Four Coupled Xmon's – Design Parameters

May 2018 – updated version
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Qubits

We wish all qubits to have the same frequency, about 6 GHz. The frequency is given by

$$f_{qubit} = \frac{1}{h}\left(\sqrt{E_c E_J \left|\cos\left(\frac{\pi\phi_{ext}}{\phi_0}\right)\right|} - E_c\right)$$

where $E_C = \frac{e^2}{2C}$, $E_J = \frac{\phi I_C}{2\pi}$. When choosing our parameters, we must consider we

- want to operate our qubits in the range where $E_J/E_C \sim 95$, to eliminate charge noise
- still don't know exactly how to determine I_c a priori.

Martinis' group usually produce $I_c \sim 40\,nA$ (Chen's thesis, 2018). This is for entire SQUID with area $\mathbf{0.3\ \mu m \times 0.2\ \mu m}$. Aiming for the same dimensions and hoping (praying) for similar a critical current. *In the last design and fab-round, the gap of 400 nm (design) gave a bridge of approximately 200 nm, so we continue with this.* This gives $\boldsymbol{E_J = 19.8\ GHz}$. Then we can choose the Xmon's *arm* length (each of the four arms) to be $\boldsymbol{l = 130\ \mu m}$ (also comparable to Martinis' group, though they try different dimensions), and with center trace and silicon-gap $\boldsymbol{S = 8\mu m}$ and $\boldsymbol{W = 4\mu m}$ respectively, the capacitance to group (C_s) of the Xmon becomes $\mathbf{91\ fF}$, resulting $\boldsymbol{E_C = 212\ MHz}$ and thus $E_J/E_c = 93.6$.

This gives a qubit frequency of $\boldsymbol{f_{qubit} = 5.6\ GHz}$, which is sufficiently central in the spectral range of the network analyzer.

Readout resonators

This design includes four readout resonators – one for each qubit. Qubit 1, 2, 3, and 4 will have readout resonators at $\boldsymbol{f_{1,2,3,4}^{read} = [6.6, 6.7, 6.8, 6.9]\ GHz}$. (again following Chen's statement that the resonator-qubit detuning be $\Delta \sim 1 GHz$). All these have (self) capacitances to ground:

$\boldsymbol{C_s^{read} = [0.360, 0.355, 0.349, 0.344] pF}$ (including horse shoe capacitors. See below).

Readout resonators are coupled *strongly* to readout line at distance $\mathbf{21\ \mu m}$ for length $\mathbf{500\ \mu m}$. This gives a coupling capacitance of $\boldsymbol{C_c \cong 2.3 fF}$ and as we have seen, this is the critical coupling (given internal losses, enabling up to $\boldsymbol{Q_c \cong 3 \times 10^4}$.

Coupling between readout resonators and qubits: Chen works with $g_{xmon}^{read} \sim 100\ MHz$. But this would require a large capacitor. For the dispersive shift, we want more like $\boldsymbol{\chi_{read} \cong \frac{g^2}{\delta} = \pm 1\ MHz}$, with $\delta = 2\pi|f_{xmon} - f_{read}|$. This requires $\boldsymbol{g_{xmon}^{read}} = [\mathbf{31.6, 33.2,\ 34.6, 36.1}]\ \boldsymbol{MHz}$. (In Barend's original paper presenting the Xmon this was 40 MHz). This coupling g_{ij} between <u>two resonators</u> with resonances f^i and f^j and self capacitances C_s^i and C_s^j coupled with given by:

$$g_{ij} = \frac{C_c^{ij}}{\sqrt{(C_s^i + C_c^{ij})(C_s^i + C_c^{ij})}}\sqrt{f^i f^j}$$

Let's assume this is true for a <u>resonator-qubit coupling</u> as well.

Readout resonators will couple to Xmon throught horse shoe resonators. These add an additional capacitance of ~20 fF to $\boldsymbol{C_c^{read}}$ (which is ~0.3 pF), resulting in a shift in f^{read} of a few hundred MHz. This will be considered here:

What should be the length of the readout capacitor?

