

Understaning Binary Black hole Merger Events through Graviational Wave Detection and Analysis

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(Dated: October 8, 2020)

In this work, we explore a system of analysing LIGO Gravitational Wave data to compute properties of a binary black hole merger event. Based on multi-parameter best-fit estimations, and currently known procedures on processing gravitational waves, we show how to extract the important signal and features in LIGO data. We show the process of using cleaning, filtering, and fitting raw data to extract a series of parameters that allow for clean representations of each signal. From these parameters, and additional known information, we also show key properties about the event that generated the waveform. We explore a control of the uncertainty

Introduction- By 1920, the formulation of the Theory of General Relativity (GR) was underway and consequently led to the prediction of the *gravitational wave* phenomena [3]. Einstein found that linearized weak-field equations had solutions that indicated a wave-like behavior of space time. When exceptionally massive objects accelerate, they cause a strain effect in space-time which produces transverse waves that propagate radially, at the speed of light through space [1]. It was not until 2016, almost 100 years after their conception, that experimental evidence of this phenomenon was detected for the first time. Two Laser Interferometer Gravitational Wave Observatory (LIGO) detectors measured an oscillating pattern in space-time that matched with the predicted in-spiral, and merger event of a binary black hole system [1][4].

There are two LIGO detection facilities in the United States, both of which are made up of a pair of orthogonal arms, roughly 4km in length. Each of the arms contains a cylindrical vacuum chamber with a coaxial laser [4]. The combination of the laser is used to produce specific interference pattern. Any changes to the shape of local space-time, such a a gravitational wave, will cause a small difference in the path of the laser, thus inducing a small phase-shift, which subsequently changes the interference pattern [1][5]. The interferometer gets it's name from the measurement of this change in interferences, which leads to the measurement of the transversal movement of space-time: a gravitational wave [4].

Since the initial measurements, much of the data from the detected event has been made publicly available for educational or personal use [2][4]. For this experiment, we use raw interference data and subject it to a series of preprocessing steps. From the resulting data, we are able to derive various key features of the merger event that enable a better comprehension of the behavior of gravitational waves and binary black hole systems. These features are includes, but not limited to (i) chirp mass, (ii) signal-to-noise ratio, (iii) total mass, (iv) relative velocities, and (v) physical quantities related to the event's ring-down.

Experimental Methodology- In order to process and analyze the data from LIGO, we follows the steps provided by the LIGO-Virgo scientific collaboration [2]. The provided Python notebook provides several packages that allows us to interact with the LIGO data using Python tools.

The analysis process begins by subjecting the signal to a high-pass filter to reduce the presence of low-frequency oscillation terms which introduce error in the time-domain representation of the signal. We additionally down sample the signal from 16 kHz to 2048 Hz. These modification introduce large amplitude spikes at both the starting and ending of the signal, which are fixed by simply removing 2 seconds of time, or 4096 data samples from either end of the array. For the purpose of this process, we shown only the steps as applied "GW150914" event, from the the Livingston observatory.

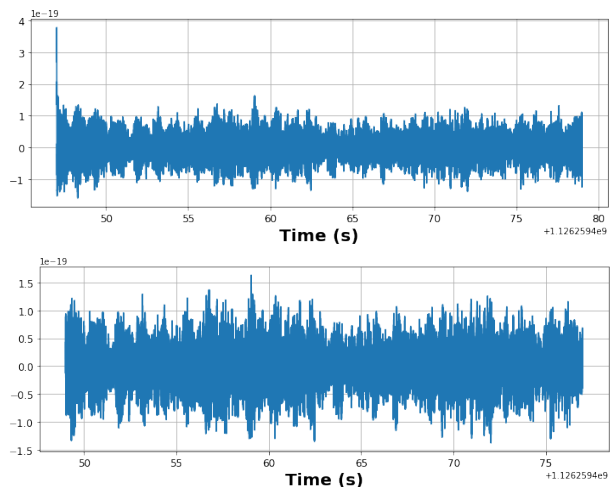


Figure 1. Time series signal (a) before and (b) after initial cleaning steps

To better understand the remaining data, we must produce a fit function and use characteristic parameters of that fit to uncover the required data. We continue by computing the power spectral density (PSD) of a 4-second or 8192 sample frame of time. The PSD density

of a signal describes the distribution of the energy of a signal in frequency-space, and is interpolated to match the size of the data [2, 6]. We then use the reciprocal of the PSD array as a filter with a window length of 8192 samples. Since the data has been previously subject to a high-pass filter, we indicate this when using an inverse spectrum truncation function provided by `pycbc`.

