Calibration

Introductory Radio Interferometry Course

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Calibration

Calibration: Determining and correcting for propagation effects in order to compute the brightness.

i.e., solve for Jones matrices $\boldsymbol{J}$ to compute $\mathbf{B}$:

$$\underbrace{\mathbf{V}_{pq}}_{\text{Visibility}} = \boldsymbol{J}_p \, \underbrace{\mathbf{B}}_{\text{Brightness}} \, \boldsymbol{J}_q^H$$

Jones matrices

$$\mathbf{B} = \boldsymbol{J}_p^{-1} \mathbf{V}_{pq} \left(\boldsymbol{J}_q^H\right)^{-1}$$

Direction-independent and direction-dependent effects

Propagation effects can be of two kinds:

- Direction-independent effects
- Direction-dependent effects

These effects can be represented by different Jones matrices:

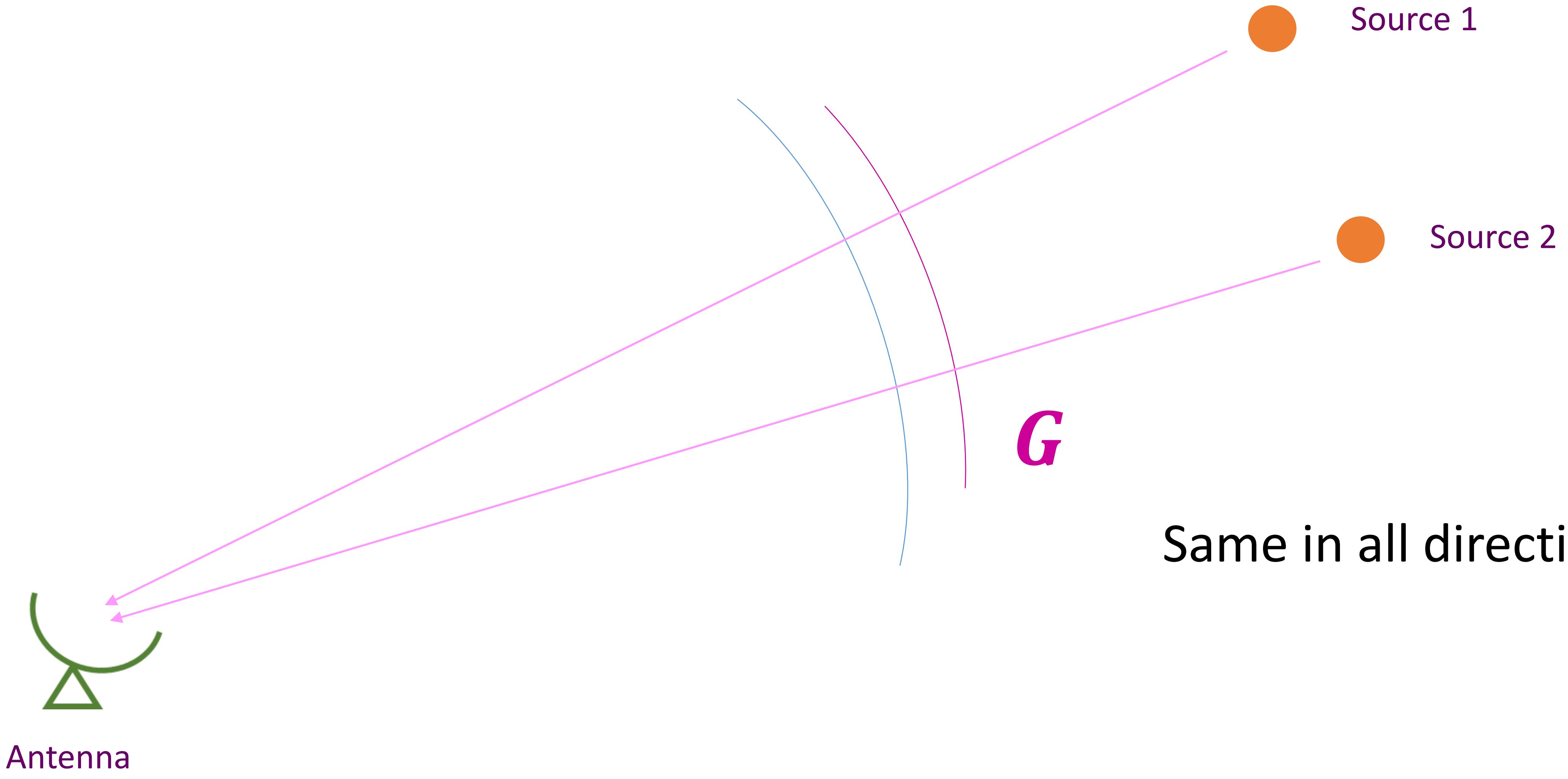
$$\boldsymbol{J} = \boldsymbol{G}\ \boldsymbol{E}$$

