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MATH 753.01
HW 04

$\boxed{2.2.2}$ Find LU Factorization. Check w/ mat.mul

a)

$2 \qquad\qquad 1 \qquad\qquad 0$

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow{R_2 = R_2 - (2)R_1} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 3 & 1 & 5 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \hat{U}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

- Check $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \rightarrow LU = A$ ✓

b)

$1 \qquad\qquad \frac{1}{2} \qquad\qquad -\frac{1}{2}$

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow{R_2 = R_2 - R_1} \begin{bmatrix} 4 & 2 & 0 \\ 0 & -2 & 2 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow{R_3 = R_3 - \frac{1}{2}R_1} \begin{bmatrix} 4 & 2 & 0 \\ 0 & +2 & 2 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_3 = R_3 - (+\frac{1}{2}) R_2}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & +2 & 2 \\ 0 & 0 & 2 \end{bmatrix} = U \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & +\frac{1}{2} & 1 \end{bmatrix} = L$$

- Check

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 2 \end{bmatrix} \rightarrow LU = A \checkmark$$

$C_{2,1} = 0$ $C_{3,1} = 1$ $C_{3,2} = 2$

c) $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2}$

$C_{3,\phi} = 1$

$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow{R_4 = R_4 - R_2} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ $C_{3,2} = 0$

$= U$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = L$

- Check:

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \rightarrow LU = A$ ✓

$\boxed{2.2.4}$ Solve using LU Factorization. Use 2-Step back

from 2.2.2

a) $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A\vec{x} = \vec{b} = LU\vec{x} = \vec{b}$, let $\vec{c} = U\vec{x}$, $\vec{b} = [0 \; 1 \; 3]^T$

i) Solve $L\vec{c} = b$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 3 \end{array}\right]$ $\begin{aligned} x &= 0 \\ y &= 1 \\ z &= 3 \end{aligned}$ $\rightarrow \vec{c} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

ii) Solve $U\vec{x} = c$

$\left[\begin{array}{ccc|c} 3 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 3 \end{array}\right] = $ $\begin{aligned} x &= -1 \\ y &= 1 \\ z &= 1 \end{aligned}$ $\rightarrow \boxed{\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}$

b) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

i) Solve $L\vec{c} = \vec{b}$ $\quad b = [2 \; 4 \; 6]^T$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 4 \\ 1/2 & 1/2 & 1 & 6 \end{array}\right] \rightarrow$ $\begin{aligned} x &= 2 \\ y &= 2 \\ z &= 4 \end{aligned}$ $\rightarrow \vec{c} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

ii) Solve $U\vec{x} = \vec{c}$

$\left[\begin{array}{ccc|c} 4 & 2 & 0 & 2 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 2 & 6 \end{array}\right] = $ $\begin{aligned} x &= 1 \\ y &= -1 \\ z &= 3 \end{aligned}$ $\rightarrow \boxed{x = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}}$

$\boxed{2.2.6}$ Given a 1000×1000 matrix, we can solve the 500 problems $A\vec{x} = b_n$ in 60 secs, w/ $A = LU$ fact.
- How much of the time is spent on the $A = LU$ factorization?

- Classical Gaussian Elimination requires $\frac{2}{3}kn^3$
- LU requires $\frac{2}{3}n^3 + 2kn^2$ $\quad n = 1000$

$\hookrightarrow Ax = b_1,\ Ax = b_2, \ldots Ax = b_{500} \leftarrow 60$ sec.

$$\underbrace{\tfrac{2}{3}n^3}_{\text{Factor}} + \underbrace{2kn^2}_{\text{Back-sub}} = \underbrace{60 \text{ sec}}_{\text{time}}$$

- Time on back-sub $= \dfrac{(\text{Factor})}{(\text{Factor} + \text{Back-Sub})} = \dfrac{?}{60}$

$$\tfrac{2}{3}(1000)^3 + 2(500)(1000)^2 = 60$$

$$\tfrac{2}{3}\times 10^9 + 1000(10)^6 \Rightarrow \tfrac{2}{3}10^9 + 10^9 = \overbrace{\tfrac{5}{3}10^9}^{\text{operations}} = \overbrace{60 \text{ sec.}}^{\text{time}}$$

$$\frac{\frac{2}{3}10^9}{\frac{5}{3}10^9} = \frac{1}{(5/3)} = \frac{2}{5} \leftarrow \text{time on Factorization}$$

$$\boxed{t \approx 24 \text{ sec.}}$$

$\boxed{2.2.8}$ If we solve a 2000 x 2000 linear System $A\vec{x} = \vec{b}$ in 0.1 second. Estimate time req. to solve 100 systems of 8000 equations in 8000 unknowns, using LU factorization.