We use this newly computed PSD in part to generate a *matched filter*. Matched filtering involves computing the inner product of the signal data and a set of appropriate weighting frequencies [2]. A large mutual projection of the two arrays shows that the template and data align properly. We proceed by computing the signal-to-noise (SNR) ratio of the time-series waveform. This is done using a matched filter function along with our template, power spectral density and the cutoff frequency in the high-pass filter. The SNR represents the signal's power to the noise's power at any given sample [6]. A large SNR is indicative of a sample that contains a large amplitude, with little background noise to interrupt the data. We show the SNR as a function of time in the figure below.

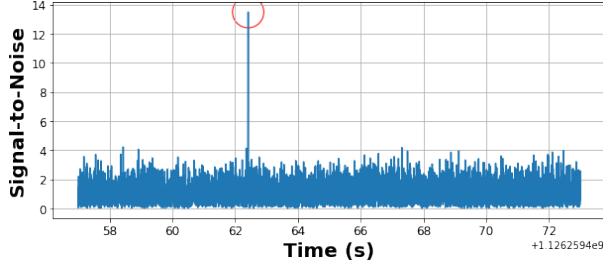


Figure 2. Signal-to-Noise ratio as a function of time. The maximum value is highlighted with a red circle.

With the SNR computed, we choose the peak value as our proposed binary black hole merger event, and align the proposal to the signal data [2]. To ensure consistency between the signal and the template waveform, we subject their frequency response curves to a band-pass filter with a range of 30 Hz to 300 kHz. We can visualize the template function compared to the raw data to understand how the two relate.

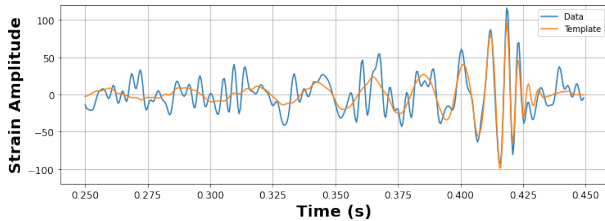


Figure 3. Raw Signal (blue) compared to the generated template signal (orange)

To model the waveform of the merger in-spiral, we choose an exponentially growing sinusoidal hypothesis

space as given in Eq. (1).

$$f(t) = Ae^{\gamma t} \cos(\omega(t)t + \delta) \quad (1)$$

This allows to account for the increasing amplitude with the exponential function, as well as standard oscillations covered by the cosine. We also choose to model the changing frequency of the signal by representing the angular frequency as a function of time: $\omega(t)$. We choose $\omega(t)$ to account for any time t in relation to the time of merger, t_m such that:

$$\omega(t) = \frac{\omega}{t - t_m} \quad (2)$$

The denominator accounts for the increase in frequency up to the merger time. We apply a curve-fitting function in python that allows for the selection of each parameter as to best model the function.

We pass Eq. (1) into a curve-fit function that optimizes the parameters in the hypothesis space to minimize the difference between the fitting function and the raw data. We apply this same process verbatim to the data collected simultaneously to from the Hanford Observatory. The results of both pre-processing, and fitting stages are shown the figure below.

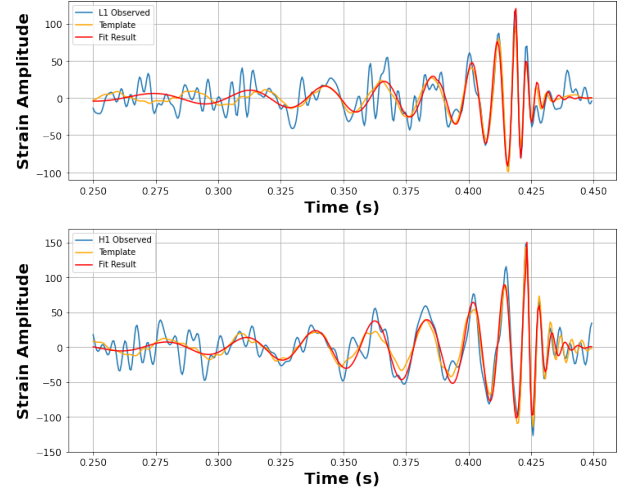


Figure 4. Raw Signal (blue) compared to the generated template signal (orange) and the curve optimizer fit (red). Results from Livingston Observatory and Hanford Observatory are labeled accordingly

