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Integrating Fuzzy Logic, Optimization, and GIS for Ecological Impact Assessments

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ABSTRACT / Appraisal of ecological impacts has been problematic because of the behavior of ecological system and the responses of these systems to human intervention are far from fully understood. While it has been relatively easy to itemize the potential ecological impacts, it has been difficult to arrive at accurate predictions of how these impacts affect popu-

lations, communities, or ecosystems. Furthermore, the spatial heterogeneity of ecological systems has been overlooked because its examination is practically impossible through matrix techniques, the most commonly used impact assessment approach. Besides, the public has become increasingly aware of the importance of the EIA in decision-making and thus the interpretation of impact significance is complicated further by the different value judgments of stakeholders. Moreover, impact assessments are carried out with a minimum of data, high uncertainty, and poor conceptual understanding. Hence, the evaluation of ecological impacts entails the integration of subjective and often conflicting judgments from a variety of experts and stakeholders.

The purpose of this paper is to present an environmental impact assessment approach based on the integration fuzzy logic, geographical information systems and optimization techniques. This approach enables environmental analysts to deal with the intrinsic imprecision and ambiguity associated with the judgments of experts and stakeholders, the description of ecological systems, and the prediction of ecological impacts. The application of this approach is illustrated through an example, which shows how consensus about impact mitigation can be attained within a conflict resolution framework.

Holling's (1978) seminal work on environmental assessments associates the difficulty in predicting the effects of human actions on the environment to the degree of complexity, uncertainty, and understanding of the interactions under scrutiny. One major problem is that ecological impacts cannot be formulated in an algorithm because the critical elements and interactions cannot be fully and unambiguously identified. This problem is typical of unstructured decision problems (see Guariso and Werthner 1989). Ecological impacts are also highly uncertain because the probability of the presumed response of the system to a specific activity is unknown and cannot be estimated. The complexity and uncertainty of ecological impacts are fur-

ther exacerbated by the spatial heterogeneity of ecological systems, and the involvement of the public in decision-making.

It is clear that a lack of theoretical body and baseline data plague the prediction of ecological impacts (Beattie 1995, Harashina 1995, Lawrence 1997). We may generalize, therefore, that the understanding about ecological impacts results from what Terano and others (1989) call "macroknowledge," defined as the understanding that results from a broad spectrum of empirical experiences, including contradictions and unfounded statements, so it lacks logical conformity and cannot be strictly defined nor systematized. Consequently, the assessment of impacts entails quantitative studies, indirect analyses, qualitative approaches, and analogies, alongside with the intuitions, perceptions, and standpoints of experts and stakeholders. Yet, given the complexities and uncertainties entailed, even experts in the same

KEY WORDS: Spatial analysis; Environmental impact assessment; Highways; Multicriteria models; Mathematical programming

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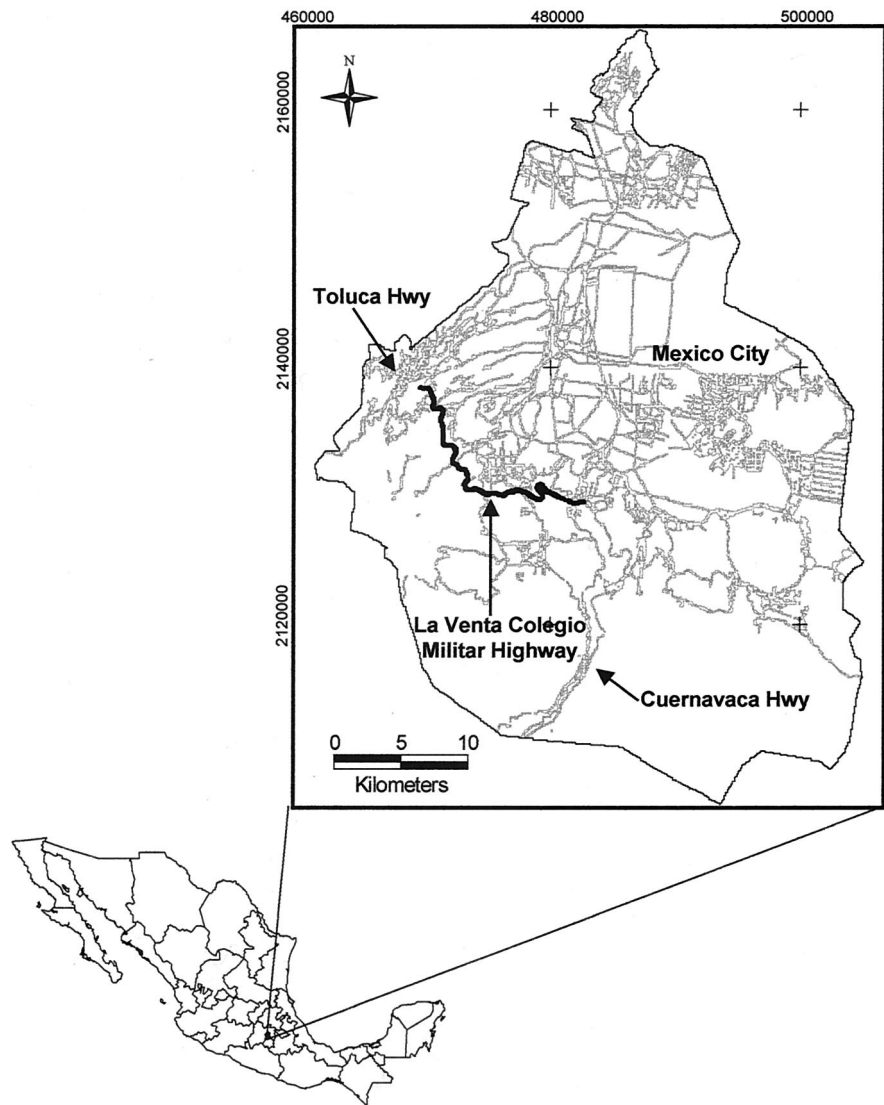


Figure 1. Location of the La Venta-Colégio Militar Highway (project: wide dark line; existing major highways, avenues, and streets in Mexico City: thin gray lines).

domain may significantly disagree about the predicted impacts.

The objective of this paper is to demonstrate how the integration of fuzzy logic, geographic information systems (GIS), and optimization models can address the problems associated with the use of macroknowledge in the prediction of extremely unstructured, highly uncertain, and poorly understood ecological impacts. Fuzzy set theory provides a useful approach to address the problems associated with the intrinsic imprecision, uncertainty and subjectivity of ecological data (Salski 1992). When linked to a GIS, the fuzzy algorithms are used to define homogeneous zones in terms of the significance of the ecological impacts.

The constraint method is then implemented through a GIS-based optimization procedure (Makczewski 1999) to find the best distribution of mitigation measures that should be implemented to attain a specific impact level.

The approach is illustrated by an example: The impact assessment of the La Venta-Colégio Militar highway project (Figure 1) on natural habitats. The example shows how planners and analysts can use this approach to deal with the intrinsic imprecision and ambiguity associated with ecological systems, take into account the spatial context of the ecological impacts, and generate mitigation measures that are acceptable by the stakeholders.

Description of the Approach

Impact assessment procedures follow a series of steps to assure that all the foreseeable effects of a project are taken into account. A typical procedure consists of three steps: (1) project description and environmental characterization, (2) identification and prediction of impacts, and (3) evaluation of impact significance. Since the details on the procedure have been described elsewhere (Bojórquez-Tapia and others 1998), the following description focuses on the development of environmental impact indices, fuzzy impact assessment, and conflict resolution.

Stage 1: Development of Impact Indices

This stage is based on an impact interaction matrix (see Bojórquez-Tapia and others 1998). It is implemented by means of a workshop in which experts and stakeholders jointly define the project activities and describe baseline conditions of the environmental factors likely to be affected. Next, each stakeholder independently assesses the matrix cells in terms of a set of environmental criteria. These are then combined through two impact indices: interaction intensity and environmental vulnerability.

