The Art of Modeling: From Concept to Math with Mass, Energy, and Momentum Balance

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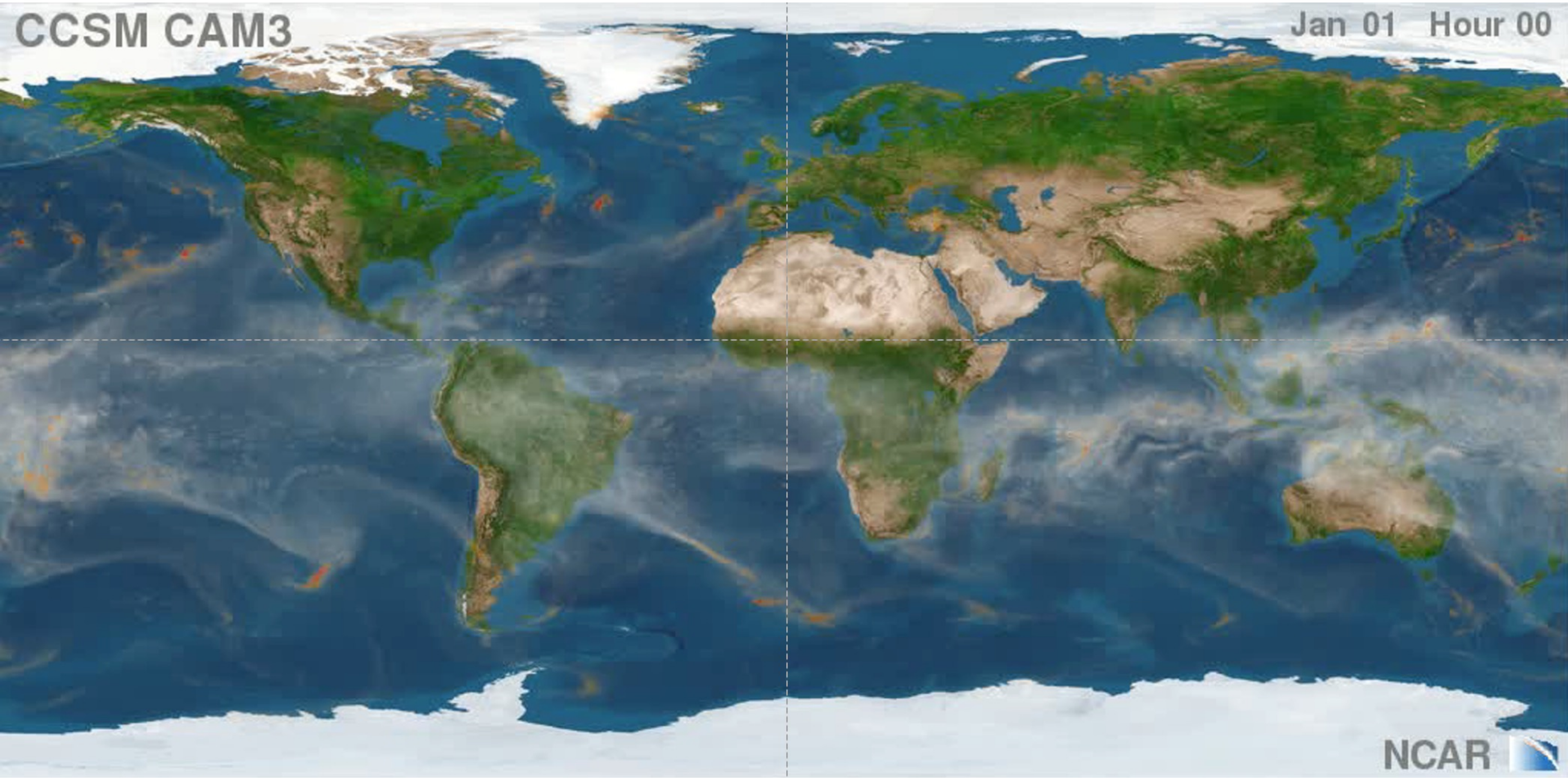
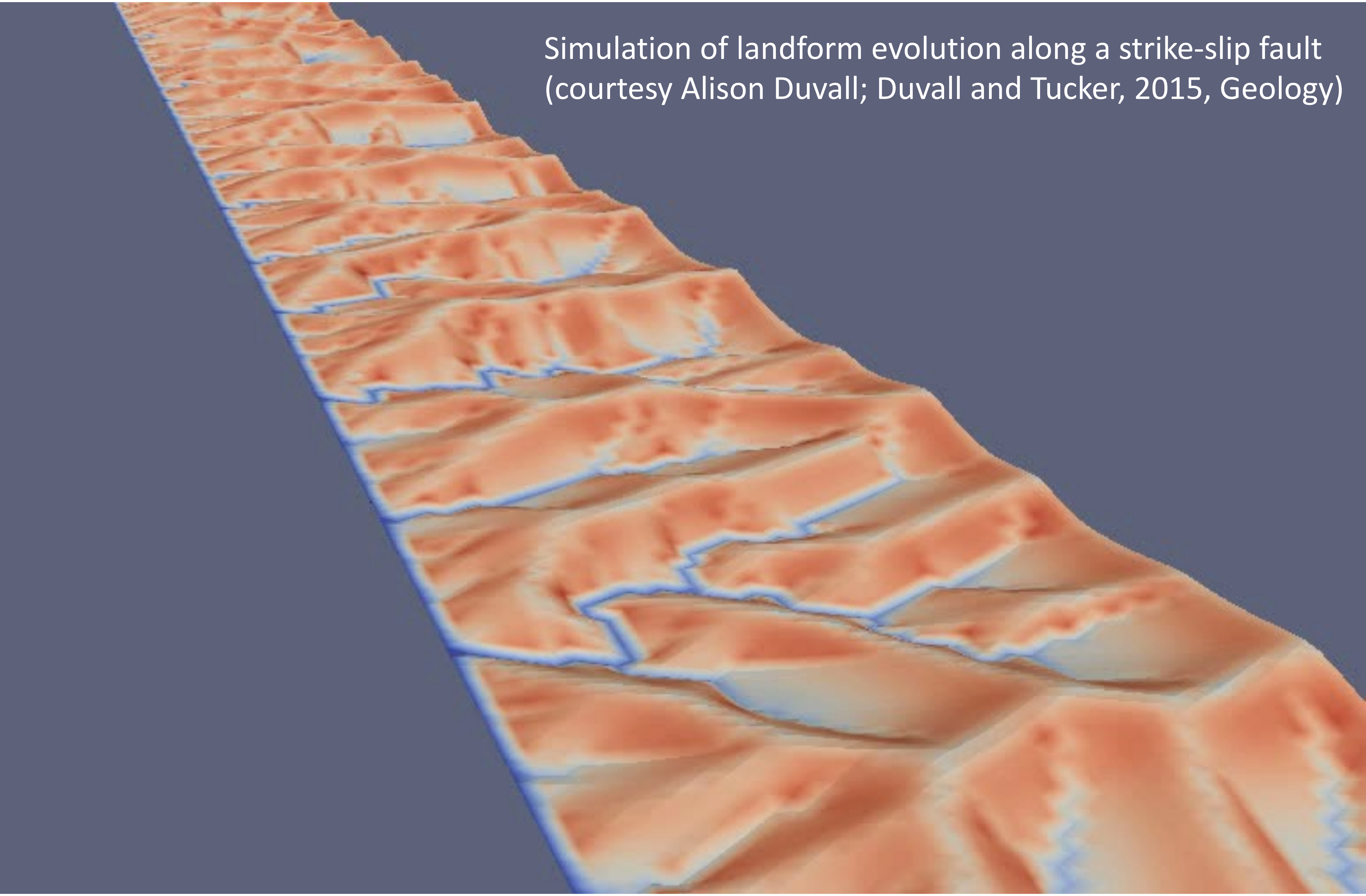
introductory notes for clinic presented at
CSDMS Annual Meeting
May 2022