Final Jones matrix

Direction-independent effects

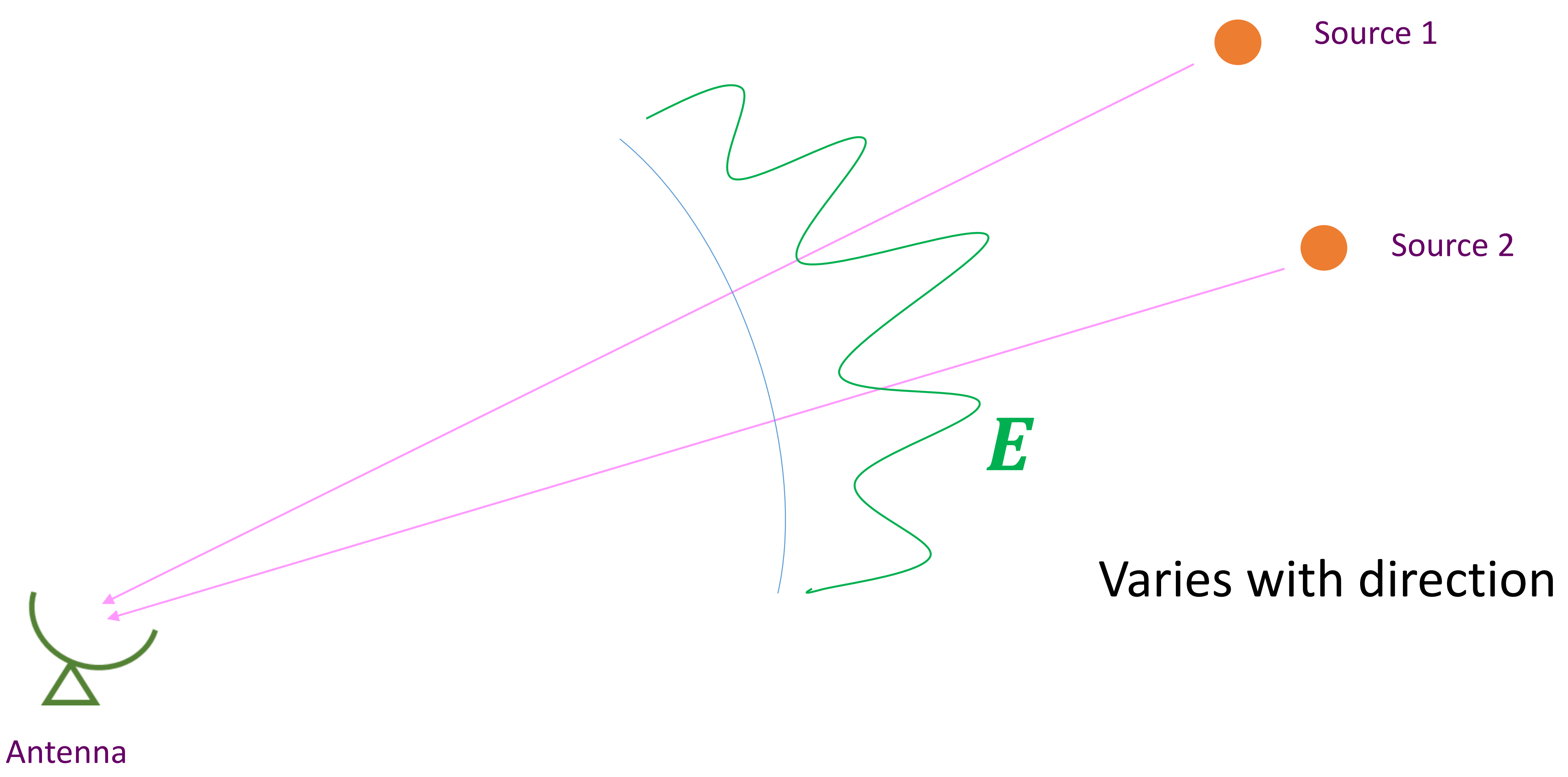
Direction-dependent effects

Direction-independent effects



Same in all directions

Direction-dependent effects



Direction-independent and direction-dependent effects

$$\boldsymbol{J} = \boldsymbol{G}\ \boldsymbol{E}$$

Final Jones matrix

Direction-independent effects

Direction-dependent effects

Visibility-plane effects

Image-plane effects

$$\mathbf{V}_{pq} = \boldsymbol{J}_p\ \mathbf{B}\ \boldsymbol{J}_q^H$$

$$\mathbf{V}_{pq} = \boldsymbol{G}_p\left(\boldsymbol{E}_p\mathbf{B}\boldsymbol{E}_q^H\right)\boldsymbol{G}_q^H$$

