Facility Location

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1 Facility Location

In a dynamic and continuously changing business and economic environment, the strategic location of facilities plays an important role in determining the success and advantage of companies. Facility location is a field that combines geography, operations research and strategic management. It focuses on optimisation of the locations of facilities such as retail stores, warehouses, distribution centres and service outlets in order to maximise profitability, market accessibility, and customer satisfaction.

The essence of facility location lies in the balance between accessibility to customers and cost efficiency. In a highly competitive market, the proximity of a facility to its target customer can significantly influence consumer choice and market share. Companies must strategically choose locations that not only minimize logistical expenses but also increase their visibility and attractiveness in order to interest potential customers. This strategic decision-making process often involves complex analyses of demographic data, traffic patterns, and competitor locations, all aimed at positioning facilities in a manner that offers a competitive advantage.

Advanced technologies and data analysis techniques have changed the way of location of competitive facilities. Geographical information systems, big data analytics and machine learning algorithms are giving companies new insights into market dynamics and consumer behaviour. These technological advances allow for more accurate facility location, enabling businesses to adapt quickly to changing market conditions and consumer preferences. As a result, the modern approach to facility location is increasingly data-driven, using complex models and simulations to predict outcomes and optimise facility location strategies.

However, the challenge of finding locations for facilities goes beyond data analysis. It involves strategic insights and the ability to adapt to market trends and disruptions. Factors such as urbanisation, technological advances and changes or uncertainties in consumer behaviour are continuously changing the market environment and require flexible and forward-looking methods to determine locations for facilities. Businesses need to identity future trends and incorporate flexibility into their location strategies in order to remain robust and competitive in a dynamic and ever-changing market environment.

Facility location problems are usually formulated as mathematical optimisation problems, where the objective is to determine the optimal location of facilities to achieve specific objectives, such as minimising costs, maximising coverage or simultaneously considering multiple criteria. These problems require the development of mathematical models that incorporate the various constraints and objectives inherent in the process of making decisions on locations of facilities. Variables may refer to possible locations of facilities, while constraints ensure that decisions are consistent with practical requirements such as budget constraints, limits for capacities if facilities, and geographical considerations. Using optimisation techniques such as linear programming, integer programming and heuristic methods, these models aim to find the solution solutions that provide a robust and quantitative basis for strategic decisions on

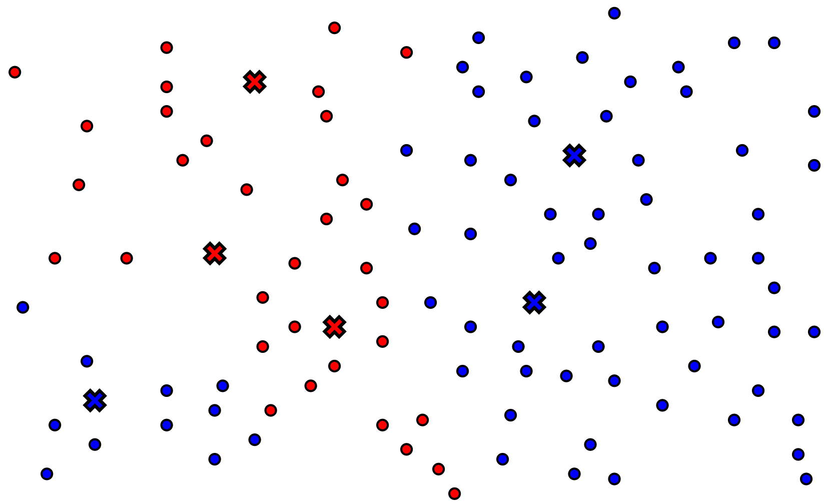

Figure 1: Illustration of a market share separated among 6 competing facilities belonging to two firms.

the location of facilities.

2 Facility Location Problem for an Entering Firm

Consider a geographical region with a demand for goods or services. All customers are considered to be aggregated to geographic demand points which are spatially separated in order to make the problem computationally tractable (see Francis et al. (2002)). It is assumed that customers' demand, qualities of the competitors' facilities and qualities of the new facilities are fixed and known.

