Micro lab 6. Questions

Cournot competition

Question - Varian 16.10 Consider an industry with 2 firms, each having marginal costs equal to zero. The (inverse) demand curve facing this industry is

$$P(Y) = 100 - Y \tag{1}$$

where $Y = y_1 + y_2$ is total output.
(a) What is the competitive equilibrium level of industry output?
(b) If each firm behaves as a Cournot competitor, what is firm 1's optimal choice given firm 2's output?
(c) Calculate the Cournot equilibrium amount of output for each firm.
(d) Calculate the cartel amount of output for the industry.
(e) If firm 1 behaves as a follower and firm 2 behaves as a leader, calculate the Stackelberg equilibrium output of each firm.

(a). For a competitive market

$$P = MC = 0 \Rightarrow 100 - Y = 0 \Rightarrow Y^* = 100$$

(b) for firm 1:

$$\max_{y_1} \pi = P \cdot y_1 - 0 \cdot y_1 = (100 - y_1 - y_2) \cdot y_1$$

FOC: $\frac{\partial \pi}{\partial y_1} = 100 - 2y_1 - y_2 = 0$

$$\Rightarrow y_1^* = \frac{100 - y_2}{2}$$

(c): 2 firms are symmetric

so. $y_1 = \frac{100 - y_2}{2}$

$y_2 = \frac{100 - y_1}{2}$

$$\Rightarrow y_1 = y_2 = \frac{100}{3} \Rightarrow y^* = \frac{200}{3}$$

(d) Carter: firms cooperate at the monopolistic output

$$\max_{Y} (100 - Y) \cdot Y$$

$$\Rightarrow Y^* = 50. \Leftrightarrow y_1^* + y_2^* = 50$$

(e) Stackelberg: firms move sequentially, first mover has the advantage.

for firm 2: take firm 1's best response as given

$$\max_{y_2} (100 - y_1 - y_2) y_2 \Leftrightarrow \max_{y_2} \left(100 - \frac{100 - y_2}{2} - y_2\right) y_2 = \left(50 - \frac{y_2}{2}\right) y_2$$

FOC: $50 - y_2 = 0 \Rightarrow y_2^* = 50$

As firm 1 is the follower, then it takes firm 2's output as given

$$\max_{y_1} (100 - 50 - y_1) \cdot y_1 \Leftrightarrow \max_{y_1} (50 - y_1) y_1$$

FOC: $50 - 2y_1 = 0 \Rightarrow y_1^* = 25$

therefore, the market output is $Y^* = 75$

Question - JR 4.13 (Cournot competition in price)

Duopolists producing substitute goods q_1 and q_2 face inverse demand schedules:

$$p_1 = 20 + \frac{1}{2}p_2 - q_1 \quad \text{and} \quad p_2 = 20 + \frac{1}{2}p_1 - q_2 \tag{2}$$

respectively. Each firm has constant marginal costs of 20 and no fixed costs. Each firm is a Cournot competitor in price, not quantity. Compute the Cournot equilibrium in this market, giving equilibrium price and output for each good.

For firm 1:

$$\max_{p_1} \; p_1 q_1 - c \cdot q_1 \;, \quad q_1 = 20 + \frac{1}{2}p_2 - p_1$$

$$\Rightarrow \max_{p_1} (p_1 - 20)(20 + \frac{1}{2}p_2 - p_1)$$

FOC: $\frac{\partial \pi}{\partial p_1} = 20 + \frac{1}{2}p_2 - 2p_1 + 20 = 0$

$$\Rightarrow p_1 = 20 + \frac{1}{4}p_2$$

For firm 2:

$$\max_{p_2} \; p_2 q_2 - c q_2$$

It is symmetric $\Rightarrow p_2 = 20 + \frac{1}{4}p_1$

Solve $\begin{cases} p_1 = 20 + \frac{1}{4}p_2 \\ p_2 = 20 + \frac{1}{4}p_1 \end{cases} \Rightarrow p_1 = p_2 = \frac{80}{3}$

$$\Rightarrow q_1 = q_2 = \frac{20}{3}$$

Bertrand competition

Question - JR 4.12

In the Bertrand duopoly of section 4.2.2, market demand is $Q = \alpha - \beta p$, and firms have fixed costs and identical marginal cost. Find a Bertrand equilibrium pair of prices, (p_1, p_2), and quantities, (q_1, q_2) , when the following hold.

(a) Firm 1 has fixed costs $F > 0$.
(b) Both firms have fixed costs $F > 0$.
*(c) Fixed costs are zero, but firm 1 has lower marginal cost than firm 2, so $c_2 > c_1 > 0$. (For this one, assume the low-cost firm captures the entire market demand whenever the firms charge equal prices.)