Direction-independent and direction-dependent effects

Example:

$$\boldsymbol{J} = \boldsymbol{B}\ \ \boldsymbol{G}\ \ \boldsymbol{D}\ \ \boldsymbol{E}\ \ \boldsymbol{P}\ \ \boldsymbol{T}$$

Bandpass gain

Instrumental gain

Polarization leakage

Primary beam

Parallactic angle feed rotation

Ionospheric Faraday rotation

Direction-dependent effect

Direction-independent effects

Structure of Jones matrices

Most Jones matrices have a simple form:

- $$J_{\text{rotation}} = \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$$
- $$J_{\text{leakage}} = \begin{bmatrix} 1 & D^R \\ D^L & 1 \end{bmatrix}$$
- $$J_{\text{gain}} = \begin{bmatrix} G^R & 0 \\ 0 & G^L \end{bmatrix}$$

(in a circularly polarized basis)

Rotation matrices

- $\boldsymbol{J}_{\text{rotation}} = \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$

Examples:

Parallactic angle

Parallactic angle feed rotation: $\boldsymbol{P} = \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$

Ionospheric Faraday rotation: $\boldsymbol{T} = \begin{bmatrix} e^{j\chi} & 0 \\ 0 & e^{-j\chi} \end{bmatrix}$

Faraday rotation angle

Leakage matrices

- $$\boldsymbol{J}_{\text{leakage}} = \begin{bmatrix} 1 & D^R \\ D^L & 1 \end{bmatrix}$$

Examples:

Polarization leakage: $$\boldsymbol{D} = \begin{bmatrix} 1 & d^R \\ d^L & 1 \end{bmatrix}$$

Polarization leakage terms

Gain matrices

- $\boldsymbol{J}_{\text{gain}} = \begin{bmatrix} G^R & 0 \\ 0 & G^L \end{bmatrix}$

Examples:

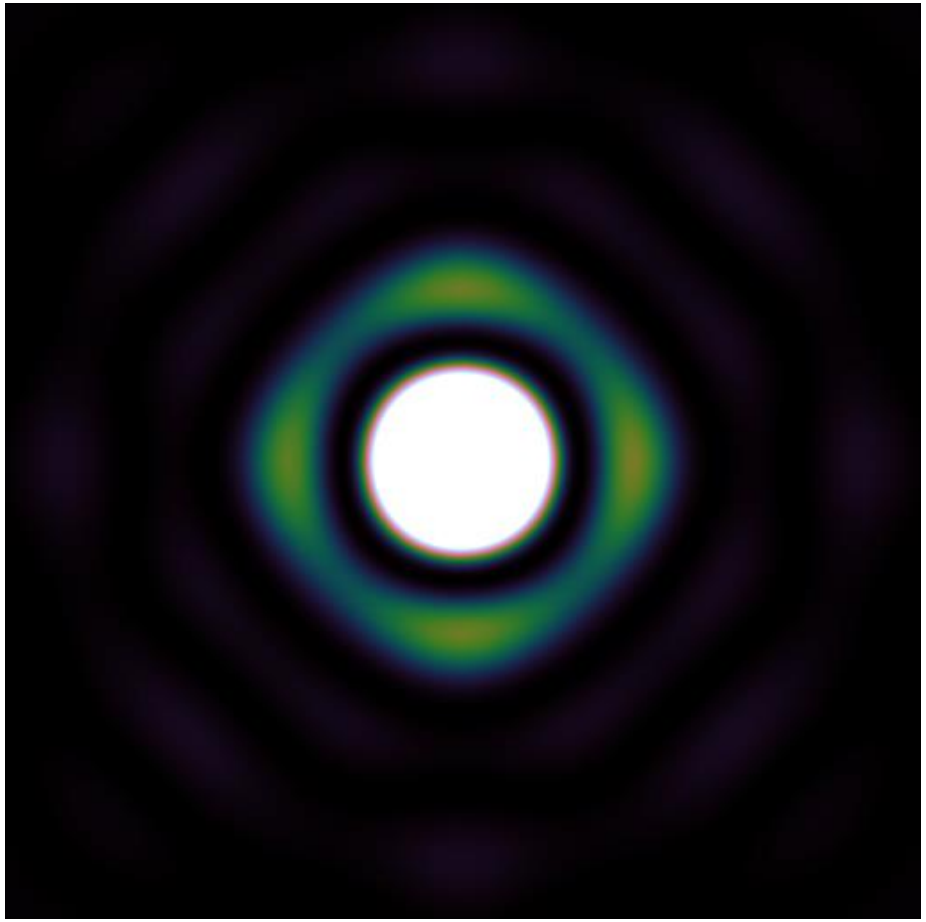
Instrumental gain: $\boldsymbol{G} = \begin{bmatrix} g^R & 0 \\ 0 & g^L \end{bmatrix} = \begin{bmatrix} a^R e^{j\phi^R} & 0 \\ 0 & a^L e^{j\phi^L} \end{bmatrix}$

