Assignment 1: Radio Interferometry Fundamentals I

January 8, 2015

1 Essentials

1.1 Orion

Part of the Orion constellation can be found in Figure 1. Betelgeuse and Rigel

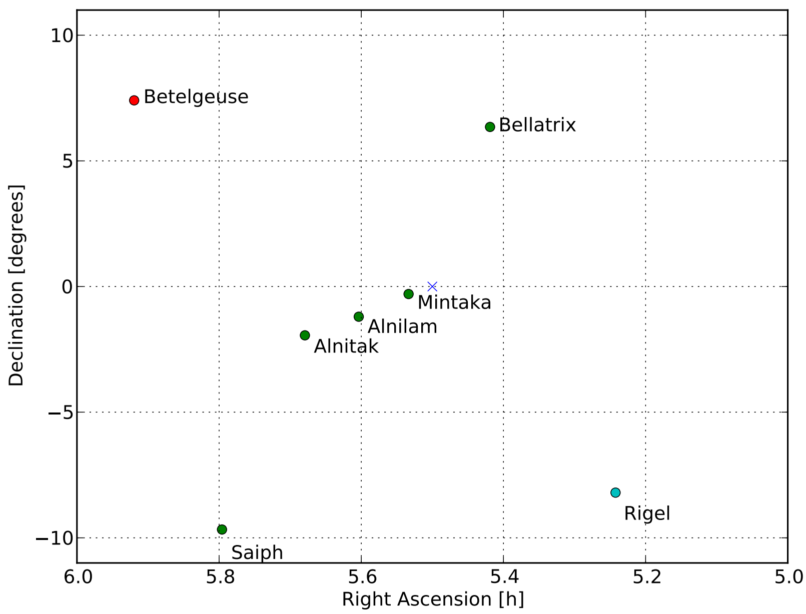

Figure 1: Part of the Orion constellation.

are the brightest stars in this constellation. The equatorial coordinates of the constellation's centre, Betelgeuse and Rigel are listed in Table 1.1.

1. Calculate the l and m coordinates of Orion's center, Betelgeuse and Rigel? Assume that your field center was chosen to coincide with Orion's center.

Table 1: Equatorial coordinates of Orion's center, Betelgeuse and Rigel.

Name	Right Ascension α	Declination δ
Center	5h 30m (α_0)	$0°$ (δ_0)
Betelgeuse	5h 55m 10.3053s	$7° \ 24' \ 25.426''$
Rigel	5h 14m 32.272s	$-8° \ 12' \ 5.898''$

[specify your answers in °]. **Hint:**

$$
\begin{aligned}
\Delta\alpha &= \alpha - \alpha_0 \\
l &= \cos\delta \sin\Delta\alpha \\
m &= \sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos\Delta\alpha.
\end{aligned}
$$

Answer:
The l and m coordinates of the constellation of Orion can be found in Figure 2.

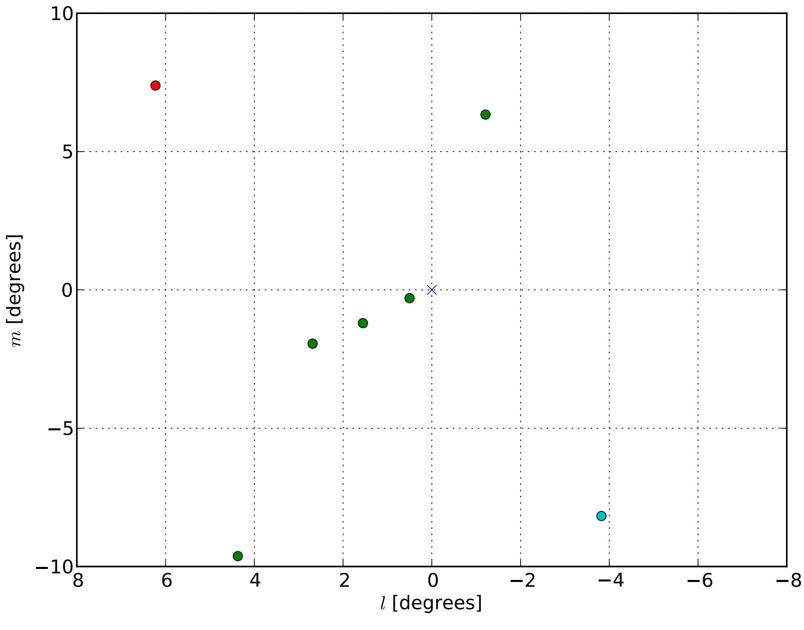

Figure 2: The l and m coordinates of the constellation of Orion.

