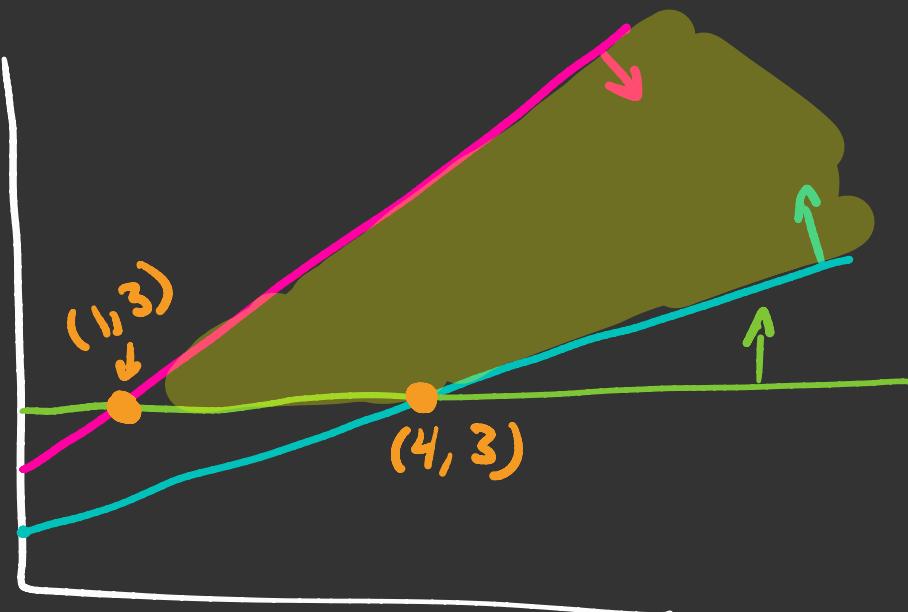


Example: Constraints : $-x + y \leq 2$ —
 $-x + 2y \geq 2$ —
 $y \geq 3$

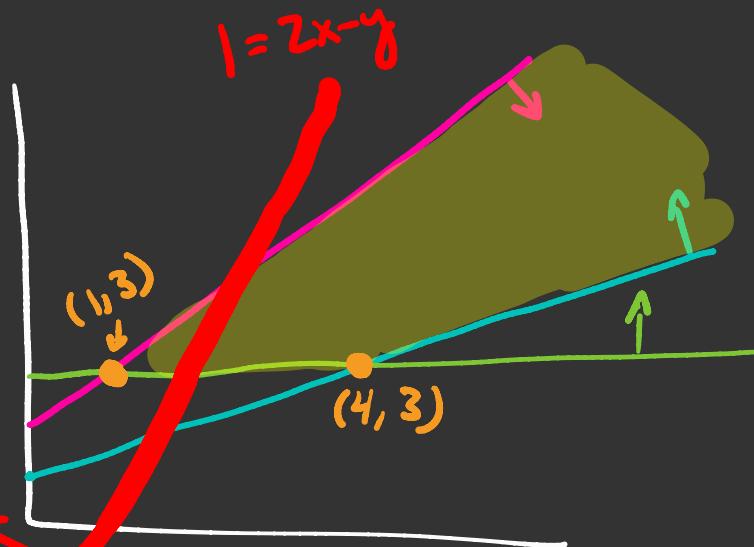


Max/min the following functions w/ these constraints

a) $z = 2x - y$

$(1, 3) : z = -1$

$(4, 3) : z = 5$



Look at $l = 2x - y$

Min: $z = -1$ at $(1, 3)$

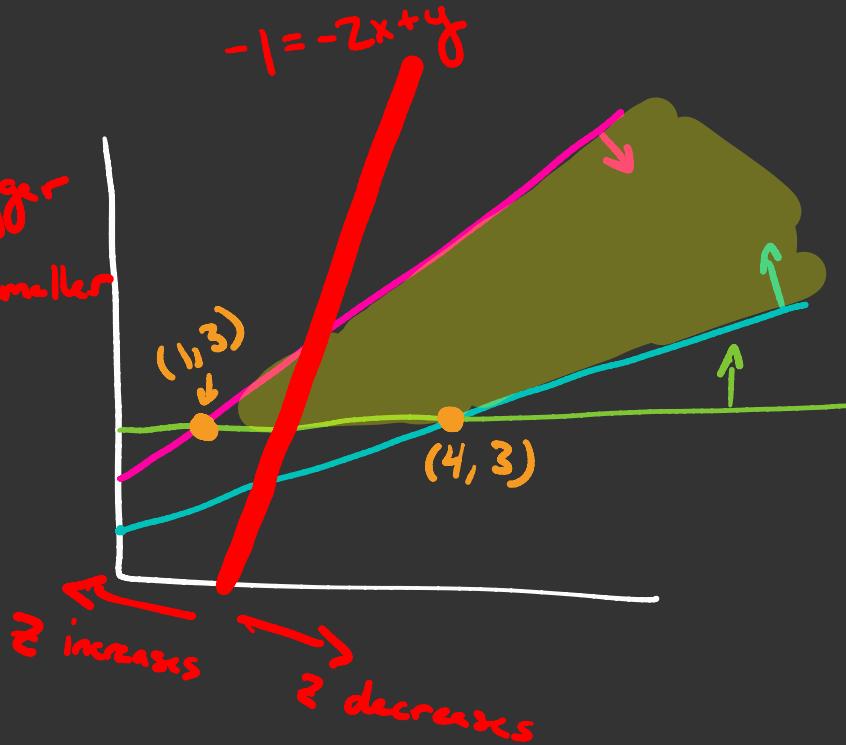
Max: None

b) $z = -2x + y$

$(1, 3) : z = 1 \leftarrow \text{Bigger}$

$(4, 3) : z = -5 \leftarrow \text{Smaller}$

Look at $-1 = -2x + y$



c) $z = 2x + y$

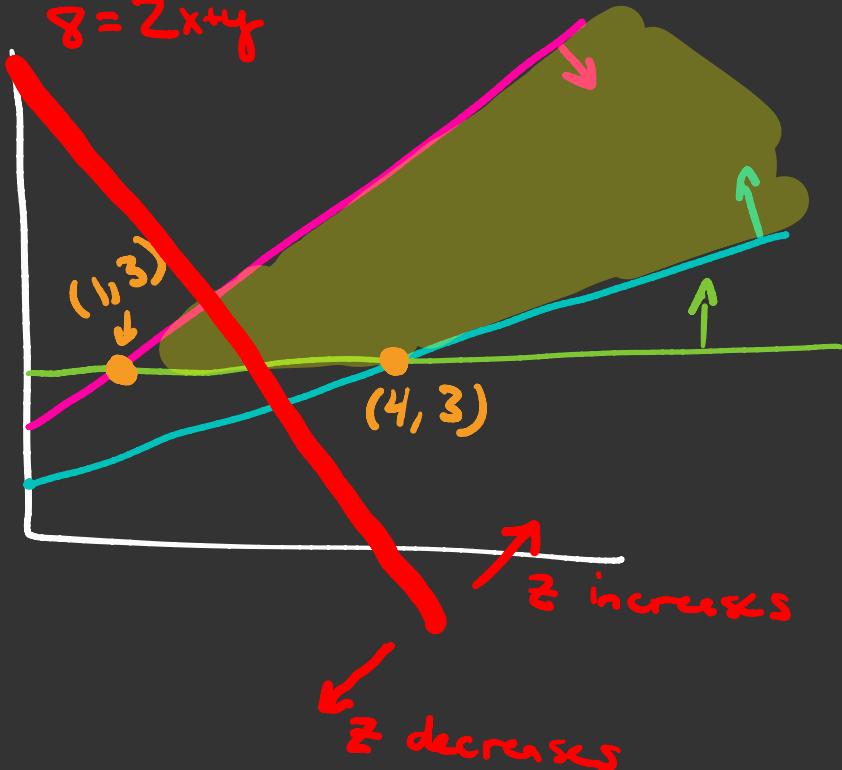
$(1, 3)$: $z = 5$

$(4, 3)$: $z = 11$

Look at $g = 2x + y$

Min: $z = 5$ at $(1, 3)$

Max: None



Matrices

Def: A matrix a rectangular array of numbers

Example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

\uparrow
size 2×3

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

\uparrow
size 4×3

The size of a matrix is (#rows) \times (#columns)

We can define addition and multiplication with matrices

Addition

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{bmatrix} = \begin{bmatrix} 1+10 & 2+11 \\ 3+12 & 4+13 \\ 5+14 & 6+15 \end{bmatrix}$$