Bandpass gain: $\boldsymbol{B} = \begin{bmatrix} B^R & 0 \\ 0 & B^L \end{bmatrix} = \begin{bmatrix} b^R(\nu) e^{j\psi^R(\nu)} & 0 \\ 0 & b^L(\nu) e^{j\psi^L(\nu)} \end{bmatrix}$

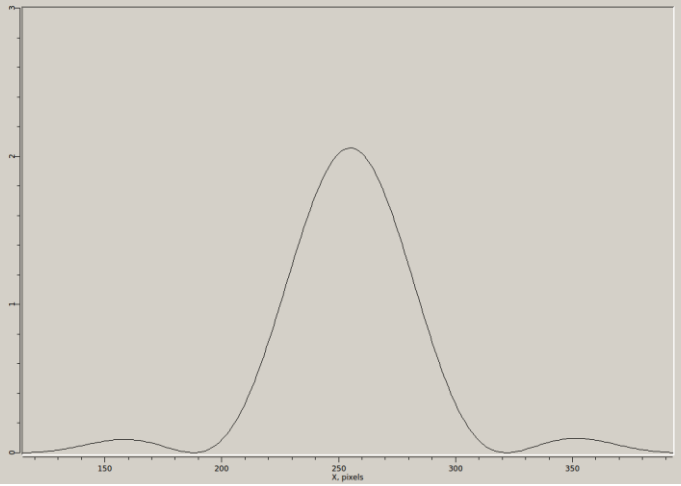
Direction dependent effects: Primary beam

- The primary beam of the antenna is the most important direction-dependent effect.
- Becomes important in wide-field, wide-band observations.
- The primary beam pattern has a multiplicative effect in the image plane, convolutional effect in the visibility plane.
- We will consider the example of a JVLA (Jansky Very Large Array) antenna here.

