QUBIT 1 - PARAMETERS

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Purpose

This write up covers the most important numbers for the design of Qubit 1, hopefully the first functional Xmon-fabrication. It also includes essential formulae for engineering of superconducting quantum circuits. The Xmon was originally suggested and implemented by Barends, and has that name because of the shape of the qubit: A large X.

The numbers in our design differ from Barends' design (and that of the Martinis' group) for two reasons.

1. We expect **our decoherence to be faster** (first try, and it is on Si and not Sapphire), meaning that **our readout must be faster**. This in turn requires **a lower Q-factor** (allowing to scan less points), which then means that the **dispersion shift must be larger**. Now, this implies that the **resonator-qubit coupling is larger** (or that the detuning is smaller, but as $\chi = g^2/\delta$, so enlarging g is more effective. This means the horse-shoe capacitor is much larger, cancelling part of the Xmons' capacitance to ground, increasing its resonance by a few percent ($\sim 2\ to\ 300\ MHz$).
2. With the oxidation parameters we tried (10 min @ 1 torr), we room temperature resistances up to $10\ \mathrm{k\Omega}$.

The qubit

The idea here is that each "arm" of the qubit can be connected to one of the following:

- XY control - excitation of the qubit ($|0> \rightarrow |1>$)
- Z-control through the SQUID (tuning the qubit frequency)
- A readout resonator (dispersive readout)
- Possible coupling to other qubits or networks

The qubit frequency f_q is given by the charge energy and the Josephson energy (of the SQUID): $\boldsymbol{E_c} = \boldsymbol{e^2/2C}$ and $\mathbf{E\_c\_}$. Note that $\boldsymbol{I_c}$ **is the SQUID's** critical current, i.e. half of that of each Junction (if identical). The desired regime is that where the charge noise is low, i.e. $E_J/E_c >> 1$.

The self-capacitance of the qubit, C is the capacitance to ground, given by coplanar geometry capacitance. Martinis wrote this document on how to calculate the capacitance, and Elisha prepared a Matlab m.file doing the math: If each finger in the **Xmon is 180 μm long, the trace is 8 μm and the gap to the surrounding grounds is 4 μm**, then $C =$1.34E-13 fF, giving $E_c = 146 MHz \times h$.

Chen used SQUIDS with $I_c \simeq 40\ nA$, meaning junctions with $I_c \simeq 20\ nA$ translating by Ambegaokar-Baratoff to around $\mathrm{R_n} \simeq 13\ \mathrm{k\Omega}$, which is a bit above what I can achieve according to the last experience… we could do another round of calibration with more intensive oxidation, or go for junctions with $\boldsymbol{I_c \simeq 30\ nA}$, i.e. each of them with $\mathrm{R_n} \simeq 8.7\ k\Omega$. We then have a combined critical current of $60\ nA$. We then get $\boldsymbol{E_J = 5.75\ GHz}$ and $E_J/E_c > 200$, which is sufficient.

The qubit frequency is:

$$f_q = \frac{1}{h}\left(\sqrt{8E_C E_J \left|\cos\left(\frac{\pi\Phi_{ext}}{\Phi_0}\right)\right|} - E_C\right)$$

Without flux, $f_q = 5.92\ GHz$.

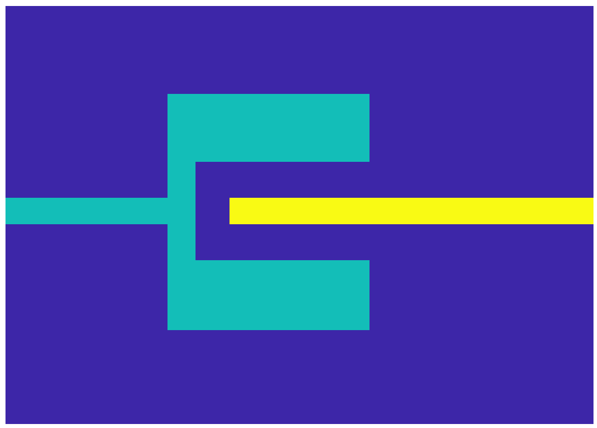
This is the energy for excitation of the qubit. What happens, when $\Phi_{ext} = \Phi_0$ and theoretically $f_q < 0$?) Also, note that Chen includes the factor 8 before $E_C E_J$ … Elisha wrote a small write-up on that. The question seems to be weather you consider $E_C$for one side of one arm only…

Quantity	Formulae	Chen's value	My suggestion
Qubit arm length	-	130 (Barends: up to 165)	180
SQUID critical current	$2I_C^{JJ}$	40 nA	60 nA
Josephson energy	$E_J = \frac{\Phi_0 I_C}{2\pi}$	19.8 GHz	29.8 GHz