There is a set of facilities located in the geographical area, which provides goods or services to satisfy customers demand. Customers from a single demand point choose the most attractive facility to satisfy their demand. See Figure 1 for the illustration of the 100 demand points (circles) and 6 facilities (crosses) belonging to two aforementioned firms: *Red* and *Blue*. Assuming that customers follow the simplest customer behavior model and choose the closest facility to buy goods or services, all buying power from the demand points marked in red goes to the firm *Red*, while all buying power from the blue demand points goes to the firm *Blue*.

If one of the firms on the market wanted to expand its market share, or a new company wanted to enter the market, it would be faced with the optimisation problem to select the optimal locations for the new facilities taking into account competition with the preexisting facilities. Figure 2 extends Figure 1 by three randomly located facilities marked in green, which attract customers from some

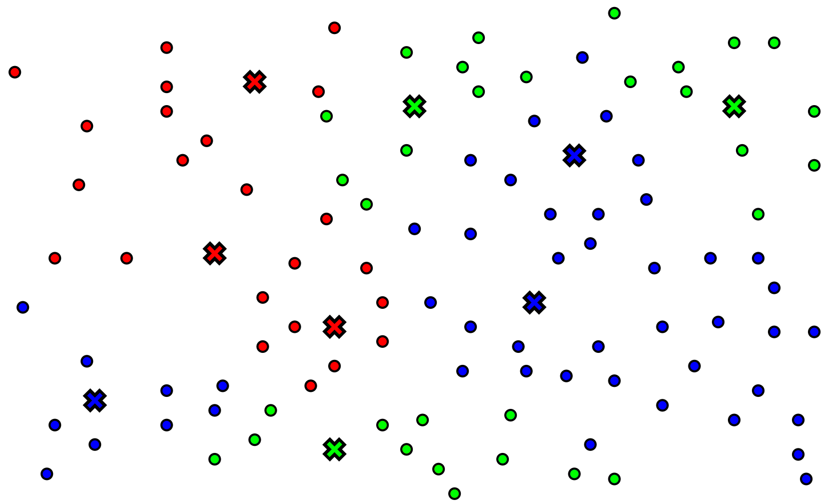

Figure 2: Illustration of a market share of the new facilities.

demand point previously served by facilities marked by red and blue.

The goal is to find the optimal locations for the new facilities in order to maximize their profit while taking into account various aspects, such as competition with other facilities on the market, expected customer behavior, or constraints for the locations.

Let us denote by

$$I = \{i_1, i_2, \dots, n_I\} \tag{1}$$

the a set of demand points and buying power w_i of the i-th demand point $(i = 1, 2, \dots, n_I)$ is fixed and known.

There is a set

$$J = \{j_1, j_2, \dots, n_J\} \tag{2}$$

of preexisting facilities which are already in the market and meet the demand of customers.

Each facility has quality indicator, which is numerically expressed and given as parameter of the problem instance. The attractiveness that customer from the i-th demand point feels by the j-th facility is calculated by

$$a_{ij} = \frac{q_j}{d(i,j) + 1}, \tag{3}$$

where q_j is the quality of the j-th facility and $d(i,j)$ is the geographical distance between i-th demand point and j-th preexisting facility. A closer facility with a moderate quality value may be more attractive than a distant facility with a higher value.

The buying power of each demand point is split between facilities that are Pareto optimal by quality of the facility and distance to the facility. The demand is split proportionally with the attraction that customer feels by these facilities. A customer will patronize a more distant facility only if it is more attractive.

A new firm wants to enter the market by opening a set

$$X = \{x_1, x_2, \dots, n_X\} \tag{4}$$

of new facilities on purpose to attract as much customers as possible taking into account the competition with the preexisting facilities. Assuming that locations for the new facilities can selected from a given set

$$L = \{l_1, l_2, \dots, n_L\} \tag{5}$$

of location candidates, the entering firm faces a mathematical optimization problem to choose a subset $X^* \subset L$ on purpose to maximize the market share of the new facilities. Considering $M(\cdot)$ as an objective function which returns a real value representing a market share of facilities located in a given set of locations ($X \subset L \rightarrow \mathbb{R}$), the optimization problem can be mathematically expressed as

$$X^* = \arg\max_{X \subset L} M(X). \tag{6}$$

Consider that (1) locations of the preexisting and the new facilities coincide with the demand points and (2) a single demand point can host several preexisting facilities, but only one new facility. This imply that X is a subset of L while L is a subset of demand point locations. The problem has a finite search space consisting of all combinations of distinct n_X elements of the set L. The number of combinations depends on the number of the new facilities and the number location candidates and can be calculated by the formula Beeler (2015)