$$= \begin{bmatrix} 11 & 13 \\ 15 & 17 \\ 19 & 21 \end{bmatrix}$$

Matrices must be same size to add !

Multiplication : Weird!

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 1 \times 10 + 2 \times 11 + 3 \times 12 \\ 4 \times 10 + 5 \times 11 + 6 \times 12 \end{bmatrix}$$

(2×3) (3×1) (2×1)

Must match

$$= \begin{bmatrix} 68 \\ 167 \end{bmatrix}$$

Go across row of 1st matrix
Go along column of 2nd matrix

To multiply : # columns of 1st matrix = # rows 2nd matrix !

Example: a)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \times 4 + 2 \times 2 & 1 \times 3 + 2 \times 1 \\ 3 \times 4 + 4 \times 2 & 3 \times 3 + 4 \times 1 \end{bmatrix}$$

(2×2) (2×2)

Match

$$= \begin{bmatrix} 8 & 5 \\ 20 & 13 \end{bmatrix}$$

b)

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 \times 1 + 3 \times 3 & 4 \times 2 + 3 \times 4 \\ 2 \times 1 + 1 \times 3 & 2 \times 2 + 1 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 20 \\ 5 & 8 \end{bmatrix}$$

Remark: It is not true in general that
 $AB = BA$ for matrices A and B