SQUID dimensions for appreciable flux

Given the $1k\Omega$ resonators on most of the DC-lines our fridge, we want $1V$ from the voltage generator to impose Φ_0. If the height of the SQUID is H, its length L, and the distance of its upper edge from the center of the flux line is d, then the flux from $1mA$ is:

$$\Phi_{ext} = \int_d^{d+H} B\, dA = L \int_d^{d+H} B\, dh = LI_{flux}\, \mu_0\, /2\pi\; \times \int_d^{d+H} dh/h$$
$$= \frac{LI_{flux}\, \mu_0}{2\pi}\, ln\left(\frac{d+H}{d}\right)$$



If $H, L, d = 20{,}30{,}20\, \mu m$, we get $\Phi_{ext} = 2\Phi_0$, which is sufficient. Here we assumed $\mu_r \simeq 1$ for aluminum. What about the screening parameter, $\beta_L = 2LI_C/\Phi_0$? Note that L is the geometric inductance of the SQUID, typically in the nano-henry scale, making $\beta_L < 0.01$.

Readout resonators

Each qubit is coupled capacitively to a resonator through a “horse shoe”, shown here to the right (yellow: resonator, green: Xmon). The given capacitance is found with the numerical script of capacitance between electrodes. The coupling frequency is

$$g_{ij} = \frac{{C_c}^{ij}\sqrt{f^i f^j}}{\sqrt{({C_s}^i + {C_c}^{ij})({C_s}^j + {C_c}^{ij})}}$$

where ${C_c}^{ij}$ is the coupling capacitance between the two, $C_s$is the self capacitance (to ground of the electrode), and f the frequency. We get ~ 2.7 fF for a distance of 5 micron, 45 micron coupling length and coupling width ("toes”) of 30 micron width.

The the parameters as shown on the image here to the right, we get a coupling capacitance of roughly $1fF$. What, then, is the coupling? This of course depends on the frequencies of the resonators. We want to perform *dispersive* readout, where the coupling between the Xmon and the resonator induces a shift in the resonator frequency, when the Xmon is excited. This is actually the Stark shift:

$$H_{JC}{}^{disp} = \hbar\left(\frac{1}{2}\omega_q\sigma_z + \left(\omega_r + \frac{g^2}{\delta}\sigma_z\right)\left(a^\dagger a + \frac{1}{2}\right)\right)$$

This is the Jaynes-Cummings Hamiltonian, describing a two-level-system coupled to a resonator in the dispersive limit, where $\delta = \omega_q - \omega_r >> g$. Here $\omega_r \rightarrow \omega_r \pm g^2/\delta$ depending on the qubits state.

Choose horse shoe capacitor, width 55 µm, length 70 µm, gap to Xmon: 5 µm

Qubit name	Archimedes	Bohr	Casimir	Debye
f_{RR} (readout res)	**6.75E+09**	**6.80E+09**	**6.85E+09**	**6.90E+09**
Required length	4.25276E-03	4.22171E-03	4.19112E-03	4.16097E-03
δ/g	1.11E+02	1.15E+02	1.20E+02	1.25E+02
g	5.67E+07	5.71E+07	5.75E+07	5.79E+07
χ	**5.11E+05**	**4.94E+05**	**4.79E+05**	**4.65E+05**

Couplings of readout resonators to readout line: **0.6 mm coupling length**, ground between the two lines: 8 µm (total distance of 18 µm between edges of transmission line and resonator). This gives a coupling of 4.8 fF, and an external Q of 14000.

Tunable resonators

XY control: Probe 20 micron wide, distance 13 micron, gives 0.11 fF coupling.

Tunable resonators

The simplest **tunable** resonator is a $\lambda/4$-resonator, as we know it, where $f = (LC)^{-1/2}$, however instead of being closed (ground) in the end not coupled to the transmission line, it is connected to a SQUID, which is then grounded. Ida-Maria Svenson, from Delsing's group at Chalmers, does all the math in her thesis, but the bottom line is presented here (in a paper without her)

$$f = \frac{f_0}{1 + \frac{L_J}{L}}$$

Here $L_J = \Phi_0/(4\pi I_c|cos(\pi\Phi_{ext}/\Phi_0|)$ is the inductance of the SQUID (stemming only from the Junctions, as the geometric inductance is negligible in comparison), and L is the transmission line's inductance to ground (typically $\simeq 0.7\ nH$ for $\lambda/4$ resonators around 6 GHz).

Note that this is true for "moderate" detunings. What happens, when $\Phi_{ext} \to \frac{\Phi_0}{2} \Rightarrow L_J \to \infty$?

Directly coupled resonators

These are qubits without XY control and without readout resonators. Instead, we simply couple the qubit to a feedline, which also serves as a readout line.

Design in four groups of 5 qubits: In each group we have the same junction size (expect the same critical current and thus E_J and we change the length of the junction arms (given a different E_C).

With the parameters:

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CouplingLength=[55]*1e-6;
CouplingGap=[20]*1e-6;
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… we get a coupling of $\sim 0.4\ fF$