$$
\begin{aligned}
C_\alpha &= \frac{\pi}{12}\left(5+\frac{30}{60}\right)\\
B_\alpha &= \frac{\pi}{12}\left(5+\frac{55}{60}+\frac{10.3053}{3600}\right)\\
R_\alpha &= \frac{\pi}{12}\left(5+\frac{14}{60}+\frac{32.272}{60}\right)
\end{aligned}
$$

$$
\begin{aligned}
C_\delta &= 0\\
B_\delta &= \frac{\pi}{180}\left(7+\frac{24}{60}+\frac{25.426}{3600}\right)\\
R_\delta &= \frac{\pi}{180}\left(-8-\frac{12}{60}-\frac{5.898}{60}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta B_\alpha &= B_\alpha - C_\alpha\\
\Delta R_\alpha &= R_\alpha - C_\alpha
\end{aligned}
$$

$$
\begin{aligned}
l_B &= \frac{180}{\pi}\cos B_\delta \sin \Delta B_\alpha\\
&= 6.22788693165^\circ\\
m_B &= \frac{180}{\pi}\sin B_\delta \cos C_\delta - \frac{180}{\pi}\cos B_\delta \sin C_\delta \cos \Delta B_\alpha\\
&= 7.38644800073^\circ
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
l_C &= 0^\circ\\
m_C &= 0^\circ\\
l_R &= -3.82309540994^\circ\\
m_R &= -8.09006178983^\circ
\end{aligned}
$$

2. What is the distance from the field center to Betelgeuse in the projected lm-plane? **Hint:** $l^2+m^2=d^2$.
   Answer:

$$d=\sqrt{l_B^2+m_B^2}=9.66158318812^\circ$$

3. What is the angular distance from the field center to Betelgeuse on the celestial sphere? **Hint:** $l^2+m^2=\sin^2\theta$.
   Answer:
   NB: From here on **we work in degrees**, we assume the trigonometric functions take degrees.

$$\theta_{CB}=\sin^{-1}\sqrt{l_B^2+m_B^2}=9.70796683775^\circ$$

4. Verify the previous question by using the equatorial coordinates directly (stay on the celestial sphere)? **Hint:** Use the spherical Pythagorean theorem.

$$\Delta B_\delta = B_\delta - C_\delta$$
$$\theta_{CB}=\cos^{-1}(\cos(\Delta B_\delta)\cos(\Delta B_\alpha))=9.70796683775^\circ$$

Table 2: Equatorial coordinates of Papino and Paperino.

Name	Flux	Right Ascension α	Declination δ
Papino	1Jy	-4h 44m 6.686s (α_0)	$-74°39'37.481''$ (δ_0)
Paperino	0.2Jy	-4h 44m 6.686	$-74°39'37.298''$

5. Why do we measure l and m in ° if they are direction cosines and therefore by definition unit-less? **Hint:** Use the previous three results in your answer.
   Answer:
   Since, $l^2 + m^2 = \sin^2\theta \approx \theta^2$ [see slides] when θ is small. The above results show that the angular distance to Betelgeuse is approximately equal to the lm-projected distance. Even at an angle of 10° this approximation works well. In a small field-of-view observation θ will be much smaller, making the approximation even better. We can therefore assume that l and m are angular distances and can therefore be measured in degrees.

6. What will the hour angle of Orion's center be when it appear above the horizon? In which direction will Orion's center appear? **Hint:** Remember Orion's center is at $\delta = 0°$. It is the same declination the sun has when it lies on one of the equinoxes.
   Answer:
   -6h due East. When the sun lies on one of the equinoxes it is in the sky for 12 hours, rises due east and sets due west.

1.2 Papino and Paperino

We will be using a fictitious piece of sky (containing only two radio sources) in the remainder of the assignment. The equatorial coordinates of this fictitious sky are given in Table 1.2.

