

# Extra reducible configurations

June 12, 2015

**Lemma 1.** *The graph in Figure 1 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 1, a string such as XXXY, represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star YZ$ ,  $\star YZZ$ ,  $YZZZ$  or  $ZZZZ$ .

**Case 1.**  $B$  is one of  $\star YZZ$ ,  $\star YYZ$ ,  $Z\star YZ$  or  $ZZZZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $YXYZ$ ,  $YZYZ$  or  $YZZZ$ .

For  $YZZZ$ , if the  $X$ -path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYYZ$  and  $ZZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the third vertex ends at the fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZYZZ$ , which is handled by Case 1.

For  $YZYZ$ , if the  $X$ -path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYZZ$  and  $ZZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the third vertex ends at the fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZYYZ$ , which is handled by Case 1. Since  $YXYZ$  has an odd number of  $Y$ 's and  $Z$ 's, there is an  $X$ -path with exactly one end in  $H$ .

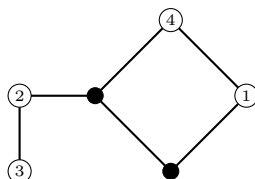


Figure 1: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

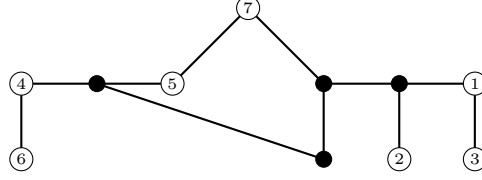


Figure 2: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

If this is the first, third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZXYZ$ ,  $XYZZ$  and  $ZYZZ$  respectively, which are handled by Case 1.

**Case 3.**  $B$  is one of  $XXYZ$  or  $XZYZ$ .

Since  $XZYZ$  has an odd number of  $Y$ 's and  $Z$ 's, there is an  $X$ -path with exactly one end in  $H$ . If this is the second, third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYYZ$ ,  $YZZZ$  and  $XYZZ$  respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by  $(4\ 1\ 3\ 2)$ , we have also handled  $XXYZ$ .

□

**Lemma 2.** *The graph in Figure 2 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 2, a string such as  $XXYZXXZ$ , represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star\star\star YZ$ ,  $\star\star\star\star\star YZZ$ ,  $\star\star\star\star YZZZ$ ,  $\star\star YZZZZ$ ,  $\star YZZZZZ$ ,  $YZZZZZZ$  or  $ZZZZZZZ$ .

**Case 1.**  $B$  is one of  $\star\star\star YZYZ$ ,  $X\star X\star YYZ$ ,  $\star Y\star YZZZ$ ,  $\star X\star YZZZ$ ,  $\star Z\star ZYZZ$ ,  $\star X\star ZYZZ$ ,  $\star Z\star ZXYZ$ ,  $\star Y\star ZXYZ$ ,  $\star XZ\star YYZ$ ,  $\star Z\star XXYZ$ ,  $\star XY\star YYZ$ ,  $\star Y\star XXYZ$ ,  $\star Z\star YYYZ$ ,  $\star ZY\star ZYZ$ ,  $\star ZX\star ZYZ$ ,  $Z\star Z\star ZYZ$ ,  $Y\star Y\star XYZ$ ,  $ZYZ\star\star YZ$ ,  $XY\star YYZ$ ,  $XYX\star\star YZ$ ,  $\star ZYZZZZ$ ,  $YXX\star XYZ$ ,  $ZXZ\star XYZ$ ,  $ZYY\star YYZ$ ,  $ZXX\star XYZ$ ,  $XY\star ZYZ$ ,  $XZZ\star YYZ$ ,  $YXY\star ZYZ$ ,  $YXX\star ZYZ$ ,  $YZZ\star YYZ$ ,  $\star YZYXYZ$ ,  $YZY\star YYZ$ ,  $\star XXYYYY$ ,  $\star YXYXYZ$ ,  $ZYY\star ZYZ$ ,  $ZXX\star ZYZ$ ,  $XYYZZZZ$ ,  $XZZYZZZZ$ ,  $XZXYZZZ$ ,  $XYYZYZZ$ ,  $XYXZYZZ$ ,  $XZZY\star YZ$ ,  $XZXY\star YZ$ ,  $YXYZZZZ$ ,  $YZZYZZZZ$ ,  $YZYYZZZ$ ,  $YZZYXYZ$ ,  $ZZZZZZZ$ ,  $ZYZZZZZ$ ,  $ZYYZZZZ$ ,  $ZYZZYZZ$  or  $ZYYZYZZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $XX\star ZXYZ$ ,  $XX\star ZYZZ$ ,  $XX\star XXYZ$ ,  $\star ZXXYZZ$ ,  $\star ZYXYZZ$ ,  $YYY\star YYZ$ ,  $YXX\star YYZ$ ,  $Z\star ZXYZZ$ ,  $XYYYXYZ$ ,  $XXXXYZZ$ ,  $XXZXZZZ$ ,  $XYXXYZZ$ ,  $XYXYZZZ$ ,  $XZXYZZZ$ ,  $XXY\star XYZ$ ,  $XXZXZZZ$ ,  $XXXXZZZ$ ,  $XZZXZZZ$ ,  $XZZZZYZ$ ,  $XYZZZZZ$ ,  $YXXXYYZ$ ,  $YYZXZZZ$ ,  $YXZXZZZ$ ,  $YZZZZYZ$ ,  $YXYXYZZ$ ,

YXXXYYZZ, YXXZYYZ, YXZZZYZ, YXYYYZZ, YZYYYZZ, YYXZYZZ, YYYZYZZ, YYZYYYZ, YYYZZZZ, YYZZZZZ, ZZXYZYZ, ZZYYXYZ, ZZXXYYZ, ZYXXYYZ, ZXYXYYZ, ZXXXYYZ, ZXYZZYZ, ZXZYZZZ, ZYZYYZZ, ZYXZYYZ, ZYYXYZZ, ZYXZYZZ, ZZXYZZZ or ZZYYZZZ.

For YYZZZZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYYYZ, ZZYYYYZ, ZYYYYYZ, ZZZYYYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, second, third or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYZYZZZ, ZYZYZZZ, YZZYZZZ, YYYYZZZ and YYZYZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second, third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZZ, ZZZZZZZ, ZYYZZZZ, ZYZZYZZ and ZYZZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZZZ and YZZZYZZ respectively, which are handled by Case 1.

For XYZZZZZ, if the X-path starting at the third vertex ends at the second, fourth, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZYZZZZ, XYYZZZZ, XYZZYZZ, XYZZYZZ and XZZYYYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XZYZZYZ, which is handled by Case 1. If the X-path starting at the fourth vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 4 2 5 3 7), (7 3 1 2 4 6 5), (7 4 1 2 3 6 5) and (7 6 2 1 3 4 5), we have also handled XZZZZYZ, YXXYYYZ, YYYXYYZ and XXXXXYZ.

For YYYZZZZ, if the X-path starting at the second vertex doesn't end in  $H$  or ends at the first, fourth, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZYZZZZ, ZZYZZZZ, ZYZZZZZ, ZYZYZZZ, ZYZZYZZ and ZYZYYYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, third or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYYYZ, YZZYYYZ, ZZYYYYZ and ZZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, third or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZ, ZYZZZZZ, YYZYZZZ and YYYYZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYYZZZZ, ZYZZYZZ and ZYYZZYZ respectively, which are handled by Case 1.

For ZZYYZZZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, second, third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZ, ZYZZZZZ, ZYYZZZZ, ZZZZZZZ, ZZYZZYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, third or fifth vertex of  $H$ , then doing an X-Kempe change there

yields ZZYZZYZ, YZYZZYZ, ZYZZYZ, ZZZYZYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the third vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZ and ZYZZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 7 6 5 4), we have also handled YZYZZZ.

For ZYZZYZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZ, ZYZZZZ, ZYZZYZ and YZYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ, ZYZZYZ and YZYZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ and YZYZZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 7 4 5), we have also handled YZYZZZ.

For ZXZZYZ, if the X-path starting at the fifth vertex ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZ, ZXZZYZ and YXZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZZYZ and YXZZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YXZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 7 4 5), (1 2 7 3 6 4 5), (2 6 7 1 4 3 5), (3 2 7 1 6 4 5), (3 6 4 1 2 5 7), (3 6 4 5 7 1 2) and (4 6 5 3 7 1 2), we have also handled YXZZYZ, ZXZZYZ, XZZZZYZ, YXZZYZ, XZZZZYZ, XXXZZYZ and XZZZZYZ.

For XZZYZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZ, XZZYZZ and YXZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZ and YXZZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZZYZZ, which is handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields YXZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 7 4 5), we have also handled YXZZYZ.

For YZZZZYZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, second, third, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZ, ZZZYZZ, YZZYZZ, YZZYZZ, YZZYZZ, YZZYZZ and ZYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth

vertex doesn't end in  $H$  or ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZZYYZ, YZYZZYZ, YZZZYZZ and ZYYYYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZYZ and YZYZZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZZZZZYZ, which is handled by Case 1. If the X-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYYYYZ, which is handled by Case 1.

For XXYZZYZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XXYZZYZ, XXYZZYZ, XXYZZYZ and XXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XXYZZYZ, XXYZZYZ and XXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XXZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 4 3 7 2 5), (4 7 6 1 2 3 5) and (5 6 4 1 7 3 2), we have also handled YXZXZYZ, XXYZZYZ and ZZZXYYZ.

For YYZXZYZ, if the X-path starting at the second vertex ends at the first, third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXZYZ, YZYXZYZ, YZZXYYZ and XZZYZZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYXZYZ and XXXYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZXZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 5 4 7 6), we have also handled ZZYXZYZ.

For ZZXZZZZ, if the Y-path starting at the second vertex ends at the first, third, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXXZZZZ, ZXZZZZZ, ZZYXZYZ and YZZXYYZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex ends at the first or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XZZYZZZ and ZZZXZYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields YZYXZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 4 5 3 7 2), we have also handled ZZZXYYZ.

For ZYZXYYZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZYZZZYZ, ZXZZXZYZ and YXYXZYZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZXZYXZYZ and XYXZYZZ respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YXYXZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 5 4 6 7), we have also handled ZXZYXZYZ.

For  $XXZXYZZ$ , if the  $Y$ -path starting at the sixth vertex ends at the first, second, third, fourth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZYZYXYZ$ ,  $YZZYXYZ$ ,  $YYYYXYZ$ ,  $YYZZXYZ$  and  $ZZXZYZZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the third vertex ends at the fourth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XXXZYZZ$  and  $ZZZZXYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 3\ 1\ 4\ 5\ 7\ 6)$  and  $(3\ 4\ 1\ 6\ 7\ 5\ 2)$ , we have also handled  $ZXXXYZZ$  and  $XXYYXYZ$ .

For  $ZZYXYZZ$ , if the  $Y$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the second, sixth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZZYZYZZ$ ,  $ZXYZYZZ$ ,  $ZZXZXYZ$  and  $YYXYXYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the seventh vertex doesn't end in  $H$  or ends at the second or sixth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YYXZXYZ$ ,  $YZXZXYZ$  and  $XXYZYZZ$  respectively, which are handled by Case 1.

For  $ZZXXYZZ$ , if the  $Y$ -path starting at the fourth vertex ends at the second, third, sixth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZXXZYZZ$ ,  $ZZZZYZZ$ ,  $ZZYZXYZ$  and  $YYZYXYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the seventh vertex ends at the second, third or sixth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YZZZXYZ$ ,  $YYZXYZ$  and  $XXZZYZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 4\ 7\ 6\ 5\ 2)$ , we have also handled  $XXXZXYZ$ .

For  $XZXYZZ$ , if the  $Y$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the second, sixth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XZYZYZZ$ ,  $XXYZYZZ$ ,  $YZXZXYZ$  and  $ZYXYXYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the seventh vertex doesn't end in  $H$  or ends at the second or sixth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZYXZXYZ$ ,  $ZZXZXYZ$  and  $ZXYZYZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 4\ 3\ 7\ 5\ 6)$ ,  $(2\ 4\ 1\ 5\ 7\ 6\ 3)$ ,  $(3\ 2\ 1\ 4\ 5\ 6\ 7)$ ,  $(4\ 2\ 1\ 3\ 7\ 6\ 5)$ ,  $(6\ 2\ 1\ 7\ 5\ 3\ 4)$  and  $(7\ 1\ 2\ 6\ 4\ 3\ 5)$ , we have also handled  $YXXZYZZ$ ,  $ZXYXYZZ$ ,  $YZXXYZZ$ ,  $ZYXXYYZ$ ,  $YXXXYZZ$  and  $YXYXYZZ$ .

For  $XYXYZZ$ , if the  $Y$ -path starting at the fourth vertex ends at the sixth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YXXZXYZ$  and  $ZXXYXYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZYYZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 5\ 4\ 6\ 7)$ , we have also handled  $XYXXYZZ$ .

For  $XXZXZYZ$ , if the  $Y$ -path starting at the seventh vertex ends at the first, second, third, fourth or fifth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XZXZXYZ$ ,  $ZXXZXYZ$ ,  $ZZZZXYZ$ ,  $ZZXXXYZ$  and  $ZZXZZYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XZXXZYZ$  and  $XZZXXYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 5\ 4\ 6\ 7)$ , we have also handled  $XXZZXYZ$ .

For ZZXZYYZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing a Y-Kempe change there yields XXZXYYZ, ZXZXYYZ, XZZXYYZ, XXXXYYZ and XXZZYYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZXZYYZ, which is handled by Case 1. If the Y-path starting at the second vertex ends at the third vertex of  $H$ , then doing a Y-Kempe change there yields ZZZZYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 4 7 5), we have also handled YYXZYYZ.

Since ZYXZYZZ has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the second, third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields ZXXZYZZ, ZYYZYZZ and ZXYZYZZ respectively, which are handled by Case 1.

For YYYZYZZ, if the Z-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXXZYZZ and XYXZYZZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields XXYZYZZ, which is handled by Case 1.

For YYXZYZZ, if the Z-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXYZYZZ and XYYZYZZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields XXXZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 4 2 7 3 5), (2 5 3 1 6 4 7), (2 6 4 1 7 3 5), (2 6 5 1 7 3 4) and (4 5 6 1 7 2 3), we have also handled ZYYXYZZ, ZYXZYYZ, ZYXYZZ, XZXXYYZ and XXXZZYZ.

For YYYZYYZ, if the Z-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, third or sixth vertex of  $H$ , then doing a Z-Kempe change there yields YYYZXYZ, XYYZXYZ, YXYZXYZ, YYXZXYZ and XXXZYYZ respectively, which are handled by Case 1. If the Z-path starting at the second vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXYZYYZ and XYXZXYZ respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XYYZYYZ, which is handled by Case 1.

For YYYYYYZ, if the Z-path starting at the fifth vertex ends at the second, third, fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXYXYYZ, YYXYXYZ, YYYXXYZ and XXXXYYZ respectively, which are handled by Case 1. If the Z-path starting at the second vertex ends at the third, fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXXYYYZ, YXYXYYZ and XYXXXYZ respectively, which are handled by Case 1.

For YYXXYYZ, if the Z-path starting at the fifth vertex ends at the first, second, third, fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields XYXXXYZ, YXXXYZ, YYYXXYZ, YYXYXYZ and XXYYYYZ respectively, which are handled by Case 1. If the Z-path starting at the second vertex ends at the third or fourth vertex of  $H$ ,

then doing a Z-Kempe change there yields YXYXYYZ and YXXYYYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 5 4 7) and (1 2 4 6 5 3 7), we have also handled XXYXXYZ and XXXYXXYZ.

For YXXXYYZ, if the Z-path starting at the fourth vertex ends at the third, fifth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXYYYYZ, YXXYXXYZ and XYYXXYZ respectively, which are handled by Case 1. If the Z-path starting at the fifth vertex ends at the third or sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXYXXYZ and XYYYYYZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the sixth vertex of  $H$ , then doing a Z-Kempe change there yields XXYXXYZ, which is handled by Case 1.

For XYYYXYZ, if the Z-path starting at the fourth vertex ends at the third, fifth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields XYXXXYZ, XYYXXYZ and YXXYYYZ respectively, which are handled by Case 1. If the Z-path starting at the fifth vertex ends at the third or sixth vertex of  $H$ , then doing a Z-Kempe change there yields XXYYYYZ and YXXXXYZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the sixth vertex of  $H$ , then doing a Z-Kempe change there yields YXYXXYZ, which is handled by Case 1.

**Case 3.**  $B$  is one of  $XX\star YYZZ$ ,  $YXZ\star XYZ$ ,  $YY\star ZZYZ$ ,  $Y\star ZXYZZ$ ,  $Z\star XYYZZ$ ,  $XYZZYZZ$ ,  $XYZXYYZ$ ,  $XYZXYZZ$ ,  $XXZYXYZ$ ,  $XXYXZYZ$ ,  $XZYXYYZ$ ,  $XYZXZYZ$ ,  $XZYXYYZ$ ,  $XYZZYZZ$ ,  $XZZXYZZ$ ,  $XZYZZZZ$ ,  $XXXXYZZ$ ,  $XXYZZZZ$ ,  $YYZXYYZ$ ,  $YYYYYYZ$ ,  $YZXZYZZ$ ,  $YZXYYZZ$ ,  $YYXXZYZ$ ,  $YZXXYYZ$ ,  $YXXYYZZ$ ,  $YXZYZZZ$ ,  $YZZYZZZ$ ,  $YYZZYZZ$ ,  $YYYXZYZ$ ,  $YZZYZZZ$ ,  $YYYXYZZ$ ,  $YZZXZYZ$ ,  $YZZZZZZ$ ,  $ZXYXXYZ$ ,  $ZZZXYYZ$ ,  $ZXXXYYZ$ ,  $ZZXYXYZ$ ,  $ZXYZZYZ$ ,  $ZXYXZYZ$ ,  $ZXYXYZZ$ ,  $ZXXZYZZ$ ,  $ZZYXYYZ$ ,  $ZXYYYZZ$ ,  $ZZYZYZZ$ ,  $ZYYYYZZ$ ,  $ZZZYXYZ$ ,  $ZZZYZZZ$ ,  $ZYXZZYZ$ ,  $ZYYYYYZ$  or  $ZXYZZZZ$ .

Since YZZZZZZ has an odd number of Y's and Z's, there is an X-path with exactly one end in  $H$ . If this is the first, second, third, fourth, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZZZ, YYZZZZZ, YZYZZZZ, YZZYZZZ, YZZZYZZ, YZZZZYZ and ZYYYYYZ respectively, which are handled by Cases 1 and 2.

For ZXYZZZZ, if the X-path starting at the fifth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZZYZZ, ZXYZYYZ and YXZYZYZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZYZZZ, ZXYYZYZ and YXZZYYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYZZZZZ, ZXZZYZZ and YXYYYYZ respectively, which are handled by Cases 1 and 2.

For YYZZYZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZYZ, YZYYZYZ, ZYYYYYZ, ZZZYZYZ, ZZYZZYZ, ZZYYYYZ and ZZYZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the second, fifth or sixth vertex of  $H$ , then



doing an X-Kempe change there yields  $YYYZYZZ$ ,  $YZYZYZZ$ ,  $YYYZZZZ$  and  $YYYZYYZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields  $YYZZZZZ$  and  $YYZYZZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $ZYZZYZZ$ , which is handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YZZZYZZ$ , which is handled by Case 1.

For  $YZZYYZZ$ , if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YZZYYYZ$ ,  $YZYYYYZ$ ,  $YZZZYYZ$ ,  $YZZYZYZ$  and  $ZYYZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYZZYZ$ ,  $ZYZZYZZ$ ,  $ZYYYZYZ$  and  $ZYYZYYZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YZYYYZZ$ , which is handled by Case 2. If the X-path starting at the fourth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YZZZYZZ$ , which is handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YZZYZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 6\ 7\ 4\ 5)$ ,  $(3\ 4\ 7\ 5\ 6\ 1\ 2)$  and  $(7\ 3\ 1\ 4\ 5\ 2\ 6)$ , we have also handled  $ZYYYYZZ$ ,  $ZZYZYYZ$  and  $YYYZZYZ$ .

For  $YXZYZZZ$ , if the X-path starting at the fifth vertex ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YXZYZZZ$ ,  $XYZZZZZ$ ,  $YXZYZZZ$  and  $ZXZYZZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YXZYZZZ$ ,  $YXZZYYZ$  and  $ZXZYZZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 6\ 7\ 4\ 5)$ ,  $(5\ 4\ 1\ 3\ 6\ 2\ 7)$  and  $(5\ 4\ 3\ 1\ 2\ 6\ 7)$ , we have also handled  $ZXYYYZZ$ ,  $ZZYXYYZ$  and  $YYZXYYZ$ .

For  $XXZYZZZ$ , if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $XXZYZZZ$ ,  $YYZZZZZ$ ,  $XXZYZZZ$  and  $XXZYZZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $XXZZYZZ$ , which is handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $XXZYYYZ$ , which is handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $XXYZZYZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 6\ 7\ 4\ 5)$ ,  $(1\ 4\ 3\ 7\ 2\ 5\ 6)$ ,  $(1\ 4\ 5\ 6\ 2\ 3\ 7)$ ,  $(1\ 5\ 3\ 7\ 2\ 4\ 6)$ ,  $(1\ 6\ 4\ 2\ 5\ 3\ 7)$ ,  $(1\ 6\ 4\ 3\ 7\ 2\ 5)$ ,  $(2\ 3\ 1\ 4\ 5\ 6\ 7)$ ,  $(2\ 3\ 1\ 7\ 6\ 4\ 5)$ ,  $(2\ 3\ 4\ 5\ 6\ 1\ 7)$ ,  $(2\ 4\ 1\ 5\ 3\ 6\ 7)$ ,  $(2\ 4\ 7\ 1\ 5\ 3\ 6)$ ,  $(2\ 4\ 7\ 6\ 3\ 1\ 5)$ ,  $(3\ 6\ 1\ 7\ 5\ 2\ 4)$ ,  $(3\ 6\ 4\ 2\ 5\ 1\ 7)$  and  $(4\ 6\ 1\ 7\ 3\ 2\ 5)$ , we have also handled  $XXYYYZZ$ ,  $XZYXYYZ$ ,  $XYZXZZZ$ ,  $XZYXYYZ$ ,  $YXZZXYYZ$ ,  $YXZZXYYZ$ ,  $ZXXYYYZZ$ ,  $YXXYYYZZ$ ,  $ZXXZYZZZ$ ,  $ZXZYXZZZ$ ,  $YXZZXYYZ$ ,  $ZXZYXZZZ$ ,  $XXYXZZZ$ ,  $ZXZYXZZZ$  and

XXZYXYZ.

For XYZXYZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the second, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYXYZZ, XZYXYZ, YXXYZZ, XYXYYZ and XXXZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YXZYZZ, which is handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZYXZY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(3\ 2\ 1\ 4\ 7\ 5\ 6)$ ,  $(4\ 3\ 1\ 6\ 5\ 2\ 7)$  and  $(5\ 3\ 1\ 7\ 4\ 2\ 6)$ , we have also handled YZXXYZ, ZZXXYZ and YYXXZY.

For ZZXYYZ, if the X-path starting at the fifth vertex ends at the second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYXYZZ, ZZYZZZ, ZZXZYZ and YYXZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the second, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYXZYZ, ZZXZYZ and YYXYZY respectively, which are handled by Cases 1 and 2.

For YZXYYZ, if the X-path starting at the fifth vertex ends at the second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYXYZZ, XZYZZZ, YZXZYZ and ZYXZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the second or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYXZYZ and ZYXYZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZXZZY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 2\ 3\ 6\ 1\ 4\ 7)$ , we have also handled YZXZYZ.

For ZYXYYZ, if the X-path starting at the sixth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYXYYZ, ZZXYYZ, ZYXZYZ, ZYXYZY and XZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the second or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZXZYZ and YZXZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 2\ 3\ 6\ 1\ 4\ 7)$ , we have also handled YYXZZY.

For YYYYYZ, if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYYYY, YZYYYY, YZYYYY, YYYZYZ, YYYZYZ and ZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZ, YZYZZZ, YYYZZZ and ZZZYZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZYYYY, which is handled by Case 2. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYYYYZ, which is handled by Case 1. If the X-path starting at the seventh vertex

doesn't end in  $H$  then doing an X-Kempe change there yields ZZZZZYZ, which is handled by Case 1.

For XXXYYZZ, if the X-path starting at the fifth vertex ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XXXYZYZ and XXXZYYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (2 3 4 1 7 5 6) and (2 5 6 1 4 3 7), we have also handled ZXXXYYZ and XYZXYYZ.

For YYZZYZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, second, third, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYZYZY, ZYZYZYZ, YZZYZYZ, YYYYZYZ, YYZYYYZ, YYZYZZZ and ZZYZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYZZZZZ, YYYZZZZ and ZZYYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the third vertex of  $H$ , then doing an X-Kempe change there yields YZZZZYZ and YZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZ, which is handled by Case 1. If the X-path starting at the fifth vertex ends at the third or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYYZ and ZZYYZZZ respectively, which are handled by Case 2.

For ZZZXYYZ, if the X-path starting at the third vertex ends at the second, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYXYYZ, ZZYXZY, ZZYXYZZ and XXZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex ends at the fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZXZY, ZYZXYZZ and XZXZZZZ respectively, which are handled by Cases 1 and 2.

For YYZXYYZ, if the X-path starting at the seventh vertex ends at the first, second, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields XZXZZZ, ZXXYZZZ, ZZYXZY and ZZYXZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the first or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYZXZY and XXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYZXYZZ, which is handled by Case 2.

For YXZYXYZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YXYXYZ, ZXYXYZ, YXYZXYZ, XYXXYZ and ZYZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZXZYXYZ, which is handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYXZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (2 3 7 4 6 5 1), we have also handled ZXYXXYZ.

Since  $XXYZZZZ$  has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, second or third vertex of  $H$ , then doing a Z-Kempe change there yields  $YXYZZZZ$ ,  $XYYZZZZ$  and  $YYYZZZZ$  respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by  $(1\ 6\ 4\ 2\ 3\ 5\ 7)$ ,  $(1\ 7\ 5\ 2\ 3\ 4\ 6)$ ,  $(2\ 6\ 3\ 1\ 4\ 5\ 7)$ ,  $(6\ 7\ 4\ 1\ 2\ 3\ 5)$  and  $(6\ 7\ 5\ 1\ 2\ 3\ 4)$ , we have also handled  $YZZXZYZ$ ,  $ZYXYXYZ$ ,  $ZYXZZYZ$ ,  $YYYXYZZ$  and  $XXXXYZZ$ .

For  $ZZZYZZZ$ , if the Y-path starting at the second vertex ends at the first, third, fifth, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $XXZYZZZ$ ,  $ZXXYZZZ$ ,  $ZYZXZZZ$ ,  $ZYZXZYZ$  and  $YZYXYYZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex ends at the fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZZYXYZZ$  and  $ZZYXZYZ$  respectively, which are handled by Cases 1 and 2.

For  $YZXYZZZ$ , if the Y-path starting at the second vertex doesn't end in  $H$  or ends at the third or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YXXYZZZ$ ,  $YXZYZZZ$  and  $XYXZYZ$  respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YZZYZZZ$  and  $XZZXZYZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XZYXYZZ$ , which is handled by Case 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XZYXZYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 4\ 5\ 6\ 7\ 2)$ ,  $(1\ 6\ 5\ 4\ 3\ 7\ 2)$ ,  $(4\ 1\ 5\ 6\ 2\ 7\ 3)$  and  $(5\ 1\ 4\ 7\ 2\ 6\ 3)$ , we have also handled  $YZZXYZZ$ ,  $XZZXYZZ$ ,  $ZZZYXYZ$  and  $YYYXZYZ$ .

For  $XYZZYZZ$ , if the Y-path starting at the third vertex doesn't end in  $H$  or ends at the first, fourth, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $XYXZYZZ$ ,  $ZYXZYZZ$ ,  $XYXXYZZ$ ,  $YXYZXYZ$  and  $ZXZYXYZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZYZZYZZ$ , which is handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZXYXYYZ$ , which is handled by Case 2.

For  $XYZZYYZ$ , if the Y-path starting at the third vertex ends at the fourth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YXXYYYZ$  and  $ZYZXYYZ$  respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZYXZYYZ$ , which is handled by Case 2. Since  $XZYZZZZ$  has an odd number of X's and Z's, there is a Y-path with exactly one end in  $H$ . If this is the first, second, fifth, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZZYYZZZ$ ,  $XXYYZZZ$ ,  $YZXXYZZ$ ,  $YZXXZYZ$  and  $ZYXXYYZ$  respectively, which are handled by Cases 1 and 2.

**Case 4.**  $B$  is one of  $XYZYYYZ$ ,  $XXYXYZZ$ ,  $XZYYYZZ$ ,  $XZYZYZZ$ ,  $XYZYZZZ$ ,  $XYZZZYZ$ ,  $XYYYYZZ$ ,  $XZZYYZZ$ ,  $YXYYYZZ$ ,  $YXXXYZZ$ ,  $YZXYXYZ$ ,  $YYZZYYZ$ ,  $YYZYZZZ$ ,  $ZYXYYYZ$ ,  $ZYXXZYZ$ ,  $ZYXXYZZ$ ,  $ZZYYYYZ$ ,  $ZZZZYYZ$  or  $ZZZYZZZ$ .

For  $ZZZYZZZ$ , if the Y-path starting at the third vertex doesn't end in  $H$  or ends at

the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYYZZ, ZZYXXYZ and YYZXXYZ respectively, which are handled by Cases 1 and 3. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZZXXYZ and XXXYYZZ respectively, which are handled by Cases 1 and 3. If the Y-path starting at the second vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZYZZ, which is handled by Case 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YYYXXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 5 6 4 7), we have also handled ZZZZYYZ.

For XYZYYZZ, if the Y-path starting at the third vertex ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields YXYXXYZ and ZXZXXYZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZYXYZZ, which is handled by Case 3. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 5 6) and (1 2 3 6 7 4 5), we have also handled XZYZYZZ and XZYYYZZ.

Since XZZYYZZ has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YZZYYZZ, XZZXYZZ and YZZXYZZ respectively, which are handled by Case 3. Since we already handled the permutation of all resulting boards by (1 2 3 6 7 4 5) and (1 3 4 2 6 5 7), we have also handled XYYYYZZ and XYZZYZ.

For YYZYZZ, if the Z-path starting at the second vertex ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXZXYZZ and XYZXYZZ respectively, which are handled by Case 3. If the Z-path starting at the fourth vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields XXZYZZ, which is handled by Case 3. Since we already handled the permutation of all resulting boards by (1 2 3 5 6 4 7) and (1 2 3 6 7 4 5), we have also handled YYZZYYZ and ZZYYYZZ.

For ZYXXYZ, if the Z-path starting at the third vertex ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields ZYYYYZZ and ZXXYYZZ respectively, which are handled by Case 3. If the Z-path starting at the fourth vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields ZXXYZZ, which is handled by Case 3. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 5 7), (1 2 3 5 7 4 6), (1 3 6 7 4 2 5) and (1 3 6 7 5 2 4), we have also handled ZYXXZYZ, YZXXYZ, YYXXYZ and XXXYZZ.

For YYXYZZ, if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the third, fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXXYYZZ, YXYYYZZ, YXXXYZ and XYXYZZ respectively, which are handled by Cases 2 and 3. If the Z-path starting at the third vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YYYYYZZ, YYYXYZZ and XXXXYZZ respectively, which are handled by Case 3. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XYXYZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (2 4 1 5 6 3 7) and (2 4 3 5 6 1 7), we have also handled XYZYYYZ and ZYXYYYZ.

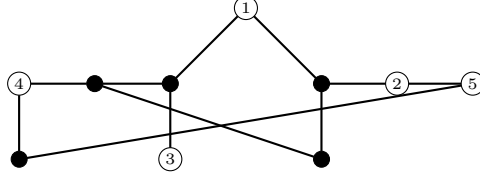


Figure 3: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

□

**Lemma 3.** *The graph in Figure 3 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 3, a string such as  $YZYYZ$ , represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -path has only one endpoint in  $H$ . We use shorthand notation like  $\mathcal{K}_{X,2}(YZYYZ, 5, 6) \Rightarrow YYXYX, YYYYYY$  (Case 1). This means the  $X$ -Kempe change on  $YZYYZ$  starting at the second vertex and ending at the fifth and sixth result in boards  $YYXYX$  and  $YYYYYY$  respectively and these are handled by Case 1. The  $\infty$  symbol means starting (or ending) outside  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star YZ$ ,  $\star\star YZZ$ ,  $\star YZZZ$ ,  $YZZZZ$  or  $ZZZZZ$ .

**Case 1.**  $B$  is one of  $\star\star ZYZ$ ,  $\star\star YYZ$ ,  $\star YZZZ$ ,  $\star ZYZZ$ ,  $\star XYZZ$ ,  $\star ZXYZ$ ,  $Z\star XYZ$ ,  $XY\star YZ$ ,  $ZZZZZ$  or  $ZYYZZ$ .

In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $XXXXYZ$ ,  $YXXYZ$ ,  $YYXYZ$ ,  $YYYZZ$  or  $YZZZZ$ .

$\mathcal{K}_{X,\infty}(YZZZZ, 1, 2, 3, 4, 5) \Rightarrow ZZZZZ, YYZZZ, YZYZZ, YZZYZ, ZYYYZ$  (Case 1).

$\mathcal{K}_{X,\infty}(YYYZZ, 1, 2, 3, 4, 5) \Rightarrow ZYYZZ, YZYZZ, YYZZZ, YYYYZ, ZZZYZ$  (Case 1).

$\mathcal{K}_{X,2}(YYXYZ, 4, 5) \Rightarrow XZYZZ, ZXYZZ$  (Case 1).

$\mathcal{K}_{X,4}(YYXYZ, 5) \Rightarrow ZZXYZ$  (Case 1).

$(12435) \Rightarrow XXXYZ$

$\mathcal{K}_{Y,\infty}(YXXYZ, 2, 3, 5) \Rightarrow YZXYZ, YXZYZ, YZZYZ$  (Case 1).

**Case 3.**  $B$  is one of  $XYZZZ$ .

$\mathcal{K}_{Y,\infty}(XYZZZ, 1, 4, 5) \Rightarrow ZYYZZ, YXXYZ, ZXXYZ$  (Case 1 and 2).

