Radio-Interferometric Measurement Equation

Introductory Radio Interferometry Course

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Radio-Interferometric Measurement Equation (RIME)

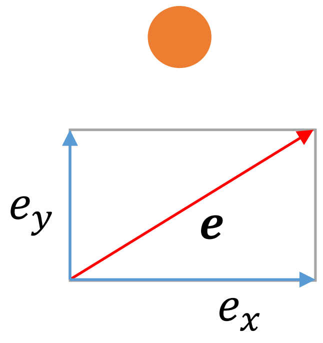
- Compact, intuitive, matrix-based way of representing propagation effects in radio interferometry.
- Useful for calibration (solving for and correcting these propagation effects).

Introduction

e'_x, e'_y : Components of electric field vector in reference frame of sky, at the observer

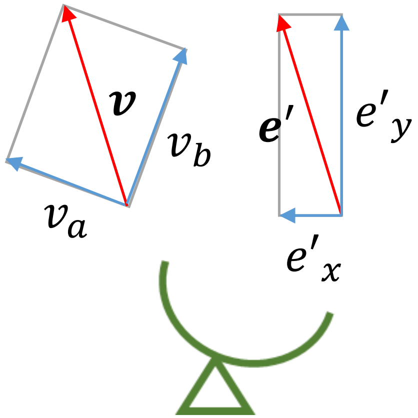
v_a, v_b : Voltages measured by antenna feed (linearly or circularly polarized)

Source



Propagation effects

e_x, e_y : Components of electric field vector in reference frame of sky, at the source



Antenna

Can be represented as vectors:

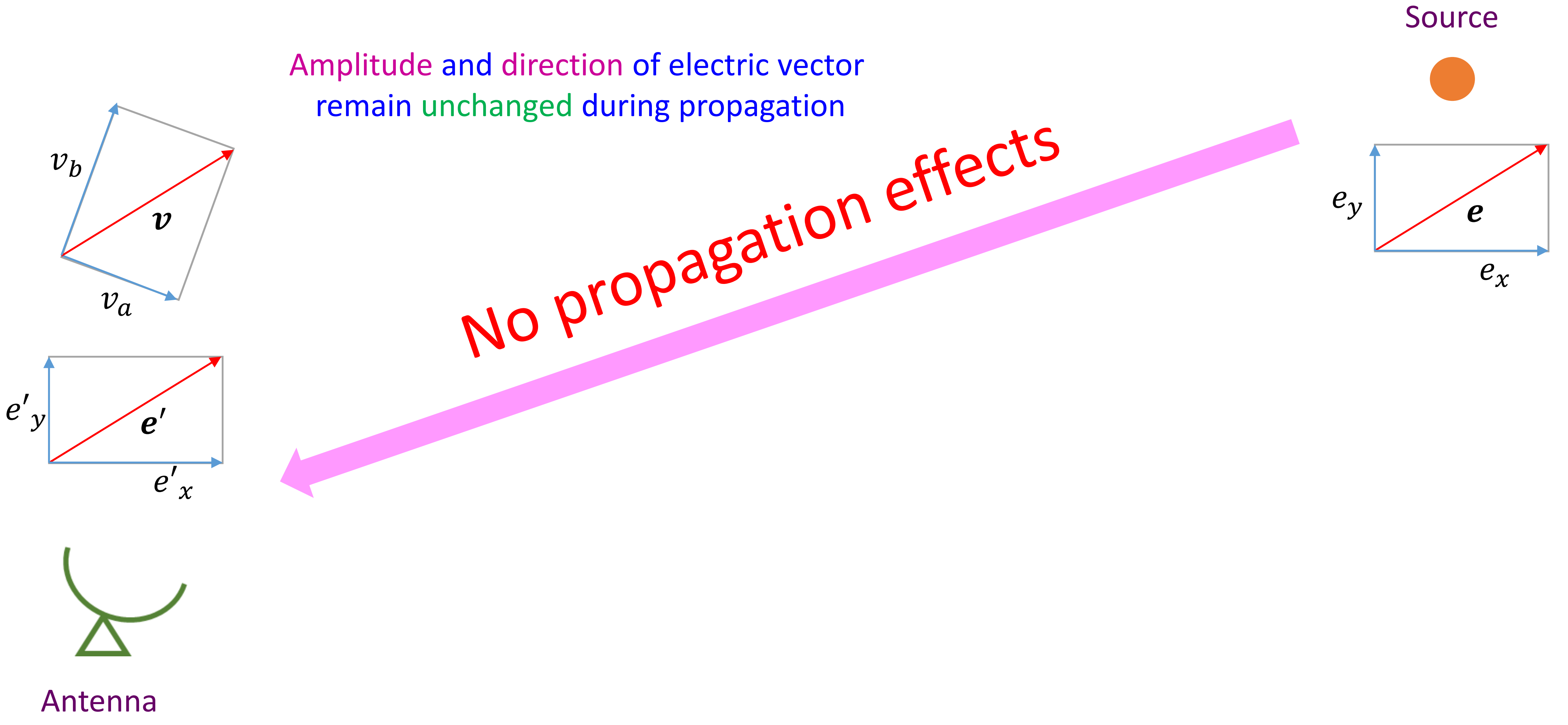
$$\boldsymbol{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$