NCAR CCSM CAM3 T341 Cloud and Precipitation Simulation



https://www.youtube.com/watch?v=n0mupl4FZsQ

Numerical models are widely used in earth-surface dynamics

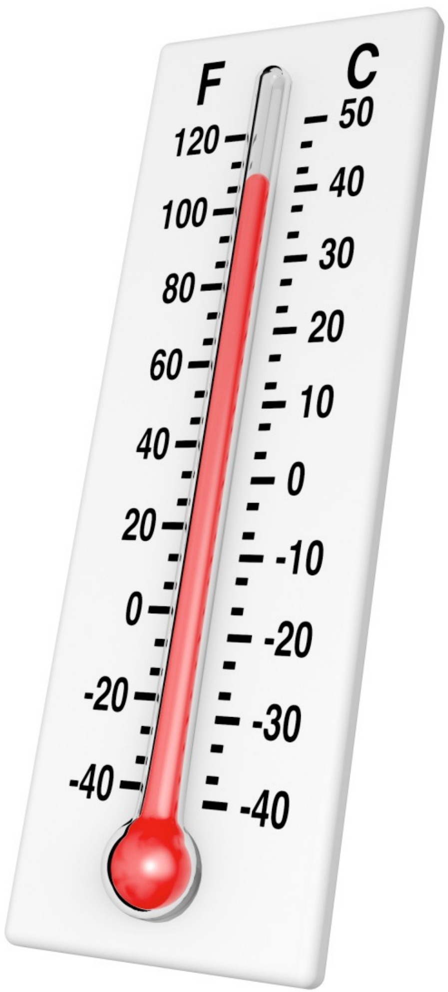


Simulation of landform evolution along a strike-slip fault
(courtesy Alison Duvall; Duvall and Tucker, 2015, Geology)

What is a model?

- A person who shows off apparel – *fashion models*
- A type of approach to something – *the open-access model*
- A scaled-down version of a real-world object – *scale model of the Empire State building*
- A scaled-down lab apparatus – *Jurassic tank*
- A illustration – *artist's conception of the early solar system*
- A concept about how something works, or worked in the past – *the snowball earth model*
- **An equation** (or many) – $F = ma$
- **A computer program** that uses numerical algorithms to represent a system – *WRF*

Nature as numbers



Nature as math

$$F = m\,a$$

$$E = m\,c^2$$

$$a^2 + b^2 = c^2$$

$$F_g = G\,m_1\,m_2 / r^2$$

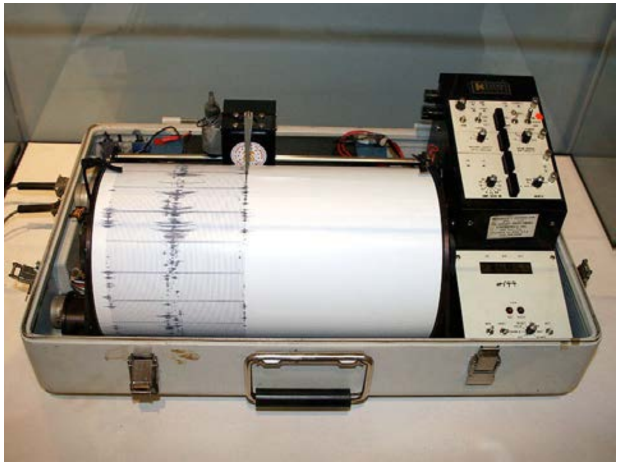
$$dN/dt = -k\,N$$

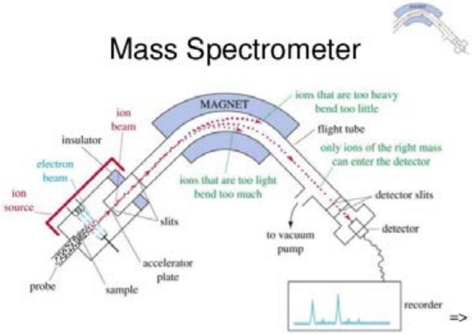
$$P\,V = n\,R\,T$$

What is a computer model?