□

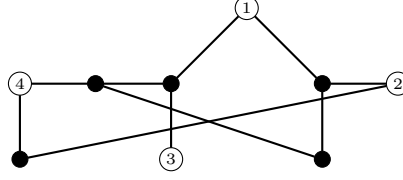


Figure 4: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

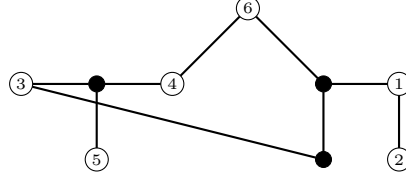


Figure 5: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

**Lemma 4.** *The graph in Figure 4 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 4, a string such as YXXX, represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -path has only one endpoint in  $H$ . We use shorthand notation like  $\mathcal{K}_{X,2}(YXXX, 5, 6) \Rightarrow XXZY, ZYZX$  (Case 1). This means the  $X$ -Kempe change on YXXX starting at the second vertex and ending at the fifth and sixth result in boards XXZY and ZYZX respectively and these are handled by Case 1. The  $\infty$  symbol means starting (or ending) outside  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star YZ$ ,  $\star YZZ$ ,  $YZZZ$  or  $ZZZZ$ .

**Case 1.**  *$B$  is one of  $\star\star YZ$ ,  $\star YZZ$ ,  $YZZZ$  or  $ZZZZ$ .*

In all these cases,  $H$  is immediately colorable from the lists.

□

**Lemma 5.** *The graph in Figure 5 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 5, a string such as YZXXYY, represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,

$Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star\star YZ$ ,  $\star\star\star YZZ$ ,  $\star\star YZZZ$ ,  $\star YZZZZ$ ,  $YZZZZZ$  or  $ZZZZZZ$ .

**Case 1.**  $B$  is one of  $\star\star YZZZ$ ,  $\star\star ZZYZ$ ,  $\star Z\star ZYZ$ ,  $\star X\star ZYZ$ ,  $XZ\star YZZ$ ,  $XX\star YZZ$ ,  $X\star ZXYZ$ ,  $Y\star ZYZZ$ ,  $YYX\star YZ$ ,  $XZX\star YZ$ ,  $XXX\star YZ$ ,  $YZ\star YZZ$ ,  $YY\star YZZ$ ,  $\star ZZXYZ$ ,  $YYZ\star YZ$ ,  $YZX\star YZ$ ,  $YYYZYZ$ ,  $YXZYYZ$ ,  $ZZZYZZ$ ,  $ZXZYZZ$ ,  $ZZZYYZ$ ,  $ZXZYYZ$  or  $ZYZXYZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $\star XYYYZ$ ,  $ZYY\star YZ$ ,  $ZY\star YZZ$ ,  $Y\star YYYZ$ ,  $XYXZYZ$ ,  $XYXYYZ$ ,  $XYXYYZ$ ,  $XYZYYZ$ ,  $XYYZYZ$ ,  $XXYXYZ$ ,  $XYZYZZ$ ,  $YXZXYZ$ ,  $YXXXYZ$ ,  $YYZZZZ$ ,  $ZYXYYZ$ ,  $ZXXXYZ$ ,  $ZZYXYZ$ ,  $ZZYYYY$ ,  $ZYXZYZ$ ,  $ZZXYYZ$ ,  $ZZXXYZ$ ,  $ZXZXYZ$ ,  $ZZXYZZ$ ,  $ZZYYZZ$  or  $ZZZZZZ$ .

For  $ZZZZZZ$ , if the  $X$ -path starting at the fifth vertex ends at the first, second, third, fourth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZYZ$ ,  $ZYZZYZ$ ,  $ZZYZYZ$ ,  $ZZZYYZ$  and  $YYYYZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the sixth vertex ends at the third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYZYYZ$  and  $YYYZYZ$  respectively, which are handled by Case 1.

For  $YYZZZZ$ , if the  $X$ -path starting at the fourth vertex ends at the second, third, fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZYZZ$ ,  $YYYYZZ$ ,  $YYZYYZ$  and  $ZZYZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the third vertex ends at the second, fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZYZZZ$ ,  $YYYZYZ$  and  $ZZZYYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(4\ 2\ 1\ 6\ 3\ 5)$  and  $(6\ 2\ 3\ 4\ 1\ 5)$ , we have also handled  $ZYZYZZ$  and  $YZYYYYZ$ .

For  $ZZYZZZ$ , if the  $X$ -path starting at the fourth vertex ends at the first, second, fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZYZZZ$ ,  $ZYYZZZ$ ,  $ZZYZYZ$  and  $YYZYYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the third vertex ends at the fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZZZYYZ$  and  $YYYZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the first vertex ends at the second or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYYYZZ$  and  $ZYZZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the second vertex ends at the sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZYZ$ , which is handled by Case 1.

For  $ZZXYYZ$ , if the  $X$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the second, fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZZYZZZ$ ,  $ZXYZZZ$ ,  $ZZXZYZ$  and  $YYXYYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the second or fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYXZYZ$ ,  $YZXZYZ$  and  $XXYZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the first vertex doesn't end in  $H$  then doing an  $X$ -Kempe change there yields  $YZXYZZ$ , which is handled by Case 1.



For ZYZZZZ, if the X-path starting at the second vertex ends at the first, third, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZZZZ, ZZZZZZ, ZZZZZZ and YZZZZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZZZZ and YZZZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 5 4 6), we have also handled ZYZZZZ.

For ZYXZZZ, if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZXZZZ, ZZXZZZ, YZXZZZ, YZXZZZ and XZZZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the second vertex of  $H$ , then doing an X-Kempe change there yields ZXYZZZ and ZZZZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZXZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 4 3 6), (1 3 5 4 2 6) and (1 5 3 4 2 6), we have also handled ZZXZZZ, ZZXZZZ and ZZXZZZ.

For ZYXZZZ, if the X-path starting at the second vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZXZZZ, ZZZZZZ and YZXZZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZZZ and YZXZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZXZZZ, which is handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZXZZZ, which is handled by Case 1.

For ZZZYYY, if the X-path starting at the fourth vertex ends at the third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYYY, ZZZYYY and YZZYYY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the third or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYYY and YZZYYY respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZYYY, which is handled by Case 1.

For ZZZYYY, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the second, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYYY, ZZZYYY, ZZZYYY and XXXYYY respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second or fifth vertex of  $H$ , then doing an X-Kempe change there yields XXZZYY, XZZZZY and YZZZZY respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 3 1 5 6 4) and (2 5 6 4 1 3), we have also handled ZXXXYY and ZYYYXX.

For ZZZYYY, if the X-path starting at the fourth vertex ends at the first, second, third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZYYY, ZZZYYY, ZZZYYY, ZZZYYY and ZZZYYY respectively, which are handled by Case 1.

ZYZZYZ, ZYYZZZ and YZZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the third or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZZ and YZZYZZ respectively, which are handled by Case 1.

For ZXYYYY, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZZY, ZXZZYZ, ZXYZZZ and YXZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZXZYYZ, ZXZYZZ and YXYZZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields YXZZYZ, which is handled by Case 1.

For ZYXYYZ, if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields XZYZZZ, ZZYZZZ, XXYZZZ, YZXYZZ and YZXZZY respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYXYYZ, which is handled by Case 1. If the X-path starting at the fourth vertex ends at the second or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZXZZY and ZXYZZZ respectively, which are handled by Case 1.

For YYYYYZ, if the X-path starting at the fourth vertex ends at the third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYZZYZ, YYYZZZ and ZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the third or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYZZYZ and ZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZ, which is handled by Case 1.

For YXYYYY, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YXYZZY, YXZZYZ, YXYZZZ and ZXZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YXZZYZ, YXZZYZ and ZXZZYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZXZZYZ, which is handled by Case 1. Since XYZZYZ has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, second or fourth vertex of  $H$ , then doing a Z-Kempe change there yields YYZZYZ, XXZZYZ and YXZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (6 3 1 5 2 4), we have also handled XXYXYZ.

For XYYZZY, if the Z-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields XXXZZY and YYXZZY respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXYZZY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 6), (6 2 3 1 4 5), (6 2 3 1 5 4), (6 2 4 1 5 3) and (6 3 4 1 5 2), we have also handled XYZZYZ, YXXXYZ,

XYXYYZ, XYXYYZ and XXYYYZ.

For XYXZYZ, if the Z-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields XXYZYZ and YYYZYZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields YXXZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 3\ 4\ 1\ 6\ 5)$ , we have also handled YXZXYZ.

**Case 3.**  $B$  is one of  $Y\star YXYZ$ ,  $XYYYYZ$ ,  $XZYXYZ$ ,  $XXZYYZ$ ,  $XYXYZZ$ ,  $XZYYYZ$ ,  $XYYYZZ$ ,  $XZZYYZ$ ,  $XYXXYZ$ ,  $XYZZZZ$ ,  $YXXYYZ$ ,  $YZZZZZ$ ,  $ZYZYYZ$ ,  $ZYXXYZ$ ,  $ZXXYZZ$  or  $ZXYZZZ$ .

For YZZZZZ, if the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second, fourth, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XZYZZZ, XYZZZZ, XZYZZZ, XZYZYZ and XYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields XZZYZZ and XYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZZZYZ, which is handled by Case 1.

For ZXYYZZ, if the Y-path starting at the second vertex ends at the fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZZXXYZ and YYXXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields XZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 6\ 2\ 4\ 3\ 5)$ ,  $(1\ 6\ 2\ 5\ 3\ 4)$  and  $(2\ 1\ 4\ 5\ 3\ 6)$ , we have also handled YXYXYZ, XYXXYZ and XZZYYZ.

Each of ZXXYZZ, XYYZZZ and ZYZYYZ have an odd number of X's and Z's, so there is a Y-path with exactly one end in  $H$ . For ZXXYZZ, if this is the first, second, third, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XXXYZZ, ZZXXYZ, ZXZYZZ, ZYXXYZ and YZZXZY respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 6\ 4\ 5)$ , we have also handled YXXYYZ.

For XYYZZZ, if this is the first, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZZZZ, YXXXYZ and ZXXXYZ respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 4\ 5\ 2\ 6)$ , we have also handled XZYZZZ.

For ZYZYYZ, if this is the first, third or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XYZYYZ, ZYXYYZ and XYXYYZ respectively, which are handled by Case 2.

For XYZZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZ, XZZZYZ, XYYZYZ, XYZZYZ and XZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZ and XZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex

doesn't end in  $H$  then doing an X-Kempe change there yields  $XYZZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(4\ 6\ 1\ 5\ 3\ 2)$ , we have also handled  $YYYXYZ$ .

For  $XYXYZZ$ , if the X-path starting at the fourth vertex ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields  $XYXZYZ$  and  $XZXYYZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields  $YZYZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 2\ 6\ 4\ 5)$ ,  $(2\ 1\ 5\ 6\ 3\ 4)$  and  $(2\ 1\ 6\ 4\ 3\ 5)$ , we have also handled  $XXZZYZ$ ,  $ZYXXYZ$  and  $XZYXYZ$ .

For  $YZYXYZ$ , if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, third or sixth vertex of  $H$ , then doing an X-Kempe change there yields  $XZXYZZ$ ,  $ZZXYZZ$ ,  $XZZYZZ$  and  $ZYZXZY$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $ZZYXYZ$ , which is handled by Case 2. If the X-path starting at the third vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YZZXZY$ , which is handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $ZXZYZZ$ , which is handled by Case 1. Since  $XYYYYYZ$  has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, second, third, fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYYYYZ$ ,  $XXYYYYZ$ ,  $XYXYYZ$ ,  $XYXYYZ$  and  $YXXXYZ$  respectively, which are handled by Case 2.

**Case 4.**  $B$  is one of  $YZZZYZZ$ ,  $YXXYZZ$ ,  $YXYZZZ$ ,  $ZXXYYZ$ ,  $ZXYXYZ$  or  $ZYZZZZ$ .

Each of  $ZYZZZZ$ ,  $YXYZZZ$  and  $YZZZYZZ$  have an odd number of X's and Z's, so there is a Y-path with exactly one end in  $H$ . For  $ZYZZZZ$ , if this is the first, third, fourth, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XYZZZZ$ ,  $ZXYZZZ$ ,  $ZXZYZZ$ ,  $ZXZZYZ$  and  $YXYYYZ$  respectively, which are handled by Cases 1, 2 and 3. For  $YXYZZZ$ , if this is the second, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YZYYZZ$ ,  $XYXXYZ$  and  $XZXXYZ$  respectively, which are handled by Cases 1 and 3. For  $YZZZYZZ$ , if this is the second, third or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YXZYYZ$ ,  $YZXYYZ$  and  $YXXYYZ$  respectively, which are handled by Cases 1 and 3.

For  $YXXYZZ$ , if the Y-path starting at the third vertex ends at the fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XYZXYZ$  and  $XZYXYZ$  respectively, which are handled by Cases 1 and 3. If the Y-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YZZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 6\ 4\ 3\ 5)$  and  $(4\ 1\ 6\ 5\ 2\ 3)$ , we have also handled  $ZXYXYZ$  and  $ZXXYYZ$ .

□

**Lemma 6.** *The graph in Figure 6 is reducible.*

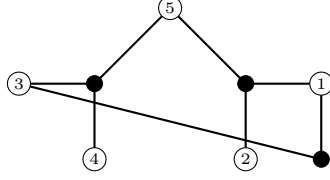


Figure 6: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 6, a string such as ZYZXX, represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star YZ$ ,  $\star\star YZZ$ ,  $\star YZZZ$ ,  $YZZZZ$  or  $ZZZZZ$ .

**Case 1.**  $B$  is one of  $X\star\star YZ$ ,  $\star X\star YZ$ ,  $X\star YZZ$ ,  $YZ\star YZ$ ,  $ZY\star YZ$ ,  $\star YYZZ$ ,  $\star XYZZ$ ,  $YYZYZ$  or  $YYXYZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $\star YZZZ$ ,  $ZZ\star YZ$ ,  $YZYZZ$ ,  $YYYYZ$ ,  $YZZZZ$  or  $ZZYZZ$ .

For  $YZZZZ$ , if the  $Y$ -path starting at the third vertex ends at the fourth or fifth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XZYYZ$  and  $XYZYZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the fourth vertex ends at the fifth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XYZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5)$ ,  $(3\ 1\ 2\ 4\ 5)$ ,  $(4\ 1\ 2\ 3\ 5)$  and  $(5\ 1\ 2\ 3\ 4)$ , we have also handled  $ZYZZZ$ ,  $ZZYZZ$ ,  $ZZZYZ$  and  $YYYYZ$ .

Since  $YYZZZ$  has an odd number of  $X$ 's and  $Z$ 's, there is a  $Y$ -path with exactly one end in  $H$ . If this is the third, fourth or fifth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XXYZZ$ ,  $XXZYZ$  and  $XXYYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 2\ 4\ 5)$  and  $(3\ 4\ 1\ 2\ 5)$ , we have also handled  $YZYZZ$  and  $ZZYYZ$ .

For  $XYZZZ$ , if the  $X$ -path starting at the third vertex ends at the fourth or fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYYYZ$  and  $XZZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the fourth vertex ends at the fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(3\ 4\ 1\ 2\ 5)$ , we have also handled  $ZZXYZ$ .

**Case 3.**  $B$  is one of  $ZZZZZ$ .

Since  $ZZZZZ$  has an odd number of  $Y$ 's and  $Z$ 's, there is an  $X$ -path with exactly one end in  $H$ . If this is the first, second, third, fourth or fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZZ$ ,  $ZYZZZ$ ,  $ZZYZZ$ ,  $ZZZYZ$  and  $YYYYZ$  respectively, which are handled by Case 2.

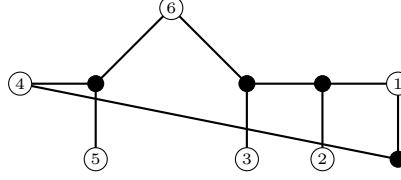


Figure 7: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

□

**Lemma 7.** *The graph in Figure 7 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 7, a string such as  $YXXYZZ$ , represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star\star YZ$ ,  $\star\star\star YZZ$ ,  $\star\star YZZZ$ ,  $\star YZZZZ$ ,  $YZZZZZ$  or  $ZZZZZZ$ .

**Case 1.**  $B$  is one of  $XZ\star\star YZ$ ,  $\star Z\star YZZ$ ,  $YZ\star\star YZ$ ,  $\star YX\star YZ$ ,  $YX\star\star YZ$ ,  $Z\star Z\star YZ$ ,  $\star\star YYZZ$ ,  $\star\star XYZZ$ ,  $Z\star\star XYZ$ ,  $XYZ\star YZ$ ,  $\star YYYYZ$ ,  $X\star YZYZ$ ,  $YYY\star YZ$ ,  $ZZ\star ZYZ$ ,  $Z\star XYYZ$ ,  $ZXY\star YZ$ ,  $XYZYZZ$ ,  $X\star XZYZ$ ,  $XXYXYZ$ ,  $X\star XXYZ$ ,  $YXZYZZ$  or  $ZXZYZZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $\star\star YZZZ$ ,  $XX\star YYZ$ ,  $\star YZZZZ$ ,  $XXZ\star YZ$ ,  $XXZYZZ$ ,  $XYXYXZ$ ,  $YYZYYZ$ ,  $YYZXYZ$ ,  $YYZYZZ$ ,  $YZZZZZ$ ,  $ZZYYYZ$ ,  $ZYZYZZ$ ,  $ZXXZYZ$  or  $ZZZZZZ$ .

For  $ZZZZZZ$ , if the  $X$ -path starting at the fifth vertex ends at the first, second, third, fourth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZYZ$ ,  $ZYZZYZ$ ,  $ZZYZYZ$ ,  $ZZZYZZ$  and  $YYYZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the fourth vertex ends at the third or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZZYYZZ$  and  $YYYZZY$  respectively, which are handled by Case 1.

For  $YYZZZZ$ , if the  $X$ -path starting at the sixth vertex ends at the first, second, third, fourth or fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZYYYY$ ,  $ZYYYYZ$ ,  $ZZZYZZ$ ,  $ZZYZYZ$  and  $ZZYYZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the third vertex ends at the fourth or fifth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYYZZZ$  and  $YYYZZY$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 2\ 5\ 6\ 4)$  and  $(2\ 3\ 1\ 4\ 6\ 5)$ , we have also handled  $YZYZZZ$  and  $ZYYZZZ$ .

For  $XYZZZZ$ , if the  $X$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYZYZZ$ ,  $XYZYYZ$

and XZYZZY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZ and XZYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZYZZY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 3 2 4 5 6) and (2 3 1 4 5 6), we have also handled XZYZZZ and ZXYZZZ.

For YYYZZZ, if the X-path starting at the fourth vertex ends at the second, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZZ, YYYZZY and ZZZZZY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the second or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZY and ZZZZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ, which is handled by Case 1.

For XYYZZZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZ, XYYZZY and XZZZZY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZY and XZZZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZZZZY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 1 3 4 5 6), (3 1 2 4 5 6), (5 2 3 1 4 6) and (6 1 4 2 3 5), we have also handled YXYZZZ, XXYZZZ, ZXXZZY and XYYZZY.

For YYZZZZ, if the X-path starting at the third vertex ends at the first, second, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZ, YZZZZZ, YYYZZY and ZZZZZY respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZZ and YZZZZY respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 5 4), we have also handled ZZYZZY.

For YYZZYZ, if the X-path starting at the third vertex ends at the first, second, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ, YZZZZY, YYYZZY and YYYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZ and ZZYZZY respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZY, YZZZZY and YZZZZZ respectively, which are handled by Case 1.

For YZZZZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XZZZZZ, XZZZZY and XYYZZY respectively, which are handled by Case 1. If the Y-path starting at the fifth

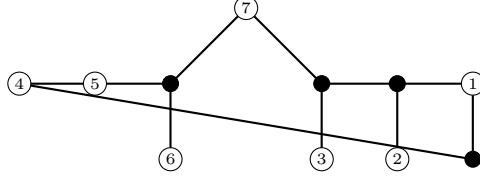


Figure 8: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZZYZ$  and  $XYYYZZ$  respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XYYYYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5\ 6)$  and  $(3\ 1\ 2\ 4\ 5\ 6)$ , we have also handled  $ZYZZZZ$  and  $ZZYZZZ$ .

For  $ZYZYZZ$ , if the Y-path starting at the third vertex ends at the fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZXYXYZ$  and  $YXZXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the fifth vertex ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XYXYZZ$ , which is handled by Case 1.

For  $XXZYZZ$ , if the Y-path starting at the second vertex doesn't end in  $H$  or ends at the third or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZYZZ$ ,  $XZXYYZ$  and  $ZYYXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXXYYZ$  and  $ZZZXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZXZYZZ$ , which is handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZZYXYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 5\ 4\ 6)$ ,  $(2\ 1\ 4\ 6\ 3\ 5)$ ,  $(3\ 6\ 1\ 4\ 5\ 2)$ ,  $(5\ 4\ 1\ 6\ 3\ 2)$  and  $(6\ 3\ 1\ 5\ 4\ 2)$ , we have also handled  $XXZZYZ$ ,  $XXYYYYZ$ ,  $YYZXYZ$ ,  $XXXYYZ$  and  $XXZXYZ$ .

For  $XXZYYZ$ , if the Y-path starting at the second vertex ends at the third or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZXYYZ$  and  $ZXXYYZ$  respectively, which are handled by Case 1. If the Y-path starting at the third vertex ends at the sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZZZYYZ$ , which is handled by Case 1.

**Case 3.**  $B$  is one of  $YYZZYZ$  or  $ZYYZYZ$ .

Since  $YYZZYZ$  has an odd number of X's and Z's, there is a Y-path with exactly one end in  $H$ . If this is the third, fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YYXZYZ$ ,  $YYZXYZ$  and  $YYXXYZ$  respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by  $(1\ 4\ 2\ 3\ 6\ 5)$ , we have also handled  $ZYYZYZ$ .

□

**Lemma 8.** *The graph in Figure 8 is reducible.*



*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 8, a string such as XZYYXXX, represents a possible list assignment on  $V(H)$  arising from a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star\star\star YZ$ ,  $\star\star\star\star YZZ$ ,  $\star\star\star YZZZ$ ,  $\star\star YZZZZ$ ,  $\star YZZZZZ$ ,  $YZZZZZZ$  or  $ZZZZZZZ$ .

**Case 1.**  $B$  is one of  $XZ\star\star XYZ$ ,  $XY\star\star XYZ$ ,  $\star Z\star ZYZZ$ ,  $\star\star YYYZZ$ ,  $\star\star XYZZZ$ ,  $\star Z\star ZZYZZ$ ,  $YX\star Z\star YZ$ ,  $\star ZZ\star YYZ$ ,  $\star ZX\star YYZ$ ,  $\star YX\star YYZ$ ,  $ZY\star\star ZYZ$ ,  $ZX\star\star ZYZ$ ,  $Z\star\star XXYZ$ ,  $Y\star Y\star YYZ$ ,  $\star\star YZYZZ$ ,  $\star\star XZYZZ$ ,  $\star ZZ\star XYZ$ ,  $\star YXZ\star YZ$ ,  $\star ZX\star\star YZ$ ,  $YX\star\star YYZ$ ,  $\star\star YXXYZ$ ,  $XYZ\star YYZ$ ,  $\star ZYYZYZ$ ,  $\star XYYXYZ$ ,  $Z\star ZYYZZ$ ,  $ZX\star ZXYZ$ ,  $ZYZ\star YYZ$ ,  $ZX\star YYYZ$ ,  $\star\star YZZYZ$ ,  $\star YXX\star YZ$ ,  $\star XYYZYZ$ ,  $\star XXYZYZ$ ,  $\star YYXZYZ$ ,  $\star ZXZXYZ$ ,  $\star YYZXYZ$ ,  $YXZ\star XYZ$ ,  $\star ZYYXYZ$ ,  $\star YYYYYZ$ ,  $\star XXXXYZ$ ,  $ZZZ\star ZYZ$ ,  $XYZZYZZ$ ,  $XYZZZYZ$ ,  $X\star XZZYZ$ ,  $XZXZYZZ$ ,  $XZY\star ZYZ$ ,  $XZYYYYYZ$ ,  $X\star XYXYZ$ ,  $YXZZYZZ$ ,  $YZXYZYZ$ ,  $YZYXZYZ$ ,  $ZYZZYZZ$  or  $ZXZYXYZ$ . In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of  $XX\star ZXYZ$ ,  $XXZ\star ZYZ$ ,  $XXX\star YYZ$ ,  $XXY\star YYZ$ ,  $XY\star YZZZ$ ,  $X\star ZYYZZ$ ,  $XZ\star YZZZ$ ,  $Y\star ZYZYZ$ ,  $Y\star ZYYZZ$ ,  $YZ\star XYZZ$ ,  $YX\star XZYZ$ ,  $YX\star XYZZ$ ,  $Y\star XYXYZ$ ,  $Y\star XYZZZ$ ,  $\star YYZZZZ$ ,  $ZZY\star YYZ$ ,  $ZX\star ZYYZ$ ,  $ZY\star YZZZ$ ,  $ZX\star YZZZ$ ,  $X\star YXYYZ$ ,  $X\star ZXZYZ$ ,  $X\star YZYYZ$ ,  $YYZ\star ZYZ$ ,  $Y\star YZZZZ$ ,  $\star ZZYZZZ$ ,  $XYXYZYZ$ ,  $XYZYZYZ$ ,  $XYXXYZZ$ ,  $XXZYYYZ$ ,  $XYZXYZZ$ ,  $XXXXZYZ$ ,  $XXZXXYZ$ ,  $XZYZZZZ$ ,  $XYZZZZZ$ ,  $YYYYXYZ$ ,  $YYYYZYZ$ ,  $YYZXXYZ$ ,  $YZYZXYZ$ ,  $YYZZYZZ$ ,  $YYYYZZZ$ ,  $YXZYZZZ$ ,  $YZZXZYZ$ ,  $YZZZZZZ$ ,  $ZYXYXYZ$ ,  $ZXYXYYZ$ ,  $ZZXYXYZ$ ,  $ZXZXYYZ$ ,  $ZYZYXYZ$ ,  $ZYZXYZZ$ ,  $ZYXXYZZ$ ,  $ZYZZXYZ$ ,  $ZZXYZZZ$ ,  $ZXYZZZZ$ ,  $ZYZZZZZ$  or  $ZZZZZZZ$ .

For  $ZZZZZZZ$ , if the  $X$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or seventh vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZZZZZYZ$ ,  $YZZZZYZ$ ,  $ZYZZZYZ$ ,  $ZZYZZYZ$ ,  $ZZZYZZZ$ ,  $ZZZZYYZ$  and  $YYYYYYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third or fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $ZZZZYZZ$ ,  $ZYZZYZZ$ ,  $ZZYZYZZ$  and  $ZZZYYZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YYYYYYZ$  and  $YYYZYYZ$  respectively, which are handled by Case 1.

For  $YZZZZZZ$ , if the  $X$ -path starting at the fifth vertex doesn't end in  $H$  or ends at the first, third, sixth or seventh vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZYZZ$ ,  $ZZZZYZZ$ ,  $YZYZYZZ$ ,  $YZZZYYZ$  and  $ZYYYZYZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the first, third or seventh vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $YZZZZYZ$ ,

ZZZZZZYZ, YZYZZYZ and ZYYYYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first or third vertex of  $H$ , then doing an X-Kempe change there yields ZYYYYYZ, YYYYYYZ and ZYZYYYYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 1 3 4 5 6 7) and (4 1 2 3 5 6 7), we have also handled ZYZZZZZ and ZZZYZZZ.

For XYZZZZZ, if the X-path starting at the fifth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZ, XYZZYYZ and XZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the third or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZ and XZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZYYYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 4 3 2 5 6 7), (2 4 3 1 5 6 7), (3 1 2 4 5 6 7), (3 2 1 4 5 6 7) and (3 4 1 2 5 6 7), we have also handled XZZYZZZ, ZXZZYZZ, XZYZZZZ, ZXYZZZZ and ZZZXYZZ.

For YYYZZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZ, ZYYZZYZ, YZYZZYZ, YYYYYZZ, YYYZZYZ and ZZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYYYYZ, YZZYYYYZ, ZYZYYYYZ, ZZZZYYZ and ZZZYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZ and YZYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 4 2 1 5 6 7), we have also handled ZYYYYZZ.

For YZZYZZZ, if the X-path starting at the fifth vertex ends at the first, third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZYZYZ, YZYZZYZ, YZZYZYZ, YZZYYYYZ and ZYYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, third, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZYZYZ, YZYZZYZ, YZZZZYZ and ZYYZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first, third or fourth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZ, ZYZZYZZ and ZYYYYYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 1 3 4 5 6 7), we have also handled ZYZYZZZ.

For XYZYZZZ, if the X-path starting at the fifth vertex ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZZ, XYZYYYZ and XZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZZ and XZYZZYZ respectively, which are handled by Case 1. If the

X-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XZYYYYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 3 2 4 5 6 7), (2 1 3 4 5 6 7), (2 3 1 4 5 6 7), (2 5 7 6 1 3 4), (3 1 2 4 5 6 7), (3 2 1 4 5 6 7), (6 1 2 4 3 5 7), (6 3 5 7 1 2 4), (6 5 3 7 1 2 4), (7 1 2 4 3 5 6), (7 1 4 2 3 5 6), (7 3 6 5 1 2 4) and (7 5 3 6 1 2 4), we have also handled XZYZZZZ, YXZYZZZ, ZXYYZZZ, ZXZZYYZ, YZXZZZZ, ZYXZZZZ, XXXXZZY, XXXXZZY, XYYXYYZ, XYYYYYZ, YYXXYYZ and XXXXYYZ.

For YYYYYZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZ, ZYYYYZZ, YZYYYYZ, YYYZZYZ, YYYYYYZ and ZZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZYZ, YZZZZYZ, ZYZZYZZ, ZZZZZYZ and ZZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields YZYYZZY and YYYZZYZ respectively, which are handled by Case 1.

For XYYYYZZ, if the X-path starting at the fifth vertex ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZ, XYYYYYZ and XZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZ and XZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (3 1 2 4 5 6 7), (6 1 4 2 3 5 7), (6 1 5 2 3 4 7), (7 2 5 3 1 4 6), (7 3 6 5 1 2 4) and (7 4 5 6 1 2 3), we have also handled YXXYZZZ, XXZXZZY, XXZZXZY, YXXYXZY, XXYXYYZ and XXXYYYZ.

For XYXZZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYXZZYZ, XYXZZYZ, XYXZZYZ and XZXZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields XZXZZYZ, XZXZZYZ and XZXZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 4 6 3 7), (1 2 6 4 5 3 7), (1 3 5 4 6 2 7), (2 1 3 4 5 7 6), (2 1 5 4 6 3 7), (2 1 7 4 3 6 5), (2 3 5 4 6 1 7), (2 3 7 5 1 6 4) and (4 1 5 2 3 6 7), we have also handled YXZXZZZ, YXZXZZY, YZXXZZZ, YXXYZZZ, XYZZXZZ, XZYXYYZ, ZYXXYZZ, YZXXYYZ and XXZZYYZ.

For YYZZZZZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZ, ZYYZZYZ, YZYZZYZ, YYYYYYZ, YYYZZYZ and ZZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, YZYZZYZ, ZYYZZYZ and ZZYZZYZ respectively, which

are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the first or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZZYZZ, ZZZZYZZ and YZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 5 6) and (1 4 3 2 5 6 7), we have also handled ZZYYYYZ and YZZZYZZ.

For YYZYZZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZ, ZYYYYZZ, YZYZZZ, YYYZZZ, YYYYYYZ and ZZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, YZYZZYZ, ZYYZZYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZZ, ZZZYZZ and ZYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YZZZYZZ and YZZZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 5 6), we have also handled ZZYZZYZ.

For YYZZYZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZ, ZYYZZYZ, YZYZZYZ, YYYZZYZ and ZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, YZYZZYZ, ZYYZZYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the first or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZZZZYZ, ZZZZZYZ and YZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 4 3 2 5 6 7), we have also handled YZZYZYZ.

For YYZYZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, YZYZZYZ, ZYYZZYZ, ZZZYZYZ and ZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYYZZYZ, YZYZZYZ, YYYZZYZ and YYYYYYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ, ZZZYZYZ and ZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZZZZYZ and YZZZYZZ respectively, which are handled by Case 1.

For XYZYZY, if the X-path starting at the fourth vertex ends at the second, third, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZZZYZ, XYZZYZ, XYZZYYZ and XZYYYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZYZZYZ, XYZZYYZ and XZZZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 5 7 6), we have also handled XZYZZYZ.

For YXZYZY, if the X-path starting at the fourth vertex ends at the first, third, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZZYZ, YXZZYZ, YXZZYYZ and ZXZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZXZYZZ and YXZYZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the fifth vertex of  $H$ , then doing an X-Kempe change there yields ZXZYZZ, which is handled by Case 1.

For YYZXZY, if the X-path starting at the second vertex ends at the first, third or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXZY, ZYXZY and YZZXYYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYXZY and YZYXZY respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYXZY, which is handled by Case 1.

For YYYZY, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZZY, ZYYZZY, YZYZZY, YYYZY and ZZZYYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, third or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZY, ZYZZYZ, ZZYZZY and ZZZZZY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYY, ZYYYYY and YYYYYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYZ and ZZYZZY respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZ, which is handled by Case 1.

For XYXYZY, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYXZZY, XZXZZY, XYXZY and XZXYYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZXZY, which is handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XYXYZY, which is handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZXZY, which is handled by Case 1. Since we already

handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5\ 7\ 6)$ , we have also handled  $ZXXZYYZ$ .

For  $YZYZZZZ$ , if the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth, fifth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $XZXZZYZ$ ,  $XYXZZYZ$ ,  $XZXZYZZ$ ,  $XZXZYZZ$  and  $XYXYZZZ$  respectively, which are handled by Case 1. If the Y-path starting at the fifth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XZXZYZZ$ , which is handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XYXYZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5\ 7\ 6)$ , we have also handled  $ZYYZZZZ$ .