1. Calculate the l and m coordinates of Papino and Paperino? Assume Papino and the field-center coincide. Express your answer in **radians**.
   Answer:
   Since the field center and Papino coincides, $l_i = 0$ rad and $m_i = 0$ rad.

$$\begin{aligned}
\delta_0 &= \frac{\pi}{180}\left(-74 - \frac{39}{60} - \frac{37.481}{3600}\right)\\
\delta &= \frac{\pi}{180}\left(-73 - \frac{39}{60} - \frac{37.298}{3600}\right)\\
\Delta\alpha &= 0
\end{aligned}$$

$$
\begin{aligned}
l_e &= \cos\delta\sin\Delta\alpha \\
&= 0 \text{ rad} \\
m_e &= \sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos\Delta\alpha \\
&= \sin(\delta-\delta_0) \\
&= 1^\circ \\
&= \frac{\pi}{180} \text{ rad}
\end{aligned} \tag{1}
$$

2. Write down an equation that completely describes this fictitious sky by assuming Papino and Paperino are perfect point sources (i.e. $I(l,m)$)? **Hint:** A point source can be represented with a delta-function, the amplitude of the delta function is equal to the flux of the point source and the translation parameters describe the position of the point source.
   Answer:

   $$I(l,m) = \delta(l,m) + 0.2\delta(l,m-m_0) = \delta(l,m) + 0.2\delta\left(l, m-\frac{\pi}{180}\right)$$

   with

   $$m_0 = 1^\circ = \frac{\pi}{180} \text{ rad}$$

3. Find the expression of the complex visibilities $V(u,v)$ that we would observe with an ideal interferometer (an interferometer that could sample the entire uv-plane). **Hint:** Take the Fourier transform of $I(l,m)$. The unit of v is rad^{-1} (per radian).
   Answer:

   $$V(u,v) = \mathcal{F}\{I(l,m)\} = 1 + 0.2e^{-2\pi i m_0 v} = 1 + 0.2e^{-\frac{\pi^2}{90}iv}$$

   Note that $\frac{\pi^2}{90} = 2\pi m_0$ and that $\frac{1}{m_0} = \frac{180}{\pi}$ rad$^{-1} \approx 52.296$ rad^{-1}.

4. Calculate $V(v) = V(0,v)$, i.e. the cross section of $V(u,v)$ with the plane $u=0$? Plot your answer. **Hint:** The unit of v is rad^{-1} (per radian).
   Answer:

   $$V(v) = 1 + 0.2e^{-\frac{\pi^2}{90}iv} = 1 + 0.2\cos\left(\frac{\pi^2}{90}v\right) - i0.2\sin\left(\frac{\pi^2}{90}v\right)$$

   The above equation is displayed graphically in Figure 3.

1.3 KAT-7

KAT-7 is a radio interferometer that consists out of 7 dishes and is located in the Karoo, South Africa. The ENU (east-north-up) coordinates of this telescope is listed in Table 3. We will assume that we observed the fictitious field from the previous section. The main aim of this section is to enable you to derive the uv-tracks of an interferometer and to improve your understanding of visibilities. Additional information about the imaginary observation that we conducted can be found in Table 4.

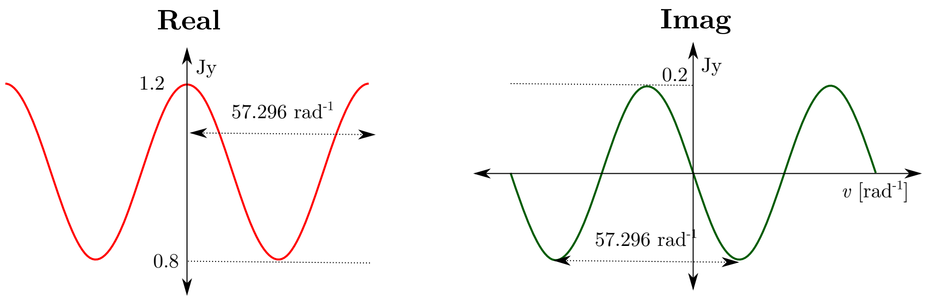

Figure 3: Graphical representation of $V(v)$.