- A computer program that calculates the (approximate) solution for a particular mathematical model, given a set of inputs
- A concept about nature that has been translated into binary digital logic (computer “instructions”)
- Nature represented as numbers that result from a series of calculations

Technology in science is often used to extend our senses





Computational models enhance our *cognition*: “Mind-enhancing machines”

```
#----------------------------------------------------------------------
# RasterModelGrid.initialize:
#
# Sets up a num_rows by num_cols grid with cell spacing dx and
# (by default) regular boundaries (that is, all perimeter cells are
# boundaries and all interior cells are active).
#   To be consistent with unstructured grids, the raster grid is
# managed not as a 2D array but rather as a set of vectors that
# describe connectivity information between cells and faces. Each
# cell in the grid has four faces. Each face has a "fromcell" and
# a "tocell"; the convention is that these always "point" up or
# right (so a negative flux across a face is either going left or
# down).
#----------------------------------------------------------------------
def initialize( self, num_rows, num_cols, dx ):

    # Debugging output flag
    self.debug = False

    # Basic info about raster size and shape
    self.nrows = num_rows
    self.ncols = num_cols
    self.ncells = num_rows * num_c
    self.dx = dx
    self.cellarea = dx*dx

    # We need at least one row or
    # side, so the grid has to be
    assert self.ncells >= 9

    # Record number of boundary an
    # of interior faces. Ultimatel
    # if using an irregular geomet
    # rectangular domain. Note tha
    # between boundary cells.
    self.n_boundary_cells = 2 * (
    self.n_interior_cells = self.n
    self.nfaces = ( num_rows - 1 )
                  ( num_rows - 2 )
    if self.debug:
        print self.nfaces
```

Common uses of computer models

1. **Make predictions** with a *trusted model*
   - Assumed to provide accurate prediction of nature in a new situation
   - Apply trusted physics/chemistry to new situation
   - Example: stresses on a planned bridge or building; law of gravitation; Newton's laws of motion

Example: an orbital mechanics model for a satellite

Common uses of computer models

2. **Test hypothesis feasibility** with a (somewhat) trusted model
   - Is my conceptual hypothesis consistent with what we know about the governing processes?

Cretaceous Western Interior Seaway: clockwise or counter-clockwise circulation?

Estuarine circulation in the Turonian Western Interior seaway of North America

Rudy Slingerland
Lee R. Kump
Michael A. Arthur
Peter J. Fawcett*
Bradley B. Sageman*
Eric J. Barron