Primary beam amplitude variation with distance from center



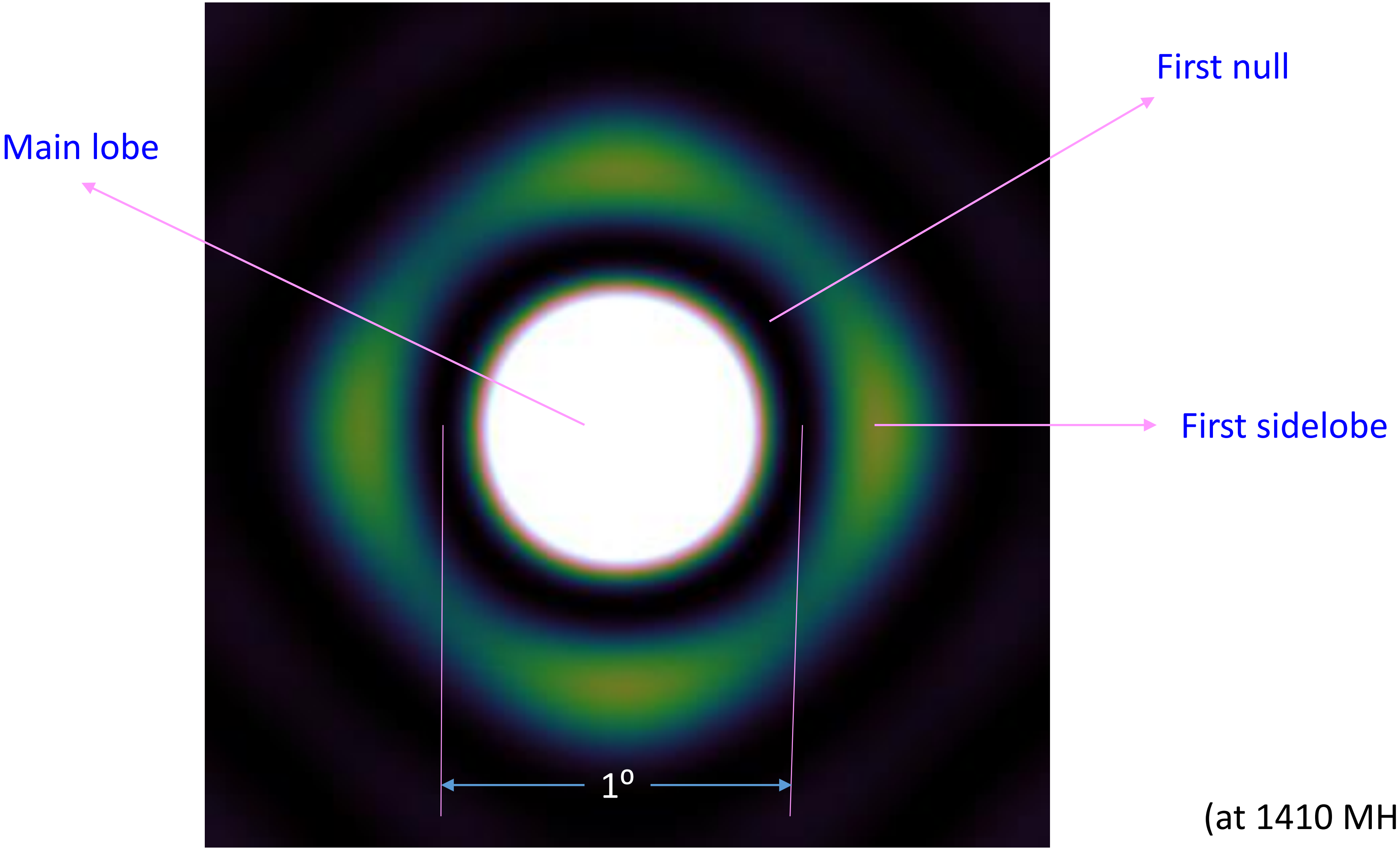
Beam gain



Distance

Horizontal cross-section of the beam through the center

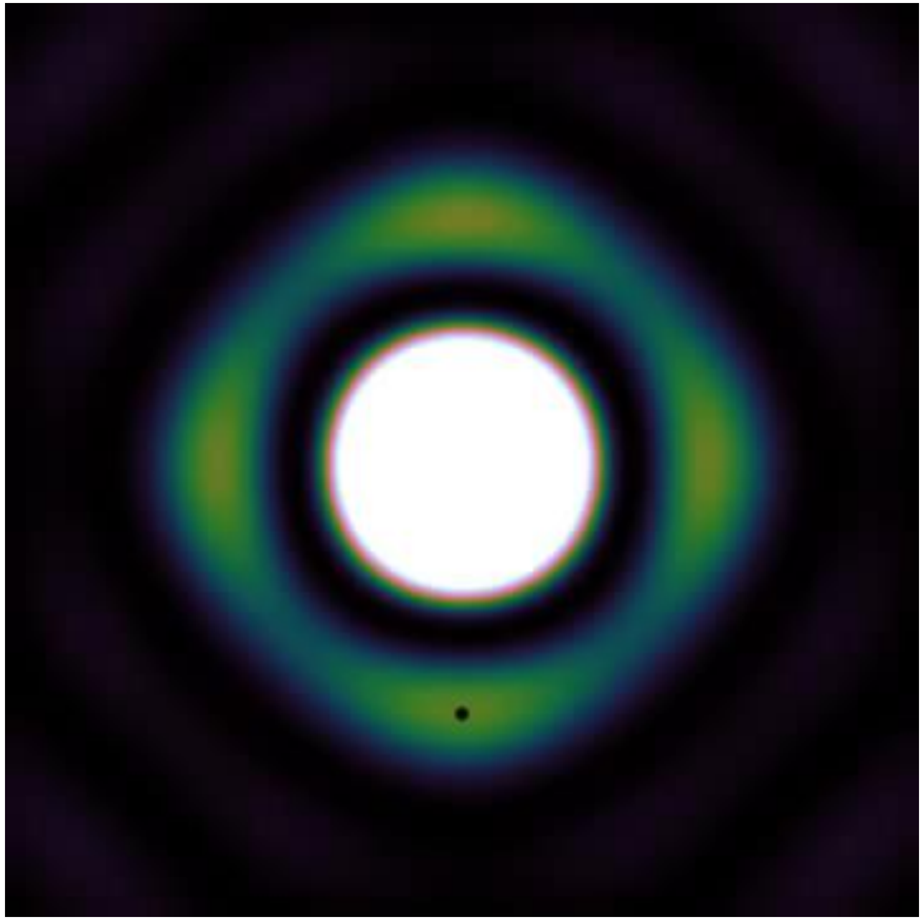