$$\binom{n_L}{n_X} = \frac{(n_L)!}{(n_X)!(n_L - n_X)!}. \tag{7}$$

Such a number identifies the amount of objective function evaluations needed to solve the problem using an explicit complete enumeration algorithm. These algorithms perform enumeration of all possible solutions and, therefore, are computationally expensive and unsuitable to larger problems. in contrast to the explicit complete enumeration, implicit complete enumeration algorithms exclude some search space parts which are definitely sub-optimal. Example of implicit complete enumeration is Branch and Bound algorithm. The third option is the incomplete enumeration which explore some parts of the search space by applying certain heuristics advantageous to a problem at hand. Incomplete enumeration algorithms do not guarantee the optimal solution but provide an approximation of the optimal solution. Well-known incomplete enumeration methods are Genetic Algorithm Holland (1975), Ant Colony Optimization, etc.

3 Customer Behavior Models

A customer behavior model in the context of facility location is a concept or mathematical representation that describes how customers choose one or other facility to satisfy their demand based on a variety of factors, such as location, quality of service, price and other attributes. This model aims to simulate or predict customer behavioural patterns in relation to the choice of a facility and plays a key role in the development of effective facility location strategies.

There are many models of customer behaviour, each reflecting different aspects of customers' decision-making processes and applied to different real-world situations. These models range from simple deterministic to complex probabilistic and stochastic models. In the following sections we will describe some of the main models, showing their basic principles, assumptions and applications.

3.1 Binary Customer Behavior Model

The binary customer behavior model assumes that only one facility, which is considered to be the most attractive to customers, satisfies the entire demand of a single demand point. This model works on the principle that all customers in a single demand point will choose a single facility which is the most attractive based on factors such as distance, quality of service, price and other relevant attributes.

Despite its limited applicability, this approach simplifies the decision-making process by assuming that customers do not split their demand between several facilities (except for certain cases), thus creating a clear and simple facility location planning, focusing on two main customer preferences: distance and quality.

Consider a simple example to illustrate the concept of the model. There are one demand point and five facilities located nearby, as illustrated in Figure 3. The black circle represents the demand point, while the red and blue circles stand for facilities owned by two different firms. The values inside the red and blue circles indicate the quality values of the respective facilities, while the values above the circles indicates the distance from the demand point (number on the left) and the attractiveness that customers feel towards each facility (number on the right), determined by a combination of distance and quality using equation (3).

Since all facilities have equal qualities, but different distances from the demand point (varying from 10 to 30 km), customer choice becomes based on the distance only and the closest facility gets 100% of the buying power from the demand point.

Figure 4 illustrates the layout of the facilities with different quality values ranging from 25 to 70. The illustration shows that the closest facility with a quality value of 30 has a lower attractiveness value (2.7) compared to the more distant facility with a quality value of 70 (3.3), which captures all the buying power of the demand point. On the other hand, the facility with the highest quality value does not attract customers due to its long distance from

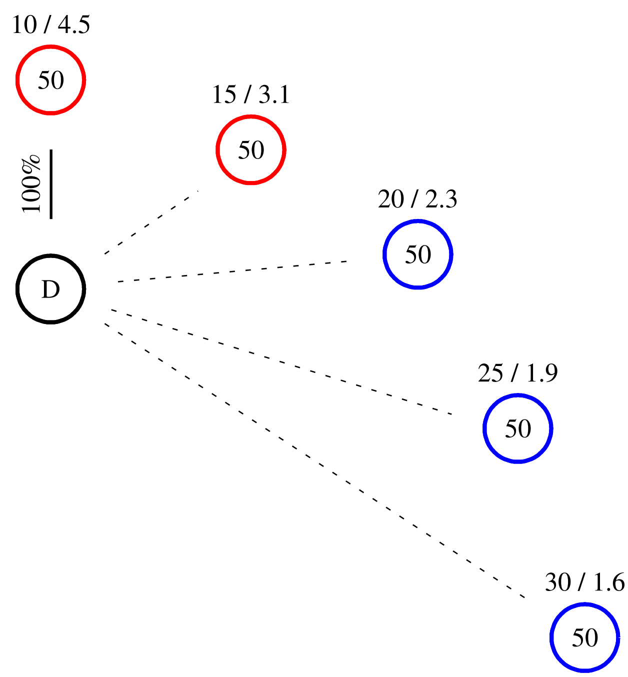

Figure 3: Illustration of distribution of demand among facilities following binary customer choice rule.