$\rightarrow$ LU-time $\rightarrow \underbrace{\frac{2}{3}n^3}_{\text{factor}} + \underbrace{2kn^2}_{\text{back-sub}}$ Classical Gauss $\rightarrow \underbrace{\frac{2}{3}kn^3}_{\text{solve}}$

$\frac{2}{3}kn^3 \rightarrow \frac{2}{3}(1)(2000)^3 = 0.1$ sec for 2000 x 2000 matrix

$'' = \frac{16}{3} \times 10^9$ [ops] $= 0.1$ [sec] $= \frac{16}{3} \times 10^{10} \left[\frac{\text{Ops}}{\text{sec}}\right]$

$\rightarrow$ LU-factor:

$\frac{2}{3}n^3 + 2kn^2 \rightarrow \frac{2}{3}(8000)^3 + 2(100)(8000)^2$

$\left(\frac{2}{3}\right)5.12 \times 10^{11} + (200)(6.4 \times 10^7) = \left(\frac{1.024}{3} \times 10^{12}\right) + \left(1.28 \times 10^{10}\right)$

$\approx 3.54 \times 10^{11}$ [ops] $\div \frac{16}{3} \times 10^{10} \left[\frac{\text{ops}}{\text{sec}}\right] = \boxed{6.64 \text{ secs}}$

$\boxed{2.7.4}$ Apply two steps of Newton's method w/ starting point $(1, 1)$

a) $\begin{cases} u^2 + v^2 = 1 \\ (u-1)^2 + v^2 = 1 \end{cases} \rightarrow \begin{array}{l} u^2 + v^2 - 1 = 0 \\ u^2 - 2u - 2 + v^2 = 0 \end{array}$

$DF(u,v) = \begin{bmatrix} 2u & 2v \\ 2u-2 & 2v \end{bmatrix} \qquad \vec{X}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- 1-st Step:

$\begin{bmatrix} 2(1) & 2(1) \\ 2(1)-2 & 2(1) \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & 2 & 1 \end{array}\right] \rightarrow \begin{array}{l} S_1 = 0 \\ S_2 = 1/2 \end{array} \rightarrow \vec{S}_0 = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$

$\vec{X}_1 = \vec{X}_0 + \vec{S}_0 = \begin{bmatrix} 1 \\ 3/2 \end{bmatrix}$

- 2nd step

$\begin{bmatrix} 2(1) & 2(3/2) \\ 2(1)-2 & 2(3/2) \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 3 & 3/2 \end{array}\right] \rightarrow \begin{array}{l} S_1 = -1/4 \\ S_2 = 1/2 \end{array} \rightarrow \vec{S}_1 = \begin{bmatrix} -1/4 \\ 1/2 \end{bmatrix}$

$\vec{X}_2 = \vec{X}_1 + \vec{S}_1 = \begin{bmatrix} 3/4 \\ 2 \end{bmatrix}$

$\boxed{2.7.4}$ (Cont.)

b) $u^2 + 4v^2 - 4 = 0$
$4u^2 + v^2 - 4 = 0$ $\rightarrow DF(u,v) = \begin{bmatrix} 2u & 8v \\ 8u & 2v \end{bmatrix}$ $\vec{X}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 2 & 8 & 1 \\ 8 & 2 & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - 4R_1} \left[\begin{array}{cc|c} 2 & 8 & 1 \\ 0 & -30 & -3 \end{array}\right] \rightarrow \begin{array}{l} S_1 = 1/10 \\ S_2 = 1/10 \end{array} \rightarrow \vec{S}_0 = \begin{bmatrix} 1/10 \\ 1/10 \end{bmatrix}$$

$$\vec{X}_1 = \vec{X}_0 + \vec{S} = [11/10,\ 11/10]^T$$

$$\left[\begin{array}{cc|c} 22/10 & 88/10 & 11/10 \\ 88/10 & 22/10 & 11/10 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 22 & 88 & 11 \\ 88 & 22 & 11 \end{array}\right] \rightarrow \begin{bmatrix} 1/10 \\ 1/10 \end{bmatrix} \leftarrow \vec{S}_1$$

$$\vec{X}_2 = \vec{X}_1 + \vec{S}_1 = [12/10,\ 12/10]^T$$

$\boxed{2.7.4}$ (Cont.)