Interaction intensity. The intensity of the interaction between the project activity and the environmental component is evaluated through seven criteria: magnitude, spatial extension, duration, synergic effects, cumulative effects, controversy among the stakeholders, and efficiency of the mitigation measure. Following Bojórquez-Tapia and others (1998), two impact indices can be generated from those criteria: basic index, b_{ij} , and supplementary index, k_{ij} .

$$b_{ij} = \left(\frac{m_{ij} + e_{ij} + d_{ij}}{27} \right) \times f_{ij},$$

$$k_{ij} = \left(\frac{s_{ij} + a_{ij} + c_{ij}}{27} \right) \times f_{ij},$$

where m_{ij} = magnitude; e_{ij} = spatial extension; d_{ij} = duration; s_{ij} = synergic effects; a_{ij} = cumulative effects; c_{ij} = controversy; f_{ij} = mitigation index; i = environmental component; and j = project activity. The mitigation index, f_{ij} , is computed as follows:

$$f_{ij} = 1 - \frac{t_{ij}}{9},$$

where t_{ij} is the mitigation efficiency measure.

For the first six criteria (m_{ij} , e_{ij} , d_{ij} , s_{ij} , a_{ij} , and c_{ij}), the relative importance of each criteria is judged through verbal expressions that correspond to a 0–9 scale: null (0), very low (1), very low-to-low (2), low (3), low-to-moderate (4), moderate (5), moderate-to-high (6),

high (7), high-to-very high (8), and very high (9). In conformity with the precautionary rationale for environmental conflicts (Cowfoot and Wondoleck 1990), whenever uncertainty exists on determining the value of a criterion, the highest value is assigned to an interaction. This lessens the chance of underestimating an impact, and it is equivalent to minimizing the public risk in statistics (Shrader-Frechette and McCoy 1993, Sokal and Rohlf 1995). The mitigation efficiency is also judged in the 0–9 scale described above, but the smallest figure is assigned whenever there is uncertainty in determining such efficiency in order to be congruent with the precautionary principle. Accordingly, the ranges for the two indices are $0 \leq b_{ij} \leq 1$, and $0 \leq k_{ij} \leq 1$.

Consider for example the two stakeholders involved in the La Venta-Colegio Militar impact assessment: conservation advocacy groups (or conservationists) and project's proponents (or developers). Suppose that the conservationists place a higher value on the impact intensity criteria ($m = 7$; $e = 5$; $d = 9$; $s = 5$; $a = 9$; $c = 9$) than the developers ($m = 3$; $e = 3$; $d = 5$; $s = 3$; $a = 4$; $c = 5$). Hence, if $f_{ij} = 1$ ($t_{ij} = 0$), these judgments result in $b_{ij} = 0.78$ and $k_{ij} = 0.85$ for the former, and $b_{ij} = 0.41$ and $k_{ij} = 0.44$ for the latter.

Ecological vulnerability. Ecological vulnerability is defined as the susceptibility of an ecological component to the effects of a project activity. The vulnerability, v_{ij} , of environmental component i , to impacts generated by project activity j is a function of the environmental risk, r_{ij} , the nearness of the source of impact, n_{ij} , and the efficiency of the mitigation measure, f_{ij} :

$$v_{ij} = r_{ij} \times n_{ij} \times f_{ij}.$$

The nearness of an environmental component to a source of impact is computed as:

$$n_{ij} = 1 - \frac{d_{ij}}{d_{max}},$$

where d_{ij} is the distance of site i to the source of impact j , and d_{max} is the maximum distance.

The ecological vulnerability index is operationalized in the GIS by creating map layers for n_{ij} and r_{ij} . The former requires the use of buffer operations from the source of impact (for example, $d_{ij} = 50$ m, 100 m, . . . 500 m), whereas the latter requires the use of reclassification operations of the thematic map layers (for example, land cover, soil, and geomorphology). The reclassification is achieved in two steps: (1) the Analytic Hierarchy Process (AHP, Saaty 1980, Malczewski 1999) or the Simple Multi-Attribute Rating Technique (SMART, Lootsma 1999) is used to estimate the relative vulnerability of each environmental component; and (2) the resulting values are rescaled into a [0,1] interval

Table 1. Criteria and importance weights, derived through the Analytical Hierarchy Process (AHP), related to the ecological vulnerability of ecological components along the track of the La Venta-Colegio Militar highway project

Hierarchy level		
1 st	2 nd	3 rd
Goal (1.000)	Geomorphology (0.189)	Convex ridges (0.120)
		Concave ridges (0.049)
		Alluvial Plains (0.020)
		Oak woodlands (0.303)
	Land cover (0.731)	Pine-pine oak forest (0.246)
		Riparian forests (0.123)
		Grasslands/agriculture (0.037)
		Urban areas (0.022)
	Soils (0.080)	Andosols (0.053)
		Cambisols (0.023)
		Litosols (0.004)

to be consistent with the range of the vulnerability index ($0 \leq v_{ij} \leq 1$).

For example, consider the AHP scores obtained in the La Venta-Colegio Militar impact assessment at the third hierarchy level (Table 1). A linear scale transformation ($x_i \div x_{max}$) of the scores for convex ridges ($0.120 \div 0.120 = 1$), riparian areas ($0.303 \div 0.123 = 0.406$), and andosols ($0.053 \div 0.053 = 0.1$) results in an aggregate value of $r_{ij} = 0.57$ (following the AHP, the aggregated score is obtained by multiplying the rescaled values by the appropriate scores of the elements at the second hierarchy level: $r_{ij} = 0.189 \times 1 + 0.731 \times 0.406 + 0.08 \times 0.1$). Thus, if $n_{ij} = f_{ij} = 1$ then $v_{ij} = 0.57$.

Stage 2: Fuzzy Impact Assessment

Fuzzy logic is a formal mathematical theory for the representation of complex, uncertain, and unstructured problems. Under fuzzy logic, the underlying mechanics of a system is represented linguistically in terms of fuzzy sets, which can be derived from either qualitative or quantitative procedures (Cox 1994, Terano and others 1989). This reduces the complexity of a problem so useful statements about the relevant impacts can be made.

In contrast to classical logic, which the only possibilities of a conclusion are true or false (also known as “crispy” values), fuzzy logic is based in the concept that different grades of truth can exist between false or true. A fuzzy set is a function that maps a value that might be a member of the set to a number between zero and one, indicating its degree of membership: a

degree of zero means that the value is not in the set, and a value of one indicates a full membership in the set; formally (Bojadsziew and Bojadsziew 1995, Cox 1994):

$$A = \{(x, \mu_A(x))\}, \forall x \in A, \mu_A(x) \in [0,1],$$

where $\mu_A(x)$ is the membership function that specifies the degree to which any element x in A belongs to the fuzzy set A .

In general, a fuzzy logic assessment involves four steps: Fuzzyfication, inference, combination, and defuzzyfication (Cox 1994, Bojadsziew and Bojadsziew 1995).

Fuzzyfication. Fuzzyfication is the process by which the degree of truth for a premise is determined through membership functions. It centers on linguistic variables and linguistic qualifiers. A linguistic variable is the name of a fuzzy set and is used to incorporate semantic meanings in the analysis (for example, “low impact”). The qualifiers change the shape of the fuzzy set in predictable ways, and function in the same way as adverbs and adjectives in the English language (for example, “very low impact”). Since the semantic partitions can overlap, the value of any of the three impact indices has greater or less membership in two fuzzy sets. Overlapping arises from the naturally occurring ambiguity associated with the intermediate states of the semantic labels (for example, the term “low impact” may imply some degree of “high impact”).