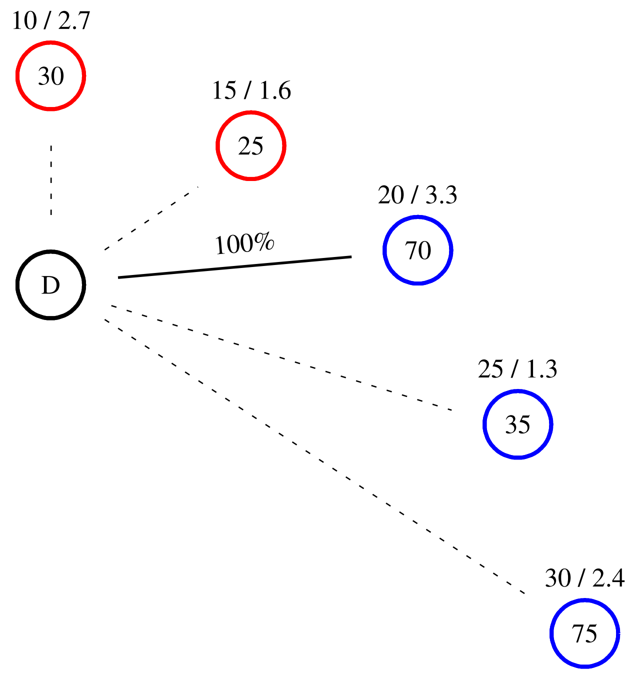

Figure 4: Illustration of distribution of demand among facilities following binary customer choice rule.

the demand point, which has significant negative impact to the attractiveness value.

It may occur that there is more than one facility with an equal maximum attractiveness. In such cases, ties are resolved by dividing the demand among the tied facilities.

The set of demand points for which the most attractive facility belongs set X of new facilities can be mathematically expressed as

$$I^{>} = \{i \in I : a_i(X) > \max\{a_i(J_k) : k \in K\}\}.$$

The most attractive new facility and the most attractive preexisting facility can have equal attractiveness values. The set of such tied facilities can be mathematically expressed as follows:

$$I^{=} = \{i \in I : a_i(X) = \max\{a_i(J_k) : k \in K\}\}.$$

The market share captured by the new facilities can be mathematically ex-

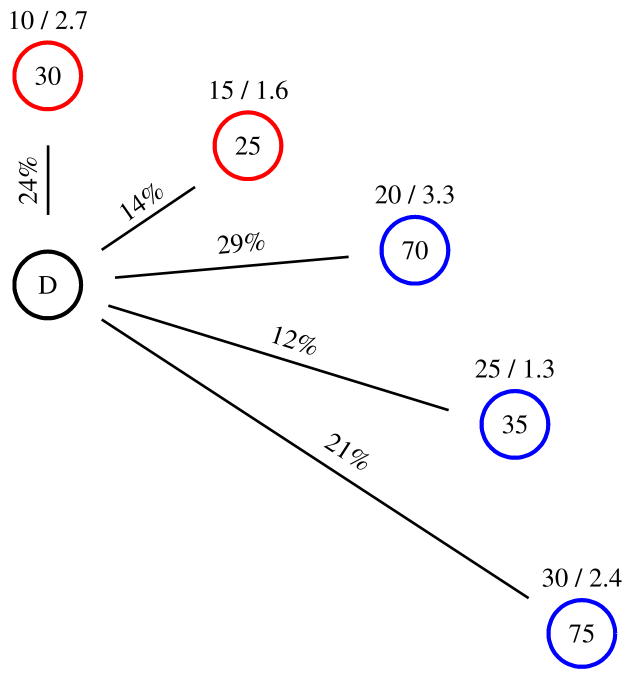

Figure 5: Illustration of distribution of demand among facilities following proportional customer choice rule.

pressed as

$$M_B(X) = \sum_{i \in I^>} w_i + \sum_{i \in I^=} \theta w_i, \tag{8}$$

where θ_i is the proportion of demand captured from the customer i in case of ties.

