Understaning Binary Black hole Merger Events through Gravitational Wave Detection and Analysis

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In this work, we explore a system of analysing LIGO Gravitational Wave data to compute properties of a binary black hole merger event. Based on multi-parameter best-fit estimations, and currently known procedures on processing gravitational waves, we show how to extract the important signal and features in LIGO data. We show the process of using cleaning, filtering, and fitting raw data to extract a series of parameters that allow for clean representations of each signal. From these parameters, and additional known information, we also show key properties about the event that generated the waveform such as total system mass, chirp mass, and luminosity distance. We compare the attained results to previously computed results to ensure proper execution and reasonably uncertainties.From the fit parameters, we have found a chirp mass of the event GW50914 to be $30.0 \pm 1.4 M_\odot$ and event GW171814 to be $25.0 \pm 1.2 M_\odot$. These result concur with previous computations in the paper that outlined their initial discovery,

Introduction- By 1920, the formulation of the Theory of General Relativity (GR) was underway and consequently led to the prediction of the*gravitational wave* phenomena [3]. Einstein found that linearized weak-field equations had solutions that indicated a wave-like behavior of space time. When exceptionally massive objects accelerate, they cause a strain effect in space-time which produces transverse waves that propagate radially, at the speed of light through space [1]. It was not until 2016, almost 100 years after their conception, that experimental evidence of this phenomenon was detected for the first time. Two Laser Interferometer Gravitational Wave Observatory (LIGO) detectors measured an oscillating pattern in space-time that matched with the predicted in-spiral, and merger event of a binary black hole system [1][4].

There are two LIGO detection facilities in the United States, both of which are made up of a pair of orthogonal arms, roughly 4km in length. Each of the arms contains a cylindrical vacuum chamber with a coaxial laser [4]. The combination of the laser is used to produce specific interference pattern. Any changes to the shape of local space-time, such a a gravitational wave, will cause a small difference in the path of the laser, thus inducing a small phase-shift, which subsequently changes the interference pattern [1][5]. The interferometer gets it's name from the measurement of this change in interferences, which leads to the measurement of the transversal movement of space-time: a gravitational wave [4].

Since the initial measurements, much of the data from the detected event has been made publicly available for educational or personal use [2][4]. For this experiment, we use raw interference data and subject it to a series of preprocessing steps. From the resulting data, we are able to derive various key features of the merger event that enable a better comprehension of the behavior of gravitational waves and binary black hole systems. These features are includes, but not limited to (i) chirp mass, (ii) total mass, (iii) relative velocities, and (iv) Luminosity distance related to the event's ring-down.

Experimental Methodology- In order to process and analyze the data from LIGO, we follows the steps provided by the LIGO-Virgo scientific collaboration [2]. The provided Python notebook provides several packages that allows us to interact with the LIGO data using Python tools.

The analysis process begins by subjecting the signal to a high-pass filter to reduce the presence of low-frequency oscillation terms which introduce error in the time-domain representation of the signal. We additionally down sample the signal from 16 kHz to 2048 Hz. These modification introduce large amplitude spikes at both the starting and ending of the signal, which are fixed by simply removing 2 seconds of time, or 4096 data samples from either end of the array. For the purpose of this process, we shown only the steps as applied "GW150914" event, from the the Livingston observatory.

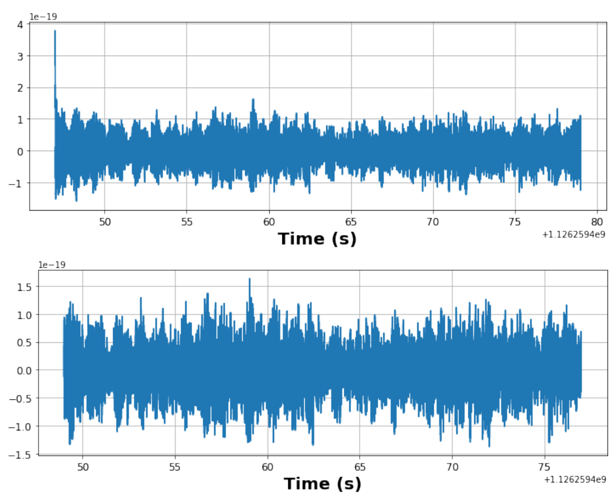

Figure 1. Time series signal (a) before and (b) after initial cleaning steps

To better understand the remaining data, we must produce a fit function and use characteristic parameters of that fit to uncover the required data. We continue by computing the power spectral density (PSD) of a 4-second or 8192 sample frame of time. The PSD density of a signal describes the distribution of the energy of a signal in frequency-space, and is interpolated to match the size of the data [2, 6].We then use the reciprocal of the PSD array as a filter with a window length of 8192 samples. Since the data has been previously subject to a high-pass filter, we indicate this when using an inverse spectrum truncation function provided by pycbc.