Nonetheless, it is important to note that fuzzy logic is a mathematical formalism, and a membership grade is a precise number, so the laws of fuzziness are not ambiguous (Kosko 1992, Terano and others 1989). In our approach, the linguistic variables for the basic index, b_{ij} , the complementary index, k_{ij} , and the ecological vulnerability index, v_{ij} , are defined as:

$$b_{ij}, k_{ij}, v_{ij} = \{VL \text{ (very low)}, L \text{ (low)},$$

$$H \text{ (high)}, VH \text{ (very high)}\}.$$

Since the three impact indices range in a $[0,1]$ interval, and being U the universal set such that $U = \{x | x \in U \text{ and } x \in [0,1]\}$, the ranges of the linguistic variables can be delimited as follows: $VL = \{x | x < 0.375\}$; $L = \{x | 0.125 \leq x \leq 0.625\}$; $H = \{x | 0.375 \leq x \leq 0.875\}$; and $VH = \{x | 0.625 \leq x \leq 1\}$. Formally, the linguistic variables are related to the following fuzzy numbers:

$$b_{ij}, k_{ij}, v_{ij} = \left\{ \begin{array}{l} \{(x, \mu_{VL}(x)) | x \in VL, \mu_{VL}(x) \in [0,1]\} \\ \{(x, \mu_L(x)) | x \in L, \mu_L(x) \in [0,1]\} \\ \{(x, \mu_H(x)) | x \in H, \mu_H(x) \in [0,1]\} \\ \{(x, \mu_{VH}(x)) | x \in VH, \mu_{VH}(x) \in [0,1]\} \end{array} \right\}$$

The membership values for b_{ij} , k_{ij} and v_{ij} are obtained by substituting x in the following membership functions:

$$\mu_b(x), \mu_k(x), \mu_v(x) = \begin{cases} \mu_{VL}(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 0.125 \\ -4x + 1.5 & \text{for } 0.125 \leq x \leq 0.375 \end{cases} \\ \mu_L(x) = \begin{cases} 4x - 0.5 & \text{for } 0.125 \leq x \leq 0.375 \\ -4x + 2.5 & \text{for } 0.375 \leq x \leq 0.625 \end{cases} \\ \mu_H(x) = \begin{cases} 4x - 1.5 & \text{for } 0.375 \leq x \leq 0.625 \\ -4x + 3.5 & \text{for } 0.625 \leq x \leq 0.875 \end{cases} \\ \mu_{VH}(x) = \begin{cases} 4x + 2.5 & \text{for } 0.625 \leq x \leq 0.875 \\ 1 & \text{for } 0.875 \leq x \leq 1 \end{cases} \end{cases}$$

Next, the fuzzy sets are “scaled down” by the corresponding membership values (also called fuzzy numbers or α -cuts):

$$\begin{aligned} \mu_{VL}(x_v) &= \mu_{VL}(x) \times \alpha_{VL}, \\ \mu_L(x_v) &= \mu_L(x) \times \alpha_L, \\ \mu_H(x_v) &= \mu_H(x) \times \alpha_H, \\ \mu_{VH}(x_v) &= \mu_{VH}(x) \times \alpha_{VH}. \end{aligned}$$

Figure 2 illustrates the fuzzyfication process as we used it in the La Venta-Colegio Militar impact assessment. Considering the four original fuzzy sets (Figure 2a), an environmental vulnerability of $v_{ij} = 0.57$ results in $\mu_L(v_{ij}) = 0.22$ (shown as α') for low impact and $\mu_H(v_{ij}) = 0.78$ (shown as α') for high impact (Figure 2b). Given these α -cuts, those fuzzy sets are scaled down to 22% and 78% of their original areas, respectively (Figure 2c).

Inference. Inference consists of the application of *if . . . and . . . then* rules or premises to combine two or more fuzzy numbers and produce new fuzzy sets. In our case, the fuzzy numbers for b_{ij} and k_{ij} are combined through the set rules summarized in Table 2 as a $[4 \times 4]$ decision table. Once applied, these rules result in a $[2 \times 2]$ induced decision table (i.e.

Table 2. Decision table showing the *if . . . and . . . then* rules used for the computation of the fuzzy interaction intensity index (VL, Very Low; L, Low; H, High; VH, Very High). The shaded cells correspond to the $[2 \times 2]$ induced decision table

		Index k			
		VL	L	H	VH
Index b	VL	VL	L	H	VH
	L	L	L	H	VH
	H	H	H	H	VH
	VH	VH	VH	VH	VH

shaded cells in Table 2), which is used to obtain the new fuzzy set through the composition conjunction

(Bojadziev and Bojadziev 1995):

$$\alpha_{nm} = \mu(x)_n \wedge \mu(x)_m = \min(\mu(x)_n, \mu(x)_m),$$

where α_{nm} is the fuzzy number corresponding to row n and column m of the induced decision table; and $\mu(x)_n$ and $\mu(x)_m$ are the membership values for the linguistic variables for b_{ij} and k_{ij} respectively.

Once each of the α -cut values have been calculated, the respective fuzzy sets are scaled as follows:

$$\begin{aligned} \mu_{VL}(x) &= \mu_{VL}(x) \times \max(\alpha_{nm}^{VL}, \alpha_{n,m+1}^{VL}, \alpha_{n+1,m}^{VL}, \alpha_{n+1,m+1}^{VL}), \\ \mu_L(x) &= \mu_L(x) \times \max(\alpha_{nm}^L, \alpha_{n,m+1}^L, \alpha_{n+1,m}^L, \alpha_{n+1,m+1}^L), \\ \mu_H(x) &= \mu_H(x) \times \max(\alpha_{nm}^H, \alpha_{n,m+1}^H, \alpha_{n+1,m}^H, \alpha_{n+1,m+1}^H), \\ \mu_{VH}(x) &= \mu_{VH}(x) \times \max(\alpha_{nm}^{VH}, \alpha_{n,m+1}^{VH}, \alpha_{n+1,m}^{VH}, \alpha_{n+1,m+1}^{VH}). \end{aligned}$$

Figure 3 illustrates graphically the inference process as we used it in the La Venta-Colegio Militar impact assessment. Considering the conservationists' judgments, fuzzyfication of the basic index ($b_{ij} = 0.78$) results in $\mu_H(b_{ij}) = 0.39$ and $\mu_{VH}(b_{ij}) = 0.61$, whereas fuzzification of the supplementary index ($k_{ij} = 0.85$) results in $\mu_H(k_{ij}) = 0.09$ and $\mu_{VH}(k_{ij}) = 0.91$ (Figure 3a–b). These α -cut values are then used to scale-down the fuzzy sets (Figure 3c). By applying the inference rules in Table 2, the composition conjunction results in the following fuzzy numbers (Figure 3d):

$$\begin{aligned} \alpha_{33} &= \mu_H(b_{ij})_3 \wedge \mu_H(k_{ij})_3 = \min(0.39, 0.09) = 0.09, \\ &\alpha_{33} \in H, \\ \alpha_{34} &= \mu_H(b_{ij})_3 \wedge \mu_{VH}(k_{ij})_4 = \min(0.39, 0.91) = 0.39, \\ &\alpha_{34} \in VH, \\ \alpha_{43} &= \mu_H(b_{ij})_4 \wedge \mu_{VH}(k_{ij})_3 = \min(0.61, 0.09) = 0.09, \\ &\alpha_{43} \in VH, \\ \alpha_{44} &= \mu_{VH}(b_{ij})_4 \wedge \mu_{VH}(k_{ij})_4 = \min(0.61, 0.91) = 0.61, \\ &\alpha_{44} \in VH, \end{aligned}$$