$$\boldsymbol{e}' = \begin{pmatrix} e'_x \\ e'_y \end{pmatrix}$$

$$\boldsymbol{v} = \begin{pmatrix} v_a \\ v_b \end{pmatrix}$$

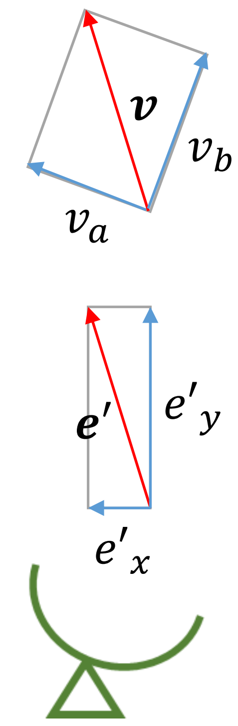
Propagation effects absent

Amplitude and direction of electric vector remain unchanged during propagation

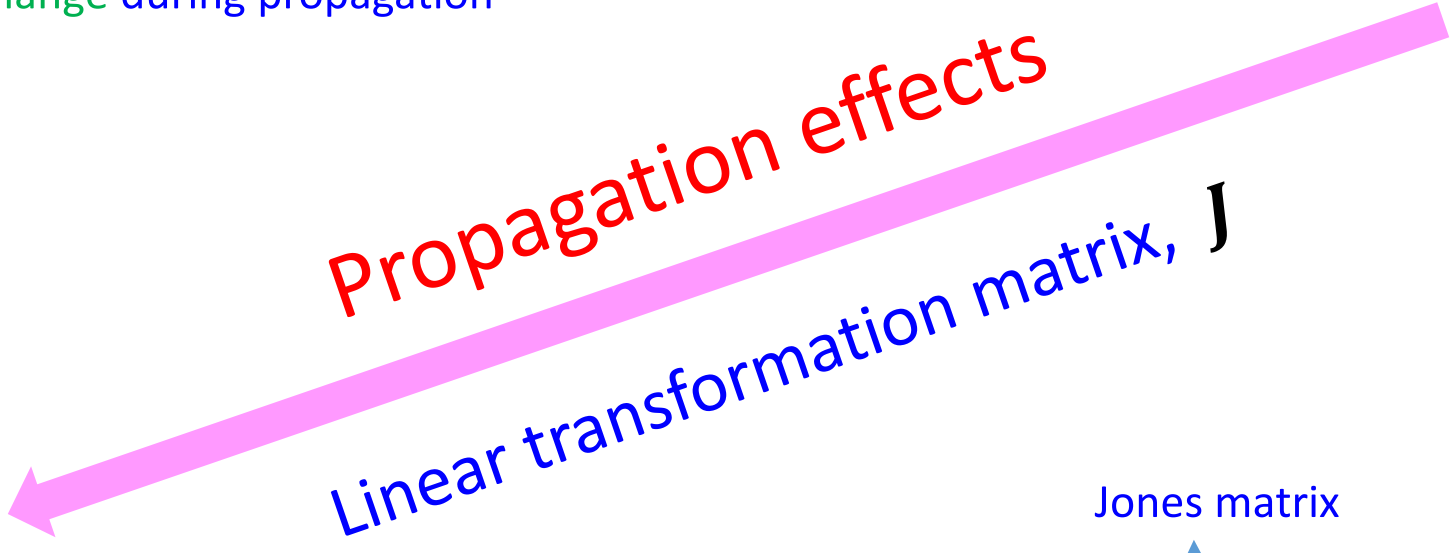


Propagation effects present

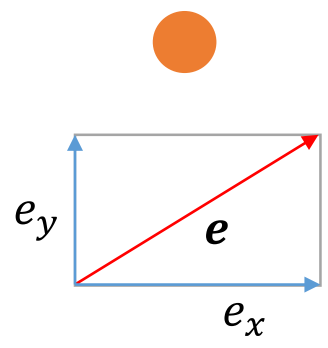
Amplitude and direction of electric vector change during propagation