c) $\begin{cases} u^2 - 4v^2 - 4 = 0 \\ u^2 - 2u - 6 + v^2 = 0 \end{cases} \rightarrow DF(u,v) = \begin{bmatrix} 2u & -8v \\ 2u-2 & 2v \end{bmatrix} \quad \vec{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2(1) & -8(1) \\ 2(1)-2 & 2(1) \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 2 & -8 & 1 \\ 0 & 2 & 1 \end{array}\right] \rightarrow \begin{matrix} S_1 = 5/2 \\ S_2 = 1/2 \end{matrix} \rightarrow \vec{S}_0 = \begin{bmatrix} 5/2 \\ 1/2 \end{bmatrix}$

$\vec{x}_1 = \vec{x}_0 + \vec{S}_0 = [7/2, 3/2]^T$

$\begin{bmatrix} 2(7/2) & -8(3/2) \\ 2(7/2)-2 & 2(3/2) \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 3/2 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 7 & -12 & 7/2 \\ 5 & 3 & 3/2 \end{array}\right] \xrightarrow{R_2 = R_2 - \frac{5}{7}R_1}$

$3 = \frac{60}{7} \Rightarrow \frac{21}{7} - \frac{60}{7} = -\frac{39}{7}$

$\frac{3}{2} = \frac{3}{2} - \frac{5}{7}\left(\frac{7}{2}\right) = \frac{3}{2} - \frac{5}{2} = -\frac{2}{2} = -1$

$\left[\begin{array}{cc|c} 7 & -12 & 7/2 \\ 0 & -\frac{39}{7} & -1 \end{array}\right] \Rightarrow$ $S_1 \rightarrow 7S_1 - 12(7/39) = 7/2$

$S_2 = 7/39$

$\Rightarrow S_1 = \left(\frac{7}{2} + 12\left(\frac{7}{39}\right)\right)\frac{1}{7}$

$S_1 = \frac{1}{2} + \frac{12}{39} = \frac{36}{78} + \frac{24}{78}$

$S_1 = \frac{60}{78}$

$\hookrightarrow \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} \frac{60}{78} \\ 7/39 \end{bmatrix}$

$\vec{x}_2 = \vec{x}_1 + \vec{S}_1$

$\hookrightarrow \begin{bmatrix} 7/2 \\ 3/2 \end{bmatrix} + \begin{bmatrix} \frac{60}{78} \\ 7/39 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} + \frac{60}{78} \\ \frac{3}{2} + \frac{7}{39} \end{bmatrix} = \vec{x}_2$

$\boxed{2.7.6}$ Apply 2 steps of Broyden II to Ex 3,
use $\vec{x}_0 = [1,1]^T$ & $B_0 = \hat{I}$

a) $\begin{cases} u^2+v^2-1=0 \\ u^2-2u \quad = 0 \end{cases}$ $\quad B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\vec{x}_1 = \vec{x}_0 - B_0 F(x_0)$, $\quad F(x_0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\vec{x}_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

$\delta_1 = x_1 - x_0 \rightarrow \delta_1 = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$

$F(x_1) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\rightarrow \Delta_1 = F(x_1) - F(x_0) \rightarrow \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$\vec{B}_1 = \begin{bmatrix} 1.4286 & 1.2857 \\ 0 & 1.0000 \end{bmatrix}$ $\approx$ w/ Matlab

$\vec{x}_2 = \vec{x}_1 - B_1 F(x_1) = \begin{bmatrix} 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 1.4286 & 1.2857 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\vec{x}_2 = \begin{bmatrix} -4.2857 \\ -2.0000 \end{bmatrix}$

