Conditional Probability

Drawing a card from a deck

Ordinary probability question:

What is the probability that the card is a King (K)?

$K = \{K(\text{spades}), K(\text{hearts}), K(\text{diamonds}), K(\text{clubs})\}$
$P(K) = \frac{n(K)}{n(SS)} = \frac{4}{52} = \frac{1}{13}$

Conditional probability question:

What is the probability that the card is a king if it is given that you dropped the card, saw a glimpse of it, saw that it was a red face card.

$\Re(\text{red face card}) = \{K(\text{hearts}), K(\text{diamonds}), Q(\text{hearts}), Q(\text{diamonds}), J(\text{hearts}), J(\text{diamonds})\}$
$P(K|\Re)) = \frac{2}{6} = \frac{1}{3}$

In general for $P(A|B)$, you can use the rule $P(A|B) = \frac{n(A \cap B)}{n(B)}$ if all of the possible results are equally likely.

Rolling a die

$SS = \{1, 2, 3, 4, 5, 6\}$
$E = \{2, 4, 6\}$
$T = 2, 3$

What is the probability that the number is even, if it is given that the number starts with a t?

$$P(E|T) = \frac{n(E \cap T)}{n(T)}$$

Bird stuff

H = happy?

x	R	W	B
H	12	6	18
H'	8	5	11

Suppose one of these birds is randomly selected.

What is the probability that the bird is white?

$$P(W) = \frac{n(W)}{n(SS)} = \frac{11}{60}$$

What is the probability that the bird is happy?

$$P(H) = \frac{36}{60}$$

What is the probability that the bird is white AND happy?

$$P(W \cap H) = \frac{6}{60}$$

What is the probability that the bird is white OR happy?

$$P(W \cup H) = \frac{11}{60} + \frac{36}{60} - \frac{6}{60} = \frac{41}{60}$$

What is the probability that the bird is not white?

$$P(W') = \frac{49}{60}$$

What is the probability that the bird is white if it is given that the bird is happy?

$$P(W|H) = \frac{n(W \cap H)}{n(H)} = \frac{6}{36} = \frac{1}{6}$$

What is the probability if the bird is happy if the bird is white?

$$P(H|W) = \frac{n(H \cap W)}{n(W)} = \frac{6}{11}$$