We use this newly computed PSD in part to generate a *matched filter*. Matched filtering involves computing the inner product of the signal data and a set of appropriate weighting frequencies [2]. A large mutual projection of the two arrays shows that the template and data align properly. We proceed by computing the signal-to-noise (SNR) ratio of the time-series waveform. This is done using a matched filter function along with our template, power spectral density and the cutoff frequency in the high-pass filter. The SNR represents the signal's power to the noise's power at any given sample [6]. A large SNR is indicative of a sample that contains a large amplitude, with little background noise to interrupt the data. We show the SNR as a function of time in the figure below.

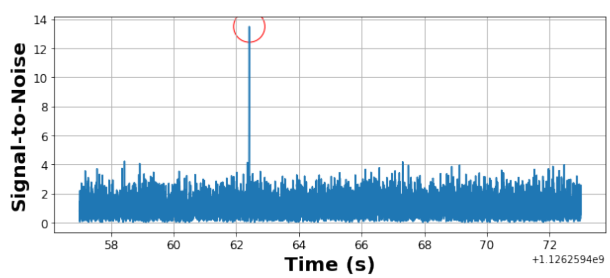

Figure 2. Signal-to-Noise ratio as a function of time. The maximum value is highlighted with a red circle.

With the SNR computed, we choose the peak value as our proposed binary black hole merger event, and align the proposal to the signal data [2]. To ensure consistency between the signal and the template waveform, we subject their frequency response curves to a band-pass filter with a range of 30 Hz to 300 kHz. We can visualize the template function compared to the raw data to understand how the two relate.

To model the waveform of the merger in-spiral, we choose an exponentially growing sinusoidal hypothesis space as given in Eq. (1).

$$f(t) = Ae^{\gamma t} \cos\big(\omega(t)t + \delta\big) \tag{1}$$

This allows to account for the increasing amplitude with the exponential function, as well as standard oscillations covered by the cosine. We also choose to model the

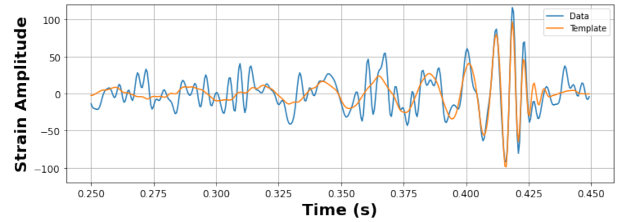

Figure 3. Raw Signal (blue) compared to the generated template signal (orange)

changing frequency of the signal by representing the angular frequency as a function of time: $\omega(t)$. We choose $\omega(t)$ to account for any time t in relation to the time of merger, t_m such that:

$$\omega(t) = \frac{\omega}{t - t_m} \tag{2}$$

The denominator accounts for the increase in frequency up to the merger time. We apply a curve-fitting function in python that allows for the selection of each parameter as to best model the function.

We pass Eq. (1) into a curve-fit function that optimizes the parameters in the hypothesis space to minimize the difference between the fitting function and the raw data. We apply this same process verabitm to the data collected simultaneously to from the Hanford Observatory. The results of both pre-processing, and fitting stages are shown the figure below.

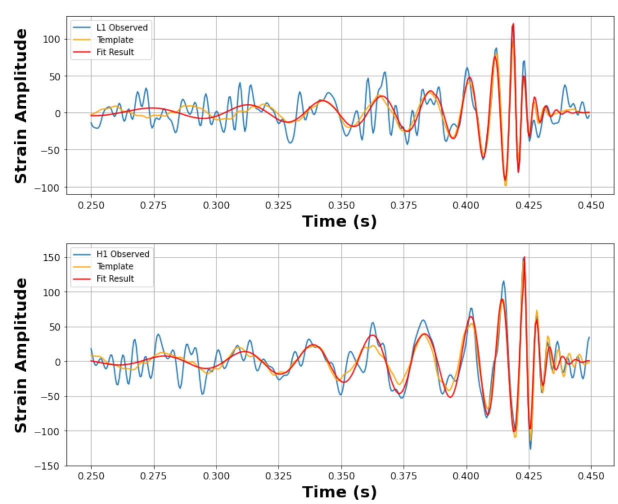

Figure 4. Raw Signal (blue) compared to the generated template signal (orange) and the curve optimizer fit (red). Results from Livingston Observatory and Hanford Observatory are labeled accordingly

The chrip mass of the signal is used to convey how the fundamental frequency of the in-spiral signal evolves with time. It relates the frequency f to the change in frequency with respect to time, $\dot{f}$, such that [1]:

$$\mathcal{M} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3}f^{-11/3}\dot{f}\right]^{3/5} \tag{3}$$

Following the standard conventions of error propagation, we can describe the uncertainty in the chirp mass measurement:

$$\delta\mathcal{M} = \left[\left(\frac{\partial M}{\partial f}\frac{\lambda}{t^2}\delta t\right)^2 + \left(\frac{\partial M}{\partial \dot{f}}\frac{2\lambda}{t^3}\delta t\right)^2\right]^{\frac{1}{2}} \tag{4}$$

Where λ gives the wavelength of the gravitational wave during the in-spiral, and δt is temporal uncertainty. We compute f by averaging the fundamental frequency in the signal over the course of the merger. Similarly, we compute $\dot{f}$ by taking the average of the first discrete time-derivative of the signal. We show the prediction and uncertainty for the chirp mass of the merger in Fig.(5) Numerical results and uncertainties are in the table in Fig. (8)

