navigating

Contents

CHAPTER 1

the naive model

A vertex-weighted digraph G, with weighting π. Imagine that for each $v \in V(G)$, there are $\pi(v)$ coins stacked at v. Given a starting location $w \in V(G)$, what is the most efficient way to amass coins? The *score* of a directed path $P := wz_1 \cdots z_r$ is

$$s(P) := \frac{1}{r} \sum_{i \in [r]} \pi(z_i).$$

For a sequence $P_1, P_2, \ldots$ of directed paths in G starting at w, we are interested in the sequence $s(P_1), s(P_2), \ldots$. Does it diverge to positive infinity, bounce around, or converge to a finite value?

1. our knowledge of π

If we know π fully, then finding an efficient way to amass coins is a static math problem; interesting, but not useful. To add our knowledge of π to the problem statement, we include a set $X \ni \pi$ of weightings of G. Now we imagine that our navigator knows X but not π. The navigator's exploration can shrink X. There is a tension between coin-greed and explore-greed; smaller X is good for coin-amass-planning, so how much coin to give up locally for exploration? Since our navigator knows X, she could perform this analysis and find efficient ways to amass coins; another static math problem. So, we need to add time and navigator limits. Now the navigator has a third greed in tension. Since our navigator knows her limits, she could perform this analysis and find efficient ways to amass coins; a fourth greed in tension.