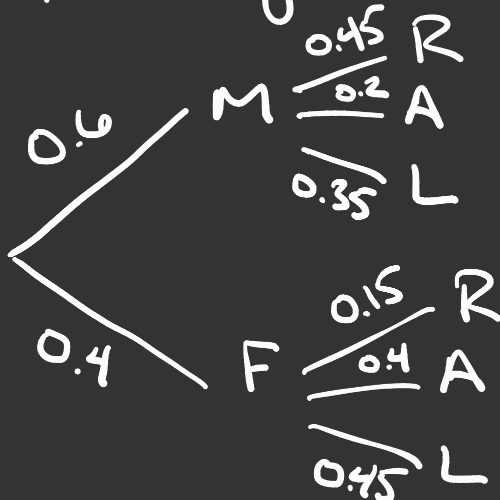
Example: City council voting on project



$$P(L|F) = 0.45$$

What about $P(F|L)$?

$$P(F|L) = \frac{P(F \cap L)}{P(L)}$$

$$P(F \cap L) = P(F) \cdot P(L|F)$$

$$P(L) = P(F \cap L) + P(M \cap L) = P(F) \cdot P(L|F) + P(M) \cdot P(L|M)$$

$$P(F|L) = \frac{P(F\cap L)}{P(L)} = \frac{P(F\cap L)}{P(F\cap L) + P(M\cap L)}$$

$$= \frac{P(F)\cdot P(L|F)}{P(F)\cdot P(L|F) + P(M)\cdot P(L|M)}$$

Bayes' Rule

$$= \frac{0.4 \times 0.45}{0.4\times 0.45 + 0.6 \times 0.35} = 0.462$$

Remark: It's important that $M \cup F$ is the entire council

(so $P(L) = P(F\cap L) + P(M\cap L)$)

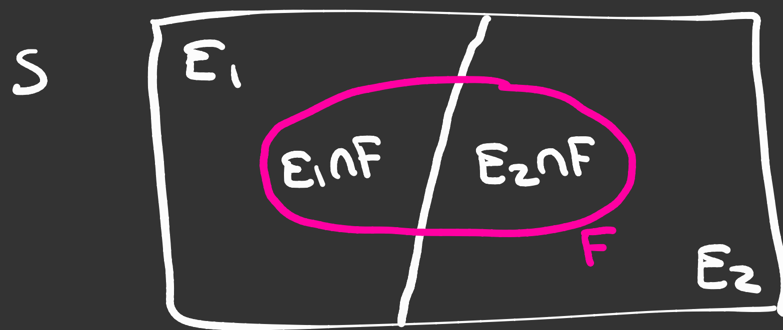
Bayes' Rule: E_1, E_2 are mutually exclusive events with $E_1 \cup E_2 = S$ (like M, F in last ex). Then for an event F with $P(F) \neq 0$

(a) $$P(E_1 | F) = \frac{P(E_1 \cap F)}{P(F)}$$

(b) $$P(E_1 | F) = \frac{P(E_1 \cap F)}{P(E_1 \cap F) + P(E_2 \cap F)}$$

(c) $$P(E_1 | F) = \frac{P(E_1) \cdot P(F | E_1)}{P(E_1) \cdot P(F | E_1) + P(E_2) \cdot P(F | E_2)}$$

Venn Diagram:



$$P(E_1 \mid F) = \frac{P(E_1 \cap F)}{P(F)} = \frac{\text{Area of } E_1 \cap F}{\text{Area of } F}$$

$$\text{Area of } F = \text{Area of } E_1 \cap F + \text{Area of } E_2 \cap F$$

$$\rightsquigarrow P(E_1 \mid F) = \frac{P(E_1 \cap F)}{P(E_1 \cap F) + P(E_2 \cap F)}$$

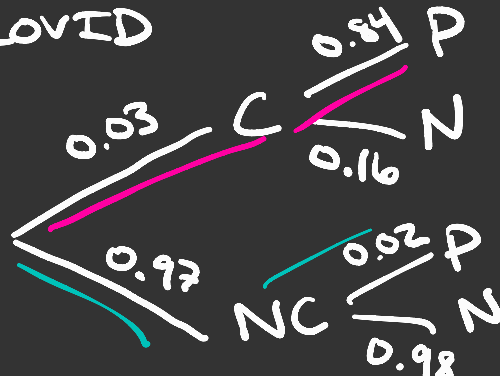
Example: COVID Testing

C = person has COVID

NC = person does not have COVID

P = test positive

N = test negative



Prevalence: % of pop having COVID $(P(C) = 0.03)$

Sensitivity: % of people who test pos when they have COVID $0.84 = P(P|C)$

Specificity: % of people who test neg when they don't have COVID $P(N|NC) = 0.98$

$$P(C \mid P) = \frac{P(C \cap P)}{P(P)} = \frac{P(C \cap P)}{P(C \cap P) + P(NC \cap P)}$$

$$= \frac{0.03 \times 0.84}{0.03 \times 0.84 + 0.97 \times 0.02} = 0.565$$