The chirp mass of the signal is used to convey how the fundamental frequency of the in-spiral signal evolves with time. It relates the frequency f to the change in frequency with respect to time, \dot{f} , such that [1]:

$$\mathcal{M} = \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f^{-11/3} \dot{f} \right]^{3/5} \quad (3)$$

Following the standard conventions of error propagation, we can describe the uncertainty in the chirp mass mea-

surement:

$$\delta\mathcal{M} = \left[\left(\frac{\partial M}{\partial f} \frac{\lambda}{t^2} \delta t \right)^2 + \left(\frac{\partial M}{\partial \dot{f}} \frac{2\lambda}{t^3} \delta t \right)^2 \right]^{\frac{1}{2}} \quad (4)$$

Where λ gives the wavelength of the gravitational wave during the in-spiral, and δt is temporal uncertainty. We compute f by averaging the fundamental frequency in the signal over the course of the merger. Similarly, we compute \dot{f} by taking the average of the first discrete time-derivative of the signal. We hypothesize that each black hole in this merger event has a mass of roughly $36 M_{\odot}$. This brings warrants a combined mass of $72 M_{\odot}$ or roughly 1.45×10^{32} Kg.

We use a software package, *Bilby*, to compute the total system mass, chirp mass, and luminosity distance using a Bayesian inference method. We provide Bilby with a set of predicted parameters, and a set of most-likely results is returned. We use the *L1* chirp mass, attained from evaluating Eq. (3) and the corresponding uncertainty, along with a predicted mass-ratio. We define this mass ratio to be m_2/m_1 with $m_2 < m_1$. As previously mentioned, we presume $m_1 \approx m_2$, therefore $m_2/m_1 \approx 1$.

After applying the appropriate values, and generating a reasonable total mass prediction, we compute the relative velocity of both bodies for the in-spiral event, using [1]

$$\frac{v}{c} = \left[\frac{GM\pi f}{c^3} \right]^{1/3} \quad (5)$$

Evaluating this expression with the data, and plotting the results, we can see how the detectors are able to measure the black holes' relative velocities. Below we visualize the relative velocities with respect to time from both the Livingston and Hanford detectors.

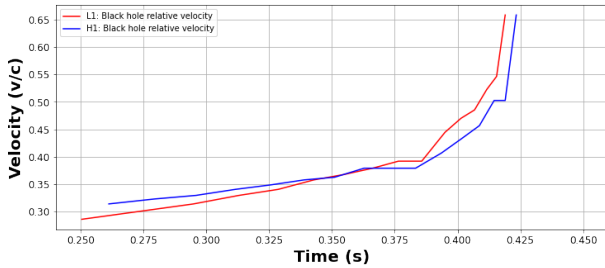


Figure 5. Relative merger velocities as measured from both LIGO facilities

Property	GW150914	GW170814
Fitted Chirp-Mass	$30.0 \pm 1.7 M_{\odot}$	$25.0 \pm 1.2 M_{\odot}$
Fitted Chirp-Mass	$30.0 \pm 1.4 M_{\odot}$	$23.0 \pm 1.5 M_{\odot}$
Bilby Chirp-Mass	$31.4 \pm 0.4 M_{\odot}$	$26.0 \pm 1.0 M_{\odot}$
System Mass	$66.5 \pm 1.4 M_{\odot}$	$55.7 \pm 1.2 M_{\odot}$
Luminosity Distance	$442 \pm 216 Mpc$	$622 \pm 157 Mpc$

Discussions- given the appropriate data and an unremarkable set of instructions provided by a series of python notebooks, we successfully derived the total system mass, our fitted chirp mass, the Bilby computed chirp mass, and the Luminosity distance of the binary black hole merger event. Using standard rules of signal processing and error propagation, we attained a series of reasonable measurements based on preexisting guesses. Using this combination of time-series wave forms and frequency-series predictors, we can show how a binary black hole merger causes a wave-like patterns in space-time which can be detected using a large interferometer observatory such as the two LIGO facilities.

This data presents us with experimental confirmation of the existence of gravitational waves. Not only can this behavior be detected, but the appropriate instructions, and tools enables us to work backwards, adhering to the postulates of general relativity to explore in detail, the events that caused these gravitational waves. Future work in this domain would benefit from a cleaner set of instructions and expectations. It would be interesting to take the results attained, and try to explore outside of the the standard physics principles, and try to use the uncertainties and variances in measurements to generate a similar, almost artificial data set. This data set could be used in the training of a neural network which could use similar predictors to generate a series of best-fit parameters to model these mergers and hopefully develop similar conclusions.

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