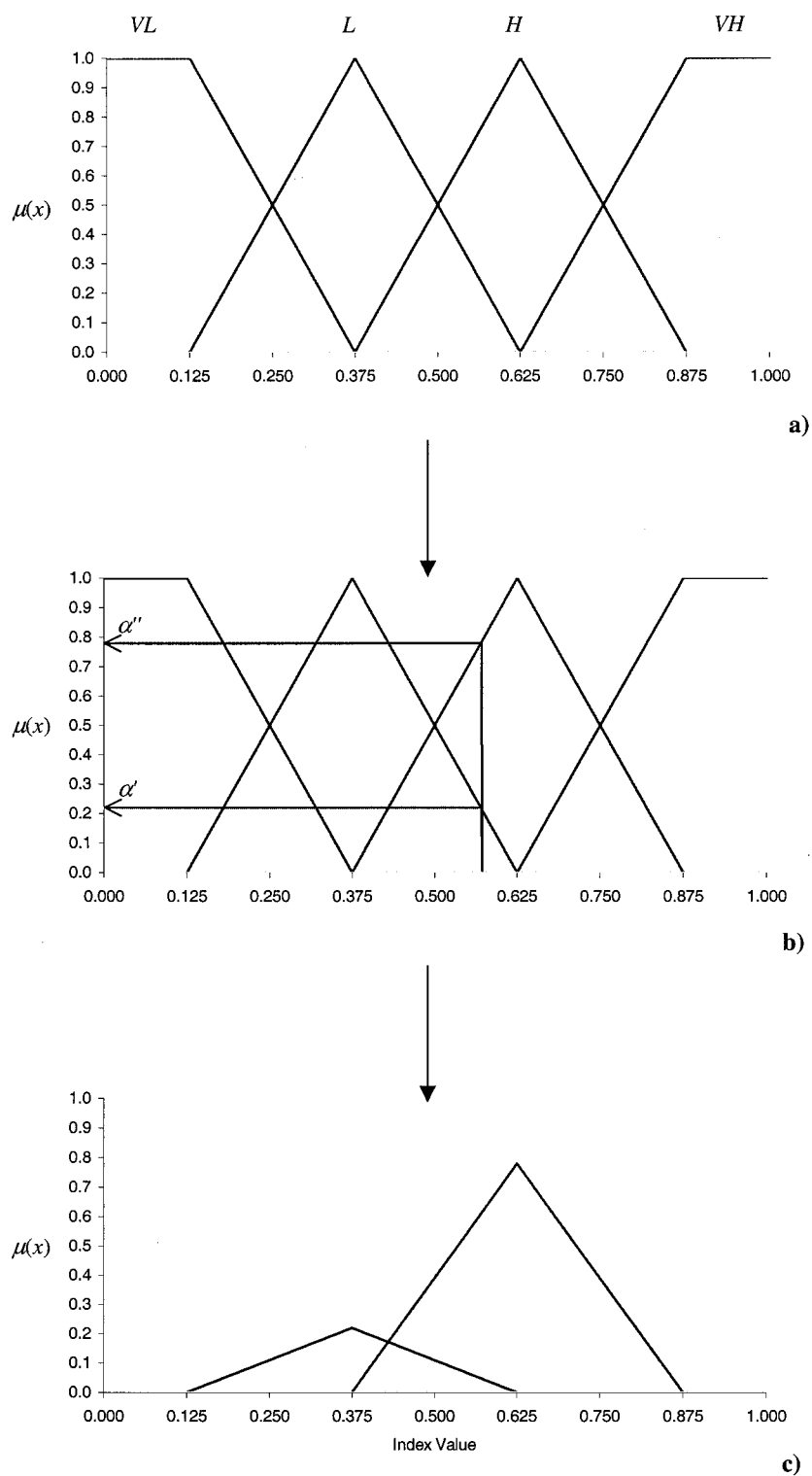


Figure 2. Fuzzification of the ecological vulnerability index $v_{ij} = 0.57$. **a**, fuzzy sets: VL , very low; L , low; H , high; VH , very high; **b**, α -cuts $\mu_L(v_{ij}) = 0.22$ (shown as α') and $\mu_H(v_{ij}) = 0.78$ (shown as α''); and **c**) rescaled fuzzy sets.

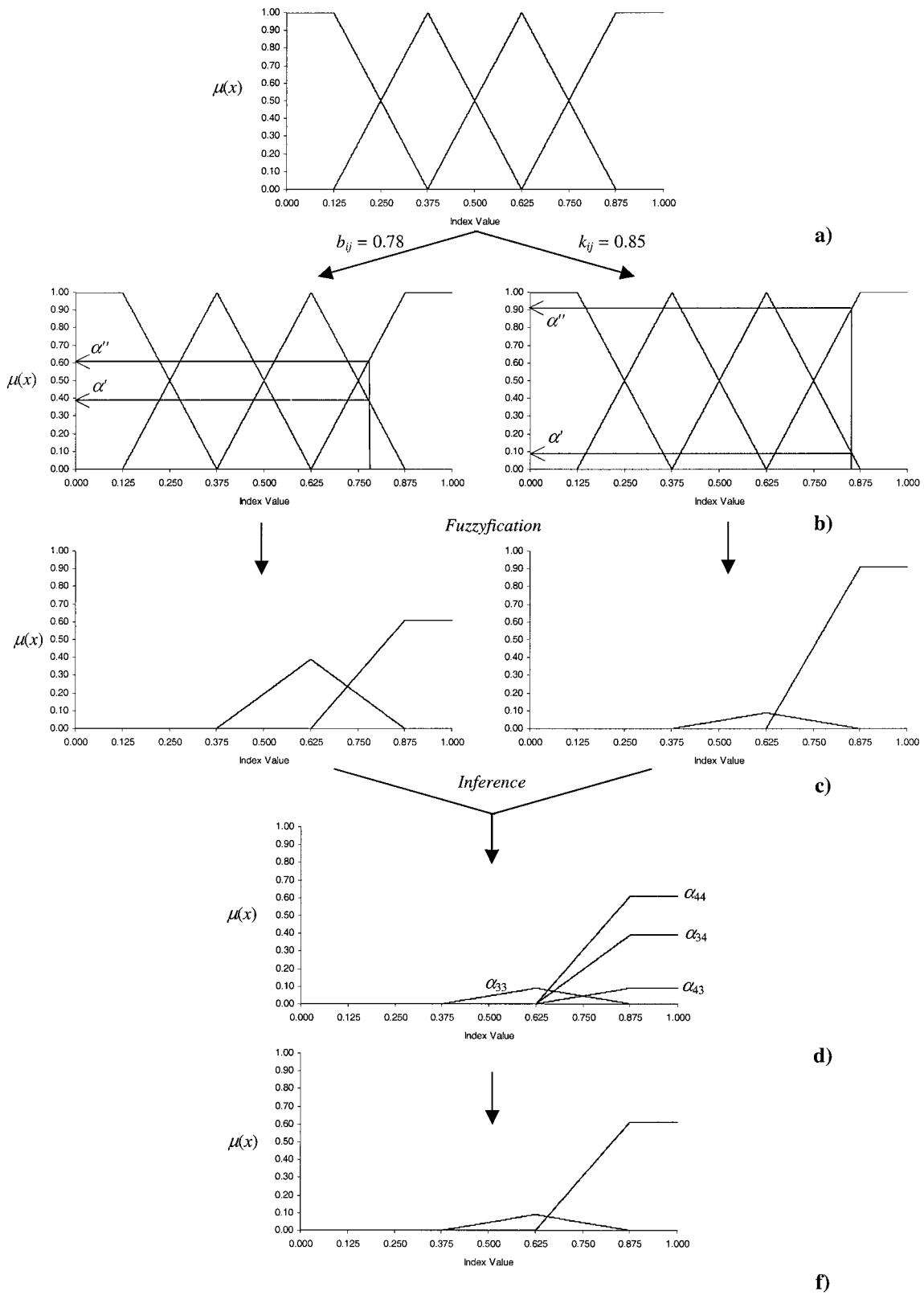


Figure 3. Inference process of the basic index ($b_{ij} = 0.78$ and $\alpha' = \mu_H(0.78) = 0.39$ and $\alpha'' = \mu_{VH}(0.78) = 0.61$); and the supplementary index of ($k_{ij} = 0.85$ and $\alpha' = \mu_H(0.85) = 0.09$ and $\alpha'' = \mu_{VH}(0.85) = 0.91$).

and the scale-down factors are found by:

$$\alpha_H = \max(0.09, 0.00, 0.00, 0.00) = 0.09,$$

$$\alpha_{VH} = \max(0.00, 0.39, 0.09, 0.61) = 0.61,$$

$$\mu_H(x_{bk}) = \mu_H(x) \times \alpha_H = \mu_H(x) \times 0.09,$$

$$\mu_{VH}(x_{bk}) = \mu_{VH}(x) \times \alpha_{VH} = \mu_{VH}(x) \times 0.61,$$

so the fuzzy sets high and very high are reduced to 9% and 61% of their original areas, respectively (Figure 3f).

