.75*L*x_g(i,2);
end

% creating icm
icm = zeros(2*N, 2*N);
for i=1:2*N
    for j=1:2*N
        % x and y distances
        x_dist = collocation_g(j,1) - panel_coords_g(i,1);
        y_dist = collocation_g(j,2) - panel_coords_g(i,2);

        % Panel Velocity (i frame)
        u_p = (1/(2*pi)) * (y_dist / ((y_dist)^2 + (x_dist-.25*L)^2));
        v_p = (-1/(2*pi)) * ((x_dist-.25) / ((y_dist)^2 + (x_dist-.25*L)^2 ));

        % Panel Velocity (global frame)
        B = inv([x_g(i,1) y_g(i,1); x_g(i,2) y_g(i,2)]);
        A = [u_p v_p];
        C = A * B;

        % updating icm matrix entry
        icm(j,i) = dot(C,y_g(j,:));
    end
end

% computing v_infinity normal and calculating gamma

v_inf_norm = zeros(2*N,1);
for i=1:2*N
    v_inf_norm(i) = -dot([v_inf_x, v_inf_y], y_g(i,:));
end
gamma = icm \ v_inf_norm;

% visualizing flow
mesh_X = linspace(0,5,5*grid_res);
mesh_Y = linspace(0,2,2*grid_res);
[X,Y]=meshgrid(mesh_X,mesh_Y);
dim_x = length(mesh_X);
dim_y = length(mesh_Y);
U = zeros(dim_y, dim_x);
V = zeros(dim_y, dim_x);
```

```
for i=1:dim_y
    for j=1:dim_x
        u_p_g = 0;
        v_p_g = 0;
        for k=1:2*N
            x = mesh_X(j) - panel_coords_g(k,1);
            y = mesh_Y(i) - panel_coords_g(k,2);

            u_p = (gamma(k)/(2*pi)) * (y/((y^2)+((x-.25*L)^2)));
            v_p = (-gamma(k)/(2*pi)) * ((x-.25*L)/((y^2)+((x-.25*L)^2)));

            B = inv([x_g(k,1) y_g(k,1); x_g(k,2) y_g(k,2)]);
            A = [u_p v_p];
            C = A * B;

            u_p_g = u_p_g + C(1);
            v_p_g = v_p_g + C(2);
        end

        U(i,j) = v_inf_x + u_p_g;
        V(i,j) = v_inf_y + v_p_g;
    end
end

quiver(X,Y,U,V);
h=gca; h.XAxis.TickLength = [0 0];
h=gca; h.YAxis.TickLength = [0 0];
axis equal
% title()
hold off
```

```
function [gamma,C_l,camber,X,Y,U,V,panel_origin_g] = computePanelData(v_inf,y_0,alpha_d,↙
N,grid_res)

    % initializing variables
    alpha_r = alpha_d*(pi/180);
    v_inf_x = v_inf*cos(alpha_r);
    v_inf_y = v_inf*sin(alpha_r);
    rho = 1.225;

    % creating the airfoil
    L = 10;
    x_i = 5; y_i = 0;
    x_f = x_i+L; y_f = y_i;
    x_0 = x_i + L/2;
    r = sqrt((x_i-x_0)^2+(y_i-y_0)^2);
    theta_i = 0.5 * (180-(2 * atand((x_0-x_i)/y_0-y_i)));
    theta_f = 90 + (90 - theta_i);
    theta = linspace(theta_i,theta_f,N+1)';
    panel_origin_g = [r * cosd(theta) + x_0, r * sind(theta)+y_0];
    camber = (max(panel_origin_g(:,2)))/L;

    % creating the panels
    panel_coords_g = zeros(N,4);

    for i=1:N
        panel_coords_g(i, 1) = panel_origin_g(i,1);
        panel_coords_g(i, 2) = panel_origin_g(i,2);
        panel_coords_g(i, 3) = panel_origin_g(i+1,1);
        panel_coords_g(i, 4) = panel_origin_g(i+1,2);
    end

    % computing panel lengths
    L = zeros(N,1);
    for i=1:N
        L(i)=sqrt((panel_coords_g(i,1)-panel_coords_g(i,3))^2 + (panel_coords_g(i,2)-↙
panel_coords_g(i,4))^2);
    end

    % computing collocation points in global frame
    x_g = zeros(N,2);
    for i=1:N
        x_g(i,1) = (panel_coords_g(i,3)-panel_coords_g(i,1))/L(i);
        x_g(i,2) = (panel_coords_g(i,4)-panel_coords_g(i,2))/L(i);
    end

    y_g = zeros(N,2);
    y_g = [0 1; -1 0]*transpose(x_g);
    y_g = -transpose(y_g);

    collocation_g = zeros(N,2);
    for i=1:N
        collocation_g(i,1) = panel_coords_g(i,1) + 0.75 * L(i) * x_g(i,1);
        collocation_g(i,2) = panel_coords_g(i,2) + 0.75 * L(i) * x_g(i,2);
    end

    % creating icm matrix
    icm = zeros(N, N);
    for i=1:N
```

```
        for j=1:N
            % x and y distances
            x_dist = collocation_g(j,1) - panel_coords_g(i,1);
            y_dist = collocation_g(j,2) - panel_coords_g(i,2);

            % Panel Velocity (i frame)
            u_p = (1/(2*pi)) * (y_dist / ((y_dist)^2 + (x_dist-.25*L(i))^2));
            v_p = (-1/(2*pi)) * ((x_dist-.25*L(i)) / ((y_dist)^2 + (x_dist-.25*L(i))^2↙
));

            % Panel Velocity (global frame)
            B = inv([x_g(i,1) y_g(i,1); x_g(i,2) y_g(i,2)]);
            A = [u_p v_p];
            C = A * B;

            % updating icm matrix entry
            icm(j,i) = dot(C,y_g(j,:));
        end
    end

    % computing v_infinity normal and calculating gamma
    v_inf_norm = zeros(N,1);
    for i=1:N
%         disp(size(y_g))
        v_inf_norm(i) = -dot([v_inf_x, v_inf_y], y_g(i,:));
    end
    gamma = inv(icm) * v_inf_norm;

    % visualizing  flow
    mesh_X = linspace(0,20,20*grid_res);
    mesh_Y = linspace(3,-3,6*grid_res);
    [X,Y]=meshgrid(mesh_X,mesh_Y);
    dim_x = length(mesh_X);
    dim_y = length(mesh_Y);
    U = zeros(dim_y, dim_x);
    V = zeros(dim_y, dim_x);
    C_l = 2*sum(gamma)/(v_inf*10);

    for i=1:dim_y
        for j=1:dim_x
            u_p_g = 0;
            v_p_g = 0;
            for k=1:N
                x = mesh_X(j) - panel_origin_g(k,1);
                y = mesh_Y(i) - panel_origin_g(k,2);

                u_p = (gamma(k)/(2*pi)) * (y/((y^2)+((x-.25*L(k))^2)));
                v_p = (-gamma(k)/(2*pi)) * ((x-.25*L(k))/((y^2)+((x-.25*L(k))^2)));

                B = inv([x_g(k,1) y_g(k,1); x_g(k,2) y_g(k,2)]);
                A = [u_p v_p];
                C = A * B;

                u_p_g = u_p_g + C(1);
                v_p_g = v_p_g + C(2);
            end

            U(i,j) = v_inf_x + u_p_g;
```

```
            V(i,j) = v_inf_y + v_p_g;
        end
    end
%     lift =
%     disp()
end
```




```
lift = sum(gamma)*v_inf*rho
```

```
lift = 17.4098
```