For  $XYYZZZZ$ , if the Y-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YXXZYZZ$ ,  $YXXZYZZ$  and  $ZXXYZYZ$  respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YXXZZYZ$  and  $ZXXYZZZ$  respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZXXYYYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 4\ 5\ 2\ 3\ 6\ 7)$ ,  $(1\ 4\ 7\ 2\ 3\ 5\ 6)$ ,  $(2\ 1\ 3\ 4\ 5\ 6\ 7)$ ,  $(4\ 1\ 3\ 2\ 5\ 6\ 7)$ ,  $(4\ 1\ 6\ 2\ 3\ 5\ 7)$ ,  $(4\ 2\ 3\ 1\ 5\ 6\ 7)$  and  $(5\ 2\ 6\ 3\ 1\ 4\ 7)$ , we have also handled  $XZZYZZZ$ ,  $XYYZZZZ$ ,  $YXYZZZZ$ ,  $XZXYZZZ$ ,  $YZZXZYZ$ ,  $ZXXYZZZ$  and  $ZYZZXYZ$ .

For  $YZZXYYZ$ , if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YZZZYZZ$ ,  $XZZXYYZ$  and  $XYYYXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZYXYZ$  and  $YXXZYZZ$  respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XYYZXZYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5\ 6\ 7)$ , we have also handled  $ZYZXYYZ$ .

For  $XYZYYZZ$ , if the Y-path starting at the third vertex ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YXYXXYZ$  and  $ZXZXXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZYXYZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 1\ 3\ 4\ 5\ 6\ 7)$ ,  $(2\ 1\ 3\ 4\ 7\ 5\ 6)$ ,  $(4\ 1\ 2\ 3\ 5\ 6\ 7)$ ,  $(4\ 1\ 3\ 2\ 7\ 5\ 6)$ ,  $(5\ 1\ 2\ 3\ 6\ 4\ 7)$  and  $(5\ 1\ 2\ 3\ 7\ 4\ 6)$ , we have also handled  $YXZYZZZ$ ,  $ZXYZYZZ$ ,  $YZYXZZZ$ ,  $ZZYXYYZ$ ,  $YZYZXZYZ$  and  $ZYZYXZYZ$ .

For  $YXYXYYZ$ , if the Y-path starting at the fourth vertex ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YXZXXYZ$  and  $XZXXYXZ$  respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $YZYZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 4\ 6\ 5\ 7)$ ,  $(1\ 2\ 3\ 4\ 7\ 5\ 6)$  and  $(4\ 1\ 5\ 2\ 6\ 3\ 7)$ , we have also handled  $YXYXZYZ$ ,  $ZXZXYYZ$  and  $XXZYYYZ$ .

For XYXXYZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XYXZYZZ, YXYZXYZ and ZXZYXYZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields YXYXYYZ and ZYZZYZZ respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 4 6 2 5 3 7), (2 1 3 4 5 6 7), (2 1 3 4 6 5 7) and (3 2 5 7 6 1 4), we have also handled YYZXXYZ, YXXXYZZ, YXXXZYZ and XYZXXYZ.

For XXZZYZZ, if the Y-path starting at the seventh vertex ends at the first, second, third, fourth or fifth vertex of  $H$ , then doing a Y-Kempe change there yields XZXXXYZ, ZXXXXYZ, ZZZXXYZ and ZZXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Y-Kempe change there yields XZXZZYZ and XZZZXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (4 7 3 1 5 6 2), we have also handled XXXZXYZ.

For XXZYZZZ, if the Y-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fifth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXXYZYZ, ZXXYZYZ, XZXYZYZ, XXXYXYZ and ZZZYXYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZYZZZ, which is handled by Case 1. If the Y-path starting at the second vertex ends at the fifth vertex of  $H$ , then doing a Y-Kempe change there yields XZZYXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 6 4 3 7 5), (3 5 1 4 2 6 7), (4 2 5 1 3 7 6), (5 3 4 1 2 7 6), (6 3 1 4 2 7 5) and (6 5 1 4 2 7 3), we have also handled XXYZZYZ, ZZXXYZZ, ZXYXYZZ, ZYXXYZZ, XXYZZYZ and XXXZZYZ.

For YYYYYYZ, if the Z-path starting at the fifth vertex ends at the first, second, third, fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields XYYYYYZ, YXYYYYYZ, YYXYYYYZ, YYXXYZZ and XXXXYZZ respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex ends at the second or third vertex of  $H$ , then doing a Z-Kempe change there yields YXYXYZZ and YYXXXYZ respectively, which are handled by Case 1.

**Case 3.**  $B$  is one of  $X\star YXYZZ$ ,  $XX\star YZZZ$ ,  $YYZ\star YYZ$ ,  $Z\star YXYZZ$ ,  $XX\star XYZZ$ ,  $XZ\star XYZZ$ ,  $ZX\star XYZZ$ ,  $XXYXZZZ$ ,  $XXZZYZZ$ ,  $XXZXYYZ$ ,  $XYYYZZZ$ ,  $XZZYZYZ$ ,  $XXZZYZZ$ ,  $XXYZZZZ$ ,  $YYZZXYZ$ ,  $YYZXXYZ$ ,  $YXYZZZZ$ ,  $YYXYZZZ$ ,  $YZYYZZZ$ ,  $YYZYZZZ$ ,  $YYYXYZZ$ ,  $YYZZZZZ$ ,  $ZYYXYZZ$ ,  $ZZZXXYZ$ ,  $ZXZZYZZ$ ,  $ZXXXXYZ$ ,  $ZZYYZZZ$ ,  $ZZYXZZZ$ ,  $ZYYZZYZ$ ,  $ZZYZXYZ$  or  $ZZYZZZZ$ .

For YZZZZZZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fourth, fifth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XXYYYYYZ, XXZYXYZ, XXYZZYZ, XXYZZYZ and XXYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing a Y-Kempe change there yields XZZZZYZ

and XXZYZZY respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by (3 7 4 1 5 6 2), we have also handled YYZYZZY.

For ZZYXYZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZYZZYZ, ZZXXYZ and YYXYXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYXYZ and XXYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YYXZZYZ, which is handled by Case 1.

For XXYXYZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXYZZYZ, YYXZZYZ and ZZXYXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields YYXYXYZ and ZZYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZZXXYZ, which is handled by Case 1.

For XXYXZZY, if the Y-path starting at the second vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing a Y-Kempe change there yields XZYXZZY, XZYZZYZ and XZYXXYZ respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the fifth vertex of  $H$ , then doing a Y-Kempe change there yields XXYZZYZ and XXYZZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXYXZZY, which is handled by Case 1. If the Y-path starting at the fifth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XXYXXYZ, which is handled by Case 1.

For ZZYZZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, ZZZZZYZ, ZZYZZYZ, ZZYZZYZ and YYZYZZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, ZZZZZYZ, ZZYZZYZ and YYZYZZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZYZ, ZZZZZYZ and YYYYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZYZZYZ, which is handled by Case 2. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYYZZYZ, which is handled by Case 2.

For YYZYZZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, third, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZ, YZYZZYZ, ZZZZZYZ, ZZYZZYZ and ZZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYYZ, ZYYZZYZ,



YYYYYZZ and YYYZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZYZZZ, which is handled by Case 2. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZZYZZZ, which is handled by Case 2. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYZYZZZ, which is handled by Case 2. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYZYZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 4 2 7 3 5 6) and (3 4 5 1 2 7 6), we have also handled ZYYZYZZ and YZYZZZ.

For XXZYZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XXZYZZZ, XXZYZZZ and XXZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XXZYZZZ and XXZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields XXZYZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (3 6 1 4 2 5 7), (5 7 2 1 3 4 6) and (5 7 6 3 1 2 4), we have also handled ZZYXZZZ, XYYZYZZ and YYXZYZZ.

For ZZYZZZZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZ, ZZYZZZZ, ZZYZZZZ, ZZYZZZZ and YZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZ, ZZYZZZZ, ZZYZZZZ and YZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZ, ZZYZZZZ and YZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZZYZZZZ, which is handled by Case 2.

For YXYYZZZ, if the X-path starting at the fifth vertex ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YXYYZZZ, YXYYZZZ, YXYYZZZ and ZYZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YXYYZZZ, YXYYZZZ and ZYZZZZZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (1 4 5 2 6 3 7), (1 4 6 7 5 2 3), (3 5 2 4 6 1 7), (6 4 5 7 2 1 3), (7 5 2 6 4 1 3) and (7 5 4 3 2 1 6), we have also handled YYZXZZZ, ZYXYYZZ, ZXXYYZZ, XXXYZZZ, XZXXYYZ and YYZZXXZ.

For ZXZZZZZ, if the X-path starting at the fifth vertex ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZZZZZ, ZXZZZZZ, ZXZZZZZ and YXYYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an

X-Kempe change there yields ZXYYYZZ, ZXZYYYZ and YXYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 4 1 3 5 6 7), we have also handled ZZZXYZZ.

For XXZXYYZ, if the X-path starting at the fifth vertex ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XXZXZYZ and XXYXYYZ respectively, which are handled by Case 2. If the X-path starting at the sixth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YYXYZZZ, which is handled by Case 2.

For XYYXYYZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YXXYZZZ, XYYXZYZ and XZZXYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYXYYZ and YZZYZZZ respectively, which are handled by Case 2. If the X-path starting at the third vertex doesn't end in  $H$  then doing an X-Kempe change there yields XYZXYZZ, which is handled by Case 2. If the X-path starting at the seventh vertex doesn't end in  $H$  then doing an X-Kempe change there yields XZZXZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 4 1 6 2 3 7) and (5 4 7 6 3 1 2), we have also handled XXZXYYZ and XXZZYYZ.

For YYZZYYZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the second, fourth, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZZ, YZYZYZZ, YYYYYYZ, YYYZZYZ, YYYZYZZ and ZZZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYZZZYZZ, YYZYZYZZ and ZZYYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYZZYZZ, YYZYZZZ and ZZYYZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZZYZZ, YZZYZZZ and ZYYYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYYZ, which is handled by Case 1.

For YYZXYYZ, if the X-path starting at the second vertex ends at the first, third, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYYZ, YZYXYYZ, YZZXZYZ, YZZXYZZ and ZXXYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex ends at the fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYXZYZ and ZZZYZZZ respectively, which are handled by Cases 1 and 2.

For ZYYXYYZ, if the X-path starting at the fifth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZXZYZ, ZXXYZZZ and YZZXYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex ends at the third or seventh vertex of  $H$ , then doing an X-Kempe change

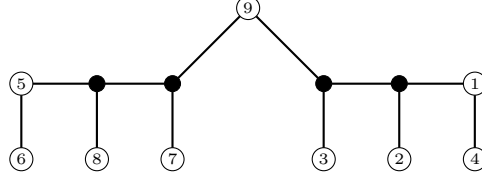


Figure 9: Solid vertices have lists of size 3 and the labeled vertices have lists of size 2.

there yields  $ZYZXYZZ$  and  $YZZXZYZ$  respectively, which are handled by Case 2. If the X-path starting at the third vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields  $XZXVZZZ$ , which is handled by Case 2. Since  $XXVZZZZ$  has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, second or third vertex of  $H$ , then doing a Z-Kempe change there yields  $YXYZZZZ$ ,  $XYVZZZZ$  and  $YVVZZZZ$  respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 5\ 3\ 4\ 6\ 7)$ ,  $(1\ 4\ 5\ 2\ 3\ 6\ 7)$ ,  $(2\ 4\ 5\ 1\ 3\ 6\ 7)$ ,  $(3\ 6\ 5\ 1\ 2\ 4\ 7)$ ,  $(4\ 6\ 1\ 2\ 3\ 5\ 7)$ ,  $(6\ 7\ 4\ 1\ 2\ 3\ 5)$  and  $(6\ 7\ 5\ 1\ 2\ 3\ 4)$ , we have also handled  $XXZZVZZ$ ,  $XZZXYZZ$ ,  $ZXZXYZZ$ ,  $ZZVZXYZ$ ,  $XZZVZYZ$ ,  $YVYXYZZ$  and  $XXXYZZ$ .

For  $XXVYZZ$ , if the Z-path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZZ$  and  $YXXVZZZ$  respectively, which are handled by Case 2. If the Z-path starting at the third vertex ends at the fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $YVVYZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 4\ 3\ 5\ 2\ 6\ 7)$ ,  $(1\ 7\ 5\ 6\ 2\ 3\ 4)$  and  $(2\ 4\ 3\ 5\ 1\ 6\ 7)$ , we have also handled  $XZYXYZZ$ ,  $ZXXVYZ$  and  $ZXYXYZZ$ .

**Case 4.**  $B$  is one of  $XXZYXYZ$ ,  $YVXXVZZ$ ,  $YVZYXYZ$ ,  $ZXXVXYZ$ ,  $ZVVYXYZ$ ,  $ZZXYZYZ$  or  $ZZXXVZZ$ .

Since  $ZZXXVZZ$  has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the third, fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $ZZVXYZZ$ ,  $ZZXYVZZ$  and  $ZZVVYZZ$  respectively, which are handled by Cases 1 and 3. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 7\ 5\ 4\ 6)$ ,  $(1\ 2\ 4\ 6\ 3\ 5\ 7)$  and  $(2\ 3\ 1\ 7\ 5\ 4\ 6)$ , we have also handled  $YVZYXYZ$ ,  $ZZXYZYZ$  and  $ZVVYXYZ$ .

Since  $YVXXVZZ$  has an odd number of Y's and Z's, there is an X-path with exactly one end in  $H$ . If this is the first, second, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $ZVXXVZZ$ ,  $YVXXVZZ$ ,  $XXVYZZZ$ ,  $YVXXVYZ$  and  $ZZXXZYZ$  respectively, which are handled by Cases 1, 2 and 3. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 7\ 5\ 4\ 6)$  and  $(2\ 3\ 1\ 7\ 5\ 4\ 6)$ , we have also handled  $XXZYXYZ$  and  $ZXXVXYZ$ .

□

**Lemma 9.** *The graph in Figure 9 is reducible.*

*Proof.* Let  $X = \{0, 1\}$ ,  $Y = \{0, 2\}$  and  $Z = \{1, 2\}$ . Then with the vertex ordering in Figure 9, a string such as  $ZYXXZZXYZ$ , represents a possible list assignment on  $V(H)$  arising from

a 3-edge-coloring of  $G - E(H)$ . By an  $X$ -Kempe change, we mean flipping colors 0 and 1 on a two-colored path in  $G - E(H)$ . We call such a path an  $X$ -path. Any endpoint of an  $X$ -path in  $H$  must end at a  $Y$  or  $Z$  vertex. The meanings of  $Y$ -Kempe change,  $Z$ -Kempe change,  $Y$ -path and  $Z$ -path are analogous. Note that if there are an odd number of  $Y$ 's and  $Z$ 's, then at least one  $X$ -Kempe change has only one endpoint in  $H$ .

We need to handle all boards up to permutations of  $\{X, Y, Z\}$ , so it will suffice to handle all boards of the form  $\star\star\star\star\star\star\star YZ$ ,  $\star\star\star\star\star\star YZZ$ ,  $\star\star\star\star\star YZZZ$ ,  $\star\star\star\star YZZZZ$ ,  $\star\star\star YZZZZZ$ ,  $\star\star YZZZZZZ$ ,  $\star YZZZZZZZ$ ,  $YZZZZZZZZ$  or  $ZZZZZZZZZ$ .

**Case 1.**  $B$  is one of

$\star\star Y\star\star YZZZ$ ,  $\star\star X\star\star YZZZ$ ,  $\star ZX\star\star\star YZZ$ ,  $\star YX\star\star\star YZZ$ ,  $\star\star Y\star ZZ\star YZ$ ,  
 $\star\star X\star ZZ\star YZ$ ,  $\star\star Y\star XX\star YZ$ ,  $\star\star X\star XX\star YZ$ ,  $\star ZY\star\star\star XYZ$ ,  $\star XY\star\star\star XYZ$ ,  
 $\star\star Y\star YZYZZ$ ,  $\star\star Y\star XZYZZ$ ,  $\star ZZY\star\star YZZ$ ,  $\star ZZX\star\star YZZ$ ,  $\star\star Y\star YYYZZ$ ,  
 $\star\star Y\star XXYZZ$ ,  $\star\star Y\star ZYZYZ$ ,  $\star\star X\star ZYZYZ$ ,  $\star\star Y\star XYZYZ$ ,  $\star\star X\star XYZYZ$ ,  
 $\star\star X\star ZYXYZ$ ,  $\star ZZY\star\star YYZ$ ,  $\star ZZX\star\star YYZ$ ,  $\star ZZY\star\star XYZ$ ,  $\star ZZX\star\star XYZ$ ,  
 $\star ZY\star Y\star YYZ$ ,  $\star XY\star Y\star YYZ$ ,  $\star\star X\star XYXYZ$ ,  $Z\star ZZ\star\star YZZ$ ,  $Z\star ZZ\star\star YYZ$ ,  
 $Z\star ZZ\star\star XYZ$ ,  $XY\star X\star\star YYZ$ ,  $YX\star Y\star\star YZZ$ ,  $YX\star X\star\star YZZ$ ,  $ZX\star X\star\star YZZ$ ,  
 $XY\star Y\star\star XYZ$ ,  $XY\star X\star\star XYZ$ ,  $ZY\star Y\star\star XYZ$ ,  $YX\star Y\star\star YYZ$ ,  $XXX\star\star\star XYZ$ ,  
 $\star YYZ\star\star YZZ$ ,  $\star YYX\star\star YZZ$ ,  $YX\star Y\star\star XYZ$ ,  $ZX\star Z\star\star YZZ$ ,  $ZY\star Z\star\star YYZ$ ,  
 $ZY\star Z\star\star XYZ$ ,  $XZ\star X\star\star YYZ$ ,  $YZ\star Y\star\star YYZ$ ,  $ZX\star Z\star\star YYZ$ ,  $Y\star YY\star\star YZZ$ ,  
 $\star XXZ\star\star XYZ$ ,  $XYZY\star\star YZZ$ ,  $XYZX\star\star YZZ$ ,  $XYZY\star\star YYZ$ ,  $XXX\star Y\star YYZ$ ,  
 $\star\star YZZZZZZ$ ,  $\star Z\star YZZZZZ$ ,  $\star ZZY\star YZZZ$ ,  $\star ZZX\star YZZZ$ ,  $YXZX\star\star YYZ$ ,  
 $YXZX\star\star XYZ$ ,  $Z\star ZZ\star YZZZ$ ,  $ZYZY\star\star YZZ$ ,  $ZYZY\star\star YYZ$ ,  $ZXZX\star\star YYZ$ ,  
 $ZXZX\star\star XYZ$ ,  $XZXZ\star\star YYZ$ ,  $XYXY\star\star YYZ$ ,  $\star ZY\star XZYYZ$ ,  $\star XY\star XZYYZ$ ,  
 $\star ZY\star ZXYYZ$ ,  $\star XY\star ZXYYZ$ ,  $YZXZ\star\star YYZ$ ,  $ZYXY\star\star YYZ$ ,  $\star\star YYZZZZZ$ ,  
 $\star\star XYZZZZZ$ ,  $YYY\star ZY\star YZ$ ,  $YYY\star XY\star YZ$ ,  $XZYZ\star\star YZZ$ ,  $XZYX\star\star YZZ$ ,  
 $XZXZ\star\star XYZ$ ,  $XZ\star X\star\star XYZ$ ,  $\star XX\star YZYZZ$ ,  $\star XX\star XZYZZ$ ,  $YZYZ\star\star YZZ$ ,  
 $\star XX\star YYYZZ$ ,  $\star XX\star XXYZZ$ ,  $\star ZX\star ZYYYZ$ ,  $\star YX\star ZYYYZ$ ,  $YZXZ\star\star XYZ$ ,  
 $\star ZX\star XYYYZ$ ,  $YZXY\star\star XYZ$ ,  $\star YX\star XYYYZ$ ,  $ZXXY\star\star XYZ$ ,  $XYZY\star YZZZ$ ,  
 $XYZX\star YZZZ$ ,  $XXX\star\star ZYYZ$ ,  $XXX\star ZXYYZ$ ,  $XYYY\star YYZ$ ,  $YXZY\star YZZZ$ ,  
 $YXZX\star YZZZ$ ,  $\star ZZYZZZYZ$ ,  $\star ZZXZZZYZ$ ,  $\star ZZYZYZYZ$ ,  $\star ZZXZYYZZ$ ,  
 $\star ZZYXYYZZ$ ,  $\star ZZXXYZYZ$ ,  $\star ZZYXXZYZ$ ,  $\star ZZXXXXZYZ$ ,  $YXXZY\star YYZ$ ,  
 $ZYZY\star YZZZ$ ,  $ZXZX\star YZZZ$ ,  $Z\star ZZZZZYZZ$ ,  $Z\star ZZZYZYZZ$ ,  $Z\star ZZZXYYZZ$ ,  
 $Z\star ZZZXXZYZ$ ,  $ZYYYY\star YYZ$ ,  $ZXXYY\star YYZ$ ,  $YX\star XZY\star YZ$ ,  $YX\star XXY\star YZ$ ,  
 $ZX\star XZY\star YZ$ ,  $ZX\star XXY\star YZ$ ,  $XYYZZY\star YZ$ ,  $XZYZ\star YYYZ$ ,  $XYYZXY\star YZ$ ,  
 $YZYZ\star YYYZ$ ,  $ZYYXZY\star YZ$ ,  $ZYYXXY\star YZ$ ,  $XYZYZZZZZ$ ,  $XYZYZZZYZ$ ,  
 $XYZXZZZYZ$ ,  $XYZYZYZYZ$ ,  $XYZXZYYZZ$ ,  $XYZYXYYZZ$ ,  $XYZXXYZYZ$ ,  
 $XYZYXXZYZ$ ,  $XYZXXXZYZ$ ,  $XYYY\star ZYYZ$ ,  $XYYYZXYYZ$ ,  
 $YXZYZZZZZ$ ,  $YXZYZZZYZ$ ,  $YXZXZZZYZ$ ,  $YXZYZYYZZ$ ,  $YXZYXYYZZ$ ,  
 $YXZYXXZYZ$ ,  $YXZXXXZYZ$ ,  $YXXZXZYYZ$ ,  $YXXZZXYYZ$ ,  $ZZZZZZZZZ$ ,  
 $ZYZZZZZZZ$ ,  $ZYZYZZZZZ$ ,  $ZYZYZZZYZ$ ,  $ZXZXZZZYZ$ ,  $ZYZYZYZYZ$ ,  
 $ZYZYXYYZZ$ ,  $ZYZYXXZYZ$ ,  $ZXZXXXZYZ$ ,  $ZYYYYXZYZ$ ,  $ZXXYXZYYZ$ ,

ZYYYZXYYZ or ZXXYZXYYZ. In all these cases,  $H$  is immediately colorable from the lists.

**Case 2.**  $B$  is one of ★★X★YZZYZ, XXZZ★★YYZ, XXZX★★YZZ, XXZZ★★YZZ, XYZZ★★XYZ, XXX★Z★YZZ, XZZZ★★YZZ, YYZZ★★XYZ, ★ZX★YYZYZ, ★YX★YYZYZ, YYZY★★XYZ, ★YX★XZZYZ, ★ZX★XZZYZ, YXZZ★★YZZ, Y★XXY★YYZ, YZZZ★★XYZ, ★YX★YZZZZ, YYYZY★★YZ, ★ZX★YZZZZ, Z★XXY★YYZ, XXZ★ZZ★YZ, XXZ★XX★YZ, YX★YY★ZYZ, ZX★ZY★ZYZ, XY★XY★ZYZ, XZ★XY★ZYZ, YYZ★ZZ★YZ, YYZ★XX★YZ, YY★★YZYYZ, YZ★YY★ZYZ, ZY★ZY★ZYZ, YY★YY★XYZ, ZZX★Y★YYZ, X★ZZXX★YZ, X★ZZZZ★YZ, Y★ZZXX★YZ, Y★ZZZZ★YZ, ★Z★YYZZYZ, Y★XXYY★YZ, YYY★Y★YYZ, YYX★Y★YYZ, Z★XXYY★YZ, XXZZ★YXYZ, XXZ★XYZYZ, XXYXZ★YZZ, XXYYZ★YZZ, XXZXZY★YZ, XYXZY★YYZ, XYYZY★YZ, XYZYY★ZYZ, XXZX★YZZZ, XXZZ★YZZZ, XYZZ★YZZZ, XXX★ZYYYZ, XZ★XZXZYZ, XY★XZXZYZ, XXX★XYYZZ, XXX★YXYZZ, XZXYY★YYZ, XXY★XYYYZ, XZZZX★YZ, XXX★XYYYZ, XYYYZ★YZZ, X★ZZZYZYZ, YX★YXZZYZ, YXXXXZ★YZ, YX★ZXYYYZ, YX★YZXZYZ, YYZZ★YYZZ, YYX★ZXXYZ, ★XYXYZZYZ, YXZX★YZ, YYZ★XXYZ, YZ★YZXZYZ, YYY★XZXYZ, YYY★ZXXYZ, YYZ★ZYZYZ, YYZ★YYZZZ, YYY★XZYYZ, YYZ★XYZYZ, Y★XXYXXYZ, ★ZZYXZZYZ, YYZ★ZYZZZ, YXZZZY★YZ, YX★YYZZZZ, YYYXY★XYZ, YX★XYZZZZ, ★YYXYZZZZ, YZZZY★YYZ, YXZZ★YZZZ, ★ZZXXZZYZ, ★YYZYZZZZ, ZXZYXX★YZ, ZY★ZZXZYZ, ZYZXZZ★YZ, ZX★ZZXZYZ, ZXZYZZ★YZ, Z★XXYXXYZ, ZXYYZ★YZZ, ZZX★ZXXYZ, ZX★ZXZZYZ, ZYZY★ZYZ, ZZY★ZYZZZ, ZYZZXX★YZ, ZXZX★YZ, ZX★XXZZYZ, ZZY★XYYYZ, ZX★XYZZZZ, ZZZ★YYZYZ, ZX★ZYZZZZ, XY★ZYXXYZ, XXZ★XYXYZ, XY★ZYZYYZ, XY★ZZXXYZ, XY★XXZZYZ, X★YYXYYYZ, XZ★XYZZZZ, XZZZZY★YZ, YXXXXZ★YZ, YXYXY★ZYZ, YX★ZZZYZZ, YXYXZ★YZ, ★ZZXYZZYZ, YXZZXY★YZ, YX★XXZZYZ, Y★XXZXXYZ, Y★YYYZZZZ, Y★ZZXZYZZ, YYZ★YYYZZ, ZXYXY★ZYZ, ZZX★YYXYZ, ZZX★YXXYZ, ZZXXX★YZ, ZY★ZZXZYZ, ZZYYZ★YZZ, ZYXXXX★YZ, Z★XXZXXYZ, Z★YYXYYYZ, ZZ★ZYZZYZ, XYYZXZYYZ, XYXYZXZYZ, XYXZYXYYZ, XYZZXYZYZ, XYZYXZZYZ, XYZZYYYZ, XXZYZYXYZ, XXZYXXYZ, XYZYXZYZ, XXZYXZYZ, XXYXZYYYZ, XXYXYXZYZ, XZXZYXZYZ, XXZXXYYZ, XXZYYZYZ, XZXZZXZYZ, XZY★YZZYZ, XXYYZYYYZ, XXZYYYYZ, XXYYYXYZZ, XXYXYXYZZ, XZXYYXXYZ, XZZYYYZYZ, XYYZYXYYZ, XXZ★ZYZYZ, XXZ★YYZZZ, XX★XXYYZZ, XX★XYXYZZ, XZXYY★YZ, XZYYZYYZZ, XZXYZXXYZ, XZYXXZZYZ, XYZZXXYZ, X★ZZZYZZZ, X★ZZYZYZ, XXXZYZZZZ, XYYXYXYZZ, XYYXYXYZZ, XY★XYZZZZ, XYZYYZZZZ, XYYYZYYYZ, X★ZZZYZZZ, XZYZYZZZZ, XYZZZZZZZ, YXXXXZYXZ, YYYZZXZYZ, YXXXYZXYZ, YXXZYXXYZ, YYYZXZZYZ, YZYXXZZYZ, YXYXZXZYZ, YYZXZYXYZ, YXXZXXYZ, YZXZYXZYZ, YYZXXYXYZ, YZXZZXZYZ, YYZZZZYZZ, YZZXYYZYZ,

YYZYXYZZZ, YXXZYXYYZ, YYZZXYZZZ, YYZYXZYYZ, YXXZZXZYZ,  
 YXXXZXYYZ, YZYXZYZZZ, YZZZZYZZZ, YZZZXXYZZ, YYYXZXYYZ,  
 YYXYZXYYZ, YZYXXYYYZ, YZZZZXYYZ, YZZZXZYYZ, YZZZXZYZZ,  
 YZZZYYYZZ, YYYYZXYYZ, YZZZZZYZZ, YZZZYZZZ, YZZZYZZZ,  
 YZZZYZZZ, YZYZZZZZ, YYZYZZZZ, YYZZZZZZ, ZYZXYZZY,  
 ZYZXZYZZ, ZYZYZXZY, ZYZXXYXYZ, ZYZXZZYZ, ZXZYXYXYZ,  
 ZYXXYZXYZ, ZYXXZXXYZ, ZYXXZXYYZ, ZYXYXZYZ, ZYXYXXYZ,  
 ZYZXXYZZ, ZYZXZYXYZ, ZXZYXXYZ, ZYXYZXZY, ZZXZXXYZ,  
 ZXXYZYZZ, ZXZYXZYZZ, ZXZYZZYZ, ZXZYYYZZ,  
 ZYZXYYYZ, ZYZXYZZZ, ZXZXZXZY, ZXZYZZYZ, ZXZYYYZZ,  
 ZZZYYXZY, ZZZYZXZY, ZYZXZYZZ, ZXZYZYXY, ZXZYXYZY,  
 ZYZXXXYZ, ZYXXZXZY, ZXXYXYYZ, ZZYYYXYZZ, ZZYXXYYZ,  
 ZXYYXYZZ, ZXYYXYZZ,  
 ZXYYZYYZ, ZZZXYZZZZ, ZZZYYZZZZ, ZYXXYYXYZ, ZYXXYYYZ,  
 ZZYZZYYZ, ZYZZZYZZ, ZYXXYYYZ, ZYZYZZZZ, ZZZZXZZYZ or  
 ZYZZYZZZ.

For YYZZZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYYYYZ, YZYYYYYZ, ZYZZYYYZ, ZZZYYYYZ, ZZYZZYYZ, ZZYZZYYZ and ZZYZZYYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZZ, ZYZZZZZZ, YZYZZZZZ, YYYZZZZZ, YYYZZZZZ and YYYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second, fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZZZ, ZZZZZZZZ, ZYZYZZZZ, ZYZZYZZZ and ZYZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 8 3 4 5 2 6 7 9), (5 2 7 6 1 3 4 8 9) and (5 8 7 6 1 2 3 4 9), we have also handled YZZZZZZYZ, ZYZZYZZZ and ZZZYZZZY.

For XYZZZZZZ, if the X-path starting at the fourth vertex ends at the second, third, sixth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZZZ, XYZZZZZZ, XYZYZZZZ, XYZYZZYZ, XYZYZZYZ and XZYZYYYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the third, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZYZ, XZYZZYZZ and XZYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the second, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZYZZZZZ, XYZZYZZZ, XYZZYZZZ and XYZZYZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 8 3 4 5 2 6 7 9), (3 5 6 7 8 1 2 4 9), (5 2 6 7 1 3 4 8 9), (5 8 6 7 1 2 3 4 9), (7 1 3 4 2 5 6 8 9), (9 3 1 2 4 5 6 7 8), (9 4 1 2 3 5 6 7 8), (9

6 1 2 3 4 7 5 8) and (9 7 1 2 3 4 6 5 8), we have also handled XZZZZZZYZ, ZZXZYZZZZ, ZXXZYZZZZ, ZZZZXZZYZ, XZZZZZYZZ, YXXYYYYYZ, YYYXYYYYZ, YYYYYXYYZ and YYYYYXYZ.

For YYZYZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZYYYYZ, YZYZYYYYZ, ZYYZYYYYZ, ZZZZYYYYZ, ZZYYYYYYZ, ZZYZYZYZZ and ZZYZYYYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZZ, ZYYZZZZZ, YZYZZZZZ, YYYZZZZZ, YYYZYZZZ, YYYZZYZZ and YYYZZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the first, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZZZ, ZZZYZZZZ, YZZYZYZZ, YZZYZYZZ and YZZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYZYZZZZ, ZYZYZYZZ and ZYZYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 8 5 6 4 7 2), (5 8 7 6 1 2 3 4 9) and (7 9 5 2 1 3 6 4 8), we have also handled YYZYYYYZZ, ZZZZYZZYZ and YZYYYYZYZ.

For ZXXZYZZZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXXZYZYZZ, ZXXZZZYZZ, ZXXZYYYZZ, ZXXZYZYZZ and YXXYZYZYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXXZYYYZ, YXXYYYYZ, YXXYZZYZZ and YXXYZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth vertex of  $H$ , then doing an X-Kempe change there yields ZYYZZZZZ and ZXXZZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZXXZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 7 8 5 1 9 3 6 4), we have also handled XZZZZZYZZ.

For YYYZYZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYZYYYZ, ZYZYZYYYZ, ZZZZYYYYZ, ZZZYYYYYZ, ZZZYZZYZZ, ZZZYZYZYZ and ZZZYZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZZ, YZYZZZZZ, YYYZZZZZ, YYYZZYZZ, YYYZZYZZ and YYYZZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZZZ, YZYZYZZZ, YYYYYZZZ and YYYZYYYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second or fourth

vertex of  $H$ , then doing an X-Kempe change there yields  $YYYZYZZZ$ ,  $YZYZYZYZ$  and  $YYYYYZYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 4\ 2\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 8\ 9\ 6\ 5\ 3\ 4\ 2\ 7)$ ,  $(5\ 2\ 9\ 4\ 1\ 3\ 6\ 8\ 7)$ ,  $(5\ 6\ 7\ 8\ 1\ 3\ 4\ 2\ 9)$  and  $(5\ 8\ 7\ 6\ 1\ 3\ 4\ 2\ 9)$ , we have also handled  $YZYZZZZZ$ ,  $ZYZZYZYZ$ ,  $ZZYZYZYZ$ ,  $YZZZYZZZ$  and  $YZZZYZZY$ .