Combination. Combination is the procedure by which fuzzy sets are aggregated to generate a fuzzy solution space P . In our approach we use a fuzzy additive system (Kosko 1992) to combine the fuzzy sets that result from the interaction intensity and the environmental vulnerability indices, formally:

$$\mu_{VL}(x_p) = \mu_{VL}(x) \times \max(\mu_{VL}(x_{bk}), \mu_{VL}(x_v)),$$

$$\mu_L(x_p) = \mu_L(x) \times \max(\mu_L(x_{bk}), \mu_L(x_v)),$$

$$\mu_H(x_p) = \mu_H(x) \times \max(\mu_H(x_{bk}), \mu_H(x_v)),$$

$$\mu_{VH}(x_p) = \mu_{VH}(x) \times \max(\mu_{VH}(x_{bk}), \mu_{VH}(x_v)),$$

$$\mu(x_p) = \mu_{VL}(x_p) + \mu_L(x_p) + \mu_H(x_p) + \mu_{VH}(x_p).$$

Figure 4 illustrates this procedure as it was used in the La Venta-Colegio Militar impact assessment. Considering the fuzzy sets for interaction intensity and ecological vulnerability (Figure 4a), the additive combination retains the highest membership values of the fuzzy sets (Figure 4b), adds the resulting membership values (Figure 4c), and produces the solution space (Figure 4d).

Defuzzification. Defuzzification converts the fuzzy solution space P to a crisp number. In our approach, defuzzification is carried out through the composite moments method (Cox 1994).

$$p_{ij} = \frac{\sum_g x_g \mu P(x)_g}{\sum_g \mu P(x)_g},$$

where $g \in G$, and G is the total number of intervals in which the range of x is divided.

Next, the crispy score is classified according with the linguistic variables described above. Considering the impact indices of our example ($b_{ij} = 0.78$, $k_{ij} = 0.85$, $v_{ij} = 0.57$, $n_{ij} = f_{ij} = 1$), defuzzification of the solution space generates a crispy score of $p_{ij} = 0.68$ (indicated by an arrow in Figure 4d). This score is interpreted as *High*

Impact Significance because it falls within the range of that linguistic variable.

Figure 5 illustrates the fuzzy space solutions and the defuzzified impact significance scores (indicated by arrows) for three sites located at different distances from the source of impact, d_{ij} , and four mitigation measures, t_{ij} . Note that as either d_{ij} or t_{ij} increases, the weight of the fuzzy sets very low or low increases and the crispy impact significant score, p_{ij} , decreases. Also, note that the impact significance is always equivalent to very low ($p_{ij} = 0.14$) when $t_{ij} = 9$.

The fuzzy impact assessment is implemented in a raster GIS for $t_{ij} = 0, 1, 2, \dots, 9$. This generates 10 map layers, each containing a crispy significance score in every pixel. By overlying these maps in the GIS, the study region is divided into homogeneous land units with respect to their response to impact mitigation, as illustrated in Figure 6a–b.

Stage 3: Conflict Resolution

Conflict resolution requires a separate GIS-based fuzzy impact assessment for each stakeholder, and the use of the constraint method (Malczewski 1999) to find a compromise result. The constraint method is implemented in two steps: (1) Based on the fuzzy assessment for the project proponents (i.e. developers), apply a first optimization model to minimize the cost to attain a desired global impact reduction, and (2) using the fuzzy assessment for another sector (i.e. conservationists), convert that cost to a constraint in a second optimization model and then maximize the impact reduction. Both optimization models are carried out through 0–1 mathematical programming (Dijkstra 1984), and result in the optimum allocation of mitigation measures in the study region.

The first optimization model is mathematically described as:

$$\text{Minimize } Z = \sum_{k=1}^K \sum_{t=1}^9 w_{kt} Y_{kt} \quad (1)$$

subject to:

$$\sum_{k=1}^K \sum_{r=1}^9 E_{kt} \geq B \quad (2)$$

$$Y_{kt} = 0, 1 \quad (3)$$

$$\sum_{r=1}^9 Y_{kt} = 1 \quad (4)$$

where: k is the land unit index; K is the number of land units; t is the mitigation measure ($t = 0, 1, 2, \dots, 9$); Y_{kt}

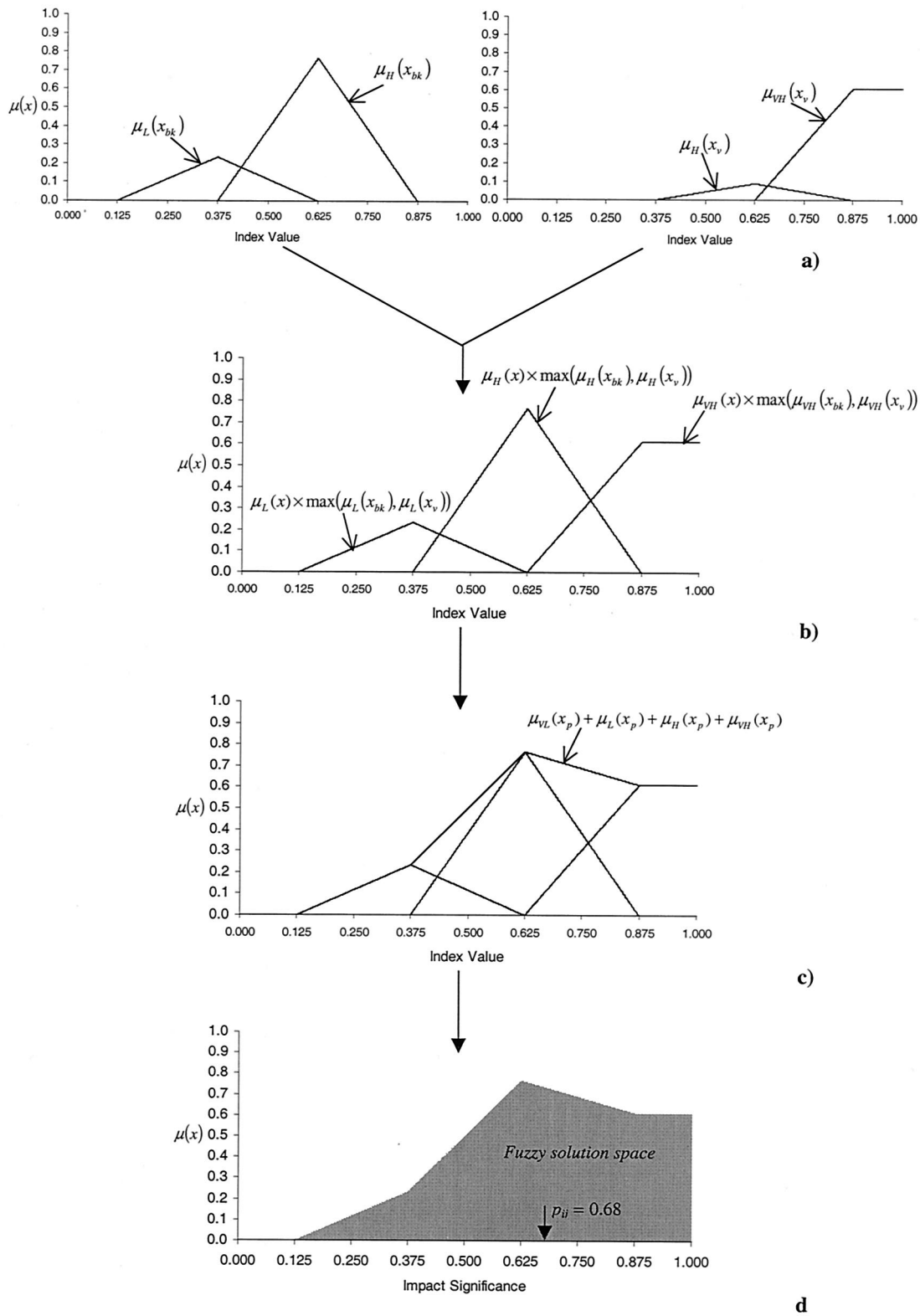


Figure 4. Combination of fuzzy sets. **a**, Interaction intensity and ecological vulnerability fuzzy sets; **b**, obtaining maximum membership values of each fuzzy set; **c**, additive combination of fuzzy sets; and **d**, solution space and defuzzified impact scores shown by the arrow.