(a), (b) $p_1 = p_2 = c \qquad q_1 = q_2 = \frac{\alpha - \beta c}{2}$

(c) we have $c_2 > c_1 > 0$

Consider the bertrand competition on prices.

dynamics:

firm 1 cannot set $p_1 > c_2$ because firm 2 can undercut and capture the market

$\Rightarrow$ firm 1 set $p_1^* = c_2$, at this price firm 2 cannot undercut.

then $\begin{cases} q_1^* = \alpha - \beta c_2 \\ q_2^* = 0 \\ p_1^* = c_2 \\ p_2^* \geq c_2 \end{cases}$

However, it is possible for firm 1 to behave as a monopolist after kicking out firm 2.

firm 1 maximizes the total revenue.

$$\max_{p_1} \; p(\alpha - \beta p) - c_1(\alpha - \beta p)$$

then $p_1^* = \frac{\alpha + \beta c_1}{2\beta}$

Hence if $c_2 \leq p_1^*$, then firm 1 cannot kick out firm 2 and become a monopolist $\Rightarrow p_1^* = c_2$.

if $c_2 > p_1^*$, then firm 1 can be a monopolist

then, $p_1^* = \frac{\alpha + \beta c_1}{2\beta}$, $q_1^* = \frac{\alpha - \beta c_1}{2}$

$p_2 \geq \frac{\alpha + \beta c_1}{2\beta} \qquad q_2 = 0$

2. Price discrimination

First-degree P.D.

> **Question - Varian 14.10.** One common way to price discriminate is to charge a lump sum fee to have the right to purchase a good, and then charge a per-unit cost for consumption of the good after that. The standard example is an amusement park where the firm charges an entry fee and a charge for the rides inside the park. Such a pricing policy is known as a two part tariff. Suppose that all consumers have identical utility functions given by $u(x)$ and that the cost of providing the service is $c(x)$. If the monopolist sets a two part tariff, will it produce more or less than the efficient level of output?

efficient level is given by $u'(x) = c'(x)$

Now consider the two part-tariff.

$$\max_{p,F}\ F + p\cdot x(p) - c(x)$$

$$\text{s.t. } u(x) - p\cdot x(p) - F \geq 0$$

$$\mathcal{L} = F + p x(p) - c(x) + \lambda(u(x) - p x(p) - F)$$

$$[p]: x + p\cdot x'(p) - c'(x)\cdot x'(p) + \lambda[u'(x)\cdot x'(p) - x(p) - p x'(p)] = 0$$

$$[F]: 1 - \lambda = 0 \Rightarrow \lambda = 1$$

then $\cancel{x} + p\,\cancel{x'(p)} - c'(x)\cdot x'(p) - p\cdot \cancel{x'(p)} - \cancel{x(p)} - u'(x)\,x'(p) = 0$

$$\Rightarrow x'(p)\,[\,c'(x) - u'(x)\,] = 0$$

$$\Rightarrow c'(x) = u'(x)$$

which implies that it will produce at the efficient level.

Second-degree P.D.

Question - Varian14.18. There are two consumers who have utility functions

$$u_1(x_1, y_1) = a_1 x_1 + y_1$$
$$u_2(x_2, y_2) = a_2 x_2 + y_2 \quad (3)$$

The price of the y-good is 1, and each consumer has a "large" initial wealth. We are given that $a_2 > a_1$. Both goods can only be consumed in nonnegative amounts.

A monopolist supplies the x-good. It has zero marginal costs, but has a capacity constraint: it can supply at most 10 units of the x-good. The monopolist will offer at most two price-quantity packages, (r_1, x_1) and (r_2, x_2). Here r_i is the cost of purchasing x_i units of the good.

(a) Write down the monopolist's profit maximization problem. You should have 4 constraints plus the capacity constraint $x_1 + x_2 \le 10$.

(b) Which constraints will be binding in optimal solution?

(c) Substitute these constraints into the objective function. What is the resulting expression?

(d) What are the optimal values of (r_1, x_1) and (r_2, x_2)?

Individual rationality - IR

Incentive compatibility IC.

Assume I_1 and I_2 are initial wealth for consumer 1 & 2

(a) $\max \; r_1 + r_2$

$$\text{s.t.} \begin{cases} a_1x_1 + (I_1 - r_1) \ge I_1 & (IR.1) \\ a_2x_2 + (I_2 - r_2) \ge I_2 & (IR.2) \\ a_1x_1 + (I_1 - r_1) \ge a_1x_2 + (I_1 - r_2) & (IC_1) \\ a_2x_2 + (I_2 - r_2) \ge a_2x_1 + (I_2 - r_1) & IC_2 \\ x_1 + x_2 \le 10 \end{cases}$$

$$\Leftrightarrow \quad \begin{aligned} & a_1x_1 - r_1 \ge 0 & ① \\ & a_2x_2 - r_2 \ge 0 & ② \\ & a_1x_1 - r_1 \ge a_1x_2 - r_2 & ③ \\ & a_2x_2 - r_2 \ge a_2x_1 - r_1 & ④ \\ & x_1 + x_2 \le 10 & ⑤ \end{aligned}$$