Qubit	f^{read}	f^* – based on experiments nov 2018, we need these to reach f^{read}	$l = \frac{c}{\sqrt{e_{eff}}4f^*}$ [m]
1	6.6 GHz	7.121597	0.004267189444968
2	6.7 GHz	7.236964	0.004200985337830
3	6.8 GHz	7.35233	0.004137987990234
4	6.9 GHz	7.467697	0.004076507311609

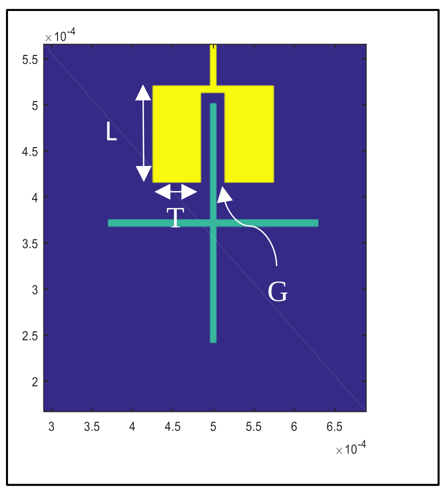
Then, as the f_{read} determines the self-capacitances, we find the necessary coupling capacitances:

Qubit	f^{xmon}	C_s^{xmon}	f^{read}	C_s^{read}	$C_c^{xmon,read}$	g_{xmon}^{read}
1	$5.6\ GHz$	$91\ fF$	$6.6\ GHz$	$\mathbf{360}\ \boldsymbol{fF}$	$\mathbf{1}\ \boldsymbol{fF}$	$33.4\ MHz$
2	$5.6\ GHz$	$91\ fF$	$6.7\ GHz$	$\mathbf{355}\ \boldsymbol{fF}$		$33.9\ MHz$
3	$5.6\ GHz$	$91 fF$	$6.8\ GHz$	$\mathbf{349}\ \boldsymbol{fF}$		$34.4\ MHz$
4	$5.6\ GHz$	$91\ fF$	$6.9\ GHz$	$\mathbf{344}\ \boldsymbol{fF}$		$34.9\ MHz$

*where we assumed the readout resonators to be $\lambda/4$.

Numbers were found numerically in matlab with the script based on Martinis integral.

These numbers (of capacitances) are very similar, and might since fabrication in any case is inaccurate, we might aim for roughly $1\ fF$, which is achieved (get 1.02) by shaping the capacitor as a "chet" (ת) with the gap $G = 14\ \mu m$, the length of the chet (not equal to coupling length) $L = 80\ \mu m$ and the thickness of the chet $T = 20\ \mu m$.



Quantum busses a.k.a. quantum walk resonators

Each Xmon is coupled to a quantum bus, which then couples to the central coupling island. The coupling between Xmon and quantum bus must be much stronger than that between the quantum busses (through the coupling island). We will again adopt Martinis' qubit-resonator coupling of $\boldsymbol{g_{xmon}^{bus} \sim 30\ MHz}$. But there are few differences from the readout resonators:

- These resonators are $\lambda/2$ (closed in both ends to a capacitor).
- All quantum busses aim for the same bare resonance frequency, $\boldsymbol{f_0^{bus} = 5\ GHz}$ This implies $\boldsymbol{C_s^{bus} = 979 fF}$

Why choose this number? It must be a bit smaller than the Xmon's bare frequency ($5.6\ GHz$), so that these are uncoupled before applying flux to the SQUID, but easily tuned into resonance, when so desired.

The dispersive shift expected off-resonance (say, when the Xmon's are not exposed to flux) will be: $\boldsymbol{\Delta_{bus} = \frac{g^2}{\delta} \sim 1.5\ MHz}$ (an order of magnitude slower than that of the readout-resonator).

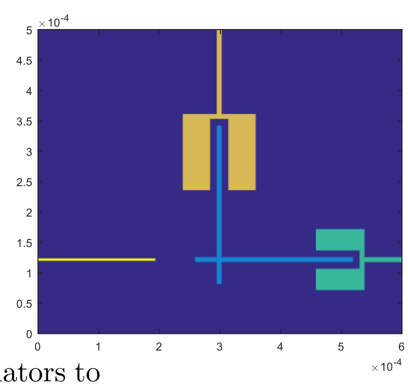
A coupling of $\boldsymbol{g_{xmon}^{bus} \sim 30\ MHz}$ is achieved by the capacitance: $\boldsymbol{C_{xmon}^{bus} = 1.710\ fF}$

This capacitance can be achieved by setting the parameters of the chet-capacitor (see image above) as: $G = 11\ \mu m$, $L = 80\ \mu m$, and $T = 35\ \mu m$.

The length of a $\lambda/2$-CPW resonator with the given C, L etc. is 11567 μm, but due to the large capacitor to the Xmon (which adds capacitance between quantum bus and ground of approximately $\boldsymbol{C_s^{HS} = 20 fF}$, we make our resonator the length 11453 μm (which would otherwise be analogue to 5.05 GHz).

