Determining the Orbital Eccentricity of the Moon With a Telescope

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Most celestial objects have elliptical orbits, and the eccentricity is used to determine how far the elliptical orbit is deviated away from a circular orbit. In this report, we used Orion StarBlast Telescope to observe the moon over the course of the moon cycle and determine the eccentricity of the moon. The eccentricity of the moon was found to be (some number) which is an elliptical orbit.

Introduction- Johannes Kepler (1571-1630) was a German astronomer, mostly commonly known for is work in quantifying planetary motion. Partially guided by his religious beliefs, he sought to develop a theory that could quantify the orbital mechanics of the planets Citation needed. Since the mathematical system that describes the motion and interaction between any arbitrary number of bodies in three dimensions as exceedingly complicated, Kepler considered a simplified version of the problem: concerning only two bodies.

Kepler considered a system of only two masses, m_1 and m_2, each with a position vector, r_1 ad r_2 respectively. Using a 2-dimensional polar coordinate system, we set $m1$ to be identically at the origin. We denote the separation between the two massive bodies as $\vec{r} \equiv r_1 - r_2$.Kepler modeled the mutual force of attraction of one body on the other as being inversely proportional to the square of the distance between them, $F \propto \frac{1}{(\vec{r})^2}$. We call this arrangement the *two-body central-force problem* [1, 2]. He derived an analytical solution to model the orbit, which describes the separation of the two bodies $\vec{r}$ as a function of angular displacements, ϕ. It was found that [1, 2]:

$$\vec{r}(\phi) = \frac{\alpha}{1 + \epsilon \cos(\phi)} \tag{1}$$

The constant α is the *semi-latus rectum* and is found by combining the reduced mass of the system, $\mu = \frac{m_1 m_2}{m_1 + m_2}$, a force constant $\gamma = G m_1 m_2$, and the orbital angular momentum, $l = \mu(r_2 - r_1)^2 \dot{\phi}$, such that [1]:

$$\alpha \equiv \frac{l^2}{\gamma \mu} \tag{2}$$

This quantity measures the separation of the two bodies when $\phi = \pm\pi/2$ radians

We should consider using something like Fig. 8.10 in Taylor to better give the geometry of the situation.

The constant ϵ measures the *orbital eccentricity* of the system. This value is a real, dimensionless measurement of how "circle-like" the orbit is [2]. By examining the extreme cases for Eq. (1), we can see how the eccentricity changes the properties of the orbit. The minimum and maximum separations are given when $\cos(\phi = 0)$ and $\cos(\phi = \pi)$. The distance of separation then becomes:

$$r_{min} = \frac{\alpha}{1 + \epsilon} \qquad r_{max} = \frac{\alpha}{1 - \epsilon} \tag{3}$$

In the case of $\epsilon = 0$, then $r_{min} = r_{max}$ and the radius of the orbit is constant, this a perfectly circular. In the case of $0 < \epsilon < 1$, $r_{min} \neq r_{max}$, but still finite and postive- thus an ellipse is created. For $\epsilon = 1$, r_{max} goes to infinity, indicating that the orbit is open, and the paths of the bodies will no longer interact. Similarly, if $\epsilon > 1$, then the $r_{min} > r_{max}$.

With these properties, we can state that the value of ϵ places the orbit into one of four categories:

Eccentricity	**Type**	**Category**
$\epsilon = 0$	Closed	Perfectly circular orbit
$0 < \epsilon < 1$	Closed	Elliptical orbit
$\epsilon = 1$	Open	Parabolic orbit
$\epsilon > 1$	Open	Hyperbolic orbit

We should consider using something like Fig. 8.11 in Taylor to show this graphically

I'm going to finish this intro tomorrow afternoon

Methodology- We used Orion StarBlast Telescope to observe the moon and attached a camera to the telescope to capture the image of the moon. We focused on the surface of the moon and marked the focus point. This allows the images to have a consistent size from session to session. We then calculate the diameter of the moon image by using pixel counter tool and converted pixel unit to meters. The diameter of the image produced on the film at the prime focus is $s = \frac{I}{M}$, where I is the diameter of the moon image in meters, and $M = \frac{f_o}{f_e} \approx 150$ is the magnification which defined as the ratio of the telescope's focal length ($f_o = 450$mm) and the eyepiece's focal length ($f_e \approx 3$mm). The moon's angular size is defined as,

$$\theta = \frac{s}{f_o}, \tag{4}$$

where s is the diameter of the moon at prime focus on the film. The distance to the moon can be calculated as,

$$r = \frac{D}{\theta}, \tag{5}$$

where D is the known diameter of the moon.

After obtaining more data, we will be able to construct the moon orbital path using the distance to the moon. From this, we can obtain the eccentricity of the moon.

Figure 1. Moon image on 10/25/2020 at 7:55pm

Results- So far we have three moon images. Here is one

Discussions-

Conclusions-

Acknowledgements-

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[1] "Two-Body Central Force Problems." *Classical Mechanics*, by John Robert Taylor, University Science Books, 2005, pp. 293–320.

[2] "Central-Force Motion." *Classical Dynamics of Particles and Systems*, by Stephen T. Thornton and Jerry B. Marion, 5th ed., Cengage Learning, 2014, pp. 287–323.