Table 3: The ENU coordinates of KAT-7

Antenna	E (x)	N (y)	U (z)
Antenna 1	25.095 m (x_1)	-9.095 m (y_1)	0.045 m (z_1)
Antenna 2	90.284 m	26.380 m	-0.226 m
Antenna 3	3.985 m	26.893 m	0.000 m
Antenna 4	-21.605 m	25.494 m	0.019 m
Antenna 5	-38.272 m	-2.592 m	0.391 m
Antenna 6	-61.595 m	-79.699 m	0.702 m
Antenna 7	-87.988 m	75.754 m	0.138 m

1. Calculate the ENU baseline difference vector $\boldsymbol{b}_{12}^{xyz}$ of baseline 12? **Hint:** $\boldsymbol{b}_{12}^{xyz} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)^T$.
   Answer:
$$\boldsymbol{b}_{12}^{xyz} = (65.189, 35.475, -0.271)^T$$
2. Calculate the length D_{12} of $\boldsymbol{b}_{12}^{xyz}$? **Hint:** $D = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$.
   Answer:
$$D_{12} = 74.22 \text{ m}$$
3. Calculate the azimuth angle A_{12} and the elevation (altitude) angle E_{12} of $\boldsymbol{b}_{12}^{xyz}$? **Hint:** Remember the azimuth angle is measured from the north towards the east.

Name	Value
Latitude L	$-30^\circ 43' 17.34''$
Starting hour angle H_0	-4h
Stopping hour angle H_1	4h
Field center δ_0	$-74^\circ 39' 37.481''$
Field center α_0	-4h 44m 6.686s
Observational Frequency f	1.4 GHz

Table 4: Additional information

Answer:

$$A_{12} = \tan^{-1}\left(\frac{65.189}{35.475}\right) = 61.44^\circ$$

$$E_{12} = -\sin^{-1}\left(\frac{0.271}{D}\right) = -0.21^\circ$$

4. Calculate $\boldsymbol{b}_{12}^{XYZ} = \begin{bmatrix} X_{12} \\ Y_{12} \\ Z_{12} \end{bmatrix}$? **Hint:** Recall that

$$\begin{bmatrix} X_{12} \\ Y_{12} \\ Z_{12} \end{bmatrix} = D \begin{bmatrix} \cos L \sin E_{12} - \sin L \cos E_{12} \cos A_{12} \\ \cos E_{12} \sin A_{12} \\ \sin L \sin E_{12} + \cos L \cos E_{12} \cos A_{12} \end{bmatrix}$$

Answer:

$$L = \left(-30 - \frac{43}{60} - \frac{17.34}{3600}\right) = -30.721^\circ$$

$$\begin{bmatrix} X_{12} \\ Y_{12} \\ Z_{12} \end{bmatrix} = \begin{bmatrix} 17.89 \text{ m} \\ 65.19 \text{ m} \\ 30.63 \text{ m} \end{bmatrix}$$

5. Calculate the observational wavelength λ? **Hint:** $\lambda f = c$.
Answer:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.4 \times 10^9} = 0.214 \text{ m}$$

6. Calculate $\sqrt{X_{12}^2 + X_{12}^2}\lambda^{-1}$, $|\sin\delta_0|\sqrt{X_{12}^2 + X_{12}^2}\lambda^{-1}$ and $\cos\delta_0 Z_{12}\lambda^{-1}$?

$$\sqrt{X_{12}^2 + X_{12}^2}\lambda^{-1} = 315.46$$

$$|\sin\delta_0|\sqrt{X_{12}^2 + X_{12}^2}\lambda^{-1} = 304.22$$

$$\cos\delta_0 Z_{12}\lambda^{-1} = 37.82 \tag{2}$$

7. Draw the uv-tracks of baseline 12 and 21 that is generated during a 24h observation? **Hint:** The baseline vector of an interferometer trace out an elliptical locus (after 24 hours it will complete one entire revolution). The values calculated in the previous question determine the shape of the elliptical locus. Moreover, $\boldsymbol{b}_{12}^{xyz} = -\boldsymbol{b}_{21}^{xyz}$.
Answer: The requested uv-tracks can be found in Figure 4. The blue ellipse was generated by baseline 12, while the red ellipse was generated by baseline 21 $[(u,v)_{12} = (-u,-v)_{21}$, since $\boldsymbol{b}_{12}^{xyz} = -\boldsymbol{b}_{21}^{xyz}]$. The center point of the blue ellipse is equal to $(0, 37.82)$, while its semi-major and semi-minor axes respectively equal 315.46 and 304.22.