Avoiding crosstalk between readout resonator and quantum bus

We want to avoid any coupling between the two resonators coupled to our Xmon. This is achieved by making the cross asymmetric: The arms to these two exits are made longer than those to the XY control (drive) and to the SQUID (tuning). But be keep the length of the Xmon constant, so that the capacitance (and thus the energy, frequency and anharmonicity) is unaltered.



With all the parameters given so far for the coupling of the resonators, we estimate the capacitance between the two resonators to be $\boldsymbol{C_{read}^{bus} \sim 7 \times 10^{-17} F}$ which is equivalent to about

$$\boldsymbol{g_{read}^{bus}} \cong \frac{C_{read}^{bus}}{\sqrt{(C_{read} C_{bus})}} f_{bus} f_{read} \cong 0.6 MHz$$

SQUID parameters

SQUIDs are fabricated by shadow evaporation, setting the Junction area to $\mathbf{0.3\ \mu m} \times \mathbf{0.2\ \mu m}$ (how does this scale with experimental numbers on the e-line?). The flux bias lines are located at distance $\mathbf{20\ \mu m}$ from the SQUID area, which is set to $\mathbf{30\ \mu m} \times \mathbf{20\ \mu m}$. With these parameters, applying $\boldsymbol{I_{flux} = 1\ mA}$ (e.g. 4 V through one of the refrigerator's 4 kΩ lines) should produce a flux according to:

$$\phi_{ext} = \int BdA = L_{SQ} \int_{dist}^{dist+H_{SQ}} BdH = \frac{L_{SQ} I_{flux} \mu_0}{2\pi} \int_{dist}^{dist+H_{SQ}} \frac{dH}{r} = \frac{L_{SQ} I_{flux} \mu_0}{2\pi} \ln\left(\frac{dist + H_{SQ}}{dist}\right)$$

Inserting the given numbers, this gives $\phi_{ext} = 60 \times \ln\left(\frac{40}{20}\right) \times 10^{-16} Wb \cong 4.2 \times 10^{-15} Wb \cong 2\phi_0$

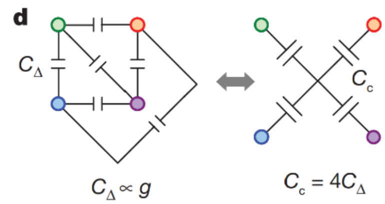
Assuming $\mu_r \cong 1$ (for aluminum), and that $\lambda = 16\ nm$, we get a negligible kinetic inductance and $\beta_L < 0.01$, which means a high modulation. And it follows that in order to tune the Xmon's frequency from its bare $f_0^{xmon} = 5.6\ GHz$ to $f^{bus} = 5\ GHz$, we must require:

$$hf^{bus} = \sqrt{E_C^{xmon} E_C^{xmon} \cos\left|\frac{\pi\phi_{ext}}{\phi_0}\right|} - E_C^{xmon}$$

i.e $\cos\left|\frac{\pi\phi_{ext}}{\phi_0}\right| = 0.0107$, which means $\phi_{ext} = 0.497\ \phi_0$, which again is equivalent to about $0.25mA$, as we found above.

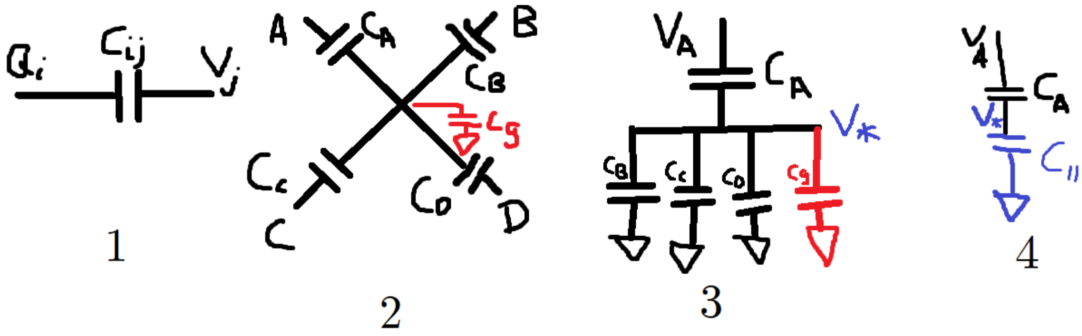
Coupling Island

Here we'll steal the design from Neeley et.al "Generation of three-qubit entangled states using superconducting phase qubits", Nature 2010, where four transmons were coupled by a central island (no quantum busses). Their figure is shown here. For completeness, I'll quote the capture:



"Capacitive coupling network to achieve symmetric coupling between all pairs of qubits (left), and simplified equivalent circuit using coupling to a central island (right). The complete network on the left requires six capacitors, and the coupling strength, g, is proportional to the qubit–qubit capacitance, C_Δ,. In the equivalent circuit on the right, the same coupling strength is attained by scaling the capacitors to C_c 5 $4C_\Delta$, but now only four capacitors are required and the circuit can be easily laid out symmetrically on a chip."