JVLA primary beam



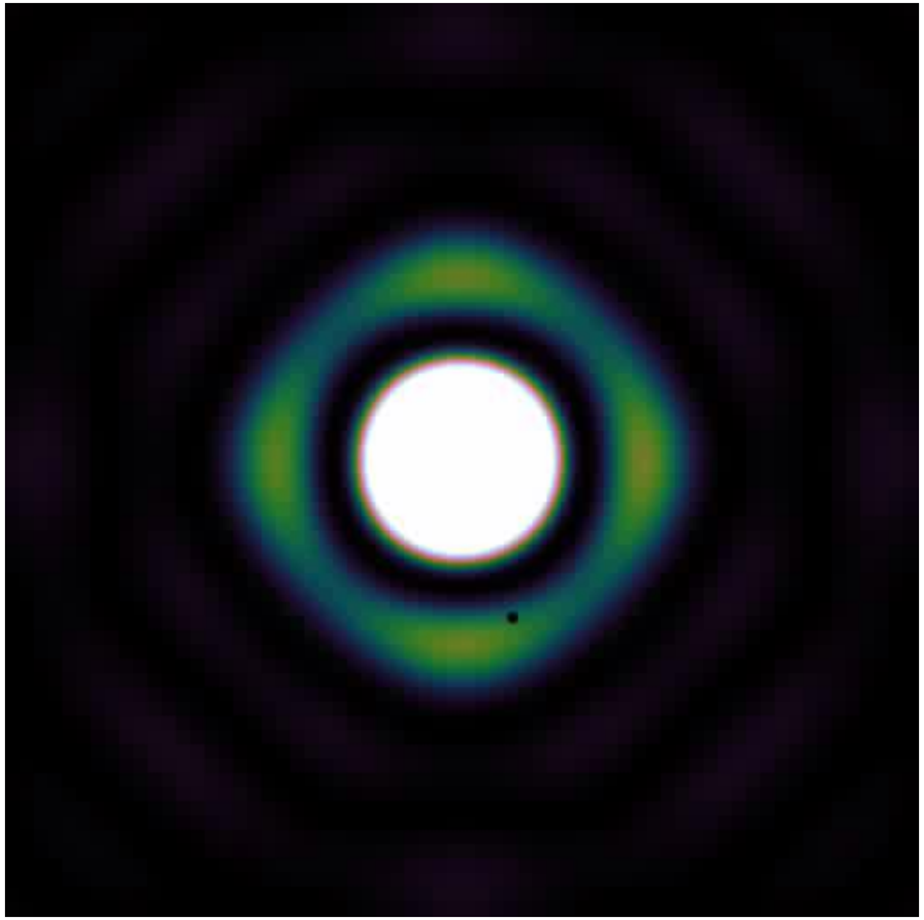
(at 1410 MHz)

Primary beam rotation

An EVLA antenna has an alt-azimuth mount;
the primary beam rotates during the course of an observation