For  $XYYZZZZZ$ , if the X-path starting at the ninth vertex ends at the fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XZZYZZYZ$ ,  $XZZYZYZY$ ,  $XZZYZYZY$  and  $XZZYZYZY$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XZYZZZZZ$ ,  $XYZZZZZZ$ ,  $XYZZZZYZ$ ,  $XYZZZZYZ$  and  $XYZZZZYZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the second, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $XZYZZYZZ$ ,  $XYZZZZYZ$  and  $XYZZZZYZ$  respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields  $XZYZZYZZ$  and  $XYZZZZYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 8\ 9\ 2\ 5\ 3\ 7\ 4\ 6)$ ,  $(3\ 1\ 4\ 2\ 5\ 6\ 7\ 8\ 9)$ ,  $(3\ 2\ 4\ 1\ 5\ 6\ 7\ 8\ 9)$ ,  $(3\ 5\ 7\ 1\ 8\ 4\ 9\ 6\ 2)$ ,  $(4\ 2\ 3\ 1\ 5\ 6\ 7\ 8\ 9)$ ,  $(5\ 2\ 9\ 4\ 1\ 3\ 6\ 7\ 8)$ ,  $(5\ 8\ 6\ 2\ 1\ 3\ 9\ 7\ 4)$ ,  $(5\ 8\ 7\ 6\ 1\ 3\ 4\ 2\ 9)$ ,  $(6\ 8\ 7\ 5\ 1\ 3\ 4\ 2\ 9)$ ,  $(7\ 1\ 3\ 4\ 2\ 5\ 9\ 8\ 6)$ ,  $(7\ 5\ 6\ 2\ 1\ 3\ 9\ 4\ 8)$ ,  $(7\ 6\ 8\ 2\ 1\ 3\ 9\ 4\ 5)$ ,  $(9\ 3\ 2\ 1\ 4\ 6\ 8\ 7\ 5)$  and  $(9\ 6\ 8\ 1\ 7\ 2\ 3\ 4\ 5)$ , we have also handled  $XYZYZZZZ$ ,  $XYZZYZYZ$ ,  $YZXYYZZZ$ ,  $ZYXYYZZZ$ ,  $ZZXZYZZY$ ,  $ZYXYYZZZ$ ,  $ZZYXXYYZ$ ,  $YZZXZYZZ$ ,  $YZZXZYZZ$ ,  $YZZXZYZZ$ ,  $XXXZZYZZ$ ,  $XZZXZYZZ$ ,  $YZZZYXYZ$ ,  $YXXXYYYZ$  and  $XXXXYYYZ$ .

For  $YYXZZZZZ$ , if the X-path starting at the ninth vertex ends at the fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZXZZYZZ$ ,  $ZZXZZYZZ$ ,  $ZZXZZYZZ$  and  $ZZXZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $ZYXZZYZZ$ ,  $YZXZZYZZ$ ,  $YYXZZYZZ$ ,  $YYXZZYZZ$  and  $YYXZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYXZZYZZ$ ,  $YYXZZYZZ$  and  $YYXZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYXZZYZZ$ ,  $YZXZZYZZ$  and  $YYXZZYZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 7\ 6\ 1\ 3\ 4\ 2\ 9)$ , we have also handled  $YZZZYZZY$ .

For  $XYXZZZZZ$ , if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XZXZZYZZ$ ,  $XZXZZYZZ$ ,  $XZXZZYZZ$  and  $XZXZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $XYXZZYZZ$ ,  $XZXZZYZZ$ ,  $XYXZZYZZ$ ,  $XYXZZYZZ$  and  $XYXZZYZZ$  re-



spectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields  $XYXZYZYZZ$ ,  $XZXZYZYZZ$ ,  $XYXYZYZZ$  and  $XYXZZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $YXYZZZZZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 2\ 9\ 3\ 5\ 6\ 7\ 8\ 4)$ ,  $(1\ 3\ 4\ 2\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 4\ 3\ 2\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 6\ 9\ 3\ 4\ 5\ 8\ 7\ 2)$ ,  $(1\ 7\ 9\ 3\ 4\ 5\ 8\ 6\ 2)$ ,  $(2\ 1\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(2\ 1\ 9\ 3\ 5\ 6\ 7\ 8\ 4)$ ,  $(2\ 3\ 4\ 1\ 5\ 6\ 7\ 8\ 9)$ ,  $(2\ 4\ 9\ 1\ 5\ 6\ 7\ 8\ 3)$ ,  $(3\ 1\ 4\ 2\ 5\ 6\ 7\ 8\ 9)$ ,  $(3\ 2\ 4\ 1\ 5\ 6\ 7\ 8\ 9)$ ,  $(5\ 3\ 9\ 4\ 6\ 1\ 8\ 7\ 2)$ ,  $(5\ 4\ 9\ 3\ 6\ 1\ 8\ 7\ 2)$ ,  $(5\ 6\ 7\ 8\ 1\ 3\ 4\ 2\ 9)$ ,  $(5\ 7\ 6\ 2\ 1\ 3\ 9\ 8\ 4)$ ,  $(5\ 8\ 6\ 2\ 1\ 3\ 9\ 7\ 4)$ ,  $(5\ 8\ 7\ 6\ 1\ 3\ 4\ 2\ 9)$ ,  $(5\ 8\ 9\ 7\ 1\ 3\ 4\ 2\ 6)$ ,  $(6\ 5\ 7\ 8\ 1\ 3\ 4\ 2\ 9)$ ,  $(6\ 5\ 8\ 2\ 1\ 3\ 9\ 7\ 4)$ ,  $(6\ 7\ 8\ 2\ 1\ 3\ 9\ 5\ 4)$ ,  $(6\ 8\ 7\ 5\ 1\ 3\ 4\ 2\ 9)$ ,  $(8\ 1\ 9\ 5\ 6\ 3\ 4\ 2\ 7)$  and  $(8\ 1\ 9\ 6\ 5\ 3\ 4\ 2\ 7)$ , we have also handled  $XYZXYZZZZ$ ,  $ZXYXXYYYZ$ ,  $XZYXYZZZZ$ ,  $XZXYYZZZZ$ ,  $ZYYXXYYYZ$ ,  $ZYYXXYYYZ$ ,  $YXZXYZZZZ$ ,  $XZYXXYYYZ$ ,  $ZXYXYZZZZ$ ,  $YZYXXYYYZ$ ,  $YZXXYZZZZ$ ,  $ZYXXYZZZZ$ ,  $YYXYZXYYZ$ ,  $YYYXZXYYZ$ ,  $XZZZYXYZZ$ ,  $YZZZXXYYZ$ ,  $YZZZXXZYZ$ ,  $YZZZXZXYZ$ ,  $YXXXZXXYZ$ ,  $XZZZXYYZZ$ ,  $XZZZXYYYZ$ ,  $XZZZZYXYZ$ ,  $YZZZZXXYZ$ ,  $XYYYXXYYZ$  and  $XYYYXXYYZ$ .

For  $ZZZYZZZZ$ , if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZYZYZZ$ ,  $YZZZYZYZZ$ ,  $ZYZZYZYZZ$ ,  $ZZYZZZYZZ$ ,  $ZZZZZYZZ$ ,  $ZZZYZZYZ$ ,  $ZZZYZZYZ$  and  $YYYZZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYYZZZYZZ$ ,  $YYYZZZYZZ$ ,  $YYYZZZYZZ$  and  $YYYZZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third, fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZYZZZZ$ ,  $ZYZYZZZZ$ ,  $ZZYZZZZZ$ ,  $ZZZZZZZZ$ ,  $ZZZYZYZZ$  and  $ZZZYZZYZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, third or fourth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZYZZZZ$ ,  $ZYZYZZZZ$ ,  $ZZYZZZZZ$  and  $ZZZYZZZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(2\ 5\ 7\ 6\ 1\ 4\ 3\ 8\ 9)$ , we have also handled  $YZZZZYZZ$ .

For  $ZZXYYZZZ$ , if the X-path starting at the ninth vertex ends at the fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYXYZYZZ$ ,  $YYXZZZYZZ$ ,  $YYXZZZYZZ$  and  $YYXZZZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YZXYZZZZ$ ,  $ZYXYZZZZ$ ,  $ZZXZYZZZ$ ,  $ZZXZYZZZ$  and  $ZZXYYZZZ$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYZZZZZ$ ,  $ZZXYYZZZ$  and  $ZZXYYZZZ$  respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields  $YZXYZYZZ$ ,  $ZYXYZYZZ$  and  $ZZXZYZYZZ$  respectively, which are handled by Case 1.

Since we already handled the permutation of all resulting boards by (5 8 7 6 1 3 4 2 9), we have also handled XZZZZXYZZ.

For YYYYYZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZYYYZ, ZYZZZYYYZ, ZZZYZYYYZ, ZZZZYYYYZ, ZZZZZYYYZ, ZZZZZYZYZ and ZZZZZYZZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZZZ, YZYZZZZZ, YYYZZZZZ, YYYZYZZZ, YYYZZYZZ and YYYZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZZ, YZYZZZZZ, YYYZYZZZ and YYYYYYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZYZZ, YZYZZYZZ and YYYZYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 6 7 8 1 3 4 2 9), we have also handled YZZZYYYYZ.

For YYXYZZZZ, if the X-path starting at the ninth vertex ends at the fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZXZYZZZ, ZZXZZYYZ, ZZXZZYZZ and ZZXZZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYXYYZZZ, YZXYYZZZ, YXXZYZZZ, YXXYZYZZ and YXXYYYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYZZZZZ, YXXYZYZZ and YXXZZYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first, second or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZYXYYZZZ, YZXYYYZZ and YXXZYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 3 9 6 4 1 8 7 5), (2 3 9 7 4 1 8 6 5), (3 8 9 6 7 1 2 4 5), (4 8 9 6 7 1 2 3 5) and (5 6 7 8 1 3 4 2 9), we have also handled YXXXYYYZ, YXXXYYXZ, XXYXXYYZ, XXXYXXYZ and YZZZYXXZ.

For YYYXYZZZZ, if the X-path starting at the ninth vertex ends at the fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYYYZ, ZZZXZZYZ, ZZZXZYZZ and ZZZXZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the first, second, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXXYZZZZ, XZXZZZZZ, YYYXZYZZ, YYYXZZYZ and YYYXZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, second or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYXYYZZ, YZYXYYZZ and YYYXYYYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first or second vertex of  $H$ , then doing an X-Kempe change there yields ZYYXZYZZ and YZYXZYZZ respectively, which are handled by Case 1. Since we already handled the

permutation of all resulting boards by (1 2 6 3 9 4 8 7 5), (1 6 3 7 2 5 9 8 4), (2 8 9 1 5 3 7 4 6), (2 8 9 4 5 6 7 1 3), (2 8 9 5 1 3 6 7 4), (2 8 9 6 1 3 4 5 7) and (5 8 7 6 1 3 4 2 9), we have also handled ZZXYYZYYZ, XXXZZXYZZ, XZYYZYYZZ, YZYXZYZZ, ZZYXYYYZ, ZZZYXYZZ and YZZZYXYZ.

For YYXXYZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZXXZYZZ, ZZXXZZYZ, ZZXXZYZZ and ZZXXZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYXXYYZZ, ZYXXYYZZ, YZXXYYZZ, YYXXZYZZ and YYXXYYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second or fifth vertex of  $H$ , then doing an X-Kempe change there yields YYXXZYZZ, ZYXXZYZZ, YZXXZYZZ and YYXXZZYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XXYYZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 2 9 3 5 8 7 4), (1 7 2 9 3 5 8 6 4), (1 7 5 9 8 3 4 2 6), (2 3 1 9 5 6 7 8 4), (3 5 8 9 7 1 2 4 6), (3 6 2 9 4 5 8 7 1), (3 6 8 9 7 1 2 4 5), (3 7 2 9 4 5 8 6 1), (4 5 8 9 7 1 2 3 6), (4 6 8 9 7 1 2 3 5) and (5 8 6 7 1 3 4 2 9), we have also handled XZXYYXYZ, XZXYYXYZ, YXXXZXYYZ, ZXXYXYYZ, XXYXYXYZ, YZXXYXYZ, XXYXXYYZ, YZXXYXYZ, XXXYXYYZ, XXXYXYYZ and YZZZYXXY.

For XYXXYZZZ, if the X-path starting at the seventh vertex ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYXXZZYZ, XYXXYYYZ, XYXXZYZZ and XZXXZYZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZXXYYYZ, XZXXZZYZ and XZXXZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 4 6 8 3 7 5 9), (1 2 5 9 8 3 4 7 6), (1 2 6 7 5 3 4 8 9), (1 2 6 7 8 3 4 5 9), (1 3 2 9 4 6 5 7 8), (1 3 4 6 8 2 7 5 9), (1 5 3 4 8 2 7 6 9), (1 5 4 6 8 2 7 3 9), (2 1 4 6 8 3 7 5 9), (2 3 4 6 8 1 7 5 9), (2 5 4 6 8 1 7 3 9), (2 9 3 4 1 6 5 7 8), (3 1 4 5 8 2 6 7 9), (3 2 4 5 8 1 6 7 9), (4 1 5 6 2 7 3 8 9), (4 1 6 8 2 5 3 7 9), (4 2 5 6 7 1 3 8 9), (4 2 5 6 8 1 3 7 9), (4 2 6 8 5 1 3 7 9), (4 2 6 8 7 1 3 5 9), (5 1 6 7 2 4 3 8 9), (5 1 6 7 8 2 3 4 9), (5 7 8 9 6 3 1 4 2) and (6 9 7 8 5 3 1 4 2), we have also handled XYZXZXZY, ZYXXZXXY, YXZZXYZZ, XYZZZXXY, ZZXXYYYZ, XZYXZXZY, XZXXYZZY, XZZXYXZY, YXZXZXZY, ZXYXZXZY, ZXZXXYZY, ZXXXYYYZ, YZXXXZZY, ZYXXXZZY, XXZYYYZZ, XXZYZYZZ, ZYZZXXYZ, ZYZZXXYZ, ZXZYXYZY, ZXZYZYXY, XXZZYYYZ, YZZZXXYZ, XXXXZYZZ and XXXXZYZZ.

For YXXXYZZZ, if the X-path starting at the ninth vertex ends at the sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXZZYZ, ZXXXZYZZ and ZXXXZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, fifth or seventh vertex of  $H$ , then doing an X-Kempe

change there yields ZXXXYYZZZ, YXXXZYZZZ and YXXXYYZZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the first or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXYZYZZ and YXXXZZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 5 8 9 6 3 4 2 7), (1 6 7 8 5 3 4 2 9), (3 5 6 7 9 2 8 4 1), (4 1 2 9 5 6 7 8 3) and (7 1 3 4 9 2 8 6 5), we have also handled XYYYZXZZZ, XZZZXYYYZ, YYZYXXXYZ, ZZYXXYYYZ and XYXXYYZYZ.

For YYZZZYZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYYZYZZ, ZYYYZYZZ, ZZZYYZYZZ, ZZYZYZYZZ, ZZYZZZYZZ, ZZYYYYYYZ and ZZYYYZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZZZ, ZYYZZYZZZ, YZYZZYZZZ, YYYZYZZZ, YYYZYZZZ, YYYZZZZZ, YYYZZYZZ and YYYZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZZYZZZ, which is handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled ZZZYZZYZ.

For XYZZZYZZZ, if the X-path starting at the ninth vertex ends at the third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZYZZ, XZYZYZYZZ, XZYZZZYZZ, XZYYYYYYZ and XZYYYZYZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the third, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XYYYZYZZZ, XYZYYYYZZ, XYZYZZZZZ, XYZYZYZZ and XYZYZYZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XYZZYZZZ, XYZZZZZZ and XYZZYZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 4 6 7 1 8 3 2 9), (9 3 1 5 2 4 6 7 8), (9 3 1 5 2 6 7 4 8), (9 3 1 5 2 7 6 4 8), (9 4 1 5 2 6 7 3 8), (9 4 1 5 2 7 6 3 8) and (9 6 1 2 3 7 4 8 5), we have also handled ZZZYXZZYZ, YYXXYYYYZ, YYXYXYYZ, YYXYYYXYZ, YYYXYXYYZ, YYYXYXYYZ and YYYYYXXYZ.

For ZYZZZYZZZ, if the X-path starting at the ninth vertex ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZXYZYZZ, YZYXZZYZ, YZYXYYYYZ and YZYXYZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYXZYZZZ, ZZYXZYZZZ, ZYXXYZZZ, ZXXYZZZZZ, ZYXXZYZZZ and ZYXXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYYZZZ, ZZZYZZZZZ and ZZZXZYZZZ respectively, which are handled by Case 1. Since we already

handled the permutation of all resulting boards by (1 4 8 3 2 9 7 5 6), (1 6 2 7 5 9 3 8 4), (1 8 7 6 2 4 3 5 9), (4 1 9 7 5 2 6 8 3) and (4 5 9 3 1 8 6 2 7), we have also handled YXZYZZZZ, ZZZZYZZZZ, XXXZZZZZZ and ZZZZZZZZ.

For YYZYZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZYZZY, ZYZZYZZY, ZZZZYZZY, ZZYZZYZZ, ZZYZZZZY, ZZYZZYYY and ZZYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZZZ, ZYYZYZZZ, YZZZYZZZ, YYYZZZZZ, YYYZZZZZ, YYYZZYZZ and YYYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZZYZZZ, YZZYYYZZ, YZZYZZZZ and YZZZYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZYZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), (9 3 1 5 7 6 4 8 2) and (9 7 5 1 2 4 6 3 8), we have also handled ZZZYYYZZ, YYYZZZZY and ZYYZZYZZ.

For YYXZYZZZZ, if the X-path starting at the ninth vertex ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYZYZZ, ZZYXZZYZ, ZZYXYYYZZ and ZZYXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYYXZYZZZ, YZYXZYZZZ, YYYXYYZZZ, XXXYZZZZZ, YYYXZYZZZ and YYYXZYZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZXYYZZZ, XZZYZZZZZ and YZZXZYZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 5 9 3 4 8 2 6 7), (1 9 5 3 2 7 6 4 8), (1 9 8 3 4 2 6 7 5), (2 5 9 3 4 8 1 6 7), (2 9 5 3 1 7 6 4 8), (3 9 1 7 5 6 4 8 2), (3 9 1 7 5 8 4 6 2), (3 9 1 7 6 5 4 8 2), (4 1 9 7 5 2 6 8 3), (4 5 9 3 1 8 6 2 7), (4 6 1 7 5 9 3 8 2), (4 6 5 3 1 9 7 2 8), (4 7 5 3 1 9 6 2 8), (4 8 7 6 1 5 3 2 9), (5 1 9 7 4 2 8 6 3), (5 9 2 7 3 8 4 6 1), (6 1 9 7 5 2 4 8 3), (6 5 9 3 1 8 4 2 7) and (8 1 9 7 4 2 5 6 3), we have also handled YZXZYZZYZ, ZYXYYYZYZ, ZZXYYYYYZ, ZYXZYZZYZ, YZXYYYZYZ, YYZYYZXYZ, XXZXXXYZZ, YYZYZYXYZ, XXZXXZYZZ, ZZXYYZZYZ, YYYZYZXYZ, YXXZYZYZZ, YXXZYZYZZ, ZZZYXZYZ, XXXZXZYZZ, XXXXZXYZZ, XXXZZXYZZ, ZZXZYZYZZ and YYZZZZXYZ.

For YZZZZYYZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZZYYZ, YZYZZZZYYZ, ZYYYZZYYZ, ZZZYZZYYZ, ZZYZZZZYYZ, ZZYYYZYYZ and ZZYYZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYYZZZ, ZYYZYZZZZ, YZYZYZZZZ,

YYYYYYZZZZ, YYYZZYZZZ and YYYZYYYZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYYZZZ, ZZZZYZZZ, ZYZYYYZZZ, ZYZZZYZZZ and ZYZZYYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 6 4 8 7 5 9), (3 4 1 2 6 7 8 9 5), (3 9 1 6 7 5 8 4 2), (5 8 7 3 2 6 4 1 9), (5 8 7 6 1 4 3 2 9) and (7 9 5 2 3 1 4 6 8), we have also handled YYZYZZZYZ, ZZYYZYZZZ, YYZYZYZYZ, ZYZZYYZYZ, YZZYYZZYZ and ZYZYYYZYZ.

For XYZZYYZZZ, if the X-path starting at the ninth vertex ends at the second, third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYYZZYYZ, XZZYZZYYZ, XZYZZZYZZ, XZYZZZYZZ and XZYZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the second, third, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZYYYZZZ, XYYYZZZZZ, XYZYZZZZZ and XYZYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZYZYZZZ, XYZZZYZZZ and XYZZYYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 1 6 7 8 4 3 2 9), we have also handled YZZYXZZYZ.

For YYZZXYZZZ, if the X-path starting at the third vertex ends at the first, second, fourth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZYYZXZZZ, YZYXZZZZ, YYYXZZZZ, YYYZXZZZ, YYYZXZZZ and ZZZYXZZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the first, second, fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZYXZZYZ, ZYYYXZZYZ, ZZYXZZYZ and ZZYXZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXZZZ and YZZYXZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields ZYZYXZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled XZZYZZYZ.

For XYZZXYZZZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the second, third, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYZYXZZZ, XZZYXZZZ, XYYYXZZZ, XYZYXZZZ, XYZYXZZZ and XZYXZZYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the third or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZYXZZYZ, XZZYXZZYZ and XZYXZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the second, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XYZZXZZZ, XZYXZZZ, XYZZXZZZ and XYZZXZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by

(1 2 4 9 7 5 8 6 3), (1 2 4 9 7 8 5 6 3), (1 8 6 7 5 4 3 2 9), (3 1 6 9 5 8 2 4 7), (3 2 6 9 5 8 1 4 7), (4 1 2 5 9 6 7 3 8), (4 1 2 5 9 7 6 3 8), (4 3 2 5 9 6 7 1 8), (4 3 2 5 9 7 6 1 8), (6 3 1 8 9 5 4 7 2), (6 3 1 8 9 7 4 5 2), (6 4 1 8 9 5 3 7 2) and (6 4 1 8 9 7 3 5 2), we have also handled YXZZXZYZZ, XYZZZZXYZ, XZZYXZZYZ, YZXZXZZYZ, ZYXZXZZYZ, XYYZYXYYZ, XYYZYXYXZ, YYXZYXYYZ, YYXZYXYXZ, YYXYXZYYZ, YYXYYZXYZ, YYYXXZYYZ and YYYXYZXYZ.

For ZYZXYZZZ, if the X-path starting at the ninth vertex ends at the first, second, third, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYXZZYYZ, YYYXZZYYZ, YZZXZZYYZ, YZYXYZZYZ and YZYXZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYXYZZZ, ZZYXYZZZ, ZYYXZYZZZ and ZYYXYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZXYZZZ, ZZZXZYZZZ and ZZZXYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 8 7 6 1 4 5 2 9), (3 8 7 6 2 5 4 1 9) and (6 2 3 4 1 8 7 5 9), we have also handled YZZYZXZYZ, ZYZZYXZYZ and YYZXXXXYZ.

For YYZYZZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZYZ, YZYZZZZYZ, ZYYZZZZYZ, ZZZZZZZYZ, ZZYYZZZZYZ, ZZYZZZZYZ and ZZYZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZZ, ZYYYYZZZ, ZYYYYYZZZ, YYYZYZZZ, YYYZYZZZ and YYYYYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the first, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZYYYZZZ, ZZZYYYZZZ, YZZYZZZZ and YZZYYYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZYYYZZZ, ZYZYZZZZ and ZYZYYYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled YZZYYYZZZ.

For YYZXYYZZZ, if the X-path starting at the ninth vertex ends at the first, second, third, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYXZZZZYZ, ZYXZZZZYZ, ZZZXZZZZYZ, ZZYXYZZYZ and ZZYXZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYXYZZZ, ZYXYYZZZ, YYYXZYZZZ and YYYXYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYZZZ, YZZXZYZZZ and YZZXYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 8 5 9 6 7 4), (1 2 3 8 6 9 5 7 4), (1 5 6 2 8 9 4 3 7), (1 6 5 2 7 3 4 9 8), (1 7 5 2 4 9 3 6 8), (1 8 7 6 5 4 3 2 9), (1 9 5 3 2 7 4 6 8), (1 9 5 3 4 7 2 6 8), (2 1 3 5 4 6 8 7 9), (2 1 3 5 8 6 4 7 9), (2 1 3 7 4 5 6 8 9), (2 1 3 7 4 8 6 5 9), (2 1 3 7 6 5 4 8 9), (2 1 3 7 6 8 4

5 9), (2 5 7 1 8 4 6 3 9), (2 9 5 3 4 7 1 6 8), (3 7 5 4 9 2 1 6 8), (3 8 7 6 4 1 5 2 9), (3 9 1 7 5 8 6 4 2), (4 5 7 3 8 1 6 2 9), (4 5 7 3 8 2 6 1 9), (5 3 1 8 6 9 7 4 2), (6 2 3 4 7 5 1 8 9), (6 5 7 1 8 4 2 3 9) and (7 3 1 6 9 8 5 4 2), we have also handled ZZXXZXXYZ, ZZZXXZXYZ, ZXYZZYZZ, YXYZZYZZ, ZXYZZYZZ, YZZYXXYZ, ZZXYYYZZ, ZYXZZYZZ, YYZYXYZZ, YYZZXYZZ, XXZXXZZZ, YYZZZZXYZ, XXZZXXYZ, YYZZZYXYZ, XYZYZZYZ, YZXZZYZZ, YZZXYZZZ, YZYZZXZZ, XXZXXZZZ, YZXYYZZZ, ZYXYYZZZ, XXZXXZZZ, ZYXYYZZZ, XZZYZZYZ and XXZXXZZZ.

For YYZZZZYZZ, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZYZZ, ZYYZZZZYZZ, YZYZZZZYZZ, YYYZZZZYZZ, YYYZZYZZZ, YYYZZYZZZ, YYYZZZZZZ and YYYZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the fifth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZZYZZ, ZZYYYYYZZ and ZZYYYYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZYZZ, ZZZZZYZZ, ZYZYZZZZ, ZYZZYZYZZ, ZYZZYZYZZ, ZYZZZZZZZ and ZYZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled ZZYZYZYZZ.

For ZZYYZZYZZ, if the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZZYZZ, ZYZZZZYZZ, ZYZZZZYZZ, ZZZYZZZYZZ, ZZZZZZZYZZ, ZZZYYZZYZZ and ZZYYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, third, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZZZZZ, ZYZZZZZZZ, ZZZYZZZZZZ and ZZYYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZYYZZYZZ, ZYZZZZYZZ, ZZZYZYZZ, ZZZYZYZZ and ZZYYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZYZYZZ, YZZYZYZZ, ZYZZYZYZZ, ZZZZZYZZ and ZZZYZYZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZYZZ, YYYZZYZZ and YZYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 8 7 6 5 4 3 2 9), we have also handled ZZYZYZYZZ.

For YYZZXZZYZZ, if the X-path starting at the ninth vertex ends at the first, sec-



ond, third, fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YZYXYZYZ$ ,  $ZYXYZYZY$ ,  $ZZZYXYZY$ ,  $ZZYZXYZY$  and  $ZZYXYZZZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the first, second, fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYZXZYZ$ ,  $YZYZXZYZ$ ,  $YYYYXZYZ$  and  $YYYZXYYZ$  respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first or fourth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZZXZYZ$  and  $YZZYXZYZ$  respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYZYXZYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 7\ 6\ 1\ 4\ 3\ 2\ 9)$ , we have also handled  $XZYZYZYZ$ .

For XXXYZYZZZ, if the X-path starting at the seventh vertex ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXXYYYZZZ, XXXYZZZZZ, XXXYZZYZZ and XXXZYZYZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fifth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXXYYZYZZ and XXXYZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XXXYYYYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 8 5 7 9), (1 2 9 4 3 5 8 6 7), (1 2 9 4 5 6 3 8 7), (1 2 9 4 5 7 3 8 6), (1 3 4 2 5 7 9 6 8), (1 3 4 2 6 5 8 7 9), (1 4 6 2 3 7 9 8 5), (1 4 6 2 7 3 8 9 5), (1 4 6 2 7 5 8 9 3), (1 5 9 3 2 4 6 8 7), (1 5 9 3 4 2 8 6 7), (1 6 9 5 2 3 4 8 7), (1 6 9 5 3 2 8 4 7), (2 3 4 1 5 6 9 8 7), (2 3 4 1 5 7 9 8 6), (2 3 4 1 6 5 8 9 7), (2 4 6 1 3 7 9 8 5), (2 4 6 1 7 3 8 9 5), (2 4 6 1 7 5 8 9 3), (4 5 6 1 2 3 9 7 8), (4 5 6 1 3 7 2 9 8), (4 5 6 1 3 8 2 9 7), (4 5 9 1 2 7 6 8 3), (4 5 9 1 6 8 2 7 3), (4 6 8 1 2 3 9 5 7), (4 6 8 1 3 5 2 9 7), (4 6 8 1 3 7 2 9 5), (5 6 7 2 3 4 1 9 8), (5 6 7 2 3 8 1 9 4), (5 7 8 6 3 2 1 4 9), (5 8 9 3 1 4 7 2 6), (5 8 9 3 1 6 7 2 4), (6 7 8 3 1 4 9 2 5), (6 7 8 3 1 5 9 2 4) and (6 7 8 3 4 1 2 9 5), we have also handled XXXYZZZYZZ, ZZXYXXXYZ, ZZXXYXYYZ, ZZXXYXXYZ, XZXXYZZYZ, XYXXYZZYZ, XZYXXYZYZ, XYXXZXZYZ, XYZXXYZYZ, ZYXXZXYYZ, ZYXXZXYYZ, ZYXXXZYZZ, ZYXXYZXXYZ, ZXXXYZYYZ, ZXXXYYZZYZ, YXXXYZZZYZ, ZYXXYZYZ.

YXYXZXZYZ, YXZXXYXZYZ, ZYZXXXYYZ, YYZXXXYYZ, YYZXXXZYZ, XYYZZXXYZ, YXZZXXYZ, ZXZYXYXZY, XXZYXYZY, XXZYZYXYZ, XXZXYYYZZ, YYZZXXXYZ, XXZZYXYYZ, XXYYZXYZZ, XXYXZYZZ, XXZZXYYYZ, XXZXZYZZ and XXXZZYYYZ.

For YZZZYZZZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZZZY, ZYZZZZZY, ZYZZZZZY, ZZZYZZZY, ZZYZZZZY, ZZYZZZYZ, ZZYZZZZY and ZZYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZZZ, ZYYZYZZZ, YZYZYZZZ, YYYZZZZZ, YYYZZZYZ, YYYZYZZZ and YYYZYZZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  or ends at the second, fourth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYZZYYYZ, ZZZZYZZZ, ZYZZYYYZ, ZYZZZYZZ and ZYZZYYYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYYYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 3 4 2 9), we have also handled YZZYYYZZ.

For XYZZZYZZ, if the X-path starting at the ninth vertex ends at the third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZYZZZY, XZYZZZZY, XZYZZZYZ, XZYZZZZY and XZYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the second, third, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZYYYZZ, XYYYZZZZ, XYZZZYZZ, XYZZZYZZ, XYZZYYYZ and XYZZYYYZ respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XYYZYZZZ and XYYZYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 1 7 6 3 4 8 2 9) and (9 3 8 1 4 6 7 2 5), we have also handled YZYXZZZY and YXXYXXYZ.

For YZZZXYYZ, if the X-path starting at the third vertex ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZXYYZ, ZYZZXYYZ, YYYXYYZ, YYYZXZZZ, YYYZXZZZ and YYYZXYYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZYXZYZZ and ZZYXZYZZ respectively, which are handled by Case 1. If the X-path starting at the first vertex ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZXYYZ, ZYZZXYYZ, ZYZZXZZZ, ZYZZXZZZ and ZYZZXYYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZXYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 9 4 7 5 8 6 3), (5 8 7 6 1 3 4 2 9) and (5 8 9 6 3 1 2 4 7), we have also handled YZZZYXZY, XZYZZZZY and YXXZYZZY.

For  $YYZXYYYYZZ$ , if the X-path starting at the ninth vertex ends at the second, third, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYXZZZZYZ$ ,  $ZZZXZZZZYZ$ ,  $ZZYXZYZZYZ$ ,  $ZZYXZZZZYZ$  and  $ZZXYZZZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the second, fifth, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YZYXYYYYZZ$ ,  $YYYYXZYZZZ$ ,  $YYYYXYZZYZ$  and  $YYYYXYZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the first, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZXYYYYZZ$ ,  $YZZXZYZZYZ$ ,  $YZZXZYZZYZ$ ,  $YZZXYYZZZZ$  and  $YZZXYYYYYZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 5\ 7\ 2\ 4\ 3\ 8\ 6\ 9)$ ,  $(1\ 8\ 7\ 6\ 3\ 4\ 5\ 2\ 9)$ ,  $(4\ 5\ 7\ 3\ 1\ 2\ 8\ 6\ 9)$ ,  $(5\ 1\ 3\ 8\ 6\ 2\ 7\ 4\ 9)$  and  $(6\ 1\ 3\ 7\ 2\ 5\ 8\ 4\ 9)$ , we have also handled  $YXYYYZZYZ$ ,  $YZYYXZYZ$ ,  $YYXYZZYZ$ ,  $XXZZXXXYZ$  and  $YYZZYXYZ$ .

For  $YYZZZZZZYZ$ , if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYYYYYYZZ$ ,  $YZYYYYYZZ$ ,  $ZYYYYYZZ$ ,  $ZZYYYYYYZZ$ ,  $ZZYZYYYYZZ$ ,  $ZZYYYYZZYZ$ ,  $ZZYYYYZZZZ$  and  $ZZYYYYYYYZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYYZZZZYZZ$ ,  $ZYYZZZZYZZ$ ,  $YZYZZZZYZZ$ ,  $YYYYZZZZYZ$ ,  $YYYZZYZYZ$ ,  $YYYZZZZYZZ$  and  $YYYZZZZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $ZYZZZZZZYZ$ , which is handled by Case 1. If the X-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields  $YZZYZZZZYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 7\ 6\ 1\ 3\ 4\ 2\ 9)$ , we have also handled  $ZYZZYZZYZ$ .

For  $XYZZZZZZYZ$ , if the X-path starting at the ninth vertex ends at the third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XZZYYYYYZZ$ ,  $XZYZYZZYZZ$ ,  $XZYZZYZYZZ$ ,  $XZYZZYZZZZ$  and  $XZYZZYYYZZ$  respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the third, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XYYYZZZZYZ$ ,  $XYZYZYZYZZ$ ,  $XYZYZZZZYZ$  and  $XYZYZZZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XYYZZYZYZZ$ ,  $XYYZZZZYZZ$  and  $XYYZZZZZZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 2\ 6\ 7\ 1\ 3\ 4\ 8\ 9)$ , we have also handled  $ZYZZXZZYZ$ .

