Radio-Interferometric Measurement Equation

Introductory Radio Interferometry Course

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Radio-Interferometric Measurement Equation (RIME)

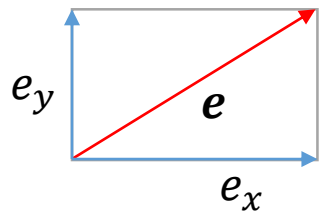
- Compact, intuitive, matrix-based way of representing propagation effects in radio interferometry.
- Useful for calibration (solving for and correcting these propagation effects).

Introduction

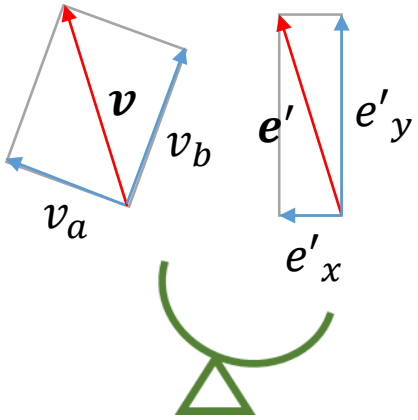
e'_x , e'_y : Components of electric field vector in reference frame of sky, at the observer

v_a , v_b : Voltages measured by antenna feed (linearly or circularly polarized)

Source



Propagation effects



Antenna

e_x, e_y : Components of electric field vector in reference frame of sky, at the source

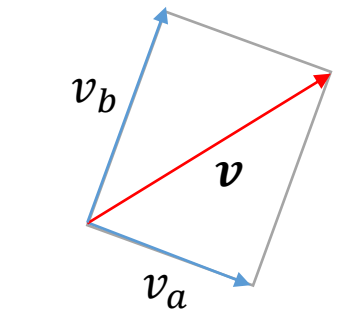
Can be represented as vectors:

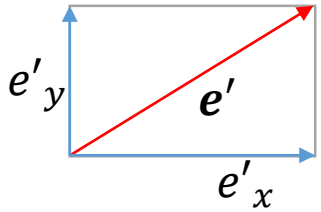
$$\boldsymbol{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$

$$\boldsymbol{e'} = \begin{pmatrix} e'_x \\ e'_y \end{pmatrix}$$

$$\boldsymbol{v} = \begin{pmatrix} v_a \\ v_b \end{pmatrix}$$

Propagation effects absent

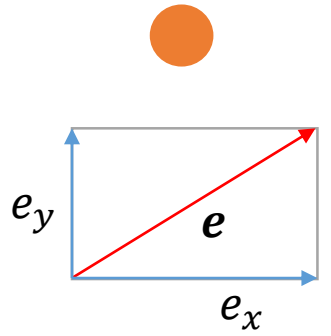




Antenna

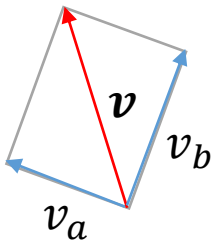
No propagation effects

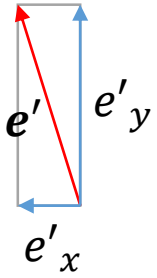
Source

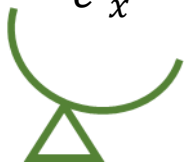


Electric vector remains unchanged during propagation: $\boldsymbol{e'} = \boldsymbol{e}$
Emitted and measured electric vector are the same.