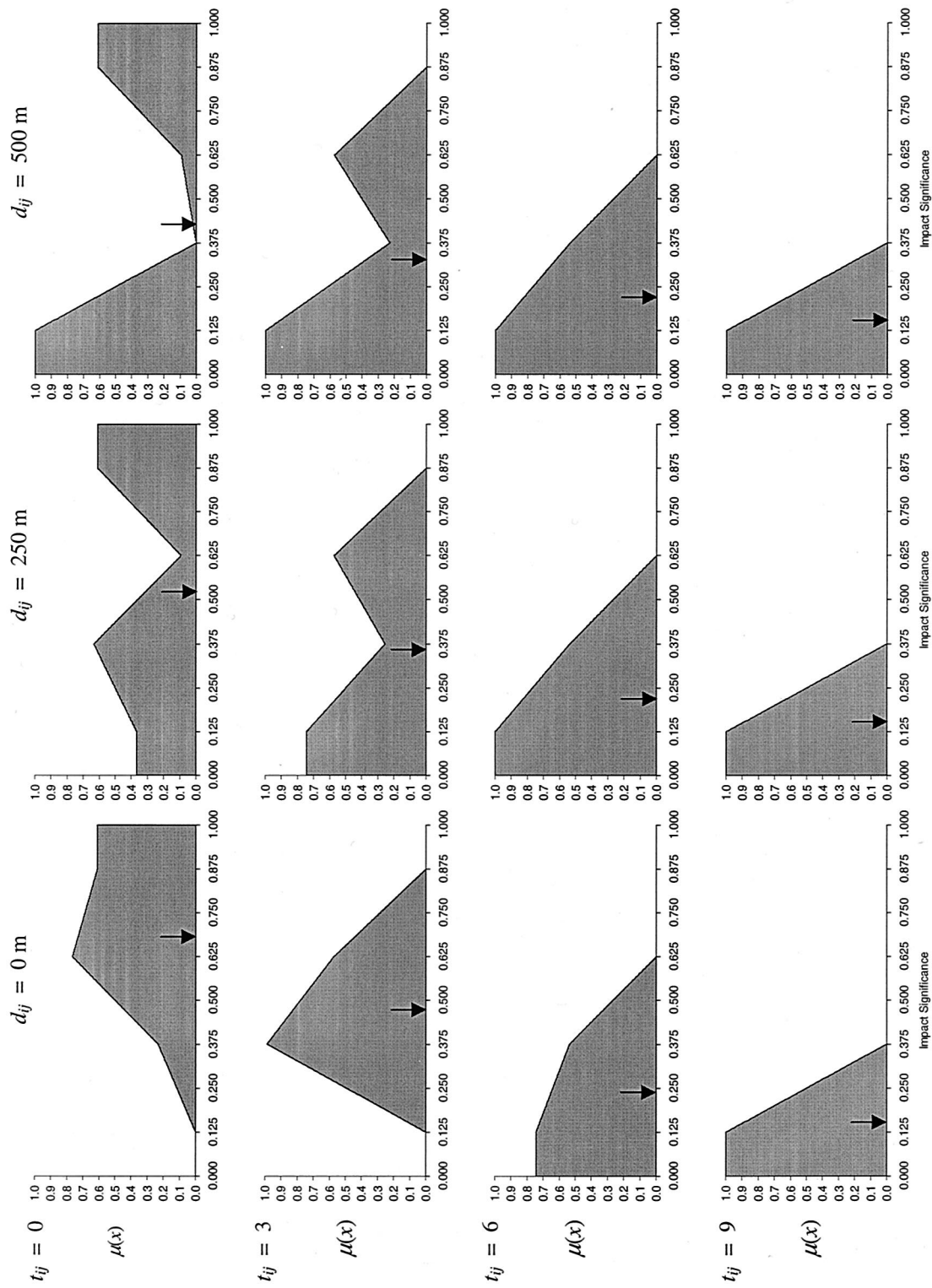


Figure 5. Fuzzy solution spaces (in gray) for selected combinations of distance of impact (d_{ij}) and mitigation measure (t_{ij}) (defuzzified impact scores shown by the arrows).

a)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	
1	k	p _{kt}				A	Δu _{ij}			Y _{kt}			E _{kt}			w _{kt}			C
2		t=0	t=3	t=6	t=9	ha	t=3	t=6	t=9	t=3	t=6	t=9	t=3	t=6	t=9	t=3	t=6	t=9	
3	1	0.33	0.27	0.20	0.14	78	0.06	0.13	0.19	0	0	1	0.0	0.0	14.8	0	0	6318	=
4	2	0.38	0.28	0.20	0.14	7	0.10	0.18	0.24	0	1	0	0.0	1.3	0.0	0	252	0	=
5	3	0.45	0.33	0.21	0.14	10	0.12	0.24	0.31	0	1	0	0.0	2.4	0.0	0	360	0	=
6	4	0.47	0.33	0.22	0.14	3	0.14	0.25	0.33	0	1	0	0.0	0.8	0.0	0	108	0	=
7	5	0.49	0.34	0.23	0.14	10	0.15	0.26	0.35	0	1	0	0.0	2.6	0.0	0	360	0	=
8	6	0.50	0.34	0.23	0.14	13	0.16	0.27	0.36	0	1	0	0.0	3.5	0.0	0	468	0	=
9	7	0.53	0.38	0.25	0.14	15	0.15	0.28	0.39	0	0	1	0.0	0.0	5.9	0	0	1215	=
10	8	0.57	0.41	0.26	0.14	1	0.16	0.31	0.43	0	0	1	0.0	0.0	0.4	0	0	81	=
11	9	0.65	0.47	0.28	0.14	2	0.18	0.37	0.51	0	1	0	0.0	0.7	0.0	0	72	0	=
12	10	0.67	0.52	0.29	0.14	5	0.15	0.38	0.53	0	0	1	0.0	0.0	2.7	0	0	405	=
13															35.0				
14															>=			Total=	\$9,639
15															35.0				

b)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	
1	k	p _{kt}				A	Δu _{ij}			Y _{kt}			E _{kt}			w _{kt}			C
2		t=0	t=3	t=6	t=9	ha	t=3	t=6	t=9	t=3	t=6	t=9	t=3	t=6	t=9	t=3	t=6	t=9	
3	1	0.53	0.38	0.23	0.14	78	0.15	0.30	0.39	0	0	1	0.0	0.0	30.4	0	0	6318	=
4	2	0.56	0.41	0.23	0.14	7	0.15	0.33	0.42	0	1	0	0.0	2.3	0.0	0	252	0	=
5	3	0.60	0.44	0.24	0.14	10	0.16	0.36	0.46	0	1	0	0.0	3.6	0.0	0	360	0	=
6	4	0.64	0.45	0.24	0.14	3	0.19	0.40	0.50	0	1	0	0.0	1.2	0.0	0	108	0	=
7	5	0.70	0.47	0.24	0.14	10	0.23	0.46	0.56	0	1	0	0.0	4.6	0.0	0	360	0	=
8	6	0.80	0.56	0.25	0.14	13	0.24	0.55	0.66	0	0	1	0.0	0.0	8.6	0	0	1053	=
9	7	0.85	0.57	0.25	0.14	15	0.28	0.60	0.71	0	1	0	0.0	9.0	0.0	0	540	0	=
10	8	0.86	0.58	0.25	0.14	1	0.28	0.61	0.72	0	0	1	0.0	0.0	0.7	0	0	81	=
11	9	0.86	0.62	0.30	0.14	2	0.24	0.56	0.72	0	0	1	0.0	0.0	1.4	0	0	162	=
12	10	0.86	0.77	0.40	0.14	5	0.09	0.46	0.72	0	0	1	0.0	0.0	3.6	0	0	405	=
13																			\$9,639
14															Total=	65.5			<=
15																			\$9,639

Figure 6. Implementation of the constraint method in *What's Best!* **a**, First optimization model minimizing the cost and setting the global impact reduction as a constraint; and **b**, second optimization model maximizing impact reduction and maintaining the total cost as a constraint.

is the decision variable; E_{kt} is the impact reduction in land unit k by the implementation of mitigation t ; B is the global impact score for the whole study area; and w_{kt} is the cost of mitigation t in land unit k .