Antenna



Source

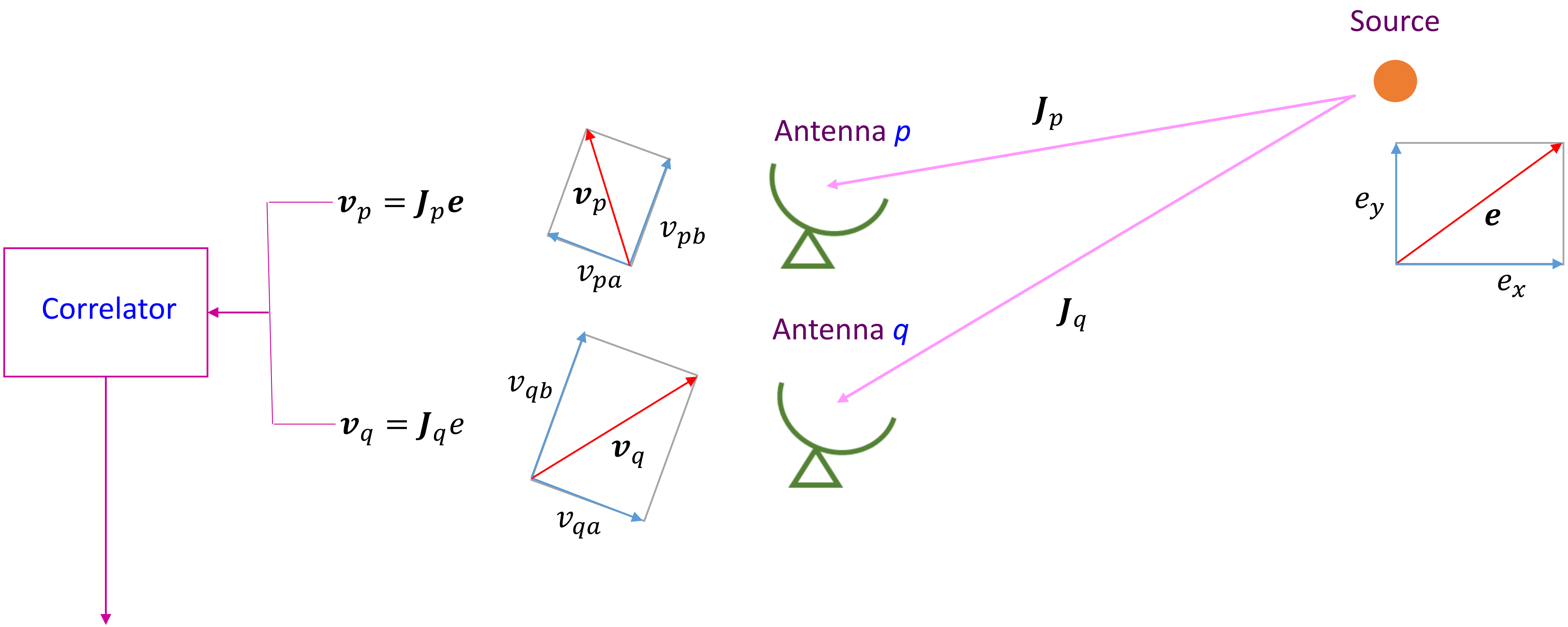


Jones matrix

Voltage vector ← $\boldsymbol{v} = \boldsymbol{J}\boldsymbol{e}$ → Electric field vector

$$\begin{pmatrix} v_a \\ v_b \end{pmatrix} = \begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$

Correlation



$$\mathbf{V}_{pq} = 2\langle \boldsymbol{v}_p \boldsymbol{v}_q^H \rangle$$

Visibility

- The correlator computes the visibility, $\mathbf{V}_{pq}$, on the baseline pq:

Hermitian conjugate

$$\mathbf{V}_{pq} = 2\underbrace{\langle \overbrace{\boldsymbol{v}_p \boldsymbol{v}_q^H}^{\text{Outer product}} \rangle}_{\text{Average}}$$

$$= 2\left\langle \begin{pmatrix} v_{pa} \\ v_{pb} \end{pmatrix} \begin{pmatrix} v_{qa}^* & v_{qb}^* \end{pmatrix} \right\rangle$$

These 4 quantities are the outputs from the correlator

$$= 2 \begin{pmatrix} \overbrace{\langle v_{pa} v_{qa}^* \rangle \quad \langle v_{pa} v_{qb}^* \rangle} \\ \langle v_{pb} v_{qa}^* \rangle \quad \langle v_{pb} v_{qb}^* \rangle \end{pmatrix}$$

Correlation

$$\boldsymbol{v}_p = \boldsymbol{J}_p \boldsymbol{e} \quad , \quad \boldsymbol{v}_q = \boldsymbol{J}_q \boldsymbol{e}$$

$$\mathbf{V}_{pq} = 2\langle\, \boldsymbol{v}_p \boldsymbol{v}_q^H \,\rangle$$