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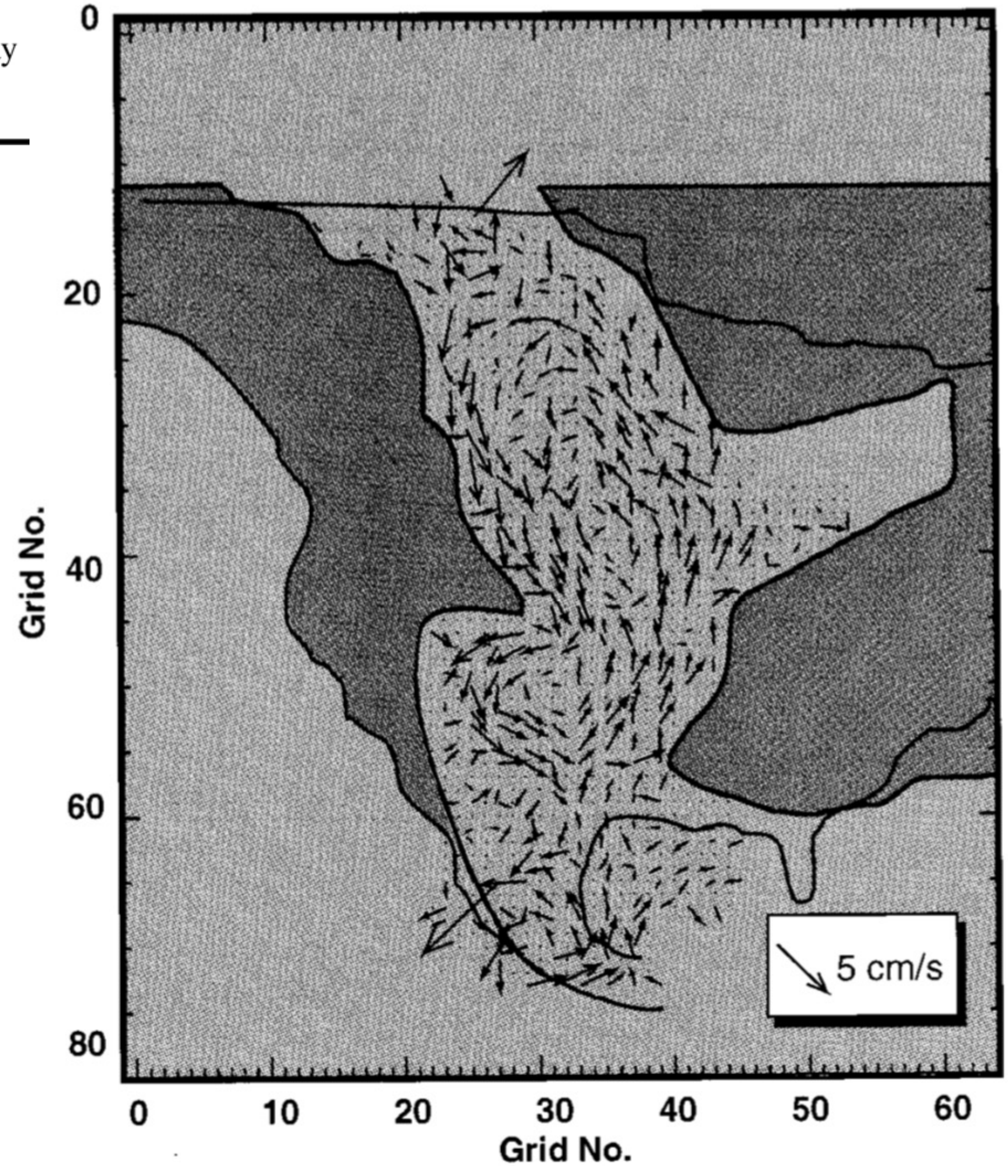

Figure 5. Mean annual steady-state circulation in top 10 m of water column as predicted by CIRC under conditions listed in Table 2. Circulation in the seaway consists of a large cyclonic gyre. CIRC is a three-dimensional formulation of turbulent flows in coastal seas developed by Leendertse and Liu (1977).

Common uses of computer models

3. **Develop theory** by iteratively comparing mathematical and/or computational model(s) with data
   - How well does model fit data?
   - Does one proposed model do better than another?
   - How can the model be improved?

How well can hillslope evolution models "explain" topography? Simulating soil transport and production with high-resolution topographic data