3.2 Proportional Customer Behavior Model

Proportional customer behavior model assumes that the probability of a customer patronizing a facility is directly proportional to an attractiveness measure and inversely proportional to a certain power of the distance to the facility. Therefore, the buying power of a single demand point is divided among all facilities in proportion to their attractiveness. This proportional model is named after its author Huff Huff (1964) and commonly referred to as the Huff model.

Consider an example with one demand point and five facilities located nearby, as illustrated in Figure 5 using the same notation as in examples of binary rule (Figures 4 and 3).

Due to the varying distances to the facilities and their differing qualities, each facility has a different attractiveness measures. Despite these differences, all facilities attract a portion of the demand point's buying power based on their attractiveness to the customers. In this example, the most attractive facility captures 29 percent of the demand point's buying power, while the least attractive facility attracts only 12 percent.

The market share captured by the set X of facilities can be mathematically expressed as

$$M(X) = \sum_{i \in I} w_i \frac{\sum\limits_{j \in X} a_{ij}}{\sum\limits_{j \in X} a_{ij} + \sum\limits_{k \in K} \sum\limits_{j \in J_k} a_{ij}}. \tag{9}$$

3.3 Partially Binary Rule for Customer Behavior

The partially binary customer behavior model assumes that while customer preferences may be split among several firms, their loyalty to individual facilities within each firm is highly selective. The total demand of the customer in this model is served by all firms, but the customers patronize only one facility per firm – the facility with the maximum attractiveness. The total demand is split between those facilities in proportion with their attractiveness.

Consider an example with one demand point and five facilities located nearby, as illustrated in Figure 6 using the same notation as in example of binary rule (Figure 3).

The customers select the most attractive facility from one firm (marked in red) and the most attractive facility from another firm (marked in blue). The total demand is divided between these two facilities, while the other facilities are ignored.

The market share captured by the set X of facilities can be mathematically expressed as

$$M_{pb}(X) = \sum_{i \in I} w_i \frac{a_i(X)}{a_i(X) + \sum_{k \in K} a_i(J_k)} \tag{10}$$

3.4 Pareto-Huff Customer Behavior Model

The proportional (or Huff) customer behavior model assumes that customers patronize very distant facilities, even when more attractive facilities are much closer, although the captured demand by these distant facilities is small. To avoid this less realistic allocation, a modified version of the Huff model, incorporating Pareto optimality, is considered. This so-called Pareto-Huff customer behavior model Peeters and Plastria (1998) assumes that customers from a single demand point will patronize only those facilities that are Pareto optimal by quality and distance.

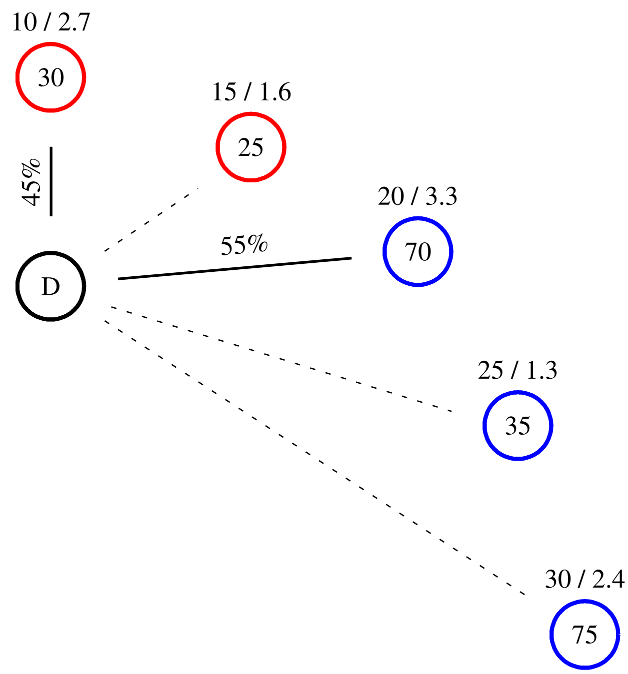

Figure 6: Illustration of distribution of demand among facilities following partially binary customer choice rule.