$$= 2\langle (\boldsymbol{J}_p \boldsymbol{e})(\boldsymbol{J}_q \boldsymbol{e})^H \rangle$$

$$= 2\langle \boldsymbol{J}_p (\boldsymbol{e}\boldsymbol{e}^H) \boldsymbol{J}_q^H \rangle$$

$$= \boldsymbol{J}_p \langle 2\boldsymbol{e}\boldsymbol{e}^H \rangle \boldsymbol{J}_q^H$$

Coherency, or Brightness

$$\mathbf{V}_{pq} = \boldsymbol{J}_p \langle 2\boldsymbol{e}\boldsymbol{e}^H \rangle \boldsymbol{J}_q^H$$

By definition, the coherency, or brightness, B, is given by:

$$\mathbf{B} = \langle 2\boldsymbol{e}\boldsymbol{e}^H \rangle = \begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix}$$

$\langle \boldsymbol{e}\boldsymbol{e}^H \rangle$ is the coherence of the electromagnetic field with itself, and is described by the Stokes parameters I, Q, U, V

$$\mathbf{V}_{pq} = \boldsymbol{J}_p \, \mathbf{B} \, \boldsymbol{J}_q^H$$

Radio-Interferometric Measurement Equation (RIME)

Visibility

Brightness

$$\mathbf{V}_{pq} = \boldsymbol{J}_p \, \mathbf{B} \, \boldsymbol{J}_q^H$$

Jones matrices

$$\begin{pmatrix} v_{aa} & v_{ab} \\ v_{ba} & v_{bb} \end{pmatrix} = \begin{pmatrix} j_{11a} & j_{12a} \\ j_{21a} & j_{22a} \end{pmatrix} \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} \begin{pmatrix} j_{11b} & j_{12b} \\ j_{21b} & j_{22b} \end{pmatrix}^H$$