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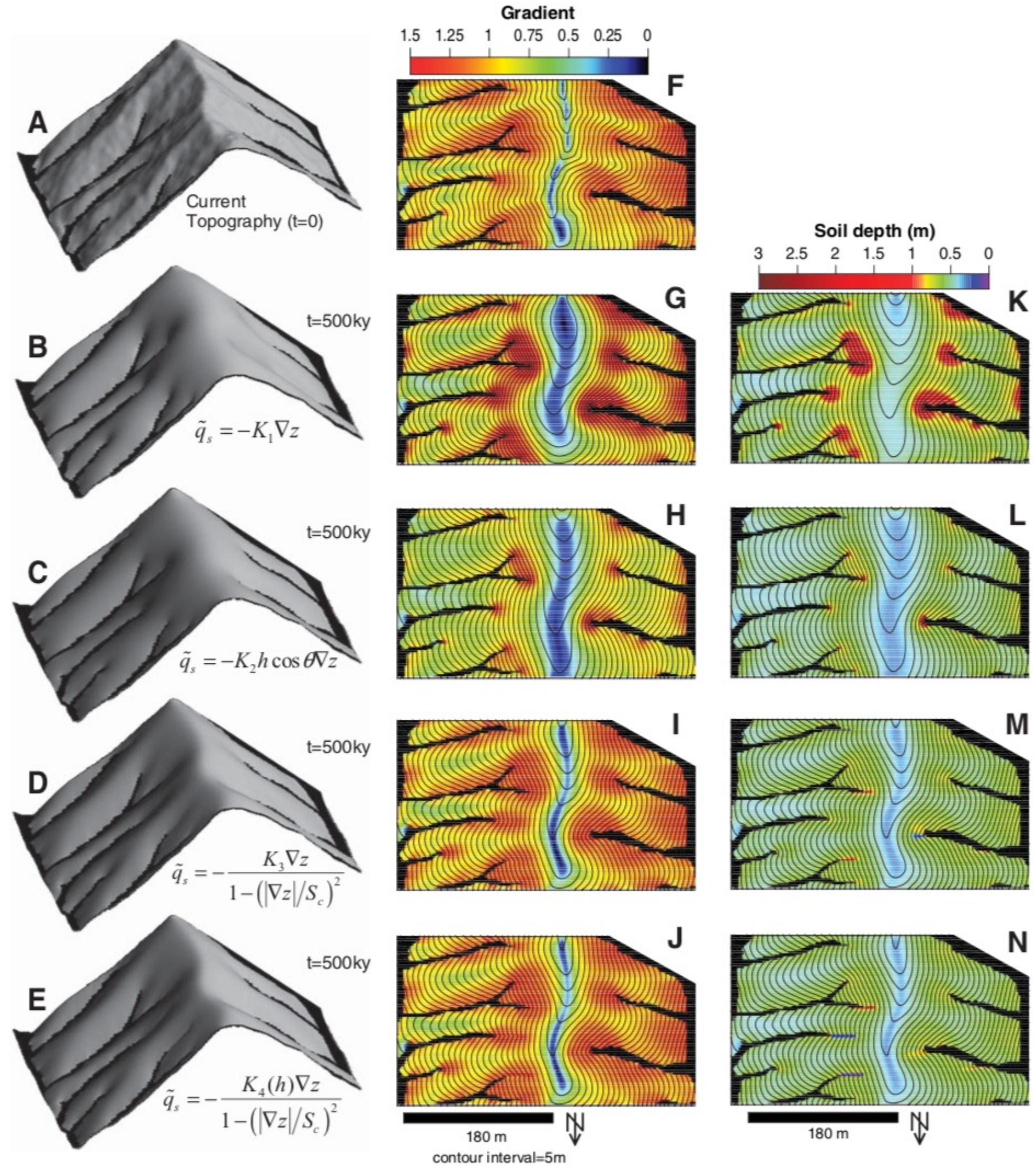


Figure 8. Comparison of simulation surfaces with current topography. (A–E) Perspective-view, shaded relief images of current and modeled topography. Modeled surfaces reflect 500,000 yr of evolution via the calibrated parameters given in Figure 7. (F–J) Spatial variation of hillslope gradient for current and modeled surfaces. The current surface is pockmarked due to bioturbation and data errors, whereas the modeled surfaces are uniformly smooth because of the continuum assumption used here. The nonlinear slope-dependent models (I and J) best represent the sharp, steep-sided slope morphology of the field site. (K–N) Spatial variation of simulated soil depth for the four transport models. Each model predicts thin soils near the ridge top and thicker soils along sideslopes.