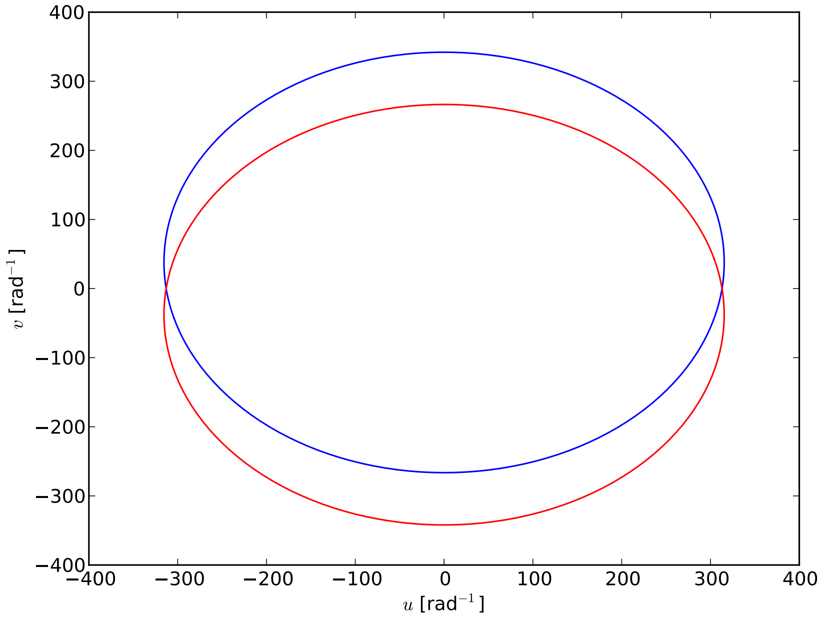

Figure 4: The uv-coverage of baseline 12 generated after 24 hours.

8. Generate the coordinate pair (u, v) associated with baseline 12 and H_0 =-4h? **Hint:** Recall that

$$\begin{bmatrix} u \\ v \end{bmatrix} = \lambda^{-1} \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta_0 \cos H & \sin\delta_0 \sin H & \cos\delta_0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Answer:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 79.81 \text{ rad}^{-1} \\ 332.15 \text{ rad}^{-1} \end{bmatrix}$$

9. Why is u and v unit-less? Why then do we measure u and v in rad^{-1}? **Hint:** In what unit do we measure length and wavelength? What is the unit of the ratio of these two quantities?
Answer:
The unit of length and wavelength are both in meters. The ratio of these two quantities are therefore unit-less. We also say that u and v are measured in λ. Since we assigned the unit of $^\circ$ to l and m, we need to measure u and v in rad^{-1}, since the lm-plane and the uv-plane are related by the Fourier transform.

10. Generate the coordinate pair (u, v) associated with baseline 12 and H_1 =4h?
Answer:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 224.41 \text{ rad}^{-1} \\ -176.00 \text{ rad}^{-1} \end{bmatrix}$$

11. Draw the uv-tracks of baseline 12 and 21 for -4h $< H <$ 4h?
Answer:
The requested uv-tracks can be found in Figure 5. The uv coordinate that we can associate with H_0 is indicated by the red marker. The uv-coordinate that we can associate with H_1 is indicated by the black marker.

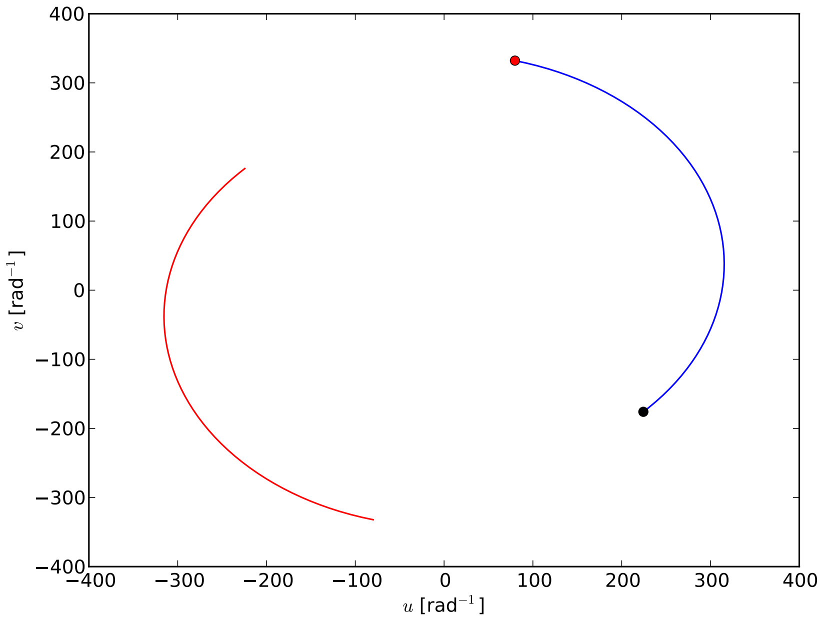

Figure 5: The uv-coverage of baseline 12 generated for -4h $< H <$ 4h.

