

1.1

$$\begin{aligned} J(\beta) &= (y - X\beta)^2 + \lambda\beta\beta \\ &= (y - X\beta)^T(y - X\beta) + \beta^T(\lambda I)\beta \\ &= y^T y - 2y^T X\beta + X^T X\beta^T \beta + \beta^T(\lambda I)\beta \end{aligned}$$

$$\begin{aligned} \nabla_{\beta} J(\beta) &= -2y^T X + 2X^T X\beta + 2\lambda I\beta \\ 0 &= -2y^T X + \beta(2X^T X + 2\lambda I) \\ 2y^T X &= \beta(2X^T X + 2\lambda I) \\ \beta &= (X^T X + \lambda I)^{-1} y^T X \end{aligned}$$

1.2

$$X = \begin{bmatrix} 1 & 2 \\ 3 & 6 \\ 5 & 10 \end{bmatrix}, Y = [1 \quad 2 \quad 3]^T$$

$$\theta = (X^T X)^{-1} X^T y$$

$$X^T X = \begin{bmatrix} 5 & 15 & 25 \\ 15 & 45 & 75 \\ 25 & 75 & 125 \end{bmatrix} \rightarrow \text{non invertable}$$

=> Cannot be solved by linear regression because  $X^T X$  is non-invertable

1.3 Lasso Regression should be used because it can generate a sparse  $\beta$  vector

1.5 Ridge regression performs better on this data set. This is probably due to the fact that after running linear regression between  $x_1$  and  $x_2$ , I found that  $\theta \approx 2$ . This means that the features are not linearly independent, and thus the  $X^T X$  matrix is non invertable, so using  $\lambda I$  in the ridge regression to regularize the function makes it so that the  $X^T X$  matrix is guaranteed to be invertable, resulting in a more accurate  $\beta$