Assignment 2: Radio Interferometry Fundamentals II

January 21, 2015

1 Essentials

1.1 FITS images

Most of the radio images that you will be working with will be fits images. Fits images are raster images. The fits images that you will be working with mostly contain a certain portion of the sky which was **projected** onto the lm-plane. Fits images are made up of $X \times Y$ **pixels/cells** (hence they are 2D digital raster images). Each cell has a certain size, which is expressed with a vector $(\Delta l_c, \Delta m_c)$, where Δl_c is its angular distance in the l direction and Δm_c its angular distance in the m direction. Each cell therefore forms a **solid angle** $\Omega_{\text{cell}} = \Delta l_c \times \Delta m_c$. The units of a fits image is Jy beam^{-1}. In Figure 1 we have a theoretical fits image (in real life you would have a lot more pixels though!).

1. Calculate the angular limits $l_{\max},\ l_{\min},\ m_{\max}$ and $m_{\min}$ of the raster image in Figure 1? Assume that the green point is the center of the lm-plane. Express your answers in arcseconds and degrees.
   Answer:
   Let $\boldsymbol{l} = [l_{\max},\ l_{\min}]$ and $\boldsymbol{m} = [m_{\max},\ m_{\min}]$. Now
   $$\boldsymbol{l} = \boldsymbol{m} = [-25'', 25''] \approx [-7 \times 10^{-3\circ}, 7 \times 10^{-3\circ}].$$

2. What is the solid angle Ω_{sphere} subtended by the celestial sphere? What is the solid angle subtended by this image Ω_{image} (assume that the field of view is small)? Express your answer in sr. What is $\frac{\Omega_{\text{image}}}{\Omega_{\text{sphere}}}$?
   Answer:
   (a) 4π sr
   (b) $50'' \times 50'' = 2500''\square = \frac{2500}{4.25\times 10^{10}}$ sr $= 5.88 \times 10^{-8}$ sr
   (c) $\frac{5.88\times 10^{-8}}{4\pi} = 4.68 \times 10^{-9}$

3. How many pixels do you require if you wanted to produce a figure that stretched from -1° to 1° in the l and m direction? Express your answer as $X \times Y$.
   Answer:
   $\frac{2\times 3600''}{5''} = 1440 \to 1440 \times 1440.$

4. How large would each cell be if the total angular dimension of your image was $720' \times 720'$ and you had 1024×1024 pixels at your disposal? Express

Table 1: Properties of interferometer

Property	Value
Antennas N_a	7
Dish diameter D	12 m
Longest baseline length $b_{\max}$	185 m
Channel bandwidth Δf	92.9121 MHz
Frequency range	1.4–1.95 GHz

your answer as: $\Delta l''_c \times \Delta m''_c$.
Answer:
$\frac{720' \times 60}{1024} = 42.1875'' \rightarrow 42.1875'' \times 42.1875''$.

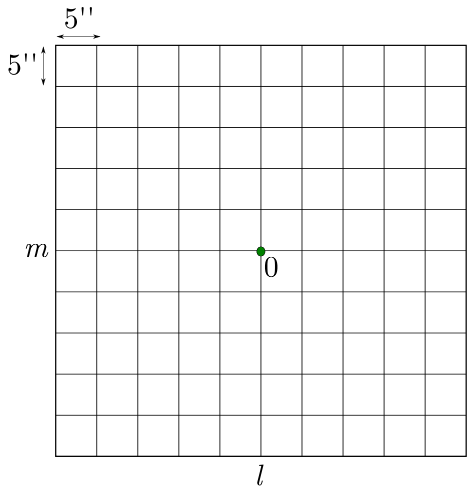

Figure 1: A theoretical fits image.

1.2 Properties of interferometer

In the table below we have some of the basic properties of an interferometer (dishes), with these basic properties we can determine some of its secondary properties. For all the questions that follow, assume we are **observing at 1.4 GHz** (f_0). All the following questions pertain to Table 1. **Hint :** Assume that all the angles in the formulae given are in **radians**. Express all your answers, those that are angles, in degrees, arcminutes and arcseconds.

1. How many baselines does the interferometer in Table 1 have? **Hint:** $\frac{N_A^2 - N_A}{2}$.
   Answer:
   21

2. Determine the approximate size of the primary beam (full width half maximum beamwidth) of the interferometer in Table 1? **Hint:** $\theta_p \approx \frac{\lambda_0}{D}$.

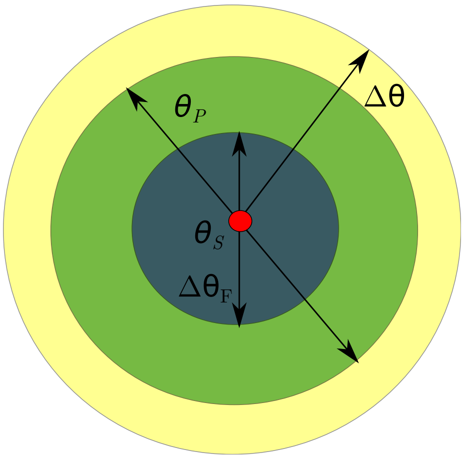

Figure 2: The angels θ_p, θ_s, $\Delta\theta$ and $\Delta\theta_F$.

Answer:
Recall that $\lambda_0 = \frac{c}{f_0} \approx 0.21$ m, from which we can calculate:

$$\theta_p \approx \frac{0.214}{12} = 1.02^\circ = 61.30' = 3678.39'' \tag{1}$$

3. Determine the angular resolution of the interferometer in Table 1? **Hint:** $\theta_s \approx \frac{\lambda_0}{b_{\max}}$.
   Answer:

$$\theta_s \approx \frac{0.214}{185} = 0.07^\circ = 3.98' = 238.91'' \tag{2}$$

4. Determine the maximum angular radius $\Delta\theta$ a source can be from the field center and not be heavily attenuated by the fringe washing function? **Hint:** $\Delta\theta\Delta f << \theta_s f_0$.
   Answer:

$$\Delta\theta = \frac{\theta_s f_0}{\Delta f} = 1^\circ = 60' = 3600'' \tag{3}$$

5. Determine the maximum integration time Δt that the correlator may use if you want to prevent time smearing (within the maximum angular radius given by your fringe washing function)? **Hint:** $\frac{2\pi\Delta\theta\Delta t}{P} << \theta_s$.
   Answer:

$$\Delta t = \frac{\theta_s}{\Delta\theta}(1.37 \times 10^4) = 15.15 \text{ min} \tag{4}$$

6. Determine the field of view $\Delta\theta_F$ in which we may assume that the w-term is negligible? **Hint:** $\Delta\theta_F < \frac{1}{3}\sqrt{\theta_s}$ **Answer:**

$$\Delta\theta_F = 3^{-1}\sqrt{\theta_s} = 0.64^\circ = 39' = 2340'' \tag{5}$$

7. Represent θ_p, θ_s, $\Delta\theta$ and $\Delta\theta_F$ graphically (in one figure)? **Hint :** Draw concentric circles and use the angles as radii or diameters.
   Answer : The answer can be found in Figure 2.