Component Jones matrices

The Jones matrix for an antenna is a product of several component Jones matrices, corresponding to different corrupting effects along the signal path

Example:

$$\boldsymbol{J} = \boldsymbol{B}\ \boldsymbol{G}\ \boldsymbol{D}\ \boldsymbol{E}\ \boldsymbol{P}\ \boldsymbol{T}$$

Bandpass gain

Instrumental gain

Polarization leakage

Primary beam

Parallactic angle feed rotation

Ionospheric Faraday rotation

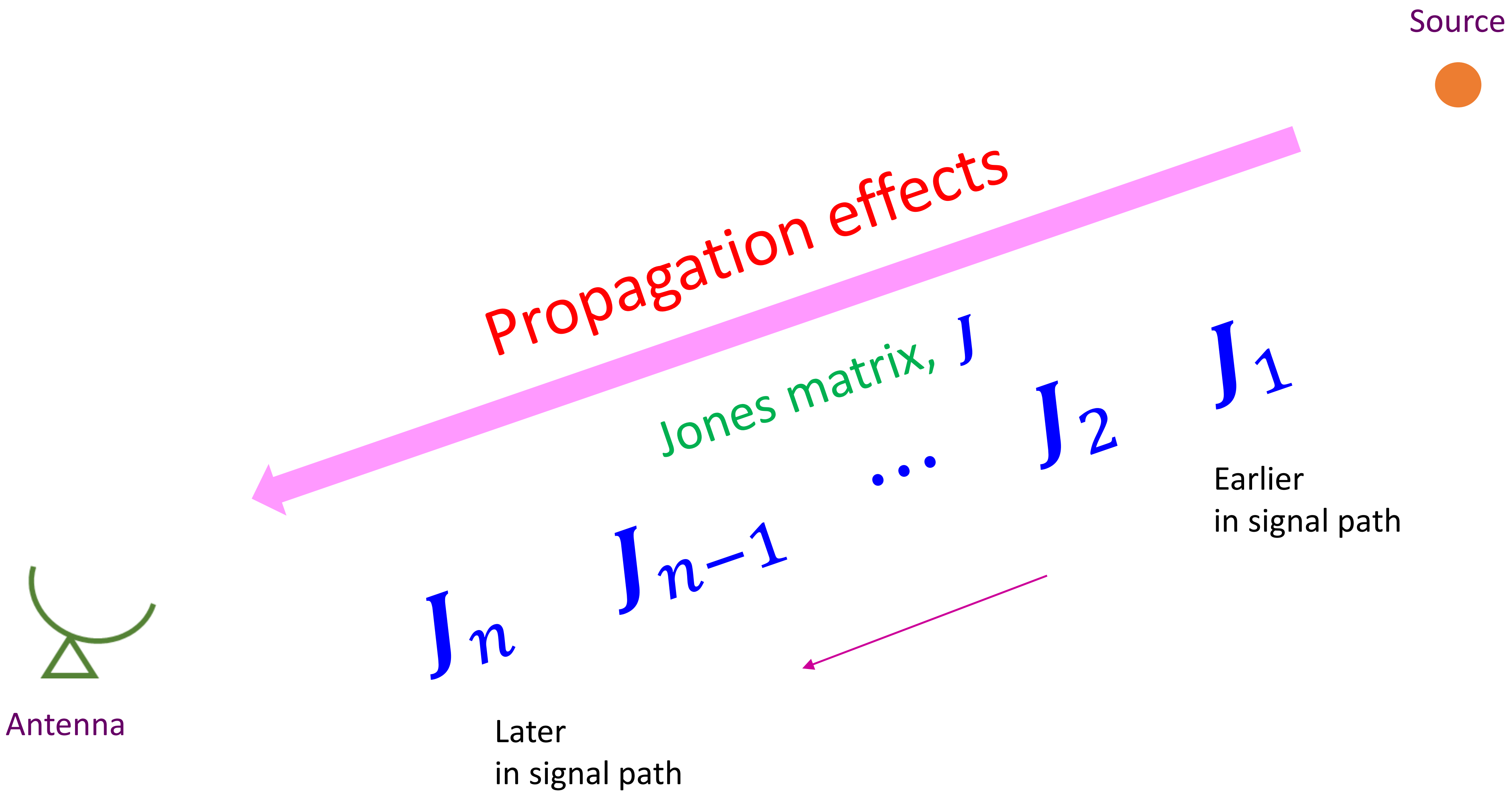
Component Jones matrices

Jones chain:

$$\boldsymbol{J} = \boldsymbol{J}_n\ \boldsymbol{J}_{n-1}\ \cdots\ \boldsymbol{J}_2\ \boldsymbol{J}_1$$

Later
in signal path ← Earlier
in signal path

Component Jones matrices



Component Jones matrices

Antenna *p* : $\boldsymbol{J}_p = \boldsymbol{J}_{pn}\,\boldsymbol{J}_{p(n-1)} \cdots \boldsymbol{J}_{p2}\,\boldsymbol{J}_{p1}$

Antenna *q* : $\boldsymbol{J}_q = \boldsymbol{J}_{qn}\,\boldsymbol{J}_{q(n-1)} \cdots \boldsymbol{J}_{q2}\,\boldsymbol{J}_{q1}$

Visibility

Brightness

$$\mathbf{V}_{pq} = \boldsymbol{J}_p\,\mathbf{B}\,\boldsymbol{J}_q^H$$

Jones matrices

$$\mathbf{V}_{pq} = \boldsymbol{J}_{pn}\,\boldsymbol{J}_{p(n-1)} \cdots \boldsymbol{J}_{p2}\,\boldsymbol{J}_{p1}\,\mathbf{B}\,\boldsymbol{J}_{q1}^H\,\boldsymbol{J}_{q2}^H \cdots \boldsymbol{J}_{q(n-1)}^H\,\boldsymbol{J}_{qn}^H$$

$$\mathbf{V}_{pq} = \boldsymbol{J}_{pn}\Big(\boldsymbol{J}_{p(n-1)}\Big(\cdots\Big(\boldsymbol{J}_{p2}\Big(\boldsymbol{J}_{p1}\,\mathbf{B}\,\boldsymbol{J}_{q1}^H\Big)\boldsymbol{J}_{q2}^H\Big)\cdots\Big)\boldsymbol{J}_{q(n-1)}^H\Big)\boldsymbol{J}_{qn}^H$$

Direction-independent and direction-dependent effects

Propagation effects can be of two kinds:

- Direction-independent effects
- Direction-dependent effects

These effects can be represented by different Jones matrices:

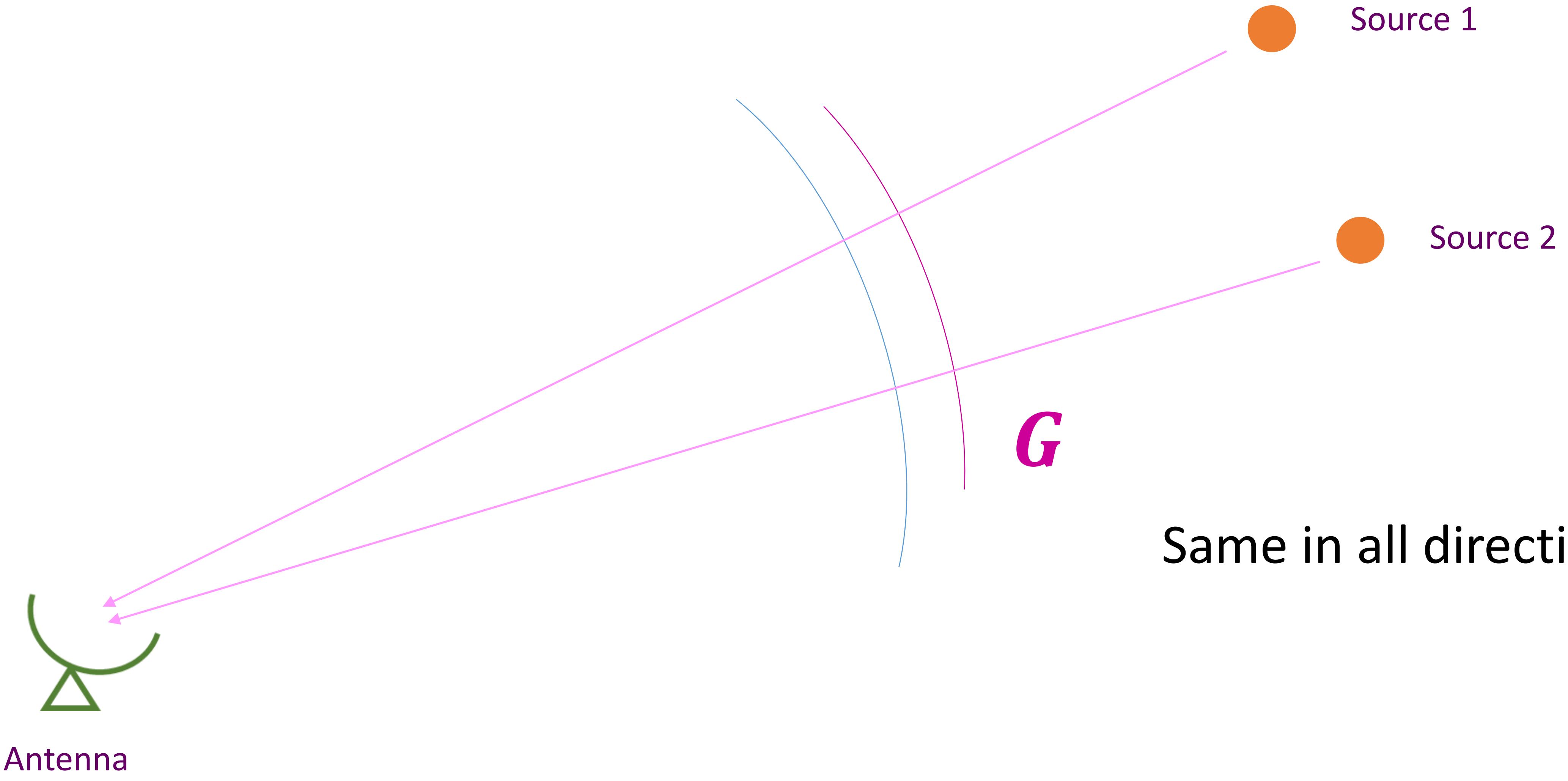
$$\boldsymbol{J} = \boldsymbol{G}\ \boldsymbol{E}$$