For  $ZYZXZZZZYZ$ , if the X-path starting at the ninth vertex ends at the second, third, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYYXYYYYZZ$ ,  $YZZXYYYYZZ$ ,  $YZYXZYZZYZ$ ,  $YZYXYYZZZZ$  and  $YZYXYYYYYZ$  respectively, which are handled by Case 1. If the X-path starting at the third vertex ends at the second, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYXZZZZYZ$ ,  $ZYYXZYZZYZ$ ,  $ZYYXZZZZYZ$  and  $ZXXYZZZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the sixth, seventh or eighth vertex of  $H$ ,

then doing an X-Kempe change there yields  $ZZZXZYZY$ ,  $ZZXZZYYZ$  and  $ZZZYZZZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 8\ 7\ 6\ 3\ 4\ 5\ 2\ 9)$ , we have also handled  $ZYZZZXZY$ .

For  $ZYYZYZZY$ , if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYZYZY$ ,  $ZZYZZY$ ,  $ZYZZZY$ ,  $ZYYYYZY$ ,  $ZYYZZZY$ ,  $ZYYZYYYY$ ,  $ZYYZYZZZ$  and  $YZZYZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YZZYZY$ ,  $YZYYZY$ ,  $YZZY$ ,  $YZZYZZ$  and  $YZZYZYYY$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third, fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYZZZY$ ,  $ZZYZZZY$ ,  $ZYZZZZY$ ,  $ZYYYZZY$ ,  $ZYYZZZY$  and  $ZYYZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZYYZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(5\ 6\ 8\ 7\ 1\ 4\ 3\ 2\ 9)$  and  $(5\ 8\ 7\ 6\ 1\ 4\ 3\ 2\ 9)$ , we have also handled  $ZYZYZZY$ ,  $YYZZZY$  and  $YZZZZY$ .

For  $ZYYZXZZY$ , if the X-path starting at the seventh vertex ends at the second, third, fourth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYZXZY$ ,  $ZYZZXZY$ ,  $ZYYYXZY$ ,  $ZYYZXYYY$ ,  $ZYYZXZZ$  and  $YZZYXZZ$  respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the second, third, fourth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYZXZY$ ,  $ZYZZXZY$ ,  $ZYYYXZY$ ,  $ZYYZXZZ$  and  $YZZYXZZ$  respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the third or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YZYYXZY$  and  $YZZYXZY$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(5\ 2\ 6\ 7\ 1\ 3\ 4\ 8\ 9)$  and  $(5\ 8\ 2\ 6\ 1\ 3\ 4\ 7\ 9)$ , we have also handled  $ZYZXZZY$ ,  $XYZZZY$  and  $XYZZZZY$ .

For  $YYYZYZZY$ , if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYZZY$ ,  $YZZYZY$ ,  $ZYZZY$ ,  $ZZZZZY$ ,  $ZZZY$ ,  $ZZYZZY$  and  $ZZYZYYY$  respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YYYZZZY$ ,  $ZYYZZZY$ ,  $YZYZZZY$ ,  $YYYZZZY$ ,  $YYYZZZY$ ,  $YYYZZZY$  and  $YYYZZZZ$  respectively, which are handled by Case 1. If the X-path starting at the eighth vertex ends at the sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YYYZYZZ$  and  $YYYZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields  $ZYZZY$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 7\ 6\ 1\ 4\ 3\ 2\ 9)$ , we have also handled

YYZZYZYYZ.

For YXXZYZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXXZYZZYZ, YXXYZZYZ, YXXZZZZYZ, YXXZYZZYZ, YXXZYZZYZ and ZXXZYZZYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXXZZZZYZ, YXXYZZYZ, YXXZZYZYZ, XYYZZZZYZ and ZXXYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields YXXZYZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 4 8 5 2 9 3 7 6), (2 7 8 4 1 9 3 5 6), (2 7 8 4 1 9 3 6 5), (3 2 6 1 8 9 7 5 4), (3 6 7 5 9 2 1 4 8), (4 2 3 1 5 6 7 8 9) and (6 3 4 1 9 8 5 7 2), we have also handled XXZYZZXYZ, XXZZXZZYZ, XXZZZXYYZ, ZXYZYXZYZ, YYZZYXXYZ, ZXXYZZYZ and YYXXYZZYZ.

For YYYZXZZYZ, if the X-path starting at the ninth vertex ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZYXYYZ, ZYZYXYYZ, ZZZZXYYZ, ZZZYXZZYZ, ZZZYXYYZ and ZZZYXYYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the first, second, fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZYYZXZZYZ, ZYZZXZZYZ, YYYXZZYZ, YYYZXZZYZ and YYYZXZZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the second or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZYXZZYZ and YYYZXZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 5 7 6 1 4 3 8 9), we have also handled XYZZYZYZ.

For YYXZXZZYZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZXYYZZYZ, ZYXYYZZYZ, ZZXZZYZYZ, ZZXYYZZYZ, ZZXYYZZYZ and ZZXYYZZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYXZZYZYZ, YZXZZYZYZ, YYXYYZZYZ, YYXZZYZYZ and YYXZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields YYXZZYZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 4 3 9 2 8 7 5), (2 5 1 6 7 4 3 8 9), (3 5 6 7 9 1 2 4 8), (3 6 4 1 9 2 8 7 5) and (3 7 6 5 9 1 2 4 8), we have also handled XYYZYXXYZ, XYZZYZXYZ, YYXXXZZYZ, YYXZYXXYZ and YYXXYZXYZ.

For YXZYZZYZ, if the X-path starting at the seventh vertex ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYZZYZ, YXZYZZYZ, YXZYZZYZ and ZXYZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex ends at the fifth, sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYZZYZ, YXZYZZYZ and ZXYZZYZYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends

at the fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYZYZYZ and ZXZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZYZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 3 4 2 9), we have also handled XXZZXXZYZ.

For YXZYXZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYXZYYZ, YXYXZYYZ, YXZYXYYYZ, YXZYXZYZZ and ZXYZXZZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZXYYZZ, ZXZZXYYZZ, ZXYZXZYZZ and ZXYZXYYYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXZYXZYZZ, YXYXZYZZ and YXZYXZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 1 7 6 8 3 4 2 9), we have also handled YXZZXXZYZ.

For ZXXXXZZYZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXXXXYYZZ, YXXXXZYZZ, YXXXXYZZZ and YXXXXYYYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXXYZYZ, ZXXXXYYYZ and ZXXXXYZZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXXZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 1 6 7 8 4 3 2 9), we have also handled YXZZZYZZZ.

For XXYXZZYZ, if the X-path starting at the sixth vertex ends at the fifth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYXZYZYZ, XXYXYYYZZ and XXYXYYZZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the fifth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYXZZYYZ and XXYXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields YYXZZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 6 7 8 1 3 4 2 9), we have also handled XXZZYYXYZ.

For XXXXZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXXXZZYYZ, XXXXZZYYZ, XXXXYYYZZ, XXXXZYZZZ and XXXXZZZZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XXXXZZYZZ, which is handled by Case 1. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XXXXYYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 6 7 8 1 4 3 2 9), we have also handled

XXZZYYYYZ.

For YXYXZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fourth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXYXZYZZ, YXZYXZYZZ, YXYZXZYZZ, YXYXYYYZ, YXYXZYZZ and ZXZZXYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the third, fourth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXYXZYZZ, YXZYXZYZZ, YXYZXZYZZ, YXYXZYZZZ and ZXZZXZYZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXZZXYYZZ and ZXZZXYYYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 1 5 6 8 3 4 7 9), we have also handled YXZZXXXYZ.

For YYXYXZZYZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZXXXYZZZ, ZYXXXYZZZ, ZZXXXYZZZ, ZZXXZYZZ, ZZXXXYZZZ and ZZXXXYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the second, fourth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYXYXZYZZ, YZXXXYZZZ, YYXXZYZZ, YYXYXYYYZ and YYXYXZYZZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields YYXYXZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 3 4 6 9 2 8 7 5), (2 5 1 6 7 4 3 8 9) and (3 5 6 7 9 1 2 4 8), we have also handled YXZZYXXYZ, XYZZYXXYZ and YYXXXZZYZ.

For XYXYXZZYZ, if the X-path starting at the seventh vertex ends at the sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYXYXYYYZ, XYXYXZYZZ and XZXZXYZZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex ends at the sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYXYXZYZZ and XZXZXYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields XZXZXYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 5 7 8 9), (1 2 4 3 5 6 7 8 9), (1 2 5 7 6 3 4 8 9), (1 2 6 5 8 3 4 7 9), (1 5 6 8 7 3 4 2 9), (1 5 7 6 8 3 4 2 9), (1 6 5 8 7 3 4 2 9), (2 1 3 4 5 6 7 8 9), (2 1 3 4 6 5 7 8 9), (2 1 4 3 5 6 7 8 9), (3 1 4 2 5 6 7 8 9), (4 5 7 6 8 1 3 2 9) and (4 6 5 8 7 1 3 2 9), we have also handled XYXYXZYZZ, XYXXZZYZ, XYZ-ZXXYYZ, YXZZXYXYZ, XYZZYXXYZ, YXZZXXYYZ, XYZZXYXYZ, YXXYXZZYZ, YXXYXZYZZ, YXYXXZZYZ, YYXXXZZYZ, ZXZYXXYYZ and ZYZXXXYYZ.

For ZYYZZXZYZ, if the X-path starting at the seventh vertex ends at the second, third, fourth, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZ-ZXYYZ, ZYZZXYYZ, ZYYYZXYYZ, ZYYZYXYYZ, ZYYZZXYZZ and XZZXXYZZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex ends at the second, third, fourth, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZZXXZYZZZ, ZXZZZYZZZ, ZXXXZYZZZ, ZXXZXYZZZ and YZZYXXYYZ.

respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the third or fifth vertex of  $H$ , then doing an X-Kempe change there yields YZYXXYZZ and YZZYZXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 6 7 8 9), (5 6 8 7 1 4 3 2 9) and (5 8 7 6 1 4 3 2 9), we have also handled ZYZYZXYZ, ZYXZYXYZ and ZYXZZXYZ.

For YYYZZXYZ, if the X-path starting at the eighth vertex ends at the first, second, fourth, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXXZZZZZ, XZZZZZZZ, XXXXZZZZ, XXXZYZZZ, YYYZZXYZ and ZZZYXXYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZYXXYZ, ZYZYXXYZ, ZZZYZXYZ, ZZZYZXYZ and ZZZXXYZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZZZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 2 4 6 5 7 8), (4 9 1 2 3 6 5 7 8), (5 8 7 1 6 4 3 2 9), (6 9 5 1 8 4 7 3 2) and (7 9 5 1 6 4 8 3 2), we have also handled ZYZYXXYZ, ZYYZYXYZ, ZYXZYXYZ, YYZZZXYZ and YYZZYXYZ.

For YXZYXZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYXXYZ, YXYXZYXZ, YXZYXXYZ, YXZYXXYZ and ZYXZYXZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the third, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYXZYXZZ, XYXXZYXZ, XYXZYXZZ and ZYXZYXXYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the third or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZYXZYXZZ, ZXXZYXZZ and ZYXZYXZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 4 1 6 7 8 3 2 9), we have also handled ZYXZYXZZ.

For YYYZYXYZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZYZZ, YZZZZYZZ, ZYZZZYZZ, ZZZZZYZZ, ZZZYXYZZ, ZZZYZXYZ, ZZZZZYZZ and ZZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZZ, ZYYZZYZZ, YZZZZYZZ, YYYZZYZZ, YYYZZYZZ, YYYZZYZZ and YYYZZYZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the second, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZYXZZ, YZYZYXZZ, YYYZYXZZ and YYYZYXZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YZYZYXZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 5 7 6 8 9) and (4 5 7 6 1 8 3 2 9), we have also handled



YYYZYZYZZ and YYZYZZYYZ.

For YYYZYXZY, if the X-path starting at the eighth vertex ends at the first, second, fourth, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXZYZZZ, XZXZXZYZZ, XXXXZYZZZ, XXXZZYZZZ, YYYZYXZY and ZZZYZZXYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the first, second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZXYZ, ZYZZZYXYZ, ZZZZZXYZZ, ZZZYXYZZ and ZZZXZYZZZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YZYZZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 5 7 6 8 4 3 2 9), we have also handled YYZYZZYYZ.

For YXZYZZYZ, if the X-path starting at the seventh vertex ends at the third, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZZZZYZ, YXZYZZYZ, YXZZZYZZ, YXZZYYZZ and ZXYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXZZZZYZ, ZXYZZYZZ, ZXYZZYZZ and ZXYZZZYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the third, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXZYZZYZ, YXZZZZYZ and YXZYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 2 3 4 9), we have also handled XXZZZZYZ.

For YYZXXZY, if the X-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYXXZY, ZYYXXZY, YZYXXZY, YYYXXZY, YYYXXZZ and ZZZXXZY respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYXXZY, ZYYXXZY and ZZYXXZY respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the first, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZXXZY, ZZZXXZY, YZZXXYY and YZZXXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 8 7 1 6 4 3 5 9), we have also handled XYZYYXZY.

For YXZYXXZY, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXZYXXZY, YXZYXXZY, YXZYXXZY, YXZYXXZY and ZYXZZZZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the third, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYZXXYZ, XYXXXYZZ, XYZXXYZ and ZXYZZXYZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the third or fifth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZZXYZ, ZXZZXYZZ and ZXYZYXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 4 7 6 1 8 3 2 9), we

For  $XYXZXYZZ$ , if the  $X$ -path starting at the eighth vertex doesn't end in  $H$  or ends at the second, fourth, sixth or ninth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYYZXZYZZ$ ,  $XZYZXZYZZ$ ,  $XYXXZYZZ$ ,  $XYYZXYZZ$  and  $XZZYXYZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the second vertex doesn't end in  $H$  or ends at the fourth, sixth or ninth vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XZYZXZYZZ$ ,  $XZYXXZYZZ$ ,  $XZYZXYYYZ$  and  $XYZYXYZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the ninth vertex doesn't end in  $H$  or ends at the third or seventh vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XZZYXYZZZ$ ,  $XZYXXYZZZ$  and  $XZZYXYZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYYZXYYYZ$  and  $XYYZXYZZZ$  respectively, which are handled by Case 1. If the  $X$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the third vertex of  $H$ , then doing an  $X$ -Kempe change there yields  $XYXXZYZZ$  and  $XYZYXZYZZ$  respectively, which are handled by Case 1.

For XZYZYZZZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YZXZXZYZZ, YZXZYZZZ, YZXZYZZZ, YZXZYZZZ and ZYXZYZZZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZYXZYZZZ, ZYXZYZZZ, ZYXZYZZZ and ZYXZYZZZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YZXZYZZZ, YZXZYZZZ and YZXZYZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 6 7 3 4 8 9), (1 2 5 7 6 3 4 8 9), (1 2 5 7 9 3 4 8 6), (1 2 6 5 7 3 4 8 9), (1 2 6 5 8 3 4 7 9), (2 1 3 4 5 6 7 8 9), (3 4 1 2 5 6 7 8 9), (3 4 6 5 9 1 2 7 8), (4 3 1 6 9 2 5 7 8), (4 3 6 5 9 1 2 7 8), (5 2 1 6 7 3 4 8 9), (5 8 1 2 3 6 7 4 9), (5 8 1 2 4 6 7 3 9), (5 8 1 2 9 4 6 3 7), (5 8 2 1 4 6 7 3 9), (5 8 3 1 4 6 7 2 9), (6 7 4 1 9 2 5 3 8), (6 7 5 3 9 1 2 4 8), (7 6 1 2 5 3 4 8 9), (7 6 4 1 9 2 5 3 8) and (8 2 1 5 7 3 4 6 9), we have also handled XZZZYZZZ, XZZZYZZZ, XYYYZZYYZ, XZZZYZZZ, XZZZYZZZ, ZXYZZZZZ, YZXZYZZZ, YXYYZZYZ,

ZYYXYYYYZ, YYYXYZYZZ, YZZZXZYZZ, XZXZYZZZZ, XZZXYZZZZ, ZYYYXYYYYZ, ZXZXZZZZ, ZZXXYZZZZ, YYYZYXYYZ, YYYZXYYZ, XZZZXZYZZ, YYYZYXYYZ and XZZZZXYYZ.

For ZYXZYZZZZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fourth, sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXYZXZYZZ, ZXZZXZYZZ, ZXYXZYZZ, ZXYZXYYZZ, ZXYZXZYZZ and YXZYXZYZZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the third, fourth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXYZXZYZZ, ZXZZXZYZZ, ZXYXZYZZ, ZXYZXZYZZ and YXZYXZYZZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing a Y-Kempe change there yields YXZYXYYZ and YXZYXYYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 4 8 3 5 6 7 1 9), (5 1 7 6 8 3 4 2 9) and (8 1 2 7 6 3 4 5 9), we have also handled ZZZXXZZYZ, YZZZZXYYZ and YXZZZYZZZ.

For XXXZYZZZZ, if the Y-path starting at the ninth vertex ends at the first, second, fourth, sixth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YZZYXYYZ, ZYZYXYYZ, ZZZXYYZ, ZZZYXZYZZ, ZZZYXZYZZ and ZZZYXYYZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the first, second, fourth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZYYZXZZZ, YZYXZZZ, YYYXZZZ, YYYZXYYZ and YYYZXZYZZ respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex ends at the second or fourth vertex of  $H$ , then doing a Y-Kempe change there yields YZYXZYZZ and YYYXZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 3 4 2 5 6 9 8 7), (3 4 6 2 9 1 8 7 5), (3 4 7 2 9 1 8 6 5), (3 6 7 2 9 1 8 4 5), (4 6 7 2 9 1 8 3 5), (5 6 7 2 1 3 9 8 4) and (5 8 7 6 1 4 3 2 9), we have also handled XZXXYZZZ, YYXXYXYYZ, YYXXYXYYZ, YYXXYXYYZ, YYYXYYZ, XZZZYYYZ and XZZZYZYZZ.

For ZZZXZZZZ, if the Y-path starting at the seventh vertex ends at the first, second, third, fourth, sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YZZYXZYZZ, ZYZYXZYZZ, ZZYXZYZZ, ZZZXZYZZ, ZZZYXYYZ, ZZZYXZYZZ and YYYZXZYZZ respectively, which are handled by Case 1. If the Y-path starting at the eighth vertex ends at the sixth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZYXZYZZ and YYYZXYYZ respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex ends at the sixth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZXZZZZ and YYYXYYZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the second or third vertex of  $H$ , then doing a Y-Kempe change there yields ZYZYXZZZ and ZZYXZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 5 7 6 1 4 3 8 9), we have also handled XZZZYZZZ.

For YXZYZZZZ, if the Y-path starting at the seventh vertex ends at the sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XYZXXYYZ, XYZXXZYZZ

and XZYXXYZYZ respectively, which are handled by Case 1. If the Y-path starting at the eighth vertex ends at the sixth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XYZZXXYZYZ and XZYXXYYZZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields XZYXXZYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 6 8 3 4 7 9), (1 2 7 5 6 3 4 8 9), (1 2 8 5 9 3 4 6 7), (1 8 2 4 5 3 6 7 9), (1 8 2 5 6 3 4 7 9), (1 8 2 5 9 3 4 6 7), (2 5 8 3 4 1 6 7 9), (2 8 1 4 5 3 6 7 9) and (6 1 2 7 8 3 4 5 9), we have also handled YXZZZYZZY, YXZZYYZZZ, ZXYYZYYYZ, XZZXZZYZ, XZZXZZYZ, ZXXXZZXYZ, ZXXXYYZZZ, ZXZXZZYZ and XZZZZYYYZ.

For YXYYYZZZZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XYXXXZYZZ, XYXXXYYZZ, XYXXXZYYZ and XZXXXYZYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZXXXYYYZ, XZXXXZYYZ and XZXXXYYZZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XYXXXYYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 6 7 3 4 8 9), (1 8 3 4 5 6 7 2 9) and (1 8 5 6 7 3 4 2 9), we have also handled YXZZYYYZ, XZXXXZZYZ and XZZZZXXYZ.

For XYXYZZZZ, if the Y-path starting at the seventh vertex ends at the sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YXYXXYYZZ, YXYXXZYYZ and ZXZXXYZZY respectively, which are handled by Case 1. If the Y-path starting at the eighth vertex ends at the sixth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YXYXXYZYZ and ZXZXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the sixth vertex ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXZXZYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 6 7 8 9), (1 2 8 4 5 3 6 7 9), (1 2 8 5 9 3 4 6 7), (2 1 3 4 5 6 7 8 9), (2 1 4 3 5 6 7 8 9), (2 1 5 6 8 3 4 7 9), (2 1 5 8 9 3 4 6 7), (2 1 6 5 7 3 4 8 9), (2 1 6 8 9 3 4 5 7), (2 1 7 5 6 3 4 8 9), (2 1 7 6 8 3 4 5 9), (2 1 8 3 5 4 6 7 9), (2 1 8 4 5 3 6 7 9), (2 1 8 5 6 3 4 7 9), (2 1 8 5 7 3 4 6 9), (3 1 8 4 5 2 6 7 9), (3 2 8 4 5 1 6 7 9), (4 1 8 3 5 2 6 7 9), (4 2 8 5 9 1 3 6 7), (5 1 6 7 8 2 3 4 9), (5 1 7 6 8 2 3 4 9), (6 1 8 5 7 2 3 4 9) and (7 1 8 5 6 2 3 4 9), we have also handled XYXYZZZZ, YXZZXZZYZ, YZXXZZXYZ, YXXYYZZZ, YXYXYZZZ, YXZ-ZXYZZ, ZXYXYZZZ, YXZZYXYZZ, ZXYYYXYZZ, XYZZXXYZ, YXZZZYXYZ, XYXZZZZYZ, XYZXXZZYZ, XYZZXXZZYZ, XYZZXZXYZ, XZYXXZZYZ, ZXYXXZZYZ, XZXYYZZYZ, XZXYYZZYZ, YZZZXXYZ, XZZZXXYZ and XZZZXYYZ.

For XXXZZZZ, if the Y-path starting at the third vertex ends at the first, second, fourth, fifth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXZZZZ, XZXXZZYZ, XXXXZZYZ, XXXZXYZZ, YYYZZXYZZ and ZZZYXXYYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex ends at the first, second, fourth or fifth vertex of  $H$ , then doing a Y-Kempe change there yields

YZYYYXYYZ, ZYXXXYYZ, ZZYZYXYYZ and ZZYZZXYYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the first or fourth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZZZYZZZ and XXXXZYZZZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex ends at the fourth vertex of  $H$ , then doing a Y-Kempe change there yields ZXZXZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 5 6 7 4 8 2), (3 9 1 7 5 8 4 6 2), (5 8 7 6 1 4 3 2 9), (7 9 5 1 2 3 4 6 8) and (7 9 5 3 1 2 4 6 8), we have also handled YZYZYXYYZ, XXZXXXYYZ, ZZZXYZZYZ, YYXYYZZY and YXYYYZZY.

For XXXXZYZZZ, if the Y-path starting at the third vertex ends at the first, second, fourth, fifth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXXZYZZZ, XZXXZYZZZ, XXXZZYZZZ, XXXXZYZZZ, YYYZYXYYZ and ZZZZYXYYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex ends at the first, second, fourth or fifth vertex of  $H$ , then doing a Y-Kempe change there yields YZYZYXYYZ, ZZYZYXYYZ, ZZYXXYYZ and ZZYZZXYYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the first, fifth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZZXZYZZZ, XZZXXYZZZ and YZZZYXYYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex ends at the fifth vertex of  $H$ , then doing a Y-Kempe change there yields ZXZXZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 6 5 3 9 4 8 7), (1 5 3 8 4 9 6 2 7), (2 1 3 5 4 6 7 8 9), (2 1 3 7 4 8 5 6 9), (3 9 1 6 8 5 4 7 2), (3 9 1 7 8 5 4 6 2), (4 3 1 7 8 6 5 9 2), (4 9 1 6 2 5 3 7 8), (5 8 7 6 1 4 3 2 9), (6 7 5 3 2 4 1 9 8), (6 9 5 4 2 1 7 3 8), (7 9 5 3 2 1 6 4 8), (7 9 5 4 2 1 6 3 8), (8 5 7 1 6 4 3 2 9) and (8 5 7 3 6 2 1 4 9), we have also handled XXYXYYYZ, YXXYXXYZ, XXZZXYZZZ, XXZZZZXYZ, YYZYXZYZZ, YZYXZYZZ, ZZYZZXYYZ, YYYZXZYZZ, ZZZXYZZYZ, ZZYXZYZZZ, XYYZYZZYZ, XYZYYYZZY, XYYZYZZYZ, YZZXYZZYZ and ZXYZZYZZ.

For XXXZYZZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fourth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXXYYZ, ZYXXYYZ, ZZZYXXYYZ, ZZYZZXYYZ, ZZYXXZYZ and ZZYXXZYZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second, fourth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XXXZYZZZ, XZXZYZZZ, XXXXZYZZZ, YYYZXZYZ and YYYZXZYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZZZZYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 8 5 7 9), (1 3 4 5 2 9 6 7 8), (3 4 5 1 2 9 6 7 8), (4 3 5 2 1 9 6 7 8), (5 7 2 1 8 9 3 4 6), (5 8 7 1 2 4 3 6 9), (5 8 7 2 1 4 3 6 9), (6 7 1 3 5 9 4 8 2), (6 7 2 1 8 9 3 4 5), (9 4 5 2 1 3 6 7 8) and (9 6 1 3 5 7 4 8 2), we have also handled XXZZZYZZZ, XZXYYYZZZ, YZXYYYZZZ, ZYXXYYYZZ, XXXXZYZZZ, ZXZXZYZZZ, XZZXYZZYZ, YYYZXZYZ, XXXXZYZZZ, XZXZYZZYZ and YYYXZYZZ.

For YXZZXYZZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends

at the second, third, fifth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YXZXXYZZZ, YZZXXYZZZ, YXXXXYZZZ, YXZXZYZZZ, XYZYYXYZZ and XZYZZXYYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third or fifth vertex of  $H$ , then doing a Y-Kempe change there yields XZYYZXYYZ, XZZYZXYYZ and XZYYYXYYZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second, fifth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields YXXZXYZZZ, YZXZXYZZZ, YXXZZYZZZ and XYYZYXYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 8 5 6 9), (2 3 6 7 8 5 1 4 9) and (4 1 6 7 8 5 3 2 9), we have also handled YXZZZZXYZ, ZXYZXZZYZ and YZZXXZZYZ.

For ZXZYYYZZZ, if the Y-path starting at the ninth vertex ends at the second, third, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YYYXXXYYZ, YZZXXXYYZ, ZYXXXZYZ and ZYXXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex ends at the second, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYYYZZZ, ZYXXXYZZ and ZYXXXZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 8 5 7 9), (1 8 6 2 3 5 4 9 7), (1 8 6 3 4 5 2 7 9), (1 8 7 2 4 6 3 5 9), (2 8 6 1 3 5 4 7 9), (2 8 7 1 4 6 3 5 9), (3 2 4 1 5 7 6 8 9), (3 2 4 1 6 7 5 8 9) and (5 2 3 1 7 8 6 9 4), we have also handled ZXZYZYZY, ZXXZXZZYZ, ZZXXXZZYZ, ZXZXZXZY, XXXZXZZYZ, XZZZXZY, YXZZYZYZ, YXZZZYYZ and YXZZZYYZ.

For XXZXYZZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fourth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYZZXYYZ, ZYZZXXYYZ, ZZZZXXYYZ, ZZYXXYYZ, ZZYZZXZY and ZZYZZXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second, fourth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XXXXYZZZ, XZXXYYZZ, XXXZYZZZ, YYYXXYZZ and YYYXXZYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZXYYZZ, which is handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZZXYZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 8 5 7 9), (1 5 3 8 2 9 6 7 4), (4 5 1 8 2 9 6 7 3), (5 1 6 2 8 9 3 4 7), (5 8 7 6 1 4 2 3 9), (5 8 7 6 2 4 1 3 9) and (6 1 5 2 8 9 3 4 7), we have also handled XXZXZYZY, YZXXYXXYZ, XZXYYXXYZ, XXYXXYZZ, XZZXYZY, ZXZXYYZY and XXYXXYZZ.

For YXZZZZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZYYYXYZ, XZZYYXYZ, XZYZYXYZ, XZYYZXYZ, XZYYYZY and YZXXXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YXZXZZYZ, YXXXXZZYZ, YXZXZYZZ, YXZXZYZZ and XYZYZZXYZ respectively, which are handled by Case 1. Since we already handled the

permutation of all resulting boards by (3 8 1 6 2 4 5 7 9), we have also handled ZZXZXZZYZ.

For XXZZYZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXXYXYZ, YZYXXYXYZ, ZYYYXXYXYZ, ZZZYXXYXYZ, ZZYZZXYXYZ, ZZYXXZXYZ and ZZXXYXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing a Y-Kempe change there yields XXXZYZYZZ, XZXZYZYZZ and XXXXYZYZZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZZYZYZZ, which is handled by Case 1. If the Y-path starting at the second vertex ends at the fourth vertex of  $H$ , then doing a Y-Kempe change there yields XZZXYZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 7 3 5 4 9 8 6), (1 2 9 4 7 5 8 6 3), (3 9 2 5 6 4 8 7 1), (3 9 2 6 5 4 7 8 1), (3 9 2 7 5 4 6 8 1), (5 8 3 6 1 2 9 7 4), (5 8 7 6 1 2 3 4 9), (5 8 9 6 2 1 3 4 7), (7 9 8 1 2 3 4 6 5), (7 9 8 2 1 3 4 6 5), (7 9 8 2 1 4 3 6 5), (9 3 2 5 7 4 8 6 1) and (9 7 8 1 2 4 3 6 5), we have also handled XXYZZYZZ, XXZZZZYZZ, XXZXXYXYZ, YYZYXXYXYZ, YYZYXXYXYZ, ZXXXYYXYZ, XZXZYZZYZ, ZXXZYZZYZ, YXYXYZZYZ, XYXYYZZYZ, XXYYYZZYZ, XXZXXYZZ and YXXYYZZYZ.

For ZXZYZZYZZ, if the Y-path starting at the ninth vertex ends at the second, third, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YYYXXYXYZ, YZZXXYXYZ, YZYXXZXYZ and XZXYYXYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the third, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYYZYZZ, ZZZYXXYZZ and ZZZXXZXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 4 7 6 8 9 3), (1 8 4 2 3 5 6 9 7) and (2 8 4 1 3 5 6 7 9), we have also handled ZXZYZZYZZ, ZXXZZXZYZ and XZXZZXZYZ.

For ZXZYXXYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YZYXZYXYZ, YYYXZYXYZ, YZZXZYXYZ, YZYXXYXYZ, YZYXXZXYZ and XZXYZXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXYXZYZZ, ZZXYYZYZZ and ZXXYXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZYXZYZZ, ZZZYZZYZZ, ZZZYXXYZZ and ZZZXYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 8 6 7 5 9), (1 8 7 3 2 4 6 5 9), (1 8 9 2 3 5 6 4 7), (1 9 8 3 6 2 4 7 5), (1 9 8 3 7 2 4 6 5), (2 8 7 3 1 4 6 5 9), (3 9 1 6 4 5 7 8 2), (4 9 1 6 3 5 7 8 2) and (5 2 9 4 7 1 8 6 3), we have also handled ZYZXZZXYZ, ZYXZZXZYZ, ZXYZZXZYZ, YYXXYZYYZ, YYXXYZYZZ, YZXZZXZYZ, YYYZYXXYZ, YYZYXXYXYZ and ZXZYZZXYZ.

For XXZYZZYZZ, if the Y-path starting at the ninth vertex ends at the second, third, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZYYXXYXYZ,

ZZZXXYXYZ, ZZYXXZXYZ and ZZXYXYZZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the third, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZXYYZYZZ, XZZYYXYZZ and YZZXXZXYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 5 4 7 6 8 9 3), (2 1 3 5 8 4 9 6 7), (2 1 3 6 7 4 8 5 9), (2 1 4 5 7 6 8 9 3), (2 8 3 1 6 5 9 7 4), (3 4 2 1 6 5 9 7 8), (3 7 2 5 8 4 9 6 1), (3 9 1 4 5 6 7 8 2), (3 9 1 4 5 7 6 8 2), (3 9 1 4 6 5 8 7 2), (3 9 2 6 7 4 8 5 1), (4 3 2 5 7 6 8 9 1), (4 7 2 5 8 3 9 6 1), (4 9 2 6 7 3 8 5 1), (5 8 1 2 3 4 6 9 7), (5 8 2 1 3 4 6 7 9), (6 3 4 1 2 7 9 8 5), (6 7 4 1 2 8 3 9 5), (6 9 1 2 3 7 4 8 5), (7 3 4 1 2 6 9 8 5), (7 6 1 4 5 3 9 8 2), (7 9 1 2 3 6 4 8 5), (7 9 1 2 4 3 6 5 8), (7 9 2 1 3 4 6 5 8), (7 9 2 1 4 3 6 5 8), (8 2 1 4 5 3 9 7 6), (8 5 1 2 3 6 4 7 9), (8 5 4 1 2 6 3 9 7), (8 5 4 1 2 6 9 7 3), (9 3 1 4 7 5 8 6 2) and (9 7 1 2 3 4 6 5 8), we have also handled XXZYZZYYZ, XXYYZYZZ, XXZZZYYYZ, XXZZYZYYZ, ZYXXXZXYZ, ZYXXYZYYZ, XXYXZXYZZ, YYZXXYXYZ, YYZXXXYYZ, XXZYXYXYZ, XXZXXYYYZ, ZZXXYZYYZ, XXXYZXYZZ, XXXZXYYYZ, ZXXZYXZYZ, XZXZYXZYZ, ZZXYXYXYZ, XXXZZYYZ, YXXXYZYZZ, ZZXYYYXYZ, YYYZZXXYZ, YXXXYZYZZ, YXYXYXZYZ, XXYXYXZYZ, XYXZZXXYZ, ZXXXYZZYZ, XXXZYZZYZ, ZZXXYXXYZ, XXZYXXYYZ and YXXYYXZYZ.

For XXZXYZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYZXYXYZ, YZYZXYXYZ, ZYYZXYXYZ, ZZZZXYXYZ, ZZYXYXYZ, ZZYXZXYZ and ZZXZYXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second or fourth vertex of  $H$ , then doing a Y-Kempe change there yields XXXXYZYZZ, XZXXYZYZZ and XXXZYZYZZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZZXYZYZZ, XZZXYXYZZ and YZZYXZXYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZXYZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 7 3 5 4 9 8 6), (1 2 7 4 5 3 9 8 6), (1 2 9 4 7 5 8 6 3), (1 7 4 6 2 8 3 9 5), (2 9 4 6 1 7 3 8 5), (3 9 2 5 6 4 8 7 1), (3 9 2 7 5 4 6 8 1), (3 9 2 7 6 4 8 5 1), (3 9 2 8 5 4 6 7 1), (4 7 1 6 2 8 3 9 5), (4 9 2 6 1 7 3 8 5), (5 3 6 4 7 2 8 9 1), (5 8 3 6 1 2 9 7 4), (5 8 3 7 1 2 9 6 4), (5 8 7 6 1 2 3 4 9), (5 8 9 6 2 1 3 4 7), (6 3 5 4 7 2 8 9 1), (6 9 2 4 5 3 7 8 1), (7 9 8 1 2 3 4 6 5), (7 9 8 2 1 3 4 6 5), (7 9 8 3 1 2 4 6 5), (7 9 8 3 2 1 4 6 5) and (8 9 2 4 5 3 7 6 1), we have also handled XXXYZYYYZ, XXYXZYYYZ, XXZXZZYYZ, YXXZZYYZZ, XZXYYZYZZ, XXZXZYXYZ, YYZYXXZYZ, XXZXXYZYZ, YYZYXXYZZ, ZXXYZYYZZ, XYXZYZYZZ, ZZXXXZYZZ, ZXXXYYXYZ, ZXXXYYXYZ, XZXZYZZYZ, ZXXZYZZYZ, ZZXXZXYYZ, YYYZXZXYZ, ZXXYXYZZ, XZYXYZZYZ, XYZXYYZYZ, YXZXYYZYZ and XXXZYXYZZ.

For XXZYXZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXZYXYZ, ZYXXZYXYZ, ZZZXZYXYZ, ZZYXYXYZ, ZZYXZZXYZ and ZZXYZXYZZ respectively, which are handled by Case 1. If the Y-path starting at the third



vertex doesn't end in  $H$  or ends at the second or sixth vertex of  $H$ , then doing a Y-Kempe change there yields XXXYXZYZZ, XZXYXZYZZ and XXXYXXYZZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZZYXZYZZ, XZZYZZYZZ, XZZYXXYZZ and YZZXYZXZY respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 8 6 7 5 9), (1 5 7 2 8 3 4 6 9), (1 8 7 3 4 5 6 2 9), (1 8 7 3 5 4 6 2 9), (1 9 8 3 7 2 4 6 5), (2 1 3 5 7 4 6 8 9), (2 1 3 5 8 4 6 7 9), (2 1 3 6 5 4 8 7 9), (2 1 3 6 7 4 8 5 9), (2 5 7 1 8 3 4 6 9), (2 8 7 3 4 5 6 1 9), (2 8 7 3 5 4 6 1 9), (2 9 8 1 7 4 3 6 5), (2 9 8 3 6 1 4 7 5), (2 9 8 3 7 1 4 6 5), (3 4 8 1 6 2 9 7 5), (3 4 8 1 7 2 9 6 5), (3 5 7 1 8 2 4 6 9), (3 5 7 2 8 1 4 6 9), (3 6 2 5 7 8 9 4 1), (3 6 8 5 4 2 9 7 1), (3 9 2 5 6 8 7 4 1), (3 9 2 5 8 6 7 4 1), (3 9 2 6 5 8 7 4 1), (3 9 2 6 8 5 7 4 1), (4 6 2 5 7 8 9 3 1), (4 8 9 1 2 5 6 3 7), (4 9 2 6 8 5 7 3 1), (4 9 8 1 7 2 3 6 5), (5 2 3 4 6 1 7 8 9), (6 2 3 4 8 1 7 5 9), (6 2 9 4 8 1 5 7 3) and (6 4 2 1 7 8 9 3 5), we have also handled YYZXZZXYZ, YXZXZZYZ, YZXZYXZY, YZXZYXZY, ZYXXYYZY, YYZZXXYZ, YYZZXXZY, XXZZXYZY, XXZZZYXY, XYZXYZZY, ZYXYZXZY, ZYXZYXZY, XZXYYYZY, YZXXYZZY, YZXYYYZY, ZYXXYYZY, ZYXXYYZY, ZYXXYYZY, XZYXYZZY, ZYXYZZY, YXYZXXY, YYXXZXYY, YYZYXZX, XXZXYYZY, YYZYXXY, XXZXYYZY, YYXXZXXY, XYZYZXZY, XXXZXYYZ, XYXZYZY, ZXZYXXYZ, ZYXZYXY, ZYXZYXY and ZYXXYYZY.

For XXZZZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ and ZZYXXYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZZZYZZ, which is handled by Case 1. If the Y-path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing a Y-Kempe change there yields XZXZZZYZZ and XZXZZZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 7 5 4 8 6 2), (5 8 6 7 1 3 4 2 9) and (7 9 3 5 1 2 6 4 8), we have also handled XXZYXXYZ, ZZXZYZZY and YXYYXZY.

For YXZZZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XZYXXYZ, XZYXXYZ, XZYXXYZ, XZYXXYZ, XZYXXYZ and YZXZZZYZZ respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YXZZZYZZ, YXZZZYZZ, YXZZZYZZ, YXZZZYZZ and XYZYZXY respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 4 1 6 2 8 5 7 9), we have also handled ZZXZYZZY.

For XXZXZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ, ZZYXXYZ and ZZYXXYZ respectively, which are handled by Case 1. If the Y-path

starting at the second vertex doesn't end in  $H$  or ends at the third, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZXZYZZ$ ,  $XZXXZYZZ$ ,  $XZZXXYZZ$  and  $YZZYZXXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZXZXZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 5 6 4 7 8 2), (3 9 1 8 5 4 7 6 2), (3 9 2 4 5 6 8 7 1), (3 9 2 4 6 5 7 8 1), (3 9 2 4 7 5 6 8 1), (5 8 1 6 2 3 4 7 9), (7 9 1 2 4 3 6 5 8), (7 9 1 6 3 2 4 8 5), (7 9 2 1 4 3 6 5 8), (7 9 2 6 3 1 4 8 5), (7 9 2 6 4 1 3 8 5), (9 3 2 4 5 7 8 6 1), (9 3 5 1 6 4 8 7 2), (9 7 1 5 3 2 6 4 8) and (9 7 1 6 4 2 3 8 5), we have also handled  $YYZXZYXYZ$ ,  $XXZYXXYZ$ ,  $XXZZXYXYZ$ ,  $YYZZXYXYZ$ ,  $YYZ-ZXXYYZ$ ,  $ZZXXYYZYZ$ ,  $YZXYXZYZ$ ,  $YXYXZZYZ$ ,  $ZYXYXZYZ$ ,  $XYXYZZYZ$ ,  $XXYYZZYZ$ ,  $XXZZXXYZ$ ,  $ZXZYXXYZ$ ,  $YXYXZZYZ$  and  $YXXYZZYZ$ .

For  $XXZZXYZZ$ , if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZZYYZXXYZ$ ,  $YZYYZXXYZ$ ,  $ZYYZXXYZ$ ,  $ZZYZXXYZ$ ,  $ZZYZZXXYZ$ ,  $ZZYYYXXYZ$  and  $ZZXXZYZZ$  respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $ZXZZXYZZ$ , which is handled by Case 1. If the Y-path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZXZYZZ$  and  $XZZXXYZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 5 7 8 9), (1 2 3 4 8 5 7 6 9), (1 2 3 4 8 5 9 7 6), (1 2 3 4 8 6 7 5 9), (5 8 6 7 1 3 4 2 9), (5 8 6 7 2 1 3 4 9), (5 8 6 7 2 1 9 3 4), (5 8 6 7 2 3 4 1 9) and (5 8 6 7 4 1 3 2 9), we have also handled  $XXZZYXYZZ$ ,  $YYZZXZXYZ$ ,  $YYXXZXXYZ$ ,  $YYZZZXXYZ$ ,  $YZXXYZZYZ$ ,  $XXYZZZYZ$ ,  $ZYXXYXXYZ$ ,  $ZYXXYZZYZ$  and  $XZXYYZZYZ$ .

For  $YXZZXZYZZ$ , if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fourth, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZYZZXZYZ$ ,  $XZZYZZXZYZ$ ,  $XZYZZXZYZ$ ,  $XZYZZXZYZ$ ,  $XZYZZXZYZ$  and  $YZXXZZYZ$  respectively, which are handled by Case 1. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $YXZXXZYZZ$ ,  $YXXXXZYZZ$ ,  $YXZXXZYZZ$ ,  $YXZXXZYZZ$  and  $XYZYXXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $YXXZXXYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 6 8 7 5 9), (1 4 9 6 2 8 5 3 7), (1 6 9 3 2 8 5 7 4), (3 1 7 6 4 8 5 2 9), (3 2 7 6 4 8 5 1 9), (4 1 8 2 3 6 9 7 5), (4 1 8 2 3 7 9 6 5), (6 3 2 1 5 7 9 4 8) and (6 4 2 1 5 7 9 3 8), we have also handled  $XYZZZYXYZ$ ,  $XYZYXZZYZ$ ,  $XYZZXZYYZ$ ,  $YZXYXZZYZ$ ,  $ZYXYXZZYZ$ ,  $YXXZYXXYZ$ ,  $YXXZYXXYZ$ ,  $YXXZYXXYZ$  and  $YYYXXZXYZ$ .

For  $YXZZZZYZ$ , if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the second, third, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $YXZXXZZYZ$ ,  $YZZXZZYZ$ ,  $YXXXXZZYZ$ ,  $YXZXXZYYZ$  and  $YZXZXXXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third or seventh vertex of  $H$ , then doing a Y-Kempe change there yields

YXXXXXXYZ, YXXXXXXYZ, YZZXXXXYZ and YZXXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the second or seventh vertex of  $H$ , then doing a Y-Kempe change there yields YXXZZZZYZ, YZXZZZZYZ and YXXZZZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (2 8 6 7 1 4 3 5 9), we have also handled ZXXXXZZYZ.

For XXZZZZYZ, if the Y-path starting at the ninth vertex ends at the first, second, third, fourth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXXXXXXYZ, ZXXXXXXYZ, ZZZXXXXYZ, ZZXZZXXYZ and ZZZXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the third vertex ends at the first, second, fourth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZXXZZZZYZ, XZXZZZZYZ, XXXZZZZYZ and XXXZZZZYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the first or fourth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZZZZZYZ and XZZZZZZYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex ends at the fourth vertex of  $H$ , then doing a Y-Kempe change there yields ZXZXXXXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled ZZZZZYZ.

For ZXZYZZYZ, if the Y-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXYZZYZ, XXYZZYZ, ZZXZZYZ, ZXXYZZYZ and XZZYXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XZXXXXXYZ, ZZXXXXXYZ, XXYXXXXYZ and XZXXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the first or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZZYZZYZ, XZZYZZYZ and ZZZYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 8 7 2 1 4 5 6 9), we have also handled ZZZZZYZ.

For XXZYZZYZ, if the Y-path starting at the third vertex doesn't end in  $H$  or ends at the first, second, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XXYZZYZ, ZXXYZZYZ, XZXZZYZ, XXYZZYZ and ZZZYXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the first, second or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZXXXXXYZ, XZXXXXXYZ, ZZXXXXXYZ and ZZXXXXXYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the first or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XZZYZZYZ, ZZZYZZYZ and XZZYZZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 7 2 1 4 3 6 9), we have also handled ZZZYXZY.

For XXZXZZYZ, if the Y-path starting at the ninth vertex ends at the first, second, third, fourth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields

$XZXZXXXXYZ$ ,  $ZXXZXXXXYZ$ ,  $ZZZZXXXXYZ$ ,  $ZZXXXXXXYZ$  and  $ZZXZXXZYZ$  respectively, which are handled by Case 1. If the Y-path starting at the third vertex ends at the first, second, fourth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZXXXZZZYZ$ ,  $XXXZZZYZ$ ,  $XXXZZZYZ$  and  $XXXXZZXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the first or seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZZZXZZZYZ$  and  $XZZXZZXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the first vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields  $ZXZXZZXYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 7\ 6\ 1\ 4\ 3\ 2\ 9)$ , we have also handled  $ZXZZYYZYZ$ .

For  $ZXYZZZZYZ$ , if the Z-path starting at the third vertex ends at the fourth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $ZXXXZZYZZ$  and  $ZYYXZZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $ZYXYZZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 8\ 3\ 6\ 2\ 4\ 7\ 5\ 9)$ , we have also handled  $ZZXZZXXYZ$ .

For  $XYYYZZYZZ$ , if the Z-path starting at the third vertex doesn't end in  $H$  or ends at the fourth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZYZZ$ ,  $XYXXZZYZZ$  and  $YXYXZZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZYZZ$  and  $YXXYZZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YYYZZYZZ$ , which is handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YXXXZZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 3\ 6\ 1\ 2\ 7\ 4\ 9)$ , we have also handled  $ZZYZXYYYZ$ .

For  $XXYYZZYZZ$ , if the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYXXZZYZZ$ ,  $XYXXZZYZZ$ ,  $YXXXZZYZZ$ ,  $YYYXZZYZZ$  and  $YXYZZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YXYZZYZZ$ , which is handled by Case 1. If the Z-path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZYZZ$  and  $XYYXZZYZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(3\ 9\ 1\ 8\ 4\ 5\ 2\ 6\ 7)$ ,  $(5\ 8\ 6\ 7\ 1\ 2\ 3\ 4\ 9)$  and  $(7\ 9\ 2\ 5\ 1\ 3\ 8\ 4\ 6)$ , we have also handled  $YYZXXXXXYZ$ ,  $ZZXZYXXYZ$  and  $XYXXYXZYZ$ .

For  $XXXZZYZZ$ , if the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYYXZZYZZ$ ,  $XYYXZZYZZ$ ,  $YXYXZZYZZ$ ,  $YYXXZZYZZ$  and  $YYYZZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the second vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZYZZ$  and  $XYXXZZYZZ$  respectively, which are handled by Case 1. If the Z-path

starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields YXXYZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 6 7 8 9), (3 9 1 8 4 5 2 6 7), (3 9 8 1 4 5 2 6 7), (5 8 6 7 1 2 3 4 9), (5 8 7 6 1 2 3 4 9), (7 9 2 5 1 3 8 4 6) and (7 9 5 2 1 3 8 4 6), we have also handled XXYXZZYZZ, ZYZXXXXYZ, XXZYZZYZZ, ZZXZYXZY, ZZXZYXYYZ, XZXZYXZY and XYXXZZYZ.

For XXXXZZYZZ, if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the third, fourth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XYXXZZYZZ, XYXXZZYZZ, XYXYZZYZZ and YXYZZYZZ respectively, which are handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields YYYZZYZZ, YXYZZYZZ and YYYZZYZZ respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields YXXXZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (3 9 1 8 4 5 2 6 7), (5 8 6 7 1 2 3 4 9) and (7 9 2 5 1 3 8 4 6), we have also handled ZXZYZZYZZ, ZZXZYZZYZ and XZXZZYZ.

For ZXYZZYZZ, if the Z-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields ZXYZZYZZ, ZXXXZZYZZ, ZXYZZYZZ and ZYXZYZZZ respectively, which are handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the third or sixth vertex of  $H$ , then doing a Z-Kempe change there yields ZYXXZZYZZ, ZYXZYZZZ and ZYXXZZYZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 5 8 9 6 7), (1 2 3 4 6 5 9 7 8), (1 2 6 7 3 8 9 4 5), (1 4 7 3 2 8 6 5 9) and (2 1 7 6 3 8 9 4 5), we have also handled XYZZXXYZ, YXZZXXYZ, XYXXZZYZ, ZZXZYXZY and YXXXZZYZ.

For YYYZZYZZ, if the Z-path starting at the eighth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XXXXZZYZ, XYXXZZYZ, XXYXXYZ, XXXYZZYZ, XXXYZZYZ and XXXXZZYZ respectively, which are handled by Case 1. If the Z-path starting at the second vertex doesn't end in  $H$  or ends at the third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXYZZYZZ, YXXYZZYZ, YXYZZYZZ, YXYZZYZZ and YXYZZYZZ respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XYYZZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 5 1 3 6 4 7 8 9), we have also handled YYYZZYZZ.

For YYYXZZYZ, if the Z-path starting at the eighth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XXXYZZYZ, XYXXYZYZ, XXYXZZYZ, XXXYZZYZ, XXXXZZYZ and XXXYZZYZ respectively, which are handled by Case 1. If the Z-path starting at the second vertex doesn't end in  $H$  or ends at the third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXYXZZYZ, YXXYZZYZ, YXYXZZYZ, YXYXZZYZ and YXYXZZYZ respectively, which are handled by Case 1. If the Z-path

starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XYYYYZYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 5 3 6 1 4 7 8 9), we have also handled XYYZYYYYZ.

For YXXXXZYYZ, if the Z-path starting at the eighth vertex doesn't end in  $H$  or ends at the third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XYYYYZXYZ, XYXYYZXYZ, XYYXYZXYZ, XYYYXZXYZ and XYYYYZYYZ respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXXYXZYYZ, YXYXZYYZ, YXXYZZYYZ and YXXYXZXYZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex doesn't end in  $H$  or ends at the fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXYXXZYYZ, YXYXZYYZ and YXYXXZXYZ respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XXXXXZYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 6 5 8 9), (2 8 6 7 1 4 3 5 9) and (2 8 6 7 3 4 1 5 9), we have also handled YXXYXZXYZ, YXXZXYYYZ and XXYZXYYYZ.

For YXXXXZXYZ, if the Z-path starting at the eighth vertex doesn't end in  $H$  or ends at the third, fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XYYYYZYYZ, XYXYYZYYZ, XYYXYZYYZ, XYYYXZYYZ and XYYYYZXYZ respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXXYXZXYZ, YXYXZXYZ, YXXYZZXYZ and YXXYXZYYZ respectively, which are handled by Case 1. If the Z-path starting at the third vertex doesn't end in  $H$  or ends at the fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXYXXZXYZ, YXYXZXYZ and YXYXXZYYZ respectively, which are handled by Case 1. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XXXXXZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 8 6 7 1 4 3 5 9), we have also handled YXYZXYYYZ.

For YYYYYYYYZ, if the Z-path starting at the eighth vertex ends at the second, third, fourth, fifth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XYXXXXXYZ, XXYXXXXXYZ, XXXYXXXXYZ, XXXYXXXXYZ, XXXXXYXYZ and XXXXXYYZ respectively, which are handled by Case 1. If the Z-path starting at the fifth vertex ends at the second, third, fourth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXYXYYYYZ, YXYXYYYYZ, YYYXXYYYZ, YYYXXYYYZ and YYYXXYXYZ respectively, which are handled by Case 1.

For XYYYYXYYYZ, if the Z-path starting at the fifth vertex ends at the third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing a Z-Kempe change there yields XYXYYYYYZ, XYYXYYYYZ, XYYYYXYYZ, XYYYYXYYZ and YXXXXXXYZ respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex ends at the third, sixth, seventh or eighth vertex of  $H$ , then doing a Z-Kempe change there yields XYXXXXYYYZ, XYYXXXXYZ, XYYXXYXYZ and YXXYXXYZ respectively, which are handled by Case 1. If the Z-path starting at the sixth vertex ends at the third or seventh vertex of  $H$ , then

doing a Z-Kempe change there yields  $XYXYXXYYZ$  and  $XYYYXXXYZ$  respectively, which are handled by Case 1. If the Z-path starting at the third vertex ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYXYXYZ$ , which is handled by Case 1.

**Case 3.**  $B$  is one of  $XX★★XZZYZ, YXZZY★★YZ, Y★★ZZY★ZYZ, XXYYY★ZYZ, XXZ★YXXYZ, XXZ★YZZYZ, XYZZ★YYYZ, XXZ★YYZYZ, XXZ★ZXXYZ, XY★ZYXZYZ, XX★XYXZYZ, XX★ZYXZYZ, XXZY★YYYZ, XXX★YYZYZ, XXZXY★YYZ, XXYZZ★YZZ, XY★ZZXYYZ, XZZZY★YZ, XXZ★XZXYZ, X★YYYZZZZ, XX★XYZZZZ, XY★ZZXZYZ, XXX★ZXZYZ, XYYYY★ZYZ, XZZZXZ★YZ, XYZZ★ZYZZ, X★ZZYZZZZ, YYZX★YYYZ, YYYYY★ZYZ, ★YYYXZZYZ, Y★XXYXZYZ, YYZXXXZ★YZ, YXZZXZ★YZ, YXZZZX★YZ, YYZ★ZXZYZ, YYZZ★YYYZ, YZYZY★ZYZ, YYZ★YZYZZ, Y★XXZXZYZ, YXYZY★ZYZ, YY★ZZXYYZ, YYZY★ZYZ, YYZXZ★YZZ, YX★ZYZZZZ, YYZ★YZZZZ, YYYXY★ZYZ, YYZ★YXYYZ, ★ZYZXZZYZ, ★ZZZZXZYZ, Z★YYYXZYZ, ZX★YXZZYZ, ZYZXY★ZYZ, ZZ★XYXZYZ, ZZX★YZXYZ, ZYYXXZ★YZ, ZYZXY★XYZ, Z★XXZXZYZ, ZYZXXXZ★YZ, ZYYXYZ★YZ, ZXZY★YYYZ, ZXZYZ★YZZ, Z★YYYZZZZ, ZZ★ZYXZYZ, XXZY★XYZ, XXZY★YYZ, XYZZY★ZYZ, XYZZZ★YZZ, YX★ZXZZYZ, YY★YZXZYZ, YZ★XYXZYZ, YYZ★XZZYZ, YYZXY★YZ, ★ZZZYZZZZ, ZXY★ZYZ, Z★XXYXZYZ, ZXZY★YYZ, ZZX★ZXZYZ, ZZY★XZZYZ, XXZYZXYYZ, XXYYZXZYZ, XXYXYYZYZ, XXZXYYXYZ, XXZZZXZYZ, XYXZXZXYZ, XYZZXZYYZ, XYYZXZZYZ, XXZZYZXYZ, XYYZYZXYZ, XXZXZXYZ, XYYZYZZYZ, XYZZXZZYZ, XZYYYYZYZ, XXZYXYYZ, XYXZXZYZ, XYXZYZXYZ, XYYZXZXYZ, XXZYXZYYZ, XXYZXYYZ, XZXYZXYZ, XZXYXZXYZ, X★YYYXZYZ, XXZYZYYZ, XXZXZYYZ, X★YZYXZYZ, XZYZYYZYZ, XZZYZXZYZ, XXZYZZYZ, XXZYZYZZ, XZZZYXYYZ, X★YYXZZYZ, XYZZXYYZ, XXXYYZZZZ, XXX★YXZYZ, XXZYXYZZ, XZYXYXYZZ, XYZZYXYZZ, XZZZYXXYZ, XZZYXZYZ, XYZZYXYYZ, XZYYZXYZZ, XZYXYXYZZ, XXYZZYYZ, XZYZZYZZ, XZYZZXZYZ, XZYYZYYYZ, XZZZXXYZ, X★ZZYZYZ, XZZZXYYZ, XZZYZZZZ, XXZYZZZZ, YYZYYYYY, YYZXZXXYZ, YYZXYXXYZ, YXXZZXZYZ, YXXZXZYZ, YYZXZYYZ, YYZXZYYZ, YYZXYZZYZ, YYZXYYZZ, YZXXXZYZ, YZXXXZYZ, YXXZXYYZ, YZXXYZXYZ, YXXZYXZYZ, YXXZXZXYZ, YYZXYXYZZ, YYZXXYYZ, YXXZYZXYZ, YXXZZXZYZ, YXYZYXYZZ, YXYZZYYZ, YXXZZXZYZ, YZYXYZZYZ, YZZZYXYZZ, YZYXZYZZ, YXYXYXZYZ, YZZXZXYZ, YZXXXZYZ, YYYXZXYZ, YXYZZXYZ, YYYXXZZYZ, YYZXXYZZ, YZYXZXYZ, YZYXXYZZ, YZYXXZZYZ, YXYZXYYZ, YXXZZYYZ, YZZZXYYZ, YXYZZYYZ, YZZZXYYZ, YZZZZXYZ, YZZZZYYZ, YZYZZXYZ, YZYZZXZYZ, YXYZZZYZ, YYZYXYYZ, YZZZXYYZ, YZZXYZZZ, YZZZXZZYZ, YZZZZYYZ, YZZZZYZZ, YZZZZZZZ, ZXXXZXYZ, ZXXYZYYZ, ZYZZYZZZ, ZXXYYXZYZ, ZXXYYXZYZ, ZYZXYYYZ, ZYZXZXYZ, ZXZYZZYZ, ZXZYXZYZ,$

$ZXZYYYZYZ$ ,  $ZXXYXYZZ$ ,  $ZYZXZXXYZ$ ,  $ZXZYXYZZ$ ,  $ZXZYXYZZ$ ,  
 $ZZYYYYZYZ$ ,  $ZYYXZXYYZ$ ,  $ZZXYXZYYZ$ ,  $ZZYYZXZYZ$ ,  $ZYZXZYZZZ$ ,  $ZYYYYZZYZ$ ,  
 $ZZYZZZYZZ$ ,  $ZYZXXYZZZ$ ,  $ZXZYXYZZZ$ ,  $ZZYXZXZYZ$ ,  $ZXXXXZYYZ$ ,  
 $ZZYXYXYZZ$ ,  $ZXXXYZXYZ$ ,  $ZZYXXYYZZ$ ,  $ZZXYXZXYZ$ ,  $ZXXYZXYZZ$ ,  
 $ZYZXYXYYZ$ ,  $ZZXYZXYYZ$ ,  $ZYYYZXYYZ$ ,  $ZZYXZYYYZ$ ,  $ZZYZXYZZZ$ ,  $ZZYXYZZYZ$ ,  
 $ZXXXXZXYZ$ ,  $ZZYZZYYYZ$ ,  $ZYYYZZYZZ$ ,  $ZYZXYZZZZ$ ,  $ZYZXXZYZZ$ ,  $ZZXZXZXYZ$ ,  
 $ZYZXYZYZZ$ ,  $ZYYYXYZZZ$ ,  $ZYYXYYZYZ$ ,  $ZXZYZZZZZ$ ,  $ZZYZYZZZZ$ ,  $ZZZXZXZYZ$ ,  
 $ZXZYZYZZZ$ ,  $ZZYZZXYZZ$ ,  $ZZYXZZYZZ$  or  $ZXZYZZZZZ$ .

For  $YZZZZZZZ$ , if the Y-path starting at the sixth vertex ends at the third, fourth, fifth, seventh, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZYZZYZZ$ ,  $XZZYZYZZ$ ,  $XZZZYZZZ$ ,  $XZZZZYZZ$ ,  $XZZZZYZYZ$  and  $XYYYZYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the eighth vertex ends at the third, fourth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZYZZZZYZ$ ,  $XZZYZZZYZ$ ,  $XZZZZZYZZ$  and  $XYYYYYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZZYZZZ$  and  $XYYYZYZZ$  respectively, which are handled by Case 2. If the Y-path starting at the fourth vertex ends at the third, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZYZZZZZ$ ,  $XZZYZZZZ$  and  $XYYZYZZZ$  respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (5 1 2 8 3 4 6 9 7), we have also handled  $ZZZZYZZZ$ .

For  $YZZZYZZZ$ , if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XYYYXYZZ$ ,  $XYZYXYZZ$ ,  $XYYZXYZZ$ ,  $XYYYXZYZZ$ ,  $XYYYXYZZ$  and  $XYYYXYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZZXZYZZ$ ,  $XZZYXZYZZ$  and  $XZZZXYYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields  $YZXZYZZZ$ ,  $YZXXYZZZ$  and  $XZYZXYYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex ends at the fourth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZYXYZZ$  and  $XZZZXZYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fourth vertex ends at the eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZZYXZZYZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 6 5 8 9) and (3 1 2 4 5 6 7 8 9), we have also handled  $YZZZZZYZZ$  and  $ZZYZYZZZZ$ .

For  $YYZZYZZZ$ , if the Y-path starting at the seventh vertex ends at the sixth, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXZZXYZZ$ ,  $XXZZXZYZZ$  and  $XXYYXYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the eighth vertex ends at the sixth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXZZXYZZ$  and  $XXYYXYZZ$  respectively, which are handled by Case 2. If the Y-path starting at the sixth vertex ends at the ninth vertex of  $H$ , then doing a Y-Kempe



change there yields  $XXYYXZYYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 5\ 6\ 7\ 8\ 2\ 3\ 4\ 9)$ ,  $(1\ 6\ 2\ 5\ 7\ 3\ 4\ 8\ 9)$  and  $(3\ 4\ 1\ 2\ 5\ 6\ 7\ 8\ 9)$ , we have also handled  $YZZZYZZYZ$ ,  $YZZZZYYZZ$  and  $ZZYYYZZZZ$ .

For  $XYZZYZZZZ$ , if the  $Y$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth, seventh or ninth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YXZ-ZXYZZZ$ ,  $YXZYXYZZZ$ ,  $YXZZXYZZZ$  and  $ZXYXZYYZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or ninth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YXZZXZYZZ$ ,  $YXZYXZYZZ$  and  $ZXYXZYYZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the ninth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XYZXYZZZZ$  and  $ZXYZXYYYZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the first vertex doesn't end in  $H$  then doing a  $Y$ -Kempe change there yields  $ZYZZYZZZZ$ , which is handled by Case 2. If the  $Y$ -path starting at the third vertex doesn't end in  $H$  then doing a  $Y$ -Kempe change there yields  $XYXZYZZZZ$ , which is handled by Case 2. If the  $Y$ -path starting at the ninth vertex doesn't end in  $H$  then doing a  $Y$ -Kempe change there yields  $ZXYXZYYZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(5\ 1\ 7\ 3\ 8\ 4\ 6\ 2\ 9)$ , we have also handled  $YZZZXZZYZ$ .

For  $YXXZYZZZZ$ , if the  $Y$ -path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, sixth, eighth or ninth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XYYZXZYZZ$ ,  $XYYYXZYZZ$ ,  $XYYZXYYZZ$ ,  $XYYZXZYYZ$  and  $XZZYXZYZZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, sixth or eighth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XZZYXYYYZ$ ,  $XZZZXYYYZ$ ,  $XZZYXZYYZ$  and  $XZZYXYYZZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth or eighth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $XYYZXZYZZ$ ,  $XYYYXZYZZ$  and  $XYYZXZYZZ$  respectively, which are handled by Case 1. If the  $Y$ -path starting at the second vertex doesn't end in  $H$  then doing a  $Y$ -Kempe change there yields  $YZXZYZZZZ$ , which is handled by Case 2. If the  $Y$ -path starting at the fourth vertex doesn't end in  $H$  then doing a  $Y$ -Kempe change there yields  $YXXXYZZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 8\ 7\ 6\ 5\ 3\ 4\ 2\ 9)$ ,  $(2\ 1\ 5\ 6\ 7\ 3\ 4\ 8\ 9)$ ,  $(2\ 1\ 6\ 9\ 8\ 4\ 5\ 3\ 7)$ ,  $(2\ 4\ 5\ 6\ 7\ 1\ 3\ 8\ 9)$ ,  $(2\ 4\ 5\ 9\ 8\ 1\ 3\ 7\ 6)$ ,  $(3\ 5\ 1\ 4\ 8\ 6\ 7\ 2\ 9)$  and  $(3\ 6\ 1\ 4\ 8\ 5\ 7\ 2\ 9)$ , we have also handled  $XZZZXZYYZ$ ,  $XYZZXZYZZ$ ,  $XYZZZXZYZ$ ,  $ZYZXXZYZZ$ ,  $ZYZXXZZYZ$ ,  $XZYXZZYZZ$  and  $XZYZZXZYZ$ .

For  $ZYZXYZZZZ$ , if the  $Y$ -path starting at the fourth vertex doesn't end in  $H$  or ends at the third, sixth, seventh, eighth or ninth vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $ZYZZYZZZZ$ ,  $ZYXZYZZZZ$ ,  $ZXZZXYZZZ$ ,  $ZXZZXZYZZ$ ,  $ZXZZXZZYZ$  and  $YXYXZYYYZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the ninth vertex doesn't end in  $H$  or ends at the third, sixth or seventh vertex of  $H$ , then doing a  $Y$ -Kempe change there yields  $YXYZXYYYZ$ ,  $YXZZXYYYZ$ ,  $YXYZXYYYZ$  and  $YXYZXYZZZ$  respectively, which are handled by Cases 1 and 2. If the  $Y$ -path starting at the third vertex doesn't end in  $H$  or ends at the seventh vertex of  $H$ , then doing a  $Y$ -

Kempe change there yields ZYXXYZZZZ and ZXYYXZYZZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XYZXYZZZ, which is handled by Case 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXZYXZYZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 1 3 6 8 4 7 2 9), we have also handled YZZZXZYZ.

For YYZYZZZZ, if the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XXZXXYZZ, XXZXXYYZ and XXYXXZYZZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields XXZXXZYZZ and XXYXXYZYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YYXYYZZZ, which is handled by Case 2. If the Y-path starting at the ninth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XXYXXYYYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 5 3 6 8 4 7 2 9), (1 6 2 7 8 3 4 5 9), (2 3 5 4 7 1 8 6 9), (2 3 7 4 5 6 8 1 9) and (3 6 1 7 8 2 5 4 9), we have also handled YZZZYZZZ, YZZZYZZZ, ZYZZZYZZ, ZYZZYZZZ and ZZYZZYYZ.

For YYZXZZZZ, if the Y-path starting at the ninth vertex ends at the third, fourth, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXZZXYYYZ, XXYXYYYZ, XXYZXZYZZ and XXYZXZYZZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing a Y-Kempe change there yields YYXZYZZZ, XXZZXYZZZ and XXZZXZYZZ respectively, which are handled by Case 2. If the Y-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing a Y-Kempe change there yields XXZYXZYZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 3 5 4 6 8 7 9), (1 2 3 6 7 5 9 4 8), (1 2 3 8 5 6 9 4 7), (1 5 3 6 8 4 7 2 9), (1 5 7 2 8 4 9 6 3), (1 6 2 5 7 3 4 8 9), (1 6 2 8 9 3 5 4 7), (3 4 1 8 6 2 5 7 9), (3 4 2 1 7 5 8 6 9), (3 4 6 1 5 7 8 2 9), (3 5 1 4 8 7 9 6 2), (3 6 1 5 7 2 4 8 9), (3 6 4 8 9 5 7 1 2), (4 5 1 2 9 7 8 6 3), (4 6 1 2 7 3 5 8 9), (4 7 1 2 9 3 6 5 8) and (5 6 2 1 8 3 4 7 9), we have also handled XXZXYZZZ, YZZZXYYZ, XXZZXZZY, YZZZYXZY, YXZZYZZY, YZZXYYZZ, ZXXXXXZY, ZZZXZXZY, XZYZZYZZ, XZYZZYZZ, ZZYXZYZZ, ZZYXYYZZ, XXZXXZY, YXZZYZZY, XZZYZZY, YXZYZZY and XZZZYZZY.

