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CS 6501 HW2

1.1

$$\begin{aligned} J(\beta) &= (y - X\beta)^2 + \lambda\beta\beta \\ &= (y - X\beta)^T(y - X\beta) + \beta^T(\lambda I)\beta \\ &= y^T y - 2yX\beta + X^T X\beta^T\beta + \beta^T(\lambda I)\beta \end{aligned}$$

$$\begin{aligned} \nabla_\beta J(\beta) &= -2y^T X + 2X^T X\beta + 2\lambda I\beta \\ 0 &= -2y^T X + \beta(2X^T X + 2\lambda I) \\ 2y^T X &= \beta(2X^T X + 2\lambda I) \\ \beta &= (X^T X + \lambda I)^{-1} y^T X \end{aligned}$$

1.2

$$X = \begin{bmatrix} 1 & 2 \\ 3 & 6 \\ 5 & 10 \end{bmatrix}, Y = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$$

$$\theta = (X^T X)^{-1} X^T y$$

$$X^T X = \begin{bmatrix} 5 & 15 & 25 \\ 15 & 45 & 75 \\ 25 & 75 & 125 \end{bmatrix} \longrightarrow \textit{non invertable}$$

=> Cannot be solved by linear regression because $X^T X$ is non-invertable

1.3 Lasso Regression should be used because it can generate a sparse β vector

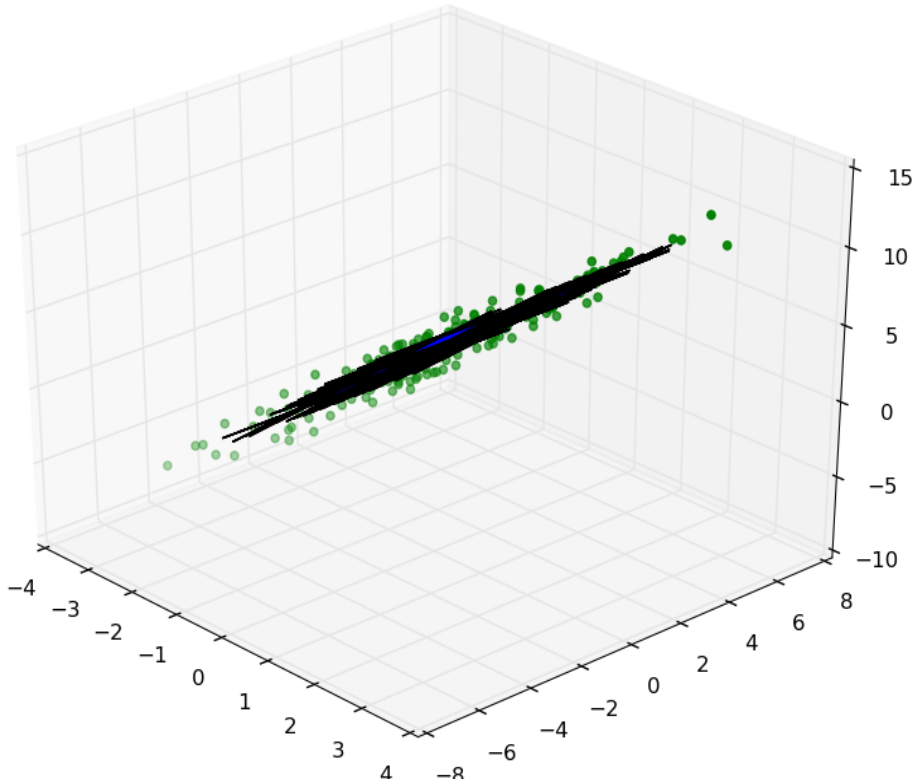
1.4 See graphs at end of document

1.5 Ridge regression performs better on this data set. This is probably due to the fact that after running linear regression between x_1 and x_2, I found that $\theta \approx 2$. This means that the features are not linearly independent ($x_2 \approx 2x_1$), and thus the $X^T X$ matrix is non invertable, so using λI in the ridge regression to regularize the function makes it so that the $X^T X$ matrix is guaranteed to be invertable, resulting in a more accurate β.

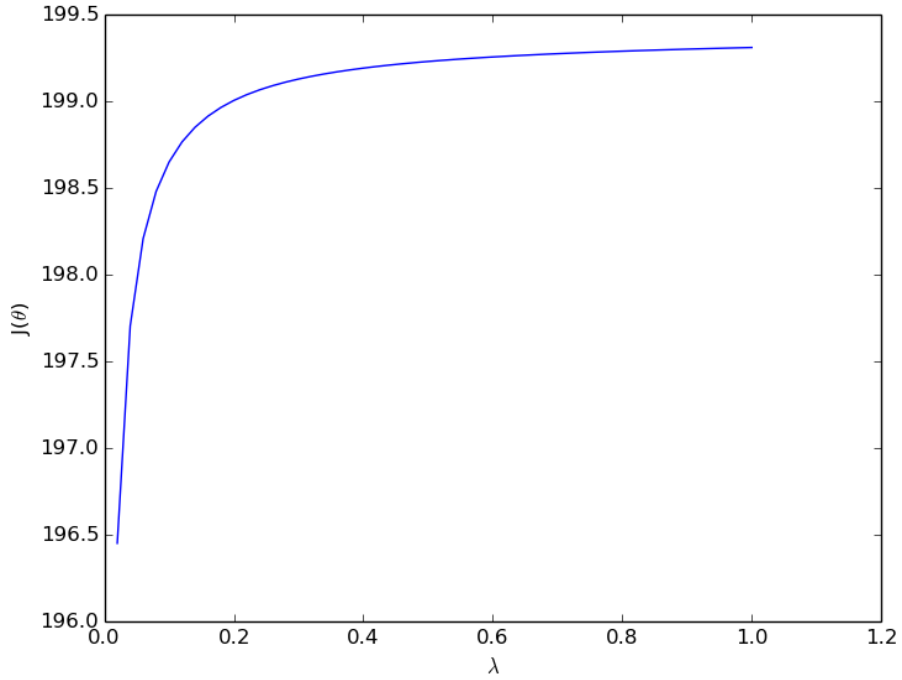
2.3 Table of Kernels and Parameters

Kernel	C	Degree	Gamma	Test Accuracy	Train Accuracy
linear	1	3	0	84.8683%	84.6414%
poly	1	3	0	78.0783%	78.2669%
poly	50	3	0	84.7457%	84.5816%
poly	50	4	0	82.3718%	82.5099%
poly	100	3	0	85.2595%	84.8605%
rbf	1	3	0	84.9976%	84.7343%
rbf	1	3	.1	91.7081%	83.3798%
rbf	50	3	0	86.1381%	85.2988%
rbf	50	3	0.1	91.708%	83.3798%
sigmoid	50	3	0	75.1077%	75.4316%

1.4.1 - linear regression learned plane and data points



1.4.2(a) λ vs J(Θ)



1.4.2(b) - ridge regression learned plane and data points