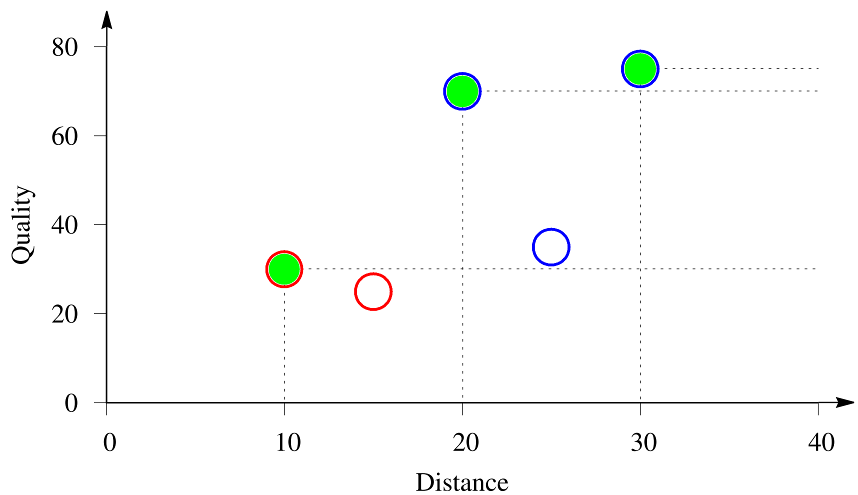

Figure 7: Illustration of distribution of demand among facilities following proportional customer choice rule.

Consider the previously discussed example of the proportional customer behavior model with one demand point and five facilities located nearby, as illustrated in Figure 5. Among the five facilities, only three are Pareto optimal in terms of quality and distance: the first, the third, and the last (counting from the top-left corner). The layout of the facilities based on their distance and quality is shown in Figure 7, where the horizontal axis stands for the distance from the demand point to a facility and the vertical axis stands for the quality of the quality of a facility.

One can see from the figure, the second and the fourth facilities are dominated by the first and the third, respectively, as they are better by both distance and quality. Customers from the demand point will ignore the dominated facilities and split their demand among Pareto optimal facilities proportionally to their attractiveness. The distribution of the demand among facilities is illustrated in Figure 8, where dominated facilities are ignored by the customers.

Lets denote by PH_i the set of facilities that are Pareto optimal to the i-th demand point. The $PH_i \cap X$ denotes the set of Pareto optimal facilities that belong to a set X of facilities. Then the market share captured by the set X of facilities can be mathematically expressed as

$$M_{PH}(X) = \sum_{i \in I} w_i \frac{\sum\limits_{j \in PH_i \cap X} a_{ij}}{\sum\limits_{j \in PH_i} a_{ij}}. \tag{11}$$

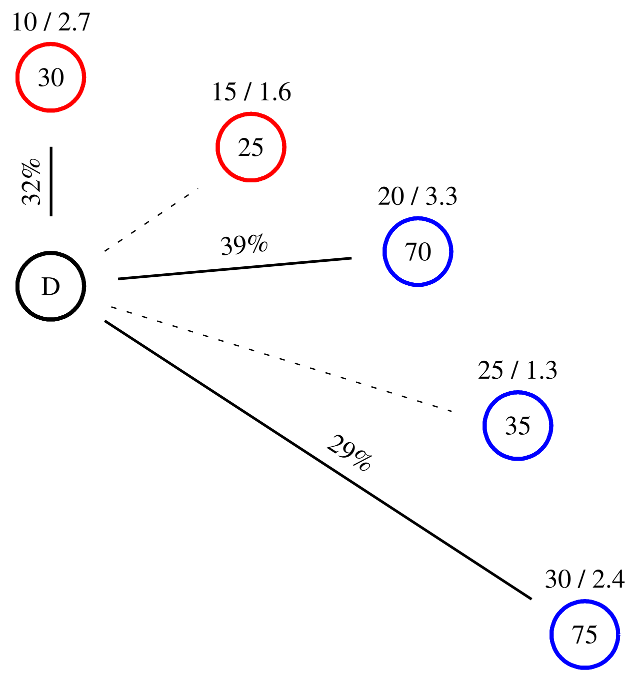

Figure 8: Illustration of distribution of demand among facilities following proportional customer choice rule.

4 Robust Facility Location

The robust facility location problem (RFLP) is a critical area of research in operations research and optimization, particularly in the context of designing resilient infrastructures such as Park & Ride hubs. This literature review synthesizes recent findings on robust facility location, its applications in green infrastructure design, and the integration of advanced optimization techniques, including artificial intelligence methods.