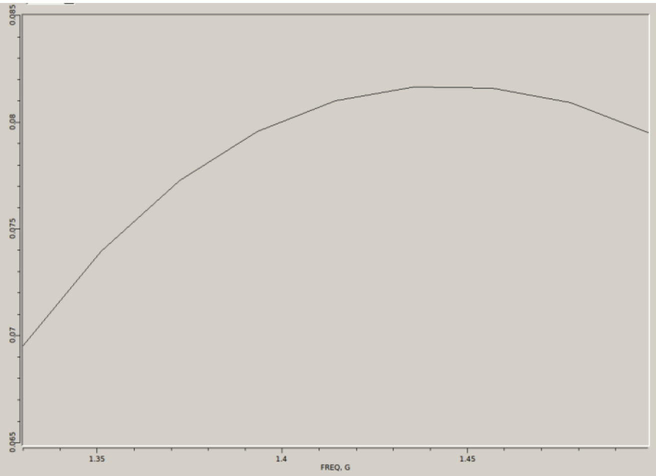


Variation of primary beam with frequency



The beam pattern becomes more compact with increasing frequency

Beam gain



Frequency

Beam-induced spectral variation for the source represented by a dot

Incorporating primary beam in calibration

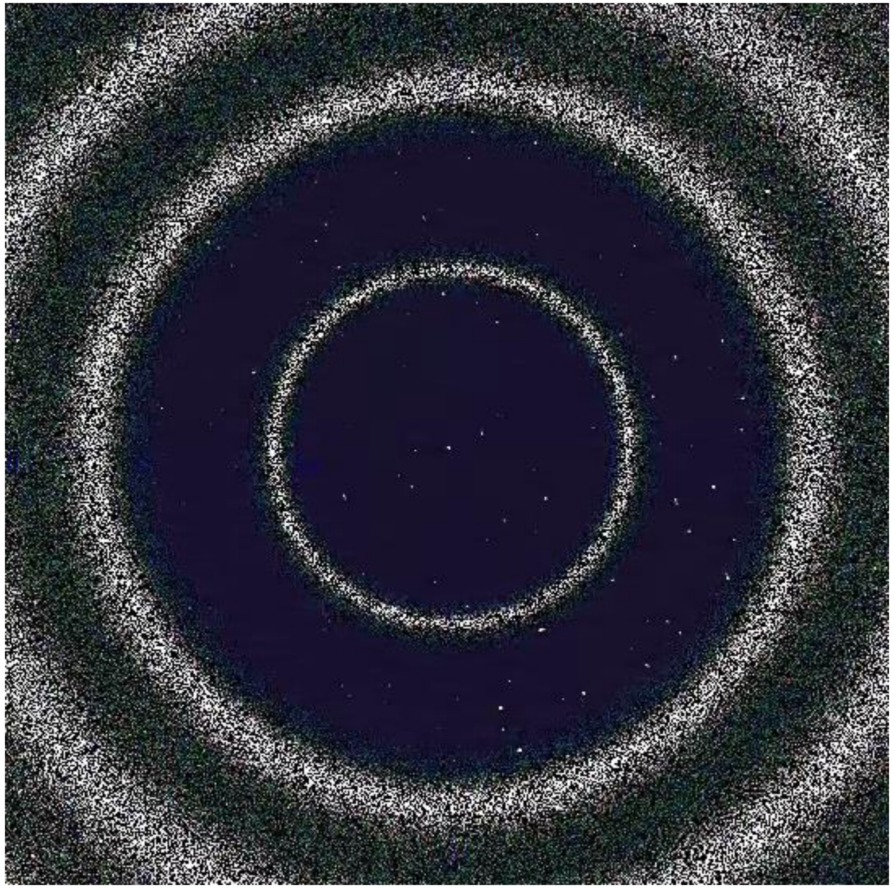
EVLA image of the field around the radio source 3C147

Calibration without primary beam included

Calibration with primary beam included

Effect of primary beam on noise over the field of view

EVLA image of the field around the radio source 3C147



Calibration procedure

1. Start with visibility data, $\mathbf{V}_{pq}$, and initial brightness model, $\mathbf{B}$.
2. Solve $\min_{\boldsymbol{J}} \left| \mathbf{V}_{pq} - \boldsymbol{J}_p \, \mathbf{B} \, \boldsymbol{J}_q^H \right|$ for $\boldsymbol{J}s$.
3. Calculate residual visibility data $\mathbf{V}_{pq}^{\mathrm{residual}} = \mathbf{V}_{pq} - \boldsymbol{J}_p \, \mathbf{B} \, \boldsymbol{J}_q^H$.
4. Image $\mathbf{V}_{pq}^{\mathrm{residual}}$ to create a residual image, I.
5. Perform a source-finding procedure to find sources in the residual image, and add these to the initial model $\mathbf{B}$ to form a new, updated model $\mathbf{B}^{\mathrm{new}}$.
6. Set $\mathbf{B}$ = $\mathbf{B}^{\mathrm{new}}$, and repeat steps 2-5 until the residual image I is noise-like.

Differential gains

- Differential gain solutions encompass the unknown and unmodeled direction-dependent effects in the signal path.
- The Jones matrix in the direction of source s is then given by:

$$\boldsymbol{J}^{(s)} = \boldsymbol{G}\ \boldsymbol{E}\ \Delta\boldsymbol{E}^{(s)}$$

Final Jones matrix in the direction of source *s*

Direction-independent effects

Known and modeled direction-dependent effects

Differential gain: Unknown/unmodeled direction-dependent effects in the direction of source *s*

Differential gains

- Differential gain solutions are computed (in the direction of a few bright sources) and applied after regular calibration in order to correct for leftover, uncalibrated effects.