Common uses of computer models

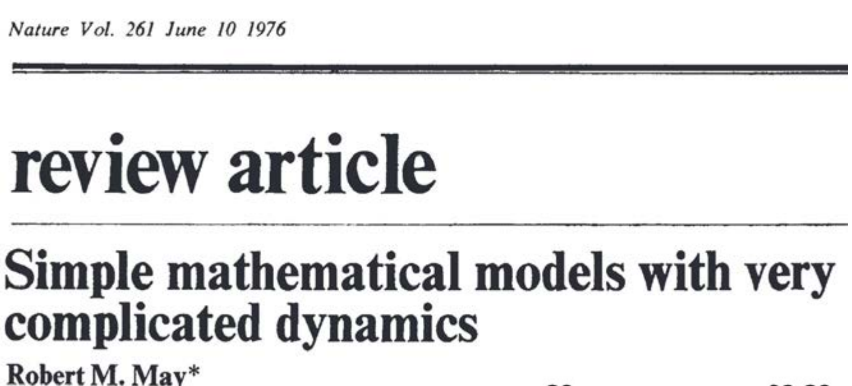
4. **Explore a new idea** with a *hypothetical model*
   - Reveal logical consequences of an idea
   - Explore “what if” scenarios; develop insight

➔ EXPLORATION AND ILLUSTRATION OF NOVEL CONCEPTS

EXAMPLE:

Logistic map as a mathematical illustration of deterministic chaos

https://en.wikipedia.org/wiki/Logistic_map

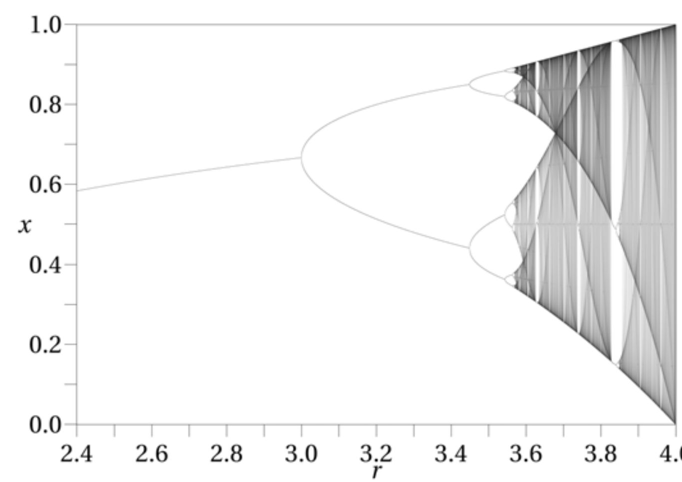

Nature Vol. 261 June 10 1976

review article

Simple mathematical models with very complicated dynamics

Robert M. May*

$$x_{n+1} = r x_n \left(1 - x_n\right)$$



Summary: some ways numerical models are used

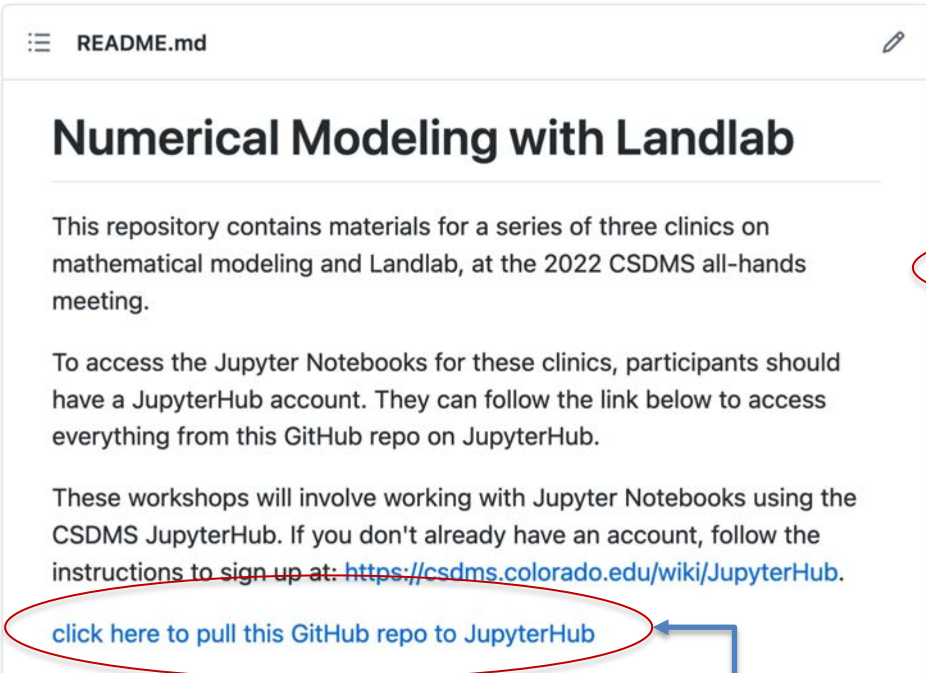
- Making predictions
- Testing hypothesis feasibility
- Developing and improving theory
- Exploring ideas