Neeley provides a more genuine treatment of the problem in his thesis, but only for the case of equal capacitances. Here, I will derive it myself for the general capacitances. As the brilliant artistic masterpiece of a paint drawing here shows you, we define the capacitance C_{ij} by the charge Q_i accumulated on node i, when applying the voltage V_j at node j (fig. 1). If so, what happens if we apply the voltage V_A at node A and keep all the other nodes grounded? This is the case in the fig. 2.



On the equivalent circuit (fig. 3), we see that the four capacitors C_B, C_C, C_D and C_g in fact shunt each other, so we can replace them with $C_{||} = C_B + C_C + C_D + C_g$. If we now assume that the central island is neutral, the charges on both capacitors are the same, or more accurately:

$$Q_A = (V_A - V_*)C_A = V_* C_{||} = V_*(C_B + C_C + C_D + C_g) \Rightarrow V_* = \frac{V_A C_A}{C_A + C_B + C_C + C_D + C_g}$$

where V_* is the charge on the central island. Now, back at fig. 3, consider for example the charge on node B. According to fig. 1, we obviously have $Q_B = V_* C_B$. But now inserting the result for V_*, we find that $Q_B = \frac{V_A C_A C_B}{C_A + C_B + C_C + C_D + C_g}$, thus relating the charge Q_B on B with the voltage V_A on A by the capacitance: $\frac{C_A C_B}{C_A + C_B + C_C + C_D + C_g}$. It should therefore appear that the capacitance between two nodes connected through the island is $\boldsymbol{C_{ij} = \frac{C_i C_j}{\sum C_k + C_g}}$, where C_i and C_j are the capacitors between the respective nodes and the island, and the sum in the denominator is over all capacitors to the island. Neeley himself neglects C_g and sets all $C_k = C_{const}$. He therefore ends up with the result that $C_{ij} = \frac{C_{const}}{4}$. So he sets all capacitors to $15fF$, and gets around $3.5\,fF$ as the actual coupling between each of his transmons.

If so, let's aim to get a couplings of $\boldsymbol{g_{bus}^{bus} \sim 10\ MHz}$ (so that the couplings are not too strong compared to the readout schemes – remember that $\boldsymbol{g_{xmon}^{bus} \sim 30\ MHz}$ and $\boldsymbol{g_{xmon}^{read} \sim 100\ MHz}$). To avoid too much symmetry, we will include some variance in the couplings.

First, remember that the island itself has a capacitance to the ground plate due to its spatial extension. More specifically, when requiring the quantum busses to be separated by at least some $\boldsymbol{350\ \mu m}$ (to avoid cheating us), the coupling island has a total "length" of about $\boldsymbol{1\ mm}$ equivalent to $\boldsymbol{\sim 80\ fF}$.

Earlier we used the very important coupling formula, i.e.

$$g_{ij} = \frac{C_c^{ij}}{\sqrt{(C_s^i + C_c^{ij})(C_s^i + C_c^{ij})}} \sqrt{f^i f^j}$$

but notice that what we couple here is effectively *quantum busses*, and therefore the islands frequency is not important. (is that true?) Moreover, all quantum busses have the same frequency $\boldsymbol{f_0^{bus} = 5\ GHz}$, and self-capacitance $\boldsymbol{C_S = 0.98pF}$ so we simplify the above to

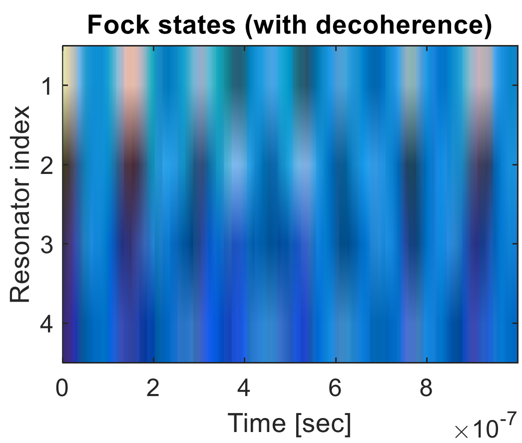
$$g_{ij} = \frac{C_c^{ij}}{C_s + C_c^{ij}} f_0^{bus} \Rightarrow C_c^{ij} = \frac{g_{ij} C_S}{f_0^{bus} - g_{ij}}$$