12. Assume we observe the exact same sky as we calculated in the previous section (i.e. the l and m coordinates of our skymodel stay the same), but now let $\delta_0 = 0^\circ$ (Papino lies on the celestial equator). Calculate the uv-coverage of baseline 12 with -4h $< H <$ 4h? Also plot the visibilities that we would observe as a function of timeslots [n] (assume 600 timeslots - we obtained the visibilities at 600 different (u, v) pairs on the elliptical locus)?
Answer:

$$\begin{bmatrix} u \\ v \end{bmatrix}_{H_0} = \begin{bmatrix} 79.81 \text{ rad}^{-1} \\ 142.96 \text{ rad}^{-1} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix}_{H_1} = \begin{bmatrix} 224.41 \text{ rad}^{-1} \\ 142.96 \text{ rad}^{-1} \end{bmatrix}$$

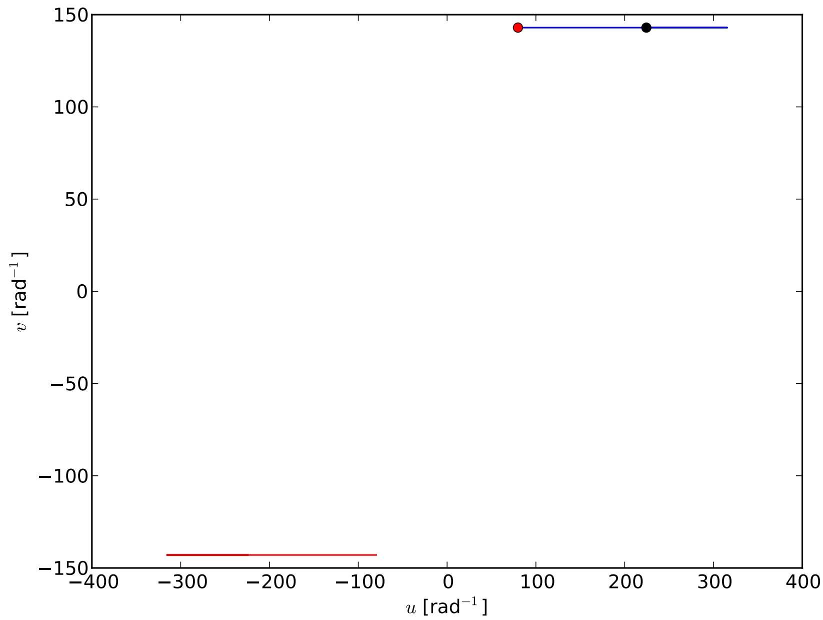

Figure 6: The uv-coverage of baseline 12 with -4h $< H <$ 4h and $\delta_0 = 0°$.

The requested uv coverage is plotted in Figure 6.

$$
\begin{aligned}
V(142.96) &= 1 + 0.2\cos\left(\frac{\pi^2}{90}(142.96)\right) - i0.2\sin\left(\frac{\pi^2}{90}(142.96)\right) \\
&= 0.8 - 0061i \text{ Jy}
\end{aligned}
$$

The visibilities observed as a function of timeslots are depicted in Figure 7

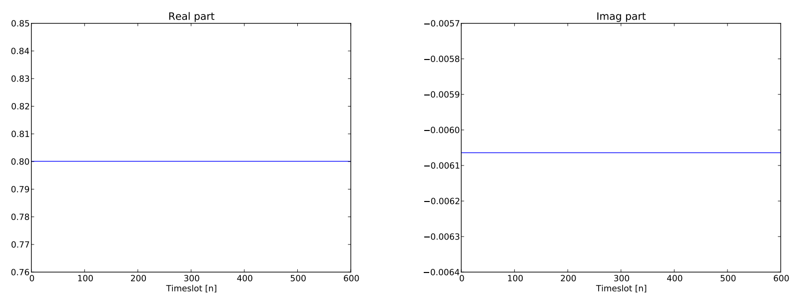

Figure 7: Visibilities observed at $\delta_0 = 0°$.