Incorporating differential gains in calibration

(Without primary beam incorporated in calibration)

Without differential gain solutions

With differential gain solutions applied

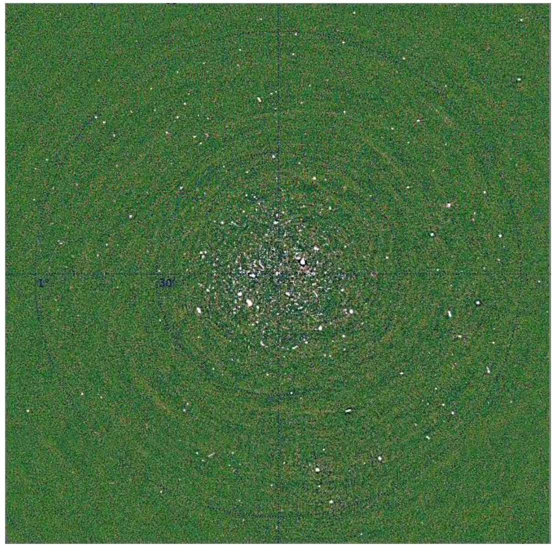
Differential gains

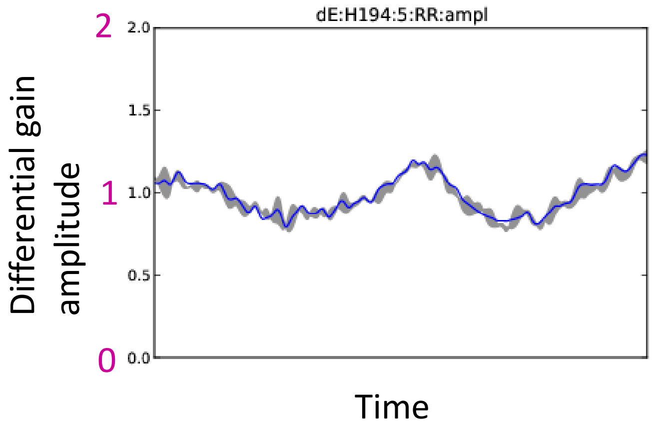
- As more corrupting effects are modeled and accounted for, the calibration becomes more comprehensive, and differential gain solutions approach unity.

Incorporating primary beam in calibration

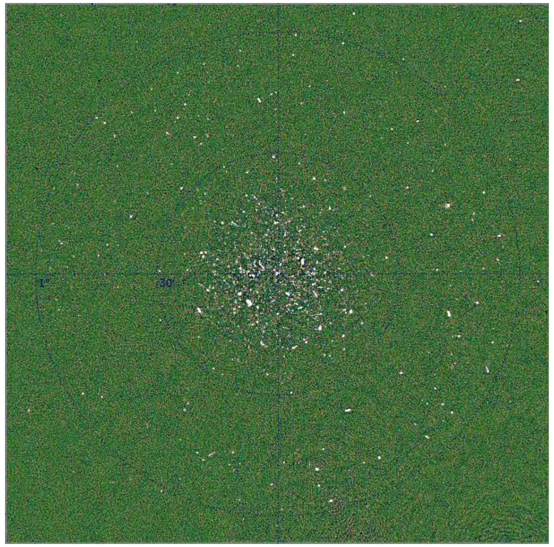
Differential gain plots

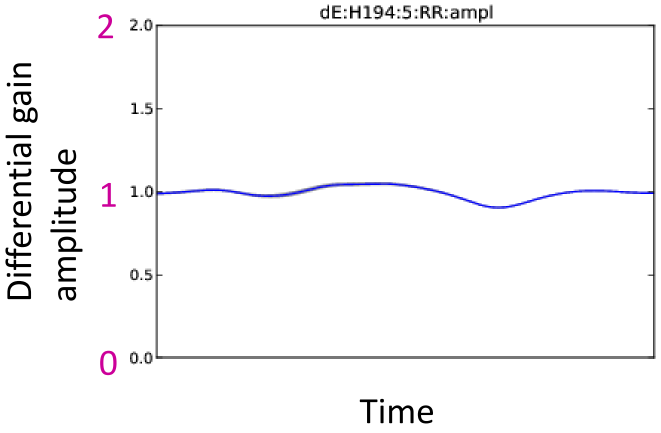
Without primary beam





With primary beam





- Flattened differential gain curves, ~1 over the whole range
- Residual variation due to remaining uncorrected direction-dependent effects (like antenna pointing errors)

References

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- *14th Synthesis Imaging Workshop* lecture slides (2014), National Radio Astronomy Observatory, Socorro, New Mexico, USA
- Oleg Smirnov's RIME lecture from *3GC3 Workshop and Interferometry School* (2013), Port Alfred, South Africa

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