For XYYYYZZZ, if the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YXXXXZYZZ, YXXXXZYZZ and ZXXXXZYZZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the eighth vertex doesn't end in  $H$  or ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields YXXXXZZY and ZXXXXYYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YXXXXYZZ, which is handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXXXXYYZ, which is handled by Case 1. Since we already handled

the permutation of all resulting boards by (1 2 5 6 8 3 4 7 9), (1 5 6 7 8 2 3 4 9), (4 1 2 5 9 7 3 6 8), (5 1 2 3 4 6 7 8 9), (5 1 2 4 8 7 3 6 9), (5 1 6 7 8 4 2 3 9) and (6 1 5 8 9 2 3 4 7), we have also handled XYZZYYZY, XZZZYYYZ, ZZYXZYYYZ, XXXXYZZZ, YYZYXZZY, YZZZXYYYZ and ZYYYZXYYZ.

For ZYZXXYZZ, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, third, fifth, seventh, eighth or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZZXYZZ, XYZZXYZZ, ZYXZXYZZ, ZYZZZYZZ, ZXZZYXYZ, ZXZZYXZY and YXYZYXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex doesn't end in  $H$  or ends at the third or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYXZYZZZ, ZYXXZYZZ and YXYZYXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XYZXXYZZ, which is handled by Case 1. If the Y-path starting at the third vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZYXXXYZZ, which is handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields YXYZZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (2 4 5 3 9 7 1 6 8), (5 4 7 6 1 8 2 3 9) and (8 3 1 7 9 6 4 5 2), we have also handled YYZXYYXYZ, XZZYZXZY and YYXYXZY.

For ZZZYZZYZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, fifth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields YYXYYYXYZ, YZXYYYXYZ, YYXZYXYZ, YYXYZYXYZ, YYXYZZXYZ and XXYXXXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the eighth vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZZXZZXYZ, ZZXZZXYZ and ZZXZZYXYZ respectively, which are handled by Case 1. If the Y-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYXZYZZ, ZZYXXZYZZ and ZZYZXXYZ respectively, which are handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing a Y-Kempe change there yields ZXYZZYZZ, ZXYXZZYZ and ZXYZZXYZ respectively, which are handled by Case 1. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZYZZYZZ, which is handled by Case 1.

For XXYZZYZZ, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, fourth, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYYYXYZ, ZYXYYYXYZ, ZZXZYXYZ, ZZXZYXYZ and ZZYXXXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the second, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields XXYXZZYZ, XZYXZZYZ, XXYXXZYZ and YXYZZXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZXYZZYZZ, which is handled by Case 1. If the Y-path starting at the second vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZYZZYZZ, which is handled by Case 1. If the Y-path starting at

the fifth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $XXYZXZYZZ$ , which is handled by Case 1. If the Y-path starting at the eighth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $YYXZZZXYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 3\ 6\ 1\ 4\ 7\ 2\ 9)$ , we have also handled  $ZZXZYXZYZ$ .

For  $YYZZYZYZZ$ , if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the third, fourth, sixth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXYXXYXYZ$ ,  $XXZYXXYXYZ$ ,  $XXYZXXYXYZ$ ,  $XXYXXZXYZ$  and  $YYXXYXYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex doesn't end in  $H$  then doing a Y-Kempe change there yields  $YYXZYZYZZ$ , which is handled by Case 1. If the Y-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXZZXYXYZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 7\ 6\ 5\ 2\ 8\ 4\ 9)$ ,  $(1\ 6\ 8\ 5\ 7\ 2\ 9\ 4\ 3)$ ,  $(2\ 4\ 3\ 1\ 7\ 5\ 9\ 6\ 8)$ ,  $(3\ 4\ 2\ 1\ 5\ 6\ 9\ 8\ 7)$  and  $(3\ 6\ 1\ 4\ 8\ 2\ 9\ 5\ 7)$ , we have also handled  $YZYZYZZYZ$ ,  $ZYYYYZZYZ$ ,  $YZYZYYZYZ$ ,  $YYZZZYYYZ$  and  $YYZYZZYZZ$ .

For  $XXZYZYZZZ$ , if the Y-path starting at the ninth vertex ends at the second, third, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZYYXXXYZZ$ ,  $ZZZXYYXXYZ$ ,  $ZZYXZZXXYZ$  and  $ZZXYXXYZZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex ends at the second, fifth or eighth vertex of  $H$ , then doing a Y-Kempe change there yields  $XZXYZYZZZ$ ,  $XXXYYXYZZ$  and  $YYYXZZXXYZ$  respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by  $(1\ 6\ 3\ 2\ 4\ 7\ 9\ 5\ 8)$ ,  $(1\ 6\ 4\ 2\ 3\ 5\ 8\ 9\ 7)$ ,  $(1\ 8\ 4\ 2\ 6\ 3\ 5\ 7\ 9)$ ,  $(1\ 8\ 7\ 3\ 2\ 4\ 6\ 5\ 9)$ ,  $(2\ 5\ 3\ 1\ 4\ 7\ 8\ 6\ 9)$ ,  $(2\ 5\ 3\ 4\ 1\ 6\ 7\ 8\ 9)$ ,  $(2\ 6\ 1\ 4\ 3\ 5\ 7\ 8\ 9)$ ,  $(2\ 6\ 1\ 4\ 3\ 8\ 9\ 5\ 7)$ ,  $(2\ 6\ 3\ 1\ 4\ 7\ 8\ 5\ 9)$ ,  $(2\ 6\ 5\ 1\ 3\ 7\ 9\ 4\ 8)$ ,  $(2\ 6\ 5\ 4\ 1\ 7\ 9\ 3\ 8)$ ,  $(2\ 8\ 1\ 4\ 3\ 5\ 7\ 6\ 9)$ ,  $(2\ 8\ 3\ 4\ 1\ 6\ 7\ 5\ 9)$ ,  $(2\ 8\ 4\ 1\ 5\ 3\ 6\ 7\ 9)$ ,  $(2\ 8\ 7\ 3\ 1\ 4\ 6\ 5\ 9)$ ,  $(3\ 6\ 4\ 1\ 7\ 2\ 8\ 9\ 5)$ ,  $(3\ 7\ 2\ 4\ 1\ 6\ 9\ 5\ 8)$ ,  $(3\ 8\ 4\ 1\ 7\ 2\ 5\ 9\ 6)$ ,  $(3\ 9\ 1\ 5\ 2\ 7\ 8\ 4\ 6)$ ,  $(4\ 5\ 1\ 3\ 2\ 8\ 9\ 6\ 7)$ ,  $(4\ 6\ 1\ 2\ 3\ 5\ 8\ 7\ 9)$ ,  $(4\ 6\ 1\ 2\ 3\ 7\ 9\ 5\ 8)$ ,  $(4\ 6\ 1\ 3\ 2\ 8\ 9\ 5\ 7)$ ,  $(4\ 6\ 3\ 1\ 5\ 2\ 8\ 7\ 9)$ ,  $(4\ 8\ 1\ 2\ 3\ 6\ 9\ 5\ 7)$ ,  $(4\ 8\ 1\ 2\ 6\ 3\ 5\ 7\ 9)$ ,  $(4\ 8\ 1\ 3\ 5\ 6\ 9\ 7\ 2)$ ,  $(4\ 8\ 1\ 3\ 6\ 5\ 9\ 7\ 2)$ ,  $(4\ 8\ 2\ 1\ 5\ 3\ 6\ 7\ 9)$ ,  $(4\ 8\ 3\ 1\ 6\ 2\ 5\ 7\ 9)$ ,  $(5\ 4\ 3\ 1\ 7\ 2\ 8\ 9\ 6)$ ,  $(5\ 8\ 7\ 3\ 1\ 4\ 6\ 2\ 9)$ ,  $(6\ 2\ 7\ 1\ 3\ 5\ 8\ 4\ 9)$ ,  $(7\ 2\ 4\ 1\ 3\ 5\ 8\ 9\ 6)$ ,  $(7\ 4\ 1\ 2\ 3\ 5\ 8\ 9\ 6)$  and  $(7\ 9\ 4\ 1\ 6\ 2\ 3\ 8\ 5)$ , we have also handled  $XZYYYXZYZ$ ,  $XYZZYXZYZ$ ,  $YXXZZZZYZ$ ,  $YZXXZXZYZ$ ,  $YXZZXZYYZ$ ,  $ZXZYXYZZZ$ ,  $ZXZYXXYZZ$ ,  $YXYZYXXYZ$ ,  $YXZZZXYYZ$ ,  $ZXYYYXZYZ$ ,  $YXYZYXZYZ$ ,  $ZYZXXZXYZ$ ,  $ZYZXXXXYZ$ ,  $XXYZZXZYZ$ ,  $ZYXXZXZYZ$ ,  $YYXZZXZYZ$ ,  $YXXZYXZYZ$ ,  $XXYZXZZYZ$ ,  $XXZXYYXYZ$ ,  $YYZXXYZZZ$ ,  $ZYZXXYXZYZ$ ,  $YZYXXYXZYZ$ ,  $YYZXYYXYZZ$ ,  $YYZXZXZYZ$ ,  $XZXYYXZYZ$ ,  $ZXXYXZZYZ$ ,  $XXZYXZXYZ$ ,  $XXZYXZXYZ$ ,  $XZXYXZXYZ$ ,  $XXZYXZZYZ$ ,  $YYZXXZZYZ$ ,  $ZZXXYXZYZ$ ,  $YXZZYXZYZ$ ,  $YXZZYZXZYZ$ ,  $ZYZXYZXYZ$  and  $XXXYYYZYZ$ .

For  $XXYZYZZYZ$ , if the Y-path starting at the fourth vertex ends at the first, second, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $ZXYXYZZYZ$ ,  $XZYXYZZYZ$ ,  $XXYXYZXYZ$  and  $ZZYZYXXYZ$  respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields  $XXYZYXXYZ$  and  $ZZYXYZXYZ$  respectively, which are handled by Case 1. If the Y-path starting at the second vertex ends at the first,

seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZZYZZZZY, XZYZZXZY and ZXYXXXYZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (1 7 4 2 6 8 3 9 5), (3 5 4 1 6 7 2 9 8) and (5 8 1 6 2 4 3 7 9), we have also handled XYYZZXZY, YYXZZZY and XXZZYZZY.

For ZXZYZZZY, if the Y-path starting at the second vertex ends at the first, third, sixth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XZZYZZZY, ZZXYYZZY, ZZZYXZY, ZZZYZZXY and XXXYYXXY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXYYZZY and XZZYXXXY respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 8 5 1 2 4 7 6 9), we have also handled XXZZXZY.

For XXZYZZZY, if the Y-path starting at the third vertex ends at the first, second, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXYYZZY, XZXYYZZY, XXXYYZZY and ZZZYXXXY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the second vertex ends at the first, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZZZYZZZY, XZZYZZXY and ZXXYYXXY respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (1 2 9 5 6 7 3 8 4), (3 9 1 5 6 2 4 8 7), (3 9 2 4 6 1 5 7 8), (3 9 4 1 2 6 5 7 8), (5 8 7 1 2 4 3 6 9), (5 8 9 1 2 3 6 4 7), (7 9 1 3 5 2 4 8 6), (7 9 5 1 2 3 6 4 8), (7 9 8 3 4 1 2 6 5), (9 3 2 4 5 7 1 8 6) and (9 7 8 1 2 3 4 6 5), we have also handled XXZZYZZY, XXZXYYZY, YYZXXXXY, XXZYYYXY, XXZZYXZY, XXZXZZZY, XXYXXYZY, XXYXXYZY, YYXXYXZY, XXZYXXXY and XXYXXYZY.

For ZYZXZZZY, if the Y-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth, sixth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZZXZZY, ZYXZZXZY, ZYZZXXZY, ZYZZZZZY, ZYZZXXXY and XYXXXZZY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZXZZZY, ZYZXZZXY and XYXZZXXY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the fifth vertex doesn't end in  $H$  or ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZXXXZY, ZYZXXXXY and XYXZZXZY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XYZXZZZY, which is handled by Case 2. If the Y-path starting at the seventh vertex ends at the third or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYXXZZXY and XYXZZZZY respectively, which are handled by Case 2. If the Y-path starting at the third vertex ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields XYZXZZZY, which is handled by Case 2.

For XXZYYYZY, if the Y-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third or seventh vertex of  $H$ , then doing a Y-Kempe change there yields ZZXYYXY, ZXXYYXY, ZZZYYXY and ZZXYYZY respectively, which are handled by Cases 1 and 2. If the Y-path starting at the second vertex doesn't end in  $H$

or ends at the third or seventh vertex of  $H$ , then doing a Y-Kempe change there yields XZZYYYZYZ, XZXYYYZYZ and XZZYYYXYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the third vertex ends at the seventh vertex of  $H$ , then doing a Y-Kempe change there yields XXXYYYXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 3 1 2 4 7 6 9), we have also handled XXZXYYZYZ.

For ZXXYZYYYZ, if the Y-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZXXYXYYYZ, XXXYXYYYZ, ZZXXYXYYZ and XZZYZYYYZ respectively, which are handled by Cases 1 and 2. If the Y-path starting at the first vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XXXYZYYYZ, which is handled by Case 2. If the Y-path starting at the second vertex doesn't end in  $H$  then doing a Y-Kempe change there yields ZZXYZYYYZ, which is handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XZZYXYYYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 8 7 2 5 3 4 6 9), we have also handled ZXXXZXYYZ.

Each of XYZZXZZYZ and YYZYYYZYZ have an odd number of X's and Z's, so there is a Y-path with exactly one end in  $H$ . For XYZZXZZYZ, if this is the first, third, fourth, fifth, sixth, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYZZXZZYZ, XYXZXZZYZ, XYZXXZZYZ, XYZZZZZYZ, XYZZXXZYZ, XYZZXZXYZ and ZYXXZXXYZ respectively, which are handled by Case 2. For YYZYYYZYZ, if this is the third, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields YYXYYYZYZ, YYZYYYXYZ and YYXYYYXYZ respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by (2 3 1 4 6 7 5 8 9), we have also handled ZYYYZYYYZ.

For ZXZYZZZZZ, if the X-path starting at the eighth vertex ends at the third, fourth, sixth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXYYZZZZZ, ZXZZZZZZZ, ZXZYZYZZZ, ZXZYZZYYZ and YXYZYYYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex ends at the third, fourth, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXYYZYZZZ, ZXZZZYZZZ, ZXZYYYZZZ and YXYZYZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the first, third, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYZZZZZZZ, ZXYZZZZZZ, ZXZZYZZZZ, ZXZZZZYZZ and YXYYYYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields ZXZYYZZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 6 3 8 4 2 5 7 9) and (2 1 3 5 4 6 8 7 9), we have also handled ZZZZZXZYZ and XZZZYZZZZ.

For YXZZYZZZZ, if the X-path starting at the ninth vertex ends at the fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXYZZZYZZ, ZXYYYYYYZ, ZXYYZZYYZ, ZXYYZYZYZ and ZXYYZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the fourth, sixth,

seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXZYZZZZZ, YXZZZYZZZ, YXZZZZYZZ and YXZZZZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex ends at the fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YXZYYYZZZ and YXZZYYYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields YXZYYZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 4 2 6 5 8 9 3 7), (1 8 5 3 6 4 7 2 9), (2 5 4 3 1 6 7 8 9), (3 8 2 1 6 4 7 9 5), (4 1 2 3 5 6 7 9 8), (5 2 1 3 4 6 7 8 9), (5 6 8 2 1 3 9 7 4), (5 8 2 3 1 4 7 9 6), (6 5 2 3 1 8 9 7 4), (7 2 8 1 4 3 6 9 5), (8 1 2 3 5 4 7 6 9), (9 1 4 2 7 3 6 8 5) and (9 5 2 4 3 7 8 6 1), we have also handled YZZXYYZZZ, XXXZZXZYZ, XXXZYZZZZ, ZZXZZXZYZ, XZZYZZZZZ, ZXZYZZZZZ, XZZZXYYZZ, XZZZXZZYZ, YZZZXYYZZ, ZXZYZZYZZ, XZZZYZZYZ, XYYYYYZYZ and YYZYXYYYZ.

For ZXZYXYZZZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, third, sixth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXXYZZZ, YXZZXYZZZ, ZXYZXYZZZ, ZYZZYZZZZ, ZXZZXYZZZ, ZXZ-ZXYZYZ and YXYXZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YXYZXZYZZ, YXYZXYYYZ and YXYZXZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXYYXYZZZ and ZXYYXYZYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YXZYXYZZZ, which is handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZXZYXYZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 3 2 8 6 5 4 7 9), (2 1 7 6 8 4 3 5 9) and (3 4 5 2 7 1 6 9 8), we have also handled ZZXZYXZYZ, YZZXZXZYZ and XXZYZZYZZ.

For XYZZZYZZZ, if the X-path starting at the seventh vertex ends at the third, fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZZZ, XYZYZYZZZ, XYZZYYZZZ, XYZZZZZZZ, XYZZZYZYZ and XZYYYZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the third, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZYZZZ, XYZYYYZZ, XYZYZZYZZ, XYZYZYYYZ and XZYZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XYZZZZYYZ, which is handled by Case 2. If the X-path starting at the fifth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields XZYZZZZYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 6 2 5 3 9 7 4 8), (4 6 1 3 5 2 7 8 9), (5 4 1 6 7 8 3 2 9), (5 4 2 1 6 9 3 7 8) and (6 4 2 7 1 8 3 5 9), we have also handled XYYYYZZYZ, ZYZZXZYZZ, ZZYXZZYZ, YZZXYYYZ and ZZYZZXZYZ.

For ZXZYZXYYZZ, if the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the first, third, fifth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXXXXXYYZZ, YXXXXXYYZZ, ZXYZZXYYZZ, ZXXXXYXYZZ, ZYZZZYZZZZ, ZXXXXXYYYZ and YXYYYXZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZYZZZXZZZZ, ZYXXXXZZZZ and YXYZYXYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fifth or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZXYYZXYYZZ, ZXYYYYXYZZ and ZXYYZXYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YXZYZXYYZZ, which is handled by Case 1. If the X-path starting at the eighth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields XYXZXYYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (4 2 9 8 5 6 1 7 3), (5 4 7 6 1 8 3 2 9) and (6 4 9 2 1 8 5 3 7), we have also handled YXXXXXZZYZ, ZZXYYXZZYZ and ZXZYXZZYZ.

For YYZYZYZZZZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third, fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYZYZYZZZZ, YYYYYZYZZZ, YYZZZYZZZZ, YYZYYYZZZZ, YYZYZZZZZZ, YYZYZYZZYZ and ZZYZYZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the third, fifth, sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYZZZYZZYZ, YYYYYZYZZZ, YYZZYYZZZZ, YYZZZZYZZZ and ZZYYYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the third or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYZYYYZZZZ, YYYYYYYZZZ and ZZYZZZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZYZZZ and ZZZZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZZYZ, which is handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZZZYZZYZ, which is handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZZYZYZZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 6 1 3 7 4 8 2 9), we have also handled ZZYYYZZYZ.

For XXYZXYYZZ, if the X-path starting at the sixth vertex ends at the fourth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXYXZZYZ, YYXZYZZZZ, XXYZXZZYZ and XXZYXYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex ends at the fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYXYYZZZ and XXYZXZZYZ respectively, which are handled by Case 1. If the X-path starting at the fourth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYXYYYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 6



9 5 8), (1 2 3 4 8 5 7 6 9), (1 2 3 5 4 9 6 8 7), (1 2 3 5 4 9 7 8 6), (1 2 3 5 8 6 4 7 9), (1 2 3 6 5 8 4 7 9), (1 2 4 5 3 7 9 6 8), (1 2 7 3 5 4 8 9 6), (1 5 3 7 8 6 2 9 4), (1 5 7 2 8 4 3 6 9), (1 5 7 3 8 2 6 4 9), (1 6 2 4 7 3 5 8 9), (1 6 2 4 9 3 8 5 7), (1 7 2 3 8 9 5 4 6), (1 7 2 3 8 9 6 4 5), (2 3 1 4 5 8 9 6 7), (2 3 1 4 8 5 7 6 9), (2 5 7 1 8 4 6 3 9), (2 5 7 4 8 1 3 6 9), (2 7 1 4 8 3 5 9 6), (2 7 1 4 8 3 6 9 5), (3 4 2 1 6 9 7 8 5), (3 5 1 4 8 2 6 7 9) and (4 6 1 3 7 2 5 9 8), we have also handled XXZYYZXYZ, YYXZXZXYZ, XXZXYYZYZ, XXZXYYZYZ, YYXXZXZYZ, XXYYXZZYZ, XXXZYYZYZ, XXZYXZYZ, YXXZYXZYZ, YZXXYZXYZ, YXZZYXXYZ, YXXZXYYZZ, ZYXXZXXYZ, YZXXZXXYZ, YZXXXZYYZ, ZXXYXXYYZ, XYYZXZXYZ, ZYZXYYXYZ, YXXZYXZYZ, XYXZZXYYZ, XYXZZXYYZ, YZXXYXZYZ, XXYZYXZYZ and XXZYXYYZZ.

For YYZYXYYZZ, if the X-path starting at the eighth vertex ends at the second, third, sixth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields YZZYXYYZZ, YYYXYYZZ, YYZYXZYZ, YYZYXZYZ and ZZXZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex ends at the second, third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYZXZZYZ, ZZZZXZZYZ, ZZYZXZYZ and ZZYZXZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the third, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YYYZXYYZZ, YYYZXZYZ and YYYZXZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex ends at the second, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZYYXYYZZ, YYYXZYZ and YYYXZZZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 2 6 1 5 4 8 7 9), (3 5 7 4 2 6 8 1 9), (5 8 7 6 1 3 4 2 9), (6 4 5 7 8 1 2 3 9) and (7 8 3 5 1 2 6 4 9), we have also handled YYYXZZYZ, ZYXXYYZYZ, XZYXXYZYZ, XXZXZXXYZ and XYZZYXXYZ.

For YYZZYZZYZ, if the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZYZ, ZYYZZYZYZ, ZZZYZYZYZ, ZZYZZYZYZ, ZZYZZYZYZ, ZZYZZYZYZ, ZZYZZYZYZ and ZZYZZYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the second, third, fourth, sixth, seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZZZZZYZ, YZZZZZZYZ, YYYZZZZYZ, YZYZZZZYZ, YZZZZZZYZ, YZZZZZZYZ and YZZZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the third or fourth vertex of  $H$ , then doing an X-Kempe change there yields YZZYZYZYZ, YYYYZYZYZ and YZYZZYZYZ respectively, which are handled by Case 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZYZ, YYYZZYZYZ and YYYZZYZYZ respectively, which are handled by Case 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYZZYZYZ, which is handled by Case 2. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZYZ, which is handled by Case 2.

For XYZZYZZYZ, if the X-path starting at the fifth vertex ends at the second, third, fourth, sixth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XZZZZZZYZ, XYYZZZZYZ, XYZYZZZZYZ, XYZZZYZZYZ, XYZZZZYYZ, XYZZZZZZZ and XZYYYYYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex ends at the fourth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYZYZZYZ and XZYZZYZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XYZYZZYZ and XYZZYYZZZ respectively, which are handled by Case 2. If the X-path starting at the fourth vertex ends at the eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYZYZZZZZ and XZYZZYZZZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (5 2 4 3 1 6 7 8 9), we have also handled YYZZXZZYZ.

For XXZZYZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXZZYZYZ, XXZZZZYZZ, XXZZYYYYZ, XXZZYZYZZ and XXYZZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYZZYZZ, XXYYYYYZZ, XXYZZYZZ and XXYZZYYYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXZZZZYZ, XXZZYZYZ and YYZZZZZZZ respectively, which are handled by Case 2. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XXZZYYZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 8 6 7 5 3 4 2 9), we have also handled YXZZXZZYZ.

For XYYZZYZZYZ, if the X-path starting at the seventh vertex ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZZYZ, XYYZZYYYYZ, XYYZZYZYZ and XZZYZYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex ends at the fifth, sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZZZZ, XYYZZYYZZ and XZZYZYYYYZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZYZYZ and XZZYZYZYZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields XZZYYYYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (1 5 9 6 3 2 7 4 8), (2 1 3 9 5 4 6 8 7), (2 1 6 3 7 4 5 9 8), (2 3 5 8 1 4 6 9 7), (2 5 6 1 4 3 7 8 9), (4 2 3 1 5 6 7 8 9), (4 3 6 8 7 1 2 9 5), (5 1 9 2 6 3 4 7 8), (5 2 7 4 1 3 6 8 9), (6 4 7 2 3 1 5 9 8), (6 7 8 4 1 3 5 2 9), (8 1 4 5 2 3 7 6 9), (8 1 7 2 5 3 4 9 6), (8 1 7 5 2 3 4 6 9) and (8 5 7 9 1 3 4 2 6), we have also handled XYZZZYZZYZ, YXYZZYZYZ, ZXYYZZYZ, ZXZYZZYZ, ZXZYZZYZ, ZYXXYZYZ, YYZXZZYZ, ZYYYXZZYZ, YYZZXZZYZ, YYZZYXZZYZ, YYZZZXZZYZ, XXZXZXZZYZ, ZXXXZXZZYZ, XXZZZXZZYZ and XXZZXZZYZ.

For  $XYYZXZZYZ$ , if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $XYYZXZYYZ$ ,  $XYYYXZYYZ$ ,  $XYYZXYYYZ$ ,  $XYYZXZYZZ$  and  $XZZYXYZZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XZZYXYZZZ$ ,  $XZZZXYYZZ$ ,  $XZZYXZYZZ$  and  $XZZYXYYYZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $XYYZXYZZZ$ ,  $XYYYXYZZZ$  and  $XYYZXYZZZ$  respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 7\ 5\ 8\ 3\ 4\ 6\ 9)$ ,  $(1\ 2\ 7\ 6\ 5\ 3\ 4\ 8\ 9)$ ,  $(1\ 3\ 6\ 4\ 8\ 5\ 7\ 2\ 9)$ ,  $(2\ 3\ 4\ 1\ 5\ 6\ 7\ 8\ 9)$  and  $(2\ 4\ 7\ 6\ 5\ 1\ 3\ 8\ 9)$ , we have also handled  $YXZZZXXYZ$ ,  $XYZZXZYYZ$ ,  $YXXZZXZYZ$ ,  $ZXYXXZZYZ$  and  $ZXZYXZYYZ$ .

For  $YXYZXZZYZ$ , if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the fourth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $YXYZXYZZZ$ ,  $YXYXXZYYZ$ ,  $YXYZXYYYZ$ ,  $YXYZXYZZZ$  and  $ZXZYXZYYZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YXYZXZYYZ$ ,  $YXYXXZYYZ$  and  $YXYZXZYZZ$  respectively, which are handled by Case 1. If the X-path starting at the fourth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields  $YXYXXZZYZ$  and  $XYXXYZZZZ$  respectively, which are handled by Case 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $ZXYZXZZYZ$ , which is handled by Case 2. If the X-path starting at the eighth vertex doesn't end in  $H$  then doing an X-Kempe change there yields  $XYXZYZZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 5\ 6\ 9\ 4\ 7\ 2\ 8)$ ,  $(4\ 2\ 9\ 1\ 8\ 5\ 3\ 6\ 7)$ ,  $(5\ 1\ 7\ 3\ 8\ 4\ 6\ 2\ 9)$  and  $(5\ 7\ 1\ 3\ 9\ 6\ 4\ 8\ 2)$ , we have also handled  $XXZYXYYYZ$ ,  $XYXZXZXYZ$ ,  $YXZZXZXYZ$  and  $YXXXYXZYZ$ .

For  $ZYZXYZZYZ$ , if the X-path starting at the fifth vertex ends at the first, second, third, sixth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields  $YYZXZZZZY$ ,  $ZZZXZZZZY$ ,  $ZYYXZZZZY$ ,  $ZYZXZYZYZ$ ,  $ZYZXZZYYZ$  and  $YZYXYYYZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex ends at the first, second, third or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $ZZYXZYYZZ$ ,  $YYYXZYYZZ$ ,  $YZZXZYYZZ$  and  $YZYXZYYZZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex ends at the second, third or sixth vertex of  $H$ , then doing an X-Kempe change there yields  $ZZZXYYZZZ$ ,  $ZYYXYYZZZ$  and  $ZYZXYYZZZ$  respectively, which are handled by Case 2. If the X-path starting at the second vertex ends at the first, sixth or seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YZZXYZZYZ$ ,  $ZZZXYYZYZ$  and  $ZZZXYYZYZ$  respectively, which are handled by Cases 1 and 2. If the X-path starting at the first vertex ends at the seventh vertex of  $H$ , then doing an X-Kempe change there yields  $YYZXZYZYZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(3$

8 7 6 1 4 5 2 9), we have also handled YYZZZXZYZ.

For YYZYZZYZ, if the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, third, fourth, sixth, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZZZYZ, ZYZYZZZZYZ, YZZYZZZZYZ, YYYZZZZYZ, YZZZZZZYZ, YZYZZYZYZ, YZYZZZYYZ, YZYZZZZZZ and ZZYZYZZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, third, fourth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YZYZZYZYZ, YZZYZZYZYZ, YYYZZYZYZ, YZZYZZYZYZ and ZZYZZYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the second, third or fourth vertex of  $H$ , then doing an X-Kempe change there yields ZZYZZYZYZ, ZYZZZYZYZ, ZZZZZYZYZ and ZZYZZYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the second vertex doesn't end in  $H$  or ends at the third vertex of  $H$ , then doing an X-Kempe change there yields YZZYZZYZYZ and YZYZZYZYZ respectively, which are handled by Case 2. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZZYZZYZYZ, which is handled by Case 2. If the X-path starting at the eighth vertex ends at the third or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZYZ and YZYZZYZYZ respectively, which are handled by Case 2. If the X-path starting at the third vertex ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZYZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 8 7 6 1 4 3 2 9), we have also handled YYZZYZYZ.

For ZYYXXZYZ, if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, third, seventh, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZYYXXYZYZ, YYYXXYZYZ, ZZYXXYZYZ, ZYZXXYZYZ, ZYYXXYZYZ, ZYYXXYZYZ and YZZXXYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the seventh or eighth vertex of  $H$ , then doing an X-Kempe change there yields YZZXXYZYZ, YZZXXYZYZ and YZZXXYZYZ respectively, which are handled by Case 1. If the X-path starting at the seventh vertex ends at the second or eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZYXXYZYZ and ZYYXXYZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields ZZXYYZZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 8 7 1 6 4 3 2 9), we have also handled XYZZXYYZ.

For YYYYYZZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZYZ, YYYZZYZYZ, YYYZZYZYZ, YYYYYYYYZ, YYYYYYZYZ and ZZZZZYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the fourth, fifth, sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYZZYZ, YYYZZYZYZ, YYYZZYZYZ, YYYYYYYYZ, YYYYYYZYZ and ZZZZZYZYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, fifth or sixth vertex

of  $H$ , then doing an X-Kempe change there yields ZZZZZYZZ, ZZZYZYZZ, ZZZZYYZZ and ZZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYZZZZY, YYYZZZZY and YYYZZZZY respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields YZZZZZZY, which is handled by Case 2. If the X-path starting at the fourth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYYZZZZY, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 6 7 1 2 4 3 8 9), we have also handled YZZZZYYY.

For XYYZZZZY, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZZY, XYYZZZZY, XYYZZZZY, XYYZZZZY and XZZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZZZZZY, XZZZZZZY, XZZZZZZY and XZZZZZZY respectively, which are handled by Case 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZZY, XYYZZZZY and XYYZZZZZ respectively, which are handled by Case 1. If the X-path starting at the sixth vertex ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZZZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 6 7 1 4 3 2 9), we have also handled XZZZZXZY.

For XYYZZXZY, if the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXXZZZZZ, YXXZZZZZ, YXXZZZZZ, YXXZZZZZ, YXXZZZZZ, XYYZZXZY and XZZZZXZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZZZXZY, XZZZZXZY, XZZZZXZY and YZZZZXZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex ends at the fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields XYYZZXZY and XYYZZXZY respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (4 7 8 1 6 5 3 2 9), we have also handled YZZZZXZY.

For YYYZZXZY, if the X-path starting at the eighth vertex ends at the second, third, fourth, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields XZZZZXZY, XZZZZXZY, XZZZZXZY, XZZZZXZY, YYYZZXZY and ZZZZZXZY respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex ends at the second, fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZXZY, ZZZZZXZY, ZZZZZXZY and ZZZZZXZY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the second, fourth or seventh vertex of  $H$ , then doing an X-Kempe change there yields YZZZZXZY, YZZZZXZY and YZZZZXZY respectively, which are handled by Case 2. Since we already handled the

permutation of all resulting boards by (5 6 7 8 1 4 3 2 9), we have also handled ZYZXYYYYYZ.

For XXYYZXZYZ, if the X-path starting at the seventh vertex ends at the fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXYYYYXYZ, XXYYZXYYZ and YYZZXYZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex ends at the fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYXXXZZZZ and XXZZYXYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields XXZZZXYYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 7 8 3 6 5 4 9), (3 6 1 8 4 9 5 2 7), (4 5 2 3 1 8 7 6 9), (4 5 6 7 1 8 3 2 9) and (4 7 2 5 1 9 3 8 6), we have also handled XXZYZXYYZ, YYZXZXXYZ, ZXXYYXZYZ, ZXZYXXXYZ and YXXZYXZYZ.

For XYYZYXZYZ, if the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, seventh or ninth vertex of  $H$ , then doing an X-Kempe change there yields YXXZXZZZ, YZXZXZZZ, YXZZXZZZ, YXXXXZZZ, YXXZZYZZ, XYYZYXYYZ and XZZYZXYYZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, fifth or seventh vertex of  $H$ , then doing an X-Kempe change there yields XZZYZXYYZ, XZZZXZZZ, XZZYXYYZ and YZZXZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing an X-Kempe change there yields XYYZYXYYZ and XYYYYYXYZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (4 1 7 6 8 5 3 2 9), we have also handled YYZXZXYYZ.