$\boxed{2.72}$ Computer Problems

Use Newton's Method to find 3 solutions of Example 2.33

$$F(\vec{x}) = \begin{bmatrix} 6x_1^3 + x_1x_2 - 3x_2^3 - 4 \\ x_1^2 - 18x_1x_2^2 + 16x_2^3 + 1 \end{bmatrix}$$

$$DF(\vec{x}) = \begin{bmatrix} (18x_1^2 + x_2) & (x_1 - 9x_2^2) \\ (2x_1 - 18x_2^2) & (-36x_1x_2 + 48x_2^2) \end{bmatrix}$$

$$\left.\begin{array}{l} \vec{x}_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \rightarrow \vec{r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \vec{x}_0 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \rightarrow \vec{r} = \begin{bmatrix} 0.8868 \\ -0.2940 \end{bmatrix} \\ \vec{x}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix} \rightarrow \vec{r} = \begin{bmatrix} 0.8654 \\ 0.4622 \end{bmatrix} \end{array}\right\}$$

file

Q272.m

$\boxed{2.7.5}$ Computer Problems

Use Multi-Var. Newton's method to find 2pts in common of the spheres in 3D space.

a) Each sphere has radius 1; w/ centers @

(i) $\vec{r}_1 = [1,1,0]$ (ii) $\vec{r}_2 = [1,0,1]$ (iii) $\vec{r}_3 = [0,1,1]$

- Eqns. of spheres

$$F(x,y,z) \begin{cases} (x-1)^2+(y-1)^2+(z-0)^2-1 = 0 \\ (x-1)^2+(y-0)^2+(z-1)^2-1 = 0 \\ (x-0)^2+(y-1)^2+(z-1)^2-1 = 0 \end{cases}$$

$$DF(x,y,z) = \begin{bmatrix} (2x-2) & (2y-2) & (2z) \\ (2x-2) & (2y) & (2z-2) \\ (2x) & (2y-2) & (2z-2) \end{bmatrix}$$

$$\vec{x}_0 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \rightarrow \vec{r} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{x}_0 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} \rightarrow \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$\boxed{2.7.5}$ Computer Problems

b) New spheres! $\vec{r}_1 = [1, -2, 0]^T$ $\vec{r}_2 = [-2, +2, -1]^T$, $\vec{r}_3 = [4, -2, 3]^T$

$$F(x,y,z) = \begin{bmatrix} (x-1)^2 + (y+2)^2 + (z-0)^2 - 25 \\ (x+2)^2 + (y-2)^2 + (z+1)^2 - 25 \\ (x-4)^2 + (y+2)^2 + (z-3)^2 - 25 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$DF(x,y,z) = \begin{bmatrix} (2x-2) & (2y+4) & (2z) \\ (2x+4) & (2y-4) & (2z+2) \\ (2x-8) & (2y+4) & (2z-6) \end{bmatrix}$$

$$\vec{X}_0 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \rightarrow \vec{r} = \begin{bmatrix} 1.8889 \\ 2.4444 \\ 2.1111 \end{bmatrix} \qquad \vec{X}_0 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \longrightarrow \vec{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\boxed{2.7.6}$ Computer Problems

Apply Newton's Method to find single point of intersection of

(i) $\vec{r}_1 = [1,0,1]$, rad $= \sqrt{8}$

$\vec{r}_2 = [0,2,2]$, rad $= \sqrt{2}$

$\vec{r}_3 = [0,3,3]$, rad $= \sqrt{2}$

$$\begin{cases} (x-1)^2 + (y+0)^2 + (z-1)^2 - 8 = 0 \\ (x-0)^2 + (y-2)^2 + (z-2)^2 - 2 = 0 \\ (x-0)^2 + (y-3)^2 + (z-3)^2 - 2 = 0 \end{cases}$$

Jacobian.

$$DF(x,y,z) \begin{bmatrix} (2x-2) & (y^2) & (2z-2) \\ (2x) & (2y-2) & (2z-4) \\ (2x) & (2y-6) & (2z-6) \end{bmatrix}$$

$$\vec{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.6905 \\ -1.8230 \\ 4.0798 \end{bmatrix}$$