Equation 1 is the objective function; Equation 2 enforces the reduction of impacts to the desired total value; Equation 3 is the binary restriction for the decision variables ($Y_{kt} = 0$ if mitigation efficiency t in land zone k is not selected, or 1 otherwise); and Equation 4 enforces the selection of only one impact mitigation level at a land unit. The variables E_{kt} and w_{kt} are computed as follows:

$$\Delta u_{kt} = p_{ij}^{kt_0} - p_{ij}^{kt}, \quad (5)$$

$$E_{kt} = x_k \Delta u_{kt} Y_{kt}, \quad (6)$$

$$w_{kt} = x_k f(t_k) Y_{kt}, \quad (7)$$

where: Δu_{kt} is the reduction in the crispy impact significance score (p) by changing the mitigation measures from t_0 (e.g. $t = 0$) to t (e.g. $t = 1, 2, \dots, 9$); and x_k is the area of land unit k ; and $f(t_k)$ is a function that relates the cost to the impact significant level.

The second optimization model is described mathematically as:

$$\text{Maximize } Q = \sum_k \sum_t E_{kt} Y_{kt}, \quad (8)$$

subject to:

$$\sum_k \sum_t x_k w_{kt} Y_{kj} \leq O, \quad (9)$$

where O is the available budget for mitigating impacts, as determined by Equation 1.

Equation 9 ensures that the cost of the allocated mitigation measures do not exceed the available budget (Equations 3–7 also apply).

Figure 6a shows the implementation of Equations 1–7 through the program *What's Best!* for *Excel* (Lindo Systems 1998). Specifically, the problem involves 10 land units ($k = 1, \dots, 10$), four impact mitigation measures ($t = 0, 3, 6, 9$), and 30 binary decision variables. The input data to the model are the impact

significance scores (cells B3:E12) and the area of the land units (cells F3:F12). The objective function (Equation 1) is implemented in cell R14, and the decision variables, Y_{kt} , are in cells J3:L12 (these cells are set to zero before optimization, so the values observed in Figure 6a are the optimization results). For example, $Y_{56} = 1$ (cell K7) indicates that impact mitigation level $t = 6$ has been selected for land unit $k = 5$, whereas $Y_{59} = 0$ (cell L7) indicates the opposite. Equation 5 is implemented in cells G3:I12; for example, cell H7 corresponds to $\Delta u_{56} = 0.49 - 0.23 = 0.26$. The values in cells M3:O12 represent the impact reduction attained at an individual land unit; for example, the value of cell N7 is obtained by $E_{56} = 10 \times 0.26 \times 1 = 2.6$. Equation 7 is operationalized in cells P3:R12, assuming a cost function $f(t_k) = t^2$ and a unitary cost of \$1/ha; for example, the value in cell Q7 is obtained by $w_{56} = 10 \times 6^2 \times 1 = \360 . Equations 2 and 4 are operationalized in cells O13:O15 and S3:S12, respectively.

The constraint method involves the search of a range of applicable solutions. The upper and lower limits of this range are called the Ideal and Anti-Ideal solutions, respectively (Malczewski 1999). The Ideal solution corresponds to a complete impact mitigation by implementing $t = 9$ in all the land units (144 ha). Given the impact significance scores of the developers, the Ideal solution results in a total cost and a global impact reduction of \$11,664 and 38.7, respectively. The Anti-Ideal solution corresponds to attaining a very low impact in all the land units. It is achieved by setting $t = 6$ for a very low impact in six land units ($k = 1, 2, 3, 4, 5, 6$) of 121 ha, and $t = 9$ for a complete mitigation in four land units ($k = 7, 8, 9, 10$) of 23 ha. Thus, the Anti-Ideal results in a total cost and total impact reduction of \$6,219 and 30.6, respectively. Hence, a set of alternative scenarios can be defined by restricting the right hand of Equation 2 to the range $30.6 \leq B \leq 38.7$ (i.e. $B = 32, 34, 35, 37$). Figure 6a shows the case of $B = 35$ that implies a cost of \$9,639 to attain a low impact in one land unit ($k = 9$) of 2 ha, a very low impact in five land units ($k = 2, 3, 4, 5, 6$) of 42 ha, and a complete mitigation in four land units ($k = 1, 7, 8, 10$) of 99 ha.

The cost of scenario is used as a constraint in the second optimization model. Figure 6b shows the implementation of this model for fuzzy assessment of the conservationists (cells B3:E13) and the total cost shown in Figure 6a. Observe that Equation 8 is now operationalized in cell O14 and Equation 9 in cells R13:15. This model results in a low impact in one land unit ($k = 7$) of 15 ha, a very low impact in four land units ($k = 2, 3, 4, 5$) of 30 ha, and a complete mitigation in five land units ($k = 1, 6, 8, 9, 10$) of 99 ha.

The results of the constraint method are presented to the stakeholders. A compromise solution is found whenever the stakeholders agree that, in spite of the differences in the impact significance scores, a given scenario produces an acceptable reduction of impacts in the study area.

Implementation

The La Venta-Colegio Militar highway was planned to be the southwestern section (22 km) of a projected four-lane transit loop for Mexico City (Figure 1). Major concerns of environmental authorities and conservationists included the effects of this project on natural habitats. The major concern of the developers was the cost of the mitigation measures.

We followed the definitions in MOPT (1989) to identify the activities of the project related to land cover clearings and the mitigation measures. We decided to used land cover as a proxy variable for ecological or habitat vulnerability, and to relate the project activities to the effects on biodiversity described in Coldwill and Thompson (1984), and Trombulak and others (2000). Following Kuitunen and others (1998), the appraisal of risk of each land cover type considered the potential species composition, total area of each cover type in the region, and conservation value (on the basis of rare and endangered species). The occurrence of species in particular habitats was established from both the literature and overall habitat descriptions.

Impact mitigation measures were defined as follows: $t_{ij} = 1$, cleaning of construction debris and garbage; $2 \leq t_{ij} \leq 3$, minor rehabilitation practices and natural regeneration; $4 \leq t_{ij} \leq 5$, natural and artificial regeneration (seeding of grassland species) and soil stabilization; $6 \leq t_{ij} \leq 7$, intensive vegetative management practices (reforestation and reclamation) and major mechanical management practices; and $8 \leq t_{ij} \leq 9$, restoration of natural habitats and compensation through the protection of areas of similar biological significance. We used a cost function $w_t = t^2$ for Equation 7, with a unitary cost of \$10/ha.

The thematic map layers (1:4,000) were compiled into the GIS for geomorphology, land cover use, soils, and distance to the highway. The maximum possible distance for the analysis ($d_{\max} = 500$ m) was determined after the "road effect zone" described by Forman and Debinger (2000). Hence, the study area was delimited by a 1 km fringe along the projected highway and encompassed 2,138 ha (Figure 7). The categories of the geomorphology, land cover, soils maps layers were reclassified using the linearly transformed AHP

