

Example: Coin matching

Two players, Rob and Chad

Each player has coin, and decide which face of coin to turn up (simultaneously)

Both H: Rob pays Chad 50¢

Both T: Rob pays Chad 25¢

One of each: Chad pays Rob 35¢

---

Can convert this to a payoff matrix

Pos : R gets paid

Neg : C gets paid

R

		C
R	H	-50¢ 35¢
	T	35¢ -25¢

Question : Is there a strategy to these games?

Answer : Not always

Example :

	H	T
R	H	$\begin{bmatrix} -1 & 1 \end{bmatrix}$
	T	$\begin{bmatrix} 1 & -1 \end{bmatrix}$

Always 50/50 chance.

Some games do have strategies.

Example :

		C	
		C <sub>1</sub>	C <sub>2</sub>
R	r <sub>1</sub>	[ 4      -9 ]	
	r <sub>2</sub>	[ 6      8 ]	

Strategy for R: r<sub>2</sub>

Value of the game

Strategy for C: C<sub>1</sub>, even though their only chance  
of winning is with C<sub>2</sub>. They  
know that R will choose r<sub>2</sub>

- The value of this game is 6
  - The location (2,1) is called the saddle point
  - $r_2, c_1$  are called solution to this game.
- 

Note: Payoff matrix  $C$  doesn't have to be  $2 \times 2$ :

$$R \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \quad C \begin{bmatrix} c_1, c_2, \dots, c_n \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

R has m choices  
C has n.

## Assumptions:

- 1) Each player wants to win as much as possible
- 2) Neither player know what other will choose beforehand (unless there's a reason for it)
- 3) Each player assumes the other player is smart and plays rationally. (C knows R will choose  $r_2$  every time in last example)

How to find strategy:

		C		Row Min
		c <sub>1</sub>	c <sub>2</sub>	
R	r <sub>1</sub>	4	-9	-9
	r <sub>2</sub>	6	8	6
		Col Max	6	8

R's perspective:  
• Look at each row, find minimum entry of each row.

- Select row with biggest min

C's perspective:  
• Look at each column, find maximum entry of each column

- Select column with smallest max

R should choose this row

If  $(\text{largest row min}) = (\text{smallest col max})$ , then  
the game is called strictly-determined.

(players should play same strategy every single  
time)

Example:

		C <sub>1</sub>	C <sub>2</sub>	Row Min
		r <sub>1</sub>	r <sub>2</sub>	r <sub>3</sub>
R	r <sub>1</sub>	1	2	1
R	r <sub>2</sub>	3	4	3
R	r <sub>3</sub>	7	5	5

(Col Max) 7 5

Strictly determined: Yes

Value: 5

Saddle Point: (3, 2)

Solution: r<sub>3</sub>, C<sub>2</sub>