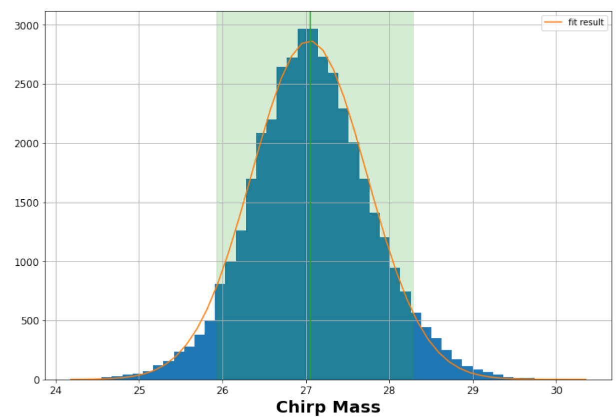

Figure 5. Chirp mass and uncertainty is measurements

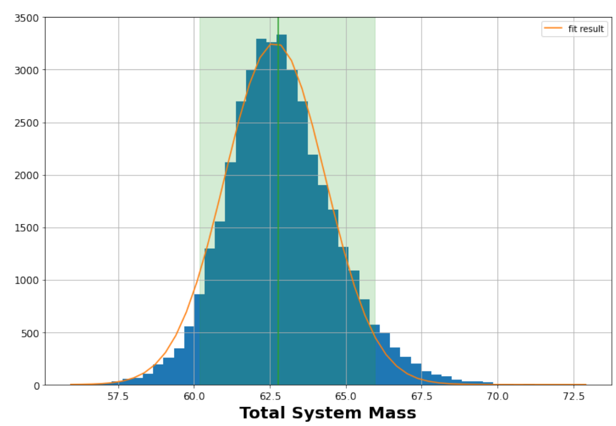

Figure 6. Total mass and uncertainty is measurements

We use a software package, *Bilby*, to compute the total system mass, chirp mass, and luminosity distance using a Bayesian inference method. We provide Bilby with a set of predicted parameters, and a set of most-likely results is returned. These results and uncertainties are shown in Fig. (5) and Fig. (6). We use the $L1$ chirp mass, attained from evaluating Eq. (3) and the corresponding uncertainty, along with a predicted mass-ratio. We define this mass ratio to be m_2/m_1 with $m_2 < m_1$. As previously mentioned, we presume $m_1 \approx m_2$, therefore $m_2/m_1 \approx 1$.

After applying the appropriate values, and generating a reasonable total mass prediction, we compute the relative velocity of both bodies for the in-spiral event, using [1]

$$\frac{v}{c} = \left[\frac{GM\pi f}{c^3}\right]^{1/3} \tag{5}$$

Evaluating this expression with the data, and plotting the results, we can see how the detectors are able to measure the black holes' relative velocities. Below we visualize the relative velocities with respect to time from both the Livingston and Hanford detectors.

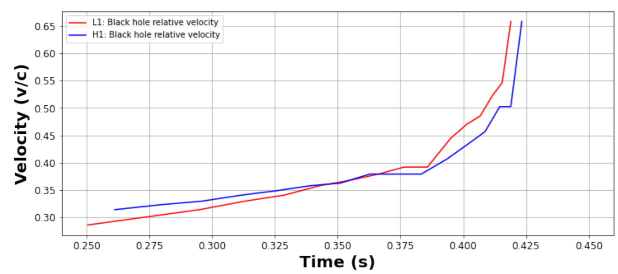

Figure 7. Relative merger velocities as measured from both LIGO faciltities

We can also use the fit parameters to estimate the total system mass.

Property	**GW150914**	**GW170814**
Fitted Chirp-Mass	$27.0 \pm 1.7M_\odot$	$25.0 \pm 1.2M_\odot$
Bilby Chirp-Mass	$31.4 \pm 0.4M_\odot$	$26.0 \pm 1.0M_\odot$
System Mass	$72.2 \pm 1.4M_\odot$	$55.7 \pm 1.2M_\odot$
Luminosity Distance	$442 \pm 216Mpc$	$622 \pm 157Mpc$

Figure 8. Experimental result and uncertainties in tabular form

Discussions & Conclusions- This data presents us with experimental evidence of the existence and behavior of the gravitational wave phenomenon as postulated by the Theory of General Relativity. The result computed and displayed concur with the results presented in [1]. We can therefore conclude the validity of the pre-processing and filter steps to extract and fit parameters to a gravitational wave. These parameters then uncover further characteristics of the merger event which can be used to estimate the chirp mass, total mass, and luminosity distance of the event. This provides quick and efficient parameter estimations which enables similar tools to be applied to future observations.

Future experimentation's on the data would warrant a more organized set of instructional jupyter notebooks and a more concise explanation and execution of data processing. Despite the non-standard data formatting, and unclear motivations for steps, it is possible to extracted the needed information for these gravitational waves. Further studies can build upon the group work laid out here, and provide a cleaner procedure, and more standard numerical processing tools to explore the data for this lab.

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