Final Jones matrix

Direction-independent effects

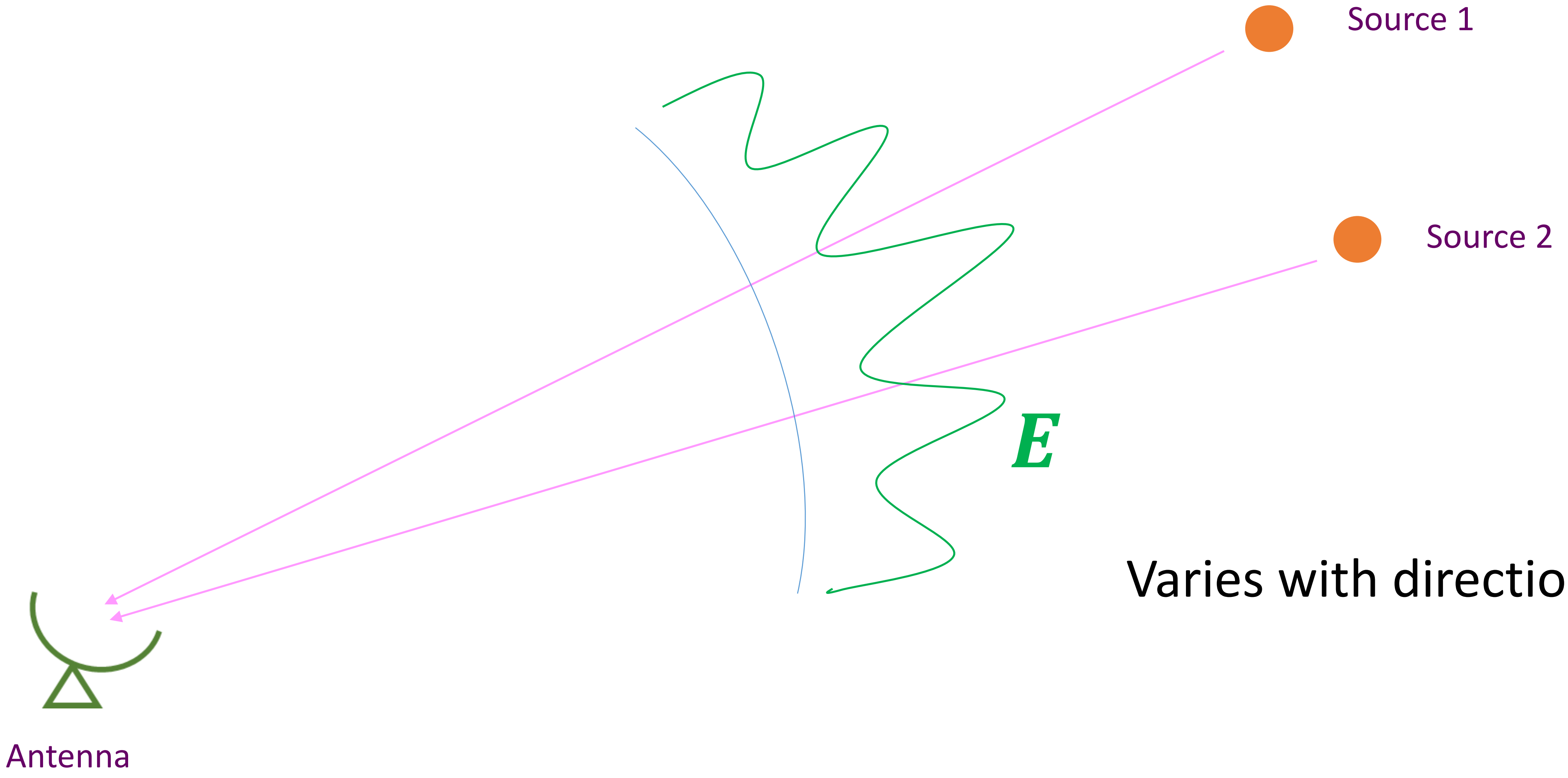
Direction-dependent effects

Direction-independent effects



Same in all directions

Direction-dependent effects



Varies with direction

Direction-independent and direction-dependent effects

$$\boldsymbol{J} = \boldsymbol{G}\ \boldsymbol{E}$$

Final Jones matrix

Direction-independent effects

Direction-dependent effects

Visibility-plane effects

Image-plane effects

$$\mathbf{V}_{pq} = \boldsymbol{J}_p\ \mathbf{B}\ \boldsymbol{J}_q^H$$

$$\mathbf{V}_{pq} = \boldsymbol{G}_p\left(\boldsymbol{E}_p\mathbf{B}\boldsymbol{E}_q^H\right)\boldsymbol{G}_q^H$$

Direction-independent and direction-dependent effects

Example:

$$\boldsymbol{J} = \boldsymbol{B}\ \ \boldsymbol{G}\ \ \boldsymbol{D}\ \ \boldsymbol{E}\ \ \boldsymbol{P}\ \ \boldsymbol{T}$$

Bandpass gain

Instrumental gain

Polarization leakage

Primary beam

Parallactic angle feed rotation

Ionospheric Faraday rotation

Direction-dependent effect

Direction-independent effects

References

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- *14th Synthesis Imaging Workshop* lecture slides (2014), National Radio Astronomy Observatory, Socorro, New Mexico, USA
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