(b). One of ① & ② must be binding,

Suppose ② is binding $\Rightarrow a_2x_2 - r_2 = 0 \ge a_2x_1 - r_1 \quad , x_1 - r_1$

$a_2 > a_1$

which contradicts to ①

So ② cannot be binding $\Rightarrow$ ① is binding

$a_1x_1 = r_1$

As one of ② & ④ has to be binding $\Rightarrow$ ④ is binding

This has an intuition that monopolist need to keep high value consumers not mimicing low value consumers.

(c) By ①: $r_1 = a_1x_1$

By ④: $r_2 = a_2x_2 - a_2x_1 + a_1x_1$

Substitute them into the objective function

$$\max_{x_1, x_2} \; a_1x_2 + 2a_1x_1 - a_2x_1$$

s.t. ② ③ & ⑤

check ②: $a_2x_2 - a_2x_2 + (a_2 - a_1)x_1 \geq 0$ automatically satisfied

check ③: $0 \geq a_1x_2 - a_2x_2 + a_2x_1 - a_1x_1$

$0 \geq (a_1 - a_2)(x_2 - x_1) \iff x_2 \geq$

& ⑤ $x_1 + x_2 \leq 10$

$$\Rightarrow \max_{x_1, x_2} \; a_1x_2 + 2a_1x_1 - a_2x_1$$

s.t. $x_2 \geq x_1$

$x_1 + x_2 \leq 10$

$$\mathcal{L} = a_2x_2 - a_2x_1 + 2a_1x_1 + \lambda(x_2 - x_1) - u(x_1 + x_2 - 10)$$

$[x_1]$: $2a_1 - a_2 - \lambda - u = 0$

$[x_2]$: $a_2 + \lambda - u = 0 \quad \Rightarrow \quad u = \lambda + a_2 > 0$

$\Rightarrow \quad x_1 + x_2 = 1$

we can substitute $x_2 = 10 - x_1$ back to the optimality problem.

$$\max_{x_1} \; a_2(10 - x_1) - a_2x_1 + 2a_1x_1$$

$$\Rightarrow \max_{x_1} \; 2(a_1 - a_2)x_1 + 10a_2$$

s.t. $\begin{cases} x_1 \geq 0 \\ x_1 \leq 5 \Leftarrow 10 - x_1 \geq x_1 \end{cases}$

Since $a_1 - a_2 < 0$, the objective function is decreasing in x_1

then $x_1^* = 0$, $x_2^* = 10$

Correspondingly. $r_1^* = 0$ and $r_2^* = 10a_2$

Third-degree P.D.

> **Question - Varian 14.19.** A monopolist sells in two markets. The demand curve for the monopolist's product is $x_1 = a_1 - b_1 p_1$ in market 1 and $x_2 = a_2 - b_2 p_2$ in market 2 , where x_1 and x_2 are the quantities sold in each market, and p_1 and p_2 are the prices charged in each market. The monopolist has zero marginal costs. Note that although the monopolist can charge different prices in the two markets, it must sell all units within a market at the same price.
> (a) Under what conditions on the parameters (a_1, b_1, a_2, b_2) will the monopolist optimally choose not to price discriminate? (Assume interior solutions.)
> (b) Now suppose that the demand functions take the form $x_i = A_i p_i^{-b_i}$, for $i = 1, 2$, and the monopolist has some constant marginal cost of $c > 0$. Under what conditions will the monopolist choose not to price discriminate? (Assume interior solutions.)

(a) In mkt 1

$$\max_{p_1} p_1(a_1 - b_1 p_1) \Rightarrow p_1^* = \frac{a_1}{2b_1}$$

In mkt 2

$$\max_{p_2} p_2(a_2 - b_2 p_2) \Rightarrow p_2^* = \frac{a_2}{2b_2}$$

No price discrimination implies that $p_1^* = p_2^*$

$$\Rightarrow \frac{a_1}{2b_1} = \frac{a_2}{2b_2} \Leftrightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2}$$

(b) for mkt i

$$\max_{p_i} (p_i - c) \cdot A_i p_i^{-b_i}$$

$$\text{FOC}: A_i p_i^{-b_i} - (p_i - c) A_i b_i p_i^{-b_i - 1} = 0$$

$$\Rightarrow 1 = \frac{(p_i - c) b_i}{p_i}$$

$$\Rightarrow 1 = \left(1 - \frac{c}{p_i}\right) b_i$$

$$\Rightarrow p_i = \frac{c}{1 - \frac{1}{b_i}}$$

therefore, if $b_1 = b_2 \Rightarrow$ No price discrimination.