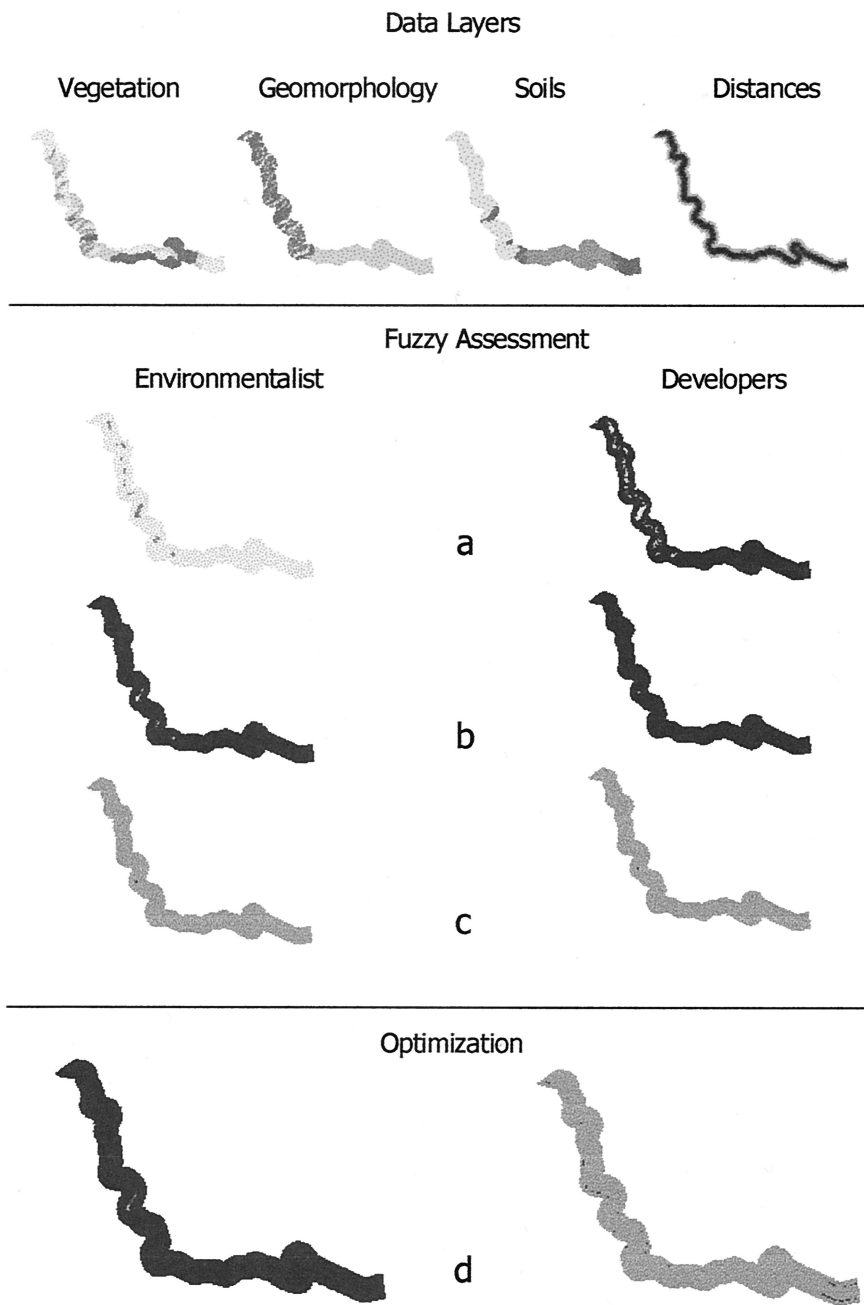


Figure 7. Map layers generated for La Venta-Colegio Militar Highway Project impact assessment. The input map layers are shown at top of the figure. Left side in **a–d**: Results for the conservationists; right side in **a–d**: results for the developers; **a**, $t_{ij} = 0$; **b**, $t_{ij} = 3$; **c**, $t_{ij} = 7$; and **d**, results of the constraint method.

scores (Table 1) to produce the corresponding environmental risk, r_{ij} , map layers (Figure 7).

The judgments of conservationists and developers about the effects of land cover clearings on biodiversity (see Interaction Intensity Index above) were submitted to the fuzzy evaluation in the GIS. In spite of the obvious differences, both sets of judgments were considered realistic according to previous experiences about the ecological effects of terrestrial transportation infrastructure and the uncertainties involved.

All the spatial analyses were conducted in the raster-based software Geographic Resource Analysis Support System, Grass 4.1 (USA-CERL 1991), operated in a Sun Ultra 10 workstation through a set of UNIX shell-scripts. The optimization models were implemented through the program What's Best!.

The spatial fuzzy assessment resulted in a larger number of land units for the developers ($K = 91$) than for the conservationists ($K = 84$). The most sensitive areas were on the central portion of the

study area within the northwestern section of the project (Figure 6). The significance scores for $t_{ij} = 0$ reflected the differences in judgment of the stakeholders (Figure 6a): For the developers, the scores ranged from low impact (92% of the study area) to high impact (remaining area), whereas for the conservationists the scores ranged from high impact (98% of the study area) to very high impact (remaining area). The spatial pattern was practically identical for $t_{ij} \geq 3$ because 99% of the study area was classified as either low or very low impact, respectively (Figure 6b–c).

The Ideal solution according with the developer's judgments resulted in a mitigation cost equivalent to 0.4% of the total cost of the project, generated by the implementation $t_{ij} = 7$ in 57 land units of 2050 ha (96% of the study area), and $t_{ij} = 8$ in the remaining land units. The Anti-Ideal solution resulted in a cost equivalent to 0.2% of the total cost of the project, generated by the implementation $t_{ij} = 5$ in 26 land units of 1766 ha (83% of the study area), $t_{ij} = 6$ in 35 land units of 296 ha (14% of the study area), $t_{ij} = 7$ in 29 land units of 72 ha (3% of the study area), and $t_{ij} = 8$ in one land unit of 2 ha.

The constraint method resulted in an agreement of a mitigation cost equivalent to 0.3% of the total cost of the project (Figure 6d). According with the developers, this budget would allow attaining a very low impact in 83% of the study area (through the implementation of $t_{ij} = 6$ in 42 land units) and complete impact mitigation in the remaining area. According with the judgments of the conservationists, this budget would allow attaining a low impact in 99% of the study area through the implementation of $t_{ij} = 3$ in four land units of 304 ha, $t_{ij} = 4$ in two land units of 0.1 ha, $t_{ij} = 5$ in 62 land units of 1819 ha, $t_{ij} = 6$ in 10 land units of 7 ha, $t_{ij} = 7$ in four land units of 4 ha, and $t_{ij} = 8$ in two land units of 2 ha. A closer examination of the results for the conservationists revealed that 85% of the study area obtained impact scores of $p_{ij} < 0.375$, indicating that the impact was between low and very low in most of the study area. The stakeholders reached the agreement of implementing the mitigation measures that resulted from the conservationists' judgments.

Discussion and Conclusions

The approach presented here effectively addresses some of the major deficiencies that have plagued ecological impact assessments (Bojórquez-Tapia and García 1998). As shown by the La Venta-Colegio Militar impact assessment, the fuzzy assessment can effectively frame the complexity of impacts into manageable

terms, taking into account the vagueness and imprecision of the information. Through the interaction and the ecological vulnerability indices, it enables planners and stakeholders to make explicit statements about the impacts.

In contrast with other impact assessment methods (such as interaction matrices), our approach makes a clear and systematic connection between data, analyses and predicted impacts. It also facilitates the settlement of disputes because the outcomes of conflicting judgments can be assessed in terms of the area affected, the impact significance, and the cost of mitigation.

When applying fuzzy logic, nonetheless, attention should be put to the operators used for combining fuzzy sets (see Enea and Salemi 2001, Malczewski 1999). The use of a compensatory operator is justified in La Venta-Colegio Militar example because the impact on habitats is assumed to decrease with the distance from the project. Therefore, a high value of the interaction intensity index has to be compensated with a low value of the vulnerability index, and vice-versa.

Additionally, experience in Mexico confirms the assertion by Cuperus and others (1999) that the cost of mitigation measures is a critical factor for reaching consensus. Consequently, contrary to most impact assessment approaches, the cost of impact mitigation is incorporated directly in the fuzzy assessment. Importantly, our results corroborate the observations of Wiehn and Pedrycz's (1996) on that fuzzy assessment help users to be aware of the ultimate financial repercussions of their judgments. Keep in mind however that a critical step in our method is the estimation of the unitary cost and the determination of the cost function.

We agree with Silvert (1997) on that a fuzzy assessment does not imply that impacts are "good" or "bad." This fact underscores the advice of Harashina (1995) about taking into account multiple viewpoints and perspectives in an evaluation. Consequently, it should be emphasized that the approach is most useful in an iterative framework that allows a comprehensive exploration of the potential solutions. However, one shortcoming of the approach presented here is that the computing time of 0–1 mathematical programming increases exponentially with the number of constraints (Dijkstra 1984). Consequently, if the number of land units is large ($k > 100$), the problem could be unmanageable in spreadsheets and thus unsuitable for a participatory impact assessment. In those situations, a practical solution is to utilize software with greater capacity

(for example, see Minkoff 1992), or to apply genetic algorithms (see Cruz-Bello 2000).

The integration of fuzzy logic, GIS and optimization is an effective tool for making the most of the macroknowledge embodied in a group of experts and stakeholders. It is a consensus-building tool because the stakeholders can be better informed of the implications of their judgments. It provides a framework for predicting ecological impacts in heterogeneous landscapes, while simultaneously enabling environmental planners and managers (*sensu* Dixon 1997) to perform a comprehensive examination of the conflicting views from experts and stakeholders.

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