For ZYXXYXZYZ, if the X-path starting at the seventh vertex ends at the first, second, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYXXYXYYZ, ZZXXYXYYZ, ZYXXZXYYZ, ZYXXYXYYZ and XZYYZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the fifth vertex ends at the eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields ZXYYZZZZ and YZXXYXYYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (5 1 4 6 8 7 3 2 9), we have also handled YYZXZXXYZ.

For YYYYYYZYZ, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth, fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYYYYZ, YYYZYYYYZ, YYYZYYYZ, YYYYYZYZZ, YYYYYYYZZ and ZZZZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the fourth, fifth, sixth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYYYYZZZ, YYYZYZZZ, YYYZYZZZ, YYYYYZZZ and ZZZZZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth, fifth or sixth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZZZYZZ, ZZZYZZYZ, ZZZYZYZZ and ZZZZZYZZ respectively, which are handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the fourth or sixth vertex of  $H$ , then doing an X-Kempe change there yields YYYZYZZYZ, YYYZZYZYZ and YYYZZZZYZ respectively, which are handled by Case 1. If the X-path starting at the second vertex doesn't end in  $H$

then doing an X-Kempe change there yields YZYYYYYZY, which is handled by Case 2. If the X-path starting at the fourth vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYYZYZY, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (5 6 7 1 2 4 3 8 9), we have also handled YYZYYYYY.

For XXYYYYZY, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the fifth, sixth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXYYYYY, XXYYZY, XXYYZY, XXYYYY and YZZZZZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fifth, sixth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XXZZZZY, XXZZZY, XXZZYY and XXZZZZY respectively, which are handled by Case 2. If the X-path starting at the fifth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing an X-Kempe change there yields XXYYZY and XXYYZZ respectively, which are handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  then doing an X-Kempe change there yields XXYYYY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 8 6 7 1 4 3 2 9), we have also handled XXZYXX.

For YYYYYXZY, if the X-path starting at the seventh vertex ends at the fourth, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYZYXY, YYYZXYY, YYYYYXZ and ZZZZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex ends at the fourth, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXXZYZZ, XXXZYZZ and ZZZZXYY respectively, which are handled by Case 1. If the X-path starting at the fifth vertex ends at the fourth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYYZXZY and ZZZZYXY respectively, which are handled by Cases 1 and 2. If the X-path starting at the fourth vertex ends at the ninth vertex of  $H$ , then doing an X-Kempe change there yields ZZZZYXY, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 6 7 1 2 4 3 8 9), we have also handled YYZXYYY.

For XYYYYYZY, if the X-path starting at the seventh vertex doesn't end in  $H$  or ends at the second, third, fourth, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XYYYYXY, XZYXY, XYZYXY, XYZYXY, XYYZYXY, XYYZYXY and YZZZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the eighth vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields YXXXXYZZ, YXXZYZZ and YXXZYZZ respectively, which are handled by Case 1. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing an X-Kempe change there yields XZZZZXY, XZZZYXY and XZZZYXY respectively, which are handled by Cases 1 and 2. If the X-path starting at the third vertex doesn't end in  $H$  or ends at the fifth vertex of  $H$ , then doing an X-Kempe change there yields XYZYXZY and XYZYXZY respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by (4 8 7 6 1 5 3 2 9), we have also handled YYZXYYY.

For ZYYXYXZY, if the X-path starting at the sixth vertex doesn't end in  $H$  or ends at the first, second, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there

yields ZYYXYYXYZ, YYYXYYXYZ, ZZYXYYXYZ, ZYXXZYXYZ, ZXXYXXYZZ and XZZYZZYZZ respectively, which are handled by Cases 1 and 2. If the X-path starting at the ninth vertex doesn't end in  $H$  or ends at the fifth or eighth vertex of  $H$ , then doing an X-Kempe change there yields XZZYZXYZZ, XZZYXXYZZ and YZZXZYXYZ respectively, which are handled by Case 1. If the X-path starting at the first vertex doesn't end in  $H$  then doing an X-Kempe change there yields YYYXYZZXYZ, which is handled by Case 2. If the X-path starting at the second vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZZYXYZZXYZ, which is handled by Case 1. If the X-path starting at the fifth vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZYXXZZXYZ, which is handled by Case 1. If the X-path starting at the eighth vertex doesn't end in  $H$  then doing an X-Kempe change there yields ZXXYXZYZZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (4 8 7 3 1 5 6 2 9), we have also handled YYXZZXYYZ.

Since XXZYZZZZZ has an odd number of X's and Y's, there is a Z-path with exactly one end in  $H$ . If this is the first, second or fourth vertex of  $H$ , then doing a Z-Kempe change there yields YXZYZZZZZ, XYZYZZZZZ and YYZYZZZZZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (2 7 3 1 4 5 6 8 9), (3 7 1 4 2 5 6 8 9), (3 7 1 6 2 4 5 8 9), (3 8 1 5 2 4 6 7 9), (3 9 1 6 2 4 5 7 8), (4 6 1 2 3 5 7 8 9), (4 6 1 8 2 3 5 7 9), (5 8 1 6 2 3 4 7 9) and (7 9 1 4 2 3 5 6 8), we have also handled XYZZZZYZZ, ZZYXZZYZZ, ZZYZZXYZZ, ZZYXZZYZ, YYZYXYYYZ, ZXZYZYZZZ, ZZZXZXZYZ, ZZZYXZYZ and YYYXYYZYZ.

For YXYZYZZZZ, if the Z-path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a Z-Kempe change there yields YYXZYZZZZ and XXXZYZZZZ respectively, which are handled by Case 2. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields XYYZYZZZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 3 4 7 5 6 8 9), (1 2 3 4 9 5 6 7 8), (1 4 2 3 7 5 6 8 9), (1 4 6 2 9 3 5 7 8), (1 4 7 2 9 3 5 6 8), (1 5 3 2 8 4 6 7 9), (1 5 8 2 9 3 4 6 7), (1 6 3 2 8 4 5 7 9), (1 8 5 2 7 3 4 6 9), (1 8 6 2 7 3 4 5 9), (2 1 5 3 7 4 6 8 9), (2 1 5 3 9 4 6 7 8), (2 4 5 1 7 3 6 8 9), (3 2 4 1 5 6 7 8 9), (3 2 4 1 9 5 6 7 8), (3 4 7 1 9 2 5 6 8), (3 6 4 1 9 2 5 7 8), (3 6 5 1 8 2 4 7 9), (3 6 5 1 9 2 4 7 8), (3 6 7 1 9 2 4 5 8), (3 8 5 1 7 2 4 6 9), (4 6 5 1 9 2 3 7 8), (5 8 7 1 9 2 3 4 6), (6 4 7 1 9 2 3 5 8) and (6 8 7 1 9 2 3 4 5), we have also handled YXYZZZYZZ, ZXZYYYYYZ, YYZXZZYZZ, ZYXXYZYYZ, ZYXXYYZYZ, YZYXZZYZ, ZYXXYYZZ, YZYZZXZYZ, XZZZXZXYZ, XZZZXXYZ, XYZZYZYZ, XZYZYYYZ, ZYZXZYZZ, ZXYYZZZZ, YXZZYYYYZ, YYZXYZYZ, YYZZXYYZ, ZZYZYXZYZ, YYZYXYYZ, YYZYXZYZ, ZZXXZXXYZ, YYYZZXYYZ, XXXXZXZYZ, YYYXYYZYZ and XXXXZZYZ.

For XXYZZZZZ, if the Z-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields YYXXYZZZZ, XYXXYZZZZ, YXXYZZZZ, YYYXZZZZ and YXXYZZZZ respectively, which are handled by Case 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields YXYZZZZZ, which is handled by Case



2. If the Z-path starting at the second vertex ends at the third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZZZ$  and  $XYXYZZZZ$  respectively, which are handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 5 9 4 3 6 7 8), (1 5 2 8 9 3 4 6 7), (1 6 3 4 7 2 5 8 9), (1 6 3 4 9 2 5 7 8), (1 6 4 5 8 2 3 7 9), (1 6 4 9 5 2 3 7 8), (2 3 4 9 5 1 6 7 8), (2 5 4 8 9 1 3 6 7), (2 6 3 4 9 1 5 7 8), (3 6 7 8 4 1 2 5 9), (3 7 1 6 2 4 5 8 9), (3 9 1 6 2 4 5 7 8), (3 9 2 4 1 5 6 7 8), (4 5 1 3 8 2 6 7 9), (4 5 2 8 9 1 3 6 7), (4 6 1 3 7 2 5 8 9), (4 6 1 3 9 2 5 7 8), (4 6 1 5 8 2 3 7 9), (4 7 2 3 6 1 5 8 9), (4 7 3 9 1 2 5 6 8), (4 8 5 7 3 1 2 6 9), (5 4 1 6 2 3 7 8 9), (5 4 3 6 7 1 2 8 9), (5 4 6 7 9 1 2 3 8), (5 4 6 9 1 2 3 7 8), (5 8 1 9 6 2 3 4 7), (5 8 6 7 1 2 3 4 9), (6 1 2 5 7 3 4 8 9), (6 1 2 8 9 3 4 5 7), (6 2 1 3 7 4 5 8 9), (6 3 7 9 5 1 2 4 8), (6 4 1 5 2 3 7 8 9), (6 4 3 5 7 1 2 8 9), (6 4 5 7 9 1 2 3 8), (7 3 4 5 8 1 2 6 9), (7 8 6 9 1 2 3 4 5), (7 9 4 5 8 1 2 3 6), (7 9 6 8 5 1 2 3 4), (8 1 2 6 9 3 4 5 7) and (8 4 6 7 9 1 2 3 5), we have also handled  $XXYZZYYZ$ ,  $XZYXYXZZ$ ,  $XZYXZYZZ$ ,  $XYZZYXYZ$ ,  $XZZYXZYZ$ ,  $XYYZZXYYZ$ ,  $YXXZZYYZ$ ,  $YXYZXYYZ$ ,  $YXZZYXYZ$ ,  $ZZXYZXYYZ$ ,  $XXYZZXYYZ$ ,  $XXZYXXYYZ$ ,  $XXZXYYYYZ$ ,  $YZYXXZZYZ$ ,  $YZYXXYYZ$ ,  $YZYXZXYYZ$ ,  $ZYXZYXYZ$ ,  $YZZXYXZYZ$ ,  $ZXXYZXYYZ$ ,  $ZYXZYXYZ$ ,  $ZZXYXZXYZ$ ,  $YYZXXYZZZ$ ,  $ZZYXXYYZZ$ ,  $YYXXZZYZ$ ,  $ZYXXZZYZ$ ,  $ZXXXYZXYZ$ ,  $XZZZYXXYZ$ ,  $XYZZYXYZ$ ,  $XZYXYXZZ$ ,  $YXYZZXYYZ$ ,  $YYXYZXZYZ$ ,  $XXZYXYZZZ$ ,  $ZZYXYXYYZ$ ,  $YYXXZXZYZ$ ,  $ZZXYYZXYZ$ ,  $ZXXXXZYYZ$ ,  $XXXYYXZYZ$ ,  $XXXXYYZYZ$ ,  $YZXXXZXYZ$  and  $XXXYXZZYZ$ .

For  $XXXYYZZZZ$ , if the Z-path starting at the fifth vertex doesn't end in  $H$  or ends at the first, second, third or fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYYXYZZZZ$ ,  $XYXYZZZZ$ ,  $YXYXYZZZZ$ ,  $YYXXYZZZZ$  and  $YYYYYZZZZ$  respectively, which are handled by Case 2. If the Z-path starting at the second vertex doesn't end in  $H$  or ends at the fourth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYXYZZZZ$  and  $XYXXYZZZZ$  respectively, which are handled by Case 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YXXYYZZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 6 7 8 9), (2 7 6 5 1 3 4 8 9), (3 8 4 1 5 2 6 7 9), (3 9 1 5 2 4 6 7 8), (3 9 1 6 2 4 5 7 8), (3 9 5 1 2 4 6 7 8), (3 9 6 1 2 4 5 7 8), (5 8 6 7 1 2 3 4 9), (5 8 7 6 1 2 3 4 9), (7 9 1 5 8 2 3 4 6), (7 9 4 5 8 1 2 3 6), (7 9 5 1 8 2 3 4 6) and (7 9 5 4 8 1 2 3 6), we have also handled  $XXYXYZZZZ$ ,  $XYZZXYYZ$ ,  $XZYXZZYZ$ ,  $ZXZYXYYYZ$ ,  $ZXZYXYYZ$ ,  $XXZYZYYYZ$ ,  $XXZYYZYYZ$ ,  $XZZZYXYZ$ ,  $XZZZYXYZ$ ,  $ZXXXYYXZYZ$ ,  $XXXZYXZYZ$ ,  $YXXXZXZYZ$  and  $XXXYZXZYZ$ .

For  $XXZYZYZZZ$ , if the Z-path starting at the second vertex ends at the fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields  $XYZXZYZZZ$  and  $YXZXZYZZZ$  respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex ends at the sixth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYZYZYZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by (1 2 4 3 5 9 6 7 8), (1 6 3 2 4 7 5 8 9), (3 7 1 4 2 6 5 8 9), (3 8 1 4 2 5 6 7 9), (3 9 1 4 2 6 5 7 8), (4 6 1 5 2 8 3 7 9), (4 6 1 7 2 9 3 5 8) and (5 8 1 7 2 9 3 4 6), we have also handled  $XXZYYYYYZ$ ,  $XYZZZXYYZ$ ,  $ZZYXZZXYZ$ ,  $ZZYXXZZYZ$ ,  $YYZXXYYYZ$ ,  $ZZZXYYXZYZ$ ,  $YYXYXZYZ$  and  $XXXXYXZYZ$ .

For  $YYZXZYZZ$ , if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $YXZXYZYZZ$ ,  $YXZYZYZZ$ ,  $YXZXXZYZZ$  and  $XYZYXZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $XXZYXZYZZ$ ,  $XXZXXZYZZ$  and  $XXZYZYZZ$  respectively, which are handled by Case 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $XYZXYZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 5\ 4\ 6\ 7\ 8\ 9)$ ,  $(1\ 3\ 2\ 4\ 7\ 5\ 9\ 6\ 8)$ ,  $(2\ 5\ 3\ 6\ 1\ 4\ 7\ 8\ 9)$ ,  $(3\ 1\ 2\ 4\ 5\ 6\ 8\ 7\ 9)$ ,  $(3\ 4\ 1\ 2\ 6\ 5\ 9\ 7\ 8)$ ,  $(3\ 5\ 2\ 1\ 6\ 4\ 8\ 7\ 9)$ ,  $(3\ 5\ 2\ 6\ 1\ 4\ 8\ 7\ 9)$ ,  $(5\ 7\ 1\ 6\ 3\ 2\ 9\ 4\ 8)$  and  $(6\ 7\ 1\ 8\ 4\ 2\ 9\ 3\ 5)$ , we have also handled  $YYZYXZYZZ$ ,  $ZYZXYYZY$ ,  $YYZZYXZY$ ,  $YZYXYZZY$ ,  $YXZZYZYZ$ ,  $XZYZYZY$ ,  $YZYZYXZY$ ,  $YYZYXZY$  and  $XXXZXZY$ .

For  $YYZXXZYZZ$ , if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the fourth, fifth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $YXZXXZYZZ$ ,  $YXZYXZYZZ$ ,  $YXZXYZYZZ$  and  $XYZYYZYZZ$  respectively, which are handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  or ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $XXZYYZZYZZ$ ,  $XXZXYZYZZ$  and  $XXZYXZYZZ$  respectively, which are handled by Case 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $XYZXXZYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(3\ 4\ 1\ 2\ 7\ 5\ 9\ 6\ 8)$ ,  $(3\ 5\ 2\ 1\ 6\ 4\ 8\ 7\ 9)$ ,  $(3\ 6\ 1\ 7\ 8\ 2\ 9\ 4\ 5)$ ,  $(7\ 4\ 1\ 2\ 3\ 5\ 9\ 6\ 8)$  and  $(7\ 6\ 1\ 3\ 8\ 2\ 9\ 4\ 5)$ , we have also handled  $YXZZYXZY$ ,  $XZYZYXZY$ ,  $XXZXXZY$ ,  $YXXZYZY$  and  $XXYXXZY$ .

For  $XXYZZYZZ$ , if the Z-path starting at the third vertex doesn't end in  $H$  or ends at the first, second or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XXXZZYZZ$ ,  $YXXZZYZZ$ ,  $XYXZZYZZ$  and  $YYYZZXYZZ$  respectively, which are handled by Cases 1 and 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YXYZZYZZ$ , which is handled by Case 2. If the Z-path starting at the second vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $XYYZZYZZ$ , which is handled by Case 1. If the Z-path starting at the seventh vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YYXZZXYZZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 7\ 6\ 4\ 5\ 3\ 2\ 8\ 9)$ ,  $(3\ 5\ 4\ 1\ 2\ 7\ 8\ 6\ 9)$ ,  $(4\ 6\ 9\ 2\ 3\ 1\ 5\ 7\ 8)$  and  $(5\ 8\ 7\ 1\ 2\ 4\ 3\ 6\ 9)$ , we have also handled  $YXXZZXYZZ$ ,  $ZZXYXZY$ ,  $ZYYXZXZY$  and  $ZZXXYZXY$ .

For  $YYZXZYZZ$ , if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the first, fourth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $YXZXZYZZ$ ,  $XXZXZYZZ$ ,  $YXZYZYZZ$ ,  $YXZXZYZZ$  and  $XYZYZXYZZ$  respectively, which are handled by Cases 1 and 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $XYZXZYZZ$ , which is handled by Case 1. If the Z-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XXZXZYZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(3\ 9\ 4\ 2\ 5\ 1\ 6\ 7\ 8)$ ,  $(4\ 9\ 1\ 2\ 5\ 3$

7 6 8), (5 8 1 6 2 4 3 7 9), (6 9 1 8 2 3 7 4 5) and (7 9 1 8 2 4 5 3 6), we have also handled ZXZYZZYYZ, YXZZYYZZ, ZZYYXXZZ, XXZXXZZY and XXXZZXZY.

For YYZXZXYZZ, if the Z-path starting at the second vertex doesn't end in  $H$  or ends at the first, fourth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXZXZXYZZ, XXZXZXYZZ, YXZYZXYZZ, YXZXZYZZ and XYZYZYZZ respectively, which are handled by Cases 1 and 2. If the Z-path starting at the first vertex doesn't end in  $H$  then doing a Z-Kempe change there yields XYZXZXYZZ, which is handled by Case 1. If the Z-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields XXZXZYZZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 1\ 6\ 2\ 4\ 3\ 7\ 9)$ , we have also handled ZZYXYXZYZ.

For XYYYYZZYZ, if the Z-path starting at the fourth vertex ends at the third, fifth or eighth vertex of  $H$ , then doing a Z-Kempe change there yields XYXXXZZYZ, XYYXYZZYZ and YXXYYZZYZ respectively, which are handled by Case 2. If the Z-path starting at the fifth vertex ends at the third or eighth vertex of  $H$ , then doing a Z-Kempe change there yields XYXYZZYZ and YXXXXZZYZ respectively, which are handled by Case 2. If the Z-path starting at the third vertex ends at the eighth vertex of  $H$ , then doing a Z-Kempe change there yields YXYXYZZYZ, which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(5\ 8\ 2\ 6\ 1\ 3\ 4\ 7\ 9)$ , we have also handled XYZZXYYYZ.

For ZYYYYXZYZ, if the Z-path starting at the fifth vertex ends at the third, fourth, sixth or eighth vertex of  $H$ , then doing a Z-Kempe change there yields ZYXYXXZYZ, ZYYXXXZYZ, ZYYYXYZYZ and ZXXXYYZYZ respectively, which are handled by Cases 1 and 2. If the Z-path starting at the eighth vertex ends at the third, fourth or sixth vertex of  $H$ , then doing a Z-Kempe change there yields ZXYXXYZYZ, ZXXYXYZYZ and ZXXXXXZYZ respectively, which are handled by Case 1. Since we already handled the permutation of all resulting boards by (3 6 7 8 1 4 5 2 9), we have also handled YYZXZYYYYZ.

**Case 4.**  $B$  is one of  $ZX\star YZXZYZ$ ,  $ZXZYZX\star YZ$ ,  $XYYYZXZYZ$ ,  $XXYZZXZYZ$ ,  $XZX YXZY YZ$ ,  $XZXYYZXYZ$ ,  $XZXYYXZYZ$ ,  $XZX YZXY YZ$ ,  $XXZY YXZY Z$ ,  $XXZY YXYZZ$ ,  $XXYZYXYZZ$ ,  $XXYXZXZYZ$ ,  $XXYZY YZY Z$ ,  $XXZX YZX YZ$ ,  $XXZXZXYYZ$ ,  $XZY YZXZY Z$ ,  $XXZY ZXZY Z$ ,  $XXZY ZX YZZ$ ,  $XZZZY ZX YZ$ ,  $XZZZZXY YZ$ ,  $XZZZYXZY Z$ ,  $XXZY YZZZZ$ ,  $XXYZYZZZZ$ ,  $YYZYZY Y YZ$ ,  $YYZX YZX YZ$ ,  $YYXZYXZY Z$ ,  $YXYZZXZY Z$ ,  $YXXZYXYZZ$ ,  $YYZX YXZY Z$ ,  $YYZXZXYY Z$ ,  $YZYXYXYZZ$ ,  $YZYXZXZY Z$ ,  $YYZY ZX YZZ$ ,  $YYZYZZZY Z$ ,  $YYZY YXYZZ$ ,  $YZYXY YZY Z$ ,  $YZYXZY Y YZ$ ,  $YZZZYXYZZ$ ,  $YZYXZZYZZ$ ,  $YZYXYZZZZ$ ,  $YZZZZXYZZ$ ,  $YXXZZZYZZ$ ,  $ZY YYY YZY Z$ ,  $ZYZXXY Y YZ$ ,  $ZYZXZXYY Z$ ,  $ZY YXZXZY Z$ ,  $ZYZXZY Y YZ$ ,  $ZY YYZXZY Z$ ,  $ZXZYXZX YZ$ ,  $ZXZY YZX YZ$ ,  $ZXXYYXYZZ$ ,  $ZZXY YXZY Z$ ,  $ZXZY YXZY Z$ ,  $ZYZXXY YZZ$ ,  $ZYZXYXYZZ$ ,  $ZXXYY YZY Z$ ,  $ZY YXYXZY Z$ ,  $ZXZY Y YXY Z$ ,  $ZZYXY YZY Z$ ,  $ZZYZY YZY Z$ ,  $ZZYZYXYZZ$ ,  $ZY YYYXYZZ$ ,  $ZYZXZZZY Z$ ,  $ZZYZZXZY Z$ ,  $ZZYXYZZZZ$ ,  $ZZYXZXZY Z$ ,  $ZZXZZXY YZ$ ,  $ZZXZXZY YZ$ ,  $ZYZXZX YZZ$ ,  $ZXXYZZZYZZ$  or  $ZXXYYZZZZ$ .

For  $XXYZYZZZZ$ , if the  $Z$ -path starting at the second vertex ends at the third or fifth vertex of  $H$ , then doing a  $Z$ -Kempe change there yields  $YXZYZZZZZ$  and  $YXXZYZZZZ$

respectively, which are handled by Cases 2 and 3. If the Z-path starting at the third vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $YYYZYZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 4\ 3\ 5\ 6\ 7\ 8\ 9)$ ,  $(1\ 6\ 5\ 2\ 8\ 3\ 4\ 7\ 9)$ ,  $(1\ 6\ 7\ 2\ 8\ 3\ 4\ 5\ 9)$ ,  $(1\ 7\ 2\ 4\ 3\ 5\ 6\ 8\ 9)$ ,  $(1\ 7\ 5\ 2\ 8\ 3\ 4\ 6\ 9)$ ,  $(2\ 3\ 4\ 1\ 5\ 6\ 7\ 8\ 9)$ ,  $(2\ 3\ 4\ 1\ 7\ 5\ 6\ 8\ 9)$ ,  $(2\ 6\ 4\ 1\ 8\ 3\ 5\ 7\ 9)$ ,  $(2\ 7\ 4\ 1\ 6\ 3\ 5\ 8\ 9)$ ,  $(3\ 5\ 7\ 1\ 8\ 2\ 4\ 6\ 9)$ ,  $(3\ 6\ 7\ 1\ 8\ 2\ 4\ 5\ 9)$  and  $(3\ 8\ 4\ 1\ 6\ 2\ 5\ 7\ 9)$ , we have also handled  $XXZYZZZZ$ ,  $XZZZYXZY$ ,  $XZZZZXYZ$ ,  $YXXZZZYZZ$ ,  $XZZZYXZY$ ,  $ZXXYYZZZ$ ,  $ZXXYZZYZ$ ,  $ZXZYXZY$ ,  $ZYZXZYZZ$ ,  $ZZXZXZY$  and  $ZZYXZXZY$ .

For  $YZYXYZZZ$ , if the Z-path starting at the third vertex ends at the fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $YZXYZZZZ$  and  $XZYZZZZZ$  respectively, which are handled by Cases 2 and 3. If the Z-path starting at the fourth vertex ends at the fifth vertex of  $H$ , then doing a Z-Kempe change there yields  $XZXXYZZZ$ , which is handled by Case 2. Since we already handled the permutation of all resulting boards by  $(1\ 2\ 3\ 4\ 7\ 5\ 6\ 8\ 9)$ ,  $(1\ 2\ 5\ 6\ 7\ 3\ 4\ 8\ 9)$ ,  $(1\ 2\ 8\ 6\ 9\ 3\ 4\ 5\ 7)$ ,  $(2\ 1\ 5\ 4\ 9\ 3\ 6\ 7\ 8)$ ,  $(2\ 1\ 7\ 4\ 9\ 3\ 5\ 6\ 8)$ ,  $(3\ 1\ 5\ 6\ 7\ 2\ 4\ 8\ 9)$  and  $(3\ 1\ 8\ 6\ 9\ 2\ 4\ 5\ 7)$ , we have also handled  $YZYXZZYZ$ ,  $YZZZYXZY$ ,  $ZYZZYXZY$ ,  $YZYXZYZZ$ ,  $ZZYZZYZZ$  and  $YYZYXYZZ$ .

For  $YYZYZZYZ$ , if the Z-path starting at the second vertex ends at the fourth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $YXZXZZYZ$  and  $XYZXZZYZ$  respectively, which are handled by Case 1. If the Z-path starting at the fourth vertex ends at the seventh vertex of  $H$ , then doing a Z-Kempe change there yields  $XXZYZZYZ$ , which is handled by Case 3. Since we already handled the permutation of all resulting boards by  $(3\ 5\ 1\ 6\ 2\ 4\ 8\ 7\ 9)$ , we have also handled  $ZZYZZYZ$ .

For  $YXYZZXZY$ , if the Z-path starting at the third vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing a Z-Kempe change there yields  $YXXZZXZY$ ,  $YXXZZYZ$  and  $XYYZZYZ$  respectively, which are handled by Cases 1 and 3. If the Z-path starting at the sixth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing a Z-Kempe change there yields  $YXYZZYZ$  and  $XYXZZXZY$  respectively, which are handled by Cases 1 and 3. If the Z-path starting at the second vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $YYYZZXZY$ , which is handled by Case 2. If the Z-path starting at the eighth vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $XYXZZYZ$ , which is handled by Case 1. Since we already handled the permutation of all resulting boards by  $(1\ 3\ 2\ 4\ 5\ 8\ 7\ 6\ 9)$ ,  $(3\ 8\ 2\ 1\ 5\ 4\ 7\ 6\ 9)$ ,  $(5\ 2\ 4\ 1\ 3\ 7\ 6\ 8\ 9)$ ,  $(5\ 4\ 2\ 1\ 3\ 8\ 6\ 7\ 9)$ ,  $(7\ 2\ 4\ 1\ 3\ 6\ 5\ 8\ 9)$ ,  $(9\ 3\ 4\ 1\ 2\ 6\ 5\ 7\ 8)$  and  $(9\ 7\ 3\ 1\ 2\ 4\ 5\ 6\ 8)$ , we have also handled  $XXYZZXZY$ ,  $ZXXYZXZY$ ,  $ZXZYXZY$ ,  $ZXZYXZY$ ,  $ZXZYXZY$ ,  $YXXZYXZY$  and  $YYZXZY$ .

For  $ZXYYZXZY$ , if the Z-path starting at the fourth vertex doesn't end in  $H$  or ends at the sixth or eighth vertex of  $H$ , then doing a Z-Kempe change there yields  $ZXYXZXZY$ ,  $ZXYXZY$  and  $ZYXYZY$  respectively, which are handled by Cases 1 and 2. If the Z-path starting at the sixth vertex doesn't end in  $H$  or ends at the eighth vertex of  $H$ , then doing a Z-Kempe change there yields  $ZXYYZY$  and  $ZYXXZXZY$  respectively, which are handled by Cases 1 and 3. If the Z-path starting at the eighth vertex doesn't end in  $H$  then doing a Z-Kempe change there yields  $ZYXXZY$ , which is handled by Case 1. Since

we already handled the permutation of all resulting boards by (1 4 8 2 5 6 7 3 9), (1 6 2 7 3 4 5 8 9), (1 8 7 2 3 4 5 6 9), (2 1 5 7 3 6 4 9 8) and (2 5 1 3 6 4 7 9 8), we have also handled ZYYXZXZYZ, ZYXZXXYYZ, ZXZYXXYZ, XYYYZXZYZ and ZYZXXYYYZ.

Each of ZZYXYZZZ, YYZYXZZ and XXZYXZZ have an odd number of X's and Y's, so there is a Z-path with exactly one end in  $H$ . For ZZYXYZZZ, if this is the third, fourth or fifth vertex of  $H$ , then doing a Z-Kempe change there yields ZZXXYZZZ, ZZYZZZZ and ZZXYYZZZ respectively, which are handled by Cases 2 and 3. Since we already handled the permutation of all resulting boards by (1 2 3 6 8 4 5 7 9), (1 3 2 4 7 5 6 8 9) and (2 3 1 6 7 4 5 8 9), we have also handled ZZYZZXZYZ, ZYZXZZYZ and YZZZXZZ.

For YYZYXZZ, if this is the first, second, fourth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields XYZYXZZ, YXZYXZZ, YYZXXYZ, YYZYZZ and XXZXZZ respectively, which are handled by Cases 1, 2 and 3. Since we already handled the permutation of all resulting boards by (3 5 1 6 2 4 8 7 9), we have also handled ZZYXYYZ.

For XXZYXZZ, if this is the first, second, fourth, sixth or seventh vertex of  $H$ , then doing a Z-Kempe change there yields YXZYXZZ, XYZYXZZ, XXZXXYZ, XXZYZZ and YYZXZZ respectively, which are handled by Cases 1, 2 and 3. Since we already handled the permutation of all resulting boards by (1 2 3 4 5 6 8 7 9), (1 3 2 4 5 8 6 7 9), (1 3 4 2 5 9 7 6 8), (1 7 2 4 3 9 6 5 8), (1 7 4 2 5 9 3 6 8), (2 5 1 4 3 7 6 8 9), (2 6 1 4 3 7 5 8 9), (2 8 1 4 3 9 6 5 7), (3 4 2 1 5 8 6 7 9), (3 5 1 4 2 9 6 7 8), (3 5 1 7 2 9 8 4 6), (3 6 1 5 2 9 8 4 7), (3 7 1 4 2 9 6 5 8), (4 5 1 2 3 8 6 7 9), (4 5 1 3 2 8 6 7 9), (4 7 3 1 5 9 2 6 8) and (5 7 1 3 2 9 8 4 6), we have also handled XXZYXZYZ, YZYXZXZYZ, ZXZYXXYZ, ZYXXYZ, ZXXYYYZ, ZYZXXYZ, ZYXZZ, YZYXXYZ, XZYXZYZ, YYZXXYZ, XXZXXYZ, XXZYXZYZ, YYZXXYZ, ZXZYXZYZ, ZZXYYXZYZ, XXYZZYZ and XXYXZZYZ.

For XXYZYXZZ, if the X-path starting at the seventh vertex ends at the fourth, fifth, eighth or ninth vertex of  $H$ , then doing an X-Kempe change there yields YYXXYZZ, YYXZZYZ, XXYZYXZYZ and XXZYXZYZ respectively, which are handled by Cases 1 and 3. If the X-path starting at the eighth vertex ends at the fourth, fifth or ninth vertex of  $H$ , then doing an X-Kempe change there yields XXYXXYZ, XXYZZYZ and YYZXZZ respectively, which are handled by Cases 1 and 2. Since we already handled the permutation of all resulting boards by (1 2 3 4 8 6 9 5 7), (1 2 5 3 8 6 4 9 7), (1 3 2 4 5 6 9 8 7), (1 3 2 4 7 6 9 8 5), (1 3 2 5 6 7 9 4 8), (1 3 2 7 6 5 9 4 8), (1 5 2 4 3 7 6 8 9) and (1 8 2 4 3 9 6 5 7), we have also handled XXZYXZZ, XXZYXZYZ, XZYXZYZ, XZYXZYZ, XZYXZYZ, YXXZYXZZ and ZXXYYXZZ.

For ZYYYXZYZ, if the Y-path starting at the sixth vertex doesn't end in  $H$  or ends at the seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYYYZZYZ, ZYYYZZYZ and XYYYXXYZ respectively, which are handled by Case 1. If the Y-path starting at the seventh vertex doesn't end in  $H$  or ends at the ninth vertex of  $H$ , then doing a Y-Kempe change there yields ZYYYXZYZ and XYYYXZZYZ respectively, which are handled by Cases 1 and 3. If the Y-path starting at the fifth vertex doesn't end in  $H$  then

doing a Y-Kempe change there yields ZYYYYXXZYZ, which is handled by Case 1. If the Y-path starting at the ninth vertex doesn't end in  $H$  then doing a Y-Kempe change there yields XYYYYXZXYZ, which is handled by Case 1. Since we already handled the permutation of all resulting boards by (5 2 6 7 1 4 3 8 9), we have also handled ZYZXZYYYYZ.

Since ZYYYYYZYZ has an odd number of X's and Z's, there is a Y-path with exactly one end in  $H$ . If this is the first, seventh or ninth vertex of  $H$ , then doing a Y-Kempe change there yields XYYYYYZYZ, ZYYYYYXYZ and XYYYYYXYZ respectively, which are handled by Cases 1 and 3. Since we already handled the permutation of all resulting boards by (3 1 2 4 6 7 5 8 9), we have also handled YYZYZYYYYZ.  $\square$