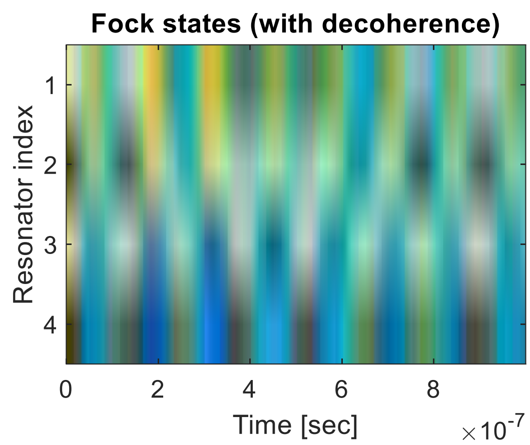
Now assuming $\boldsymbol{g_{bus}^{bus} \sim 10\ MHz}$, we get: $C_c^{ij} \sim 2fF$. Remember, this is the order of magnitude of couplings we want *effectively* between quantum busses – and that a $\lambda/2$ resonators with $f_0^{bus} = 5\ GHz$ has a capacitance to ground of $0.98\ pF$. If so, we can try the following physical capacitors.

Capacitances to island		1	2	3	4
		7.00E-15	2.00E-14	3.00E-14	1.00E-14
1	7.00E-15	3.33E-16	9.52E-16	1.43E-15	4.76E-16
2	2.00E-14	9.52E-16	2.72E-15	4.08E-15	1.36E-15
3	3.00E-14	1.43E-15	4.08E-15	6.12E-15	2.04E-15
4	1.00E-14	4.76E-16	1.36E-15	2.04E-15	6.80E-16

G's (in Hz)	1	2	3	4
1	**0.00E+00**	**4.85E+06**	**7.28E+06**	**2.43E+06**
2	**4.85E+06**	**0.00E+00**	**2.07E+07**	**6.93E+06**
3	**7.28E+06**	**2.07E+07**	**0.00E+00**	**1.04E+07**
4	**2.43E+06**	**6.93E+06**	**1.04E+07**	**0.00E+00**

How would this actually look in a simulation? Let's assume dephasing and amplitude damping with $\boldsymbol{T_1 = T_\phi = 25\ \mu s}$. The simulations below show what we can expect for initially 1 and 2 excitations with these couplings





XY drive

Now it becomes exciting (...). The drive must be coupled in a way that won't allow too high dissipation. Chen writes in his thesis that the Q-factor of a transmon due to drive-coupling is: $\boldsymbol{Q_d = C/RC_d^2\omega}$, where R is the impedance of the drive line, C the qubit's capacitance (to ground) C_d the coupling capacitance between drive and qubit, and ω the qubit's bare frequency. Sank, on the other hand, expresses this as: $\boldsymbol{Q_d = \left(\frac{C}{C_d}\right)^2}$, since he assumes that the impendance of the circuit and the internal resistance of the source are both $\sim 50\Omega$. In any case it is clearly necessary to *limit* C_d to preserve the coherence of the qubit, and the Martinis'es couple with $\sim 50\, aF$. This capacitance will then dominate the impedance of the drive ($Z_{C_d} \gg R_d$), and with our numbers: $\boldsymbol{Q_d \sim 3 \times 10^6}$, equivalent to $\boldsymbol{T_1 \cong 90\, \mu sec}$ for $f = 5.6\, GHz$.

We need a $\pi/2$ pulse to "induce" a photon in the Xmon.

$$\frac{1}{2\hbar}\frac{V_0 Q_{zpf}}{1 + C/C_d}\int dte(t) \sim \pi$$

where $V_d = V_0 f(t) = V_0 e(t)\sin(\omega_d t + \phi_d)$ is the drive voltage, and Q_{zpf} the charge zero point fluctuation (according to Chen: for Xmons, $Q_{zpf} \sim 2.5\, e \sim 4 \times 10^{-19} C$). We see, that if $C_d/C \sim 5.5 \times 10^{-4}$ as in our case, we will need $V_0 \int dte(t) \cong 1.5 \times 10^{-12} J \cdot s$.

Is this a reasonable number? Say $e(t) = e = 1$, and that we want to excite our qubit within $10\, nsec$. Then $V_0 \sim 10^{-4} V$ which is an order of magnitude or two *above* what attenuators in the refrigerator allows. But since this signal is not returned (and therefore won't meet the amplifiers), this might be okay?

The drive coupling C_d is achieved by a coupling gap of $15\, \mu m$, given the thickness of the drive "finger" to be $8\, \mu m$. This gives a coupling of $\boldsymbol{C_{XY}^{xmon} \cong 54\, aF}$