Four not-so-simple steps

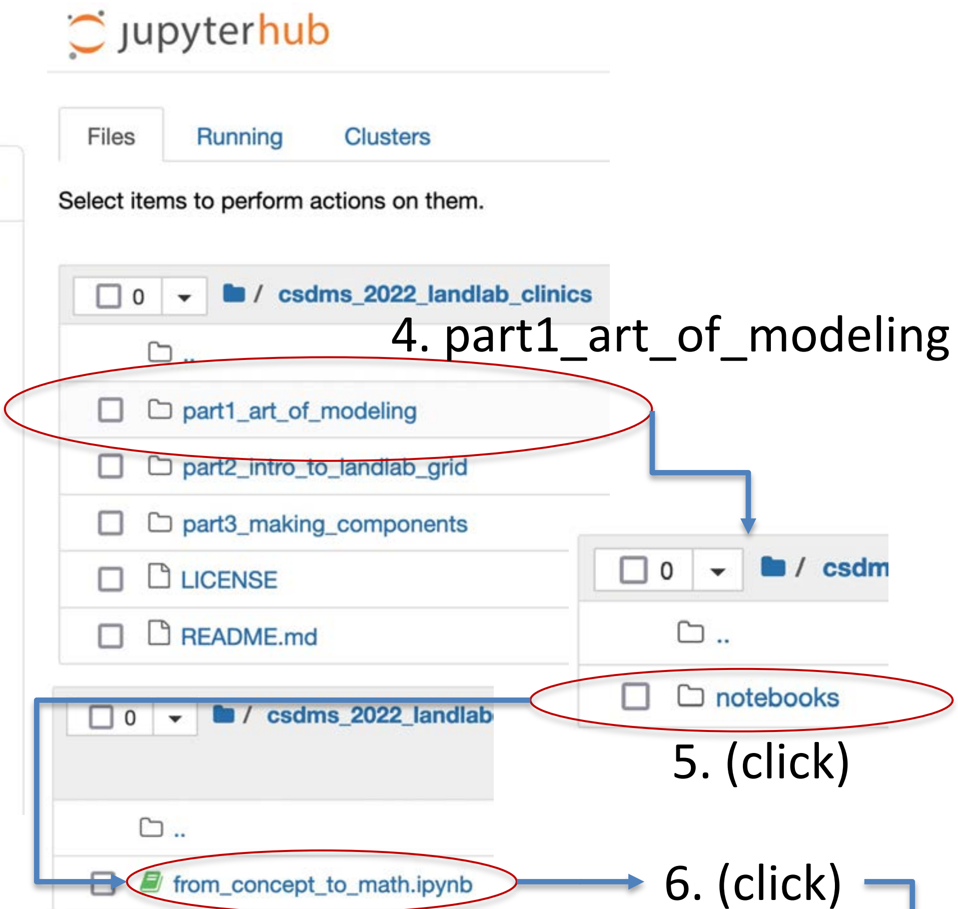
- concept ➔ math
- math ➔ algorithms
- algorithms ➔ computer code
- code ➔ insight and discovery

1. Go to: https://github.com/landlab/csdms_2022_landlab_clinics

2. Page down to:



3. Click here



4. part1_art_of_modeling

5. (click)

6. (click)