Robust facility location models are designed to address uncertainties in demand and facility disruptions, making them particularly relevant for applications in urban planning and green infrastructure. For instance, Xu et al. propose hybrid approaches that incorporate multi-objective programming and analytic hierarchy processes to optimize the location of emergency medical facilities under uncertain conditions, showcasing the versatility of robust optimization in facility location problems (Xu et al., 2022). Similarly, Cheng et al. explore robust facility location under demand uncertainty and facility disruptions, emphasizing the need for reliable systems that can withstand various operational challenges (Cheng et al., 2021). These studies underscore the importance of developing robust models that can adapt to changing urban dynamics, particularly in the context of green infrastructure, which aims to enhance urban resilience and sustainability.

5 Green Infrastructure Design

The integration of socio-ecological factors into the design and implementation of green infrastructure is essential for its success. Tayouga and Gagné highlight that collaborative efforts among urban planners, developers, and scientists are crucial for articulating the linkages between ecological and social systems (Tayouga and Gagné, 2016). This collaborative approach can enhance the effectiveness of robust facility location models by ensuring that they account for the diverse needs and behaviors of urban populations. Furthermore, the work of Oijstaeijen et al. emphasizes the importance of valuation toolkits in urban planning, which can aid in demonstrating the multiple benefits of green infrastructure to stakeholders (Oijstaeijen et al., 2020). By incorporating these socio-ecological considerations, robust facility location models can be better aligned with the goals of sustainable urban development.

Customer behavior models play a significant role in optimizing the location of facilities such as Park & Ride hubs. Understanding how users interact with these facilities can inform the design of more effective transportation networks. For example, multi-agent systems and reinforcement learning techniques can be employed to simulate user behavior and optimize facility placement dynamically. This approach is supported by the findings of Li et al., who discuss the use of robust optimization techniques to address uncertainties in user demand and facility performance (Li et al., 2022). By leveraging artificial intelligence methods, planners can create adaptive systems that respond to real-time data, thereby enhancing the efficiency and reliability of facility networks.

Moreover, the application of robust optimization in the context of green infrastructure design is gaining traction. Luan et al. argue for a resilient design paradigm that incorporates adaptive approaches to urban green infrastructure, which can be enhanced through robust facility location models (Luan et al., 2021). This perspective aligns with the findings of Karsten et al., who emphasize the need for dynamic facility location models that can accommodate changes in urban environments, such as population shifts and evolving transportation needs Karsten et al. (2023). By integrating robust optimization with green infrastructure planning, urban areas can develop more sustainable and resilient transportation networks that effectively serve the community.

The problem of finding locations for Park & Ride hubs can be formulated as a discrete competitive facility location problem. The goal is to determine the optimal locations for Park & Ride hubs, considering the competition between different hubs for customers and the objective to maximize the efficiency of these facilities while minimizing congestion, environmental impact, etc.

5.1 Locations for Park & Ride Hubs

The primary objective is to select the best locations for Park and Ride hubs in Vilnius from a set of predefined potential sites. These hubs will compete with one another for users who choose to park their cars at the hubs and switch to public transport or other forms of sustainable mobility, such as bicycles or scooters, to reach their final destination.

Constraints and Considerations

- **Predefined set of location candidates**. There is a finite set of potential locations (discrete points) where the Park & Ride hubs can be located.
- **Competition between hubs**. Each hub competes with other hubs for users. The attractiveness of a hub depends on factors such as proximity to high-traffic areas, connectivity to public transport, convenience, and pricing. Users will choose the hub that minimizes their travel time or cost.
- **Customer behavior and demand**. Demand is influenced by the population density, traffic patterns, and public transport accessibility. The hubs must be placed where the potential demand for Park & Ride services is highest.
- **Capacity limitations**. Each hub has a limited capacity to accommodate vehicles. Overloading one hub can lead to reduced service quality, while underutilized hubs result in inefficient resource use.
- **Budgetary constraints**. There are financial limits regarding the number of hubs that can be established and maintained. This imposes a constraint

on the total number of hubs that can be selected from the set of potential sites.

- **Environmental impact**. The placement of hubs should help reduce overall traffic congestion and carbon emissions by encouraging more users to switch from private car usage to public transport or other sustainable means of transportation.

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