Propagation effects present

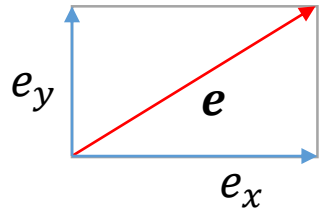






Antenna

Source



Propagation effects

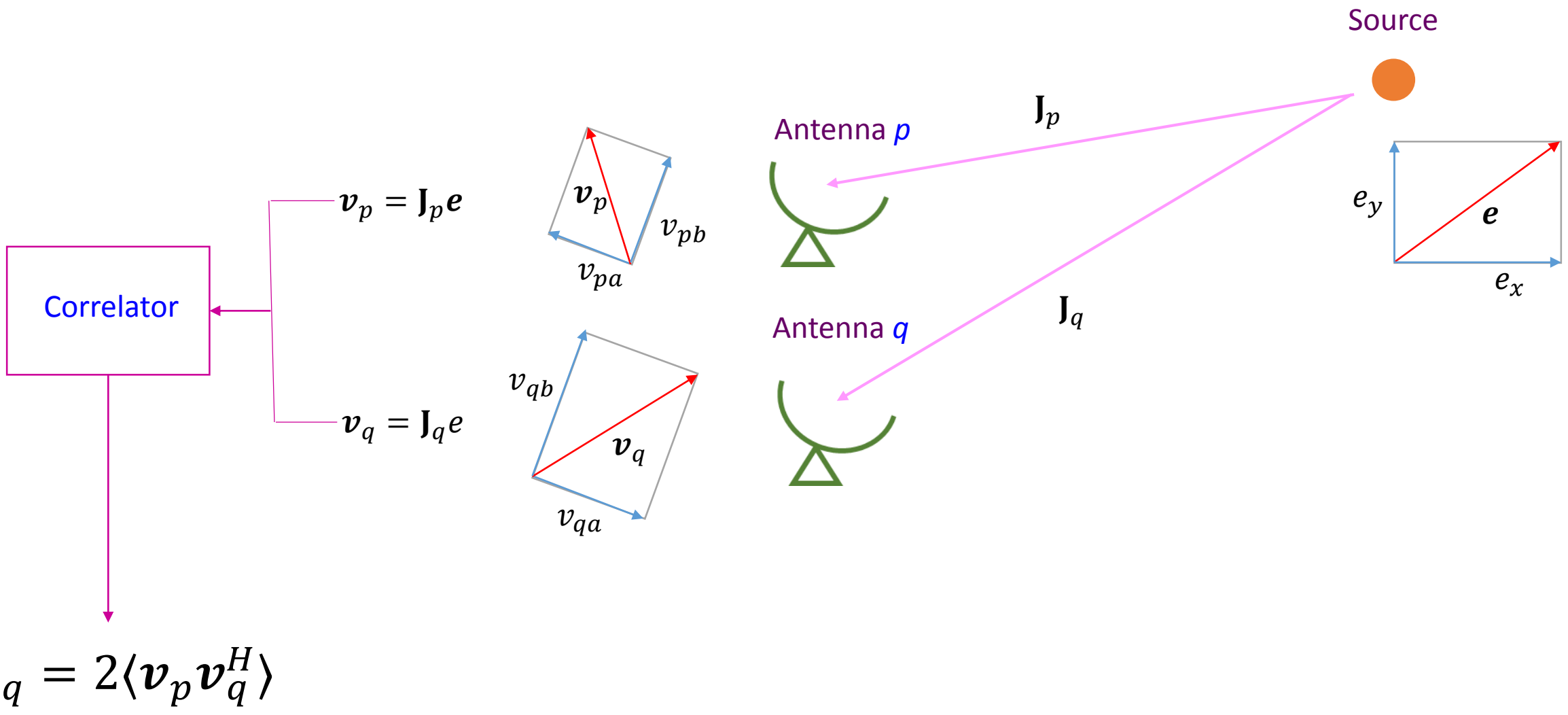
Linear transformation matrix, **J**

Jones matrix

Voltage vector ← $\boldsymbol{v} = \mathbf{J}\boldsymbol{e}$ → Electric field vector

$$\begin{pmatrix} v_a \\ v_b \end{pmatrix} = \begin{pmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{pmatrix} \begin{pmatrix} e_x \\ e_y \end{pmatrix}$$

Correlation



$$\mathbf{V}_{pq} = 2\langle \boldsymbol{v}_p \boldsymbol{v}_q^H \rangle$$

Visibility

- The correlator computes the visibility, $\mathbf{V}_{pq}$, on the baseline pq:

Hermitian conjugate

Outer product

$$\mathbf{V}_{pq} = 2\langle \boldsymbol{v}_p \boldsymbol{v}_q^H \rangle$$

Average

These 4 quantities are the outputs from the correlator

$$= 2\left\langle \begin{pmatrix} v_{pa} \\ v_{pb} \end{pmatrix} \begin{pmatrix} v_{qa}^* & v_{qb}^* \end{pmatrix} \right\rangle$$

$$= 2 \begin{pmatrix} \langle v_{pa} v_{qa}^* \rangle & \langle v_{pa} v_{qb}^* \rangle \\ \langle v_{pb} v_{qa}^* \rangle & \langle v_{pb} v_{qb}^* \rangle \end{pmatrix}$$

Correlation

$$\boldsymbol{v}_p = \mathbf{J}_p \boldsymbol{e} \quad , \quad \boldsymbol{v}_q = \mathbf{J}_q \boldsymbol{e}$$

$$\mathbf{V}_{pq} = 2\langle \boldsymbol{v}_p \boldsymbol{v}_q^H \rangle$$

$$= 2\langle (\mathbf{J}_p \boldsymbol{e})(\mathbf{J}_q \boldsymbol{e})^H \rangle$$

$$= 2\langle \mathbf{J}_p (\boldsymbol{e}\boldsymbol{e}^H) \mathbf{J}_q^H \rangle$$

$$= \langle \mathbf{J}_p (2\boldsymbol{e}\boldsymbol{e}^H) \mathbf{J}_q^H \rangle$$

Coherency, or Brightness

$$\mathbf{V}_{pq} = \langle \mathbf{J}_p (2\boldsymbol{ee}^H) \mathbf{J}_q^H \rangle$$

By definition, the coherency, or brightness, B, is given by:

$$\mathbf{B} = \langle 2\boldsymbol{ee}^H \rangle = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}$$

I, Q, U, V: Stokes parameters

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{B} \mathbf{J}_q^H$$

Radio-Interferometric Measurement Equation (RIME)

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{B} \mathbf{J}_q^H$$

$$\begin{pmatrix} v_{aa} & v_{ab} \\ v_{ba} & v_{bb} \end{pmatrix} = \begin{pmatrix} j_{11a} & j_{12a} \\ j_{21a} & j_{22a} \end{pmatrix} \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} \begin{pmatrix} j_{11b} & j_{12b} \\ j_{21b} & j_{22b} \end{pmatrix}^H$$