Burning Candle Seesaw

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We analyze the oscillatory motion of a candle supported by a transversely punctured sewing needle as it is being burnt from both ends. From our analysis we determine that the oscillations are initialized by the asymmetric drip rate of the wax from either end of the candle, and are driven by the conversion of gravitational potential energy to rotational kinetic energy as the liquefied wax drips off either end of the candle.

A seesaw is a specific type of lever. As most children know, one can make a seesaw oscillate by placing different masses away from the fulcrum. To bring the seesaw back into balance, one needs to place the same mass at the same distance away from the fulcrum, on the other side of the seesaw. The oscillations of the seesaw are due to a non-zero net torque acting on either end. If one wanted to keep the seesaw oscillating indefinitely (assuming no friction or air resistance), there would need to be a way to ensure the torque remained non-zero, while also keeping either end relatively in balance so as to keep the seesaw from striking the ground or going vertical. One way to accomplish this would be to slowly lose mass from either end as they reach the bottom point of the oscillation. This loss of gravitational potential energy would be converted into rotational kinetic energy, propelling the now lighter end upwards and prompting the now heavier end to fall downward, where it too will have some of its mass siphoned off. A candle supported by a sewing needle stuck transversely through the center with both ends lit will accomplish this. The needle is supported on either end by wine glasses, so there is very little friction keeping the candle from rotating freely and the glasses are tall enough that if the rotational amplitude becomes large the candle will not come into contact with the surface beneath it, as seen in Fig. 1. The ends of the candle are then lit. No matter the starting angle of the candle, once the ends are lit it will eventually level off into a more horizontal alignment (still with oscillatory motion). This is because the lower end of the candle will lose wax more quickly, thus exposing more of the wick [1]. Since the lower end is losing mass more rapidly than the higher end, they will eventually come closer into balance with each other, but due to the low friction on the sewing needle the candle will keep oscillating, and during the instant one of the ends reaches the bottom of the oscillation it will lose slightly more wax than the more upright end, thus driving the oscillation. We therefore expect the oscillations to grow in time until the point at which the length of the candle shortens to the point that the amplitude begins to decrease. A candle starting in a perfectly balanced horizontal position before lighting the ends will still eventually display the oscillating behavior. We hypothesize this is because the two sides of our candle are

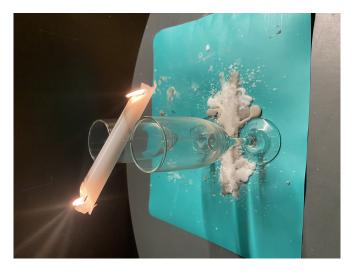


Figure 1. A standard candle is pierced approximately through the center by a sewing needle, balanced between two champagne flutes and allowed to rotate freely.

not completely symmetric. This will cause both ends to still burn at a slightly different rate, even with the candle perfectly horizontal, which is what begins the oscillations as already explained. A thought provoking question is, "What will happen to a perfectly symmetric candle which begins in a perfectly balanced horizontal position?" We hypothesize that if everything about the candle is symmetric the wax from either end will melt away at the exact same rate and no oscillations will occur. We attempt to answer this by creating a simulation mimicking the oscillatory behavior of the candle (not done yet). We also set out to calculate the average angular velocity of the oscillating candle. We know the equation for angular velocity is given by

$$\omega = \frac{\Delta \theta}{\Delta t} \tag{1}$$

with $\Delta\theta$ the rotation angle and Δt the time. We will then first have to calculate the rotation angle $\Delta\theta$ by using

$$\Delta\theta = \frac{\Delta s}{r} \tag{2}$$

with Δs being the arc length and r being the radius. In order to calculate rotational velocity then, we need to

know the arc length. The arc length can be calculated using

$$\Delta s = 2r \cdot \arcsin\left(\frac{L}{2r}\right) \tag{3}$$

with L being the chord length of the oscillation given by the x and y coordinates. Using Eqs. 1, 2 and 3 we can eventually find the rotational velocity of the oscillating candle. As mentioned above, we believe the onset of instability (oscillations) is caused by the candle not being perfectly symmetric. We have not yet come up with a way to test this, since we have no way of making a perfectly symmetric candle. (One potential way to test this would be to attempt to make different candles of varying degrees of symmetry, and see if the "more symmetric" candles had a delayed onset of oscillatory behavior)

We recorded the burning candle's motion on a standard cell phone camera, analyzed the motion using a tracking software package, then used Python to plot the data points in order to obtain physical values such as rotational position and velocity. We were able to determine distances by including a ruler in the frame of the video, then on the tracker software could use the ruler in frame to indicate how many pixels a certain length was. There is some uncertainty introduced here; in the actual distance that the tick marks appear on the video, how much does the screen distort these markings?(not answered vet) The tracker software itself also had some bugs that could affect the motion analysis. We needed to mark a spot on the side of the candle to give the software a "point" to track and we chose to mark the flame on one end as this point. The issue with doing this is that the flame was not one fixed-size point, it changed shape as the candle moved and this caused the software to misinterpret what it was "seeing" every so often. This is why in Fig. 2 there is the occasional spike in the plot, these correspond to points where the software lost track of the point. We do not think these introduce much of a problem however, as we can still clearly see the basic growth of the oscillations. A simple way to fix this problem (which we plan on doing before the final report) is rather than track the flame, mark a spot on the side of the candle with a marker. This may be an improvement because this mark would not change size or shape and may be easier for the software to follow. As can be seen from Fig. 2, the oscillations grow in time and then, once the candle loses too much of its length, the amplitude of oscillations goes down. The y-coordinates in Fig. 2 become more negative because of how the axes were set. Before starting the experiment, the candle was held horizontally, and the end we were going to be tracking is where the origin was set. Another quirk of the tracking software is that the axes are rotated 90° from what one would normally call "x" and "y". This means that in Fig. 2 the y-coordinate corresponds to the horizontal coordinate and it gets more negative in time because the

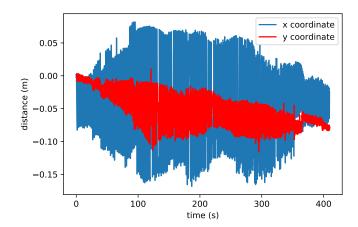


Figure 2. A plot showing the oscillatory motion of a burning candle allowed to rotate freely with very little friction. We can see the oscillations grow in time before dying out as more of the candle is burned away. The narrow gaps throughout the image are from a lack of data for those points, as occasionally the tracker software would encounter 'bad frames' that had to be ignored.

length of the candle is decreasing (so the flame is shifting to more and more negative of a y value). Using the plotted data points to determine the chord length L during a given oscillation and from Eqs. 1, 2 and 3 we can determine the average angular velocity value of (insert value here once computed).

discussion not written yet as we are not done all the experimentation and data analysis. Will include how the computer modeling of the candle's behavior agreed or disagreed with our predictions, as well as how testing candle's with varying degrees of symmetry went.

conclusion not yet written since we are not done the experimental aspect of the lab yet

[1] NASA, Candle Flames, https://www.nasa.gov.